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Nonlinear response of laterally loaded rigid piles in sand

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Abstract. This paper investigates nonlinear response of 51 laterally loaded rigid piles in sand. Measured response of each pile test was used to deduce input parameters of modulus of subgrade reaction and the gradient of the linear limiting force profile using elastic-plastic solutions. Normalised load - displacement and/or moment - rotation curves and in some cases bending moment and displacement distributions with depth are provided for all the pile tests, to show the effect of load eccentricity on the nonlinear pile response and pile capacity. The values of modulus of subgrade reaction and the gradient of the linear limiting force profile may be used in the design of laterally loaded rigid piles in sand.

Keywords: piles; lateral loading; shear modulus; modulus of subgrade reaction; ultimate soil resistance

1. Introduction

Extensive theoretical studies, in-situ full-scale tests and laboratory model tests have been carried out on laterally loaded rigid piles in cohesionless soils (Poulos and Davis 1980, Scott 1981, Dickin and Nazir 1999, Laman *et al.* 1999, Guo 2008, Zhang *et al.* 2005, Zhang 2009, Chen *et al.* 2011). Several methods have been developed for predicting lateral capacity of rigid piles based on an assumed profile of soil resistance per unit length along a pile (Brinch Hansen 1961, Broms 1964, Petrasovits and Awad 1972, Meyerhof *et al.* 1981, Fleming *et al.* 2009, Prasad and Chari 1999). The capacity was also determined as the load at a certain displacement from a measured lateral load - displacement or the moment with reference to a specified pile rotation angle from a measured moment - rotation curve (Broms 1964, Haldar *et al.* 2000, Chen *et al.* 2011). These methods, nevertheless offer different lateral capacities for same measured data. To resolve the issue, Guo (2008) established elastic-plastic solutions for analysing laterally loaded rigid piles, assuming a constant modulus of subgrade reaction or a linearly increasing modulus of subgrade reaction with depth together with a linear limiting force profile (LFP). Presented in explicit expressions in terms of the slip depths mobilised from the ground line and pile tip, the solutions enable nonlinear response of piles and displacement-based capacity to be estimated. The estimations are satisfactory against the pile responses in model tests presented by Prasad and Chari (1999) and the experimental and numerical analysis results by Laman *et al.* (1999).

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Significant research effort has also been made to study passive piles subjected to lateral soil movements based on field monitoring and analysis, centrifuge and laboratory model tests, analytical and numerical analysis as reviewed by Qin (2010). The study indicates the analysis of the piles requires the modulus of subgrade reaction or Young's modulus of the soil and limiting force p_u profile (Poulos et al., 1995, Guo 2006, 2013a), which may be related to those for laterally loaded piles discussed herein (Guo 2013b).

In this paper, elastic-plastic solutions were used to study the measured response of 51 laterally loaded pile tests in sand, including 16 full-scale field tests, 12 centrifuge tests and 23 laboratory model tests. This is illustrated in light of a full-scale field test to demonstrate the calculation and its reliability. The study examines the impact of load eccentricity on the nonlinear pile response, range of modulus of subgrade reaction, average shear modulus and limiting force profile for laterally loaded rigid piles in sand.

2. Elastic-plastic solutions

A free-headed pile with a lateral load T_t applied at an eccentricity e above the ground line is schematically shown in Fig. 1(a). The pile is defined as rigid if the pile-soil relative stiffness, E_p/G_s exceeds a critical ratio $(E_p/G_s)_c$, where $(E_p/G_s)_c = 0.052(l/r_0)^4$ (Guo and Lee 2001), E_p is the effective Young's modulus, defined as $E_p = (EI)_p/(\pi r_0^4/4)$, $(EI)_p$ is the pile bending rigidity, G_s is the shear modulus of the soil, l is the pile embedded length and r_0 is the outer radius of the pile.

2.1 Load transfer model

Guo (2008) provides a pile-soil interaction model characterised by a series of springs distributed along the shaft. Each spring has an idealised elastic-plastic p - $y(u)$ curve at any depth shown in Fig. 1(b). The soil resistance per unit length p is proportional to the local displacement u at that depth and to the modulus of subgrade reaction kd by

$$p = kdu \quad (\text{Elastic state}) \quad (1)$$

Fig. 1 Schematic analysis for a rigid pile (after Guo 2008)

The magnitude of k is related to the average shear modulus $\bar{G}_s$ by

$$kd = \frac{3\pi\bar{G}_s}{2} \left\{ 2\gamma \frac{K_1(\gamma)}{K_0(\gamma)} - \gamma^2 \left[\left(\frac{K_1(\gamma)}{K_0(\gamma)} \right)^2 - 1 \right] \right\} \quad (2)$$

where d is the outer diameter of the pile, $\bar{G}_s$ is an average shear modulus of the soil over the pile embedded length, $K_i(\gamma)$ is the modified Bessel function of second kind of i^{th} order ($i = 0, 1$), γ is a non-dimensional factor given by $\gamma = k_1 r_0 / l$, $k_1 = 2.14$ and 3.8 for pure lateral load ($e = 0$) and pure moment loading ($e = \infty$), respectively. The value of k_1 can be approximately estimated by $k_1 = 2.14 + e/l / (0.2 + 0.6e/l)$, increasing from 2.14 to 3.8 as e increase from 0 to ∞ (Guo 2012). The k may be written as $k_0 z^m$ [k_0 , FL $^{-m-3}$], with $m = 0$ and 1 being referred to as constant k ($k = k_0$) and Gibson k ($k = k_0 z$) hereafter. For the constant k and Gibson k , the k and k_0 have a unit of MN/m³ and MN/m⁴, respectively.

Once the local pile displacement u exceeds a threshold value of u^* as seen in Fig. 1(b), p reaches the limiting value p_u and the pile-soil relative slip is initiated. It is assumed that the p_u

increases linearly with depth z as shown by the dashed line in Fig. 1(c) and may be described by

$$p_u = A_r z d \quad (\text{Plastic state}) \quad (3)$$

where $A_r z$ is the net limiting pressure on the pile surface and A_r may be expressed as

$$A_r = N_g \gamma'_s K_p^2 \quad (4)$$

where γ'_s is the effective unit weight of the soil, i.e. bulk unit weight above water table and buoyant unit weight below, $K_p = \tan^2(45^\circ + \phi'_s/2)$ is the coefficient of passive earth pressure, ϕ'_s is an effective frictional angle of the soil, N_g is a non-dimensional parameter. The actual N_g can be back-calculated from the measured pile responses as shown later.

2.2 Explicit expressions for the solutions

Typical pile-soil interaction states and pile displacement modes have been defined as follows. The pile has a displacement $u = \omega z + u_0$. It rotates about a depth $z_r (= -u_0 / \omega)$ at which deflection $u = 0$, note u_0 is the pile displacement at ground line, ω is the rotational angle in Fig. 1(d). The soil resistance per unit length p attains the limiting force per unit length p_u once the deflection u exceeds u^* [$= A_r/k_0$ (Gibson k) or $= A_r z_0/k$ (constant k)]. The soil resistance p along the pile, i.e., the on-pile force distribution is illustrated in Fig. 1(c). The on-pile force per unit length p follows the positive p_u profile given by Eq. (3) to a slip depth z_0 from ground line. In other words, the pile soil interaction is in plastic state. Below the z_0 , it is described by Eq. (1) since the pile-soil interaction is still in elastic state. In particular, once the pile tip-displacement u ($z = l$) touches $-u^*$ (Gibson k) or $-u^*/z_0$ (constant k), or the soil resistance p ($z = l$) at the pile-tip touches $A_r l d$, the pile is said at *tip-yield state*. After the pile-tip yields, increasing loading will also result in pile-soil relative slip initiating from the pile-tip and expanding upwards to another slip depth z_1 as illustrated in Fig. 1(c). The two plastic zones will merge eventually and the pile reaches the ultimate state, i.e. *yield at rotation point* ($z_0 = z_1 = z_r$).

The solutions are presented in explicit expressions characterized by the slip depths. Their non-dimensional forms for pre-tip yield and tip yield states are presented in Table 1 in form of normalised lateral load $T_t/(A_r d l^2)$, ground line displacement $u_0 k_0/A_r$ (Gibson k) or $u_0 k/(l A_r)$ (constant k), rotation angle $\omega k_0 l/A_r$ (Gibson k) or $\omega k/A_r$ (constant k), depth of maximum bending moment z_m , and maximum bending moment $M_{\max}/(A_r d l^3)$. The reader is referred to Guo (2008) for details of the solutions.

The solutions were entered into a spreadsheet program, which adopts user-defined macros in Microsoft Excel VBA. The input parameters are as follows: (1) pile dimensions d and l , and soil parameters ϕ'_s and γ'_s , (2) loading eccentricity e , and (3) parameters A_r and k (or k_0). Given a set of input parameters, nonlinear response and ultimate lateral capacity of the pile can be predicted. Conversely, the parameters A_r and k (or k_0) may be deduced from measured responses of laterally loaded piles using the closed-form solutions.

Table 1 Solutions for pre-tip and tip yield state (Guo 2008)

$u = \omega z + u_0$ and $z_r/l = -u_0/\omega l$	
$p = kdu$, $p_u = A_r dz$, kd is the modulus of subgrade reaction, k is written as $k_0 z^m$.	
Gibson k ($m = 1$)	Constant k ($m = 0$)
$\frac{T_t}{A_r dl^2} = \frac{1}{6} \frac{1 + 2\bar{z}_0 + 3\bar{z}_0^2}{(2 + \bar{z}_0)(2\bar{e} + \bar{z}_0) + 3}$	$\frac{T_t}{A_r dl^2} = \frac{\bar{z}_0}{2(2 + 3\bar{e} + \bar{z}_0)}$
$\frac{u_0 k_0}{A_r} = \frac{3 + 2(2 + \bar{z}_0^3)\bar{e} + \bar{z}_0^4}{[(2 + \bar{z}_0)(2\bar{e} + \bar{z}_0) + 3](1 - \bar{z}_0)^2}$	$\frac{u_0 k}{A_r l} = \frac{(2 + 3\bar{e})\bar{z}_0}{(2 + 3\bar{e} + \bar{z}_0)(1 - \bar{z}_0)^2}$
$\omega \frac{k_0 l}{A_r} = \frac{-2(2 + 3\bar{e})}{[(2 + \bar{z}_0)(2\bar{e} + \bar{z}_0) + 3](1 - \bar{z}_0)^2}$	$\omega \frac{k}{A_r} = \bar{z}_0 \frac{\bar{z}_0^2 + 3(\bar{z}_0 - 2)\bar{e} - 3}{[2 + 3\bar{e} + \bar{z}_0](1 - \bar{z}_0)^2}$
$\bar{z}_m = \sqrt{2T_t/(A_r dl^2)} \quad (z_m \leq z_0)$	$\bar{z}_m = \sqrt{2T_t/(A_r dl^2)} \quad (z_m \leq z_0)$
$M_{\max} = (2z_m/3 + e)T_t \quad (z_m \leq z_0)$	$M_{\max} = (2z_m/3 + e)T_t \quad (z_m \leq z_0)$
$(\bar{z}_0^y)^3 + (2\bar{e} + 1)(\bar{z}_0^y)^2 + (2\bar{e} + 1)\bar{z}_0^y - (\bar{e} + 1) = 0$ (Solving numerically)	$\bar{z}_0^y = -(1.5\bar{e} + 0.5) + 0.5\sqrt{5 + 12\bar{e} + 9\bar{e}^2}$
Note: T_t , u , u_0 , ω , z , z_0 , z_r , e and l are defined in Fig 1. z_m is the depth of maximum bending moment $M_{\max}$, z_0^y is the slip depth z_0 at tip yield state. $\bar{z}_0 = z_0/l$, $\bar{z}_m = z_m/l$, $\bar{e} = e/l$, $\bar{z}_0^y = z_0^y/l$.	

3. Analysis of measured pile responses

51 pile tests in horizontal ground were studied, comprising 16 full-scale field tests, 12 centrifuge tests and 23 model tests. The pile diameter d , embedded length l and loading eccentricity e are summarised in Table 2. The properties of sand including the relative density D_r , the angle of internal friction ϕ'_s and effective unit weight γ'_s are presented in Table 3. The measured pile responses for selected tests are plotted as symbols in Figs. 2-9.

3.1 Back calculation

Back calculations were carried out by best matching (via visual comparison) between the elastic-plastic solutions and the measured responses of the 51 test piles. This is sufficiently accurate as shown by the sensitivity analysis by Qin (2010). Theoretically, two measured load-displacement $T_t - u_0(u_t)$ and moment-rotation $M_0 - \omega$ curves are required to uniquely deduce the two parameters A_r and k (or k_0). With only one measured curve, either $T_t - u_0(u_t)$ or $M_0 - \omega$, back calculations were still carried out by fitting the initial elastic portion through adjusting k (or k_0), and the last nonlinear portion of the curve by adjusting the A_r , as discussed later.

The deduced parameters A_r , k_0 and k for each pile are presented in Table 3. Furthermore, the statistical analysis of the pile characteristics, soil properties and analysis results is presented in Qin (2010). The calculated pile responses with a Gibson k and constant k were plotted in Figs. 2-9

as dotted and solid lines, respectively, and as hollow dot points $\circ$ and solid dots $\bullet$ for those at tip-yield. This is illustrated next for the field test F1.

3.2 An example calculation– Field tests of steel pole foundations in loose sand

Haldar *et al.* (2000) conducted eight full-scale field tests on fully instrumented steel transmission pole foundations. Each pole consisted of top and bottom sections with diameters of 0.779 m and 0.740 m (an average diameter d of 0.76 m). The two parts were joined together by bolted connections. The typical cross section of the pole was a 12-sided polygon. The embedded length l of the pole varied from 2.36 m to 3.2 m. The lateral loads were applied at an eccentricity e of approximately 23.0 m to investigate the responses of pole foundations under a large moment. Each pole was instrumented to measure the applied load at the top of pole and deflections near the ground line. The rotation of the pole was determined from the deflection of the pole at two different distances. Ten strain gauges were installed at different sections of the pole to measure distribution of the bending moment at selected depths. Lateral load was applied in an incremental manner until it reached the safe structural capacity of the pole or it induced a large deflection at ground line.

The poles were tested in four different types of backfills, namely, sand, in-situ gravelly sand, crushed stone and flowable material, respectively. The loose to medium dense sand backfill (F1-F5) had a relative density D_r of 22%-56%, an effective unit weight γ'_s of 16.4-17.6 kN/m³ and an effective internal frictional angle ϕ'_s of 32.6°-39.2°, respectively. The dense crushed stone (F6) and in-situ gravelly sand (F7) have a relative density of 85% with larger effective internal frictional angles of 49.8° and 42.7°.

The pole test F1 (with $d = 0.7545$ m, $l = 3.2$ m, and $e = 22.25$ m) was tested in loose sand backfill. The measured $M_\theta - \omega$ curve is plotted in Fig. 2(a). The measured bending moment distribution with depth and pole displacement at a ground line moment M_θ of 245 kNm, 365 kNm, 485 kNm, and 685 kNm are plotted in Figs. 2(b)-(c). The measured soil pressure on the pole using pressure cells at $M_\theta = 685$ kNm is plotted in Fig. 2(d).

Table 2 Characteristics of pile tests

Test No.	Reference	Pile No. in reference	Pile type	e (m)	l (m)	d (m)	Measured M_u or T_u	Measured curves
Full-scale field tests								
F1	Haldar <i>et al.</i> (2000)	4	Steel 12-sided steel polygonal pole	22.25	3.20	0.755	855 kNm	M_0 - ω , $M(z),u(z)$
F2		1		22.93	2.52	0.760	137 kNm	M_0 - ω
F3		2		22.94	2.52	0.760	253 kNm	
F4		2A		22.86	2.59	0.761	615 kNm	
F5		3		22.86	2.59	0.759	726 kNm	
F6		5		22.88	2.57	0.759	654 kNm	
F7		7		22.86	2.59	0.759	674 kNm	
F8	Bhushan <i>et al.</i> (1981)	4	Cast-in-place drilled piers	0	5.50	0.610		T - u_0
F9		5		0	5.50	0.915		
F10		6		0	5.50	0.915		
F11		7		0	5.50	1.220		
F12	Ismael and Klym (1981)	Footing 1	Cased Augered pile	0	6.40	0.9144	530kN	T - u_0 , $M(z)$, $u(z)$
F13	Pender and Matuschka (1988)	Field test	Bored piers	5.4	1.97	0.750	21.8kN	T - u_t , T - ω
F14	Lee <i>et al.</i> (2010)	T1	Bored piles	2.0	1.20	0.40	22kN	T - u_0
F15		T2		2.0	2.40	0.40	50kN	
F16		T3		0.15	2.40	0.40	210kN	
Centrifuge tests								
C1	Georgiadis <i>et al.</i> (1992)	P1	Stainless steel pipe pile	1.25	9.05	1.092		$M(z)$, $T(z)$, $u(z)$
C2		P2		1.25	9.05	1.224		T - u_t , $M(z),T(z)$, $u(z)$
C3	Laman <i>et al.</i> (1999)	1	Circular pier	6	2	1	400 kNm	M_0 - ω
C4	Dickin and Laman (2003)	$d/l=0.33$	Rough steel rectangular pier	6	3	1		M_0 - ω
C5		$d/l=0.6$		6	3	1.8		
C6		$d/l=1$		6	3	3		
C7		$d/l=1.33$		6	3	4		
C8		$d/l=2$		6	3	6		
C9		$d/l=0.33$		6	3	1		
C10		$d/l=1$		6	3	3		
C11		$d/l=1.33$		6	3	4		
C12		$d/l=2$		6	3	6		

Table 2 Characteristics of pile tests (continued)

Test No.	Reference	Pile No. in reference	Pile type	e (m)	l (m)	d (m)	Measured M_u or T_u	Measured curves
Model tests								
M1	Petrasovits and Awad (1972)	$l/d=14.3$	Smooth pile	0.14	0.5	0.035	553.7N	$T- u_t$
M2		$l/d=25.0$		0.14	0.5	0.020	301.4 N	
M3		$l/d=38.5$		0.14	0.5	0.013	245 N	
M4	Adams and Radhakrishna (1973)	$d=101.6\text{mm}$	Steel pipe pile	0.3175	0.4445	0.1016		$T- u_0$
M5		$d=101.6\text{mm}$		0.3175	0.4445	0.1016		
M6		$d=76.2\text{mm}$	Steel pipe pile filled with cement grout	0.3175	0.4445	0.0762		
M7		$d=50.8\text{mm}$		0.3175	0.4445	0.0508		
M8	Meyerhof <i>et al.</i> (1981)	Loose sand	Rough steel pile	0	0.2	0.0125	11N	$T- u_0$
M9		Dense sand		0	0.2	0.0125	40N	
M10	Chari and Meyerhof (1983)		Smooth steel pipe pile	0.075	0.991	0.075	2050N	$T- u_t$
M11	Swane (1983)	DSSU1	Aluminum circular pile	0.05	0.4	0.024		$T- u_0$
M12		DSSU2		0.05	0.4	0.024		
M13		LSSU1		0.05	0.4	0.024		
M14	Prasad and Chari (1996)	1	Smooth steel pipe pile	24	0.612	0.102	425 Nm	$M_0- \omega$
M15		2		24	0.612	0.102	1200 Nm	
M16		5		24	0.51	0.102	325 Nm	
M17		6		24	0.51	0.102	800 Nm	
M18	Prasad and Chari (1999)	$D_f=25\%$	Smooth steel pipe pile	0.15	0.612	0.102	620N	$T- u_t$
M19		$D_f=50\%$		0.15	0.612	0.102	1040N	
M20		$D_f=75\%$		0.15	0.612	0.102	1790N	
M21	Qin and Guo (2007)	TS1	Aluminum pipe pile	0.115	0.5	0.032	740N	$T- u_0,$ $M(z), u(z)$
M22		TC1		0.115	0.5	0.032	810 N	
M23		TC2		0.115	0.5	0.032	820 N	

Table 3 Soil properties and deduced parameters

Test No.	Soil type	D _r (%)	ϕ_s' (°)	γ_s' (kN/m ³)	A _r (kN/m ³)	N _g	k ₀ (MN/m ⁴)	k (MN/m ³)
Full-scale field tests								
F1	Loose sand	45	37.1	17.2	400	1.43	26.1	41.0
F2	Very loose sand	22	32.6	16.4	150	0.82	3.0	4.5
F3	Loose sand	31	34.4	16.7	278	1.29	5.1	9.3
F4	Medium dense sand	56	39.2	17.6	512	1.48	31.1	49.5
F5	Loose sand	43	36.7	17.1	575	2.13	31.1	49.0
F6	Dense crushed stone	86	49.8	19.2	600	0.56	20.1	30.5
F7	Dense gravelly sand	85	42.7	19.7	560	1.05	31.1	49.0
F8	Silt sand (0~0.9m) and silty sand with gravely layers (0.9~5.5m)	77 (0~0.9m)	40 [†]	16.5	480	1.38	26.5	94.0
F9		88 (0.9~5.5m)	40 [†]	16.5	350	1.00	25.0	94.0
F10	Silt sand (0~1.8 m) and silty sand with gravely layers (1.8~5.5m)	38 (0~1.8m)	38 [†]	16.5	360	1.23	25.0	99.0
F11		92 (1.8~5.5m)	38 [†]	16.5	360	1.23	50.0	129.0
F12	Fine to medium sand with silt		34	11.0	140	1.00	7.5	26.0
F13	Silty sand		30	8.2	222.5	3.00	38.0	34.0
F14	Clayey sand	30~35	35.4	14.5	950.0	4.65	220.0	184.0
F15			35.4	14.5	365	1.80	200.0	234.0
F16			35.4	14.5	800	3.90	200.0	304.0
Centrifuge tests								
C1	Uniform fine grained dry medium dense to dense sand	60	36	16.3	340	1.41	3.0	10.0
C2		60	36	16.3	280	1.16	3.0	10.0
C3	Dry dense sand		46.1	16.4	621.7	1.00	25.0	34.4
C4	Fine clean dry dense sand	85	49	16.4	532	0.63	45.0	55.5
C5		85	49	16.4	490	0.58	45.0	85.5
C6		85	49	16.4	385	0.46	35.0	50.5
C7		85	49	16.4	375	0.45	35.0	48.5
C8		85	49	16.4	365	0.44	35.0	48.5
C9		Fine clean dry loose sand	37	39	14.6	180	0.64	6.0
C10	37		39	14.6	145	0.51	6.0	12.5
C11	37		39	14.6	125	0.44	6.0	12.5
C12	37		39	14.6	120	0.43	6.0	9.4

Table 3 Soil properties and deduced parameters (continued)

Test No.	Soil type	D_r (%)	ϕ'_s (°)	γ'_s (kN/m ³)	A_r (kN/m ³)	N_g	k_0 (MN/m ⁴)	k (MN/m ³)
Model tests								
M1	Cohesionless soil	84	37.2	17.6	820.5	2.83	90.0	34.0
M2		84	37.2	17.6	850	2.93	95.0	30.0
M3		84	37.2	17.6	1050	3.62	97.0	33.0
M4	Uniformly graded silica sand	89	31	15.7	112.5	0.73	100.0	30.0
M5		100	45	17.6	416	0.70	450.0	140.0
M6		100	45	17.6	450	0.75	500.0	160.0
M7		100	45	17.6	610	1.02	800.0	240.0
M8	Well graded medium to coarse angular sand	35	35	14 [‡]	250	1.31	51.2	4.8
M9		70	50	15.2 [‡]	950	1.09	231.2	28.0
M10	Coarse uniform angular dry sand	82	46	15	410	0.73	20.0	10.0
M11	Dry medium grained quartz sand	88	38	16.12	1210	4.25	107	33
M12		88	38	16.12	1360	4.77	107	33
M13		44	33.5	15.12	380	2.09	87	23
M14	Well graded angular medium dry sand	45	36	17	420	1.66	32.1	12.0
M15		80	44.5	18.6	630	1.25	32.1	15.0
M16		45	36	17	630	2.50	40.0	15.0
M17	Crushed stone	80	49	18.5	755	0.80	65.0	28.0
M18	Well graded angular dry sand	25	35	16.5	306	1.36	9.24	3.88
M19		50	41	17.3	340	0.85	48.2	12.05
M20		75	45.5	18.3	890	1.36	47.5	16.96
M21	Dry medium grained quartz sand	89	38	16.27	1050	3.65	91.0	30.0
M22		89	38	16.27	1200	4.17	91.0	36.5
M23		89	38	16.27	1225	4.26	91.0	36.5

[†] Average values of the two layers. [‡] Reported by Prasad and Chari (1999).

Fig. 2 Predicted and measured (Haldar *et al.* 2000) response of pile F1

Fig. 3 Predicted and measured (Georgiadis *et al.* 1992) response of pile C1

Fig. 4 Predicted and measured (Georgiadis *et al.* 1992) response of pile C2

Fig. 5 Predicted and measured (Ismael and Klym 1981) response of pile F12

Fig. 6 Predicted and measured (Pender and Matuschka 1988) response of pile F13

Fig. 7 Predicted and measured (Qin and Guo 2007) response of pile M21

Fig. 8 Predicted and measured (Qin and Guo 2007) response of pile M22

Fig. 9 Predicted and measured (Qin and Guo 2007) response of pile M23

The back-calculated pole curves are also plotted in Figs. 2(a)-(d), which are based on $A_r = 400$ kN/m³, $k_0 = 26.1$ MN/m⁴, and $k = 41.0$ MN/m³. The following features are observed.

1. Taking the same value of A_r , back calculation using the solutions with a constant k gives a better match with the measured $M_0 - \omega$ relationships (see Fig. 2(a)).

2. Pile deflections are well predicted (see Fig. 2(a), (c)), while the bending moment distributions are slightly overestimated (see Fig. 2(b)) especially at high-load levels using either k .

3. The calculated $M_0 = 682.4$ kNm (Gibson k) is close to the measured value of 685 kNm at ground line, and the calculated M_0 is 751.65 kNm (constant k) at the tip-yield state. The measured soil pressure profile and the on-pile force profiles for both k at tip-yield state are plotted in Fig. 2(d). The soil pressure distribution proposed by Prasad and Chari (1999) was included for comparison as well. The measured data fall within the zones enclosed by the individual soil pressure profile, further confirming that the pole was at pre-tip yield or close to tip-yield state.

4. The ultimate ground line moment of the pole was calculated as 875.7 kNm, which is 2.4% greater than the reported ultimate moment of 855 kNm at 5° rotation of the pole.

4. Discussions

4.1 Reliability of the back calculation

The 51 pile tests are divided into three groups based on the number of measured pile response curves: (1) eight tests (F1, F12-13, C1-2 and M21-23) with two or more curves; (2) thirteen tests (F14-16 and C3-12) with the $T_t - u_0(u_t)$ or $M_0 - \omega$ curve ranging from elastic to a clear ultimate state; and (3) the remaining thirty tests having only $T_t - u_0(u_t)$ or $M_0 - \omega$ curve, but without clear indication of ultimate state. In order to investigate the effect of e/l on the pile responses, the measured $T_t - u_0(u_t)$ and $M_0 - \omega$ data for each test were normalised by $A_r dl^2$, $A_r l/k$, $A_r dl^3$ and A_r/k , respectively, using the deduced A_r and constant k in Table 3. The normalised lateral load versus ground line displacement or pile-head displacement data are plotted in Fig. 10(a) and normalised moment versus ground line rotation data in Fig. 10(b). The deduced A_r , k and k_0 for the 21 tests in the first and second groups are warranted and reliable because of the good agreement between the back-calculated curves with the measured ones shown in Figs. 2-9. The back-calculated results in the third group may vary if additional measured responses are available.

The back calculation shows that the solution with constant k generally offers a better match against the measured responses of the piles than that based on Gibson k , in light of the linear limiting force profile with the same gradient A_r . However, tests M3, M8, M10 and M18 were not well predicted, owing to stress hardening characteristics (Guo 2008). The following discussions are limited to back calculation using the solution with constant k .

4.2 Effect of e/l on nonlinear pile response, pile capacity T_0 and M_0

The non-dimensional $T_t/(A_r dl^2) - u_0 k/(A_r l)$ and $M_0/(A_r dl^3) - (-\omega k/A_r)$ curves at the e/l ratios calculated from Table 2 were obtained from the solution with constant k and are plotted as solid lines in Figs. 10(a)-(b). It can be seen that at a specific e/l , the normalised measured $T_t - u_0(u_t)$ or $M_0 - \omega$ curves merge or fall within a very narrow band around the solid lines, regardless of soil properties. The ratio e/l has a significant impact on the normalised load $T_t/(A_r dl^2)$, which reduces with the increase of e/l . For instance, at $u_0 k/(A_r l) = 2$, the $T_t/(A_r dl^2)$ reduces about 40% from 0.09 to 0.053 as e/l increases from 0 to 0.8. On the other hand, the normalised moment $M_0/(A_r dl^3)$ increases with the increasing e/l . At $-\omega k/A_r = 2$, $M_0/(A_r dl^3)$ increases by 35% from 0.052 to 0.07 with e/l increasing from 2 to 47.

The measured ultimate lateral capacities of 29 tests were reported in terms of either lateral load T_u or groundline moment M_u and are presented in Table 2. These ultimate capacities were determined as: (1) the load at which the lateral load - pile head displacement curve becomes linear or substantially linear (Meyerhof *et al.* 1981, Chari and Meyerhof 1983, Prasad and Chari 1996, 1999, Lee *et al.* 2010); or (2) the lateral load/moment at a rotation angle of 3.5° - 5.5° (Laman *et al.* 1999, Dickin and Laman 2003) or 5° (Haldar *et al.* 2000). Figs. 11(a)-(b) show the normalised measured pile capacity $T_0/(A_r dl^2)$ and moment $M_0/(A_r dl^3)$ against normalised eccentricity e/l , respectively, in which the theoretical curves by Guo (2008) at tip-yield and yield at rotation point (YRP) are also plotted. Fig. 11(a) shows that the measured ultimate lateral load T_u is generally less than the calculated capacity at tip-yield state. By contrast, Fig. 11(b) shows that the measured ultimate ground line moment M_u falls in the range of the capacity between tip-yield state and yield at rotation point, except tests M14 and M16. As reported the measured values of M_u for

the two tests were obtained at a much lower pile rotation angle ω of around 1.5° . Overall the pile capacity at the yield at rotation point provides a good upper bound.

Fig. 10 Normalised load, displacement and rotation: measured versus predicted

Fig. 11 Normalised pile capacity at critical yield states

4.3 Estimation of average shear modulus $\bar{G}_s$

The modulus of subgrade reaction kd is related to the average shear modulus $\bar{G}_s$ of the sand over the embedded length of the pile via Eq. (2). Conversely, the shear modulus of the sands can be deduced from the back-calculated modulus of subgrade reaction. On the other hand, the small strain shear modulus G_{max} (for which many empirical equations are available) may be used as a universal reference or benchmark value of stiffness when applied to foundation systems (Poulos *et al.* 2001). For instance, Seed and Idriss (1970) and Seed *et al.* (1986) proposed the following

$$G_{max} = 218.8 K_{2,max} (\sigma'_m)^{0.5} \quad (5)$$

where G_{max} is in kPa, σ'_m is the effective mean stress in kPa, which is related to the vertical effective stress σ'_v by $\sigma'_m = [(1 + 2K_0)/3] \sigma'_v$, and to the coefficient of earth pressure at rest $K_0 = 1 - \sin \phi'_s$ (Jaky 1944). In this study, the σ'_v is taken as the average vertical effective stress along the embedded length of the pile. $K_{2,max}$ is a dimensionless modulus coefficient that depends on the relative density D_r in percent (Seed and Idriss (1970) and Yan and Byrne (1992))

$$K_{2,max} = 3.5 (D_r)^{2/3} \quad (6)$$

Seed *et al.* (1986) stated that the values of $K_{2,max}$ range from about 30 for loose sands to about 75 for dense sands and they are 1.35 - 2.5 times greater for gravels than for sands. Therefore, the values of $K_{2,max}$ calculated from Eq. (6) for tests F6 - F11 (piles tested in dense crushed stone, gravelly sand and gravelly silty sand) are doubled (the approximate average value of 1.35 - 2.5).

The ratio $kd/\bar{G}_s$ was calculated from Eq. (2), which depends only on the loading characteristics, loading eccentricity, pile diameter and embedded length. The average shear modulus $\bar{G}_s$ was subsequently obtained from the back-calculated k for each pile. Likewise, the G_{max} was calculated from Eqs. (5) and (6). The second (MTD2) and fourth methods (MTD4) presented by Wichtmann and Triantafyllidis (2009) (see footnote of Table 4) were also used to calculate the G_{max} and to provide an order-of-magnitude check of the deduced G_{max} from Eqs. (5) and (6). These results are presented in Table 4 and summarised as follows.

Table 4 Average shear modulus $\overline{G}_s$ and G_{max}

Test No	γ	$K_1(\gamma)$	$K_0(\gamma)$	$kd/\overline{G}_s$	K_0	σ'_m (kPa)	MTD2*	MTD4‡		Eqs. (5) and (6)		$\overline{G}_s$ (MPa)	$\overline{G}_s / G_{\max}^{\dagger}$
							$G_{\max}$ (MPa)	$K_{2,\max}$	$G_{\max}$ (MPa)	$K_{2,\max}$	$G_{\max}$ (MPa)		
Full-scale field tests and centrifuge tests													
F1	0.440	1.946	1.033	5.49	0.397	16.453	38.014	40.850	36.254	44.280	39.299	5.639	0.143
F2	0.565	1.412	0.825	6.21	0.461	13.242	28.047	33.382	26.578	27.480	21.880	0.551	0.025
F3	0.565	1.412	0.825	6.21	0.435	13.117	30.298	36.254	28.729	34.539	27.370	1.138	0.042
F4	0.550	1.462	0.846	6.13	0.368	13.189	37.263	44.573	35.417	51.230	40.707	6.147	0.151
F5	0.549	1.468	0.848	6.12	0.402	13.322	33.796	40.183	32.090	42.958	34.306	6.077	0.177
F6	0.553	1.452	0.842	6.14	0.236	12.109	88.431	55.258	84.144	68.192	103.840	3.767	0.036
F7	0.549	1.468	0.848	6.12	0.322	13.978	94.113	54.888	89.800	67.662	110.698	6.077	0.055
F8	0.119	8.263	2.259	3.27	0.357	25.931	127.451	55.258	123.134	68.192	151.956	17.536	0.115
F9	0.178	5.408	1.864	3.76	0.357	25.931	127.451	55.258	123.134	68.192	151.956	22.878	0.151
F10	0.178	5.408	1.864	3.76	0.384	26.751	149.408	50.996	144.276	61.858	175.006	24.095	0.138
F11	0.237	3.967	1.590	4.19	0.384	26.751	149.408	50.996	144.276	61.858	175.006	37.528	0.214
F12	0.153	6.350	2.012	3.56	0.441	22.078							
F13	0.690	1.072	0.671	6.91	0.500	5.385							
F14	0.588	1.338	0.793	6.34	0.421	5.340	19.943	36.739	18.576	35.644	18.023	11.605	0.644
F15	0.278	3.336	1.444	4.47	0.421	10.680	27.816	36.739	26.270	35.644	25.488	20.946	0.822
F16	0.200	4.769	1.751	3.93	0.421	10.680	27.816	36.739	26.270	35.644	25.488	30.964	1.215
C1	0.159	6.111	1.976	3.61	0.412	44.855	69.108	45.952	67.338	53.642	78.606	3.028	0.039
C2	0.178	5.417	1.866	3.76	0.412	44.855	69.108	45.952	67.338	53.642	78.606	3.258	0.041
C3	0.910	0.704	0.480	8.08	0.279	8.522							
C4	0.595	1.318	0.784	6.38	0.245	12.223							
C5	1.071	0.535	0.381	8.92	0.245	12.223	44.122	54.888	41.987	67.662	51.758	17.253	0.333
C6	1.784	0.187	0.149	12.51	0.245	12.223	44.122	54.888	41.987	67.662	51.758	12.111	0.234
C7	2.379	0.086	0.072	15.42	0.245	12.223	44.122	54.888	41.987	67.662	51.758	12.580	0.243
C8	3.569	0.021	0.018	21.16	0.245	12.223	44.122	54.888	41.987	67.662	51.758	13.754	0.266
C9	0.595	1.318	0.784	6.38	0.371	12.712	31.434	38.204	29.803	38.863	30.317	1.701	0.056
C10	1.784	0.187	0.149	12.51	0.371	12.712	31.434	38.204	29.803	38.863	30.317	2.998	0.099
C11	2.379	0.086	0.072	15.42	0.371	12.712	31.434	38.204	29.803	38.863	30.317	3.242	0.107
C12	3.569	0.021	0.018	21.16	0.371	12.712	31.434	38.204	29.803	38.863	30.317	2.666	0.088

Table 4 Average shear modulus $\overline{G}_s$ and G_{max} (continued)

Test No	γ	$K_1(\gamma)$	$K_0(\gamma)$	$kd/\overline{G}_s$	K_0	σ'_m (kPa)	MTD2*	MTD4†		Eqs. (5) and (6)		$\overline{G}_s$ (MPa)	$\overline{G}_s/G_{\max}^\dagger$
							$G_{\max}$ (MPa)	$K_{2,\max}$	$G_{\max}$ (MPa)	$K_{2,\max}$			
Model tests													
M1	0.102	9.702	2.412	3.11	0.395	2.627	20.952	54.520	19.333	67.131	23.804	0.382	0.016
M2	0.058	17.136	2.966	2.65	0.395	2.627	20.952	54.520	19.333	67.131	23.804	0.227	0.010
M3	0.038	26.444	3.395	2.37	0.395	2.627	20.952	54.520	19.333	67.131	23.804	0.181	0.008
M4	0.374	2.364	1.173	5.09	0.485	2.291	20.279	56.371	18.670	69.769	23.107	0.599	0.026
M5	0.374	2.364	1.173	5.09	0.293	2.068	20.704	60.524	19.042	75.405	23.724	2.795	0.118
M6	0.281	3.292	1.433	4.49	0.293	2.068	20.704	60.524	19.042	75.405	23.724	2.715	0.114
M7	0.187	5.126	1.816	3.83	0.293	2.068	20.704	60.524	19.042	75.405	23.724	3.184	0.134
M8	0.067	14.842	2.825	2.75	0.426	0.865	8.505	37.551	7.640	37.450	7.619	0.022	0.003
M9	0.067	14.842	2.825	2.75	0.234	0.744	10.387	49.460	9.333	59.447	11.217	0.127	0.011
M10	0.093	10.655	2.502	3.02	0.281	3.868	24.896	53.787	23.146	66.061	28.428	0.248	0.009
M11	0.078	12.724	2.675	2.87	0.384	1.901	18.419	55.999	16.892	69.245	20.888	0.276	0.013
M12	0.078	12.724	2.675	2.87	0.384	1.901	18.419	55.999	16.892	69.245	20.888	0.276	0.013
M13	0.078	12.724	2.675	2.87	0.448	1.911	13.417	40.516	12.256	43.622	13.195	0.192	0.015
M14	0.316	2.880	1.325	4.72	0.412	3.164	17.228	40.850	15.897	44.280	17.232	0.259	0.015
M15	0.316	2.880	1.325	4.72	0.299	3.032	21.853	53.057	20.214	64.982	24.758	0.324	0.013
M16	0.379	2.326	1.161	5.12	0.412	2.636	15.784	40.850	14.512	44.280	15.731	0.299	0.019
M17	0.379	2.326	1.161	5.12	0.245	2.344	19.313	53.057	17.773	64.982	21.768	0.558	0.026
M18	0.237	3.970	1.591	4.19	0.426	3.118	14.405	34.332	13.265	29.925	11.562	0.094	0.008
M19	0.237	3.970	1.591	4.19	0.344	2.978	17.416	42.530	16.060	47.502	17.937	0.293	0.016
M20	0.237	3.970	1.591	4.19	0.287	2.937	20.798	51.247	19.217	62.246	23.341	0.413	0.018
M21	0.090	10.943	2.528	3.00	0.384	2.398	20.728	56.371	19.100	69.769	23.639	0.320	0.014
M22	0.090	10.943	2.528	3.00	0.384	2.398	20.728	56.371	19.100	69.769	23.639	0.389	0.016
M23	0.090	10.943	2.528	3.00	0.384	2.398	20.728	56.371	19.100	69.769	23.639	0.389	0.016

Note: *MTD2: $G_{max} = A_D \frac{1 + D_r / 100}{(a_D - D_r / 100)^2} p_{atm}^{1-n} (\sigma'_m)^n$ (MPa), $A_D=177$, $a_D=17.3$, $n=0.48$, $p_{atm}=100$ kPa, atmospheric pressure, and

†MTD4: $K_{2,max} = A_{KD} \frac{1 + D_r / 100}{(a_{KD} - D_r / 100)^2}$, $A_{KD}=6900$, $a_{KD}=16.1$. $G_{max} = 218.8 K_{2,max} (\sigma'_m)^{0.5}$, (kPa). Wichtmann and Triantafyllidis (2009)

† G_{max} calculated from Eqs. (5) and (6).

1. The full-scale field and centrifuge tests C1 and C2 have $kd/\bar{G}_s = 3.27 \sim 6.91$, with an average of 5.0. The model tests have $kd/\bar{G}_s = 2.37 - 5.12$, with an average of 3.7. High values of $kd/\bar{G}_s$ (an average value of 13.32) for the centrifuge tests C4 - C12 were obtained for the rectangular piers. Strictly speaking, Eq. (2) obtained from a cylindrical pile is not suitable for the rectangular pier (Basu and Salgado 2008). Therefore, the back-calculated values of the shear modulus from tests C4 - C12 with a width of 1 - 6 m were not included in the later analysis. This may partly explain the relatively high values of $kd/\bar{G}_s$ gained from tests F1-F7 with the 12-sided polygonal pole.

2. With constant pile diameter and embedded length, an increasing loading eccentricity generally results in an increased ratio $kd/\bar{G}_s$. For instance, the $kd/\bar{G}_s$ increases from 3.93 to 4.47 as the eccentricity increases from 0.15 m in test F16 to 2 m in test F15. The ratio $kd/\bar{G}_s$ appears to increase with the pile diameter. For example, in the series of tests M5 - M7, when the pile diameter is doubled from 0.0508 m to 0.1016 m, the $kd/\bar{G}_s$ increases by 33% from 3.83 to 5.09.

3. The values of G_{max} calculated using the methods proposed by Wichtmann and Triantafyllidis (2009) are within $\pm 25\%$ and $\pm 20\%$ of those calculated by Eqs. (5)-(6).

4. The ratios of $\bar{G}_s/G_{max}$ for the three tests F14 - F16 (bored piles in clayey sand) are much larger than those of the other full-scale field tests. The back-calculated $\bar{G}_s$ for test F16 is even 22% higher than the calculated G_{max} , owing to high plasticity (Vucetic and Dobry 1987). Thus, Eqs. (5) and (6) are not suitable for the clayey sand. The model tests M5 - M7 in extremely dense sand ($D_r = 100\%$) are associated with a ratio of $\bar{G}_s/G_{max}$ of 0.12, which is about 8.6 times the average value of 0.014 for the $\bar{G}_s/G_{max}$ obtained from the other model tests. The G_{max} might be underestimated.

5. The results of the 15 tests (F14 - F16, C4 - C12 and M5 - M7) were excluded in statistical analysis due to the reasons mentioned above, so were tests F12, F13 and C3 without D_r values. The deduced ratios of $\bar{G}_s/G_{max}$ are plotted against the relative density D_r for the remaining 33 tests in Fig. 12. The back-calculated $\bar{G}_s$ is approximately (3-20) % of G_{max} (with an average of 11.3%) for the 11 full-scale field tests (F1 - F11) and 2 centrifuge tests (C1 - C2), and (0.8-2.6) % of G_{max} (with an average of 1.4%) for the 20 model tests, indicating the impact of scale (Poulos *et al.* 2001), stress and strain level (Pestana and Salvati 2006, Guo 2012). The variation for the field tests may reflect the impact of installation for bored piles, cast-in-place piers and drilled piers as noted by Dyson and Randolph (2001) and Kim *et al.* (2004).

6. The current correlation of G_{max} with relative density is less accurate than that with void ratio (Wichtmann and Triantafyllidis 2009). Nevertheless, it is sufficiently accurate for practical purpose.

Fig. 12 $\bar{G}_s/G_{max} \sim D_r$ relationship

4.4 Estimation of N_g

The value of the dimensionless parameter N_g was calculated from the deduced A_r for each test using Eq. (4) and is presented in Table 3. The comparative study shows:

1. Excluding the five pile tests of F2 (in very loose sand), F6 (in dense crushed stone) and F14 - F16 (in clayey sand), the N_g is obtained as 1.0 - 3.0 (with an average of 1.41) for the 11 full-scale field tests and the three centrifuge tests C1, C2 and C3. This average N_g is 41% higher than that obtained from Eq. (4) with $N_g=1$. The current value is consistent with that obtained for 20 flexible piles in sand (Guo and Zhu 2010, Guo 2013a). The latter shows $N_g = 0.4$ -2.8 (with an average of 1.29) but for p_u varying with $z^{1.7}$ owing to the pile flexibility. The value of N_g varies from 0.70 to 4.77 (an average of 2.0) for the 23 model tests.

2. The N_g decreases with increase in pile diameter or width d . In particular, N_g reduces from 0.63 to 0.44 as the width of the rectangular pier increases from 1 m (test C4) to 6 m (test C8). The large pier behaves more as a rigid wall than a pile.

3. Excluding the three tests F14, F15, and F16 in clayey sand, the back-calculated N_g from the 48 tests in sand and crushed stones were plotted against the normalised pile diameter d/d_{ref} ($d_{ref}=1.0$ m) in Fig. 13. The N_g may be correlated with diameter by

$$N_g = (0.4-1.8)(d/d_{ref})^{-0.25} \quad (7)$$

5. Conclusions

The measured responses of 51 laterally loaded rigid piles in sand have been studied using the elastic-plastic solutions by Guo (2008). The analysis provides the critical parameters A_r , k and k_0 for the limiting force profile and modulus of subgrade reaction. These results are useful in conducting nonlinear design of lateral piles. The study shows:

1. The elastic-plastic solution based on a constant k and a linear limiting force profile generally gives good estimation against measured nonlinear response rather than that with a Gibson k . Generally, the solution with a constant k should be used to design the lateral piles.

2. The normalised load capacity reduces while the normalised moment capacity increases, as the ratio e/l increases.

3. The ratio of $kd/\bar{G}_s$ is 3.27 - 6.91 (with an average of 5.0) for the 16 full-scale field tests and 2 centrifuge tests; and it is 2.37- 5.12 (with an average of 3.7) for the 23 laboratory model tests.

4. The ratio of $\bar{G}_s/G_{max}$ is (3-20)% for the 11 full-scale and 2 centrifuge tests and (0.8-2.6)% for 20 model tests, with the G_{max} being calculated from Eqs. (5)- (6) using the relative density D_r . The $\bar{G}_s$ is only a small fraction of the small-strain modulus G_{max} .

5. The N_g may be estimated by $N_g = (0.4-1.8)(d/d_{ref})^{-0.25}$. The ultimate pile capacity increases with the increasing N_g .

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