

1-1-2005

## Friction in the roll bite under various hot rolling conditions

Weihua Sun

A. K. Tieu

*University of Wollongong, ktieu@uow.edu.au*

Hongchun Li

hl087@uow.edu.au

Zhengyi Jiang

*University of Wollongong, jiang@uow.edu.au*

Guodong Wang

*Northeastern University*

*See next page for additional authors*

Follow this and additional works at: <https://ro.uow.edu.au/engpapers>



Part of the [Engineering Commons](#)

<https://ro.uow.edu.au/engpapers/2637>

---

### Recommended Citation

Sun, Weihua; Tieu, A. K.; Li, Hongchun; Jiang, Zhengyi; Wang, Guodong; and Liu, Xianghua: Friction in the roll bite under various hot rolling conditions 2005, 110-120.

<https://ro.uow.edu.au/engpapers/2637>

---

**Authors**

Weihua Sun, A. K. Tieu, Hongchun Li, Zhengyi Jiang, Guodong Wang, and Xianghua Liu

# Friction in the Roll Bite Under Various Hot Rolling Conditions

W. Sun<sup>1,2,3</sup>, K. Tieu<sup>2</sup>, H. Li<sup>2</sup>, Z. Jiang<sup>2</sup>, G. Wang<sup>3</sup>, X. Liu<sup>3</sup>

(1. Technical Centre, Jinan Iron and Steel Ltd., Jinan China 250101; 2. Faculty of Engineering, University of Wollongong, NSW2522 Australia; 3. The State Key Laboratory of Rolling and Automation, Northeastern University, Shenyang China 110004)

**Abstract:** In this paper, the effect of hot rolling parameters on the friction coefficient was studied by inverse calculation on the experimental roll loads where the hot rolling experiments were carried out at 7.5-45.0% reductions, 0.12-0.72m/s speeds and temperature of 850-1025°C. Dry rolling, water, oil/water mixture emulsion and pure oil were used as lubricants in the experiments. In carrying out the calculation, a flow stress model for the present test material was first obtained experimentally. Hot rolling parameters, including reduction, roll speed, work piece entry temperature and lubrication conditions, were investigated to determine their effect on friction coefficient and mill loads. The effect of oxide scale thickness after rolling on friction was also examined.

**Keywords:** friction coefficient, oxide scale, lubrication, hot rolling of steel

## 1 Introduction

Friction in the roll bite during steel rolling has always been a topic of interest. As reported in reference [1-2], friction affects significantly the rolling load, roll wear, and strip shape, and, the friction requirement and roll speed control are important during threading and rolling on a hot strip mill. In fact, the steel processing parameters, such as reductions, rolling speeds, rolling temperature, surface roughness and scaling, affect the interfacial conditions in the roll bite and hence the friction coefficient [3]. Therefore, many studies on the effects of hot rolling parameters on friction have been carried out in the past two decades [4-18]. However, almost all the studies employed the primary scales that were resulted from reheating in the furnace, even though oxide scale thickness were concerned in some cases [9, 14, 16-17, 19]. What is more, the studies on the effect of hot rolling parameters were often focused on individual parameters. A comprehensive understanding of the influence from as many parameters on friction still needs to be made further.

The objective of this paper is to study how the hot rolling parameters affect the friction and mill loads. The study is to be carried out in two new areas. One is that all the investigation on friction focus on the so called 'secondary oxide scale' surface, on which the oxide scales created in the reheating furnace were removed. The second is that as many as the rolling parameters, such as reduction, roll speed, rolling temperature and the oxide scale thickness, are taken into account to make an integrate analysis on friction. Influence of test material original surface roughness was also taken into account in analyzing the dependence of friction

coefficient on hot rolling conditions.

## 2 Materials and experiments

**Material.** Steel for rolling experiments is a mild steel. Table 1 shows the chemical compositions of the material. The steel bar was 100mm wide and 450mm long. Surface of the rolling test coupons were as-supplied and ground to 0.075 $\mu$ m, 0.30 $\mu$ m and 3.0 $\mu$ m roughness along rolling direction before heating.

**Table 1** Chemical composition of the steel (mass, %)

C	Si	Mn	P	S	Cr	Ni	Cu	Mo	Al-T	Ti
0.18	0.18	0.95	0.026	0.027	0.10	0.067	0.13	0.19	0.004	<0.003

**Experiment procedure.** Hot rolling experiments were carried out on a 2-high Hille 100 experimental rolling mill. Detailed description of the rolling mill can be found in Ref. [23]. The reheating furnace was preliminarily soaked for 2-3 hours at 1200°C before heating of samples. Due to the sensitivity of oxide scale to heating time, samples were heated one by one in the furnace and soaked for 5 minutes at 1200°C so that all the samples would have same thermal history in the furnace and that the influence of heating time on the scale thickness would be minimized. Samples were descaled before rolling operation. Reductions for hot rolling were 7.5, 15.0, 25.0, 35.0 and 45.5% at 0.12m/s speed to investigate the effect of deformation on the friction coefficient at four temperatures as of 850, 900, 950 and 1025°C. There were five lubrication conditions, which included water, 1/100 oil/water mixture and 1/200 oil/water mixture, pure oil and without lubrication, were applied to exam the effect of lubrication on the coefficient of friction. On account of the effect of speed on the friction coefficient, four speeds were selected 0.12, 0.24, 0.48 and 0.72m/s. All samples were cooled in a cooling box with inert gas protection. The mentioned lubrication oil was donated by Quaker Chemicals Ltd., Australia.

**Hot tensile tests.** In order to accomplish the friction coefficient assessment, a flow stress model for the present tested steel material was set up by regressing the hot tensile test results from the GLEEBLE-3500 Thermal Mechanical Simulation Machine. The hot tension tests temperatures were from 800-1100°C and strain rates were from 0.5-10s<sup>-1</sup>.

**Oxide scale thickness.** Oxide scale thicknesses of samples after rolling were measured by using an optical microscope on the as-rolled sample surface along the rolling direction.

## 3 Inverse calculation for friction coefficient and flow stress model

A numerical model by Alexander was used to determine friction coefficient by an inverse calculation, in which the calculated rolling force was matched till it was less than 1% error with the measured one by varying the friction coefficient value. This program was developed

on the basis of Orowan's rolling model. The features and theory adapted in this model were detailed in [20]. According to [3], the yield stress of a steel can be described as

$$Y = \sigma_{p0} \cdot e^{-aT} \cdot k_1 \varepsilon^{m_1} \cdot k_2 \dot{\varepsilon}^{m_2} \quad (1)$$

where  $Y$  the yield stress,  $\sigma_{p0}$  the base yield stress,  $\varepsilon$  and  $\dot{\varepsilon}$  the strain and strain rate ( $s^{-1}$ ),  $T$  temperature in K,  $a$ ,  $k_1$ ,  $k_2$ ,  $m_1$  and  $m_2$  are all constants. In the Alexander's program, the flow stress is modified in the following shape [20]:

$$Y = Y_0 e^{-aT} (1 + B\varepsilon)^{n_1} \times (1 + D\dot{\varepsilon})^{n_2} \quad (2)$$

where  $Y_0$ ,  $a$ ,  $B$ ,  $D$ ,  $n_1$  and  $n_2$  are constant. Assuming  $B=D=1000 \gg 1$ , equation (2) still complies to equation (1). Then the constants  $Y_0$ ,  $a$ ,  $n_1$  and  $n_2$  in equation (7.2) are easily to be determined by multiple-regression. The regression result of the mentioned hot tensile tests is shown in equation (3):

$$Y = 158.44 \times \exp(-0.002532T) \times (1 + 1000\varepsilon)^{0.3695} \times (1 + 1000\dot{\varepsilon})^{0.1097} \quad (3)$$

in which  $T$ ,  $\varepsilon$  and  $\dot{\varepsilon}$  are the same as those in equations (1) and (2). Fig.1 illustrates comparison between the present flow stress model and Shida's model [30] in two cases. According to the chemical analysis of the present testing material,  $C_{eq}$  equals 0.34 when calculating yield stress with Shida's equation in Fig.1. The result in figure 1 demonstrates that the present model is consistent with Shida's model. The range of application for equation (3) is limited to temperatures between 800-1100°C, strain rate from 0.5-10 $s^{-1}$ , a strain below 0.6 and a constant  $C_{eq}$  equals 0.34 for a mild steel. The item  $Y_0 e^{-aT}$  in equation (3) had to be pre-calculated as an input data as well as  $n_1$  and  $n_2$  in equation (2) for each calculation.

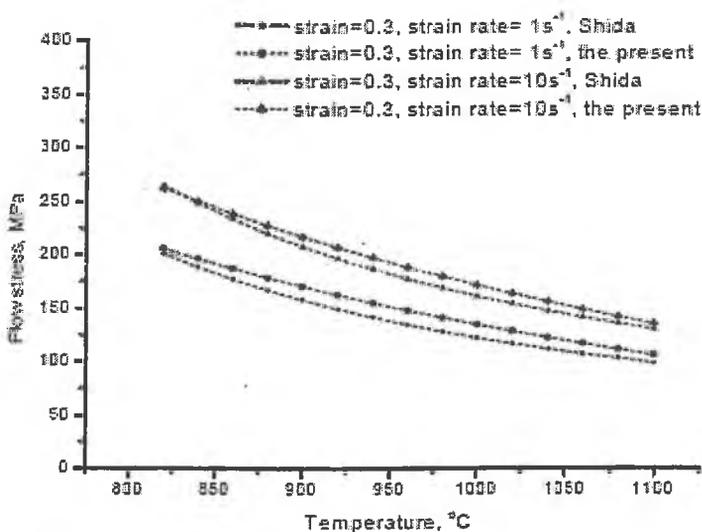


Fig. 1 Comparison of the present strain-stress curve with Shida's model

## 4 Results and Discussion

**Effect of entry temperature on the friction coefficient at various reductions.** There are two lubrication conditions considered. Fig.2 illustrates the effect of hot rolling temperature on

the friction coefficient at various reductions and at a certain rolling speed when rolling was carried out without lubrication and with oil lubrication. The original sample surface was  $R_a=0.30\mu\text{m}$ . In the cases without lubrication, as shown in Fig. 2 (a), the rolling temperatures were from 835-1029°C that were grouped into four series as 850, 900, 950 and 1025°C respectively. The nominal reductions were catalogued into four groups as 7.5, 15, 25 and 35% that were controlled in 6.0-35%. Rolling speed for the experiments in Fig. 2 was 0.12m/s. The coefficient of friction increases with reduction or as temperature decreases. From Fig. 2, it can be seen that friction coefficient increases more rapidly with temperature decreases without lubrication than with lubrication, consistent with Roberts<sup>[31]</sup>. However, the inverse calculation results show that friction coefficient of the present study is smaller than the published ones in [1-2, 31] where for an industrial hot mill or hot strip mill, the speed is much higher. In [31], Roberts also reports that the friction coefficient is 0.25-0.50 when the hot mill is cooled only by water, whilst friction coefficient decreases to 0.22-0.28 with a typical lubricant is applied in hot rolling.

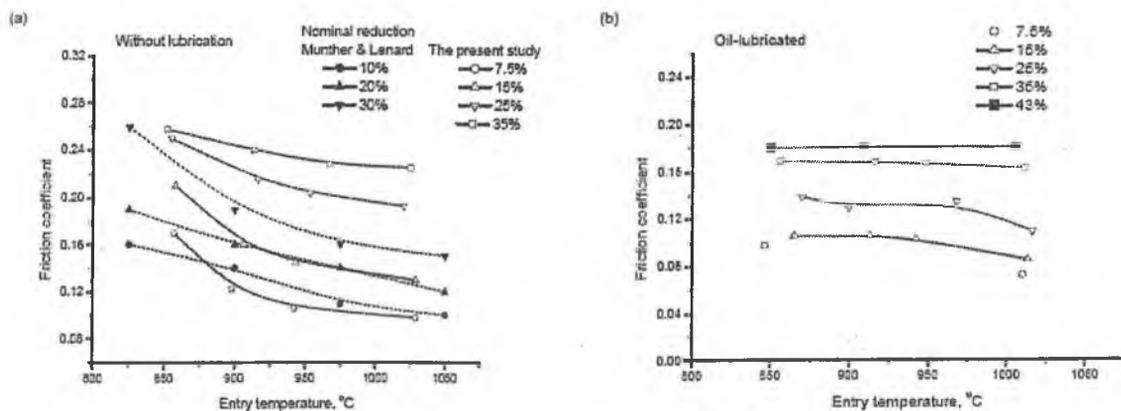


Fig. 2 Effect of rolling temperature on the friction coefficient and at various reduction. Rolling speed=0.10-0.12m/s for the present study and 0.196m/s for Munther and Lenard's<sup>[22]</sup>

In Fig. 2(a), the results were compared with those of Munther and Lenard's<sup>[22]</sup> whose data were applicable to low carbon steel AISI1018 on an experimental hot mill. From Fig. 2, it can be seen that the present friction coefficient values are approximately 10-30% larger than those in Ref. [22]. The difference between the present results in Fig.2 and Munther's was that the hot rolling speed in the latter study was 0.196m/s while it was about 0.12m/s in the present. In the meantime, sample surface was controlled as a uniform "secondary" oxide scale layer about 40-80 $\mu\text{m}$  in the present study while in [22] "the specimens was rolled with the scale on" after they were preheated for 90 minutes at 1200°C. There should be much difference in scale thickness between the present study and Munther and Lenard's, with the latter expected to be much thicker. The difference in rolling speeds and oxide scale thickness between the

two sets of results may be responsible for the difference of friction coefficient. For oil-lubricating conditions, Fig.2 (b) shows the influence of temperature on friction coefficient for various reductions. Generally, the coefficient of friction increases with reduction. However, the effect of temperature on friction coefficient was modified by lubrication. Although it increases slightly when temperature decreases at each reduction, the change of friction coefficient due to temperature is not significant. The heavier is the reduction, the smaller the change will be. At reductions of 35% and 43%, it seems that friction coefficient changes little when the entry temperature decreases from 1025 to 850°C. As a whole, the friction coefficient for lubricated condition is smaller than without lubricating. The value of friction coefficient is between 0.16-0.17 when reduction is 35% for example compared with 0.225-0.26 when there is no lubrication applied. This also complies for other reductions.

**Effect of entry temperature on the friction coefficient at various rolling speed.** Fig.3 illustrates the effect of temperature on friction coefficient when rolling was carried out at various speeds and at 15% reduction without lubrication. The nominal rolling speeds are 0.12, 0.24, 0.48 and 0.72m/s that were controlled in a range from 0.10-0.72m/s. As it can be seen from Fig. 3 that values for the coefficient of friction decreases with entry temperature when rolling was carried out without lubrication in the present study. Rolling speed is another important factor on friction coefficient. The higher is the rolling speed, the smaller the friction coefficient will be. For example, the friction coefficient changes from 0.16 to 0.14 at approximately 900°C when rolling speed increases from 0.12m/s to 0.72m/s. According to Fig.3, the friction coefficient at 15% reduction and 0.196m/s speed from Munther and Lenard's are quite close to that at 0.24m/s in the present study, indicating reasonable calculation result.

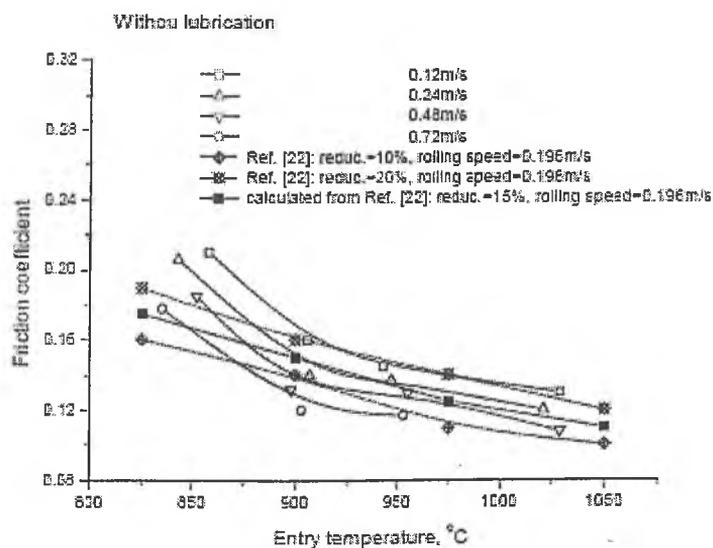
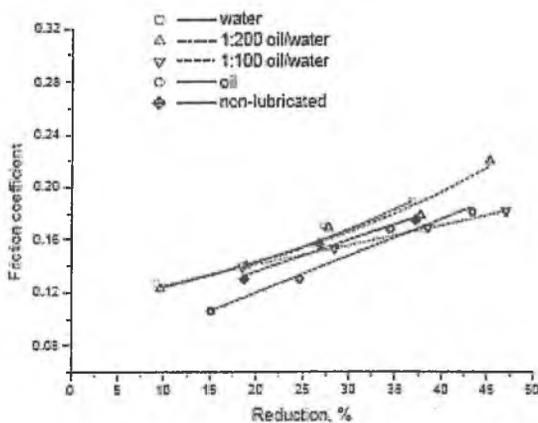
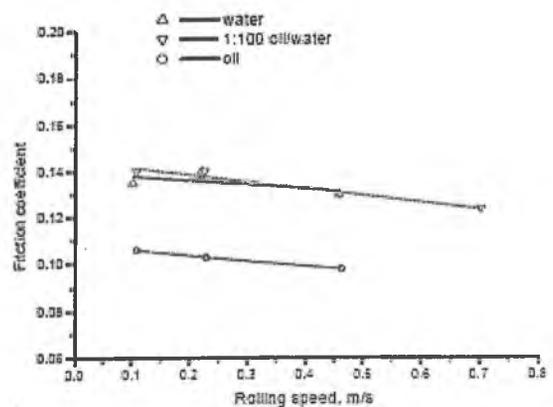


Fig. 3 Effect of entry temperature on friction coefficient at various rolling speeds without lubrication, comparing with [22]

**Influence of lubrication conditions on the friction coefficient.** The effect of lubrication on friction was further examined at 900°C entry temperature with five lubrication conditions: dry-lubricated, water, 1:200 and 1:100 oil/water emulsions, and pure oil. Fig. 4 displays the results of the effect of lubrication conditions on friction coefficient at different reductions. All the sample original surface roughness before reheating was 0.30 $\mu\text{m}$ . Nominal rolling speed was 0.12m/s and entry temperatures was 900°C. From Fig. 4, there is little difference in the coefficients of friction between water and the 1:200 oil/water mixed emulsion. The values of friction coefficient with no lubricant sit next to these two lubricants. For all the reduction conditions, the friction coefficient under pure oil lubrication were smaller than those without lubrication. The effect from the 1:100 oil/water mixed emulsion on friction coefficient was changing with reduction. At a lower reduction which is smaller than 27.5%, the friction coefficient for 1:100 oil/water emulsions is among the values for water, 1:200 oil/water mixture and dry-lubrication. As reduction increases, its value becomes smaller than these three cases. When heavy reduction was applied, 37.5% for example, the friction coefficient of the 1:100 emulsion case becomes even smaller than that at oil-lubricated condition. Increasing rolling speed brings about very limited reduction of friction coefficient with water, 1:100 oil/water mixture and oil lubrication, as shown in Fig.5.



**Fig. 4** Effect of lubricating conditions on friction coefficient: Rolling speed=12m/s and entry temperatures= 900°C



**Fig. 5** Effect of lubrications on friction coefficient and mill loads at various rolling speed: reduction=15.2%, temperature=900°C

**Effect of sample original sample surface roughness on friction.** The influence of surface roughness on friction in cold metal forming process has frequently been reported in the past few years [24-28], whilst it is seldom reported in hot rolling. The development of oxide scale layers on hot steel modifies the steel sample surface during preheating and exposed to atmosphere during the rolling and hence, giving rise to a more complex circumstances than in cold rolling. The interface between the tool and the work piece is expected to involve the oxide scale and the work-roll surface. Fresh hot metal may be pushed through cracks in the

oxide scale layer and contacted with the cold roll surface.

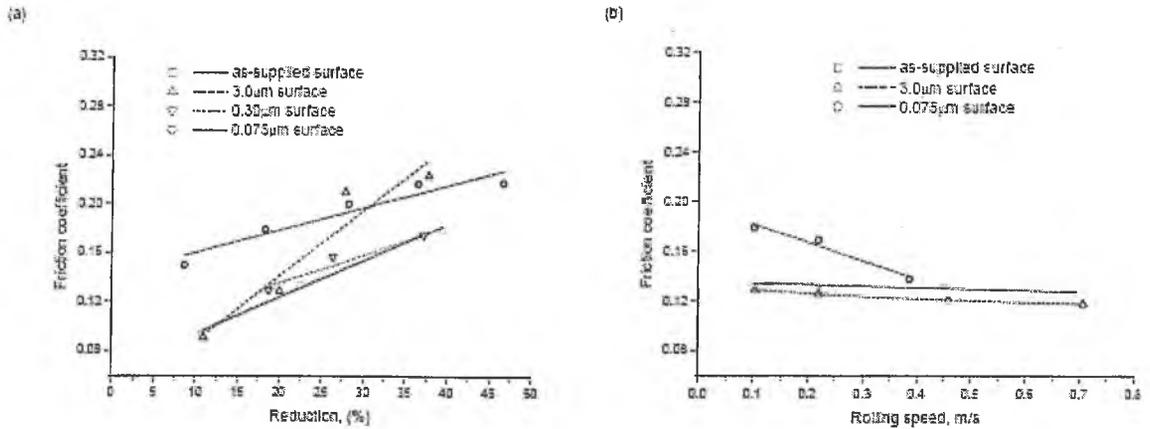


Fig. 6 Effect of sample original surface roughness on friction coefficient: rolling speed=12m/s and entry temperature=900°C: (a) at various reductions; (b) at various rolling speeds

Fig.6 displays the effect of the sample original surface roughness on the friction coefficient without lubrication. From Fig. 6, there exists significant difference in the values of the friction coefficient due to original sample surface roughness. Even though the values of the friction coefficient increase as reduction increases, the order in which the original surface roughness affects friction coefficient is unpredictable. However, the sensitivity of friction coefficient to reduction seems to be larger for a ground surface, as can be seen in Fig.6. Friction on samples with as-supplied surface increases moderately with reduction compared with the three sample groups that were mechanically ground. Friction coefficient decreases with rolling speed but not significant on the samples with as-supplied surface and the 0.30 μm machined surface cases.

**Effect of oxide scale thickness on friction.** In the case of the effect of oxide scale thickness on friction, it was that “thin scale promotes sliding friction with smooth rolls but sticking friction with rough rolls [3].” A linear relationship between the friction coefficient  $\mu$  and oxide scale thickness

after rolling  $\xi_{\text{exit}}$  (in  $\mu\text{m}$ ) was presented by Yu and Lenard [19]:

$$\mu = 0.369 - 0.0006\xi_{\text{exit}} \quad (4)$$

In equation (4), value of  $\xi$  is from 10-80 μm, working out an influence value of 0.0048 on the friction coefficient. However, in the present it is not easy to find any significant relationship between the coefficient of friction and the scale layer thickness for both ‘without lubrication’ and ‘with lubrication’. Fig.7 summarizes the friction coefficient as function of oxide scale thickness without lubrication and with oil-lubrication.

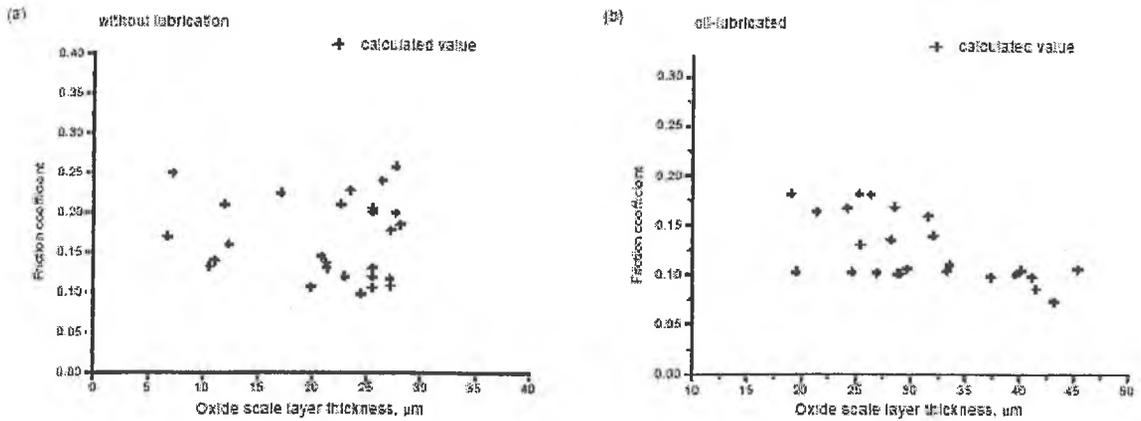


Fig. 7 Friction coefficient as function of oxide scale layer thickness. Entry temperature=835-1030°C; Reduction=6.2-44.2%; Speed=0.09-0.72m/s: (a) without lubrication; (b) at oil-lubrication

Many authors presented formulae to calculate friction coefficient for hot rolling of flat steel. Of all the results, linear relationship is usually used in accounting for the influence of rolling parameters. Roberts<sup>[31]</sup> found the friction coefficient increases with temperature, as shown in equation 5, where the author analyzed the data from a 2-high experimental rolling mill, an 84-inch hot strip mill and a 132-inch hot strip mill.

$$\mu = 2.7 \times 10^{-4} T - 0.08 \quad (5)$$

in which T is the temperature in °F. However, many others gave opposite results from the effect of work piece temperature. Rowe<sup>[28]</sup> and Underwood<sup>[29]</sup> who presented similar equation in describing the influence of temperature on friction coefficient, as illustrated in equations (6) and (7).

$$\mu = 0.84 - 0.0004T \quad (6)$$

$$\mu = 1.05 - 0.0005T \quad (7)$$

where the temperature T (°C) is in excess of 700°C. According to Geleji's work that was quoted by Lenard<sup>[3]</sup>, friction coefficient was described as linear functions of rolling speed and work-piece temperature depending on work roll materials:

$$\mu = 1.05 - 0.0005T - 0.056v \quad \text{for steel roll} \quad (8)$$

$$\mu = 0.94 - 0.0005T - 0.056v \quad \text{for double poured and cast roll} \quad (9)$$

$$\mu = 0.82 - 0.0005T - 0.056v \quad \text{for ground roll} \quad (10)$$

where T is the work-piece temperature in °C and v is the rolling velocity in m/s. According to equations (5)-(10), an effort of multiple regressions to correlate friction coefficient with all the rolling parameters was made using the following model:

$$\mu = a \times \varepsilon + b \times v + c \times T + d \times \xi + e \quad (11)$$

where a, b, c, d and e are constant,  $\varepsilon$  reduction in %, v the roll circumferential velocity in m/s, T entry temperature in °C and  $\xi$  scale thickness after rolling, in  $\mu\text{m}$ , for which the ranges for the parameters are in 6.2-44.2%, 0.09-0.72m/s, 835-1030°C and 14.7-45.4 $\mu\text{m}$  respectively. The regression results have been shown as equations (12) and (13).

$$\mu_{non-lub} = 0.404 + 0.0047\varepsilon - 0.056\nu - 0.00033T - 7.25 \times 10^{-5}\xi \quad (12)$$

$$\mu_{oil-lub} = 0.138 + 0.0028\varepsilon - 0.017\nu - 8.17 \times 10^{-5}T - 6.20 \times 10^{-5}\xi \quad (13)$$

According to the regression results, roll circumferential velocity and entry temperature, whose absolute values of t-ratio are larger than 1, exhibit significant influence on the friction coefficient [21]. The maximum influence of scale thickness on the friction coefficient is 0.0033 for without lubrication and 0.0028 for lubrication respectively. When lubrication is applied, the effect oxide scale thickness on the friction coefficient is much smaller than without lubrication. However, absolute t-ratio value of parameter  $\xi$  is less than 1, indicating that insignificant effect of scale thickness at exit of roll bite on the friction coefficient. Thus, the effect of scale thickness on the coefficient of friction is neglectable. Equations (14) and (15) are the friction coefficient at both without and with lubrication as functions of the indicated rolling parameters except for scale thickness at the roll bite exit.

$$\mu_{non-lub} = 0.405 + 0.0047\varepsilon - 0.057\nu - 0.00033T \quad (14)$$

$$\mu_{oil-lub} = 0.138 + 0.0028\varepsilon - 0.017\nu - 8.17 \times 10^{-5}T \quad (15)$$

Even though scale thickness is not included in equations (14) and (15), it is assumed that its effect may lie in the term of temperature, which displays a significant effect on scale thickness. In the present model, the effects of work-piece temperature and the roll speed on friction coefficient are quite close to Geleji's [3], Rowe [28] and Underwood [29], as illustrated in equations (6)-(10). Fig. 8 illustrates the comparison between the inverse calculated friction coefficient and the one predicted by the present models in equations (14)-(15). From Fig. 8, it demonstrates that the present models have reasonable accuracy. The results were summarized in Fig.8 for both 'without lubrication' and 'with lubrication' respectively, where the work-piece entry temperature, work roll circumferential velocity and reduction vary linearly with friction.

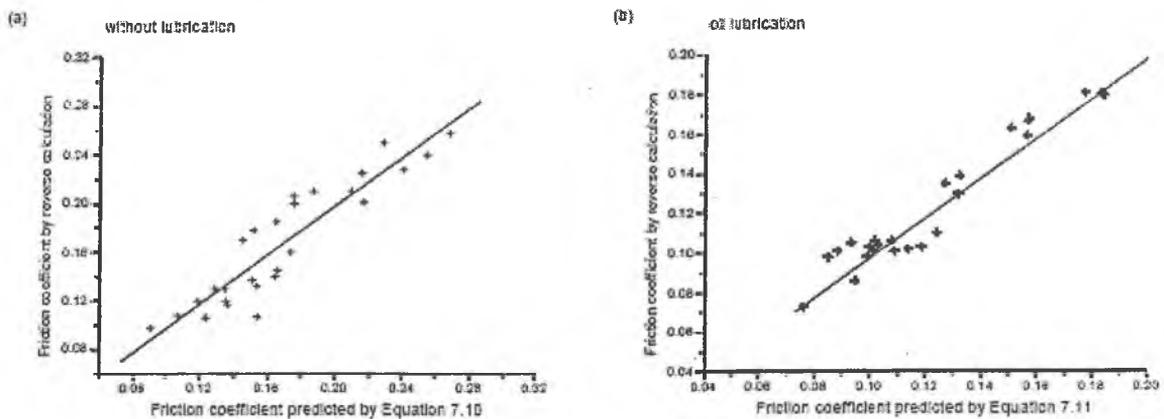


Fig. 8 Comparison between the inverse calculated and the predicted by the present models: (a) without lubrication; (b) oil-lubrication

## 5 Conclusions

In the present study, the effect of hot rolling parameters on the friction coefficient was studied by inverse calculation on the experimental roll loads. In carrying out the calculation, a flow stress model for the present test material was first obtained experimentally. Hot rolling parameters, including reduction, roll speed, work piece entry temperature and lubrication conditions, were investigated to determine their effect on friction coefficient and mill loads. The influence of sample original surface roughness and roll surface state were also examined.

- (1) Generally, coefficient of friction increases with reduction increases and as temperature decreases for rolling without lubrication. With oil as lubricant, the influence of temperature on the friction coefficient is insignificant.
- (2) For all temperatures, friction coefficient decreases as rolling speed increases. However, effect of entry temperature on friction at each rolling speed is insignificant.
- (3) Examination on the effect of emulsion lubricant on friction coefficient indicates the effectiveness of oil-lubrication at reduction less than 35%. At a higher reduction, the 1:100 oil/water mixed emulsion proves to be more effective.
- (4) The original sample surface roughness also displays a significant effect on friction coefficient. In the present study, it seems that the rougher the original sample surface is the more sensitive is the friction coefficient on reduction.
- (5) Linear regression results of friction coefficient as a function of relative rolling parameters can be written as:

$$\mu_{non-lub} = 0.405 + 0.0047\varepsilon - 0.057\nu - 0.00033T$$

$$\mu_{oil-lub} = 0.138 + 0.0028\varepsilon - 0.017\nu - 8.17 \times 10^{-5}T$$

However, it has been found that the oxide scale layer thickness does not have a significant influence on friction coefficient.

## Acknowledgments

The first author would like to thank Australian Government for scholarship support (IPRS and UPA) to undertake this research and acknowledge Bluescope Steel, Australia for permission to public the paper.

## References

- [1] W. Y. D. Yuen, First Australasia Con. on Appl. Mech., Feb. 1996, Melbourne: 927-932.
- [2] W. Y. D. Yuen, in: Proc. 1995 Int. Mech. Engng Conf. And Expo., San Francisco, Nov. 1995.
- [3] L. G. Lenard, M. Pietrzyk and L. Cser, Mathematical and Physical Simulation of the properties of Hot Rolled Product, Oxford, UK, 1999.
- [4] L. H. Luong and T. Heijkoop, Wear, 1981, 71: 93-102.
- [5] Y. H. Li, M. Krzyzanowski, J. H. Beynon and C. M. Sellars, ACTA METALLURGICA SINICA, 2000,

13: 359-368.

- [6] J. H. Beynon, Y. H. Li, M. Krzyzanowski and C. M. Sellars, in: Pietrzak, J. Kusiak, J. Majta, P. Hartley and I. Pillinger (eds), *Metal Forming 2000*, Balkema, Rotterdam, 2000: 3-10.
- [7] Y. H. Li and C. M. Sellars, in: J. H. Beynon, P. Ingham, H. Teichert and K. Waterson (eds), *Proc. 2nd Int. Conf on Modelling of Metal Rolling Processes*, London, 1996: 192-206.
- [8] J. Mascia, O. C. Marini, and E. Ubici, *Iron Steel Engineer*, 1998, 75: 48.
- [9] Per A. Munther, J. G. Lenard, *J. Mater. Proces. Technol.*, 1999, 88: 105-113.
- [10] D. T. Blazevic, in: *Proce. of the 37th MWSP Conf., ISS-AIME*, 1996, 37: 33-38.
- [11] M. Krzyzanowski and J. H. Beynon, *Mat. Sci. Techn.*, 1999, 15: 1191-1198.
- [12] M. Krzyzanowski, W. Yang, C. M. Sellars and J. H. Beynon, *Mat. Sci. Techn.*, 1999, 19: 109-116.
- [13] A. K. E. H. A. El-Kalay, L. G. M. Sparling, *Factors affecting friction and their effect upon load, torque, and spread in hot flat rolling*, *J. Iron Steel Inst.* 1968, 43: 152-168.
- [14] P. A. Munther, J. G. Lenard, *Scand. J. Metall.* 1997, 26: 231-240.
- [15] Y. H. Li, C. M. Sellars, in J. H. Beynon, P. Ingham, P. Kern and K. Waterson (eds): *Proc. of the 2nd Intern. Conf. on Model. of Metal Rolling Proces*, London, 1999: 178-186.
- [16] A. Shirizly and J. G. Lenard, *J. Mater. Proces. Technol.*, 2000, 101: 250-259.
- [17] A. Shirizly and J. G. Lenard, *J. of Mater. Proces. Technol.*, 2000, 97: 61-68.
- [18] J. Robertson and M. I. Manning, *Mater. Sci. Technol.* 1990, 6: 81-91.
- [19] Y. Yu and J. G. Lenard, *J. Mater. Proces. Technol.*, 2002, 121: 60-68.
- [20] J. M. Alexander, R. C. Brewer, G. W. Rowe, *Manufacturing Technology, Volume 2: engineering processes*, Ellis Horwood Ltd. Published, West Sussex, 1987.
- [21] J. Zhou, *Regression Analysis*, Normal University of Eastern China, Mar. 1993: 206-215.
- [22] P. A. Munther, J. G. Lenard, *CIRP Ann.* 1995, 44: 213-216.
- [23] W. Sun, A. K. Tieu et al, *Steel GRIPS*, 2 (2004) *Metal Forming*, 2004: 579-583.
- [24] S. Huart, M. Dubar., R. Deltombe, A. Dubois, L. Dubar, *Wear*, 2004, 257: 471-480.
- [25] H. Saiki, Y. Marumo, *J. Mater. Proces. Technol.*, 2003, 140: 25-29.
- [26] B. H. Lee, Y. T. Keuma, R. H. Wagoner, *J. Mater. Proces. Technol.*, 2002, 130: 60-63.
- [27] O. Mahrenholtz, N. Bontcheva, R. Iankov, *J. Mater. Proces. Technol.*, 2005, 159: 9-16.
- [28] G. W. Rowe, *Principles of industrial metal working process*, Edward Arnold, London, 1977.
- [29] L. R. Underwood, *The rolling of metals*, John Wiley & Sons, Inc., New York, 1950.
- [30] S. Shida, *Journal of JSTP*, 1969, 10: 610-617 (in Japanese).
- [31] W.L. Roberts, *Hot Rolling of Steel*, Marcel Dekker, New York, 1983.