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Nonlinear response of laterally loaded rigid piles in sliding soil

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Abstract

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Nonlinear response of laterally loaded rigid piles in sliding soil

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ABSTRACT

This paper proposes a new, integrated 2-layer model to capture nonlinear response of rotationally restrained laterally loaded rigid piles subjected to soil movement (sliding soil, or lateral spreading). First, typical pile response from model tests (using an inverse triangular loading profile) is presented, which includes profiles of ultimate on-pile force per unit length at typical sliding depths, and the evolution of pile deflection, rotation, and bending moment with soil movement. Second, a new model and closed-form expressions are developed for rotationally restrained passive piles in 2-layer soil, subjected to various movement profiles. **Third**, the solutions are used to examine the impact of the rotational restraint on nonlinear response of bending moment, shear force, on-pile force per unit length, and pile deflection. **And finally, they are** compared with measured response of model piles in sliding soil, or subjected to lateral spreading, and that of an in-situ test pile in moving soil.

The study indicates that (1) nonlinear response of rigid passive piles is owing to elastic pile-soil interaction with a progressive increase in sliding depth, whether in sliding soil or subjected to lateral spreading. (2) Theoretical solutions for a uniform movement can be used to model other soil movement profiles upon using a modification factor in the movement and its depth. And (3) A triangular and a uniform pressure profile on piles are theoretically deduced along lightly head-restrained, floating-base piles, and restrained-base piles, respectively, once subjected to lateral spreading. Nonlinear response of an in-situ test pile in sliding soil and a model pile subjected to lateral spreading is elaborated to highlight the use and the advantages of the proposed solutions, along with the ranges of four design parameters deduced from ten test piles.

Key words: passive piles, analytical solutions, non-linear response, soil-structure interaction

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1. INTRODUCTION

Passive piles are known as these piles that are subjected to soil movement and are commonly used for stabilizing a sliding slope, supporting bridge abutments, and providing a lateral pressure barrier adjacent to a pile driving or an excavation operation. Design of these passive piles may alter with pile-slide relative position, and pile-soil relative stiffness (Guo 2003; Guo 2008). More importantly, vertically loaded piles need to be checked against passive loading, induced by lateral spreading in earthquake zone.

Elastic solutions were proposed to simulate slope stabilising piles subjected to a uniform soil movement (Fukuoka 1977), and to model piles under an inverse triangular profile of moving soil (Cai and Ugai 2003). The later solutions compare well with measured response of six in-situ piles, albeit using measured sliding thrust and gradient of soil movement with depth for each pile. All the predictions are unfortunately not related to magnitude of the soil movement (Ito and Matsui 1975; De Beer and Carpentier 1977; Viggiani 1981; Chmoulian 2004). Guo (2003) proposed to gain a fictitious load on a passive pile for each magnitude of soil movement (w_s). The load is subsequently employed to predict response of the passive pile using the elastic-plastic solutions for a laterally loaded pile underpinned by the limiting force per unit length (p_u), and modulus of subgrade reaction (k_s). The closed-form solutions well capture non-linear response of two infinitely long, passive piles, and six upper rigid (in sliding layer) and low flexible (in stable layer) piles (Guo 2012) against measured data using a progressively increasing ‘slip’ (equivalent to loading) depth. Nevertheless, they are not applicable to piles rigid in both sliding and stable layers, for which new solutions are required to avoid overestimating bending moment in passive piles (Chen and Poulos 1997) by considering nonlinear response.

Numerical analyses have been extensively conducted (Stewart et al. 1994; Poulos 1995; Chow 1996; Bransby and Springman 1997), which demonstrate the dominant impact of the p_u profile and pile-soil relative stiffness on the pile response (Guo 2012). Several p - y curves (p = force per unit length, y = local pile displacement) for liquefied soil are suggested, such as those using an average p -multiplier (Brandenberg et al. 2005), an average residual strength (Seed and Harder 1990; Wang and Reese 1998; Olson and Stark 2002; Idriss and Boulanger 2007), and a dilation-based liquefaction model (Rollins et al. 2005). These p - y curves, while useful for some pertinent circumstances (Franke and Rollins 2013), offer values of on-pile force per unit length (thus p_u)

different by up to an order of magnitude. Naturally, the existing methods such as the p - y curve based analysis (Chen et al. 2002; Smethurst and Powrie 2007; Frank and Pouget 2008) are not sufficiently accurate. The p_u and the modulus k can be effectively and uniquely deduced using measured nonlinear response and elastic-plastic solutions, as has been done recently for about 70 laterally loaded piles (Guo 2006; Guo 2008).

An extensive experimental and numerical analysis has been conducted over the past decades on response of piles subjected to lateral spreading (Jakrapiyanun 2002; Boulanger et al. 2003; Kagawa et al. 2004; Cubrinovskia et al. 2006; Juirnarongrit and Ashford 2006). The response is generally characterised by rigid pile-liquefied sand interaction, as the liquefied sand is of very low stiffness and strength. The impact of soil movement on the pile is largely captured using a stipulated uniform, or a linearly distributed limiting pressure, from which simple solutions were developed using equilibrium of force and bending moment of the pile (Dobry et al. 2003; He et al. 2009). The solutions for rigid passive piles (Fukuoka 1977; Viggiani 1981; Cai and Ugai 2003; Dobry et al. 2003; Brandenburg et al. 2005) may work well for certain cases, but they generally break down theoretically without compatible displacement between piles and the moving soil. For instance, some measured data indicate a linear variation of bending moment along piles, which generally do not support a uniform or a triangular distributed p profiles as stipulated (Dobry et al. 2003; He et al. 2009), neither support ~ 10 times different average p over typical piles observed in previous study.

Guo (2014) recently developed a concentrated load (P -) based model, a power-law pressure (p -based) model and 2-layer model to capture the impact of soil movement on rigid piles using the load P or the distributed pressure p . New closed-form solutions were developed for each model, in light of equilibrium of force and moment, and displacement compatibility (rigorous) for the pile-soil system. In particular, the solutions for the 2-layer model yield a limiting pressure on the passive pile about one-third that on active piles, which is in accord with measured data. Nevertheless, the model application domain is confined to a uniform soil movement, free rotational constraint along pile (e.g. head-rotational stiffness $k_A = 0$, base-rotational stiffness $k_B = 0$), and no head constrained force $H (= 0)$, nor bending moment $M_o (= 0)$. Experimental data (Dobry et al. 2003; He et al. 2009) indicate the on-pile p (thus pile response) varies with distance, stiffness and profile of soil movement. A non-liquefied layer may cause dragging on a lateral spreading layer, which may be encapsulated as a rotational stiffness (thus moment M_A), a concentrated thrust H at the top of underlying layer, and a moment M_o due to loading eccentricity of H (Dobry et al. 2003; Brandenburg et al. 2005). To predict the pile response, new solutions are required.

This paper presents new **displacement-compatible solutions** to capture nonlinear response of laterally loaded rigid piles subjected to moving soil. First, the response (on-pile force per unit

length, deflection, shear force, and bending moment) of five model piles is highlighted, subjected to an inverse ‘triangular’ profile of soil movement to sliding depths of $(0.18\sim0.5)l$ (l = pile embedment). Second, an advanced 2-layer model for laterally loaded rigid piles in sliding soil is proposed, including the constraints on the top-layer ($k_A \neq 0$, $H \neq 0$, and $M_o \neq 0$, with a subgrade modulus k_s), and the base-layer ($k_B \neq 0$, with mk_s), respectively. New closed-form expressions are developed for the model, which are illustrated in non-dimensional charts. Third, the solutions are employed to capture nonlinear evolution of bending moment, shear force, on-pile force per unit length, and pile deflection by using a gradually increased sliding depth and on-pile pressure. They are elaborated, respectively, for one pile in sliding soil and another one subjected to lateral spreading. Finally, the solutions are used to predict response of four model piles and one in-situ test pile in sliding soil, and that of six model piles subjected to lateral spreading. Input parameters of the model are deduced against the measured data to facilitate the use of the new solutions.

2. MODEL TESTS ON PASSIVE PILES

Guo and Ghee (2006) devised a square shear apparatus with $1 \times 1 \text{ m}^2$ in plan and 0.8 m in height to simulate response of passive piles (see Fig. 1a). Horizontal force was applied laterally (via the lateral jack) on a loading block to translate the aluminum frames of the upper portion of the shear box (thus the adjacent sand). The loading block was made to a uniform (U), an inverse triangular (T) (as shown in Fig. 1a) and an arc (A) shape. It generates a U, T or A profile of soil movement (thus referred to as U, T or A profiles) at the loading location, respectively, but an unknown sand movement across the shear box and around the test pile. The model piles tested, referred to as d_{32} or d_{50} piles, were all made of aluminum tube with 1,200 mm in length. The d_{32} piles are featured by d (diameter) = 32 mm, t (wall thickness) = 1.5 mm, and $E_p I_p$ (calculated bending stiffness) = $1.28 \times 10^6 \text{ kNm}^2$; whereas the d_{50} piles have $d = 50 \text{ mm}$, $t = 2.0 \text{ mm}$, and $E_p I_p = 5.89 \times 10^6 \text{ kNm}^2$. The d_{50} and d_{32} piles were tested to model rigid and flexible piles, respectively in a sand that has a unit weight of 16.27 kN/m^3 , and an angle of internal friction of 38° . During the shearing, the sand surface was free of loading, the pile was thus only subjected to lateral pressure caused by the moving sand, apart from the overburden pressure (typically, $\sim 11.4 \text{ kPa}$ at pile-tip level, and with an average of $3.25\sim6.5 \text{ kPa}$) due to self-weight. Advancing the lateral T block horizontally (see Fig. 1a), for instance, the frames (thus the sand) was displaced downwards (to a maximum depth l_m) with each 10 mm horizontal movement (measured on the top frame), until a total lateral (frame) movement w_f (see Fig. 1a) of $110 \sim 150 \text{ mm}$ was achieved. The model sand samples are predominantly sheared under an overburden stress of $3.25\sim6.5 \text{ kPa}$ (at $l_m = 200\sim400$

mm). The lateral shear force (measured in the loading jack) increased by about 10% for each additional test pile (Guo and Qin 2010).

Five tests of T32-0 on the d_{32} piles (without vertical load on pile-head) using T block are reviewed herein. They were conducted to a final sliding depth l_m of 125, 200, 250, 300, and 350 mm, respectively for a pile embedment l of 700 mm (Guo and Qin 2010). Each test provides readings of ten pairs of strain gauges (along the pile length), two LVDTs (for displacements at pile-head level, and pile rotation), and the force on the lateral jack under each frame movement. They were input into a spreadsheet program (via Microsoft Excel VBA) to obtain the profiles of (1) bending moment; (2) inclination and deflection, respectively (from 1st and 2nd order numerical integration of the bending moment, respectively); and (3) shear force, and soil reaction (by using single and double numerical differentiation of the bending moment, respectively) (Guo and Qin 2006). Typical response is presented here, including (i) The profile of the net force per unit length p on the pile to a final sliding depth l_m shown in Fig. 2a; (ii) The evolution of pile deflection w_g ($\approx 0.72w_f - 42$ mm) at ground-line with the total soil movement w_f in Fig. 2b; (iii) The normalised rotation angle $\omega_r k_s / p$ (ω_r = rotation angle, k_s = modulus of subgrade reaction) versus pile-head displacement $w_g k_s / p$ in Fig. 2c; and (iv) The maximum bending moment M_m for each displacement w_g in Fig. 2d. Similar response of d_{50} pile is noted, which is presented here in Fig. 2c only.

These tests reveal (i) a progressive increase in the on-pile force per unit length p with the sliding depth l_m , which is described by $p = p_l l_m / l$ with p_l being the maximum p at pile-tip level (see Fig. 2a); (ii) the pile-deflection w_g (at ground-line) being a fraction of the shear frame (soil) movement w_f (see Fig. 2b); (iii) a linear correlation (thus elastic pile-soil interaction) between w_g and ω_r for typical sliding depths of l_m (see Fig. 2c); (iv) A highly nonlinear dependence between the pile deflection w_g and the maximum bending moment M_m (see Fig. 2d). A gradually increased l_m and the on-pile p (with depth) in Fig. 2a with soil movement thus render the nonlinear relationship between w_g and M_m in Fig. 2d. It is worthy to stress that the on-pile p for a sliding depth of 350 mm (with $l_m / l = 0.5$) in Fig. 2a should be the largest (see later discussion) for a uniform soil movement. The peak p at a reduced 0.3 m indicates the impact of soil movement profile (e.g. via a factor α) around the test piles.

3. ADVANCED 2-LAYER MODEL AND SOLUTIONS

A pile is classified as rigid, once the pile-soil relative stiffness, $E_p / \tilde{G}_s$, exceeds $0.052(l/r_o)^4$, as with a laterally loaded free-head pile (Guo 2006; Guo 2008). Note that E_p = Young's modulus of an equivalent solid pile; r_o = an outside radius of a cylindrical pile; and $\tilde{G}_s$ = average shear modulus

over the embedment l .

The passive pile addressed here is illustrated Fig. 3a: A rotationally restrained, rigid pile (with embedment of l) is subjected to an upper, moving layer (of a thickness l_m), and is stabilised by a lower layer (of λl_m in thickness). The pile-soil interaction (active or passive loading) is modelled by a series of springs distributed along the pile shaft (Guo 2008), which has a modulus of subgrade reaction k_s and mk_s in the sliding layer, and the stable layer respectively. The rotational restraint can be a distributed or a concentrated moment at any position along the pile, although it is plotted as the lumped springs k_A and k_B at the pile-top and bottom, respectively, in Fig. 3c. As shown in Fig. 3c, the impact of a uniform soil movement $w_s (= p/k_s)$ is replaced with a uniform force per unit length p to a depth of c on the pile. The pile rotates rigidly about a depth $z_r (= -w_g/\omega_r)$ to an angle ω_r and a mudline deflection w_g ; and has a deflection $w(z) (= \omega_r z + w_g)$ at depth z and $w(z_r) = 0$. The resistance per unit length $p(z)$ on the pile is proportional to the modulus of subgrade reaction $k_s (= kd$, a constant within each layer; d = outside diameter or width) and the local displacement, $w(z) (= w)$ with $p(z) = k_s w(z)$ in the sliding layer and $p(z) = mk_s w(z)$ in the stable layer, respectively. The modulus k_s is equal to $(2.2 \sim 2.85) \tilde{G}_s$, for instance, for a model pile having $l = 0.7$ m, and $d = 0.05$ m (Guo 2008).

3.1 Advanced 2-layer Model for Piles with H , M_o , and $k_\theta (= k_A + k_B)$

As reviewed earlier, Guo (2014) developed the 2-layer model shown in Fig. 3c and its solutions, concerning the pile without any constraints and force but for the soil resistance. As a further step, an **advanced 2-layer model** is proposed here to incorporate the impact of (1) any moment induced by rotational restraint ($= k_\theta \omega_r$) over the pile embedment [such as the head-constraint moment $M_A (= k_A \omega_r$, and $k_A > 0$), the base constraint moment $M_B (= k_B \omega_r$, and $k_B > 0$), etc]; (2) the lateral shear force H at the head level ($H > 0$); (3) the **ground-level** bending moment M_o (**due to eccentric loading**); and (4) the soil movement profile and loading distance from the pile(s). Note the impact of (4), as explained later, is incorporated through use of the factor α **in the** on-pile $p [= p_l l_m / (\alpha l)]$. The on-pile resistance force per unit length $p(z)$ is proportional to the corresponding subgrade modulus k_s or mk_s , respectively. The net force per unit length of $p_1(z)$ or $p_2(z)$ has an upper limit of the on-pile p at l_m .

Incorporating the conditions of $k_A \neq 0$, $k_B \neq 0$, $H \neq 0$, and $M_o \neq 0$, (see Fig. 3c), new explicit expressions for the advanced 2-layer model were deduced in the same manner as that shown previously by Guo (2014) in light of force and bending moment equilibrium (see Appendix A). Typical expressions are as follows:

(1) The pile-deflection at depth z , $w(z)$ is given by

$$[1] \quad w(z) = (\bar{\omega}_r \bar{z} + \bar{w}_g) p / k_s$$

where $\bar{\omega}_r [= w'(z)k_s l / p]$ and $\bar{w}_g (= w_g k_s / p)$ are given by

$$[2] \quad \bar{\omega}_r = \frac{-6[(2m\lambda + m\lambda^2 + 1)(\bar{c} + \bar{H})\bar{l}_m - (m\lambda + 1)(\bar{c}^2 - 2\bar{M}_o)]}{(1 + 4m\lambda + 6m\lambda^2 + m^2\lambda^4 + 4m\lambda^3)\bar{l}_m^3 + 12(1 + m\lambda)\bar{k}_\theta}$$

$$[3] \quad \bar{w}_g = \frac{4(3m\lambda + 3m\lambda^2 + m\lambda^3 + 1)(\bar{c} + \bar{H})\bar{l}_m^3 + (6\bar{M}_o - 3\bar{c}^2)(2m\lambda + m\lambda^2 + 1)\bar{l}_m^2 + 12(\bar{c} + \bar{H})\bar{k}_\theta}{[(1 + 4m\lambda + 6m\lambda^2 + m^2\lambda^4 + 4m\lambda^3)\bar{l}_m^3 + 12(1 + m\lambda)\bar{k}_\theta]\bar{l}_m}$$

where $\bar{k}_\theta = k_\theta / (k_s l^3)$, $\bar{c} = c / l$, $\bar{l}_m = l_m / l$, $\bar{H} = H / (pl)$, and $\bar{M}_o = M_o / (pl^2)$. The k_θ is equal to the total rotational stiffness along the pile. For instance, it is the sum of the top stiffness $k_A (= M_A / \omega_r)$ and bottom stiffness $k_B (= M_B / \omega_r)$ of non-liquefied layers (i.e. $k_\theta = k_A + k_B$). The values (e.g. k_A and k_B) of the stiffness may be different, but the associated angle of rotation ω_r is identical along the rigid pile.

(2) The maximum bending moment M_{m2} is given by

$$[4] \quad \begin{aligned} M_{m2} / (pl^2) = & \left[\frac{m}{6} \bar{z}_{m2}^3 + (1 - m)\bar{l}_m^2 \left(\frac{\bar{z}_{m2}}{2} - \frac{\bar{l}_m}{3} \right) + \bar{k}_A \right] \bar{\omega}_r - \bar{z}_{m2} \bar{H} \\ & - \bar{M}_o + [0.5m\bar{z}_{m2}^2 + (1 - m)\bar{l}_m (\bar{z}_{m2} - 0.5\bar{l}_m)] \bar{w}_g - 0.5\bar{c} (2\bar{z}_{m2} - \bar{c}) \end{aligned}$$

where $\bar{M}_{m2} = M_{m2} / (pl^2)$, $\bar{k}_A = k_A / (k_s l^3)$, and $\bar{z}_{m2} = z_{m2} / l$. Note that the impact of pile cross-section shape and any vertical load P (see Fig. 2c) on the pile is accommodated through a modified value of the force per unit length p_l . As will be published elsewhere, a vertical load normally induces a higher value of p_l , and additional bending moment (due to P - δ effect). Other expressions are provided in Table 1, which encompass the normalised depth $\bar{z}_{m2}$ of the M_{m2} , the maximum shear force T_{m2} , the shear force $T_i(z)$ and the bending moment $M_i(z)$ at depth z ($= 0 \sim c$, with subscript 1) and those at $z = c \sim l$ (with subscript 2). At $\bar{k}_\theta = 0$, $\bar{H} = 0$, and $\bar{M}_o = 0$, the current solution reduces to the 2-layer solution proposed by Guo (2014), as expected. In using the solutions, it should be stressed that (1) the net resistance per unit length $p_l(z)$ within the loading depth l_m is the difference between p and $k_s w_1(z)$; (2) Loading depth c is equal to sliding depth l_m ($< l$) for piles in a two-layer soil; (3) c is less than l_m for full-length ($l_m = l$) lateral spreading case; and (4) Four input parameters m , k_s , p (via p_l), and k_θ are required. The use of the solutions to rigid piles subjected to other soil movement profiles are discussed subsequently.

3.2 Salient Features of 2-layer Models

The evolution of normalised rotation $\bar{\omega}_r$, displacement $\bar{w}_g$, maximum bending moment $M_m / (pl_m l)$,

and maximum shear force $T_m/(pl_m)$ with the normalised sliding depth $\bar{l}_m$ was obtained using 2-layer model (Guo 2014) and the current advanced 2-layer model for a few typical m values. Some salient features of the two 2-layer models are noted, such as

- (i) The calculated on-pile pressure is close to the measured values on passive piles in clay (Viggiani 1981), which reveals an elastic pile-soil interaction. The estimated maximum shear force, however, is higher than the measured values in the model piles (Guo 2014) in sliding sand (and on the safe side).
- (ii) The normalised maximum bending moment M_mk_s/pl_m at various normalised displacements of $\bar{w}_g$ compares well with the boundary element solution (BEM) (Chen and Poulos 1997) upon using a pile deflection $w_g = w_s (= p/k_s)$ for a uniform soil movement (Guo 2012); and
- (iii) The nonlinear pile response (e.g. the moment M_m , the pile-displacement w_g) is originated from a gradual increase in the sliding depth l_m and the associated increase in the on-pile force per unit length $p (= p_l l_m/l)$.

Equations [1] – [4] and those expressions in Table 1 are deduced for a uniform movement of sliding soil, but they can be used to predict response of piles subjected to other shapes of soil movement, as explained below:

- The current solutions for a uniform soil movement $w_s (= \text{pile displacement } w_g)$ are obtained first. The movement w_s and its depth l_m are then modified as w_s/α (i.e. $w_g/\alpha = w_s$) and l_m/α , respectively. They then become these for an inverse triangular moving soil (i.e. IT w_s), for instance, by taking $\alpha = 0.72$, and match well with the corresponding BEM solution (see Fig. 4a). The use of $w_g/\alpha = w_s$ is also justified for all piles as elaborated subsequently.
- The current model tests show $w_g = 0.72w_s$ ($\alpha = 1.39$, see Fig. 1b, $w_s \approx w_f - 42$ mm, ignoring the 42 mm ineffective movement.). The high α value may be attributed to other profiles (e.g. a trapezoid) of soil movement under the T-block loading. The α value in later examples is equal to 0.59 (in-situ test piles) and 1.39-1.5 (for the model tests in sliding soil or subjected to lateral spreading).

The use of α is convenient to capture the overall impact of soil movement on passive piles. In practice, a pile may be embedded in a sandwiched liquefied layer with an upper and a lower non-liquefied layer (see Fig. 4b₁). As mentioned previously, the impact of the upper non-liquefied layer on the pile is encapsulated as a shear force (H), and a rotational moment $M_A (= k_A \omega_r)$ that exerts at the top of the liquefied layer (see Fig. 4b); whereas that of the lower layer on the pile is captured using a rotational constraint $M_B (= k_B \omega_r)$. The modelling of the pile-soil interaction during lateral

spreading thus becomes resolving the advanced 2-layer model in Fig. 3c but for the following salient features:

- The total soil movement w_s is equal to the displacement w_g of the rigid pile subjected to lateral spreading, which consists of rotational and translational components. The relative (rotational) pile displacement between the top and base displacements is equal to $w'(z)l$ (rotation $w'(z) = \omega_r$). The net local displacement y between the pile and the surrounding soil at depth z is equal to $\omega_r z$ ($= \bar{\omega}_r \bar{z}$) after deducting the translation component. The associated resistance force per unit length $p(z)$ is equal to $\omega_r z p$ ($= \bar{\omega}_r \bar{z} p$) after deducting the translational resistance $w_g k_s$ (see eq. [1]). The displacement $w(z)$ and the force per unit length $p(z)$ constitute the p - y (w) curve at the depth z .
- The net pressure gradually increases to a maximum and subsequently reduces with the lateral movement. A translational resistance may stay at a very large soil movement, and holds a residual bending moment if $k_B \neq 0$.

4. PARAMETRIC ANALYSIS ($H = 0$, $M_0 = 0$)

Out of the four input parameters m , k_s , p (via p_l), and k_θ , the two parameters k_s and p are used as normalisers. Parametric analysis was thus only focused on the impact of rotational stiffness and the modulus non-homogeneity m on pile response, and is presented in form of

- normalised soil movement ($= \alpha w_g k_s / p$) induced by increasing normalised sliding depths [$= l_m / (\alpha l)$] (see Fig. 5);
- normalised pile-soil relative displacement ($= \omega_r k_s l / p$) with the normalised soil displacement ($= \alpha w_g k_s / p$) (Fig. 6);
- normalised bending moment [$= M_m / (p l_m l)$] with the normalised soil movement (Fig. 7);
- normalised thrust [$= T_m / (p l_m)$] at sliding depth (Fig. 8) and that at true depth (Fig. 9), respectively; and
- normalised profiles of bending moment $M(z) / (p l_m l)$, shear force $T(z) / (p l_m)$, on-pile force per unit length $p(z) / p$, and pile-displacement $w(z) / l$ for a normalised sliding depth $\bar{l}_m$ of 0.75 (Fig. 10).

Figs. 5a and 5c indicate a linear increase in $\alpha w_g / w_s$ with the ratio $l_m / (\alpha l)$ for a perfectly head-rotationally restrained pile. At a movement w_s , an average pressure of $w_s k_s l_m / (\alpha^2 l)$ is induced over the pile embedment. The transitional movement w_g is thus equal to $w_s l_m / (\alpha^2 l)$ ($=$ the pressure over the k_s), or $\alpha w_g / w_s = l_m / (\alpha l)$. As the modulus ratio m increases, the base resistance becomes apparent, which reduces the ratio $\alpha w_g / w_s$ significantly (see Fig. 5c).

Fig. 6 shows an upper limit ratio $-\omega_r/l/w_g$ of 1.5 (= pile-soil relative displacement over soil movement $w_s = w_g$) for $l_m/l < 0.5$. This ratio and its displacement mode are independent of loading properties, and thus are identical to a laterally loaded rigid pile (Guo 2012). At a high $l_m/l (> 0.5)$, the normalised displacement $\bar{w}_g^*$ (* denotes the lower bound) shows an invert mirror image of that for $l_m/l < 0.5$, as is illustrated in the inset of Fig. 6a. The $\bar{w}_g^*$ is thus equal to the normalised base displacement w_b/w_s for $l_m/l < 0.5$. Therefore, $\bar{w}_g^* = w(l)/w_s = \bar{w}_r + \bar{w}_g$ is obtained in light of eq. [1]. As $-\omega_r/l/w_g = 1.5$, it follows $\bar{w}_g^* = -\bar{w}_r/3$, the lower bound for $l_m/l > 0.5$. The two extreme (bold) lines in Fig. 6a intersect at the point $(\bar{w}_g = 2, -\bar{w}_r = 3)$, which implies $\bar{w}_g \leq 2$ and $|\bar{w}_r| \leq 3$ for any rigid piles. For a highly rotational restrained pile, the moment at pile base $M_m (= k_B \omega_r = \omega_r \bar{k}_\theta k_s l^3)$ is equal to $pl_m/2$ at a negligible displacement $w_g/w_s (\approx 0)$. The normalised angle $-\bar{w}_r$ should be equal to $1/(2\bar{k}_\theta)$. In other words, the normalised pile relative-displacement converges towards $1/(2\bar{k}_\theta)$ as the m increases (e.g. $-\bar{w}_r = 0.05$ for $\bar{k}_\theta = 10$ at $w_g/w_s = 0$), which is illustrated in Fig. 6c.

The maximum bending moment (M_m) generally occurs at the depth l_m for piles in a sandwiched liquefied layer (which differs from that for a free-head laterally loaded pile). Irrespective of the head restrained conditions, the bending moment was calculated using $z = l_m$ in $M_1(z)$ (see Table 1) for typical $\bar{k}_\theta$ and m . The normalised $\bar{M}_m$ obtained is plotted in Fig. 7. In particular, for a fixed-head pile ($\bar{k}_\theta = 10$), the l_m/l (at $m = 1$) is 0.5, which offers the p distribution profiles shown in the insert of Fig. 7a. The M_m at l_m is thus deduced as $pl^2/16$, or $M_m/(pl_m) = 0.125$. The normalised M_m increases by 2.6 times from 0.124 ($m = 1, \bar{l}_m = 0.5$) to 0.32 ($m = 18, \bar{l}_m = 0.8$) for fully base-restrained piles, and converges towards 0.5 (see Fig. 7c). This is comparable with the moment of laterally loaded, fixed-head piles, of $0.5Hl$ (floating base) to $0.6Hl$ (fully restrained bases) (Guo 2012), and converges towards Hl (considering that $T_m \approx 0.5H$ for restrained head and base piles).

The normalised thrust $T_m/(pl_m)$ should not exceed the limit value of 0.333 (Viggiani 1981; Guo 2014), see Fig. 8 at sliding level. This is seen for a $\bar{l}_m$ below 0.4 [at a lightly head-restrained piles with $\bar{k}_\theta = 0.05$] to a $\bar{l}_m$ below 0.7 (fixed-head piles) (Fig. 8b). A high value of $\bar{T}_m (> 0.333$, dash lines) is difficult to achieve in practice. It should be mobilised instead, at a different depth from the l_m , which exhibits as dragging or formation of a translation layer (indicated by a high m value) (Fig. 9b). The normalised $\bar{T}_m$ reduces, see Fig. 9a (for $m = 1$), with the increase in the

normalised stiffness $\bar{k}_\theta$, which is not realistic for the head-constrained piles. The fact is that at a high $\bar{k}_\theta$, the T_m normally occurs at sliding level, and should be based on Fig. 8. In addition, at a high sliding depth, a much lower, normalised thrust will be induced, as it is governed by Mode A ($\bar{l}_m > 0.4 \sim 0.7$) in Fig. 9a, as discussed previously (Guo 2014). Finally, Figs. 5 through to 9 are for elastic response by using the on-pile force per unit length p .

The impact of base-rotational stiffness ($k_A = 0$, and $k_B \neq 0$) on the distribution profiles along a typical pile subjected to lateral spreading is evident (see Fig. 10). A free-head and floating-base pile will induce these profiles in dashed lines, whereas a fully fixed-base pile ($\bar{k}_\theta =$ infinitely large) may induce a uniform $p(z)$ in i^{th} layer and a uniform pile-displacement $w(z)$ with depth z . The assumed triangular and uniform p profiles (Dobry et al. 2003; He et al. 2009) are thus justified for a lightly head-restrained pile (e.g. $\bar{k}_\theta = 0.1$), and a fully fixed-base pile, respectively.

Finally, the impact of the applied shear force H and bending moment M_0 on the prediction can be examined through eq. [2]. It is not discussed here, but illustrated through the next example.

5. CASE PREDICTIONS

The 2-layer model (i.e. the current advanced model with $k_A = k_B = 0$, $H = 0$, and $M_0 = 0$) well predicts the nonlinear response of all model piles in sliding soil (Guo 2014) but for overestimating the maximum shear force. As will be published elsewhere, the overestimation can be avoided by introducing a transitional layer into either 2-layer model and using slightly different values of k_s , m and p_1 (see Fig. 2d, for instance). The predictions adopt a linearly increasing force per unit length p [$= p_1 l_m / (\alpha l)$] with the normalised sliding depth (l_m / l) in the elastic solution. Assuming a uniform p to a sliding depth of $i l_m / 10$ [$l_m =$ an assumed final sliding depth, say, $l_m = (0.7 \sim 0.9)l$ for full length lateral spreading], calculation is made for step $i = 1$, and for $i = 2, \dots, 10$, respectively. At the final sliding depth l_m ($i = 10$), for instance, the model pile-soil system is illustrated in Fig. 11a (upper figure). The uniform p (applied) should become a triangular increase (for a number of steps), as is depicted in Fig. 11a (lower figure), and is different from the net on-pile $p_1(z)$. The new features of the advanced model is examined, respectively, next by analysing an in-situ pile in sliding soil with $H \neq 0$, and $M_0 \neq 0$ ($k_A = k_B = 0$), a model pile subjected to lateral spreading with $k_A \neq 0$, and $k_B \neq 0$ ($H = 0$), and base rotationally-constrained ($k_B \neq 0$) piles subjected to full length, lateral spreading ($k_A = H = M_0 = 0$).

5.1 An In-situ Test Pile in Sliding Soil ($H \neq 0, M_o \neq 0$)

Frank and Pouget (2008) reported response of an pipe pile installed in downslope of an ‘sliding’ embankment. The pile (11.0 m in length, 0.915 m in diameter, and 19 mm in wall thickness) was instrumented with strain gauges. The soil movement was monitored using inclinometers and piezometers, which shows a trapezoidal movement profile to a sliding depth of 6.8 m. The soil has an average undrained shear strength s_u of 88 kPa, a unit weight γ_s of 17.0 kN/m³, and an effective angle of internal friction ϕ of 24.5°. During the 16- years-long test, the pile was pulled back by applying force H and moment M_o (at ~ 0.5 m above ground level) four times, while the soil sliding continued (thus the p exerted). The measured response by Frank and Pouget (2008) is plotted in Figs.11b -11d, including (b) the time-evolution of maximum bending moment M_{m1} at a depth of 3.75m and the shear load T_{m1} at pile-head level plotted as the dash line of M_{m1} [$= 0.25T_{m1}l$ (Guo and Qin 2010) **using the measured load T_{m1}**]; (c) the five profiles of force per unit length along the pile p after each ‘pulling back’ and at year 1999; And (d) The four pile-deflection profiles prior to **and after** each pulling-back. The applied bending moment M_o , and shear load H are provided in Table 2, along with the measured values of the M_{m1} and the ground-line displacement w_g . The measured bending moment profiles **during and after** each of the four pulling-back are plotted in Fig. 12. The displacement profiles exhibit the feature of laterally loaded, fixed-head piles during each pulling-back; whereas the linearly decreased displacement after each pre-pulling-back (from the ground-line to the sliding depth of 6.8 m) resembles that of a rigid pile subjected to passive loading. The theory for laterally loaded piles and the advanced 2-layer models are thus employed for the predictions, respectively.

To conduct the 2-layer prediction, the pile and soil properties were as follows: $l = 11.0$ m, $d = 0.915$ m, and $c = l_m = 6.8$ m ($\lambda = 0.618$). The $p_l = p_u$ at $l = 11.0$ m was estimated as 749.7 kN/m [$= 0.75\gamma_s K_p^2 dz$] (see Fig. 11c), in light of $\gamma_s = 17.0$ kN/m³, $\phi = 24.5^\circ$, and $d = 0.915$ m. The ultimate p_l increases with the repetition of the pulling-backs (see Table 2).

Taking $p_l = 0.9p_u$ for the 1986 pulling-back, **for instance**, the p ($= p_l l_m / l$) was estimated as 417.1 kN/m at the sliding level. The applied moment M_o ($= -94$ kNm), and the pile-head load $H = 0$ (see Table 2) offer $\bar{M}_0 = -1.863 \times 10^{-3}$ [$= -94 / (417.1 \times 11)$], and $\bar{H} = 0$. With $\bar{k}_\theta = 0$, $k_A = 0$, and taking $m = 4.5$, and $k_s = 2.86$ MPa (lower than the k for lateral loading due to a large pile-soil relative movement), the normalised ratios of $\bar{\omega}_r = -1.458$, and $\bar{w}_g = 1.251$ were obtained, respectively, using eqs. [2] and [3]. These values allow profiles of displacement, bending moment, shear force to be predicted using the expressions in Table 1. The predicted and measured displacement and moment profiles are plotted in Figs. 11d and 12a, respectively. Furthermore, the depths z_{m1} and z_{m2} of

maximum bending moment M_{m1} and M_{m2} were estimated as 3.729 m ($\bar{z}_{m1} = 0.345$), and 8.217 m ($\bar{z}_{m2} = 0.717$) in the sliding and the stable layer, respectively. The moment M_{m1} and M_{m2} were estimated as -345.23 kNm, and 659.23 kNm, respectively using $M_1(z_{m1})$ and $M_2(z_{m2})$ (see Table 1, and eq. [4]). As for the 1986-pulling stage, the input values were $M_o = -209$ kNm, $H = 310$ kN, $m = 5.5$ (high value for large dragging), and $k_s = 2.86$ MPa. The predictions were made, and are also shown in the figures, respectively.

As with the analysis of 1986 measurement, the predictions were repeated for other three stages (in 1988, 1992 and 1995) using the values of M_o , H , m , and k_s (see Table 2), and are shown in Figs. 11d and 12. Overall the predicted and the measured bending moment profiles agree with each other for each stage (see Figs. 12a-12d) on 5 Nov. 1986, 11 Nov. 1988, 1 Oct. 1992 and 6 July, 1995, respectively, so do the deflection profiles of the pre-pulling backs. Note the deflection and bending moment profiles during the pulling-back (solid symbols) should be predicted using the solutions for a laterally loaded pile, which are not pursued herein. In contrast, the profiles of bending moment during pull-backs depend solely on the ultimate on-pile pressure (at a sufficiently large pile-soil movement), and thus were estimated using the advanced 2-layer model.

The variations of the bending moments M_{m1} and M_{m2} with the pile-head displacement w_g during the loading cycles are illustrated in Figs. 13a and 13b for the sliding layer and the stable layer, respectively. A simplified loading of $M_o = 0$, and $H = 0$ kN, along with $m = 4.5$, $k_s = 2.86$ MPa, and $p_1 = 900$ kN/m ($= 1.2 \times 749.68$ kN/m) were used to predict the evolution of the maximum bending moments with the overall soil movement (with $\alpha = 0.588$), and that with the pile-head displacement over the 16 years, respectively. They are plotted in Figs. 13c and 13d, respectively. The predictions compare well with the measured data after the swap between $-M_{m1}$ at a depth of 3.75 m with the M_{m2} at depth 8-9 m. The predicted base displacement w_b versus the moment M_m curve does compare well with the measured w_g versus M_m curve at 8-9 m as expected. The swaps between the moments M_{m1} and M_{m2} at the depths, and between the displacements w_b and w_g thus verify the impact of the deep sliding ($l_m/l > 0.5$) on the displacement depicted in the insert of Fig. 6a. Furthermore, the impact of non-homogeneity m and any dragging ($\bar{k}_\theta > 0$) may be assessed against Figs. 7a and 7b.

5.2 Piles Subjected to Lateral Spreading

Abdoun et al (2003) conducted 8 centrifuge tests on 9 models of single piles and pile groups, at a centrifugal acceleration of 50g ($g = \text{gravity}$). The models were excited in flight with an input base acceleration that has 40 cycles of uniform acceleration, a prototype amplitude of 0.3g

and frequency of 2 Hz. Accelerometers and pore pressure transducers were installed in the soil to measure lateral accelerations and excess pore pressures; lateral LVDTs were mounted on the flexible walls of the laminar box to monitor the free-field soil lateral deformations; and strain gauges were used to measure bending moments in the piles.

Their Model 3 for a single pile tested in a two-layer soil profile is simulated herein, as an example. The 8-m-long pile was embedded in a 6-m-thick liquefiable sand layer (with a relative density D_r of 40%) overlying a 2-m-thick layer of slightly cemented sand (with a cohesion of 5.1 kPa, and an internal friction angle of 34.5°). The pile test measured ground movement (w_s), the pile-soil relative displacement ($l\omega_r$), and the maximum bending moments (M_m) (Abdoun et al. 2003). They are plotted in Fig. 14, which encompass a cyclic and a permanent component. The moment M_m was measured at a depth of 5.75 m in the liquefied layer. It increased to 113 kNm at a maximum pile-head deflection of 270 mm, and subsequently decreased (together with the deflection), despite the continual increase in the free-field (lateral spreading) movement. The ultimate measured profiles of the bending moments, and the soil movements are plotted in Figs. 15a and 15c, respectively.

(a) 2-layer Model Prediction

The current prediction for the Model 3 test, renamed as C1-M3 (see Table 3, ‘M3’ denotes ‘Model 3 test’) utilises $m = 1.9$, $k_s = 23$ kPa, $p_l = 30$ kN/m, $l = 8$ m, $H = 0$, and $k_B = k_\theta = 3.821$ MNm/radian ($\bar{k}_\theta = 0.317$). The $p_l = p_u$ at $l = 8.0$ m was estimated as $0.9\gamma_s K_p^2 dz$, in light of γ_s (effective) = 9.0 kN/m³, $\phi = 0^\circ$, and $d = 0.475$ m. The k_θ value is only two-third of 5.738 MNm/radian adopted previously (Dobry et al. 2003), owing to incorporating the impact of the soil modulus k_s (ignored previously). The value of modulus k_s was obtained from tests on model rigid piles in sliding sand, which is 15~60 kPa (Guo and Qin 2010). The calculation is done in three steps: First, specifying a sliding depth l_m ($= c = 0.1l < \text{final sliding depth}$), the normalised rotation $\bar{\omega}_r$ and displacement $\bar{w}_g$ were calculated using eqs. [2] and [3], respectively. Second, the maximum bending moment M_m (at a depth of 5.75 m), shear force T_m and on-pile force per unit length p were calculated using the expressions in Table 1 (see Table 4). Third, the first and second calculation steps are repeated for a series of new $c = l_m$ (say, $0.2l$, $0.4l$, $0.6l$, $0.8l$, and l), which enable the results shown in Table 4. The obtained w_s and M_m values for each l_m/l , for instance, are plotted together to formulate the $w_s \sim M_m$ (bold, solid) curve (see Fig. 14b). Likewise, the w_s and $l\omega_r$ values, and the $l\omega_r$ and M_m values for each l_m/l are plotted as bold, solid curves in Figs. 14c and 14d, respectively.

Importantly, it should be stressed that (i) The pile movement is the relative displacement between the pile-head and toe, to be consistent with the measured data; (ii) The effective soil movement w_s around the pile location is equal to $0.667w_g$ ($\alpha = 1.5$); and (iii) Increasing the sliding depth $l_m (= c)$ and the on-pile force per unit length p allow nonlinear response to be captured. The M_m and $l\omega_r$ predicted compare well, respectively, with the measured evolution of the M_m (see Fig. 14b), and the pile-head displacement (Fig. 14c) with the (ground-level free-field displacement) w_s . The bending moment M_m eventually drops to 27 kN-m (?). It would drop further without the stable layer ($k_B > 0$), as noted in other centrifuge tests (Motamed and Towhata 2010). The predicted $l\omega_r \sim M_m$ curve shows an increase and decrease cycle, which agrees with the measured relationship as well (Fig. 14d). In the same manner, the calculations were repeated by taking $m = 1$ and the predictions are plotted in Figs. 14b and 14c as well, which serve well as a lower bound for the bending moment, and the pile displacement, respectively.

With the same parameters of $H = 0$, $c = l_m = 6$ m, $l = 8$ m, $\lambda = 0.333$, $k_s = 23$ kPa, $p_l = 30$ kN/m, and $k_\theta = 3.821$ MNm/radian, the following were predicted using the expressions in Table 1: the profiles of bending moment $M(z)$, shear force $T(z)$, pile displacement $w(z)$, the net force per unit length $p_l(z)$ at ultimate state; and the p - $y(w)$ curves at depths of 2 m, 3 m, and 4 m, and 5.75 m. They are plotted in Figs. 15a through 15e, respectively. A good prediction of the $M(z)$ is noted against the measured data, so is the force per unit length $p_l(z)$ against similar centrifuge tests (González et al. 2009). The predicted average $p_l(z)$ over the 6-m liquefied layer is 7.23 kN/m (increasing linearly from 4.46 to 10 kN/m). The associated on-pile pressure is 9.47 ~20.4 kPa, which agrees well with the previous suggestions, so do the p - y curves. Finally, the impact of selected $\bar{k}_\theta (= 0.326)$ on the prediction can be ascertained from Fig. 10.

(b) Prediction for Case C2-M5a

In the same test series as the Model 3 test, Abdoun et al (2003) presented Model 5a test (i.e. C2-M5a in Table 3). The test was identical to the Model 3 (C1-M3) test, but for having a rectangular pile cap [$2 \times 2.5 \times 0.5$ m (in thickness)] rigidly connected to the top of the pile. The C2-M5a test thus has a 2.5×0.5 m side area exposed to the soil pressure pushing on the cap during lateral spreading. The experiment indicates a prototype $M_{\max}$ of 170 kNm at a pile-head deflection of 350 mm. The measured data allow the parameters k_s , m , k_θ and p_l for the pile to be deduced, which are provided in Table 3. This deduced p_l for the C2-M5a test (with a cap) was 33% higher than for the C1-M3 test (without a cap). The response is not detailed herein owing to limited space.

5.3 Piles (with known k_B) in Single Layer Subjected to Lateral Spreading

He et al (2009) investigated the response of single piles in Models 1, 2, 3 and 6 tests (or C3-M1 through to C6-M6 in Table 3, respectively) subjected to liquefaction-induced lateral soil flow (with ground sloping up to 6 degrees). The piles were ‘fixed’ to the base before construction of the soil stratum (which had a relative density of 40–50%, and saturated density of 19 kN/m³). Each pile was instrumented with strain gauges along the shaft, and with a displacement transducer at the pile head, to allow for estimating bending moments and deformation in the pile due to lateral soil flow. Each model was instrumented with accelerometers and pore pressure sensors in a sand stratum.

As with the above-calculation, the single, base rotationally restrained piles C3-M1 through to C6-M6 subjected to lateral spreading were studied. The measured maximum bending moment and ground-line pile-deflection at an ‘ultimate’ soil movement for each pile are tabulated in Table 3; and the response profiles are plotted in Fig. 16. The measured data allow the parameters k_s , m , k_θ and p_l (see Table 3) for each pile to be deduced using the current advanced 2-layer solutions.

In using the 2-layer model for the base-restrained piles in a full-length liquefied soil ($l_m = l$), the loading depth c is taken as $(0.75 \sim 0.9)l$, as a reduced bending moment at a distance of $(0.1 \sim 0.25)l$ about the base (e.g. in Fig. 16d) is observed, resembling that along retaining walls. The exact loading depth c was deduced by fitting current solutions to measured bending moment profile for the known base rotational stiffness k_B . This is briefly described next for each test.

Case C3-M1: The original Model 1 test (He, et al, 2009) on a flexible pile having a base stiffness k_B of 185.0 MNm/rad and on a rigid pile with $k_B = 8.5$ MNm/rad was tested in Kasumigaura saturated sand (5.0 m in thickness) using a large laminar soil container [$\sim 12 \times 3.5 \times 6$ m (high)]. The sand (Kagawa et al. 2004) has $D_{50} = 0.31$ mm, fines content $F_c = 3\%$, and uniformity coefficient $C_u = 3$. Displacement transducers were mounted on the laminar container exterior wall to measure free-field lateral displacement.

Cases C4-M2~C6-M6: The Model 2, 3, and 6 tests adopted silica sand (from a San Diego, CA quarry), which has the properties of (He et al. 2009) $D_{50} = 0.32$ mm, a fines content F_c below 2%, and a uniformity coefficient C_u of 1.5. The tests were conducted in the sand saturated in a medium laminar container [$4 \text{ m} \times 1.8 \text{ m} \times 2 \text{ m}$ (high)] (Jakrapiyanun 2002). The pile-base stiffness k_B was reported as 0.11 MNm/rad (C4-M2), and 0.2 MNm/rad (C5-M3), respectively. As with Model 1(C3-M1), a single, vertical pile in each test was installed in the container with a 2° inclined (to the horizontal) ground surface. Model 6 (C6-M6) was conducted using a levelled, rigid-wall container [$4 \text{ m} \times 1.8 \text{ m} \times 2 \text{ m}$ (high)], within which the soil surface was inclined at a slope of 6%. The Model C6-M6 has a $k_B = 0.3$ MNm/rad, for a single, concrete pile.

During the tests, the pile-head and soil displacements were found alike prior to the onset of liquefaction. Thereafter, the pile-displacement increases to its peak and decreases slightly, as the ground continues to displace laterally. The bending moment exhibits a similar increase-decrease pattern. Pertinent moment and displacement profiles are plotted in Fig. 16, and a maximum bending moment M_m generally attains the value of $k_\theta \omega_r$ around the pile-base. Typical maximum pile-head displacements and moments induced in model tests are provided in Table 3.

Using the 2-layer model and the parameters in Table 3, the predictions using Table 1 expressions were made concerning (a₁) the bending moment profile $M(z)$ and (a₂) the pile displacement profile $w(z)$ for test C3-M1; (b) the $M(z)$ for test C4-M2; (c) the $M(z)$ and $w(z)$ for test C5-M3, and (d) the $M(z)$ for test C6-M6. The predicted profiles of $M(z)$ agree with the measured data in Figs. 16a₁, 16b, 16c₁ and 16d, respectively, so do the predicted profiles of $w(z)$ against the available data in Figs. 16a₂ and 16c₂.

Overall given measured response, the modulus k_s may be adjusted to fit evolution of soil movement w_s ; the values of m and k_θ adjusted to match maximum bending moment, rotational angle and displacement of a pile (base stiffness of lower layer); and the p_l adjusted to fit on-pile pressure (thus distribution of bending moment with depth). The current model warrants force, moment equilibrium and displacement compatibility. The deduction is thus rigorous. Nevertheless, the deduced parameters for full-length lateral spreading may vary with soil movement profile, which is unknown without the M_m versus w_s curve etc. The parameters deduced are thus provided here for reference only.

The k_θ values deduced are consistent between C1-M3 and C2-M5a tests (Group 1). The normalised stiffness $\bar{k}_\theta$ deduced is close to the pile-base stiffness $\bar{k}_b$ for C4 and C5 piles (He et al. 2009); whereas the values of k_B for C4-C6 tests are also in good agreement with reported data. As for the C4 test, the stiffness k_B is lower than the reported of 18.5 MN-m/rad, indicating the impact of other rotational constraint along the pile. As $\bar{k}_\theta = 0.32 \sim 1.1$, the piles may exhibit the features of fixed-head piles ($\bar{k}_\theta = 10$). For instance, the ratio $\alpha w_g/w_s$ may increase linearly with the sliding depth (see Fig. 5c).

The calculation of p_l for the fixed-base piles in a single layer is rather new. The p_l would be estimated as 19.5 kN/m ($= \gamma_s' K_p^2 dz$) for C3 pile using $z = 4.8$ m, $\gamma_s' = 9.0$ kN/m³, $\phi = 5^\circ$, and $d = 0.318$ m, which is far below the deduced 50 kN/m. The p_l would be estimated as 29 kN/m ($= \gamma_s dz$, 50% the deduced p_l) using the overburden pressure (He et al. 2009). The on-pile force per unit length p_l on C4 and C5 piles was deduced 7.3, and 9.9 kN/m, which are close to 7.72 kN/m (C4), and 7.52 kN/m (C5) estimated using $p_l = \gamma_s dz$, respectively; whereas the deduced p_l of 8.8 kN/m for

C6 is about twice the estimated value of 4.33 kN/m ($= \gamma_s dz$) The estimated on-pile pressures ($\approx p/d$) was 9.5~30 kPa (C1-C2), and 2.7~4.7 kPa (C4-C6), which are in good accord with reported values (He et al. 2009). The C3 test induced a pressure about twice that on C2, which may be attributed to the large k_B value. The average on-pile pressure (over pile embedment) and the pile-base level pressure seem to increase with the base rotational stiffness k_B , as is seen in Fig. 17a for the investigated tests C1-C6. In contrast, the pile-head level pressure seems to increase with the pile diameter (see Fig. 17b).

Finally, the response of the model piles C7-C10 was predicted in the manner described previously (Guo 2014) using the parameters provided in Table 3. The predicted normalised rotational displacement is plotted in Fig. 2c against normalised displacement. The bending moment versus displacement relationship is plotted in Fig. 2d. The predictions are satisfactory against the measured data and the previous p_u -based solutions (Guo 2012), but for the shear force.

6 COMMENTS

The above predictions assume (1) a linear increase $p [= p_l l_m / (\alpha l)]$ with sliding depth to capture nonlinear response; (2) The p_l being the measured value of the net on-pile force per unit length (thus ignoring the impact of sliding resistance). The assumptions are examined for the in-situ test pile in sliding layer. The net on-pile pressure profiles were predicted for a sliding depth of 0.68, 1.36, ..., 6.8 m (increased by $l_m/10$ m to a final sliding depth l_m of 6.8 m), respectively, and are plotted in Fig. 18a as thin dash lines. The predicted pressure increases to a maximum at $0.5l_m (= 3.4$ m), and decreases subsequently with increase in the sliding depth. This seems to be supported by the increase in the measured values of the p to a maximum in years 92-95 (see Fig. 11c) and the decrease afterwards. The on-pile pressure should evolve along the '(red) bold, dash lines' (see Fig. 18a), and attain the '(blue) bold, solid' lines at the l_m . The pressure is overestimated against the measured data, in particular in stable layer.

Likewise, the pressure on the C1-M3 pile was predicted during lateral spreading, and is depicted in Fig. 18b. The pressure in sliding layer increases from 0 to AB (at $0.5l_m = 3.0$ m), and the profile follows AB, BC and CD curves. As the sliding depth increases from 3 to 6 m, the pressure decreases slightly to A'B' in sliding layer, whereas the resistance pressure (in stable layer) increases from CD to C'D'. This prediction may alter, as a general form of $p = p_l [l_m / (\alpha l)]^n$ and $n \neq 1$ may be seen as noted in the p_u profiles for active piles (Guo 2013). The exact value of the power n can be determined by comparing measured on-pile pressures with the current theoretical solution.

7 CONCLUSIONS

An advanced 2-layer model and closed-form solutions are developed to capture nonlinear response of rotationally restrained, rigid passive piles subjected to soil movement (sliding soil or lateral spreading). In particular, the pile-head displacement is generally measured as relative displacement w_r/l during lateral spreading, which is different from w_g for piles in sliding soil, but both cases have a soil moment w_g/α . The model has been successfully used to capture the response of all model piles and one in-situ test pile in sliding soil, two piles in 2-layer soil and four fixed-base single piles in single layer subjected to lateral spreading. The study reveals a dominant elastic pile-soil interaction around the piles, which causes nonlinear response through a progressive increase in sliding depth l_m and the on-pile force per unit length $p [= p_l l_m / (\alpha l)]$. The impact of profile and source of the movement w_s on passive piles is effectively incorporated using a modified sliding depth of l_m/α and movement w_s/α , respectively. Other conclusions are drawn as follows:

- The predicted pile response (for a uniform movement) can be converted into that under an inverse triangular soil movement by factoring the w_s and its depth l_m as w_s/α and l_m/α ($\alpha = 0.72$), respectively. The α values are deduced as 0.59 and 1.39~1.5, respectively, for an in-situ test pile (in sliding soil) and nine model piles (in sliding soil or subjected to lateral spreading).
- A triangular and a uniform p profile (Dobry et al. 2003; He et al. 2009) may be induced along a lightly head-restrained, floating-base pile and fixed-base piles, respectively. The pressure increases with the base rotational stiffness.

The good predictions can be achieved using four parameters k_s , m , k_θ and p_l (or p_u), and a series of stipulated sliding depths. In particular, they (e.g. k_s and p_u) may be determined using the low-cost model shear tests (rather than shaking tables). Nevertheless, more experiment are required to verify the impact of rotational stiffness k_θ on the normalised thrust $\bar{T}_m$, as the $\bar{T}_m$ at $k_\theta = 0$ is overestimated without considering the dragging impact for the model piles in sliding soil. The study on exact variation of p with l_m/l is also recommended.

REFERENCES

- Abdoun, T., Dobry, R., O'Rourke, T. D. and Goh, S. H. 2003. Pile response to lateral spreads: Centrifuge modeling. *Journal of Geotechnical and Geoenvironmental Engineering*, ASCE **129**(10): 869-878.
- Boulanger, R. W., Kutter, B. L., Brandenburg, S. J., Singh, P. and Chang, D. 2003. Pile foundations in liquefied and laterally spreading ground during earthquakes: Centrifuge experiments and analyses. Rep. No. UCD/CGM-03/01, Center for Geotechnical Modeling, Univ. of California, Davis. California, USA.

- Brandenberg, S. J., Boulanger, R. W. and Kutter, B. L. 2005. Discussion of "Single piles in lateral spreads: field bending moment evaluation" by R. Dobry, T. Abdoun, T. D. O'Rourke, and S. H. Goh. *Journal of Geotechnical and Geoenvironmental Engineering*, American Society of Civil Engineers **131**(4): 529-534.
- Brandenberg, S. J., Boulanger, R. W., Kutter, B. L. and Chang, D. 2005. Static pushover analyses of pile groups in liquefied and laterally spreading ground in centrifuge tests. *J. Geotech. and Geoenviron. Engrg. Div., ASCE* **133**(9): 1055-1066.
- Bransby, M. F. and Springman, S. M. 1997. Centrifuge modelling of pile groups adjacent to surcharge loads. *Soils and Foundations* **37**(2): 39-49.
- Cai, F. and Ugai, K. 2003. Response of flexible piles under laterally linear movement of the sliding layer in landslides. *Canadian Geotechnical Journal* **40**(1): 46-53.
- Chen, L. T. and Poulos, H. G. 1997. Piles subjected to lateral soil movements. *Journal of Geotechnical and Geoenvironmental Engineering*, ASCE **123**(9): 802-811.
- Chen, L. T., Poulos, H. G., Leung, C. F., Chow, Y. K. and Shen, R. F. 2002. Discussion of "Behavior of pile subject to excavation-induced soil movement.". *Journal of Geotechnical and Geoenvironmental Engineering*, American Society of Civil Engineers **128**(3): 279-281.
- Chmoulian, A. 2004. Briefing: Analysis of piled stabilization of landslides. *Proceedings of the Institution of Civil Engineers, Geotechnical Engineering* **157**(2): 55-56.
- Chow, Y. K. 1996. Analysis of piles used for slope stabilization. *International Journal for Numerical and Analytical Methods in Geomechanics* **20**(9): 635-646.
- Cubrinovskia, M., Kokushob, T. and Ishihara, K. 2006. Interpretation from large-scale shake table tests on piles undergoing lateral spreading in liquefied soils. *Soil Dynamics and Earthquake Engineering* **26**(275-286).
- De Beer, E. and Carpentier, R. 1977. Discussion on 'Methods to estimate lateral force acting on stabilising piles' By Ito, T., and Matsui, T. (1975). *Soil and Foundations* **17**(1): 68-82.
- Dobry, R., Abdoun, T., O'Rourke, T. D. and Goh, S. H. 2003. Single piles in lateral spreads: field bending moment evaluation. *Journal of Geotechnical and Geoenvironmental Engineering*, American Society of Civil Engineers **129**(10): 879-889.
- Frank, R. and Pouget, P. 2008. Experimental pile subjected to long duration thrusts owing to a moving slope. *Geotechnique* **58**(8): 645-658.
- Franke, K. W. and Rollins, K. M. 2013. Simplified hybrid p-y spring model for liquefied soils. *Journal of Geotechnical and Geoenvironmental Engineering*, ASCE **139**(4): 564-576.
- Fukuoka, M. 1977. The effect of horizontal loads on piles due to landslides. *In Proceedings 9th International Conference on Soil Mechanics and Foundation Engineering*, Speciality session 10, Tokyo: 27-42.
- González, L., Abdoun, T. and Dobry, R. 2009. Effect of soil permeability on centrifuge modeling of pile response to lateral spreading. *Journal of Geotechnical and Geoenvironmental Engineering*, ASCE **135**(1): 62-73.
- Guo, W. D. 2003. A simplified approach for piles due to soil movement. *In Proc. 12th Panamerican conference on soil mechanics and geotechnical engineering*, Cambridge, Massachusetts, USA, Verlag Gluckauf GMBH. Essen (Germany). 2: 2215-2220.
- Guo, W. D. 2006. On limiting force profile, slip depth and lateral pile response. *Computers and Geotechnics* **33**(1): 47-67.
- Guo, W. D. 2008. Laterally loaded rigid piles in cohesionless soil. *Canadian Geotechnical Journal* **45**(5): 676-697.
- Guo, W. D. 2012. *Theory and practice of pile foundations*. Boca Raton, London, New York, CRC press.
- Guo, W. D. 2013. Simple model for nonlinear response of fifty-two laterally loaded piles. *Journal of Geotechnical and Geoenvironmental Engineering*, ASCE **139**(2): 234-252.

- Guo, W. D. 2014. Elastic models for nonlinear response of rigid passive piles. *International Journal for Numerical and Analytical Methods in Geomechanics* (in press): DOI: 10.1002/nag.2292.
- Guo, W. D. and Qin, H. Y. 2006. Vertically loaded piles in sand subjected to triangular profiles of soil movements. *In Proceedings of 10th International Conference on Piling and Foundations*, Amsterdam, Netherland 1: paper no. 1371.
- Guo, W. D. and Qin, H. Y. 2010. Thrust and bending moment of rigid piles subjected to moving soil. *Can. Geotech. J.* **47**(2): 180-196.
- Guo, W. D., Qin, H. Y. and Ghee, E. H. 2006. Effect of soil movement profiles on vertically loaded single piles. *In Proc. International Conf. on Physical Modelling in Geotechnics Hong Kong, China*, Taylor & Francis Group plc, London, UK. 2: 841-846.
- He, L., Elgamal, A., Abdoun, T., Abe, A., Dobry, R., Hamada, M., Menses, J., Sato, M., Shantz, T. and Tokimatsu, K. 2009. Liquefaction-induced lateral load on pile in a medium Dr sand layer. *Journal of Earthquake Engineering* **13**(9): 916-938.
- Idriss, I. M. and Boulanger, R. W. 2007. SPT- and CPT-based relationships for the residual shear strength of liquefied soils. *In Proc., 4th Int. Conf. on Earthquake Geotechnical Engineering*, K. D. Pitilakis, ed., Dordrecht, Netherlands. 1: 1-22.
- Ito, T. and Matsui, T. 1975. Methods to estimate lateral force acting on stabilizing piles. *Soils and Foundations* **15**(4): 43-59.
- Jakrapiyanun, W. 2002. Physical modeling of dynamic soil-foundation-structure-interaction using a laminar container. *Civil Engineering*. San Diego, La Jolla, University of California. Ph.D.
- Juirmarongrit, T. and Ashford, S. A. 2006. Soil-pile response to blast-induced lateral spreading. II: Analysis and assessment of the p-y method. *Journal of Geotechnical and Geoenvironmental Engineering*, American Society of Civil Engineers **132**(2): 163-172.
- Kagawa, T., Sato, M., Minowa, C., Abe, A. and Tazoh, T. 2004. Centrifuge simulations of largescale shaking table tests: Case studies. *J. Geotech. and Geoenviron. Engrg Div., ASCE* **130**(7): 663-672.
- Motamed, R. and Towhata, I. 2010. Shaking table model tests on pile groups behind quay walls subjected to lateral spreading. *Journal of Geotechnical and Geoenvironmental Engineering*, ASCE **136**(3): 477-489.
- Olson, S. M. and Stark, T. D. 2002. Liquefied strength ratio from liquefaction flow failure case histories. *Canadian Geotechnical Journal* **39**(2): 629-647.
- Poulos, H. G. 1995. Design of reinforcing piles to increase slope stability. *Canadian Geotechnical Journal* **32**(5): 808-818.
- Qin, H. Y. 2010. Response of pile foundations due to lateral force and soil movements. *School of Engineering*. Gold Coast, Griffith University. Ph.D.
- Rollins, K. M., Gerber, T. M., Lane, J. D. and Ashford, S. 2005. Lateral resistance of a full-scale pile group in liquefied sand. *J. Geotech. and Geoenviron. Engrg Div., ASCE* **131**(1): 115-125.
- Seed, R. B. and Harder, L. F. 1990. SPT-based analysis of cyclic pore pressure generation and undrained residual strength. *In Proc., H. Bolton Seed Memorial Symp., BiTech., Richmond, BC, Canada*. 2: 351-376.
- Smethurst, J. A. and Powrie, W. 2007. Monitoring and analysis of the bending behaviour of discrete piles used to stabilise a railway embankment. *Geotechnique* **57**(8): 663-677.
- Stewart, D. P., Jewell, R. J. and Randolph, M. F. 1994. Design of piled bridge abutment on soft clay for loading from lateral soil movements. *Geotechnique* **44**(2): 277-296.
- Viggiani, C. 1981. Ultimate lateral load on piles used to stabilise landslide. *In Proc. 10th Int. Conf. Soil Mech. and Found. Engrg., Stockholm, Sweden*. 3: 555-560.
- Wang, S.-T. and Reese, L. C. 1998. Design of pile foundations in liquefied soils. *In Proc., Geotechnical Earthquake Engineering and Soil Dynamics III*, P. Dakoulas, M. Yegian, and R. Holtz, eds., Reston, VA. 2: 1331-1343.

NOTATION

The following symbols are used in the paper:

- c = loading depth
 d = diameter of an equivalent solid cylinder pile;
 E_s = Young's modulus of soil;
 G_s ($\tilde{G}_s$) = average soil shear modulus over the pile length, l ;
 H = shear force just about a liquefied layer, induced by an upper, non-liquefied layer; or shear force at pile-head level;
 k = modulus of subgrade reaction for lateral piles;
 k_A = rotational stiffness of a pile-cap, or an upper non-liquefied layer;
 k_B = rotational stiffness of a stable layer underlying a liquefied layer;
 k_i = coefficient for limiting resistance for sliding layer ($i = 1$) and stable layer ($i = 2$);
 k_s = modulus of subgrade reaction for piles in moving soil, $k_s < k$;
 k_θ = rotational stiffness of a pile-cap, or a non-liquefied layer on liquefied layer;
 l = embedded pile length;
 l_m = thickness of an upper moving soil layer;
 M_A, M_B = constraint moment at the top and bottom of a liquefied layer, respectively.
 $M_i(z)$ = bending moment at depth z ;
 M_{mi} = maximum bending moment within a pile for sliding layer ($i = 1$) and stable layer ($i = 2$);
 M_o = applied bending moment at ground level;
 m = ratio of the subgrade modulus of the stable layer over that of the upper sliding layer; $m = K_p/K_a$, ratio of coefficient of passive earth pressure (K_p) over that of active earth pressure (K_a) for progressively sliding soil;
 P_t = vertical load on passive piles during model tests;
 p = on-pile force per unit length, and $p = p_l l_m / l$;
 p_l = value of limiting force per unit length p_u at the depth of pile-tip level;
 p_u = limiting (maximums) force per unit length;
 $p(z)$ = resistance force per unit length at the depth z ;
 $p_i(z)$ = net force per unit length at the depth z ($i = 1, 2$ for upper and lower layer, respectively);
 s_u ($\tilde{s}_u$) = undrained shear strength of soil (average s_u over a maximum slip depth anticipated);
 T_m = maximum shear force induced in a passive pile;
 $T(z), T_i(z)$ = shear force at depth z ;
 w_g = pile-deflection at ground level;
 w_s = soil movement in model pile tests;
 $\bar{w}_g$ = $w_g k_s / p$, normalised pile-displacement at ground level;
 $w(z), w'(z)$ = deflection and rotation at depth z ;
 $z, \bar{z}$ = depth and the normalised depth z/l , respectively;
 z_m, z_{mi} = depth of maximum bending moment ($i = 1, 2$);
 z_{mt} = depth for maximum shear force T_m ;
 α = a parameter to cater for impact of soil movement profile and distance from piles;
 γ_s = a unit weight of the soil;
 λ = ratio of thickness of lower stable layer over sliding layer;
 ϕ = angle of internal friction;
 ω_r = rotation angle of pile at ground-level

765 $\bar{\omega}_r = w'(z)k_s l/p$, normalised rotation angle;

766 Bar ‘-’ for normalised parameters and variables. Depths c , z , l_m are all normalised by pile
 767 embedment length l .

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Appendix A Solutions for A Passive Pile in 2-Layer Soil

In this appendix, derivation of the elastic solutions for the pile in two-layered in the paper is elaborated. All of the symbols used are of identical meanings to those defined earlier.

The force per unit length p stipulated allows the horizontal force equilibrium of the rigid pile (see Fig. 3a) to be written as

$$[A1] \quad \int_0^{l_m} (\bar{\omega}_r s + \bar{w}_g) p ds + \int_{l_m}^l (\bar{\omega}_r s + \bar{w}_g) p m ds - \int_0^c p ds = H$$

The integration is made with respect to 's'. The moment equilibrium about the pile-base offers

$$[A2] \quad \int_0^{l_m} (\bar{\omega}_r s + \bar{w}_g) p (l-s) ds + \int_{l_m}^l (\bar{\omega}_r s + \bar{w}_g) p m (l-s) ds - \int_0^c p (l-s) ds = p l^2 \bar{k}_\theta \bar{\omega}_r + H l + M_0$$

Equations [A1] and [A2] allow the $\bar{\omega}_r$ and $\bar{w}_g$ to be determined as eqs. [2] and [3].

The expressions for the $T(z)$ and $M(z)$ are provided in Table 1. By $T_1'(z) = 0$, the depth of maximum shear force T_{mi} , z_{mti} is determined; whereas with $M_1'(z) = 0$, the depth of the maximum bending moment M_{mi} , z_{mi} is gained.

Table 1 2- layer theoretical model for response profiles

Depth	$z \leq l_m$	$z > l_m$
$w(z)$	$w(z) = (\bar{\omega}_r \bar{z} + \bar{w}_g) p / k_s$	
$p_i(z)$	$p_1(z) = (\bar{\omega}_r \bar{z} + \bar{w}_g - 1) p$	$p_2(z) = (\bar{\omega}_r \bar{z} + \bar{w}_g) m p$
Shear force $T(z)$	$\frac{T_1(z)}{pl} =$ $0.5 \bar{\omega}_r \bar{z}^2 + \bar{w}_g \bar{z} - \bar{z} - \bar{H}$	$\frac{T_2(z)}{pl} =$ $0.5[(1-m)\bar{l}_m^2 + m\bar{z}^2]\bar{\omega}_r + [(1-m)\bar{l}_m + m\bar{z}]\bar{w}_g - \bar{c} - \bar{H}$
Bending moment $M(z)$	$\frac{M_1(z)}{pl^2} = -\bar{M}_o + \frac{1}{6} \bar{\omega}_r \bar{z}^3$ $+ \frac{1}{2} (\bar{w}_g - 1) \bar{z}^2 - \bar{H} \bar{z} + \bar{k}_A \bar{\omega}_r$	$\frac{M_2(z)}{pl^2} = [(\frac{1}{2} \bar{l}_m^2 \bar{z} - \frac{1}{3} \bar{l}_m^3)(1-m) + \frac{m}{6} \bar{z}^3 + \bar{k}_A] \bar{\omega}_r - \bar{H} \bar{z}$ $+ [(\bar{l}_m \bar{z} - 0.5 \bar{l}_m^2)(1-m) + 0.5 m \bar{z}^2] \bar{w}_g - 0.5 \bar{c} (2 \bar{z} - \bar{c}) - \bar{M}_o$
z_m	$\bar{z}_m = \frac{-1}{\bar{\omega}_r} \left\{ (\bar{w}_g - 1) + [(\bar{w}_g - 1)^2 + 2 \bar{\omega}_r \bar{H}]^{0.5} \right\} \quad (0 \leq z \leq l_m)$ $\bar{z}_m = \bar{z}_{m2} = \frac{1}{\bar{\omega}_r} \left\{ -\bar{w}_g + \left\{ \frac{m-1}{m} \bar{\omega}_r^2 \bar{l}_m^2 + 2 \left[\frac{(m-1)}{m} \bar{l}_m \bar{w}_g + \frac{\bar{H} + \bar{l}_m}{m} \right] \bar{\omega}_r + \bar{w}_g^2 \right\}^{0.5} \right\} \quad (l_m < z \leq l)$	
T_{m1} T_{m2}	$\bar{z}_{m1} = (1 - \bar{w}_g) / \bar{\omega}_r, \quad T_{m1} = T_1(z_{m1}) \quad (0 \leq z \leq l_m)$ $\bar{z}_{m2} = -\bar{w}_g / \bar{\omega}_r, \quad T_{m2} = T_2(z_{m2}) \quad (l_m < z \leq l)$	

786 Table 2 Calculated versus measured (Frank and Pouget 2008) response of an in-situ tested pile

Year	Applied		Measured		Input		Calculated		
	$-M_o$ (kNm)	H (kN)	w_g (mm)	$-M_{m1}$ (kNm)	m	p_l/p_u	$-T_{m1}/T_{m2}$ (kN)	$-T_m@l_m$ (kN)	$-M_{m1}$ (kNm)
1984.01.01	0	0	0	0				0	0
1986.11.4	94	0	17.8		4.5	0.9	99.4/302.8	565.3	345.2
1986.11.5	209	-310	-2.4	901.9	5.5		310.0/230.0	705.8	973.2
1988.11.10	154	0	16.5	536.9	3.9	1.1	133.0/302.	655.5	505.3
1988.11.11	262	-273	-0.9	1102.3	4.6		285.4/250.8	787.3	1050.0
1992.9.30	138	0	35.2	544.3	4.3	1.73	200.8/552.0	1063	656.8
1992.10.01	313	-321	-2.0	1473.5	4.3		375/405.0	1171	1403
1995.07.05	182	0	29.8	756.1	4.0	1.48	177.0/422.3	888.4	646.5
1995.07.06	295	-347	-1.8	1434.2	4.3		372.0/314.6	1023	1358
1999.07.20	184	--	21.7	932.7					

787 Note: $l = 11.0$ m, $l_m = 6.8$ m, $d = 0.915$ m, $k_s = 2.86$ MPa (1986, 1988 after pulling backs), and 2.5
788 MPa (1992, 1995, after pulling backs), $p = p_l l_m / l$, and $m = K_p / K_a$, ratio of coefficient of passive
789 earth pressure (K_p) over that of active earth pressure (K_a).
790

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Table 3 Model predictions for 10 piles

Cases	l or c^a/l_m (m)	$m/p_1/p$ (kN/m)	k_s (kPa) $/d$ (cm)	k_θ (MN $m/rad)$ / $\bar{k}_\theta$	Mea/Pre w_s (cm)	Mea/Pre M_m (kNm)	Mea/Pre. w_g (cm)	Reference
2-layer model for piles subjected to lateral spreading								
C1-M3 ^c	8/6	1.9/30 $/4.5-9.7$	23.5 $/47.5$	3.82/ 0.317	78 $/76.8$	113 $/114$	27 $/23.9$	(Dobry et al. 2003)
C2-M5a ^c	8/6	5.3/40 $/6.7-14.4$	23.5 $/47.5$	3.93/ 0.327	77 $/67.2$	170 $/170.3$	35 $/35.2$	
C3-M1 ^c	3.6 ^a $/4.8$	7/50. $/10.5-14.4$	40 $/31.8$	<u>4.90</u> / 1.108	105 $/98.7$	132 $/131$	9.8 ^b $/9.8$	(He et al. 2009)
C4-M2 ^c	1.28 ^a $/1.6$	8/7.3 $/1.3-2.8$	50 $/25.4$	0.10/ .407	12.5 $/12.0$	2.65 $/2.59$	3.4 ^b $/3.0$	
C5-M3 ^c	1.2 ^a $/1.56$	8/9.9 $/1.5-3.0$	120 $/25.4$	0.200/ 0.439	7.8 $/7.1$	3.0 $/2.2$	1.2 ^b $/1.4$	
C6-M6 ^c	1.2 ^a $/1.54$	8/8.8 $/1.7-2.3$	50 $/15.2$	0.200/ 0.913	-/14.1	1.86 $/2.26$	1.4 $/1.3$	
2-layer and 3-layer models for model piles in sliding soil								
C7- TS32-294 ^d	0.7 $/\sim 0.7$	11/12.5 $/2\sim 7.0$	60 $/3.2$	0/0	0~14 $/-$	0~0.175 $/0\sim 0.175$	0~6. $/0\sim 6.$	(Guo and Qin 2010)
C8-TS32-0 ^d	0.7 $/\sim 0.7$	7/12.5 $/2\sim 6.0$	60 $/3.2$	0/0	0~14 $/-$	0~0.14 $/0\sim 0.14$	0~8. $/0\sim 8.$	
C9- TS50-0 ^e	0.7 $/\sim 0.7$	5.5/12.5 $/1.5\sim 3.$	280 $/5.0$	0/0	0~14 $/-$	0~0.08 $/0\sim 0.08$	0~0.8 $/0\sim 0.8$	(Guo et al. 2006)
C10- TS50-294 ^e	0.7 $/\sim 0.7$	7.5/12.5 $/2.0\sim 3.5$	280 $/5.0$	0/0	0~14 $/-$	0~0.11 $/0\sim 0.11$	0~0.8 $/0\sim 0.8$	

792 Note in the second through to fifth columns, the values of m , p_1 , and k_s , along with the given values
793 of l , l_m , d , and k_θ are all input values. Values of p are all calculated from p_1 , and p_2 (Table 1). ^a The
794 loading depth $c = (0.70 \sim 0.9)l_m$ for the single-layer, **restrained-base** piles. ^b At soil surface level. ^c
795 Letters 'Mi' and 'M5a' denotes original test name of 'Model i' and 'Model 5a'. ^d **Model tests on**
796 **d_{32} pile to a sliding depth $l_m = 200$ mm with $P = 0$ or $P = 294$ N, respectively;** ^e **Model tests on d_{50}**
797 **pile to a sliding depth $l_m = 200$ mm, with $P = 0$ or $P = 294$ N, respectively.**
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800

801
Table 4 Response of Model 3 (C1-M3) pile subjected to lateral spreading ^a

l_m/l	$\bar{w}_g$	$\bar{\omega}_r$	$\omega_r l$ (cm) ^b	w_g/w_s (cm) ^c	M_m (kNm) ^d	T_m (kN) ^d	p (kN/m) ^d
0.2	0.222	-0.024	5.1	5.8/3.9	23.18	-1.39	0.84
0.4	0.452	-0.042	17.7	23.7/15.8	75.88	-8.33	2.90
0.5	0.561	-0.047	24.5	36.8/24.5	101.0	-15.36	4.03
0.6	0.663	-0.048	30.1	52.2/34.8	116.4	-25.71	4.95
0.8	0.844	-0.037	31.1	88.6/59.1	90.0	-36.19	5.11
1.0	1.0	0	0	131.1/87.5	0	0	0

802 ^a $m = 1.9$, $k_A = 0$, $k_B = k_\theta = 3.821$ MNm/radian, and $H = 0$;
803 ^b relative pile displacement between head and toe;
804 ^c w_g estimated from eq. [3], $w_s = w_g/\alpha$ and $\alpha = 1.5$;
805 ^d Bending moment M_m , shear force T_m and the on-pile force per unit length p ($= \omega_r z_c p_l l_m/l$) at a
806 depth of $z_c = 5.75$ m.

Figure Captions

- Fig. 1 Schematic diagram of shear box (a) elevation view; (b) plan view (A-A)
- Fig. 2 (a) Measured p_u profiles (Guo 2012); (b) pile deflection $w_g \sim$ soil movement w_f (Qin 2010); predictions versus measured data (Guo and Qin 2010): (c) $w_g \sim \omega_r$. (d) $w_g \sim M_m$
- Fig. 3 2-layer models for rigid, passive pile: (a) pile - soil system; (b) p applied & $p(z)$ induced; (c) model with k_A & k_B
- Fig. 4 (a) equivalent of inverse triangular w_s using uniform soil movement (constant k_s); (b₁) a capped pile in 3-layer soil, (b₂) schematic model for b₁ pile subjected to lateral spreading
- Fig. 5 Normalized uniform soil movement w_g (constant k_s) with loading depth owing to (a) normalized cap stiffness ($m = 1$); (b) modulus ratio m and $\bar{k}_\theta = 0.05$; (c) modulus ratio m and $\bar{k}_\theta = 10$
- Fig. 6 Normalized uniform soil movement w_s (constant k_s) with loading depth owing to (a) normalized cap stiffness ($m = 1$); (b) modulus ratio m and $\bar{k}_\theta = 0.05$; (c) modulus ratio m and $\bar{k}_\theta = 10$
- Fig. 7 Normalized maximum bending moment at sliding depth with uniform soil movement w_s (constant k_s) owing to (a) normalized cap stiffness ($m = 1$); (b) modulus ratio m and $\bar{k}_\theta = 0.05$; (c) modulus ratio m and $\bar{k}_\theta = 10$
- Fig. 8 Normalized thrust at sliding depth owing to (a) normalized cap stiffness ($m = 1$); (b) modulus ratio m and $\bar{k}_\theta = 0.05$; (c) modulus ratio m and $\bar{k}_\theta = 10$
- Fig. 9 Normalized thrust T_m (not @ l_m) owing to (a) normalized cap stiffness ($m = 1$); (b) the modulus ratio m ($\bar{k}_\theta = 0.05$)
- Fig. 10 Normalized (a) $M(z)$, (b) $T(z)$, (c) $w(z)$ and (d) $p_i(z)$ for a normalized cap stiffness of 0~0.2 ($m = 1.9$, $\bar{l}_m = 0.75$).
- Fig. 11 An in-situ pile (Frank and Pouget 2008): (a) 2- model for the in-situ pile; (b)-(d) evolution of the response of: (b) maximum bending moment at a depth of 3.75 m and pile-head shear load; (c) force per unit length p ; (d) profiles of pile deflections.
- Fig. 12 Predicted versus measured (Frank and Pouget 2008) bending moment profiles in (a) 1986; (b) 1988; (c) 1992; (d) 1995 (with $p = p_l l_m / l$, $l_m = 6.8$ m and $l = 11$ m)
- Fig. 13 Measured (Frank and Pouget 2008) maximum bending moment (a) - M_{m1} at depth 3.75m, and (b) M_{m2} at 8~9 m, respectively; predicted ($m = 4.5$, $k_s = 2.4$ MPa, $p_l = 900$ kN/m, $H = M_o = 0$) versus measured (Frank and Pouget 2008) (c) $w_s \sim M_{mi}$, and (d) $w_g \sim M_{mi}$ respectively
- Fig. 14 Predicted versus measured (Abdoun et al. 2003) response of Model 3 (C1-M3) subjected to lateral spreading: (a) pile-soil interaction model; (b) $w_s \sim M_m$; (c) $w_s \sim$ pile-soil relative movement; (d) pile-soil relative movement versus M_m
- Fig. 15 Predicted profiles of Model 3 (C1-M3) pile ($p = 30 l_m / l$ kN/m, $l_m = 6$ m, $l = 8$ m, $k_s = 22.9$ kPa, $k_\theta = 3821$ kNm/radian): (a) $M(z)$; (b) $T(z)$; (c) $w(z)$; (d) $p_i(z)$; (e) p - y curves
- Fig. 16 Prediction versus measured (He et al. 2009) response for piles subjected to lateral spreading
- Fig. 17 On-pile pressure versus (a) pile-base rotational stiffness k_B ; (b) pile diameters
- Fig. 18 On-pile pressure for (a) the pile in sliding soil by Frank and Pouget (2008); (b) C1-M3 pile subjected to lateral spreading by Abdoun et al (2003)

Fig. 1 Schematic diagram of shear box (a) elevation view; (b) plan view (A-A)

Fig. 2 (a) Measured p_u profiles (Guo 2012); (b) pile deflection $w_g \sim$ soil movement w_f (Qin 2010); predictions versus measured data (Guo and Qin 2010); (c) $w_g \sim \omega_r$. (d) $w_g \sim M_m$

Fig. 3 2-layer models for rigid, passive pile: (a) pile - soil system; (b) p applied & $p(z)$ induced; (c) model with k_A & k_B

Fig. 4 (a) equivalent of inverse triangular w_s using uniform soil movement (constant k_s); (b₁) a capped pile in 3-layer soil, (b₂) schematic model for b₁ pile subjected to lateral spreading

Fig. 5 Normalized uniform soil movement w_g (constant k_s) with loading depth owing to (a) normalized cap stiffness ($m = 1$); (b) modulus ratio m and $\bar{k}_\theta = 0.05$; (c) modulus ratio m and $\bar{k}_\theta = 10$

(b) Normalised soil displacement, $\alpha w_g/w_s$ (c) Normalised soil displacement, $\alpha w_g/w_s$

Fig. 6 Normalized uniform soil movement w_s (constant k_s) with loading depth owing to (a) normalized cap stiffness ($m = 1$); (b) modulus ratio m and $\bar{k}_\theta = 0.05$; (c) modulus ratio m and $\bar{k}_\theta = 10$

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