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© 2020 Elsevier Inc. The consensus tracking of singular multi-agent systems (MASs) with Lipschitz-type nonlinearities and exogenous disturbances is researched in this paper. Governed by a Markov chain, the network interaction randomly switches in a directed graph set, where the directed spanning tree is not contained in each graph while exists in the union rooting at the leader node. By utilizing a collection of in-neighbors' information that involves communication delay, the intention is to design a protocol such that the resultant consensus error system is stochastic admissible with an H∞ disturbance attenuation level. Based on algebraic graph theory, stochastic admissibility analysis and linear matrix inequality (LMI) technique, tracking consistency is first regulated in the concerned MAS by considering the case of completely known transition probabilities. Then, thanks to a group of free-connection weighting matrices, the obtained result is extended to the case that transition probabilities are partially known. Finally, the theoretical analysis is confirmed by some numerical examples.

Keywords

topology, h∞, tracking, delayed, protocol, design, nonlinear, singular, multi-agent, systems, under, markovian, switching

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H_∞ delayed tracking protocol design of nonlinear singular multi-agent systems under Markovian switching topology

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Abstract

The consensus tracking of singular multi-agent systems (MASs) with Lipschitz-type nonlinearities and exogenous disturbances is researched in this paper. Governed by a Markov chain, the network interaction randomly switches in a directed graph set, where the directed spanning tree is not contained in each graph while exists in the union rooting at the leader node. By utilizing a collection of in-neighbors' information that involves communication delay, the intention is to design a protocol such that the resultant consensus error system is stochastic admissible with an H_∞ disturbance attenuation level. Based on algebraic graph theory, stochastic admissibility analysis and linear matrix inequality (LMI) technique, tracking consistency is first regulated in the concerned MAS by considering the case of completely known transition probabilities. Then, thanks to a group of free-connection weighting matrices, the obtained result is extended to the case that transition probabilities are partially known. Finally, the theoretical analysis is confirmed by some numerical examples.

Keywords: Communication delay, Consensus tracking, Lipschitz nonlinearity, Markovian switching topology, Singular agent network.

1. Introduction

With the rapid development of broadband wireless access network technology, a group of identical agents can be collected to cooperatively fulfill certain tasks (e.g., collaborative resource management, public safety guarding and intelligent traffic scheduling) which are not capable of being finished by a single agent [4, 26, 27]. This promotes the scientific research on multi-agent systems (MASs). And a great number of results have been developed to study their collective behaviors, including synchronization [23], rendezvous [5], formation [9] and so on.

Since the consensus was first systematically investigated in [11], it has been well-recognized as an important and fundamental issue in studying other dynamic behaviors of MASs. According to whether there is a leader in the agent network, two categories on achieving consensus are roughly classified. One is leaderless consensus [14], the other is leader-following consensus which is also known as consensus tracking [43]. As is pointed out in [21], compared to leaderless consensus, the leader-following one plays a more effective role in reducing energy consumption and increasing practical applicability of MASs. For some real-world MASs, in addition, each agent is preferably modeled as a high-order system rather than the lower-order one (i.e., first-order and second-order integrators) [34]. With consideration given to existences of packet losses, channel fadings and data congestions, the assumption on fixed network topology may not be practically satisfied [27]. In this regard, it is more suitable to consider that the network topology is time-varying and switches in a finite set [31]. Up to date, on the topic of consensus tracking design for high-order MASs with

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switching topologies, much attention has been paid. To mention a few, by collecting the information from in-neighboring agents under switching interaction topologies, a tracking protocol has been designed in [20] to regulate consistency in a group of high-order dynamic agents. In [2], with the aid of an event-triggered-based protocol, the consensus tracking has been achieved for high-order MASs when taking into account eliminations of continuous interagent interactions and time-varying communication topologies. In [33], to tackle the leader-following consensus issue for high-order agent network, an estimator-based event-triggered control strategy has been proposed by considering cases of fixed and switching topologies.

It should be noted that, in above-reviewed literature, each agent is modeled as a general linear high-order system and interagent interactions do not encounter any time-delays. Owing to the uniformly bounded Jacobions, a tremendous number of engineering systems (e.g., industrial manipulators) inherently contain Lipschitz nonlinear dynamics [40]. It may be difficult to regulate consistency of nonlinear MASs since some behaviors, such as chaos, limited cycles and almost-periodic oscillations, are hard to be analyzed and controlled [24]. Caused by the limited data transmission capability of physical interaction channels in MASs, communication delay is unavoidably encountered in the information exchange among in-neighboring agents [30, 38]. It is widely-known that, however, time-delay is often regarded as a provenance of deteriorating the system performance or even destabilizing the system [42]. Under these scenarios, it makes practical sense in the consensus design of high-order MASs with Lipschitz nonlinear dynamics and communication delay. Few results have been reported to tackle this issue. More specifically, for a class of Lipschitz nonlinear MASs with switching topologies, a leader-following consensus controller has been designed in [41] by taking into account the actuator saturation and communication delay. In [15], by employing a delayed data sampling scheme under switching topology, the non-fragile consensus tracking has been concerned for a collection of Lipschitz nonlinear agents. In [16], when regulating consistency in a group of identical agents subject to Lipschitz nonlinearities, the delayed in-neighbors' information has been collected to characterize the desired resilient controller.

Compared to the normal MASs in above-reviewed literature, the singular ones are capable of providing more flexibility and generality in describing a variety of practical systems, such as car-like robots, electrical circuits and high-speed vehicles [8, 13]. Recently, many researchers have devoted to the consensus design of singular MASs. And some interesting results have been reported. In [35], under switching topology, the leaderless consensus has been achieved in linear singular MASs by considering the control energy consumption. In [28], conditions on output regulation of linear singular MASs with jointly connected switching topologies have been derived. In [12], with consideration given to the tracking performance regulation, a protocol has been designed for Lipschitz nonlinear singular MASs subject to time-varying interaction topologies. It is noteworthy that, in almost all aforementioned literature, the network topology evolutes in a deterministic switching manner. In practice, owing to the existences of sudden link failures, abruptly environment changes and unknown communication obstacles, the interagent interaction may randomly switch in a finite set, which can be governed by a Markov chain [3, 6, 17, 18]. Moreover, the dynamic agent is inevitably affected by exogenous disturbances whose stochastic characteristics are grasped few or even completely unknown. As is well-known, the H_∞ control scheme plays an effective role in obtaining disturbance attenuation [10, 39]. It is, accordingly, of significant importance to design the H_∞ consensus protocol for noise-disturbed singular MASs under Markovian switching topology. However, little attention has been paid to this issue, which essentially motivates this paper.

By collecting the delayed in-neighbors' information, this paper aims to design an H_∞ tracking protocol for singular MASs with Lipschitz-type nonlinearities and exogenous disturbances. Governed by a Markov chain, the network interaction randomly switches among a set of directed graphs whose union contains a directed spanning tree rooting at the leader node. On the basis of algebraic graph theory, stochastic admissibility analysis and free-connection weighting matrix technique, some sufficient conditions are derived to characterize the desired protocol by considering cases of completely known and partially known transition probabilities. Then, the following points are organized to highlight key novelties of this paper. First, from system model's point of view, each dynamic agent concerned in this paper is modeled by a singular system subject to Lipschitz-type nonlinearities and exogenous disturbances, which is more representative and complex than the normal system in [15, 16, 41]. Second, compared to deterministic switching topologies without any communication delay in [12, 28, 35], this paper gives a more general and practical interaction framework, that is Markovian switching topologies with time-varying communication delay. Third, to relax restrictions that each topology is connected in [35] or contains a directed spanning tree in [29], this paper merely requires that the union of all possible interaction graph contains a directed spanning tree rooting at the leader node. Fourth, in addition to considering the Markov chain with completely known transition probabilities in [3], the case of partially known transition probabilities is also concerned in this paper.

Notations: The space $\mathbb{R}^{n\times m}$ ($\mathbb{R}^n$) is constituted by all $n\times m$-dimensional real matrices (n-dimensional vectors). For an n-dimensional square summable function, it lies in the set $\mathfrak{L}_2([0,+\infty),\mathbb{R}^n)$. Appropriately dimensional identity and zero matrix are respectively represented by I and 0. diag$\{\cdot\}$ denotes a diagonal matrix. $\|\cdot\|$ is the Euclidean norm of a vector. The Kronecker product operator is $\otimes$. In a symmetric matrix, $*$ represents the symmetric term. For matrix $\mathcal{Q}$, its transpose is $\mathcal{Q}^{\mathrm{T}}$. If real matrix $\mathcal{Q}$ is symmetric and positive (negative) definite, furthermore, one denotes that $\mathcal{Q}>0$ ($\mathcal{Q}<0$). Define $\mathrm{col}(\mathcal{Q}_1,\mathcal{Q}_2)=[\mathcal{Q}_1^{\mathrm{T}}\ \mathcal{Q}_2^{\mathrm{T}}]^{\mathrm{T}}$, where $\mathcal{Q}_1$ and $\mathcal{Q}_2$ are matrices (vectors) with compatible dimensions. For an event, let $\mathbb{P}\{\cdot\}$ and $\mathbb{E}\{\cdot\}$ respectively denote its probability and mathematical expectation.

2. Problem formulation and preliminaries

This section begins with reviewing some concepts on algebraic graph theory. Then, the concerned consensus tracking problem is mathematically formulated.

2.1. Graph theory

The concerned singular MAS consists of $N+1$ dynamic agents. Designate agent 0 as the leader and agent i ($i=1,2,\cdots,N$) as the ith follower. By virtue of algebraic graph theory, an agent is represented by a node which belongs to the node set $\mathcal{V}=\{v_0,v_1,\cdots,v_N\}$. If a node pair (v_ι,v_j) ($\iota,j=0,1,\cdots,N$) lies in the edge set $\mathcal{D}(\subseteq\mathcal{V}\times\mathcal{V})$, node v_j is accessible to node v_ι while not vice versa. For the scalar $a_{\iota j}$ in weighted adjacency matrix $\mathcal{A}=[a_{\iota j}]\in\mathbb{R}^{(N+1)\times(N+1)}$, one defines $a_{\iota j}>0$ ($\iota\neq j,(v_j,v_\iota)\in\mathcal{D}$) and $a_{\iota j}=0$ otherwise. Hence, we describe the MAS as directed graph $\mathcal{G}(\mathcal{V},\mathcal{D},\mathcal{A})$. With distinct nodes v_{ι_k} ($k=1,2,\cdots,\lambda$), there exists a directed path from node v_ι to node v_j if a sequence of node pairs $(v_\iota,v_{\iota_1}),(v_{\iota_1},v_{\iota_2}),\cdots,(v_{\iota_\lambda},v_j)\in\mathcal{D}$. A directed spanning tree in directed graph $\mathcal{G}$ indicates a directed path from a node (i.e., the root) to any other nodes. By defining $\mathcal{H}=\mathrm{diag}\{h_\iota\}$ with $h_\iota=\sum_{\iota=0}^{N}a_{\iota j}$, the Laplacian matrix $\mathcal{L}=[l_{\iota j}]\in\mathbb{R}^{(N+1)\times(N+1)}$ can be calculated as $\mathcal{L}=\mathcal{H}-\mathcal{A}$.

Remark 1. *As for the consensus tracking issue, the leader's motion is not affected but needs to be tracked by each follower [29]. Accordingly, one can get that* $l_{0j}=0$ ($j=0,1,\cdots,N$). *For scalars* l_{i0} ($i=1,2,\cdots,N$), *one defines* $l_{i0}=-a_{i0}$ *if follower* i *can access to the leader and* $l_{i0}=0$ *otherwise. Let* $L_\ell=\mathrm{diag}\{a_{10},a_{20},\cdots,a_{N0}\}\in\mathbb{R}^{N\times N}$ *and* $L_g=-\mathrm{col}(a_{10},a_{20},\cdots,a_{N0})\in\mathbb{R}^N$. *Corresponding to the interaction topology among all followers, the Laplacian matrix is* L_f. *Hence, by denoting* $L=L_\ell+L_f$, *the Laplacian matrix* $\mathcal{L}$ *takes the following partitioned form:*

$$\mathcal{L}=\begin{bmatrix}0 & 0\\ L_g & L\end{bmatrix}. \tag{1}$$

Remark 2. *In [33], the interaction topology is considered to be fixed or switches in a deterministic manner. This may run counter to a more general and practical case, i.e., Markovian switching topologies since the realistic interagent communication may be subject to random failures and establishments. In this regard, the topology switching scheme is reasonably governed by a Markov chain* $\{\nu_t,t\geq\}$, *whose state space is* $\mathcal{M}=\{1,2,\cdots,M\}$ *and transition rate matrix is* $\mathcal{T}=[\pi_{rs}]\in\mathbb{R}^{M\times M}$ *with*

$$\mathbb{P}\{\nu_{t+\Delta t}=s|\nu_t=r\}=\begin{cases}\pi_{rs}\Delta t+\mathrm{o}(\Delta t), & s\neq r,\\ 1+\pi_{rr}\Delta t+\mathrm{o}(\Delta t), & s=r,\end{cases} \tag{2}$$

where $\Delta t>0$ *and* $\mathrm{o}(\Delta t)\to 0$ ($\Delta t\to 0$). *For* $\forall r,s\in\mathcal{M}$, *one defines* $\pi_{rs}\geq 0$ ($r\neq s$) *and* $\pi_{rr}=-\sum_{s=1,s\neq r}^{M}\pi_{rs}$ ($r=s$).

Denote a directed graph set $\mathcal{S} = \{\mathcal{G}_1, \mathcal{G}_2, \cdots, \mathcal{G}_M\}$. For $\nu_t = r \in \mathcal{M}$, the interagent communication of the concerned singular MAS is described by $\mathcal{G}_r \in \mathcal{S}$. By denoting $L_{\ell,r} = \text{diag}\{a_{10,r}, a_{20,r}, \cdots, a_{N0,r}\} \in \mathbb{R}^{N\times N}$ and $L_{g,r} = -\text{col}(a_{10,r}, a_{20,r}, \cdots, a_{N0,r}) \in \mathbb{R}^N$, it thus follows from Remark 1 that

$$\mathcal{L}_r = \begin{bmatrix} 0 & 0 \\ L_{g,r} & L_r \end{bmatrix}. \tag{3}$$

where $L_r = L_{\ell,r} + L_{f,r}$.

Then, the following assumption is given to switching graphs $\mathcal{G}_r$ $(r \in \mathcal{M})$.

Assumption 1. *The union graph $\mathcal{G}^U = \bigcup_{r=1}^{M} \mathcal{G}_r$ contains a directed spanning tree rooting at the leader node v_0.*

Remark 3. *In [28, 29], each possible graph is assumed to be connected or contain a directed spanning tree. Indeed, this may not be practically satisfied due primarily to the limited sensing capability caused by, for instance, sensor failures, communication obstacles and unknown disturbances. To get a relaxation, it is suitable to consider the directed spanning tree is not contained in each graph while exists in the union rooting at the leader node. It has been pointed out in [3] that, within the communication framework given in Assumption 1, all followers can access to the leader as time evolves.*

2.2. Problem description

This paper considers a singular MAS of the following form:

$$\begin{cases} E\dot{x}_i(t) = Ax_i(t) + f(x_i(t)) + Bu_i(t) + Dw_i(t), i = 1, 2\cdots, N, & (4)\\ E\dot{x}_0(t) = Ax_0(t) + f(x_0(t)), & (5)\end{cases}$$

where $x_0(t)$ and $x_i(t) \in \mathbb{R}^n$ are leader and follower i's state. $u_i(t) \in \mathbb{R}^p$ is the control input of follower i. $w_i(t) \in \mathfrak{L}_2([0, +\infty), \mathbb{R}^q)$ is the noise disturbance. A, B, D and E with $\text{rank}E = \sigma < n$ are appropriately dimensional matrices. The nonlinear function $f : \mathbb{R}^n \times [0, +\infty) \to \mathbb{R}^n$ is Lipschitz continuous, which further implies the following inequality holds for a scalar $\varrho > 0$:

$$\|f(\varsigma_1) - f(\varsigma_2)\| \leq \varrho\|\varsigma_1 - \varsigma_2\|, \forall\varsigma_1, \varsigma_2 \in \mathbb{R}^n. \tag{6}$$

Assumption 2. *The matrix triplet (E, A, B) is stabilizable.*

Remark 4. *Compared to normal system in [16, 41] and linear singular system in [28, 35] without exogenous disturbances, this paper considers a more representative and complex agent model, that is noise-disturbed singular system with Lipschitz nonlinear dynamics. This is primarily caused by reasons that (i) some real-world systems (e.g., transistor circuits) may preserve special modes (i.e., static mode and impulsive mode), which merely arise in a singular control system; (ii) the Lipschitz-type nonlinearities are inherently existent in a variety of engineering systems (e.g., industrial manipulators), whose undesirable effects on performance analysis and control synthesis may be difficult to eliminate; (iii) each practical agent works in realistic environment which is full of various noise signals.*

By collecting the delayed in-neighbors' information, the following protocol is constructed for MAS (4) and (5):

$$u_i(t) = K_r \textstyle\sum_{j=1}^{N} a_{ij,r}(x_i(t - \tau(t)) - x_j(t - \tau(t))) + K_r a_{i0,r}(x_i(t - \tau(t)) - x_0(t - \tau(t))), \tag{7}$$

where $K_r \in \mathbb{R}^{p\times n}$ is the desired protocol parameter. Given scalars $d_a > 0$ $(a = 1, 2, 3)$, $\tau(t)$ is the time-varying communication delay satisfying

$$0 \leq d_1 \leq \tau(t) \leq d_2, \dot{\tau}(t) \leq d_3, \tag{8}$$

where $d_3 \in [0, 1)$.

Substituting (7) into (4), the closed-loop system of follower i takes the following form:

$$\begin{aligned} E\dot{x}_i(t) &= Ax_i(t) + f(x_i(t)) + BK_r \textstyle\sum_{j=1}^{N} a_{ij,r}(x_i(t-\tau(t)) - x_j(t-\tau(t))) \\ &+ BK_r a_{i0,r}(x_i(t-\tau(t)) - x_0(t-\tau(t))) + Dw_i(t). \end{aligned} \tag{9}$$

Let $z_i(t) = x_i(t) - x_0(t)$ denote the consensus error of follower i. Accordingly, it follows from (5) and (9) that

$$\begin{aligned} E\dot{z}_i(t) &= Az_i(t) + f(z_i(t)) + BK_r \textstyle\sum_{j=1}^{N} a_{ij,r}(z_i(t-\tau(t)) - z_j(t-\tau(t))) \\ &+ BK_r a_{i0,r} z_i(t-\tau(t)) + Dw_i(t), \end{aligned} \tag{10}$$

where $f(z_i(t)) = f(x_i(t)) - f(x_0(t))$.

Before conducting the Kronecker product, define

$$\begin{aligned} z(t) &= \mathrm{col}(z_1(t), z_2(t), \cdots, z_N(t)), \\ F(z(t)) &= \mathrm{col}(f(z_1(t)), f(z_2(t)), \cdots, f(z_N(t))), \\ w(t) &= \mathrm{col}(w_1(t), w_2(t), \cdots, w_N(t)). \end{aligned}$$

Then, the resultant consensus error systems (10) can be written as the following global form:

$$\bar{E}\dot{z}(t) = \bar{A}z(t) + F(z(t)) + \bar{L}_r z(t-\tau(t)) + \bar{D}w(t), \tag{11}$$

where $\bar{E} = I_N \otimes E$, $\bar{A} = I_N \otimes A$, $\bar{L}_r = L_r \otimes BK_r$ and $\bar{D} = I_N \otimes D$.

Remark 5. *In [12, 28, 35], consensus protocols have been characterized by considering that the interagent interaction is not subject to any communication delay. Indeed, this may not be the practical case since the in-neighbors' information is transmitted via physical channels whose data transmission capability is limited. As a result, it makes practical sense to consider the communication delay when designing the consensus protocol.*

Remark 6. *It has been pointed out in [3] that $\rho(t) = \{(z(t), \nu_t), t \geq 0\}$ is not a Markov process. Over $[-d_2, 0]$, we first supplement $z(t)$ of the resultant consensus error system (11) as $z(t) = \phi(t)$, where $\phi : [-d_2, 0] \to \mathbb{R}^{Nn}$ is a continuous vector-valued function. Then, denote $z_t(\varepsilon) = z(t+\varepsilon)$, $\forall \varepsilon \in [-d_2, 0]$. For the newly-defined process $\bar{\rho}(t) = \{(z_t, \nu_t), t \geq 0\}$, one can get that it is a Markov process with the weak infinitesimal generator $\mathscr{J}$.*

Under Markovian switching topology, this paper aims to design the delayed tracking protocol (7) for noise-disturbed singular MAS (4) and (5) such that the resultant consensus error system (11) is stochastic admissible with an H_∞ disturbance attenuation level. Cases of completely known and partially known transition probabilities are both concerned. Before proceeding further, some useful definitions and lemmas are presented as follows.

For $r \in \mathcal{M}$, consider the following delayed Markovian jump singular system:

$$\mathcal{E}\dot{y}(t) = \mathcal{A}_r y(t) + \mathcal{A}_{dr} y(t-\tau(t)), \tag{12}$$

where $y(t) \in \mathbb{R}^n$ is the state. $\mathcal{A}_r$, $\mathcal{A}_{dr}$ and $\mathcal{E}$ with $\mathrm{rank}\mathcal{E} = \bar{\sigma} < n$ are appropriately dimensional matrices.

Definition 1. *[36] The system (12) is said to be*

(i) *regular and impulse-free, if the pairs $(\mathcal{E}, \mathcal{A}_r)$ and $(\mathcal{E}, \mathcal{A}_{dr})$ are regular and impulse-free;*

(ii) *stochastically stable, if*

$$\lim_{t\to\infty} \mathbb{E}\{\int_0^t \|y(\alpha)\|^2 \mathrm{d}\alpha | y(0), \nu_0\} < +\infty;$$

(iii) *stochastically admissible, if it is regular, impulse-free and stochastically stable.*

Lemma 1. *[32, 37] For $r \in \mathcal{M}$, the pair $(\mathcal{E}, \mathcal{A}_r + \mathcal{A}_{dr})$ is regular and impulse-free if*

$$\begin{cases} \mathcal{E}^{\mathrm{T}}\mathcal{P}_r = \mathcal{P}_r^{\mathrm{T}}\mathcal{E} \geq 0, \\ (\mathcal{A}_r + \mathcal{A}_{dr})^{\mathrm{T}}\mathcal{P}_r + \mathcal{P}_r^{\mathrm{T}}(\mathcal{A}_r + \mathcal{A}_{dr}) + \sum_{s=1}^{M} \pi_{rs}\mathcal{E}^{\mathrm{T}}\mathcal{P}_s \leq 0, \end{cases}$$

hold for matrices $\mathcal{P}_r$.

Lemma 2. *[7] For matrix $\mathcal{E} \in \mathbb{R}^{n\times n}$ with $\mathrm{rank}\mathcal{E} = \bar{\sigma} < n$, there exist full row rank matrix $H \in \mathbb{R}^{(n-\bar{\sigma})\times n}$ and full column rank matrix $F \in \mathbb{R}^{n\times(n-\bar{\sigma})}$ satisfying $H\mathcal{E} = 0$ and $\mathcal{E}F = 0$. Let $\mathcal{E} = \mathcal{E}_L\mathcal{E}_R^{\mathrm{T}}$, where matrices $\mathcal{E}_L$ and $\mathcal{E}_R \in \mathbb{R}^{n\times(n-\bar{\sigma})}$ are of full column rank. Then, two equivalent set are given as follows:*

$$\mathbb{S}_1 = \{\mathcal{P} \in \mathbb{R}^{n\times n} | \mathcal{E}^{\mathrm{T}}\mathcal{P} = \mathcal{P}^{\mathrm{T}}\mathcal{E} \geq 0, \mathcal{P}\,\text{is nonsingular}\},$$
$$\mathbb{S}_2 = \{\mathcal{P} = X\mathcal{E} + H^{\mathrm{T}}\Lambda F^{\mathrm{T}} | X = X^{\mathrm{T}} \in \mathbb{R}^{n\times n}, \mathcal{E}_L^{\mathrm{T}}X\mathcal{E}_L > 0, \Lambda \in \mathbb{R}^{(n-\bar{\sigma})\times(n-\bar{\sigma})}\,\text{is nonsingular}\}.$$

Definition 2. *By protocol (7), the concerned singular MAS (4) and (5) achieves H_∞ consensus tracking if*

(i) *for $w(t) = 0$, the resultant consensus error system (11) is stochastically admissible;*

(ii) *under zero initial condition, the following constraint*

$$\lim_{t\to\infty} \mathbb{E}\{\int_0^t \left(z^{\mathrm{T}}(\alpha)z(\alpha) - \gamma^2 w^{\mathrm{T}}(\alpha)w(\alpha)\right) \mathrm{d}\alpha\} < 0$$

holds for a given scalar $\gamma > 0$ and any $0 \neq w(t) \in \mathfrak{L}_2([0, +\infty), \mathbb{R}^{Nq})$.

Lemma 3. *[25] Given a continuously differentiable function $\varphi : [a_1, a_2] \to \mathbb{R}^n$, let $\varphi_1 = \varphi(a_2) - \varphi(a_1)$ and $\varphi_2 = \varphi(a_2) + \varphi(a_1) - \frac{2}{a_2-a_1}\int_{a_1}^{a_2} \varphi(\alpha)\mathrm{d}\alpha$. Then, the following inequality*

$$\int_{a_1}^{a_2} \dot{\varphi}^{\mathrm{T}}(\alpha)\mathcal{Q}\dot{\varphi}(\alpha)\mathrm{d}\alpha \geq \frac{1}{a_2 - a_1}\varphi_1^{\mathrm{T}}\mathcal{Q}\varphi_1 + \frac{3}{a_2 - a_1}\varphi_2^{\mathrm{T}}\mathcal{Q}\varphi_2$$

holds for a matrix $\mathcal{Q}(\in \mathbb{R}^{n\times n}) > 0$.

Lemma 4. *[25] Given a function $\varphi : [a_1, a_2] \to \mathbb{R}^n$, the following inequality*

$$(a_2 - a_1)\int_{a_1}^{a_2} \varphi^{\mathrm{T}}(\alpha)\mathcal{Q}\varphi(\alpha)\mathrm{d}\alpha \geq \left(\int_{a_1}^{a_2} \varphi(\alpha)\mathrm{d}\alpha\right)^{\mathrm{T}} \mathcal{Q} \left(\int_{a_1}^{a_2} \varphi(\alpha)\mathrm{d}\alpha\right)$$

holds for a matrix $\mathcal{Q}(\in \mathbb{R}^{n\times n}) > 0$.

Lemma 5. *[22] Given a open subset $\mathcal{O}$ of $\mathbb{R}^n$, let $h_1, h_2, \cdots, h_N : \mathbb{R}^n \to \mathbb{R}$ have positive values in $\mathcal{O}$. Over $\mathcal{O}$, one has*

$$\min_{\{\alpha_i | \alpha_i > 0, \sum_i \alpha_i = 1\}} \sum_i \frac{1}{\alpha_i}h_i(t) = \sum_i h_i(t) + \max_{g_{i,j}(t)} \sum_{i\neq j} g_{i,j}(t)$$

subject to

$$\left\{g_{i,j} : \mathbb{R}^n \to \mathbb{R}, g_{j,i}(t) = g_{i,j}(t), \begin{bmatrix} h_i(t) & g_{i,j}(t) \\ g_{j,i}(t) & h_j(t) \end{bmatrix} \geq 0\right\}.$$

Lemma 6. *[1] Given matrices Z and W of appropriate dimensions, the following matrix inequality*

$$Z^{\mathrm{T}}W + W^{\mathrm{T}}Z \leq \delta Z^{\mathrm{T}}Z + \delta^{-1}W^{\mathrm{T}}W$$

holds for any scalar $\delta > 0$.

3. Main results

In this section, by considering cases of completely known and partially known transition probabilities, some sufficient conditions are first derived to ensure stochastic admissibility of the resultant consensus error system. Then, based on the derived results, parameters of the desired H_∞ tracking protocol are characterized.

3.1. Stochastic admissibility analysis

This subsection is organized to give stochastic admissibility analysis of the resultant consensus error system (11) with $w(t) = 0$. Cases of completely known and partially known transition probabilities are both concerned.

To begin with, the following theorem is given to handle the case of completely known transition probabilities.

Theorem 1. *For $r \in \mathcal{M}$, given scalars $d_a > 0$ $(a = 1, 2, 3)$ and $\varrho > 0$, if there exist $Nn \times Nn$ matrices $Q_a > 0$, $R_a > 0$, $X_r = X_r^{\mathrm{T}}$, R, $(Nn - N\sigma) \times (Nn - N\sigma)$ nonsingular matrix Φ_r and $p \times n$ matrix K_r such that the following matrix constraints hold:*

$$\Xi \geq 0, \tag{13}$$

$$E_L^{\mathrm{T}} X_r E_L > 0, \tag{14}$$

$$\mathcal{R} = \begin{bmatrix} R_3 & * \\ R^{\mathrm{T}} & R_3 \end{bmatrix} > 0, \tag{15}$$

$$\Psi_r = \begin{bmatrix} \Psi_{1,r} & * \\ \Psi_{2,r} & \Psi_{3,r} \end{bmatrix} < 0,, \tag{16}$$

with

$$\begin{aligned}
\Xi &= Q_1 + Q_2 + d_3 Q_3 + d_{12} R_3 + \varrho^2 I - \bar{E}^{\mathrm{T}} \left(\tfrac{4}{d_1} R_1 + \tfrac{4}{d_2} R_2 \right) \bar{E}, \\
\Psi_{1,r} &= \begin{bmatrix} \Psi_{11,r} & * & * & * & * & * & * \\ \Psi_{21,r} & -I & * & * & * & * & * \\ \Psi_{31,r} & 0 & \Psi_{33,r} & * & * & * & * \\ \Psi_{41,r} & 0 & 0 & \Psi_{44,r} & * & * & * \\ \Psi_{51,r} & 0 & 0 & 0 & \Psi_{55,r} & * & * \\ \Psi_{61,r} & 0 & 0 & \Psi_{64,r} & 0 & \Psi_{66,r} & * \\ \Psi_{71,r} & 0 & 0 & 0 & \Psi_{75,r} & 0 & \Psi_{77,r} \end{bmatrix}, \\
\Psi_{2,r} &= \begin{bmatrix} R_d \bar{A} & R_d & R_d \bar{L}_r & 0 & 0 & 0 & 0 \end{bmatrix}, \\
\Psi_{3,r} &= -R_d, P_r = X_r \bar{E} + U^{\mathrm{T}} \Phi_r V^{\mathrm{T}},
\end{aligned}$$

where matrices E_L, U and V corresponding to $\bar{E}$ are defined in Lemma 2 and

$$\begin{aligned}
\Psi_{11,r} &= \bar{A}^{\mathrm{T}} P_r + P_r^{\mathrm{T}} \bar{A} + \textstyle\sum_{s=1}^{M} \pi_{rs} \bar{E}^{\mathrm{T}} P_s + \sum_{a=1}^{3} Q_a + d_{12} R_3 + \varrho^2 I - \bar{E}^{\mathrm{T}} \left(\tfrac{4}{d_1} R_1 + \tfrac{4}{d_2} R_2 \right) \bar{E}, \\
\Psi_{21,r} &= P_r, \Psi_{31,r} = \bar{L}_r^{\mathrm{T}} P_r, \Psi_{33,r} = -(1 - d_3) Q_3, \Psi_{41,r} = -\tfrac{2}{d_1} \bar{E}^{\mathrm{T}} R_1 \bar{E}, \Psi_{44,r} = -Q_1 - \tfrac{4}{d_1} \bar{E}^{\mathrm{T}} R_1 \bar{E}, \\
\Psi_{51,r} &= -\tfrac{2}{d_2} \bar{E}^{\mathrm{T}} R_2 \bar{E}, \Psi_{55,r} = -Q_2 - \tfrac{4}{d_2} \bar{E}^{\mathrm{T}} R_2 \bar{E}, \Psi_{61,r} = \tfrac{6}{d_1^2} R_1 \bar{E}, \Psi_{64,r} = \Psi_{61,r}, \Psi_{66,r} = -\tfrac{12}{d_1^3} R_1, \\
\Psi_{71,r} &= \tfrac{6}{d_2^2} R_2 \bar{E}, \Psi_{75,r} = \Psi_{71,r}, \Psi_{77,r} = -\tfrac{12}{d_2^3} R_2, R_d = d_1 R_1 + d_2 R_2, d_{12} = d_2 - d_1.
\end{aligned}$$

Then, with $w(t) = 0$ and completely known transition probabilities, the resultant consensus error system (11) is stochastically admissible.

Proof. It can be easily deduced from [19] and Definition 1 that, with $w(t) = 0$, the resultant consensus error system (11) is stochastically admissible if (i) the pairs $(\bar{E}, \bar{A})$ and $(\bar{E}, \bar{A} + \bar{L}_r)$ are regular and impulse-free; (ii) the system (11) is stochastically stable. Subsequently, we are going to prove that the above-mentioned two points can be ensured by LMIs (13)-(16).

Firstly, on the basis of LMIs (13), (14) and (16), the regularity and non-impulsiveness of pairs $(\bar{E}, \bar{A})$ and $(\bar{E}, \bar{A}+\bar{L}_r)$ are confirmed as follows.

Owing to rank$\bar{E} = N\sigma < Nn$, further calculation on the matrix $\bar{E} \in \mathbb{R}^{Nn\times Nn}$ gives

$$N_1 \bar{E} N_2 = \begin{bmatrix} I_{N\sigma} & 0 \\ 0 & 0 \end{bmatrix}, \tag{17}$$

where N_1 and $N_2 \in \mathbb{R}^{Nn\times Nn}$ are nonsingular matrices.

According to Lemma 2, one can easily get that the following matrix inequality

$$\bar{E}^{\mathrm{T}} P_r = P_r^{\mathrm{T}} \bar{E} \geq 0 \tag{18}$$

holds for a matrix $P_r (\in \mathbb{R}^{Nn\times Nn})$.

Let

$$N_1^{-\mathrm{T}} P_r N_2 = \begin{bmatrix} P_{11,r} & P_{12,r} \\ P_{21,r} & P_{22,r} \end{bmatrix}, N_1 \bar{A} N_2 = \begin{bmatrix} \bar{A}_{11} & \bar{A}_{12} \\ \bar{A}_{21} & \bar{A}_{22} \end{bmatrix}. \tag{19}$$

For $\forall r \in \mathcal{M}$, integrating (17), (18) and (19) derives $P_{12,r} = 0$. From (16), we have $\Psi_{11,r} < 0$, which further implies the following matrix inequality holds:

$$\bar{A}^{\mathrm{T}} P_r + P_r^{\mathrm{T}} \bar{A} + \sum_{s=1}^{M} \pi_{rs} \bar{E}^{\mathrm{T}} P_s - \bar{E}^{\mathrm{T}} \left(\frac{4}{d_1} R_1 + \frac{4}{d_2} R_2 \right) \bar{E} < 0. \tag{20}$$

In view of (17) and (19), using N_2^{T} and N_2 to pre- and post-multiply (20) derives

$$\begin{bmatrix} \star & \star \\ \star & \bar{A}_{22}^{\mathrm{T}} P_{22,r} \end{bmatrix} + \begin{bmatrix} \star & \star \\ \star & P_{22,r}^{\mathrm{T}} \bar{A}_{22} \end{bmatrix} < 0, \tag{21}$$

where the element represented by $\star$ is not involved in the subsequent derivation process.

By (21), we have $\bar{A}_{22}^{\mathrm{T}} P_{22,r} + P_{22,r}^{\mathrm{T}} \bar{A}_{22} < 0$. Then, one can get that $\bar{A}_{22}$ is a nonsingular matrix, which confirms regularity and non-impulsiveness of the pair $(\bar{E}, \bar{A})$.

In addition, one deduces from (16) that

$$\begin{bmatrix} \Psi_{11,r} & * \\ \Psi_{31,r} & \Psi_{33,r} \end{bmatrix} < 0. \tag{22}$$

Conducting Schur Complement Lemma on (22) yields

$$\Psi_{11,r} - \Psi_{31,r}^{\mathrm{T}} \Psi_{33,r}^{-1} \Psi_{31,r} < 0. \tag{23}$$

On the basis of (13) and (23), we have

$$(\bar{A} + \bar{L}_r)^{\mathrm{T}} P_r + P_r^{\mathrm{T}} (\bar{A} + \bar{L}_r) + \sum_{s=1}^{M} \pi_{rs} \bar{E}^{\mathrm{T}} P_s < 0. \tag{24}$$

From (18) and (24), one can conclude from Lemma 1 that the matrix pair $(\bar{E}, \bar{A}+\bar{L}_r)$ is regular and impulse-free.

Then, we will give stochastic stability analysis of the resultant consensus error system (11). For $r \in \mathcal{M}$, consider the stochastic Lyapunov candidate of the following form:

$$V(z_t, r) = \sum_{a=1}^{3} V_a(z_t, r), \tag{25}$$

with

$$\begin{array}{rcl} V_1(z_t,r) &=& z^{\mathrm{T}}(t)\bar{E}^{\mathrm{T}}P_r z(t),\\ V_2(z_t,r) &=& \int_{t-d_1}^{t} z^{\mathrm{T}}(\alpha)Q_1 z(\alpha)\mathrm{d}\alpha + \int_{t-d_2}^{t} z^{\mathrm{T}}(\alpha)Q_2 z(\alpha)\mathrm{d}\alpha + \int_{t-\tau(t)}^{t} z^{\mathrm{T}}(\alpha)Q_3 z(\alpha)\mathrm{d}\alpha,\\ V_3(z_t,r) &=& \int_{-d_1}^{0}\int_{t+\theta}^{t} \dot{z}^{\mathrm{T}}(\alpha)\bar{E}^{\mathrm{T}}R_1\bar{E}\dot{z}(\alpha)\mathrm{d}\alpha\mathrm{d}\theta + \int_{-d_2}^{0}\int_{t+\theta}^{t} \dot{z}^{\mathrm{T}}(\alpha)\bar{E}^{\mathrm{T}}R_2\bar{E}\dot{z}(\alpha)\mathrm{d}\alpha\mathrm{d}\theta\\ &+& \int_{-d_2}^{-d_1}\int_{t+\theta}^{t} z^{\mathrm{T}}(\alpha)R_3 z(\alpha)\mathrm{d}\alpha\mathrm{d}\theta, \end{array}$$

where the matrix P_r satisfies (18).

By utilizing the weak infinitesimal generator $\mathscr{J}$ in Remark 6, one derives that

$$\mathscr{J}V_1(z_t,r) = \dot{z}^{\mathrm{T}}(t)\bar{E}^{\mathrm{T}}P_r z(t) + z^{\mathrm{T}}(t)\bar{E}^{\mathrm{T}}P_r\dot{z}(t) + z^{\mathrm{T}}(t)\left(\sum_{s=1}^{M}\pi_{rs}\bar{E}^{\mathrm{T}}P_s\right)z(t), \tag{26}$$

$$\begin{array}{rcl} \mathscr{J}V_2(z_t,r) &=& z^{\mathrm{T}}(t)\left(\sum_{a=1}^{3}Q_a\right)z(t) - z^{\mathrm{T}}(t-d_1)Q_1 z(t-d_1) - z^{\mathrm{T}}(t-d_2)Q_2 z(t-d_2)\\ &-& (1-\dot{\tau}(t))z^{\mathrm{T}}(t-\tau(t))Q_3 z(t-\tau(t)), \end{array} \tag{27}$$

$$\begin{array}{rcl} \mathscr{J}V_3(z_t,r) &=& \dot{z}^{\mathrm{T}}(t)\bar{E}^{\mathrm{T}}R_d\bar{E}\dot{z}(t) + d_{12}z^{\mathrm{T}}(t)R_3 z(t) - \int_{t-d_1}^{t}\dot{z}^{\mathrm{T}}(\alpha)\bar{E}^{\mathrm{T}}R_1\bar{E}\dot{z}(\alpha)\mathrm{d}\alpha\\ &-& \int_{t-d_2}^{t}\dot{z}^{\mathrm{T}}(\alpha)\bar{E}^{\mathrm{T}}R_2\bar{E}\dot{z}(\alpha)\mathrm{d}\alpha - \int_{t-d_2}^{t-d_1} z^{\mathrm{T}}(\alpha)R_3 z(\alpha)\mathrm{d}\alpha. \end{array} \tag{28}$$

For the R_1-dependent integral term in (28), it follows from Lemma 3 that

$$-\int_{t-d_1}^{t}\dot{z}^{\mathrm{T}}(\alpha)\bar{E}^{\mathrm{T}}R_1\bar{E}\dot{z}(\alpha)\mathrm{d}\alpha \le -\frac{1}{d_1}\eta_1^{\mathrm{T}}(t)R_1\eta_1(t) - \frac{3}{d_1}\bar{\eta}_1^{\mathrm{T}}(t)R_1\bar{\eta}_1(t), \tag{29}$$

where $\eta_1(t) = \bar{E}(z(t) - z(t-d_1))$ and $\bar{\eta}_1(t) = \bar{E}(z(t) + z(t-d_1) - \frac{2}{d_1}\int_{t-d_1}^{t}\bar{E}z(\alpha)\mathrm{d}\alpha)$.

Similarly, with defining $\eta_2(t) = \bar{E}(z(t) - z(t-d_2))$ and $\bar{\eta}_2(t) = \bar{E}(z(t) + z(t-d_2) - \frac{2}{d_2}\int_{t-d_2}^{t}\bar{E}z(\alpha)\mathrm{d}\alpha)$, the R_2-dependent integral term in (28) can be derived as

$$-\int_{t-d_2}^{t}\dot{z}^{\mathrm{T}}(\alpha)\bar{E}^{\mathrm{T}}R_2\bar{E}\dot{z}(\alpha)\mathrm{d}\alpha \le -\frac{1}{d_2}\eta_2^{\mathrm{T}}(t)R_2\eta_2(t) - \frac{3}{d_2}\bar{\eta}_2^{\mathrm{T}}(t)R_2\bar{\eta}_2(t). \tag{30}$$

In accordance with Lemmas 4 and 5, we calculate the R_3-dependent integral term in (28) as

$$\begin{array}{rcl} -\int_{t-d_2}^{t-d_1} z^{\mathrm{T}}(\alpha)R_3 z(\alpha)\mathrm{d}\alpha &=& -\int_{t-d_2}^{t-\tau(t)} z^{\mathrm{T}}(\alpha)R_3 z(\alpha)\mathrm{d}\alpha - \int_{t-\tau(t)}^{t-d_1} z^{\mathrm{T}}(\alpha)R_3 z(\alpha)\mathrm{d}\alpha\\ &\le& -\frac{1}{d_2-\tau(t)}\left(\int_{t-d_2}^{t-\tau(t)} z(\alpha)\mathrm{d}\alpha\right)^{\mathrm{T}} R_3\left(\int_{t-d_2}^{t-\tau(t)} z(\alpha)\mathrm{d}\alpha\right)\\ &-& \frac{1}{\tau(t)-d_1}\left(\int_{t-\tau(t)}^{t-d_1} z(\alpha)\mathrm{d}\alpha\right)^{\mathrm{T}} R_3\left(\int_{t-\tau(t)}^{t-d_1} z(\alpha)\mathrm{d}\alpha\right)\\ &\le& -\frac{1}{d_{12}}\varsigma^{\mathrm{T}}(t)\begin{bmatrix} R_3 & * \\ R^{\mathrm{T}} & R_3 \end{bmatrix}\varsigma(t), \end{array} \tag{31}$$

where $\varsigma(t) = \mathrm{col}(\int_{t-d_2}^{t-\tau(t)} z(\alpha)\mathrm{d}\alpha, \int_{t-\tau(t)}^{t-d_1} z(\alpha)\mathrm{d}\alpha)$ and the matrix R satisfies LMI (15).

With consideration given to (6), it is easy to get that the following inequality:

$$F^{\mathrm{T}}(z(t))F(z(t)) \le \varrho^2 z^{\mathrm{T}}(t)z(t), \tag{32}$$

holds for a scalar $\varrho > 0$.

By denoting $\xi(t) = \mathrm{col}(z(t), F(z(t)), z(t-\tau(t)), z(t-d_1), z(t-d_2), \int_{t-d_1}^{t}\bar{E}z(\alpha)\mathrm{d}\alpha, \int_{t-d_2}^{t}\bar{E}z(\alpha)\mathrm{d}\alpha)$, some algebraic manipulations on (11) and (26)-(32) give

$$\mathscr{J}V(z_t,r) \le \xi^{\mathrm{T}}(t)\Psi_r\xi(t) - \frac{1}{d_{12}}\varsigma^{\mathrm{T}}(t)\mathcal{R}\varsigma(t). \tag{33}$$

Following from (15), (16) and (33), it is easy to deduce that, for a scalar $\kappa > 0$, the following inequality holds:

$$\mathcal{J}\{V(z(t),\nu_t = r)|w(t) = 0\} \leq -\kappa z^{\mathrm{T}}(t)z(t). \tag{34}$$

According to Dynkin's formula, one derives from (34) that

$$\begin{array}{rcl}\mathbb{E}\{V(z(t),\nu_t = r)\} - \mathbb{E}\{V(z(0),\nu_0)\} & = & \mathbb{E}\{\int_0^t \mathcal{J}V(z(\alpha),\nu_\alpha = r)\mathrm{d}\alpha|w(t) = 0, z(0),\nu_0\} \\ & \leq & -\kappa\mathbb{E}\{\int_0^t \|z(\alpha)\|^2\mathrm{d}\alpha|w(t) = 0, z(0),\nu_0\}.\end{array} \tag{35}$$

Owing to $\mathbb{E}\{V(z(t),\nu_t = r)\} \geq 0$, we have

$$\lim_{t\to\infty} \mathbb{E}\{\int_0^t \|z(\alpha)\|^2\mathrm{d}\alpha|w(t) = 0, z(0),\nu_0\} \leq \frac{1}{\kappa}\mathbb{E}\{V(z(0),\nu_0)\} < +\infty, \tag{36}$$

which further implies that the resultant consensus error system (11) is stochastically stable.

Hence, by Definition 1, one can conclude that the resultant consensus error system (11) is stochastically admissible. This completes the proof. □

Before moving on, we define

$$\mathcal{M} = \mathcal{M}_1^r + \mathcal{M}_2^r, \tag{37}$$

where $\mathcal{M}_1^r = \{s|\pi_{rs}\,\text{is known}, s \in \mathcal{M}\}$ and $\mathcal{M}_2^r = \{s|\pi_{rs}\,\text{is unknown}, s \in \mathcal{M}\}$.

Then, a theorem is given to handle the case of partially known transition probabilities.

Theorem 2. *For $r \in \mathcal{M}$, given scalars $d_a > 0$ $(a = 1,2,3)$ and $\varrho > 0$, if there exist $Nn \times Nn$ matrices $Q_a > 0$, $R_a > 0$, $X_r = X_r^{\mathrm{T}}$, $F_r = F_r^{\mathrm{T}}$, R, $(Nn - N\sigma) \times (Nn - N\sigma)$ nonsingular matrix Φ_r and $p \times n$ matrix K_r such that LMIs (13)-(15) and*

$$\hat{\Psi}_r = \begin{bmatrix} \hat{\Psi}_{1,r} & * \\ \Psi_{2,r} & \Psi_{3,r} \end{bmatrix} < 0, \tag{38}$$

$$\bar{E}^{\mathrm{T}}P_s - F_r \leq 0, s \in \mathcal{M}_2^r, s \neq r, \tag{39}$$

$$\bar{E}^{\mathrm{T}}P_s - F_r \geq 0, s \in \mathcal{M}_2^r, s = r, \tag{40}$$

hold with

$$\hat{\Psi}_{1,r} = \begin{bmatrix} \hat{\Psi}_{11,r} & * & * & * & * & * & * \\ \Psi_{21,r} & -I & * & * & * & * & * \\ \Psi_{31,r} & 0 & \Psi_{33,r} & * & * & * & * \\ \Psi_{41,r} & 0 & 0 & \Psi_{44,r} & * & * & * \\ \Psi_{51,r} & 0 & 0 & 0 & \Psi_{55,r} & * & * \\ \Psi_{61,r} & 0 & 0 & \Psi_{64,r} & 0 & \Psi_{66,r} & * \\ \Psi_{71,r} & 0 & 0 & 0 & \Psi_{75,r} & 0 & \Psi_{77,r} \end{bmatrix},$$

where

$$\hat{\Psi}_{11,r} = \bar{A}^{\mathrm{T}}P_r + P_r^{\mathrm{T}}\bar{A} + \sum_{s\in\mathcal{M}_1^r} \pi_{rs}\left(\bar{E}^{\mathrm{T}}P_s - F_r\right) + \sum_{a=1}^{3} Q_a + d_{12}R_3 + \varrho^2 I - \bar{E}^{\mathrm{T}}\left(\frac{4}{d_1}R_1 + \frac{4}{d_2}R_2\right)\bar{E}$$

and other elements are defined in Theorem 1. Then, with $w(t) = 0$ and partially known transition probabilities, the resultant consensus error system (11) is stochastically admissible.

Proof. By considering (38)-(40), it is easy to deduce from Theorem 1 that the resultant consensus error system (11) is regular and impulse-free.

Owing to $\sum_{s=1}^{M}\pi_{rs} = 0$, the following zero equation holds for any matrix $F_r = F_r^{\mathrm{T}}$:

$$-z^{\mathrm{T}}(t)\left(\sum_{s=1}^{M}\pi_{rs}F_r\right)z(t) = 0. \tag{41}$$

Following from (33) and (41), one derives

$$\mathcal{J}V(z_t,r) \le \xi^{\mathrm{T}}(t)\hat{\Psi}_r\xi(t) + z^{\mathrm{T}}(t)\left(\sum_{s\in\mathcal{M}_2^r}\pi_{rs}\left(\bar{E}^{\mathrm{T}}P_s - F_r\right)\right)z(t) - \frac{1}{d_{12}}\varsigma^{\mathrm{T}}(t)\mathcal{R}\varsigma(t). \tag{42}$$

Based on the similar proof line of Theorem 1, one can obviously deduce that the resultant consensus error system (11) is stochastically stable.

Recalling Definition 1, we thus get the conclusion of Theorem 2. This completes the proof. □

3.2. Consensus racking protocol design

On the basis of above-derived results, this subsection aims to design a protocol such that the concerned singular MAS achieves consensus tracking with an H_∞ disturbance attenuation level. For the case of completely known transition probabilities, we first present the following theorem.

Theorem 3. *For $r\in\mathcal{M}$, given scalars $d_a>0$ $(a=1,2,3)$, $\delta>0$, $\varrho>0$ and $\gamma>0$, if there exist $Nn\times Nn$ matrices $Q_a>0$, $R_a>0$, $X_r=X_r^{\mathrm{T}}$, R, $(Nn-N\sigma)\times(Nn-N\sigma)$ nonsingular matrix Φ_r and $p\times n$ matrix K_r such that LMIs (13)-(15) and*

$$\Theta_r = \begin{bmatrix} \Theta_{1,r} & * & * \\ \Theta_{2,r} & \Theta_{3,r} & * \\ \Theta_{4,r} & \Theta_{5,r} & \Theta_{6,r} \end{bmatrix} < 0, \tag{43}$$

hold with

$$\begin{aligned}
\Theta_{1,r} &= \begin{bmatrix} \Theta_{11,r} & * & * & * & * & * & * \\ \Psi_{21,r} & -I & * & * & * & * & * \\ 0 & 0 & \Psi_{33,r} & * & * & * & * \\ \Psi_{41,r} & 0 & 0 & \Psi_{44,r} & * & * & * \\ \Psi_{51,r} & 0 & 0 & 0 & \Psi_{55,r} & * & * \\ \Psi_{61,r} & 0 & 0 & \Psi_{64,r} & 0 & \Psi_{66,r} & * \\ \Psi_{71,r} & 0 & 0 & 0 & \Psi_{75,r} & 0 & \Psi_{77,r} \end{bmatrix}, \\
\Theta_{2,r} &= \begin{bmatrix} \bar{D}^{\mathrm{T}}P_r & 0 & 0 & 0 & 0 & 0 & 0 \\ R_d\bar{A} & R_d & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \Theta_{3,r} = \begin{bmatrix} -\gamma^2 I & * \\ R_d\bar{D} & -R_d \end{bmatrix}, \\
\Theta_{4,r} &= \begin{bmatrix} \delta P_r & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \bar{L}_r & 0 & 0 & 0 & 0 \end{bmatrix}, \Theta_{5,r} = \begin{bmatrix} 0 & \delta R_d^{\mathrm{T}} \\ 0 & 0 \end{bmatrix}, \Theta_{6,r} = -\delta I,
\end{aligned}$$

where

$$\Theta_{11,r} = \bar{A}^{\mathrm{T}}P_r + P_r^{\mathrm{T}}\bar{A} + \sum_{s=1}^{M}\pi_{rs}\bar{E}^{\mathrm{T}}P_s + \sum_{a=1}^{3}Q_a + d_{12}R_3 + (\varrho^2+1)I - \bar{E}^{\mathrm{T}}\left(\frac{4}{d_1}R_1 + \frac{4}{d_2}R_2\right)\bar{E}$$

and other elements are defined in Theorem 1. Then, with nonzero $w(t)\in\mathfrak{L}_2([0,+\infty),\mathbb{R}^{Nq})$ and completely known transition probabilities, the singular MAS (4) and (5) with protocol (7) achieves consensus tracking with an H_∞ disturbances attenuation level γ.

Proof. By Theorem 1, it can be easily deduced from LMIs (13)-(15) and (43) that the resultant consensus error system (11) with $w(t)=0$ is stochastically admissible. To analyze the H_∞ disturbance attenuation performance, a cost function is constructed as follows:

$$J_{zw} = \mathbb{E}\{\int_0^t \left(z^{\mathrm{T}}(\alpha)z(\alpha) - \gamma^2 w^{\mathrm{T}}(\alpha)w(\alpha)\right)\mathrm{d}\alpha\}. \tag{44}$$

Denote $\bar{\xi}(t)=\mathrm{col}(\xi(t),w(t))$. Under zero initial condition, one derives from (11) and (26)-(31) that

$$\begin{aligned} J_{zw} &\le \mathbb{E}\{\textstyle\int_0^t \left(z^{\mathrm{T}}(\alpha)z(\alpha) - \gamma^2 w^{\mathrm{T}}(\alpha)w(\alpha) + \mathcal{J}V(z(\alpha),\nu_\alpha=r)\right)\mathrm{d}\alpha\} \\ &\le \mathbb{E}\{\textstyle\int_0^t \bar{\xi}^{\mathrm{T}}(\alpha)\bar{\Theta}_r\bar{\xi}(\alpha)\mathrm{d}\alpha\}, \end{aligned} \tag{45}$$

with

$$\bar{\Theta}_r = \begin{bmatrix} \bar{\Theta}_{1,r} & * \\ \bar{\Theta}_{2,r} & \Theta_{3,r} \end{bmatrix},$$

where

$$\bar{\Theta}_{1,r} = \begin{bmatrix} \Theta_{11,r} & * & * & * & * & * & * \\ \Psi_{21,r} & -I & * & * & * & * & * \\ \Psi_{31,r} & 0 & \Psi_{33,r} & * & * & * & * \\ \Psi_{41,r} & 0 & 0 & \Psi_{44,r} & * & * & * \\ \Psi_{51,r} & 0 & 0 & 0 & \Psi_{55,r} & * & * \\ \Psi_{61,r} & 0 & 0 & \Psi_{64,r} & 0 & \Psi_{66,r} & * \\ \Psi_{71,r} & 0 & 0 & 0 & \Psi_{75,r} & 0 & \Psi_{77,r} \end{bmatrix},$$

$$\bar{\Theta}_{2,r} = \begin{bmatrix} \bar{D}^{\mathrm{T}} P_r & 0 & 0 & 0 & 0 & 0 & 0 \\ R_d \bar{A} & R_d & R_d \bar{L}_r & 0 & 0 & 0 & 0 \end{bmatrix}.$$

Defining $\mathcal{Z} = \mathrm{col}(P_r^{\mathrm{T}}, 0, 0, R_d)^{\mathrm{T}}$ and $\mathcal{W} = \mathrm{col}(0, 0, \bar{L}_r^{\mathrm{T}}, 0)^{\mathrm{T}}$, we equivalently decompose $\bar{\Theta}_r$ in (45) as

$$\bar{\Theta}_r = \begin{bmatrix} \Theta_{1,r} & * \\ \Theta_{2,r} & \Theta_{3,r} \end{bmatrix} + \mathcal{Z}^{\mathrm{T}}\mathcal{W} + \mathcal{W}^{\mathrm{T}}\mathcal{Z}. \tag{46}$$

According to Lemma 6 and Schur Complement Lemma, one can obviously derive from (43) that $\bar{\Theta}_r < 0$, which further implies that the following inequality

$$J_{zw} = \mathbb{E}\{\int_0^t \left(z^{\mathrm{T}}(\alpha)z(\alpha) - \gamma^2 w^{\mathrm{T}}(\alpha)w(\alpha)\right) \mathrm{d}\alpha\} < 0 \tag{47}$$

holds for a given scalar $\gamma > 0$ and $0 \neq w(t) \in \mathfrak{L}_2([0, +\infty), \mathbb{R}^{Nq})$.

By Definition 2, one can conclude that the singular MAS (4) and (5) with protocol (7) achieves consensus tracking with an H_∞ disturbance attenuation level γ. This completes the proof. □

Then, we extend the result in Theorem 3 to the case of partially known transition probabilities.

Theorem 4. *For $r \in \mathcal{M}$, given scalars $d_a > 0$ ($a = 1, 2, 3$), $\delta > 0$, $\varrho > 0$ and $\gamma > 0$, if there exist $Nn \times Nn$ matrices $Q_a > 0$, $R_a > 0$, $X_r = X_r^{\mathrm{T}}$, $F_r = F_r^{\mathrm{T}}$, R, $(Nn - N\sigma) \times (Nn - N\sigma)$ nonsingular matrix Φ_r and $p \times n$ matrix K_r such that LMIs (13)-(15), (39), (40) and*

$$\hat{\Theta}_r = \begin{bmatrix} \hat{\Theta}_{1,r} & * & * \\ \Theta_{2,r} & \Theta_{3,r} & * \\ \Theta_{4,r} & \Theta_{5,r} & \Theta_{6,r} \end{bmatrix} < 0, \tag{48}$$

hold with

$$\hat{\Theta}_{1,r} = \begin{bmatrix} \hat{\Theta}_{11,r} & * & * & * & * & * & * \\ \Psi_{21,r} & -I & * & * & * & * & * \\ 0 & 0 & \Psi_{33,r} & * & * & * & * \\ \Psi_{41,r} & 0 & 0 & \Psi_{44,r} & * & * & * \\ \Psi_{51,r} & 0 & 0 & 0 & \Psi_{55,r} & * & * \\ \Psi_{61,r} & 0 & 0 & \Psi_{64,r} & 0 & \Psi_{66,r} & * \\ \Psi_{71,r} & 0 & 0 & 0 & \Psi_{75,r} & 0 & \Psi_{77,r} \end{bmatrix},$$

where

$$\hat{\Theta}_{11,r} = \bar{A}^{\mathrm{T}} P_r + P_r^{\mathrm{T}} \bar{A} + \sum_{s \in \mathcal{M}_1^r} \pi_{rs} \left(\bar{E}^{\mathrm{T}} P_s - F_r\right) + \sum_{a=1}^{3} Q_a + d_{12} R_3 + (\varrho^2 + 1)I - \bar{E}^{\mathrm{T}} \left(\frac{4}{d_1} R_1 + \frac{4}{d_2} R_2\right) \bar{E}$$

and other elements are defined in Theorem 3. Then, with nonzero $w(t) \in \mathfrak{L}_2([0, +\infty), \mathbb{R}^{Nq})$ and partially known transition probabilities, the singular MAS (4) and (5) with protocol (7) achieves consensus tracking with an H_∞ disturbances attenuation level γ.

Proof. By applying the similar analysis of Theorem 2 to Theorem 3, one can easily get the conclusion of Theorem 4. Accordingly, details are omitted. □

Remark 7. *By virtue of free-weighting matrix technique, a few integral inequalities (e.g., free-matrix-based integral inequality) have been promisingly developed to reduce the conservatism on estimating bounds of some integral terms. As is illustrated in [32], the calculation burden on performance analysis and control synthesis may increase with the number of the introduced free-weighting matrices. It is noteworthy that, to achieve a tradeoff between calculation burden and conservatism reduction, the Wirtinger-based integral inequality combined with reciprocally convex combination has been encouragingly utilized in [25], which also inspires us for this study.*

4. Numerical examples

This section gives two numerical examples to verify the theoretical analysis.

Example 1. This example concerns the H_∞ delayed tracking protocol (7) design for singular MAS (4) and (5) consisting of one leader and three followers. The system parameters are given as

$$E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, A = \begin{bmatrix} 0 & 1 & 0 \\ 9.8 & 0 & 1 \\ -20 & -3 & -1 \end{bmatrix}, B = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}, D = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, f(x_i(t)) = \begin{bmatrix} \varrho\eta(x_i(t)) \\ 0 \\ 0 \end{bmatrix},$$

where $\eta(x_i(t)) = |x_{i1}(t) + 1| - |x_{i1}(t) - 1|$.

Regulated by $\mathcal{T} = \begin{bmatrix} -0.3 & 0.3 \\ 0.6 & -0.6 \end{bmatrix}$, Fig. 1 depicts the evolution of all switching topologies visualized in Fig. 2. Each edge's weight is prescribed as 1. It can be obviously observed in Fig. 2 that, rooting at the leader, the union $\mathcal{G}^U = \mathcal{G}_1 \bigcup \mathcal{G}_2$ contains a directed spanning tree.

Choose

$$(d_1, d_3, \delta, \varrho) = (0.1, 0.1, 0.9, 0.333), U = V^{\mathrm{T}} = I_3 \otimes \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}, E_L = E_R = I_3 \otimes \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}.$$

For various d_2, solving LMIs (13)-(15) and (43) in Theorem 3 yields the results of optimal H_∞ performance level γ, which have been organized in Table 1. From Table 1, one can easily get that the optimal H_∞

d_2	0.3	0.5	0.7	0.9	1.2
γ	0.8933	1.1394	1.4187	1.6745	2.0388

Table 1: Optimal γ for various d_2.

performance level γ increases proportionally to delay upper bound d_2.

Furthermore, we choose $\tau(t) = 0.7 + 0.5\sin(t)$ and $w(t) = 0.3\sin(t)$. From (8), it is easy to get that $d_1 = 0.2$, $d_2 = 1.2$ and $d_3 = 0.5$. Then, some feasible solutions of LMIs (13)-(15) and (43) are obtained as

$$\gamma = 2.1736, K_1 = \begin{bmatrix} 1.4122 & 0.7004 & -0.0815 \end{bmatrix}, K_2 = \begin{bmatrix} 1.9719 & 0.8525 & -0.0936 \end{bmatrix}.$$

Choosing the initial states as $x_0(0) = [-1\ 1\ 0]^{\mathrm{T}}$, $x_1(0) = [1\ 2\ -2]^{\mathrm{T}}$, $x_2(0) = [-1\ -2\ 1]^{\mathrm{T}}$ and $x_3(0) = [-2\ -1\ 2]^{\mathrm{T}}$, Figs. 3, 4 and 5 plot state trajectories of singular MAS (4) and (5). As is shown in aforementioned figures, the state trajectories of all followers asymptotically approach to those of the leader, which intuitively verifies the conclusion of Theorem 3.

Example 2. The singular MAS (4) and (5) concerned in this example consists of one leader and five followers, whose system parameters are

$$E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, A = \begin{bmatrix} 0 & 1 & 0 \\ -3 & -2 & 1 \\ 0 & 0 & -1 \end{bmatrix}, B = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}, D = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, f(x_i(t)) = \begin{bmatrix} \varrho\eta(x_i(t)) \\ 0 \\ 0 \end{bmatrix},$$

where $\eta(x_i(t))$ is given in Example 1.

The switching graphs $\mathcal{G}_b$ $(b = 1, 2, \cdots, 4)$ are visualized in Fig. 6, where each edge's weight is prescribed as 1. Rooting at the leader, one can obviously observe in Fig. 6 that the spanning tree is not contained in each possible graph $\mathcal{G}_b$ while exists in the union $\mathcal{G}^U = \bigcup_{b=1}^{4} \mathcal{G}_b$.

The transition rate matrix $\mathcal{T}$ is chosen as

$$\mathcal{T} = \begin{bmatrix} ? & 0.1 & 0.2 & ? \\ 0.2 & -0.7 & 0.3 & 0.2 \\ ? & ? & ? & 0.3 \\ 0.4 & 0.3 & 0.2 & -0.9 \end{bmatrix},$$

where ? represents the unknown element.

Generating from $\mathcal{T}$, a Markov process is utilized to govern the evolution of all switching graphs $\mathcal{G}_b$, whose state trajectory is depicted in Fig. 7.

Choose the communication delay $\tau(t)$ as $\tau(t) = 0.3+0.2\sin(t)$. It follows from (8) that $d_1 = 0.1$, $d_2 = 0.5$ and $d_3 = 0.2$. Before solving LMIs (13)-(15), (39), (40) and (48) in Theorem 4, one prescribes

$$(\delta, \varrho, \gamma) = (0.8, 0.333, 1.27), U = V^{\mathrm{T}} = I_5 \otimes \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}, E_L = E_R = I_5 \otimes \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}.$$

Then, some feasible solutions are calculated as

$$\begin{aligned} K_1 &= \begin{bmatrix} 1.1836 & -0.0319 & 0.8962 \end{bmatrix}, K_2 = \begin{bmatrix} 0.5407 & 0.1975 & 0.3524 \end{bmatrix}, \\ K_3 &= \begin{bmatrix} 0.5915 & 0.1986 & 0.3751 \end{bmatrix}, K_4 = \begin{bmatrix} 0.4806 & 0.1023 & -0.2042 \end{bmatrix}. \end{aligned}$$

Moreover, the exogenous disturbance $w_i(t)$ is chosen as $w_i(t) = 0.1\sin(t)$. Under initial conditions $x_0(0) = [1.5\ 1\ -1.5]^{\mathrm{T}}$, $x_1(0) = [-1\ -1.5\ 0.5]^{\mathrm{T}}$, $x_2(0) = [1\ 1.5\ -1]^{\mathrm{T}}$, $x_3(0) = [1.5\ 1\ 1.5]^{\mathrm{T}}$, $x_4(0) = [-1.5\ -1\ 1]^{\mathrm{T}}$ and $x_5(0) = [0.5\ -1.5\ -0.5]^{\mathrm{T}}$, Figs. 8, 9 and 10 plot consensus error trajectories of singular MAS (4) and (5), where all curves asymptotically converge to zero. This verifies the conclusion of Theorem 4.

5. Conclusion

In this paper, the tracking consistency has been regulated for Lipschitz nonlinear singular MASs with exogenous disturbances. The network interaction randomly switches among a set of directed graphs, whose evolution is governed by a Markov chain. Assumption is that the directed spanning tree is not necessarily contained in each graph while exits in the union rooting at the leader node. By collecting the delayed in-neighbors' information, a protocol has been constructed to ensure stochastic admissibility of the resultant consensus error system with an H_∞ disturbance attenuation level. In terms of LMIs, sufficient conditions have been firstly derived to characterize the protocol parameter by considering the case of completely known transition probabilities. By virtue of free-connection weighting matrix technique, the derived results have been extended to the case that transition probabilities are partially known. Numerical simulations have verified the proposed design method. Since the public safety network can be abstractly considered as a MAS [27], our future attention will be paid to extend the proposed analytical framework to public safety systems.

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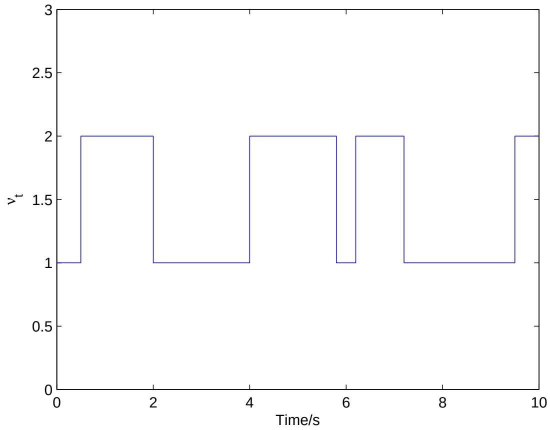

Figure 1: Markovian jump modes in Example 1.

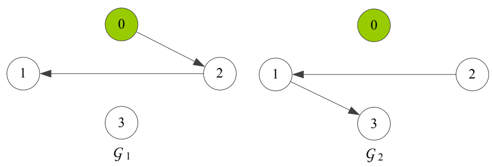

Figure 2: Interaction topologies in Example 1.

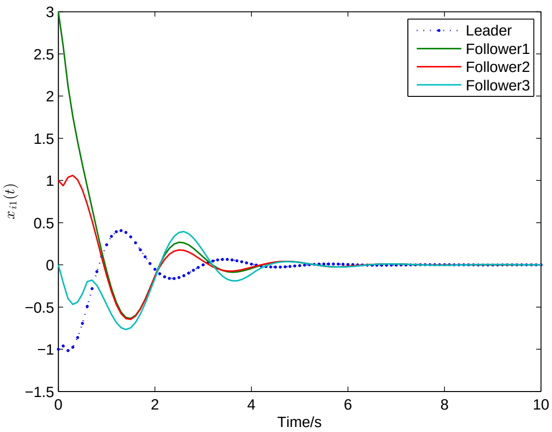

Figure 3: Consensus states $x_{j1}(t)$, $j = 0, 1, \cdots, 3$, in Example 1.

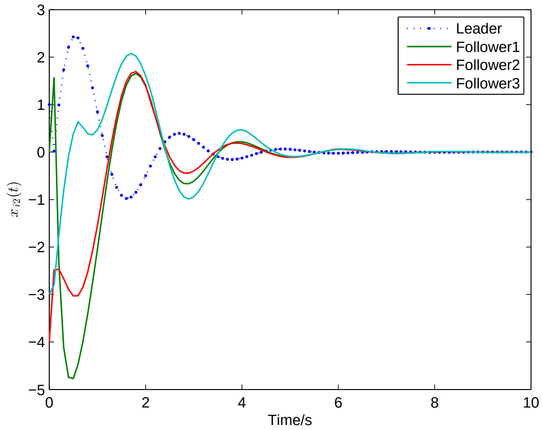

Figure 4: Consensus states $x_{j2}(t)$, $j = 0, 1, \cdots, 3$, in Example 1.

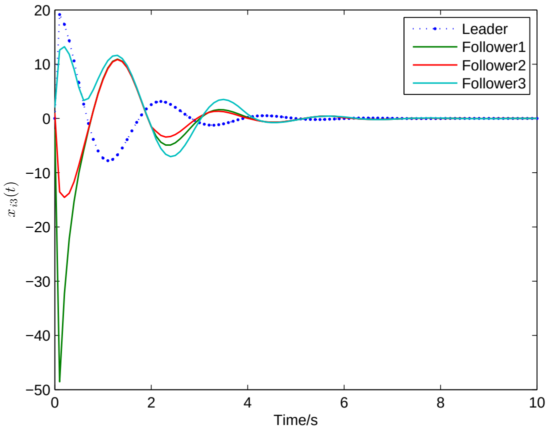

Figure 5: Consensus states $x_{j3}(t)$, $j = 0, 1, \cdots, 3$, in Example 1.

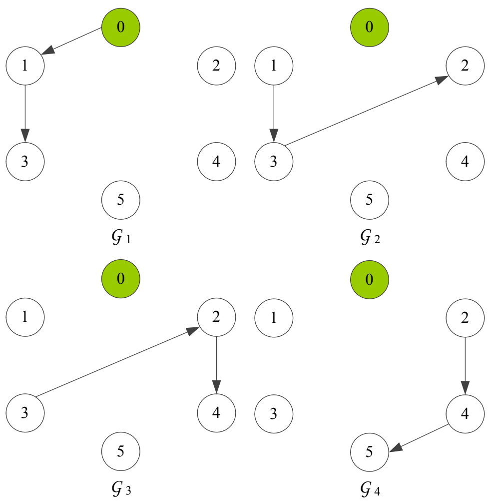

Figure 6: Interaction topologies in Example 2.

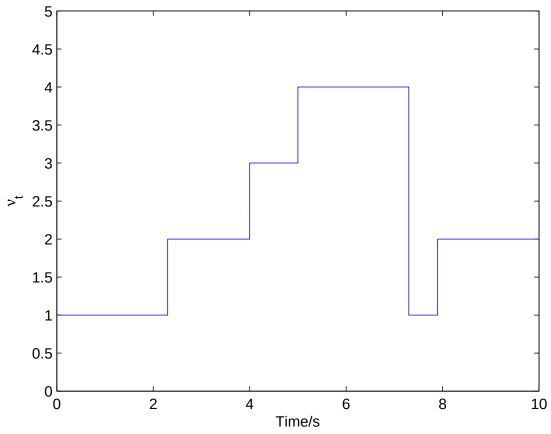

Figure 7: Markovian jump modes in Example 2.

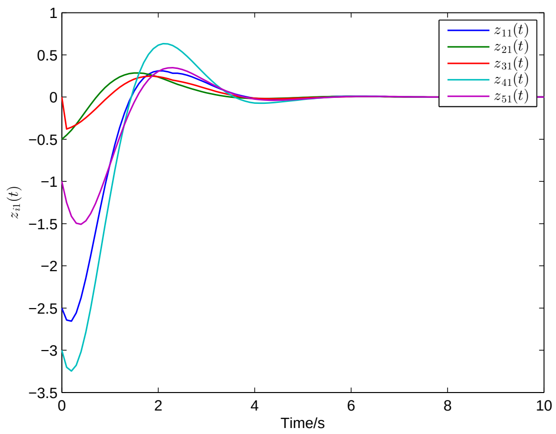

Figure 8: Consensus errors $z_{i1}(t)$, $i = 1, 2 \cdots, 5$, in Example 2.

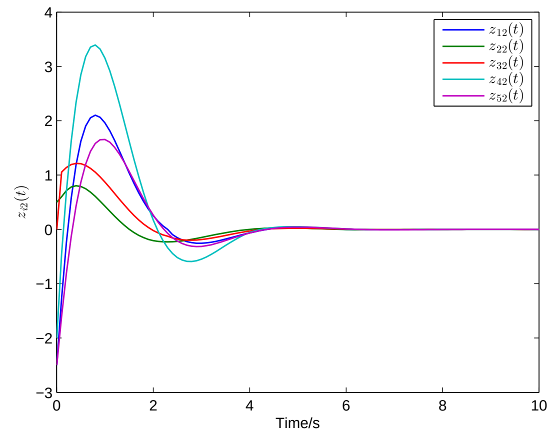

Figure 9: Consensus errors $z_{i2}(t)$, $i = 1, 2 \cdots, 5$, in Example 2.

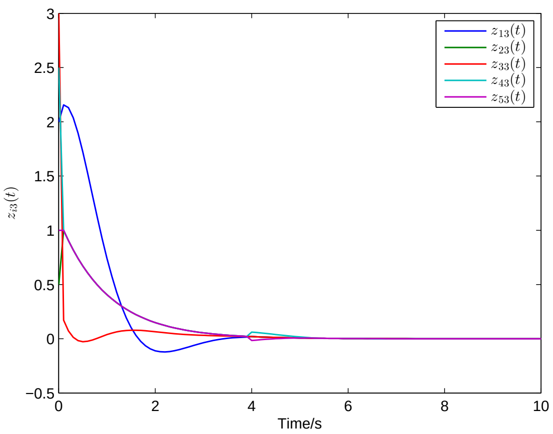

Figure 10: Consensus errors $z_{i3}(t)$, $i = 1, 2 \cdots, 5$, in Example 2.