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Keywords

multi, request, input, automatic, output, repeat, efficiency, systems, antenna, selection, energy, over, nakagami, m, fading, channels, analysis

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Energy Efficiency Analysis of Antenna Selection MIMO ARQ Systems over Nakagami-*m* Fading Channels

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Abstract

In this paper, we investigate energy efficiency in antenna selection multi-input multi-output automatic repeat request (MIMO ARQ) wireless systems. We first derive an approximate expression for the average frame-error rate (FER) in antenna selection MIMO systems over quasi-static Nakagami-*m* fading channels. The FER approximation is then used to obtain an analytical expression of an energy-efficiency metric that is defined as the total energy required to successfully deliver one information bit. We prove that this energy-efficiency metric is a quasi-convex function with respect to the average signal-to-noise ratio (SNR) value. Based on this analysis, we obtain the optimal value of the average energy per transmitted data symbol such that the total energy consumption in the system is minimized. Our results show that the energy efficiency in antenna selection MIMO ARQ systems is improved when the number of equipped antennas is increased. Simulation results are provided to validate the analysis.

Index Terms

Antenna selection, energy efficiency, MIMO, automatic repeat request (ARQ), Nakagami-*m* fading channel.

I. INTRODUCTION

Multi-input multi-output (MIMO) has been considered as a key technique to improve system capacity and/or link reliability in wireless communications [1], [2]. In fact, MIMO has been incorporated into many standards, such as WLAN IEEE 802.11n, WiMAX IEEE 802.16e, or LTE (Long-Term Evolution). A main drawback of the use of multiple antennas at transceivers is that additional radio frequency (RF) chains are required, which increases implementation cost and power consumption. To deal with this issue, antenna selection techniques, in which only a small number of antennas among available antennas are selected for transmission, were proposed [3], [4]. Antenna selection is a simple but powerful scheme as it could attain the benefits of the MIMO technique with only a small number of RF chains [4], [5].

Many research works considered antenna selection systems in the literature [3]-[10]. These works investigated various aspects of antenna selection, such as antenna selection criteria (e.g.,

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maximizing ergodic capacity or minimizing error probability) [3], analyzing antenna selection systems under imperfect scenarios (e.g., spatial correlation [6]), or designing low-complexity antenna selection algorithms [7], etc. Antenna selection was also extended to OFDM (orthogonal frequency division multiplexing) systems [8], cross-layer antenna selection systems [9], or cooperative/relay systems [10]. Despite the existing of numerous works on antenna selection, it is noted that the efficacy of antenna selection schemes has only been investigated from either error-performance or capacity perspective. Due to an increasing concern about energy consumption in future wireless networks [11], [12], it is necessary to consider antenna selection systems from an energy efficiency perspective.

Recently, energy-efficient antenna selection systems were studied in [13]-[15]. In [13], the authors jointly optimized the transmit power and the number of selected antennas to maximize the energy efficiency. Antenna selection strategies for improved energy efficiency were also considered in large-scale antenna selection systems [14] and antenna selection OFDM systems [15]. However, in these works, energy efficiency is defined as a ratio between the ergodic capacity and the total consumed power. We note that this energy-efficiency metric does not take into account many important system parameters, such as channel codes, modulation schemes, and detection methods. From a practical viewpoint, it is important to consider energy efficiency which involves those system parameters.

In order to achieve reliable transmission over fading channels, automatic repeat request (ARQ) protocols can be employed at the medium access control (MAC) layer [16]. ARQ protocols for MIMO systems were considered in the literature, see, e.g., [17]-[20]. Also, to have an insight into the energy efficiency in ARQ systems, some research works investigated the trade-off between the energy efficiency and delay, i.e., the energy-delay trade-off (EDT) [21]-[23]. In [21], the authors examined energy efficiency in non-collaborative and collaborative hybrid ARQ (HARQ) scenarios. The minimum total energy consumption required for reliable transmission is determined for various HARQ protocols, including HARQ Type I (HARQ-TI), HARQ Chase Combining (HARQ-CC), and HARQ Incremental Redundancy (HARQ-IR). In [22], the authors analysed the EDT in one-way and two-way relay systems with the HARQ-IR protocol. The EDT trade-off in multiuser systems was also considered in [23]. We note that these works, i.e., [21]-[23], focused only on single-input single-output (SISO) ARQ systems.

In this paper, we investigate the energy efficiency in antenna selection MIMO ARQ systems for the first time. We define an energy-efficiency metric as the total energy required to successfully deliver one information bit, which takes into account the impacts of important system parameters. The main contributions of this work are summarized as follows.

i) An analytical expression that can accurately approximate a frame-error rate (FER) in antenna selection MIMO systems over quasi-static Nakagami-*m* fading channels is derived.

ii) An energy-efficiency metric, which is defined as the total energy required to successfully deliver one information bit, is shown to be quasi-convex with respect to the average signal-to-noise ratio (SNR).

iii) The optimal value of the average energy per transmitted symbol to minimize the energy consumption in the antenna selection MIMO ARQ system is determined, which is important from an energy efficiency viewpoint.

iv) Antenna selection MIMO ARQ systems are shown to outperform SISO ARQ systems from an energy efficiency perspective. Moreover, the energy efficiency in antenna selection MIMO ARQ systems is improved when the number of equipped antennas is increased.

The remainder of the paper is organized as follows. In Section II, a system model for an antenna selection MIMO ARQ system is described. In Section III, we derive an approximation expression for FER over Nakagami-*m* fading channels. In Section IV, we analyze energy efficiency in the antenna selection MIMO ARQ system. Simulation results are provided in Section V. Finally, Section VI concludes the paper.

II. SYSTEM MODEL

We consider an antenna selection MIMO ARQ system with n_T transmit antennas and n_R receive antennas over Nakagami-*m* fading channels as shown in Fig. 1. Each frame of L_f bits, consisting of L_d information bits and $L_{oh} = (L_f - L_d)$ overhead bits, is first encoded by a rate-r_c channel encoder, and then mapped into a *M*-ary quadrature amplitude modulation (*M*-QAM) constellation. At any time instant, only one out of n_T transmit antennas and only one out of n_R receive antennas are selected for data transmission. Note that there is only one transmit RF chain and one receive RF chain in this system. Assuming that the flat fading channel is quasi-static (i.e., fading coefficients remain constant during one frame, and vary from one frame to another), the received signal can be expressed as

$$y = \sqrt{E_r}\, h_{i,j} x + n_j, \tag{1}$$

where x is a modulation symbol drawn from a unit-energy constellation, E_r is the average received energy per symbol, $h_{i,j}$ is the flat Nakagami-*m* fading channel coefficient between the i^{th} transmit antenna and the j^{th} receive antenna with $\varepsilon\{|h_{i,j}|^2\} = 1$, where $\varepsilon\{.\}$ denotes an expectation operator, and n_j is the additive white Gaussian noise at the j^{th} receive antenna with power spectral density (PSD) of N_0.

Denote E_s to be the average transmitted energy per symbol. Note that this energy E_s is the actual average transmitted energy (i.e., after a power amplifier). Then, we have a relation $E_s = E_r G_d$, where G_d is a factor that represents antenna gain, the path-loss, noise figure, etc. This factor can be expressed as $G_d = G_0 d^{\chi} G_M$, where G_0 is the path-loss at a unit distance, d is the transmission distance, χ is the path-loss exponent, and G_M stands for other parameters such as antenna gain, receiver noise figure, and the link margin compensating the variations of hardware process [24]. The average signal-to-noise ratio (SNR) in terms of E_b/N_0 at the receiver can be defined as

$$\bar{\gamma} = \frac{E_b}{N_0} = \frac{E_r}{N_0 r_c \log_2 M} = \frac{E_s}{G_d N_0 r_c \log_2 M}, \tag{2}$$

where $E_b = E_r / (r_c \log_2 M)$ is the average received energy per uncoded bit. Also, the instantaneous SNR between the i^{th} transmit and the j^{th} receive antennas can be expressed as

$$\gamma_{i,j} = \frac{E_s |h_{i,j}|^2}{G_d N_0 r_c \log_2 M}. \tag{3}$$

In this work, we consider a maximum SNR criterion for simplicity. Accordingly, the transmit and receive antennas are selected to maximize the instantaneous SNR value $\gamma_{i,j}$, i.e.

$$(\hat{i}, \hat{j}) = \arg \max_{\substack{i=1,2..,n_T \\ j=1,2..,n_R}} \gamma_{i,j}. \tag{4}$$

Assuming that the channel state information (CSI) is available at the receiver, the receiver can select the optimal transmit and receive antennas based on (4). The transmitter is then informed of the selected transmit antenna index via a low-rate feedback link. Note that the CSI can be obtained by using pilot symbols. In addition, in the system with a time-division duplex (TDD) mode, the transmitter can estimate the CSI due to the channel reciprocity. In such case, there is no need to feedback the selected transmit antenna index.

With respect to the employed ARQ protocol, we consider type-I ARQ in this system. This type of protocol is simple, which is important for low cost and low energy systems. For each transmission round, the receiver decodes data only based on the received signal in that round. If the receiver recovers the data frame successfully, it sends a positive acknowledgement (ACK) to the transmitter. In case the data frame cannot be recovered, it will be discarded, and a negative acknowledgement (NACK) will be sent to the transmitter. The data frame will be retransmitted if the transmitter receives a NACK. In this work, we assume that there is no limit on the number of retransmissions, i.e., delay tolerant systems. As the retransmissions are independent, the average number of transmissions can be obtained as [16]

$$\upsilon = \frac{1}{1 - FER}, \tag{5}$$

where FER is the average frame-error rate. Note that as the fading channel is assumed quasi-static, the selected antennas during the first transmission round and retransmission rounds (if required) of the same information data frame are not necessarily the same.

III. Approximation of Frame-Error Rate in Antenna Selection Systems

In this section, we derive an expression for FER approximation in antenna selection systems. It is well-known that a Nakagami-*m* distribution can model a wide range of fading channel conditions depending on the parameter *m*. Thus, in this work, an expression for FER approximation is derived directly in Nakagami-*m* fading channels. The obtained result will be used to analyze the energy-efficiency in Section IV. As the exact expression for the average FER over quasi-static fading channels is hard to derive, a threshold-based FER approximation approach was considered in the literature, i.e., in turbo codes based single-antenna systems [25], or non-iterative decoded single-antenna systems [26]. In antenna selection systems, we can express the average FER as

$$FER(\bar{\gamma}) = \int_0^{\infty} FER_G(\gamma) p_{AS}(\bar{\gamma},\gamma) d\gamma \approx \int_0^{\gamma_{th}} p_{AS}(\bar{\gamma},\gamma) d\gamma = F_{AS}(\bar{\gamma},\gamma_{th}), \tag{6}$$

where $FER_G(\gamma)$ is the FER over the Gaussian channel, $p_{AS}(\bar{\gamma},\gamma)$ is the probability density function (PDF) of the received SNR in antenna selection systems, γ_{th} is a threshold SNR with an assumption that $FER_G(\gamma \mid \gamma \le \gamma_{th}) \approx 1$ and $FER_G(\gamma \mid \gamma > \gamma_{th}) \approx 0$, and $F_{AS}(.)$ is the cumulative distribution function (CDF) of the received SNR, i.e., $F_{AS}(\bar{\gamma},\gamma_0) = \Pr(\gamma \le \gamma_0) = \int_0^{\gamma_0} p_{AS}(\bar{\gamma},\gamma) d\gamma$.

In Nakagami-*m* fading channels, the PDF and CDF of the received SNR can be expressed, respectively, as $p_{\bar{\gamma}}(\gamma) = (m/\bar{\gamma})^m \gamma^{m-1} e^{-m\gamma/\bar{\gamma}} / \Gamma(m)$, $\gamma \ge 0$, and $F_{\bar{\gamma}}(\gamma) = \psi(m, m\gamma/\bar{\gamma})/\Gamma(m)$, $\gamma \ge 0$, where $\Gamma(x) = \int_0^{+\infty} e^{-t} t^{x-1} dt$ and $\psi(a,x) = \int_0^x e^{-t} t^{a-1} dt$, $\mathrm{Re}\{a\} > 0$, denote the Gamma function and the incomplete Gamma function, respectively [27], [28]. Moreover, when the parameter *m* is an integer value, we can rewrite the CDF of the received SNR as $F_{\bar{\gamma}}(\gamma) = 1 - e^{-m\gamma/\bar{\gamma}} \sum_{k=0}^{m-1} (m\gamma/\bar{\gamma})^k / k!$, $\gamma \ge 0$ [28, Eq.(8.352.6)]. Thus, $p_{AS}(\bar{\gamma},\gamma)$ in antenna selection systems can be calculated by means of order statistics [29] as

$$p_{AS}(\bar{\gamma},\gamma) = n_T n_R \times p_{\bar{\gamma}}(\gamma) \times [F_{\bar{\gamma}}(\gamma)]^{n_T n_R - 1}$$
$$= n_T n_R \left(\frac{m}{\bar{\gamma}}\right)^m \frac{\gamma^{m-1} e^{-m\gamma/\bar{\gamma}}}{\Gamma(m)} \left(1 - e^{-m\gamma/\bar{\gamma}} \sum_{k=0}^{m-1} \frac{(m\gamma/\bar{\gamma})^k}{k!}\right)^{n_T n_R - 1}, \gamma \geq 0. \quad (7)$$

Also, the CDF of the received SNR in antenna selection systems is calculated as

$$F_{AS}(\bar{\gamma},\gamma) = \left(F_{\bar{\gamma}}(\gamma)\right)^{n_T n_R} = \left(1 - e^{-m\gamma/\bar{\gamma}} \sum_{k=0}^{m-1} \frac{(m\gamma/\bar{\gamma})^k}{k!}\right)^{n_T n_R}, \ \gamma \geq 0. \quad (8)$$

By substituting (8) into (6), we obtain the FER approximation as

$$FER(\bar{\gamma}) \approx F_{AS}(\bar{\gamma},\gamma_{th}) = \left(1 - e^{-m\gamma_{th}/\bar{\gamma}} \sum_{k=0}^{m-1} \frac{(m\gamma_{th}/\bar{\gamma})^k}{k!}\right)^{n_T n_R}. \quad (9)$$

Note that (9) can also be obtained by substituting (7) into the integral in (6). When the parameter *m* = 1 (i.e., Rayleigh fading channel), (9) can be simplified to as

$$FER(\bar{\gamma}) \approx \left(1 - e^{-\gamma_{th}/\bar{\gamma}}\right)^{n_T n_R}. \quad (10)$$

We now consider the calculation of the SNR threshold γ_{th} in (9). For diversity systems, the threshold γ_{th} can be obtained by using a criterion proposed in [30]. However, this method only offers a good FER approximation at the high SNR region. It was shown in [31] that SISO ARQ systems achieve the optimal energy efficiency at the low SNR region. Thus, it is important to obtain γ_{th} that could offer an accuracy approximation of FER at the low SNR region. For diversity systems operating over Nakagami-*m* fading channels, a derivation of a mathematical expression of γ_{th} at the low SNR region is hard. Consequently, in this paper, we investigate the following two approaches to obtain γ_{th}.

Least-square (*LS*) *matching* (*i.e., curve fitting*): In this method, the threshold γ_{th} is obtained by matching (i.e., fitting) the FER curve based on the approximation expression in (9) with the simulation FER curve using a least-square criterion.

Minimum sum-error (*MSE*) *criterion*: This criterion was considered to obtain the threshold γ_{th} over Rayleigh fading channels in SISO systems [26], and orthogonal space-time block codes based (OSTBC) MIMO systems [32]. Although this criterion is not necessarily optimal at the low SNR region, it was shown in [26], [30] that it can offer a good FER approximation. Moreover, this method requires less complexity compared to the least-square matching approach. In our antenna selection MIMO system operating in the Nakagami-m fading channels, the optimal SNR threshold γ_{th} is derived as (see Appendix A)

$$\gamma_{th} = \frac{A}{B}\left(\int_0^\infty \frac{1-FER_G(\gamma)}{\gamma^2} d\gamma\right)^{-1}, \tag{11}$$

where $A = \sum_{q=1}^{n_T n_R} (-1)^q C_q^{n_T n_R} \sum_{u=0}^{(m-1)q} \omega_{u,q} u! \big/ q^{u+1}$, $B = (\sum_{q=1}^{n_T n_R} (-1)^q C_q^{n_T n_R} \sum_{u=0}^{(m-1)(q-1)} \omega_{u,q-1} (u+m)! \big/ q^{u+m}) \big/ \Gamma(m)$, C_a^b and $\omega_{u,q}$ denote the binomial coefficient and multinomial coefficient, respectively. We note that it is easy to calculate coefficients A and B. When m = 1 (i.e., Rayleigh fading channels), it is readily observed that $A = B$. Therefore, (11) can be simplified to as

$$\gamma_{th} = \left(\int_0^\infty \frac{1-FER_G(\gamma)}{\gamma^2} d\gamma\right)^{-1}. \tag{12}$$

Note that, $FER_G(\gamma)$, the FER over the Gaussian channel, can be expressed in a closed-form in uncoded systems (see e.g., [26]). For coded system, it can be numerically calculated using Monte-Carlo methods. The accuracy of the two considered approaches, i.e., LS matching and MSE, will be provided in Section V.

IV. Optimal Transmit Energy for Maximum Energy Efficiency

In this section, we focus on the energy-efficiency optimization in antenna selection MIMO ARQ systems. The total energy E required to successfully deliver one information bit is considered as a metric to measure the energy efficiency of the system, i.e.,

$$E = \upsilon E_0, \tag{13}$$

where υ is the average number of transmissions per successful bit (cf. (5)) and E_0 is the energy required to transmit one information bit for each transmission attempt. The energy per information bit E_0 can be expressed via the energy per symbol E_t as

$$E_0 = \frac{L_f}{L_d} \times \frac{E_t}{r_c \log_2 M}, \tag{14}$$

where $E_t = E_s + E_c$ is the energy per symbol, which consists of the actual transmitted energy per symbol E_s and the energy consumed by transceiver circuitry E_c. Note that a factor (L_f / L_d) is included in (14) in order to take into account the energy waste due to the transmission of $L_{oh} = (L_f - L_d)$ overhead bits (i.e., non-information data bits). Also, the circuit energy consumption E_c is given as $E_c = (\xi/\eta - 1)E_s + \varphi/R_s$ [24], [31], where $\xi \approx 3(\sqrt{M} - 1/\sqrt{M} + 1), M \geq 4,$ is the peak-to-average power ratio of M-QAM signals, η is the drain efficiency of power-amplifier (PA), φ is a power consumption constant dependent on the transceiver structure (i.e., baseband processing power consumption and RF power consumption excluding the PA), and R_s is the symbol rate that is related to the information bit rate R_b as $R_b = R_s r_c \log_2 M\, L_d / L_f$. Thus, we can rewrite E_0 in (14) as

$$\begin{aligned} E_0 &= \frac{L_f}{L_d} \times \frac{1}{r_c \log_2 M} \times \left(\frac{\xi}{\eta} E_s + \frac{\varphi}{R_s} \right) \\ &= \alpha \bar{\gamma} + \beta, \end{aligned} \tag{15}$$

where $\bar{\gamma} = E_s / (G_d N_0 r_c \log_2 M)$ is the average SNR value (cf. (2)), $\alpha = (L_f / L_d) \times (\xi / \eta) \times G_d N_0$, and $\beta = \varphi / R_b$. Substituting (5), (9), and (15) into (13) results in

$$E(\bar{\gamma}) = \frac{1}{1 - \left(1 - e^{-m\gamma_{th}/\bar{\gamma}} \sum_{k=0}^{m-1} \frac{(m\gamma_{th}/\bar{\gamma})^k}{k!} \right)^{n_T n_R}} \times (\alpha\bar{\gamma} + \beta). \tag{16}$$

To achieve energy-efficient transmission, the total energy $E(\bar{\gamma})$ in (16) should be as small as possible. For fixed values of n_T, n_R, α and β, we could find the optimal value of $\bar{\gamma}$, denoted as $\bar{\gamma}^{opt}$, so that the total energy $E(\bar{\gamma})$ is minimized. This is based on the following theorem.

Theorem 1: *The energy-efficiency metric $E(\bar{\gamma})$ defined in (16) is a quasi-convex function with respect to the average SNR $\bar{\gamma}$.*

Proof: A proof is provided in Appendix B.

As the energy efficiency $E(\bar{\gamma})$ is quasi-convex, it has a unique minimum value. Due to the complexity of the function $E(\bar{\gamma})$, the closed-form expression for $\bar{\gamma}^{opt}$ is difficult to derive. However, we note that it is easy to obtain $\bar{\gamma}^{opt}$ via numerical optimization, e.g., using the Newton's method [33].

Corollary 1: *The optimal value $\bar{\gamma}^{opt}$ that minimizes $E(\bar{\gamma})$ is the root of the following equation:*

$$\partial E(\bar{\gamma}) / \partial \bar{\gamma} = 0, \tag{17}$$

or, equivalently,

$$\begin{aligned} &\alpha \left(1 - \left(1 - e^{-m\gamma_{th}/\bar{\gamma}} \sum_{k=0}^{m-1} \frac{(m\gamma_{th}/\bar{\gamma})^k}{k!} \right)^{n_T n_R} \right) - \\ &\qquad (\alpha\bar{\gamma} + \beta) n_T n_R \frac{(m\gamma_{th})^m}{(m-1)!} \left(\frac{1}{\bar{\gamma}^{m+1}} \right) e^{-m\gamma_{th}/\bar{\gamma}} \left(1 - e^{-m\gamma_{th}/\bar{\gamma}} \sum_{k=0}^{m-1} \frac{(m\gamma_{th}/\bar{\gamma})^k}{k!} \right)^{n_T n_R - 1} = 0. \end{aligned} \tag{18}$$

Proof: The proof is obtained directly from Theorem 1 due to the quasi-convexity of $E(\bar{\gamma})$.

Note that given a particular value of the parameter *m*, it is easy to solve (18) numerically using Newton's method. Once the optimal value $\bar{\gamma}^{opt}$ is obtained, we can get the optimal average transmitted energy per symbol E_s^{opt} using Eq. (2), i.e.,

$$E_s^{opt} = \bar{\gamma}^{opt} r_c G_d N_0 \log_2 M. \tag{19}$$

So far, we have derived the threshold-based FER approximation expression in Eq. (9), where the SNR threshold γ_{th} can be obtained by using either Eq. (11) or the curve-fitting approach. Based on Eq. (9), we have formulated the analytical expression that calculates the total energy consumption $E(\bar{\gamma})$ required to successfully deliver one information bit in Eq. (16). The optimal value $\bar{\gamma}^{opt}$ to minimize $E(\bar{\gamma})$ is determined based on *Theorem* 1 and *Corollary* 1. To realize the optimal value $\bar{\gamma}^{opt}$, the optimal average transmitted energy per symbol follows Eq. (19). In addition, the minimum energy consumption $E^{opt}(\bar{\gamma})$ is calculated by substituting $\bar{\gamma}^{opt}$ into $E(\bar{\gamma})$ in Eq. (16). Numerical and simulation results that corroborate these analyses are provided in the next section.

V. Simulation Results

In our simulations, we use the following parameters, which follows those in [24], [31]. In particular, frame length L_f = 1000 bits, overhead bits L_{oh} = 48 bits, bit rate R_b = 300 kbps, PA's efficiency η = 0.35, circuit power consumption φ = 310 mW, PSD of noise $N_0/2$ = - 174 dBm/Hz, and G_0 = 30 dB, G_M = 40 dB, path-loss exponent χ = 3.5, transmission distance d = 100 m. Also, a 4-QAM modulation and a convolutional code with a rate r_c = 1/2, generator polynomial $[5,7]_8$ and soft Viterbi decoding are adopted. We assume that the feedback link is zero-delay and error-free.

Figure 2 plots the frame-error rates (FER) of the antenna selection systems over Nakagami-*m* channels. It can be seen that the analytical curves based on the FER approximation expression in (9) agree well with the simulation curves in the low SNR region in all scenarios. Also, the analytical curve based on the MSE criterion is very close to that with the least-square matching approach. Therefore, using the simple FER approximation expression in (9) facilitates the analysis of energy efficiency in the antenna selection MIMO ARQ systems.

The energy consumption per information bit $E(\bar{\gamma})$ versus the average SNR is shown in Fig.3. First, it can be seen that the antenna selection MIMO ARQ system outperforms the SISO ARQ system (i.e., $n_T \times n_R$ = 1) from an energy efficiency perspective. The energy saving of the antenna selection system over the SISO system comes from diversity gain offered by the use of multiple antennas. In fact, a larger diversity gain leads to a lower FER, which in turn reduces the number of transmissions υ (cf. (5)). On the other hand, as only one RF chain is equipped at the transmitter and receiver in all systems, the energy E_0 in (13) is constant regardless of how many antennas are equipped. Consequently, the total energy $E = \upsilon E_0$ defined (13) is reduced. Second, the optimal average SNR values $\bar{\gamma}^{opt}$ (marked as '×' in the figure) calculated using Newton's method match exactly with the analytical curves. Third, the optimal SNR value $\bar{\gamma}^{opt}$ and optimal (minimum) energy $E^{opt}(\bar{\gamma})$ are reduced when the number of equipped antennas is increased. This behavior can be explained by an additional diversity gain that is achieved when increasing the number of antennas as mentioned above. Similar observations can be made in the Nakagami-m channels with $m = 2$ as shown in Fig. 4.

It is also worth noting that the optimal SNR value $\bar{\gamma}^{opt}$ results from the energy-delay trade-off (EDT) in the systems. Specifically, when $\bar{\gamma} < \bar{\gamma}^{opt}$, the frame-error rate (FER) is high. Therefore, a large number of retransmissions is required to guarantee reliable transmission. As a result, the energy consumption is high (i.e., $E(\bar{\gamma})$ is dominated by retransmissions in this case). When $\bar{\gamma} > \bar{\gamma}^{opt}$, FER is small enough, and thus the impact of retransmissions is negligible. Also, the energy consumption $E(\bar{\gamma})$ increases when the average SNR value $\bar{\gamma}$ increases (cf. (16)). Consequently, it is important to select the optimal operating point (i.e., obtain $\bar{\gamma}^{opt}$) for energy saving.

Fig. 5 shows the minimum energy consumption required to successfully deliver one information bit, i.e., $E(\bar{\gamma}^{opt})$, versus the transmission distance $d(m)$ in different systems. It can be seen that when the distance d increases (i.e., the path-loss is increasing), the energy consumption $E(\bar{\gamma}^{opt})$ increases. However, it is worth noting that the antenna selection MIMO ARQ system requires lower energy consumption compared to the SISO ARQ system for all values of the distance d. For example, at $d = 150m$, the values of $E(\bar{\gamma}^{opt})$ in the antenna selection MIMO ARQ system with $n_T = 2$, $n_R = 1$ and the SISO ARQ system are 2.77×10^{-4} Joule and 4.63×10^{-4} Joule, respectively. Thus, an energy saving of about 40% can be achieved by the antenna selection system.

Finally, we plot in Fig. 6 and Fig. 7 the energy consumption $E(\bar{\gamma})$ versus the average SNR $\bar{\gamma}$ under different values of L_f, L_{oh}, and R_b. These results demonstrate the impact of those system parameters on the energy efficiency $E(\bar{\gamma})$ and the optimal SNR value $\bar{\gamma}^{opt}$. For instance, it can be seen from Fig. 6 that a larger frame length L_f results in a higher value $\bar{\gamma}^{opt}$. It is also worth mentioning that in this paper, we focus on performing a mathematical analysis of the energy efficiency $E(\bar{\gamma})$ with respect to (w.r.t.) the average SNR value $\bar{\gamma}$. To this end, we optimize $E(\bar{\gamma})$ w.r.t. $\bar{\gamma}$, while assuming that the other parameters, such as L_f and R_b, are fixed. However, it would be possible to perform a joint optimization approach which optimizes the SNR value $\bar{\gamma}$ and other parameters simultaneously for improved energy efficiency. Analysis on these matters is beyond the scope of this work.

VI. Conclusions

In this paper, an antenna selection MIMO ARQ system has been investigated from an energy efficiency perspective. An analytical expression that can accurately approximate the FER over quasi-static Nakagami-m fading channels has been derived. Moreover, we have proved that the energy efficiency is a quasi-convex function with respect to the average SNR value. Thus, we have obtained the optimal value of the average energy of the transmitted symbols so that the energy consumption in the antenna selection MIMO ARQ system is minimized. In addition, it has been shown analytically and numerically that the antenna selection MIMO ARQ system offers a significant improvement in energy efficiency, compared to the SISO ARQ system. Our future work includes considering the system with adaptive modulation as well as with other ARQ mechanisms.

Appendix A

Derivation of the SNR Threshold γ_{th}

In this appendix, we derive the optimal SNR threshold γ_{th} for antenna selection systems over Nakagami-m fading channels. According to the minimum sum-error criterion [26], the optimal SNR value γ_{th} is the one that satisfies the following condition (cf. (6))

$$\lim_{\Lambda\to\infty}\left(\int_0^{\Lambda} F_{AS}(\lambda,\gamma_{th})d\lambda - \int_0^{\Lambda} FER(\lambda)d\lambda\right) = 0, \tag{20}$$

where $\lambda = 1/\bar{\gamma}$. The first integral in (20) can be expressed as

$$\begin{aligned}\int_0^{\Lambda} F_{AS}(\lambda,\gamma_{th})d\lambda &= \int_0^{\Lambda}\left(1 - e^{-\lambda m\gamma_{th}}\sum_{k=0}^{m-1}\frac{(\lambda m\gamma_{th})^k}{k!}\right)^{n_T n_R} d\lambda \\ &= \int_0^{\Lambda}\left(\sum_{q=0}^{n_T n_R} C_q^{n_T n_R}(-1)^q e^{-q\lambda m\gamma_{th}}\left(\sum_{k=0}^{m-1}\frac{(\lambda m\gamma_{th})^k}{k!}\right)^q\right)d\lambda,\end{aligned} \tag{21}$$

where $C_a^b = b!/a!(b-a)!$ is the binomial coefficient. By performing a multinomial expansion of $(\sum_{k=0}^{m-1} x^k/k!)^q = \sum_{u=0}^{(m-1)q}\omega_{u,q}x^u$, where a coefficient $\omega_{u,q}$ is the u^{th} element of a vector $\boldsymbol{\omega}_q$ that is defined as $\boldsymbol{\omega}_0 = 1$, $\boldsymbol{\omega}_1 = [1\ \ 1/1!\ \ 1/2!\ \ldots.\ \ 1/(m-1)!]$, and $\boldsymbol{\omega}_q = \boldsymbol{\omega}_{q-1} \otimes \boldsymbol{\omega}_1$, where $\otimes$ denotes a discrete convolution [34], we can rewrite (21) as

$$\int_0^{\Lambda} F_{AS}(\lambda,\gamma_{th}) d\lambda = \sum_{q=0}^{n_T n_R} \left((-1)^q C_q^{n_T n_R} \sum_{u=0}^{(m-1)q} \left(\omega_{u,q} \int_0^{\Lambda} (\lambda m \gamma_{th})^u e^{-q\lambda m \gamma_{th}} d\lambda \right) \right)$$
$$= \Lambda + \sum_{q=1}^{n_T n_R} \left((-1)^q C_q^{n_T n_R} \sum_{u=0}^{(m-1)q} \left(\omega_{u,q} \frac{1}{m\gamma_{th}} \left(\frac{u!}{q^{u+1}} - e^{-q\Lambda/(m\gamma_{th})} \sum_{s=0}^{u} \frac{u!}{s!} \times \frac{(\Lambda/(m\gamma_{th}))^s}{q^{u-s+1}} \right) \right) \right), \quad (22)$$

where, in the last equality, we use the integral of $\int_0^a x^n e^{-bx} dx = n!/b^{n+1} - e^{-ab} \sum_{s=0}^{n} \left(n! a^s / s! b^{n-s+1} \right)$ [28, Eq. (3.351.1)].

The second integral in (20) can be expressed as

$$\int_0^{\Lambda} FER(\lambda) d\lambda = \int_0^{\Lambda}\int_0^{\infty} FER_G(\gamma) p_{AS}(\lambda,\gamma)\, d\gamma d\lambda$$
$$= \int_0^{\Lambda}\int_0^{\infty} p_{AS}(\lambda,\gamma) d\gamma d\lambda - \int_0^{\Lambda}\int_0^{\infty} \left(1 - FER_G(\gamma)\right) p_{AS}(\lambda,\gamma)\, d\gamma d\lambda$$
$$= \Lambda - \int_0^{\infty} \left(\left(1 - FER_G(\gamma)\right) \int_0^{\Lambda} p_{AS}(\lambda,\gamma)\, d\lambda \right) d\gamma, \quad (23)$$

where the first term in the second equality is evaluated based on the fact that $\int_0^{\infty} p_{AS}(\lambda,\gamma) d\gamma = 1$. By performing similar calculations as done to obtain (22), we can express the inner integral in the second term in (23) as follows (cf. (7))

$$\int_0^{\Lambda} p_{AS}(\lambda,\gamma) d\lambda = \int_0^{\Lambda} n_T n_R (m\lambda)^m \frac{\gamma^{m-1} e^{-m\lambda\gamma}}{\Gamma(m)} \left(1 - e^{-m\lambda\gamma} \sum_{k=0}^{m-1} \frac{(m\lambda\gamma)^k}{k!} \right)^{n_T n_R - 1} d\lambda$$
$$= \int_0^{\Lambda} \left(n_T n_R (m\lambda)^m \frac{\gamma^{m-1} e^{-m\lambda\gamma}}{\Gamma(m)} \sum_{q=0}^{n_T n_R - 1} \left((-1)^q C_q^{n_T n_R - 1} e^{-qm\lambda\gamma} \sum_{u=0}^{(m-1)q} \omega_{u,q} (m\lambda\gamma)^u \right) \right) d\lambda$$
$$= \frac{n_T n_R}{\Gamma(m)} \sum_{q=0}^{n_T n_R - 1} \left((-1)^q C_q^{n_T n_R - 1} \sum_{u=0}^{(m-1)q} \omega_{u,q} m^{u+m} \gamma^{u+m-1} \int_0^{\Lambda} \lambda^{u+m} e^{-(q+1)m\lambda\gamma} d\lambda \right)$$
$$= \frac{n_T n_R}{\Gamma(m)} \sum_{q=0}^{n_T n_R - 1} \left((-1)^q C_q^{n_T n_R - 1} \sum_{u=0}^{(m-1)q} \omega_{u,q} \left(\frac{(u+m)!}{m(q+1)^{u+m+1}\gamma^2} - \right.\right.$$
$$\left.\left. m^{u+m} \gamma^{u+m-1} e^{-\Lambda(q+1)m\gamma} \sum_{s=0}^{u+m} \frac{(u+m)!}{s!} \times \frac{\Lambda^s}{\left((q+1)m\gamma\right)^{u+m-s+1}} \right) \right). \quad (24)$$

By substituting (22), (23) and (24) into (20), and noting that all terms containing $e^{-\Lambda}$ are eliminated as $e^{-\Lambda} \to 0$ when $\Lambda \to \infty$, we arrive at

$$\sum_{q=1}^{n_T n_R}\left((-1)^q C_q^{n_T n_R}\sum_{u=0}^{(m-1)q}\left(\omega_{u,q}\frac{1}{m\gamma_{th}}\times\frac{u!}{q^{u+1}}\right)\right)+$$
$$\int_0^\infty\left(\left(1-FER_G(\gamma)\right)\frac{n_T n_R}{\Gamma(m)}\sum_{q=0}^{n_T n_R-1}\left((-1)^q C_q^{n_T n_R-1}\sum_{u=0}^{(m-1)q}\omega_{u,q}\left(\frac{(u+m)!}{m(q+1)^{u+m+1}\gamma^2}\right)\right)\right)d\gamma=0, \tag{25}$$

or, equivalently,

$$\frac{1}{\gamma_{th}}\sum_{q=1}^{n_T n_R}\left((-1)^q C_q^{n_T n_R}\sum_{u=0}^{(m-1)q}\omega_{u,q}\frac{u!}{q^{u+1}}\right)=$$
$$-\frac{n_T n_R}{\Gamma(m)}\sum_{q=0}^{n_T n_R-1}\left((-1)^q C_q^{n_T n_R-1}\sum_{u=0}^{(m-1)q}\omega_{u,q}\frac{(u+m)!}{(q+1)^{u+m+1}}\right)\int_0^\infty\frac{1-FER_G(\gamma)}{\gamma^2}d\gamma. \tag{26}$$

For notational convenience, let us denote

$$A=\sum_{q=1}^{n_T n_R}\left((-1)^q C_q^{n_T n_R}\sum_{u=0}^{(m-1)q}\omega_{u,q}\frac{u!}{q^{u+1}}\right), \tag{27}$$

and

$$\begin{aligned}B&=-\frac{n_T n_R}{\Gamma(m)}\sum_{q=0}^{n_T n_R-1}\left((-1)^q C_q^{n_T n_R-1}\sum_{u=0}^{(m-1)q}\omega_{u,q}\frac{(u+m)!}{(q+1)^{u+m+1}}\right)\\&=\frac{1}{\Gamma(m)}\sum_{q=1}^{n_T n_R}\left((-1)^q C_q^{n_T n_R}\sum_{u=0}^{(m-1)(q-1)}\omega_{u,q-1}\frac{(u+m)!}{q^{u+m}}\right),\end{aligned} \tag{28}$$

where the equality in (28) is obtained by changing a variable (q+1) by q, and use a binomial identity of $C_q^{n_T n_R}=(n_T n_R/q)C_{q-1}^{n_T n_R-1}$. From (26), we can express the optimal SNR threshold as

$$\gamma_{th}=\frac{A}{B}\left(\int_0^\infty\frac{1-FER_G(\gamma)}{\gamma^2}d\gamma\right)^{-1}. \tag{29}$$

APPENDIX B

PROOF OF THEOREM 1

As the energy efficiency $E(\bar{\gamma})>0$, to prove that $E(\bar{\gamma})$ is a quasi-convex function w.r.t. $\bar{\gamma}$, we will show that its reciprocal $1/E(\bar{\gamma})$ is a quasi-concave function w.r.t. $\bar{\gamma}$. It is well-established that if $f(x)$ is a sigmoid function (or S-shaped, i.e., it is initially convex and then concave), then $f(x)/x$ is a quasi-concave function [35]. Therefore, in what follows, we will show that

$$f(\bar{\gamma})=1-\left(1-e^{-m\gamma_{th}/\bar{\gamma}}\sum_{k=0}^{m-1}\frac{(m\gamma_{th}/\bar{\gamma})^k}{k!}\right)^{n_T n_R} \tag{30}$$

is a sigmoid function for all $\bar{\gamma} > 0$. It then follows that $E(\bar{\gamma})$ is a quasi-convex function.

Let us denote $g(\bar{\gamma}) = e^{-m\gamma_{th}/\bar{\gamma}} \sum_{k=0}^{m-1} (m\gamma_{th}/\bar{\gamma})^k / k!$, then (30) can be rewritten as

$$f(\bar{\gamma}) = 1 - (1 - g(\bar{\gamma}))^{n_T n_R}. \tag{31}$$

The first-order and second-order derivatives of $g(\bar{\gamma})$ with respect to $\bar{\gamma}$ are calculated, respectively, as

$$g'(\bar{\gamma}) = \frac{(m\gamma_{th})^m}{(m-1)!} \left(\frac{1}{\bar{\gamma}^{m+1}} \right) e^{-m\gamma_{th}/\bar{\gamma}}, \tag{32}$$

and

$$g''(\bar{\gamma}) = \frac{(m\gamma_{th})^m}{(m-1)!} \left(\frac{m\gamma_{th} - (m+1)\bar{\gamma}}{\bar{\gamma}^{m+3}} \right) e^{-m\gamma_{th}/\bar{\gamma}}. \tag{33}$$

As $g'(\bar{\gamma}) > 0, \forall \bar{\gamma} > 0$, the function $g(\bar{\gamma})$ is increasing over its domain. Also, we have $g''(\bar{\gamma}) > 0$ when $\bar{\gamma} < m\gamma_{th}/(m+1)$, and $g''(\bar{\gamma}) < 0$ when $\bar{\gamma} > m\gamma_{th}/(m+1)$. Thus, $g(\bar{\gamma})$ is initially convex and then concave with the inflection point of $\bar{\gamma}_0 = m\gamma_{th}/(m+1)$, i.e., $g(\bar{\gamma})$ is a sigmoid function.

We now prove that $f(\bar{\gamma}) = 1 - (1 - g(\bar{\gamma}))^{n_T n_R}$ is also a sigmoid function w.r.t. $\bar{\gamma}$. We note that it is very hard, if not impossible, to obtain an explicit solution of $f''(\bar{\gamma}) = 0$. Therefore, in what follows, we will show that $f(\bar{\gamma})$ satisfies all the properties of a sigmoid function described in [35].

1) It is clear that its domain is the interval [0, ∞).

2) We have $f'(\bar{\gamma}) = n_T n_R g'(\bar{\gamma})(1 - g(\bar{\gamma}))^{n_T n_R - 1} > 0$ because $g'(\bar{\gamma}) > 0, \forall \bar{\gamma} \in [0, \infty)$ (cf. (32)). Thus, $f(\bar{\gamma})$ is increasing.

3) We have $\lim_{\bar{\gamma} \to 0} g(\bar{\gamma}) = 0$, $\lim_{\bar{\gamma} \to +\infty} g(\bar{\gamma}) = 1$, and $g(\bar{\gamma})$ is increasing. Thus, the range of $g(\bar{\gamma})$ is the interval [0,1). As $f(\bar{\gamma})$ is increasing, it is readily from (31) that the range of $f(\bar{\gamma})$ is also [0,1).

4) ("Eventually concavity") The second-order derivative of $f(\bar{\gamma})$ is calculated as

$$f''(\bar{\gamma}) = n_T n_R [g''(\bar{\gamma})(1 - g(\bar{\gamma})) - (n_T n_R - 1)(g'(\bar{\gamma}))^2](1 - g(\bar{\gamma}))^{n_T n_R - 2}. \tag{34}$$

Recall that $g''(\bar{\gamma}) < 0$ when $\bar{\gamma} > \bar{\gamma}_0$, and the range of $g(\bar{\gamma})$ is the interval [0,1). Thus, it is clear from (34) that $f''(\bar{\gamma}) < 0$ when $\bar{\gamma} > \bar{\gamma}_0$. In other words, in the interval $(\bar{\gamma}_0, \infty)$, $f(\bar{\gamma})$ is concave (i.e., eventually concave).

5) ("Initially convexity") We first note that

$$\lim_{\bar{\gamma}\to 0} f'(\bar{\gamma}) = \lim_{\bar{\gamma}\to 0}\left\{n_T n_R g'(\bar{\gamma})\left(1-g(\bar{\gamma})\right)^{n_T n_R -1}\right\}$$
$$= \lim_{\bar{\gamma}\to 0}\left\{n_T n_R \frac{(m\gamma_{th})^m}{(m-1)!}\left(\frac{1}{\bar{\gamma}^{m+1}}\right)e^{-m\gamma_{th}/\bar{\gamma}}\left(1-e^{-m\gamma_{th}/\bar{\gamma}}\sum_{k=0}^{m-1}\frac{(m\gamma_{th}/\bar{\gamma})^k}{k!}\right)^{n_T n_R -1}\right\} = 0. \qquad (35)$$

As mentioned earlier, $f'(\bar{\gamma}) > 0$. Thus, $f'(\bar{\gamma}_0) = \theta, \theta > 0$. By applying the mean value theorem, we have $f''(\bar{\gamma}_c) = \left(f'(\bar{\gamma}_0) - f'(0)\right)/(\bar{\gamma}_0 - 0) = \theta/\bar{\gamma}_0 > 0$ at some point $\bar{\gamma}_c \in (0, \bar{\gamma}_0)$. Recall that $f''(\bar{\gamma}) < 0, \forall \bar{\gamma} \in (\bar{\gamma}_0, +\infty)$. This implies that when $\bar{\gamma}$ decreases from $\bar{\gamma}_0$ toward zero, $f''(\bar{\gamma})$ increases from a negative value to a positive value. Moreover, it can be shown that $f''(\bar{\gamma}) = 0$ has a unique solution, denoted as $\bar{\gamma}_z$, in the interval $(0, \bar{\gamma}_0)$ (see Appendix C). Therefore, $f''(\bar{\gamma}) > 0, \forall \bar{\gamma} < \bar{\gamma}_z$, and $f''(\bar{\gamma}) < 0, \forall \bar{\gamma} > \bar{\gamma}_z$. In other words, the function $f(\bar{\gamma})$ is convex in $(0, \bar{\gamma}_z)$ and concave in $(\bar{\gamma}_z, \infty)$, with the unique inflection point of $\bar{\gamma}_z$.

6) It is readily that $f(\bar{\gamma})$ has a continuous derivative.

This completes the proof.

APPENDIX C

PROOF THAT $f''(\bar{\gamma}) = 0$ HAS A UNIQUE SOLUTION

It is noted from (34) that $f''(\bar{\gamma}) = 0$ is equivalent to

$$g''(\bar{\gamma})(1-g(\bar{\gamma})) - (n_T n_R - 1)(g'(\bar{\gamma}))^2 = 0. \qquad (36)$$

By using (32) and (33), we can explicitly express the left hand side in (36) as

$$g''(\bar{\gamma})(1-g(\bar{\gamma})) - (n_T n_R - 1)(g'(\bar{\gamma}))^2 =$$
$$\frac{(m\gamma_{th})^m}{(m-1)!}\left(\frac{m\gamma_{th} - (m+1)\bar{\gamma}}{\bar{\gamma}^{m+3}}\right)e^{-m\gamma_{th}/\bar{\gamma}}\left(1-e^{-m\gamma_{th}/\bar{\gamma}}\sum_{k=0}^{m-1}\frac{(m\gamma_{th}/\bar{\gamma})^k}{k!}\right) - (n_T n_R - 1)\left(\frac{(m\gamma_{th})^m}{(m-1)!\bar{\gamma}^{m+1}}e^{-m\gamma_{th}/\bar{\gamma}}\right)^2. \qquad (37)$$

Thus, the equation $f''(\bar{\gamma}) = 0$ is now equivalent to

$$\left(m\gamma_{th} - (m+1)\bar{\gamma}\right)\left(1-e^{-m\gamma_{th}/\bar{\gamma}}\sum_{k=0}^{m-1}\frac{(m\gamma_{th}/\bar{\gamma})^k}{k!}\right) - (n_T n_R - 1)\frac{(m\gamma_{th})^m}{(m-1)!\bar{\gamma}^{m-1}}e^{-m\gamma_{th}/\bar{\gamma}} = 0, \qquad (38)$$

or

$$h(\bar{\gamma}) := m\gamma_{th} - (m+1)\bar{\gamma} + \left[\left(-m\gamma_{th} + (m+1)\bar{\gamma}\right)\sum_{k=0}^{m-1}\frac{(m\gamma_{th}/\bar{\gamma})^k}{k!} - (n_T n_R - 1)\frac{(m\gamma_{th})^m}{(m-1)!\bar{\gamma}^{m-1}}\right]e^{-m\gamma_{th}/\bar{\gamma}} = 0. \qquad (39)$$

It is straightforward to show that

$$\lim_{\bar{\gamma}\to 0} h(\bar{\gamma}) = m\gamma_{th} > 0, \tag{40}$$

and

$$h(\bar{\gamma}_0) = -(n_T n_R - 1)\frac{m\gamma_{th}}{(m-1)!}(m+1)^{m-1}e^{-m-1} < 0, \tag{41}$$

where $\bar{\gamma}_0 = m\gamma_{th}/(m+1)$ is the inflection point of $g(\bar{\gamma})$ as mentioned in Appendix B. Therefore, to prove that $f''(\bar{\gamma}) = 0$ has a unique solution in the interval $(0, \bar{\gamma}_0)$, we only need to show that $h'(\bar{\gamma}) < 0, \forall \bar{\gamma} \in (0, \bar{\gamma}_0)$ (i.e., $h(\bar{\gamma})$ is strictly decreasing).

The first-order derivative of $h(\bar{\gamma})$ is calculated as

$$\begin{aligned} h'(\bar{\gamma}) = -(m+1) + \Bigg[& \left(-m\gamma_{th} + (m+1)\bar{\gamma}\right)\frac{m\gamma_{th}}{\bar{\gamma}^2}\frac{(m\gamma_{th}/\bar{\gamma})^{m-1}}{(m-1)!} - (n_T n_R - 1)\frac{(m\gamma_{th})^{m+1}}{(m-1)!\,\bar{\gamma}^{m+1}} + \\ & (m+1)\sum_{k=0}^{m-1}\frac{(m\gamma_{th}/\bar{\gamma})^k}{k!} + (n_T n_R - 1)(m-1)\frac{(m\gamma_{th})^m}{(m-1)!\,\bar{\gamma}^m}\Bigg] e^{-m\gamma_{th}/\bar{\gamma}} \\ = -e^{-m\gamma_{th}/\bar{\gamma}}\Bigg[& (m+1)e^{m\gamma_{th}/\bar{\gamma}} - (m+1)\sum_{k=0}^{m-1}\frac{(m\gamma_{th}/\bar{\gamma})^k}{k!} + \\ & n_T n_R \frac{(m\gamma_{th}/\bar{\gamma})^{m+1}}{(m-1)!} - \left((n_T n_R - 1)(m-1) + m + 1\right)\frac{(m\gamma_{th}/\bar{\gamma})^m}{(m-1)!}\Bigg]. \end{aligned} \tag{42}$$

We note that the Maclaurin series of the function $e^{m\gamma_{th}/\bar{\gamma}}$ is expressed as $e^{m\gamma_{th}/\bar{\gamma}} = \sum_{k=0}^{+\infty}(m\gamma_{th}/\bar{\gamma})^k/k!$ [28, Eq. (0.318.2)]. Thus, it is clear that

$$(m+1)e^{m\gamma_{th}/\bar{\gamma}} - (m+1)\sum_{k=0}^{m-1}\frac{(m\gamma_{th}/\bar{\gamma})^k}{k!} = (m+1)\sum_{k=m}^{+\infty}\frac{(m\gamma_{th}/\bar{\gamma})^k}{k!} > 0, \forall \bar{\gamma} > 0. \tag{43}$$

On the other hand, as $\bar{\gamma} \in (0, \bar{\gamma}_0)$, we have $m\gamma_{th}/\bar{\gamma} > m+1$. Therefore, it is readily that

$$n_T n_R \frac{(m\gamma_{th}/\bar{\gamma})^{m+1}}{(m-1)!} - \left((n_T n_R - 1)(m-1) + m + 1\right)\frac{(m\gamma_{th}/\bar{\gamma})^m}{(m-1)!} > 2(n_T n_R - 1)\frac{(m\gamma_{th}/\bar{\gamma})^m}{(m-1)!} > 0. \tag{44}$$

By combining (42), (43) and (44), we obtain the desired result of $h'(\bar{\gamma}) < 0, \forall \bar{\gamma} \in (0, \bar{\gamma}_0)$. Therefore, $f''(\bar{\gamma}) = 0$ has a unique solution in the interval $(0, \bar{\gamma}_0)$. Also, recall that $f''(\bar{\gamma}) < 0, \forall \bar{\gamma} \in (\bar{\gamma}_0, +\infty)$. Consequently, $f''(\bar{\gamma}) = 0$ has a unique solution for all $\bar{\gamma} \in (0, +\infty)$.

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Fig. 1. Block diagram of an antenna selection MIMO ARQ wireless system.

Fig. 2. Comparison of the simulated FER and approximated FER.

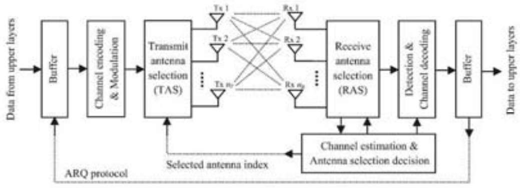
Fig. 3. Energy consumption per information bit $E(\bar{\gamma})$ versus the average SNR $\bar{\gamma}$, (m = 1).

Fig. 4. Energy consumption per information bit $E(\bar{\gamma})$ versus the average SNR $\bar{\gamma}$, (m = 2).

Fig. 5. Minimum energy consumption per information bit versus the transmission distance d (m).

Fig. 6. Energy consumption $E(\bar{\gamma})$ versus the average SNR $\bar{\gamma}$ under different values of L_f and R_b. (L_{oh} = 48 bits, n_T = 2, n_R = 1, m = 1).

Fig. 7. Energy consumption $E(\bar{\gamma})$ versus the average SNR $\bar{\gamma}$ under different values of L_{oh} and R_b. (L_f = 1000 bits, n_T = 2, n_R = 1, m = 1).



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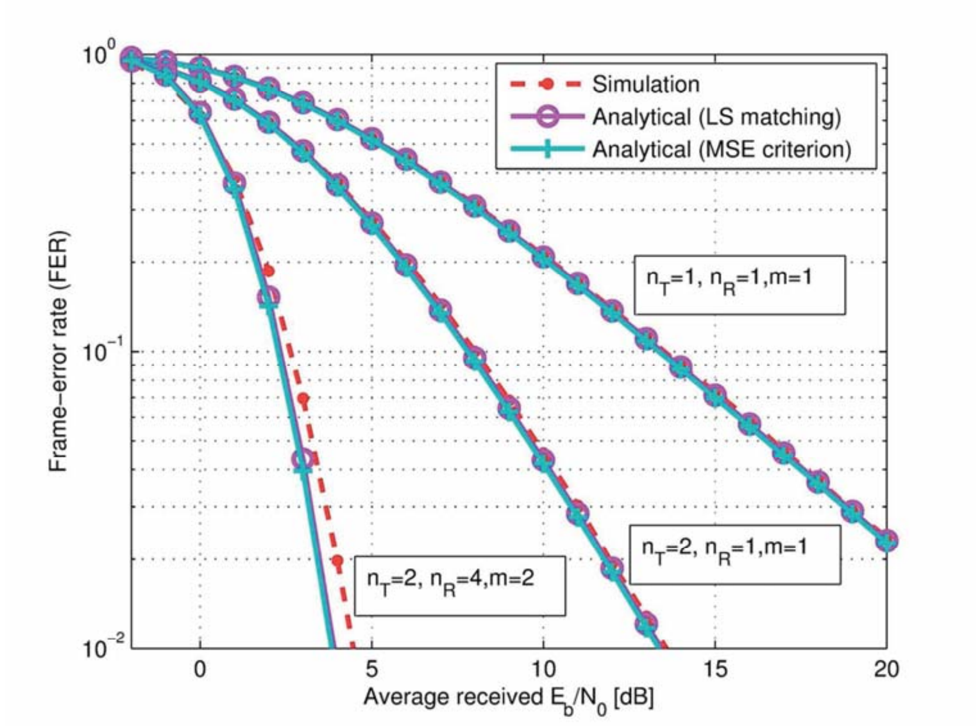

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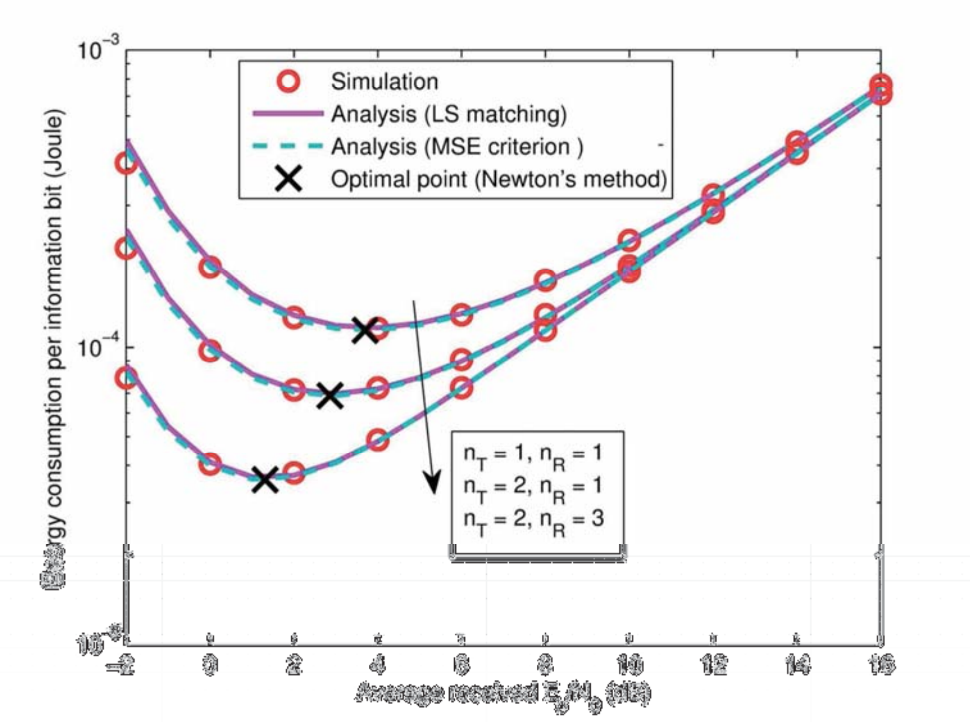

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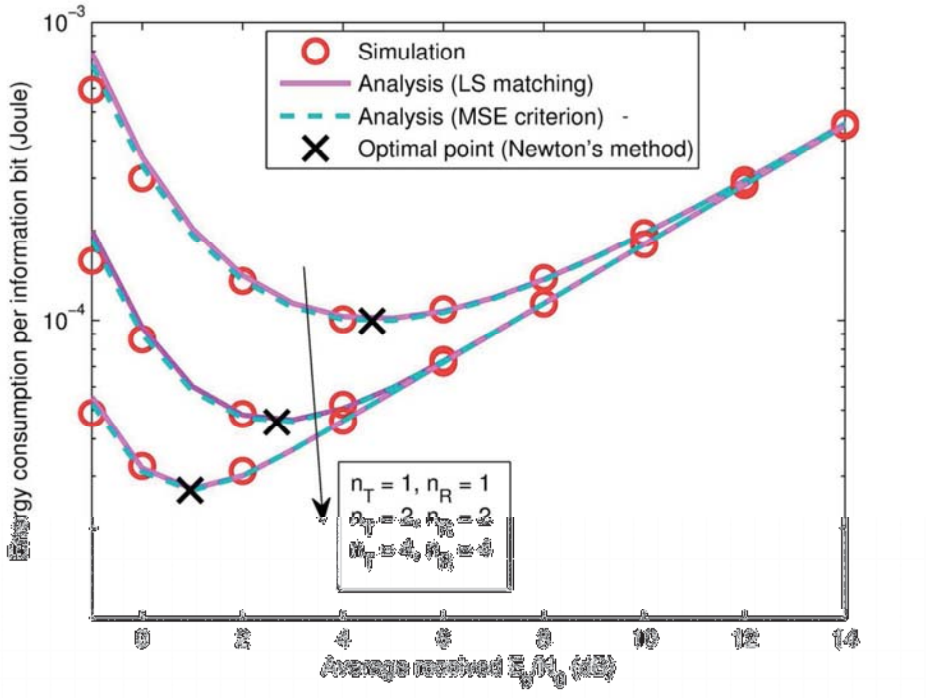

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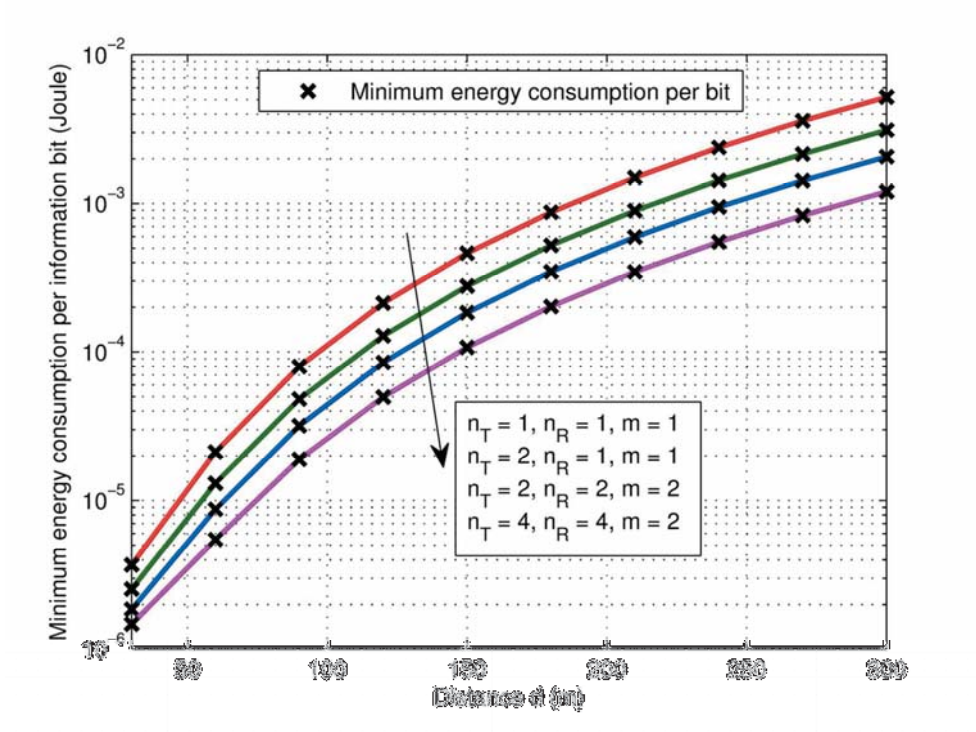

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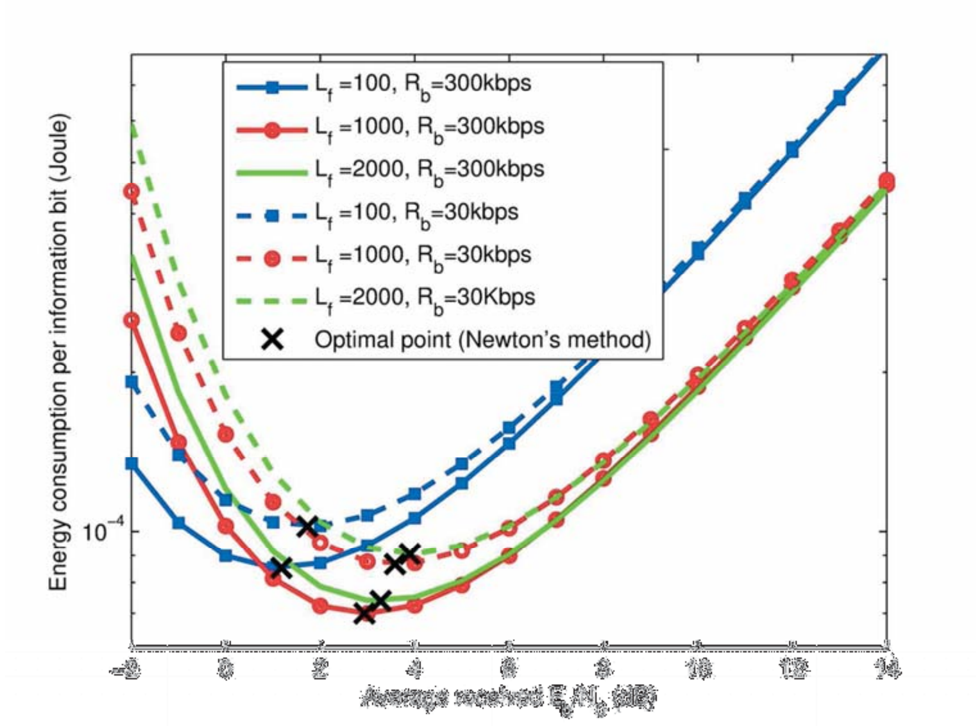

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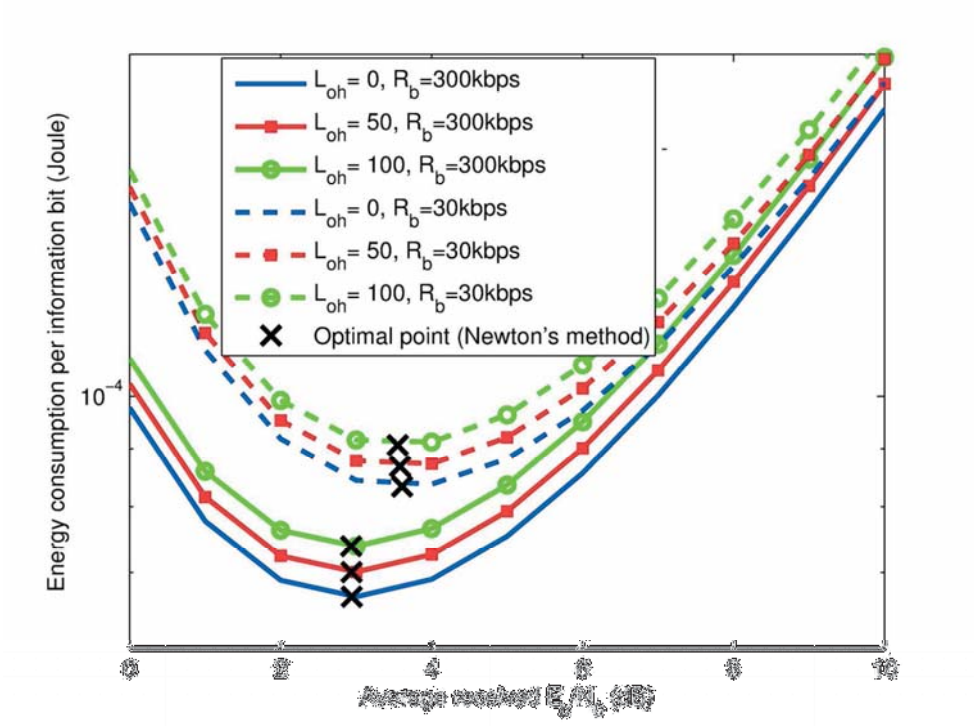

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