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## Abstract

In this paper, we consider a novel optimal design for per-subcarrier antenna selection multi-input multi-output orthogonal frequency division multiplexing systems from an energy-efficiency perspective. The optimal number of antennas, i.e., the number of radio frequency chains, that needs to be built-in on a transmitter to achieve the maximum energy-efficiency is determined. Specifically, an optimization problem for maximizing the average energy-efficiency with respect to the number of antennas is formulated. To solve this optimization problem, we first derive a closed-form formula expressing the cost function (i.e., the average energy-efficiency) as a function of the number of antennas. Search algorithms are then designed to obtain the optimal number of antennas. It is shown that the optimal number of equipped antennas depends on particular values of the circuit power consumption, the actual transmitted power, and the channel characteristics. Simulation results are provided to validate the analysis.



## Keywords

energy, efficient, per, subcarrier, antenna, design, selection, optimal, mimo, ofdm, wireless, systems

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# Optimal Design for Energy-Efficient Per-Subcarrier Antenna Selection MIMO-OFDM Wireless Systems

Ngoc Phuc Le, Le Chung Tran, and Farzad Safaei

**Abstract** In this paper, we consider a novel optimal design for per-subcarrier antenna selection MIMO-OFDM (multi-input multi-output orthogonal frequency division multiplexing) systems from an energy-efficiency perspective. The optimal number of antennas, i.e., the number of radio frequency chains, that needs to be built-in on a transmitter to achieve the maximum energy-efficiency is determined. Specifically, an optimization problem for maximizing the average energy-efficiency with respect to the number of antennas is formulated. To solve this optimization problem, we first derive a closed-form formula expressing the cost function (i.e., the average energy-efficiency) as a function of the number of antennas. Search algorithms are then designed to obtain the optimal number of antennas. It is shown that the optimal number of equipped antennas depends on particular values of the circuit power consumption, the actual transmitted power, and the channel characteristics. Simulation results are provided to validate the analysis.

**Keywords** Per-subcarrier antenna selection, energy efficiency, MIMO, OFDM systems.

## 1 Introduction

In recent years, energy-efficiency has emerged as one of the main concern for future high-speed wireless networks [1]. To offer high-rate transmissions, a combination of MIMO (multi-input multi-output) techniques and OFDM (orthogonal frequency division multiplexing) has been considered as a key technique [2]. Among various MIMO schemes, antenna selection appears to be a promising approach for OFDM systems. This is because the implementation of antenna selection is low-cost and requires small amount of feedback information, compared to other precoding or beamforming techniques [3, 4]. Therefore, it is important to consider the design of antenna selection OFDM systems from an energy-efficiency perspective.

The application of antenna selection in OFDM systems has been considered in the literature, e.g., see [5-8]. In general, the proposed systems can be categorized into two approaches: bulk selection (i.e., choosing the same antennas for all subcarriers) [5, 6] and per-subcarrier selection (i.e., selecting antennas independently for each subcarrier) [6-8]. It is shown in [6] that the per-subcarrier selection system can achieve a much better performance than its counterpart. Thus, per-subcarrier selection is very attractive for wideband communications. However, all of these research works, i.e., [5-8], only investigated antenna selection from either capacity or error-performance perspective. Recently, we investigated energy-efficiency in per-subcarrier antenna subset selection OFDM systems with the objective of peak-power reduction [9]. In this work, for a given number of transmit and receive antennas, a strategy for data-subcarrier allocation was proposed to improve power efficiency of power amplifiers, which in turn improves the energy-efficiency of the system. However, we note that [9] only examined the system from a viewpoint of power-amplifier efficiency. In other words, it did not consider the impact of the number of equipped antennas on the energy-efficiency, which is the focus of the present work.

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In per-subcarrier antenna selection, antennas are selected independently for each subcarrier. Thus, all equipped antennas as well as RF (radio frequency) chains are active. We note that this scheme can achieve an improved performance at a cost of additional power consumption due to multiple active RF chains. Therefore, to achieve high energy-efficiency, we need to find out the optimal number of antennas (i.e., the number of RF chains) for transmissions. One approach to deal with the impact of RF power consumption is adaptively selecting the optimal number of active RF chains among the available equipped RF chains, dependent on the channel condition. This approach was investigated in single-carrier MIMO systems [10], and MIMO-OFDMA systems [11]. However, in this approach, the number of the equipped RF chains that affects the energy-efficiency of the systems is not specified. Specifically, the larger the available number of RF chains, the higher energy-efficiency may be achieved. In contrast to this approach, we are interested in a new design problem that takes into consideration the fact that RF chains are expensive. Thus, the number of equipped RF chains should be as small as possible. Also, all equipped RF chains are active during transmissions. To this end, an important question is *how many RF chains need to be equipped so that the system can achieve the maximum energy-efficiency*?

In this paper, we consider the new design problem for energy-efficient antenna selection MIMO-OFDM systems as mentioned above. To determine the optimal number of antennas, we formulate an optimization problem for maximizing the average energy-efficiency with respect to the number of antennas. For solving this optimization problem, we first derive an explicit (closed-form) formula expressing the cost function as a function of the number of antennas, given the fading channel distribution and the transmission range. Search algorithms are then designed to obtain the optimal number of equipped antennas. Results obtained using Monte-Carlo simulations are provided to validate the analytical method. The results also show that the system with the optimal number of antennas achieve maximal energy-efficiency.

The remainder of the paper is organized as follows. In Section 2, an energy-efficiency based antenna selection MIMO-OFDM system is described, which includes a system model, a defined energy-efficiency metric, and an antenna selection criterion. In Section 3, an optimization problem for maximizing the energy-efficiency is formulated and a method to solve it is proposed. Simulation results are provided in Section 4. Finally, Section 5 concludes the paper.

**Notation** Throughout this paper, a bold letter denotes a vector or matrix, whereas an italic letter denotes a variable. $(.)^*$, $(.)^T$, $(.)^H$, $\varepsilon\{.\}$, and $\|.\|^2$ indicate complex conjugation, transpose, Hermitian transpose, expectation, and a squared norm, respectively.

## 2 Energy-Efficiency based Antenna Selection MIMO-OFDM Systems

### 2.1 System Model

We consider a MIMO-OFDM system with $K$ subcarriers, $n_T$ transmit antennas, and $n_R$ receive antennas as shown in Fig. 1. For the $k^{\text{th}}$ subcarrier, the data symbol $u(k)$ is assigned to the selected antenna based on feedback information (i.e., the selected antenna indices) from the receiver. Hence,

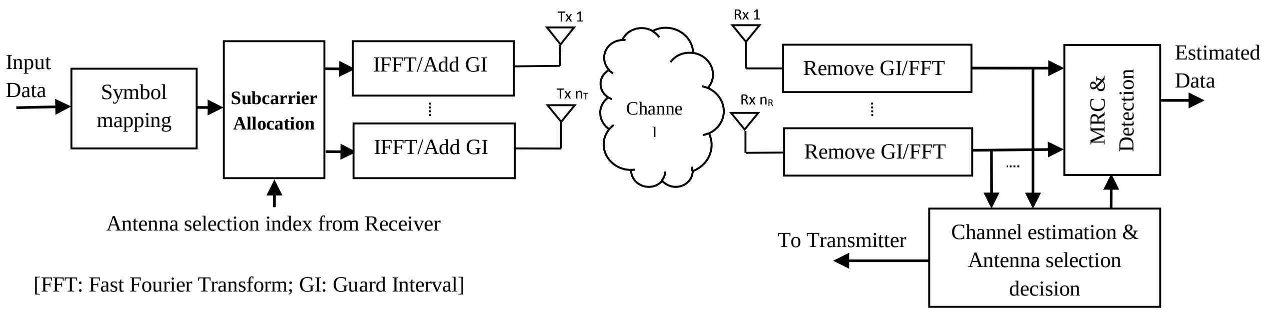



**Fig. 1** A simplified block diagram of a MIMO-OFDM system with per-subcarrier antenna selection

only one element in a vector $\mathbf{x}(k) = [x_1(k), x_2(k),..., x_{n_T}(k)]^T$ is allocated the data symbol, whereas the others are zeros. The received signal in the frequency domain at the $k^{\text{th}}$ subcarrier can be expressed as [2]

$$\mathbf{y}(k) = \sqrt{P_r}\mathbf{H}(k)\mathbf{x}(k) + \mathbf{n}(k) = \sqrt{P_r}\mathbf{h}_{\hat{i}}(k)u(k) + \mathbf{n}(k), \tag{1}$$

where $\mathbf{H}(k)$ denotes the $n_R \times n_T$ subchannel matrix associated with the $k^{\text{th}}$ subcarrier, $\mathbf{h}_{\hat{i}}(k)$ indicates the effective channel corresponding to the selected antenna, denoted as $\hat{i}$, on the $k^{\text{th}}$ subcarrier, and $P_r$ is an average per-subcarrier signal power on each receive antenna. Also, $\mathbf{y}(k) = [y_1(k), y_2(k),..., y_{n_R}(k)]^T$ and $\mathbf{n}(k) = [n_1(k), n_2(k),..., n_{n_R}(k)]^T$, where $y_j(k)$ and $n_j(k)$ denote the received signal and the noise at the $j^{\text{th}}$ receive antenna, respectively. Here, the noise is modeled as a Gaussian random variable with zero-mean. Assume that a MRC (maximum ratio combining) method is used at the receiver, the detected signal is given as

$$z(k) = \sqrt{P_r} \,\|\mathbf{h}_{\hat{i}}(k)\|^2 u(k) + \mathbf{h}_{\hat{i}}^H(k)\mathbf{n}(k). \tag{2}$$

In addition, in this system, transmit power adaptation and power loading across subcarriers are not available (i.e., fixed equal power allocation among subcarriers) as we assume that the feedback information contains only the selected antenna indices to reduce the number of feedback bits.

## 2.2 Energy-Efficiency Metric in OFDM-based Systems

We consider an energy-efficiency metric defined as a ratio between the ergodic capacity and the consumed power, i.e., [10, 11]

$$EE = C/P_{total}, \quad (bits/Joule) \tag{3}$$

where $C$ denotes the ergodic capacity (bits/s) and $P_{total}$ is the consumed power (watt). Let us denote $I_i(k)$ to be the instantaneous normalized capacity (with respect to the bandwidth) associated with the $k^{\text{th}}$ subcarrier and the $i^{\text{th}}$ transmit antenna, i.e., [12]

$$I_i(k) = \log_2(1 + \rho\|\mathbf{h}_i(k)\|^2), i = 1,2,..., n_T; k = 0,1,..., K-1, \tag{4}$$

where $\rho = P_r/\sigma_n^2$ is an average pre-processing SNR (signal-to-noise ratio) per subcarrier, and $\sigma_n^2$ is the noise power. The ergodic capacity achieved in this system is evaluated by

$$C(n_T, \rho) = W\varepsilon_{\mathbf{H}}\left\{\frac{1}{K}\sum_{k=0}^{K-1} I_{\hat{i}(k)}(k)\right\}, \tag{5}$$

where $W$ is the system bandwidth (Hz). Also, the power consumption $P_{total}$ in (3) is given as [13]

$$P_{total} = (\xi/\eta)P_T + n_T P_{RFtx} + n_R P_{RFrx} + P_{sp}, \tag{6}$$

where $\eta$ is the drain efficiency of power amplifier (PA), $\xi$ is a power back-off level, $P_T$ is the total transmit power, $P_{RFtx}$ is the circuit power consumption per transmit branch (excluding the PA), $P_{RFrx}$ is the power consumption per receive branch, and $P_{sp}$ is the baseband processing power consumption. Note that $P_T = KP_t$, where $P_t$ is the transmit power per subcarrier. Also, given a distance $d$ between the transmitter and the receiver (*Tx-Rx*), we have $P_t = LP_r$, where $L = L_0 d^{\chi} M$ with $L_0$ is the path loss at a unit distance, $\chi$ is a path-loss exponent, and $M$ stands for other factors, such as a link margin compensating the hardware process variation, a receiver noise figure, etc [13].

**Remark:** A parameter $\xi$ in (6) was originally defined as the PAPR (peak-to-average-power ratio) of an $M$-ary modulation signal in single-carrier systems in [13]. To achieve distortionless transmission, a power back-off value that is equal to $\xi$ is required, which reduces the PA's efficiency. In OFDM systems, it is well-known that the PAPR of OFDM signals is very large. However, the probability of occurrences of a very large PAPR is small [14]. Hence, to achieve high PA's efficiency at an acceptable level of nonlinear distortions, practical OFDM systems perform a back-off level that is much smaller than the PAPR value, especially if a PAPR reduction technique [13] is used. Therefore, in our energy-efficiency metric, we define $\xi$ as a back-off value. Also, for analytical simplicity, we assume that nonlinear distortions are negligible. This assumption is valid when a back-off level is quite large, and of course, is still much smaller than the PAPR value.

### 2.3 Energy-Efficiency based Antenna Selection Criterion

As antennas are selected independently for each subcarrier, all equipped antennas and RF chains are active. It is clear from (3)-(6) that, given a fixed power $P_t$, the $EE$ value is maximized when antennas are selected to maximize $I_i(k)$, $k$ = 0, 1,..., $K$-1. Thus, the optimal antenna at the $k^{\text{th}}$ subcarrier is selected as

$$\hat{i}(k) = \arg \max_{i=1,\dots,n_T} I_i(k) = \arg \max_{i=1,\dots,n_T} \|\mathbf{h}_i(k)\|^2 . \quad (7)$$

## 3 Optimal Number of Equipped Antennas for Maximal Energy-Efficiency

### 3.1 Optimization Problem Formulation

An energy-efficiency based antenna selection MIMO-OFDM system has been considered in Section 2. Assuming that the subcarriers are independent, the energy-efficiency (in bits/Joule) achieved in this system is calculated as[1]

$$EE(n_T,\rho) = \frac{C(n_T,\rho)}{P_{total}} = \frac{W\varepsilon_{\mathbf{H}}\{\log_2(1+\rho\|\mathbf{h}_{\hat{i}}\|^2)\}}{(\xi/\eta)P_T + n_T P_{RFtx} + n_R P_{RFrx} + P_{sp}} . \quad (8)$$

Note that the value $EE$ in (8) is the energy-efficiency corresponding to a $Tx$-$Rx$ distance of $d$, i.e., $\rho = \rho(d) = P_t / L_0 d^{\chi} M\sigma_n^2$ . Let $\rho_{\max} = \rho(d_{\min})$ and $\rho_{\min} = \rho(d_{\max})$ be the largest and smallest average received SNR within the coverage area, respectively. To find out the optimum number of equipped antennas, the achieved energy-efficiency needs to be evaluated for all possible values of $\rho$, i.e., $\forall\rho \in [\rho_{\min}, \rho_{\max}]$. Thus, the cost function in our optimization problem can be expressed as

$$EE(n_T) = \varepsilon_\rho\{EE(n_T,\rho)\} = \frac{W\varepsilon_\rho\{\varepsilon_{\mathbf{H}}\{\log_2(1+\rho\|\mathbf{h}_{\hat{i}}\|^2)\}\}}{(\xi/\eta)P_T + n_T P_{RFtx} + n_R P_{RFrx} + P_{sp}} . \quad (9)$$

Denote $n_T^{\max}$ to be the maximum possible number of built-in transmit antennas, which depends on physical size of the devices, an operating frequency, etc. Then, we have $n_T \le n_T^{\max}$ . With respect to the minimum number of equipped antennas, it is obvious that $n_T \ge 1$. By convention, the link budget planning during the design phase guarantees that the average received SNR value $\gamma$ is well above the required SNR value $\gamma_0$. In a situation that, due to a limited transmit power, the improved SNR resulting from the use of antenna selection needs to be taken into account to obtain $\gamma \ge \gamma_0$, then the minimum number of antennas is larger than one. This utilizes the fact that a larger number of

[1] The subcarrier index $k$ is dropped for simplicity.

equipped antennas leads to a higher $\gamma$ (cf.(7)). Thus, $n_T^{\min}$ is determined via the following condition

$$\gamma = \varepsilon_{\mathbf{H}}\{\rho_{\min} \| \mathbf{h}_{\hat{i}} \|^2\} = \rho_{\min}\varepsilon_{\mathbf{H}}\{\| \mathbf{h}_{\hat{i}} \|^2\} \geq \gamma_0. \quad (10)$$

Suppose that $\varepsilon_{\mathbf{H}}\{\| \mathbf{h}_{\hat{i}} \|^2\}$ is explicitly expressed as a function of $n_T$, we can easily determine $n_T^{\min}$ from the condition in (10). Note that re-planning the link budget is inevitably required if (10) is not satisfied even with $n_T = n_T^{\max}$. After all, the optimization problem can be expressed as

$$n_T^{opt} = \arg \max_{n_T^{\min} \leq n_T \leq n_T^{\max}} \frac{W\varepsilon_{\rho}\{\varepsilon_{\mathbf{H}}\{\log_2(1+\rho \| \mathbf{h}_{\hat{i}} \|^2)\}\}}{(\xi/\eta)P_T + n_T P_{RFtx} + n_R P_{RFrx} + P_{sp}}. \quad (11)$$

3.2 Solving the Optimization Problem

In order to solve (11), we first need to explicitly express the term $\varepsilon_{\rho}\{\varepsilon_{\mathbf{H}}\{\log_2(1+\rho \| \mathbf{h}_{\hat{i}} \|^2)\}\}$ as a function of $n_T$. The optimal value of $n_T^{opt}$ is then obtained by a search algorithm.

3.2.1 *Exact calculation of* $\varepsilon_{\rho}\{\varepsilon_{\mathbf{H}}\{\log_2(1+\rho \| \mathbf{h}_{\hat{i}} \|^2)\}\}$

We assume that the fading coefficients $|h_{j,i}|$, $i$ = 1, 2,..., $n_T$, $j$ = 1, 2,..., $n_R$, follow the Rayleigh distribution and the average SNR $\rho$ is uniformly distributed on [$\rho_{\min}$, $\rho_{\max}$] for simplicity. The obtained result is stated in the following proposition:

**Proposition 1**: *A closed-form expression of the expected value* $\varepsilon_{\rho}\{\varepsilon_{\mathbf{H}}\{\log_2(1+\rho \| \mathbf{h}_{\hat{i}} \|^2)\}\}$ *with* $n_R \geq 2$ *is given as*

$$\varepsilon_{\rho}\{\varepsilon_{\mathbf{H}}\{\log_2(1+\rho \| \mathbf{h}_{\hat{i}} \|^2)\}\} = \frac{n_T}{(n_R-1)!(\rho_{\max}-\rho_{\min})\ln 2}\sum_{u=0}^{n_T-1}\Bigg\{ (-1)^u \mathrm{C}_u^{n_T-1} \times \sum_{q=0}^{(n_R-1)u}\Bigg( \alpha_{u,q}\big[Q(n_R+q-2,\rho_{\max}) - Q(n_R+q-2,\rho_{\min})$$
$$+\rho_{\max} Q(n_R+q-1,\rho_{\max}) - \rho_{\min} Q(n_R+q-1,\rho_{\min}) - (\rho_{\max}-\rho_{\min})(n_R+q-1)!(u+1)^{-n_R-q}\big]\Bigg)\Bigg\}, \quad (12)$$

*where* $\mathrm{C}_a^b = b!/a!(b-a)!$ *is the binomial coefficient,* $\alpha_{u,q}$ *denotes the multinomial coefficient, and*

$$Q(n,\rho) = \frac{n!}{\rho^{n+1}} e^{(u+1)/\rho} \sum_{s=1}^{n+1}\left(\frac{u+1}{\rho}\right)^{-s} \Gamma\left(-n-1+s, \frac{u+1}{\rho}\right), \quad (13)$$

*where* $\Gamma(a,x) = \int_x^{+\infty} e^{-t} t^{a-1} dt$ *is the incomplete gamma function.*

**Proof**: A proof is provided in the Appendix.

By substituting (12) into (9), we obtain $EE(n_T)$ that is explicitly expressed as a function of $n_T$.

3.2.2 *Searching for the optimal value of* $n_T^{opt}$

The cost function $EE(n_T)$ in (9) is now explicitly expressed as a function of $n_T$. Thus, the optimal number of antennas can be obtained by exhaustive search using the proposed algorithm in Table 1. We have some remarks on this search algorithm:

**Table 1.** A search algorithm for $n_T^{opt}$.

| |
|---|
| 1: Initial setting: $EE_{opt} = 0$. |
| 2: Find $n_T^{\min}$ that satisfies $\gamma(n_T^{\min}, n_R) \geq \gamma_0$ using Eq. (10) and Eq. (20). |
| 3: **for** $n_T = n_T^{\min} : n_T^{\max}$ **do** |
| 4: Calculate $EE(n_T)$ using Eq. (9) and Eq. (12) . |
| 5: **if** $EE(n_T) > EE_{opt}$ |
| 6: $EE_{opt} \leftarrow EE(n_T),\ n_T^{opt} \leftarrow n_T$. |
| 7: **end if** |
| 8: **end for** |
| 9: The optimal number of equipped antennas is $n_T^{opt}$. |

**Table 2.** Values of $\varepsilon_{\mathbf{H}}\{\| \mathbf{h}_{\hat{i}} \|^2\}$ versus $(n_T, n_R)$.

| $\varepsilon_{\mathbf{H}}\{\| \mathbf{h}_{\hat{i}} \|^2\}$ | $n_T = 1$ | $n_T = 2$ | $n_T = 3$ | $n_T = 4$ | $n_T = 5$ | $n_T = 6$ |
|---|---|---|---|---|---|---|
| $n_R = 2$ | 2.00 | 2.75 | 3.21 | 3.54 | 3.80 | 4.02 |
| $n_R = 3$ | 3.00 | 3.93 | 4.49 | 4.89 | 5.19 | 5.44 |

- Given the expression of $\varepsilon_{\mathbf{H}}\{\| \mathbf{h}_{\hat{i}} \|^2\}$ in (20) (see Appendix), it is easy to obtain $n_T^{\min}$ in step (2). Some values of $\varepsilon_{\mathbf{H}}\{\| \mathbf{h}_{\hat{i}} \|^2\}$ versus the number of antennas are shown in Table 2.
- A search space in the algorithm is $(n_T^{\max} - n_T^{\min})$, which requires low complexity given that $n_T^{\max}$ is not very large in reality, e.g., in wireless local area network WLAN 802.11.n/ac. Moreover, the complexity of this algorithm may not a main concern as this algorithm is used only in the design phase.
- As mentioned above, an exhaustive search using the algorithm in Table 1 is efficient to find out $n_T^{opt}$ in many practical scenarios. Therefore, a mathematical derivation to obtain an explicit expression of $n_T^{opt}$, while very complicated, is not necessary. Here, we note that the cost function $EE(n_T)$ is a quasi-concave function. This is because the ergodic capacity $C(n_T,\rho)$ in antenna selection systems is a concave function with respect to $n_T$ [15]. Thus, the numerator of $EE(n_T)$, which is an average of $C(n_T,\rho)$ over a distribution of $\rho$, is also a concave function. Meanwhile, the denominator of $EE(n_T)$ is an affine function with respect to $n_T$.
- When $n_T^{\max}$ is large, e.g., in large-scale (massive) MIMO systems, a low-complexity search algorithm might be preferred. To efficiently search for $n_T^{opt}$ in those systems, we propose a bisection-based search algorithm shown in Table 3. This algorithm is designed by exploiting the fact that the function $EE(n_T)$ is quasi-concavity as discussed above. It is worth noting that both proposed search algorithms in Table 1 and Table 3 attain the same value of $n_T^{opt}$. However, the bisection-based search algorithm requires lower complexity in large-scale MIMO systems. This is because the exhaustive search algorithm calculates $EE(n_T)$ for all possible values of $n_T$, i.e., $n_T \in [n_T^{\min}, n_T^{\max}]$. Meanwhile, in the bisection-based search algorithm, only a few of values of $n_T$ are selected for a calculation of $EE(n_T)$.

**Table 3.** A bisection-based search algorithm for $n_T^{opt}$.

Find $n_T^{\min}$ that satisfies $\gamma(n_T^{\min}, n_R) \geq \gamma_0$ using Eq. (10) and Eq. (20).
Setting $l = n_T^{\min}$ , $u = n_T^{\max}$
**while** $u - l > 1$ **do**
    $m = \lfloor (v+u)/2 \rfloor$, ( where $\lfloor . \rfloor$ denotes a floor function)
    **if** $EE(m) > EE(m+1)$
        $u = m,$
    **else**
        $l = m,$
    **end if**
**end while**
The optimal number of equipped antennas is $n_T^{opt} = \arg\max_{n_T = l,u} EE(n_T)$.

**Fig. 2** Energy-efficiency *EE* versus the number of transmit antennas $n_T$ in Scenario A ($\gamma_0$ = 15 dB)

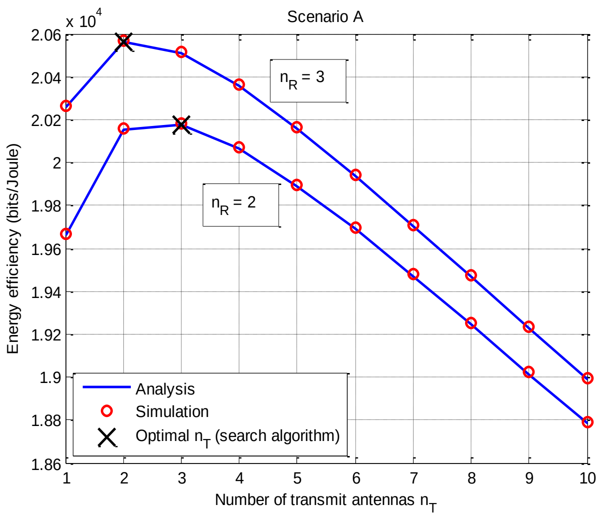



**Fig. 3** Energy-efficiency *EE* versus the number of transmit antennas $n_T$ in Scenario B ($\gamma_0$ = 15 dB)

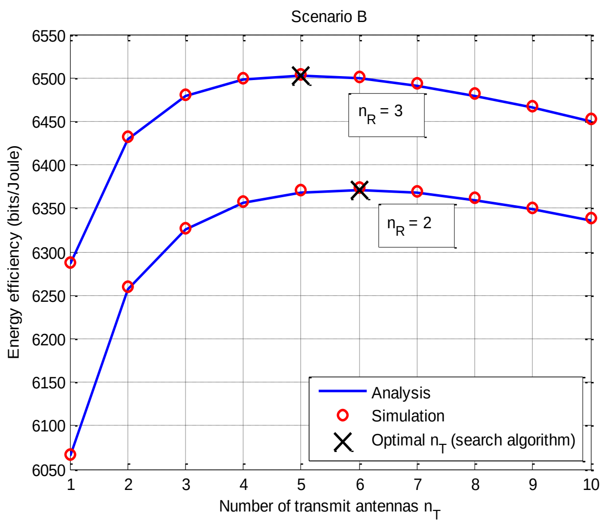
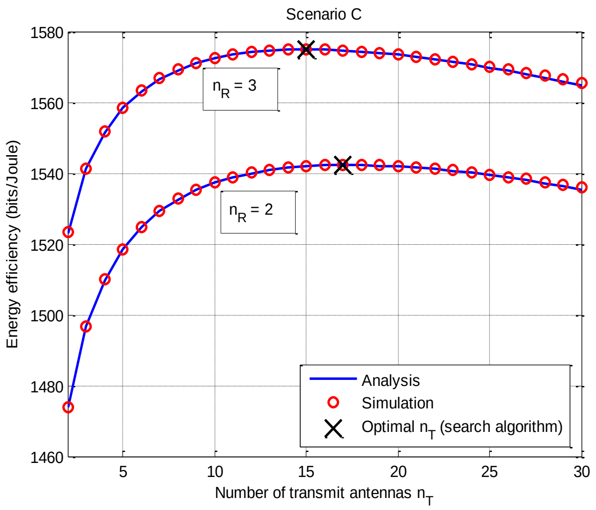


**Fig. 4** Energy-efficiency *EE* versus the number of transmit antennas $n_T$ in Scenario C ($\gamma_0$ = 15 dB)




- It is more efficient to find out $n_T^{opt}$ by using the proposed analysis method compared to a time-consuming Monte-Carlo simulation based method. Particular values of $n_T^{opt}$ for different design scenarios are provided in Section 4.

## 4 Simulation Results

We consider the system with the following parameters in our simulations: bandwidth of 10 kHz, carrier frequency of 2.5 GHz, FFT size of 64, circuit power consumption ($P_{RFtx}$ = 130 mW, $P_{RFrx}$ = 130 mW, and $P_{sp}$ = 100 mW), $\eta$ = 0.35, $\xi$ = 10 dB, $M$ = 50 dB, and thermal noise PSD (Power Spectral Density) of -174 dBm/Hz. Three design scenarios are illustrated: Scenario A ($P_T$ = 251 mW (~24 dBm), $\chi$ = 3.5, and [$d_{min}$, $d_{max}$] = [3m, 30m]), Scenario B ($P_T$ = 1W (~30 dBm) , $\chi$ = 2.5, and [$d_{min}$, $d_{max}$] = [4m, 200m]), and Scenario C ($P_T$ = 4W (~36 dBm), $\chi$ = 2.5, and [$d_{min}$, $d_{max}$] = [10m, 500m]).

The energy-efficiency versus the number of transmit antennas in Scenario A, Scenario B, and Scenario C, are plotted in Fig. 2, Fig. 3, and Fig. 4, respectively. The obtained results demonstrate the following. First, it can be seen that the analytical curves agree very well with the simulation curves, which validates the accuracy of the analytical method in Section 3. Second, the maximum *EE* value occurs at any $n_T$ ($n_T \geq 1$) depending on a particular system scenario, which reflects the fact that *EE* is a quasi-concave function. Also, when $n_T$ becomes very large, an increased power consumption due to the RF chains has more impact on *EE* than the capacity improvement does, which reduces the *EE* value (cf.(9)). Third, as expected, the maximum *EE* is achieved in the systems deploying the optimal numbers of antennas that are obtained by the search algorithms in Table 1 and Table 3 (marked as "x" in the figures). Note that the values $n_T^{opt}$ in the figures are obtained when $\gamma_0 = 15\text{dB}$. If the required SNR is $\gamma_0 = 18\text{dB}$, then, in Scenario A, $n_T^{opt} = 4$ when $n_R$ = 2, due to the constraint of $\gamma \geq \gamma_0$ (cf. (10)). Finally, the relation between the transmitted power and the circuit power consumption has a significant impact on the optimal number of equipped antennas. In particular, if the transmitted power dominates the circuit power consumption, e.g., Scenario B in Fig. 3, the maximum *EE* is achieved when a large number of antennas is equipped. Similar observations can be made in large-scale MIMO-OFDM systems where the number of equipped antennas is large as shown in Fig. 4.

## 5 Conclusions

This paper has considered a novel design problem for energy-efficient antenna selection MIMO-OFDM systems. The optimal number of equipped antennas to achieve the maximum energy-efficiency is obtained analytically. Several design scenarios have been investigated. It has been shown that, given the channel characteristics, a large number of antennas should be equipped to achieve a high energy-efficiency when the transmitted power significantly dominates the circuit power consumption, and vice versa. Simulation results show that the systems equipped with the optimal number of antennas achieve maximal energy efficiency. It is also worth mentioning that the proposed method in this paper can be used to design other MIMO schemes as well as multiuser MIMO systems with an OFDMA multiple access.

**Appendix** Proof of Proposition 1

Suppose that $|h_{j,i}|$ follows a Rayleigh distribution with $\varepsilon\{|h_{j,i}|^2\}=1$, it is clear that $\|\mathbf{h}_i\|^2$ is a chi-square distribution with $2n_R$ degrees of freedom. Thus, its pdf (probability distribution function) and cdf (cumulative distribution function) are given as $f(x)=e^{-x}x^{n_R-1}/(n_R-1)!, \forall x>0$, and $F(x)=1-e^{-x}\sum_{v=0}^{n_R-1}x^v/v!, \forall x>0$, respectively [12]. Therefore, we can express the pdf of $\|\mathbf{h}_{\hat{i}}\|^2$ using order statistics [16] as

$$f_M(x)=n_T f(x)F^{n_T-1}(x)=n_T\frac{e^{-x}x^{n_R-1}}{(n_R-1)!}\left(1-e^{-x}\sum_{v=0}^{n_R-1}\frac{x^v}{v!}\right)^{n_T-1}$$

$$=\frac{n_T}{(n_R-1)!}e^{-x}x^{n_R-1}\sum_{u=0}^{n_T-1}\left\{(-1)^u\,\mathrm{C}_u^{n_T-1}\,e^{-ux}\left(\sum_{v=0}^{n_R-1}\frac{x^v}{v!}\right)^u\right\}, \tag{14}$$

where $\mathrm{C}_a^b=b!/a!(b-a)!$ is the binomial coefficient. By performing a multinomial expansion as $(\sum_{v=0}^{n_R-1}x^v/v!)^u = \sum_{q=0}^{(n_R-1)u}\alpha_{u,q}x^q$, where $\alpha_{u,q}$ is the coefficient resulting from the multinomial expansion corresponding to $x^q$, we have

$$f_M(x)=\frac{n_T}{(n_R-1)!}e^{-x}x^{n_R-1}\sum_{u=0}^{n_T-1}\left\{(-1)^u\,\mathrm{C}_u^{n_T-1}e^{-ux}\sum_{q=0}^{(n_R-1)u}\alpha_{u,q}x^q\right\}. \tag{15}$$

On the other hand, assuming that the average SNR $\rho$ is uniformly distributed on $[\rho_{\min}, \rho_{\max}]$, the pdf of $\rho$ is given as

$$f(\rho)=1/(\rho_{\max}-\rho_{\min}),\ \forall\rho\in[\rho_{\min},\rho_{\max}]. \tag{16}$$

Thus, $\varepsilon_\rho\{\varepsilon_{\mathbf{H}}\{\log_2(1+\rho\|\mathbf{h}_{\hat{i}}\|^2)\}\}$ can now be calculated as

$$\begin{aligned}
&\varepsilon_\rho\{\varepsilon_{\mathbf{H}}\{\log_2(1+\rho\|\mathbf{h}_{\hat{i}}\|^2)\}\}\\
&=\int_{\rho_{\min}}^{\rho_{\max}}\int_0^{+\infty}\log_2(1+\rho x)f_M(x)f(\rho)\,dx\,d\rho=\int_0^{+\infty}\left[\int_{\rho_{\min}}^{\rho_{\max}}\log_2(1+\rho x)f(\rho)d\rho\right]f_M(x)dx\\
&=\frac{1}{(\rho_{\max}-\rho_{\min})\ln 2}\int_0^{+\infty}\left[\left(\frac{1}{x}+\rho_{\max}\right)\ln(1+\rho_{\max}x)-\left(\frac{1}{x}+\rho_{\min}\right)\ln(1+\rho_{\min}x)-(\rho_{\max}-\rho_{\min})\right]f_M(x)dx\\
&=\frac{n_T}{(n_R-1)!(\rho_{\max}-\rho_{\min})\ln 2}\sum_{u=0}^{n_T-1}\left\{(-1)^u\,\mathrm{C}_u^{n_T-1}\sum_{q=0}^{(n_R-1)u}\left(\alpha_{u,q}\times\left[Q(n_R+q-2,\rho_{\max})+\rho_{\max}Q(n_R+q-1,\rho_{\max})\right.\right.\right.\\
&\qquad\left.\left.\left.-Q(n_R+q-2,\rho_{\min})-\rho_{\min}Q(n_R+q-1,\rho_{\min})-(\rho_{\max}-\rho_{\min})T\right]\right)\right\},
\end{aligned} \tag{17}$$

where

$$Q(n,\rho) := \int_0^{+\infty} \ln(1+\rho x) x^n e^{-(u+1)x} dx = \frac{n!}{\rho^{n+1}} e^{(u+1)/\rho} \sum_{s=1}^{n+1} \left( \frac{u+1}{\rho} \right)^{-s} \Gamma\left( -n-1+s, \frac{u+1}{\rho} \right), n \geq 0, \quad (18)$$

where $\Gamma(a,x) = \int_x^{+\infty} e^{-t} t^{a-1} dt$ is the incomplete gamma function [17, Eq. (8.350.2)], and

$$T := \int_0^{+\infty} x^{n_R+q-1} e^{-(u+1)x} dx = (n_R + q - 1)!(u+1)^{-n_R-q}. \quad (19)$$

Note that (18) is obtained by using the integral result in [18, Eq. (78)]. Meanwhile, (19) is based on the integral of $\int_0^{+\infty} x^n e^{-\mu x} dx = n! \mu^{-n-1}$ [17, Eq. (3.351.3)].

In addition, the expression of $\varepsilon_{\mathbf{H}}\{\| \mathbf{h}_{\hat{i}} \|^2\}$ can be calculated as

$$\begin{aligned} \varepsilon_{\mathbf{H}}\{\| \mathbf{h}_{\hat{i}} \|^2\} &= \int_0^{+\infty} x f_M(x) dx = \frac{n_T}{(n_R-1)!} \sum_{u=0}^{n_T-1} \left\{ (-1)^u \mathrm{C}_u^{n_T-1} \sum_{q=0}^{(n_R-1)u} \alpha_{u,q} \int_0^{+\infty} x^{n_R+q} e^{-(u+1)x} dx \right\} \\ &= \frac{n_T}{(n_R-1)!} \sum_{u=0}^{n_T-1} \left\{ (-1)^u \mathrm{C}_u^{n_T-1} \sum_{q=0}^{(n_R-1)u} \alpha_{u,q} (n_R+q)!(u+1)^{-n_R-q-1} \right\}. \quad (20) \end{aligned}$$

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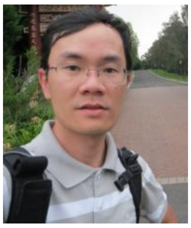
## Biographies



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