Magneto-optical imaging and current profiling on superconductors

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MAGNETO-OPTICAL IMAGING AND CURRENT PROFILING ON SUPERCONDUCTORS

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3464234

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ABSTRACT

Magneto-optical imaging (MOI) is a useful and highly versatile technique for the investigation of magnetic and current-carrying properties of superconductors. High-speed imaging is particularly important as superconductors often exhibit interesting magnetic behaviour over short timescales, such as the behaviour of individual flux vortices during magnetic flux penetration. The following method was developed to facilitate the high-speed acquisition of magneto-optical images of superconducting samples, and the determination of magnetic field and current flow data from these images. This method may be the first to allow current mapping of short timescale magnetic events in superconductors.

Magneto-optical images were acquired using a high-speed camera while monitoring sample conditions. These images were calibrated to obtain quantitative magnetic field data, which was then analysed to create images of the supercurrent distribution in the sample. For this purpose, an inverse Biot-Savart law process was applied, followed by an iterative in-plane field correction. The most efficient and accurate mathematical formulae for current calculation were derived from critical analysis and testing of formulae described in the literature. Image acquisition, sample monitoring, quantification and current calculation were all undertaken using purpose-built LabVIEW programs.

The imaging procedure was then applied to model images and to samples of YBa$_2$Cu$_3$O$_{7-\delta}$ to test the method developed. Upon application, magnetic fields in the samples were observed and analysed, and current behaviour was mapped visually from the magnetic field images.
### LIST OF SYMBOLS AND ABBREVIATIONS

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<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>α</td>
<td>Faraday Rotation Angle</td>
</tr>
<tr>
<td>$B, B_x, B_y, B_z$</td>
<td>Magnetic Field, x, y and z Magnetic Field Components</td>
</tr>
<tr>
<td>$B_A$</td>
<td>Indicator Film Anisotropy Field</td>
</tr>
<tr>
<td>$B_z^{(n)}$</td>
<td>The $n^{th}$ iterative approximation for $B_z$</td>
</tr>
<tr>
<td>β</td>
<td>Indicator film absorption coefficient</td>
</tr>
<tr>
<td>c</td>
<td>Indicator film rotation parameter</td>
</tr>
<tr>
<td>CCD</td>
<td>Charge-Coupled Device</td>
</tr>
<tr>
<td>d</td>
<td>Thickness of superconducting sample</td>
</tr>
<tr>
<td>$d_{ind}$</td>
<td>Thickness of indicator film,</td>
</tr>
<tr>
<td>E</td>
<td>Electric Field</td>
</tr>
<tr>
<td>$E_A$</td>
<td>Anisotropy Energy of indicator film</td>
</tr>
<tr>
<td>$f_d$</td>
<td>Driving Force</td>
</tr>
<tr>
<td>$f_p$</td>
<td>Pinning Force</td>
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<tr>
<td>$\tilde{F}$</td>
<td>Tilde indicates Fourier Transform of a quantity $F$</td>
</tr>
<tr>
<td>GGG</td>
<td>Gadolinium Gallium Garnet</td>
</tr>
<tr>
<td>GSV</td>
<td>Greyscale Value</td>
</tr>
<tr>
<td>h</td>
<td>Height of indicator film above sample</td>
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<tr>
<td>$H_c$</td>
<td>Critical Field</td>
</tr>
<tr>
<td>$H_{c1}$</td>
<td>First Critical Field</td>
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<tr>
<td>$H_{c2}$</td>
<td>Second Critical Field</td>
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<tr>
<td>I</td>
<td>Light Intensity</td>
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<tr>
<td>$I_0$</td>
<td>Incident Light Intensity</td>
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<tr>
<td>ISEM</td>
<td>Institute for Superconducting and Electronic Materials</td>
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<tr>
<td>$J, J_x, J_y$</td>
<td>Current Density, x and y current density components</td>
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<tr>
<td>$J_c$</td>
<td>Critical Current Density</td>
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<tr>
<td>$k, k_x, k_y$</td>
<td>Fourier spatial frequency (wavevector), x and y Fourier components</td>
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\( k_{\text{max}} \) Cut-off Wavevector

LUT Look-Up Table

\( \lambda \) London Penetration Depth

\( M, M_z \) Magnetisation, z Magnetisation Component

MO Magneto-Optical

MOI Magneto-Optical Imaging

MPMS Magnetic Property Measurement System® sample magnetometer

\( M_0 \) Spontaneous Magnetisation of indicator film

\( \mu_0 \) Permittivity of Free Space

\( \phi_0 \) Magnetic Flux Quantum

\( t \) Time

\( T_c \) Critical Temperature

\( \theta \) Deviation of the angle between polariser and analyser from full extinction (90°)

\( V \) Verdet Constant

\( W \) Window Function for filtering

\( \xi \) Coherence Length

YBCO Yttrium Barium Copper Oxide superconductor: \( \text{YBa}_2\text{Cu}_3\text{O}_{7-\delta} \)
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1 INTRODUCTION

1.1 MOTIVATION

Magneto-Optical Imaging (MOI) is the one of the leading techniques for analysis of the localised electromagnetic properties of superconductors. It allows the magnetic field around a superconductor to be seen visually, using a camera or even by eye through a microscope.

The aim of this project was to develop and test a procedure for high-speed magneto-optical imaging and current mapping on superconducting samples at the University of Wollongong's Institute for Superconducting and Electronic Materials (ISEM). The procedure developed includes a streamlined process for fast acquisition of magneto-optical (MO) images of superconductors, as well as a method for mathematical determination of current flow information in the samples from these images.

The procedure developed may be utilised in many future projects at ISEM. It is useful for quantitative investigation of the magnetic field and current in interesting superconducting samples, including wires, Josephson junctions, samples containing defects and samples of irregular geometry. The procedure may readily be used to extend the technique of magneto-optical imaging to higher spatial and temporal resolution than has been previously achieved. Upon such implementation, this would be the first procedure to allow high-speed current mapping in superconductors from MO images.

1.2 OBJECTIVES

This project aims to facilitate future use of the high-speed magneto-optical imaging apparatus at ISEM, and to allow quantitative analysis of images. In order to achieve this, the major objectives were:

• Integrating a new high-speed camera into the existing MOI microscopy system.

• Developing a simple desktop user interface, which allows as much of the imaging procedure as possible to be controlled remotely.

• Acquiring magneto-optical images of superconducting samples with various morphologies.

• Determining quantitative information about the magnetic field in the sample from these images.

• Mapping the flow of supercurrents in the sample by application of an inverse Biot-Savart law procedure to the magnetic field data.

• Optimising this mathematical procedure for accuracy and efficiency through literary analysis and empirical testing.
1.3 Thesis Outline

The sections of this thesis are presented in an increasingly technical manner. The document begins with qualitative discussions of the various topics under research, then moves on to the most technically specific information toward the end. Often a topic is discussed briefly at first, and then returned to in more detail in later sections. This writing style was adopted for accessibility to those not familiar with the subject material, though it may hinder readers if they are only interested in a single topic, such as quantification or the in-plane field correction. To remedy this, extensive self-referencing has been included to allow one to easily jump between the relevant sections on a particular topic.

Section 2 of this thesis discusses the theoretical basis for magneto-optical imaging, beginning in section 2.1 with the interesting magnetic properties of superconductors as a motivation for MOI. Section 2.2 discusses the physics behind the technique. It explains how a magnetic field can actually be seen visually by rotation of polarised light (the Faraday Effect). 2.3 describes how magnetic field images can be used to calculate the flow of current in the sample by inversion of the Biot-Savart law, and a correction to this technique for in-plane magnetic fields is given in 2.4. A few examples of applications of MOI are given in section 2.5.

Section 3 goes into more detail on the experimental application of the techniques described in section 2. 3.1 describes how images are acquired, including the microscope apparatus and the computer program used to control the imaging procedure. 3.2 outlines the methods used to obtain quantitative magnetic field data from magneto-optical images, and then 3.3 continues to explain how this data is used to calculate current distributions in the superconductor.

The results are presented in section 4. 4.1 shows a few example images of the magnetic behaviour of superconductors. 4.2 then shows the application of the current mapping procedure to a simple model. 4.3 gives characteristics of the indicator films, needed to apply an in-plane field correction. The correction process is then tested in section 4.4. Finally, 4.5 presents the current maps which have been created of various samples, which was the desired goal of this research.

Section 5 is the conclusion, which includes discussion of the completed MOI procedure, the images and other results acquired. Section 6 proposes several promising avenues for future work. References are listed in section 7.

Appendix A discusses the inconsistencies in the literature regarding the inversion of the Biot-Savart law. Appendix B describes the process and the actual equations used in the LabVIEW program for current calculation.
2 THEORY

2.1 SUPERCONDUCTOR MAGNETIC PROPERTIES

Besides their complete lack of electrical resistance below a critical temperature $T_c$, the most striking property of superconductors is the total expulsion of external magnetic fields a superconducting sample. A weak external magnetic field $B$ produces no magnetic field inside a superconductor. This is known as the Meissner effect, and is shown in figure 1. It occurs due to supercurrents flowing without resistance in a thin layer near the surface of the material, producing a magnetic shielding effect. This also occurs only below $T_c$, which is much colder than room temperature for all known superconductors.

![Fig. 1. The Meissner Effect. Magnetic fields around superconductors and other materials.](image)

Magnetic flux does penetrate a short distance into a superconductor at any interface between the bulk superconductor and non-superconducting material (or vacuum). A constant magnetic field outside the superconductor leads to an exponentially decaying field inside (Kittel 8th ed.). The field decays away with a characteristic length $\lambda$, called the London penetration depth.

In type I superconductors, the Meissner effect is complete up to a critical magnetic field value $H_c$, up to which no field enters the sample, and above which the sample abruptly returns to a normal (non-superconducting) state.

In type II superconductors, the magnetic field is completely excluded from the sample up to a critical value $H_{c1}$, after which magnetic penetration into the sample increases with increasing field up to a value of $H_{c2}$ above which the sample returns to a normal state. All further discussion relates to type II superconductors, as they can have higher critical temperatures and magnetic fields than type I, and also have a variety of unique magnetic properties which can be investigated using magneto-optical imaging.
The increased flux penetration between \( H_{c1} \) and \( H_{c2} \) occurs due to the presence of Abrikosov vortices, which are small isolated cylindrical regions of normal state material in equilibrium with the surrounding superconducting material. One quantised unit of magnetic flux may enter the sample for each such vortex. The equilibrium of a vortex is maintained by the flow of supercurrents around the surface of this cylindrical region, the name "vortex" being given due to this current flow pattern (Kittel 8th ed.). Magneto-optical imaging allows investigation of flux penetration and of these vortices.

The magnetic field due to a single flux vortex is concentrated in the normal core, and extends a distance \( \lambda \) out into the surrounding superconducting material in a cylindrical shape. Thus, the flux \( \phi_0 \) introduced into a superconductor by a single vortex is a fixed value, dependent on the penetration depth. As more vortices are introduced, the flux in the sample increases in quantised units of \( \phi_0 \).

The coherence length, \( \xi \) is a quantity describing the range of electron-electron interactions within a superconductor, and is "the minimum spatial extent of a transition layer between normal and superconductor" (Kittel 8th ed.). As such, it also defines the minimum allowed distance between vortices before the sample returns to a normal state (at field \( H_{c2} \)). It can therefore be seen that if \( \xi \) is greater than \( \lambda \), no vortices can exist within a superconductor, and it is type I. If \( \xi \) is smaller than \( \lambda \), it is type II, and as \( \lambda/\xi \) increases, more vortices may exist before the normal state is reattained, increasing the gap between \( H_{c1} \) and \( H_{c2} \).

Once vortices appear in a superconductor, they are driven toward the centre of the sample by a driving force \( f_d = J \times B \) (Jooss et al. 1998), where \( J \) is the current flowing around the material edges and \( B \) is the applied magnetic field. However, this flow is restricted by pinning forces \( f_p \) occurring when vortices are unable to move past defects in the sample (Roussel 2007). As vortices are driven further from the edges of a sample, more defects are encountered and the effect of pinning forces increases. For a fixed magnetic field, the penetration front is the quasi-equilibrium point at which \( f_p + f_d = 0 \) and the flux is not driven any further into the sample.

The penetration front is not a true stationary equilibrium even for a fixed magnetic field, since the pinning force \( f_p \) relaxes over time (Jooss et al. 1998). This leads to the phenomenon of flux creep, where a magnetic flux penetration front slowly moves further into the sample at a constant magnetic field. Flux creep occurs over timescales of \( 10^{-5} \) to \( 10^{-7} \)s (Ren et al. 1996), and may be due to vortex "hopping" from one defect to another (Feigel'man et al. 1989), but has also been shown to be dependent on sample size and other macroscopic properties (Ren et al. 1996).

A simple model for current flows in a superconductor is the Bean model, shown in figure 2. It considers a type II superconductor as a material “capable of sustaining lossless macroscopic current up to a critical current density, \( J_c \)” (Bean 1961). In the vortex state, current therefore flows at uniform density \( J_c \) around the sample. It is uniform over the penetration depth (from the sample edge to the penetration front), after which the current density drops to zero.
When the applied magnetic field is removed from a superconducting sample, the flux which has penetrated the sample is driven in the opposite direction, toward the outside of the sample. In the absence of vortex pinning, the flux fronts might recede until the sample returned to the Meissner state, as the field was reduced below $H_{c1}$. However, this is not observed. Instead, superconductors exhibit magnetic hysteresis; once flux has penetrated a superconductor, some residual flux will remain inside even after the applied field is removed. This residual flux effect is caused by the same pinning forces as discussed above, which restrict vortex motion around sample defects.

No amount of magnetic manipulation can remove all of the vortices from a penetrated sample. The only way to reattain the virgin superconductor case (before magnetic penetration) is to heat the sample above $T_c$ so that it is no longer superconducting, and then re-cool the sample to a superconducting state in the absence of applied field.

Overall, superconductors are seen to have many unique electromagnetic properties, and so a technique is desired to further explore these. The mysterious nature of short timescale magnetic behaviour such as vortex motion during flux creep leads naturally to the application of a high-speed magnetic field investigation technique. Magneto-optical imaging has become one of the most promising and widely used techniques (Jooss et al. 2002) for the investigation of the finer properties of magnetic vortex penetration into type II superconductors, and the analysis of current flows in relation to these vortices.
2.2 Visualising the Magnetic Field

Magneto-optical imaging allows the magnetic field around a superconductor to be seen visually and in real time through an optical polarising microscope. This is done by passing linearly polarised light through a highly specialised material known as a magneto-optical (MO) indicator film. Indicator films make use of the Faraday Effect (figure 2) to rotate the polarisation of linearly polarised light as it passes through the film.

Circularly birefringent materials have different refractive indices for left- and right-hand circularly polarised light. Passing linearly polarised light through the material therefore slows one of its circular polarisation components more than the other, leading to a rotation of the polarisation of the light. For a Faraday-active material, the angle of rotation is proportional to the component of magnetic field along a certain axis of the material (Roussel 2007, Jooss et al. 2002).

Magneto-optical indicators are composed of a film of Faraday-active material, with this magnetic axis (chosen to be the z axis) directed normally to the plane of the film. The film is placed on top of a thin film superconducting sample and parallel to it. When polarised light is passed through the indicator film and reflected off the sample it will then experience a rotation which is ideally proportional to the z-component of the magnetic field near the sample surface. The rotation angle $\alpha$ (also known as the Faraday angle) is given by:

$$\alpha = 2VB_zd_{\text{ind}}$$  \hspace{1cm} (1)

Here $d_{\text{ind}}$ is the thickness of the indicator film and $V$ is a constant of proportionality called the Verdet constant. The factor of two arises from the fact that light is reflected from the base of the indicator film, hence passing through it twice and being doubly rotated (Polyanskii et al. 2004).
Equation (1) is not entirely valid. It has been discovered that the other components of the magnetic field also affect the rotation of light (Laviano et al. 2002). See section 2.4.

The polarisation is rotated by an amount proportional to the magnetic field strength at a point in the film. The photons whose polarisation has been less rotated (passing through weaker magnetic fields) are eliminated by a suitably oriented analyser, leading to a light intensity that is proportional to magnetic field strength at the film. The experimental apparatus is discussed further in section 3.1.

A filter is often inserted before the MO indicator film to select the wavelength of light for which the Faraday Effect is strongest in the film (Laviano et al. 2002).

Magneto-optical indicator films can have a spatial resolution down to 1µm (Jooss et al. 1998; Goa et al. 2001). MOI also has higher temporal resolution any other known method of magnetic field visualisation, being limited only by the light detection equipment used.

Quantitative magnetic field data can be attained from magneto-optical images by calibration of the light intensity with the magnetic field (Roussel 2007). Calibration can be undertaken by attempting to calculate an exact theoretical relation between light intensity and magnetic field, though practically it is impossible to account for all sources of light loss with a great degree of precision. Instead, a more direct empirical calibration procedure is used in this research. See section 3.2.
2.3 Determining Current Profiles

We wish to acquire information about the current flow in superconductors from this magnetic field information. Currents \( J \) and magnetic fields \( B \) are intricately related, as evidenced by Ampère's law:

\[
\mu_0 J = \nabla \times B
\]  

(2)

\( \mu_0 \) is the permeability of free space. This law alone is not sufficient to determine the current distribution in a sample, since only the z-component of the \( B \) field is determined from MOI. Since this field equation proves useless here, techniques must be employed to calculate the current by integrating the \( B \) field across the entire sample (Jooss et al. 1998).

Historically, current profiles were determined by the fitting of current flow models to the \( B \) field data (Frankel et al. 1979). Assumptions were made about the general current behaviour, and important quantities such as the critical current density \( J_c \) were calculated as parameters.

Roth et al. (1998) demonstrated that current distributions \( J(r) \) in a flat plane (such as a thin film superconductor) could be accurately reproduced using a magnetometer at position \( r' \) which is a fixed height \( h \) above the plane (such as an indicator film above the sample), by measuring only the component of the magnetic field strength perpendicular to the plane of the current. Roth’s reconstruction algorithm forms the basis of supercurrent mapping from magneto-optical images. This algorithm begins by considering the Biot-Savart law:

\[
B(r') = \frac{\mu_0}{4\pi} \int \frac{J(r) \times (r' - r)}{|r' - r|^3} d^3r
\]

(3)

We take \( J_z = 0 \) to consider only the current components in a flat plane. Then the component form of the Biot-Savart law is reduced to equations (4).

\[
B_x(x, y, h) = \frac{\mu_0}{4\pi} \int \int \int \frac{J_y(x', y', z')(h - z')}{[(x - x')^2 + (y - y')^2 + (h - z')^2]^{3/2}} dx' dy' dz'
\]

(4.1)

\[
B_y(x, y, h) = \frac{\mu_0}{4\pi} \int \int \int \frac{-J_x(x', y', z')(h - z')}{[(x - x')^2 + (y - y')^2 + (h - z')^2]^{3/2}} dx' dy' dz'
\]

(4.2)

\[
B_z(x, y, h) = \frac{\mu_0}{4\pi} \int \int \int \frac{J_x(x', y', z')(y - y') - J_y(x', y', z')(x - x')}{[(x - x')^2 + (y - y')^2 + (h - z')^2]^{3/2}} dx' dy' dz'
\]

(4.3)

Roth et al. (1998) attempted to find the inverse of equation (4.3); to find \( J_x \) and \( J_y \) from \( B_z \) using a Fourier transform technique, as well as applying the charge continuity equation in the quasi-static case:

\[
\nabla \cdot J = 0
\]

(5)

However, they were unable to find a unique expression for the sheet current \( J(r) = (J_x, J_y, 0) \) in terms of the perpendicular magnetic field component \( B_z(x, y) \), calling this the "bulk current inverse problem".
Johansen et al. (1996), applied this technique to magneto-optical imaging of superconductors, and solved a one-dimensional version of the inverse problem for a thin superconducting strip. This was the first model-independent determination of supercurrent flows from MOI, though it was only useful for the specific sample geometry of a thin strip.

Johansen et al. solved the inverse problem for a two-dimensional case one year later, in 1997, allowing current mapping in thin film superconductors. The final equations he obtained were similar to those which will be used in this research. Equations (6.1) and (6.2) are derived from Fourier transforms of (4.3) and (5) respectively.

\[ 
\tilde{B}_z(k_x, k_y, h) = i \frac{\mu_0}{2} \frac{e^{-hk}}{k_y} \sinh \left( \frac{d}{2}k \right) \tilde{J}_x(k_x, k_y) 
\]  

(6.1)

\[ 
\tilde{J}_y(k_x, k_y)k_y + \tilde{J}_x(k_x, k_y)k_x = 0 
\]  

(6.2)

The tilde here indicates the Fourier transform of a quantity, i.e. \( \tilde{B}_z \) indicates the Fourier transform of the magnetic field component \( B_z \), and similarly with other quantities. \( k_x \) and \( k_y \) are the spatial frequency components in the x and y directions, \( d \) is the thickness of the sample and \( h \) is the height of the film above the sample.

The currents that flow in a superconductor are magnetisation currents, related to the local magnetisation \( M \) in the sample by equation (7).

\[ 
J = \nabla \times M 
\]  

(7)

The currents flowing in a thin film sample are two dimensional. Such two dimensional currents are produced by a magnetisation in the z direction, given by:

\[ 
\tilde{J}_x = \frac{\partial M_z}{\partial y} 
\]  

(8.1)

\[ 
\tilde{J}_y = -\frac{\partial M_z}{\partial x} 
\]  

(8.2)

Because of these relations, contour plots of the magnetisation \( M_z \) in the sample can be interpreted as current flow maps (Johansen et al. 1997). This can be demonstrated in a simple case by substituting \( \frac{\partial M_z}{\partial x} = 0 \) into equations (8). This describes a straight magnetisation contour line in the x direction, and from the equations it also produces a current flow solely in the x direction.

The magnetisation can be determined from the z component of the magnetic field using a similar Fourier technique as was used to determine the current components.

\[ 
\tilde{M}_z = \frac{2\tilde{B}_z e^{hk}}{\mu_0 \sinh \left( \frac{d}{2}k \right)} 
\]  

(9)

The technique of inversion of the Biot-Savart law is now well established in the literature (Jooss et al. 1998, 2002; Laviano et al. 2002; Roussel 2007), though many papers published on the subject have equations that are inconsistent with one another. An analysis of the equations presented by various groups is given in appendix A.
There are several other practical considerations to take into account when determining current profiles from magneto-optical images:

- Roth *et al.* (1988) determined that the spatial resolution of the image is very important in determining the quality of the final current map. This is one reason that their inversion technique lends itself to magneto-optical imaging. MOI allows for very high resolution magnetic measurements, as mentioned in section 2.2.

- Johansen *et al.* (1997) state that the current mapping technique is highly susceptible to high-frequency noise, due to the $e^{hk}$ term in (6.1). High-frequency shot noise is often present in MO images whenever there is a low light level, due to the random nature of photon capture in the camera used. The magnetic field data must therefore be passed through a low-pass filter before calculating the current profile. The filter used suggested by Roth *et al.* (1988) is a Hanning window, which is given by equation (10).

\[
W(k) = \begin{cases} 
0.5 \left( 1 + \cos \left( \frac{\pi k}{k_{\text{max}}} \right) \right) & k < k_{\text{max}} \\
0 & k > k_{\text{max}} 
\end{cases}
\]  

(10)

Where the cut-off wavevector $k_{\text{max}}$ is a parameter that must be optimised to reduce noise while preserving spatial resolution. The larger the value of $k_{\text{max}}$ used, the more spatial resolution is retained, but this also retains more high frequency noise. Despite the importance of spatial resolution, it is therefore limited by the optimal value of $k_{\text{max}}$ that can be taken, and so ultimately is determined by the noise in the image.

- The height of the indicator film $h$ is difficult to determine, since the film is placed directly onto the sample. Johansen *et al.* (1997) used a standard value of 10µm when simulating the MOI process, to account for roughness in the film and sample surfaces. Jooss *et al.* (1998) undertook MOI by evaporating a magneto-optical indicator layer onto the sample, with a gold layer of differing thickness between the indicator layer and the sample. With this accurate reference for film heights, Jooss calculated a film height of 5 to 10µm for films simply placed on top of the sample. Thus, the standard value of $h = 10\mu m$ was adopted in the present procedure.

- While the applied magnetic field is purely perpendicular to the magneto-optical film, the currents flowing in the superconductor will induce parallel $B$ field components, which are generally neglected. These are taken into account using Laviano’s in-plane field correction method, described below.
2.4 In-Plane Field Correction

Up until 2002, it had been assumed that polarisation rotation in indicator films was dependent only on the z component of magnetic field at the film, as described by equation (1). It was assumed that the effects of other components were negligible, and that MO images would be a perfect representation of the magnetic field z component. However, Laviano et al. (2002) showed that MO images obtained are influenced significantly by magnetic field components parallel to the indicator film as well as those that are perpendicular. They showed that $B_x$ and $B_y$ can affect the light intensity passed through the film by up to 40% locally.

Therefore, ignoring these magnetic field components will lead to highly inaccurate current mapping. In particular, it gives a calculated critical current that is higher than the physical value, with the peak current value occurring outside the sample edge.

Laviano proposed an iterative method to correct for these in-plane fields, allowing more accurate current profiles to be created. This method begins once again with the application of a Fourier transformation to the Biot-Savart law, simply considering the $B_x$ and $B_y$ components in place of $B_z$. Equations (11.1) and (11.2) are obtained from (4.1) and (4.2) respectively (Roth et al. 1988, Johansen et al. 1997).

$$B_x(k_x, k_y, h) = \frac{\mu_0}{2} e^{-hk} \sinh \left( \frac{d}{2} k \right) J_y(k_x, k_y)$$

(11.1)

$$B_y(k_x, k_y, h) = -\frac{\mu_0}{2} e^{-hk} \sinh \left( \frac{d}{2} k \right) J_x(k_x, k_y)$$

(11.2)

An inverse Fourier transform then gives the magnetic field components $B_x$ and $B_y$. These in-plane field components are then used to calculate a better approximation for $B_z$ using equation (12), which is derived from the response of the indicator film to different magnetic field components (Laviano et al. 2002).

$$B_z^{(n+1)} = \left( 1 + \frac{B_x^2 + B_y^2}{B_A} \right) B_z^{(n)}$$

(12)

Here $B_z^{(n)}$ is the nth approximation for $B_z$. $B_A$ is the anisotropy field of the indicator film, which was determined by magnetic analysis of the film, as discussed in section 4.3.

Overall, the corrective procedure is as follows: The first approximation considers the MO image obtained to be solely dependent on $B_z$ and calculates $J_x$ and $J_y$ from these $B_z$ values (as occurs normally, without the correction), then $J_x$ and $J_y$ are used to calculate $B_y$ and $B_x$, which are in turn used to calculate a better approximation for $B_z$. This process is then repeated until the desired level of accuracy is attained.

By application of this iterative procedure, Laviano was able to improve the calculated current profiles significantly. After applying the procedure, current no longer appeared outside the sample. Also, the current within the sample was at a near-constant value throughout the
penetration depth, which is consistent with the Bean model (Bean 1962). Empirical and mathematical analysis of Laviano’s in-plane field correction is presented in section 4.4.

Other groups have also made attempts to correct for the effects of in-plane field components. Paturi (2005) created an alternative technique using the fact that the magnetic field data is discretised into pixels to reduce the integrals in equations (4) to discrete sums over the total number of pixels, assuming a constant magnetic field and current across a single pixel. These equations were then reduced to matrix multiplications, and could be solved for $B_x$, $B_y$ and $B_z$. The in-plane components could then be removed from the indicator film data in a single calculation. Paturi’s technique was not applied in this research, simply due to time restrictions.

2.5 APPLICATIONS OF MAGNETO-OPTICAL IMAGING

Magneto-optical imaging is often used to identify grain boundaries and other defects in superconducting samples. This is especially important in analysis of a particular sample for its usefulness in applications. It is used to map transport currents (Song et al. 2009), and to identify and assess current limiting defects in a sample (Feldmann et al. 2001; Jiang et al. 2001, Pan et al. 2009).

MIO is also used for general analysis of superconducting properties. Because of its high resolution, magneto-optical imaging can be used to observe the behaviour of individual magnetic flux vortices (Goa et al. 2001). The rapid response of these vortices to changes in external magnetic field is one major factor in the move toward high-speed magneto-optical imaging.

There are many aspects of the study of vortices that are specifically suited to high-speed visual analysis, such as the manipulation of vortices in relation to domain walls (Goa et al. 2002) which may lead to new superconducting nanotechnology. Another high-speed application is visualising the current and field redistributions as a superconducting sample returns to its normal state after a “quench” or thermal disturbance. Interesting features such as spatial non-uniformity have been observed in this application using MOI (Song et al. 2009).

Other applications of MOI include analysis of high-current superconducting wires, both coated and non-coated and superconducting tapes for critical current, current flow and current-limiting defects. From such analysis, methods have been developed such as healing cracks in superconducting tapes, leading to even higher critical currents (Jiang et al. 2001) and even analysis of the flow of over-critical currents in coated superconducting wires to show how currents larger than $J_c$ may be produced (Pan et al. 2003).

The MOI technique has been applied to many superconducting samples of irregular geometries, where flux penetration cannot be easily predicted. One example is periodic superconducting networks, where it revealed non-uniform vortex penetration possibly due to defects, as well as dendritic penetration along long superconducting legs (Tamegai et al. 2009). Another is samples deposited on “mis-cut” substrates, whose surface is tilted with respect to a regular crystallographic direction (Yurchenko et al. 2010). The critical current in such samples was observed to be anisotropic. Using MOI, it was also seen that the flux penetrates most easily
along certain channels in the material, and that flux penetration may occur along any of these channels at random, causing sporadic flux jumps which could have a detrimental effect on applications.

Magneto-optical imaging has also been used to create and verify new models for magnetic behaviour in superconductors, such as uneven flux penetration and flux pinning around grain boundaries (Pan et al. 2009). The current flow patterns and magnetic field distribution predicted by such models must always be compared to experimental data for verification and to identify limitations in the model. MOI can provide this experimental data, and in doing so, the visualisation of magnetic fields aids in improving the theoretical understanding of superconductivity.

MOI is an increasingly widely used technique for the investigation of superconductors, and many other various and novel applications are continually being developed. It is beyond the scope of this thesis to detail the many research avenues currently being undertaken. For a broader (though less recent) overview of the various areas of MOI research on superconductors, see Jooss et al. (2002).

High-speed magneto-optical imaging is also used for investigation into the magnetic behaviour of non-superconducting materials, such as the dynamic behaviour of small ferromagnetic structures (Freeman 1998), though this lies beyond the scope of this thesis.
3 EXPERIMENTAL PROCEDURE

3.1 IMAGE ACQUISITION

To acquire magneto-optical images, superconducting samples are first placed in the field of view of a purpose-built polarising microscope, shown in figure 3, and cooled to below their critical temperature with cryogenic liquid. The superconductors used in this research were YBa$_2$Cu$_4$O$_{7−δ}$ (YBCO), with a maximum critical temperature of 92K (Wu et al. 1987). As such, they could be made superconducting using either liquid nitrogen (boiling point 77K) or helium (4.2K). However, helium was used exclusively in this research as lower temperatures lead to a stronger expulsion of magnetic fields from a superconductor.

A variable external magnetic field is then applied to the samples using a solenoidal electromagnet around the sample. The current through the solenoid is varied to vary the field according to a known linear relation. The temperature and applied magnetic field are regulated continuously throughout the cooling and measurement process.

![Polarising microscope used for magneto-optical imaging](image)

Fig. 3. Polarising microscope used for magneto-optical imaging, with key components and direction of light flow indicated. A solenoidal electromagnet is usually placed around the sample mount, but has been removed for this picture.
The magnetic field strength (and relative direction) near the sample's surface is visualised using an indicator film, which is placed on top of the samples. The indicator films used in this apparatus are ferrite garnets grown on a substrate of gadolinium gallium garnet (GGG), which is one of several commonly used compositions.

The direction of light flow through the apparatus is indicated in figure 2; Light is passed through a polariser and then directed onto the indicator film. The film rotates the polarisation of the light by an angle proportional to the local magnetic field strength. The light is then reflected off the base of the indicator film, into the microscope's objective lens and passed through an analyser before the image is viewed. The magnetic field strength around the sample can then be seen visibly through the microscope eyepiece, and images can be captured using the Leica DC 300F and Photron Fastcam SA3 MOI cameras (the Photron camera is pictured). Use of the latter camera allows for high-speed imaging, up to 120,000 frames per second.

An attempt was made at filtering the light to select only the region of the spectrum for which the indicator film is most sensitive. A broad green filter was used, since green light is most useful for this purpose. However, any increase in sensitivity due to this filter was offset by the reduction in light intensity. Therefore, the final procedure used unfiltered white light from a bright microscope bulb.

A program was created using the LabVIEW development system to control the MO imaging procedure and to process the images. The program both controls the Photron camera and regulates the sample conditions for imaging. The camera control section of the program was based largely on a sample LabVIEW camera controller provided by Photron. It allows live MO images to be viewed and saved. The sample conditioning section of the program includes temperature monitoring and heating using a LakeShore 331 digital temperature controller; as well as control and monitoring of the applied magnetic field by regulation of current to the solenoid using a Xantrex XKW 40-75 power supply. The user interface (front panel) is shown in figure 4.
Fig. 4. Front panel of the image acquisition LabVIEW program. The applied magnetic field is regulated from the top right section, temperature from the middle right section, and the camera is controlled from the bottom right section. Live MO video or saved images are displayed in the large space on the left. The image currently visible shows a square indicator film, which appears bright on a dark background. Defects in the film appear as dark spots and scratches. The film is seen to visualise a light and dark pattern of near-vertical lines across its centre, which are magnetic data on a credit card.
3.2 MAGNETIC FIELD QUANTIFICATION

The greyscale value (GSV) of a pixel is directly related to the light incident on the corresponding photodetector in the camera’s CCD (charge coupled-device) array. The Photron Fastcam SA3 has a highly linear relation between detected light intensity and image greyscale value. Other cameras often have a logarithmic light response, or some other relation designed to enhance bright features for visually appealing images. The linear relation of the Fastcam is preferable for quantification.

The light intensity in turn is directly related to the magnetic field at the indicator film. There are precise mathematical formulae relating light intensity to the magnetic field at a point on the film’s surface (Roussel 2007, Laviano et al. 2002), such as equation (13):

\[
I = I_0 \exp \left( -2 \beta d_{\text{ind}} \right) \sin^2 \left( \frac{c M_s B_z}{\sqrt{B_A^2 + (B_x^2 + B_y^2)^2 + B_z^2}} + \theta \right)
\]

Here \(I_0\) is the incident light intensity and \(I\) is the intensity of light after passing through the indicator film. \(M_s\) is the saturation magnetisation of the film, \(\beta\) is the coefficient of absorption in the film, \(d_{\text{ind}}\) is the film thickness, \(\theta\) is the deviation of the angle between polariser and analyser from full extinction (90°) and \(c\) is a parameter describing the amount of rotation, dependent on the particular film used.

Despite the apparent complexity of this equation, it is not sufficient for real-world quantitative analysis of magnetic fields with respect to light intensity. There are many factors influencing light intensity that are difficult to measure: such as ambient light; defects in the MO indicator; light loss and aberrations in the optics; reflections from the top surface of the indicator film or from the cryostat window (before the light is rotated); the values of the parameters \(\beta, c, I_0\) and the actual sensitivity of the camera.

A more practical calibration procedure involves direct comparison between magnetic field and GSV over a range of values. The external magnetic field is varied in a set number of steps over the measured range and a calibration image captured for each. This range must cover at least the range of magnetic fields which will be applied to the superconducting sample. The average greyscale value is calculated for each of these images over an appropriate non-superconducting region in the film. The average grey scale values are then plotted against the corresponding magnetic field values to create a calibration curve, see figure 5.
The GSV of each pixel in the image of the sample is then converted into a quantitative magnetic field value by interpolation with respect to the calibration curve. This process occurs in a second LabVIEW program, which has been developed to analyse the captured images. It is able to quantitatively calculate the magnetic field at each pixel in the image, and use this information to map supercurrent flows in the sample.

A separate calibration procedure must be undertaken for each sample in each imaging session because different films are used, the illumination is at a different level and the angle $\theta$ is different for each set of images taken.

Calibration can be done in two different ways; either the GSV is averaged over the whole film at room temperature or it is taken over a smaller section of the film at the temperature of measurement (usually 4 to 10 K). Calibration over the whole film gives a more accurate average of the greyscale value at any point and removes any systematic error arising from inhomogeneous illumination, though it neglects the temperature dependence of the film, which may be quite significant. Alternatively, calibration over a smaller section can be undertaken immediately after measurements are taken, ensuring that the polariser angle and illumination are consistent with those used for sample imaging. A smaller section may also be preferable since the indicator films used contain defects which appear as dark spots on the image, and a relatively defect-free section can be selected for calibration. However, the section of the image used is at a different position to that of the sample, which could lead to errors due to inhomogeneity in the film or in illumination.

For the purposes of this research, calibration over a small section was preferred, especially because the illumination level cannot be fixed at a constant value due to heating issues in the bulb, and the illumination used for imaging cannot be later reproduced exactly. Calibration was therefore taken immediately after each imaging session. It was not done before imaging, as the fields applied for calibration would then leave a residual field in the...
superconductor, meaning that the virgin flux penetration could not be observed. Also, for practical acquisition of the best image, various adjustments are made during imaging, such as the microscope focus, polariser angle and illumination. Calibration is taken after imaging, when these variables are fixed at their optimal values.

When calibration data is not available for imaging, the program used allows for a simple linear fit using two data points: the average GSV in a region outside the sample (at the applied field value) and the average GSV in the centre of the film (zero field). This linear approximation assumes $I \propto B_z$, which is reasonably accurate with respect to equation (12), which has $I \propto \sin^2\left(\frac{B_z}{\sqrt{1 + B_z^2}}\right)$. The closeness of the approximation can be seen from the graph of $\sin^2\left(\frac{x}{\sqrt{1 + x^2}}\right)$ in figure 6.

![Graph of $\sin^2\left(\frac{x}{\sqrt{1 + x^2}}\right)$ and a linear fit from $x=0$ to $\pi/2$, showing the accuracy of a linear approximation to the magnetic field - intensity calibration curve. The curve is most linear for small angles.](image)

Of course, this linear approximation is not possible when the applied field is zero, as the field outside the sample would be zero. There would therefore be only one reference point where GSV and magnetic field are known, which is not sufficient to plot a line. Therefore, the full calibration procedure is necessary for all residual field images, and preferable for all MO images for the best accuracy.
3.3 CURRENT PROFILING

From the quantified magnetic field image, the supercurrent distribution in the sample is calculated by an analytical technique involving inversion of the Biot-Savart law (see section 2.3). The equations are applied to the data in the same program as is used for magnetic field quantification. Images are produced of the total supercurrent density and the x and y components, as well as a contour map of the magnetisation $M_z$, which shows the flow pattern of the current in the sample. The contour map may be superimposed on the original MO image for a good visual indication of the supercurrent behaviour.

To begin the current mapping process, the image is first quantified using the procedure discussed above. A low pass-filter is applied to the image, as the inversion technique is quite susceptible to high-frequency distortion (Johansen et al. 1997). The filter chosen is a Hanning window, as this was seen to give optimal noise reduction without excessive blurring (see section 4.2). A Fourier transform is then applied to the array of quantified magnetic field values, and the current profile $J(x, y)$ is calculated from the image of the magnetic field component $B_z$ using equations (14).

$$
\tilde{J}_x(k_x, k_y) = -\frac{2i\tilde{B}_z(k_x, k_y, h)e^{i\hbar k_y}}{\mu_0 \sinh\left(\frac{d}{2}k\right)} \\
\tilde{J}_y(k_x, k_y) = \frac{2i\tilde{B}_z(k_x, k_y, h)e^{i\hbar k_x}}{\mu_0 \sinh\left(\frac{d}{2}k\right)}
$$

Equations (14) are obtained simply from equations (6). These were found to be preferable to (6) in an actual current mapping procedure, since there is no need to divide by $k_x$ or $k_y$, which results in division by zero for the central Fourier components.

The magnetisation in the sample is also determined in Fourier space, using equation (9):

$$
\tilde{M}_z = \frac{2\tilde{B}_z e^{i\hbar k}}{\mu_0 \sinh\left(\frac{d}{2}k\right)}
$$

After applying the inverse Fourier transformation, a contour plot is created of the magnetisation $M_z$, which can be interpreted as a map of current flow in the sample, as explained in section 2.3. The direction of current flow is equivalent to lines of constant $M_z$.

Finally, an inverse Fourier procedure is applied to find the final arrays of values for the x and y current components. Images are then created of the x and y current components and of the total current density from these values.

Laviano’s iterative procedure, discussed in section 2.3, can then be applied to correct the current profiles for magnetic fields parallel to the indicator film. However, upon implementation it was seen that successive applications of the procedure produced increasingly unphysical images of the current distribution and magnetic field, see section 4.4. Therefore, the in-plane field correction technique was not used for any of the final current calculations.
A LabVIEW program was created to enable both the quantification and current profiling procedures. Saved MO images are loaded by the program and cropped to the size of the sample and immediate surrounds. Then each pixel’s GSV is compared to a saved calibration curve for quantification. A Fourier transform is then applied to the array of quantified pixel data, before equations (14) and (9) are applied. An inverse Fourier transform is applied to the calculated $\hat{J}$ and $\hat{M}_z$, and then the $J$ components are displayed as images and $M_z$ as a contour plot. Finally, the $M_z$ contour plot can be superimposed on the $B_z$ image to show the current flow pattern. The user interface for this program is shown in figure 7.

A more in-depth discussion of the workings of the current calculator program is given in appendix B.

![LabVIEW program](image)

**Fig. 7.** Front panel of the current calculator (and quantification) LabVIEW program. Loaded magneto-optical images are displayed at the top left after cropping. Calculated images of the x and y current components are displayed in the two right images, and the calculated total current is displayed at the bottom left. An explanation of the current images is given in section 4.2. Magnetisation contours showing the current flow pattern may be added to the magnetic field image using the toggle at the far left.
4 RESULTS TO DATE

4.1 IMAGES ACQUIRED

Many magneto-optical images have been acquired using the experimental apparatus described in section 3.1. A selection of images from a thin film $\text{YBa}_2\text{Cu}_3\text{O}_{7-\delta}$ (YBCO) superconducting sample is presented in figure 8, showing the characteristic behaviour of magnetic fields within a superconductor as the applied magnetic field is varied. These images represent those typically acquired whenever imaging of a sample is undertaken. For a description of the structure and properties of samples such as those used in this research, see Pan et al. (2006).

The colour images in figure 8 were acquired using a Leica DC 300F camera. These images are quite visually appealing; though for the purposes of quantification, a Photron Fastcam SA3 is used in preference. The Fastcam produces higher resolution greyscale images with a more linear response to light intensity, as discussed in section 3.2.

In a typical imaging procedure, the sample is zero-field cooled until it reaches a stable temperature in a superconducting state. Flux penetration is then observed at particular values of the applied magnetic field as is increased from zero. Slight flux penetration is seen in figure 8a, most of the field still being excluded. A larger degree of penetration is seen in 8b. The penetration of flux around sample defects, as seen at the top and bottom of this sample, is of primary importance in magneto-optical analysis. After reaching a maximum value, the applied field is reduced to zero, and a residual magnetic field is observed due to flux pinning, as seen in figure 8c.
Fig. 8. Magneto-optical images taken with the Leica camera, of a rectangular sample of zero-field cooled thin film YBa$_2$Cu$_3$O$_{7-\delta}$ (YBCO) under applied magnetic fields (a) 0.02T and (b) 0.09T, and (c) the residual flux after switching off the 0.09T applied field.

- Inside of sample repels field
- Brighter image implies higher magnetic field
- Flux penetrates from edges and around defects
- Field builds up around edges
- Indicator film defects
- Residual flux remains in sample
4.2 SIMPLE MODELLING

Current profiling was first carried out using a very simple model image. The model simulates a magneto-optical image of a uniform magnetic field $B_z$ perfectly excluded from a superconducting sample, with some field build up at the sample edges. Using this model, it was possible to visualise Meissner currents flowing close to the sample edges, as seen in figure 9.

The equations discussed in section 3.3 were applied to the image in order to calculate current components $J_x$, $J_y$, the total current $J$ and the magnetisation $M_z$. Images were created of the simulated current components and total current, and a contour plot of the magnetisation. Each of these gives a visual indication of the current distribution which would produce the model magneto-optical image.

The model image simulates only the z-component of the magnetic field. As such, Laviano’s method to correct for in-plane magnetic field components was not yet considered. However, the first evidence against the validity in-plane correction procedure is seen here: The magnetisation contour plot shows unphysical current flow outside the sample, even for this model image which does not contain in-plane fields. This is significant, since it was claimed that such currents appearing outside the sample were caused by in-plane fields, and could be removed by the correction. It is apparent that some other factor is responsible for these unphysical currents, though it could be an issue with the very simplistic model MO image considered here.

This first visualisation was then used for testing of the various filters which had been built into the current calculation program to remove high-frequency noise from the image. The best filter was found to be the Hanning window discussed in section 2.3. Further tests were undertaken to find the optimal $k_{\text{max}}$ value for this image, with the main results shown in figure 10. If the value taken for $k_{\text{max}}$ is too small, the final current image becomes blurred as all high-frequency components are removed (fig. 10d). If the value taken is too large then the current calculation procedure succumbs to high frequency noise, creating spurious lines of alternating high and low current. These spurious effects are seen most clearly when no filtering is applied to the image (fig. 10b). The optimal $k_{\text{max}}$ value was found to be 80 for this image (fig. 10c).

Linear current profiles are also shown in figure 10. Each of these was produced from the corresponding current image, showing the calculated current distribution around the edge of the sample for each value of $k_{\text{max}}$. Non-smoothness in the current profile is caused by the high frequency noise, while broadening of the current profile is evidence of image blurring. Therefore, the optimal $k_{\text{max}}$ value was taken to be that which produced the narrowest current profile which remained smooth.

These early calculations used rough parametric values for the thickness $d$ of the sample and the height $h$ of the indicator film above it. After testing with more standard values of $d = 100\mu$m and $h = 10\mu$m (as discussed in section 2.3), the value of $k_{\text{max}} = 100$ was taken to be optimal. Thus the default value was set to 100 for further imaging, though the actual optimal value will differ for each image to which the current calculation procedure is applied. This depends on the extent of the noise in the image, the spatial frequency composition of the
current in the sample and the height of the indicator film above the sample. Practically, the differences were minimal, so $k_{max} = 100$ was used for all current calculations.

Fig. 9. (a) Model MO image of a uniform magnetic field $B_z$ perfectly excluded by a rectangular superconductor. Brighter regions indicate higher magnetic fields.

(b) Visualisation of the current density in the $x$ direction. (c) Current density in the $y$ direction. Bright regions in images (b) and (c) indicate current in the positive $x$ and $y$ directions respectively, and dark regions indicate current in the negative direction.

(d) Visualisation of the total supercurrent density $|J|$ calculated from this image. The brighter regions indicate higher currents, while black indicates no net current.

(e) Magnetisation contour map superimposed onto the model magnetic field image. The magnetisation contour lines also show the current flow direction in the sample. Paint.NET was used to add colour. Unphysical current is observed to flow outside the sample, despite the fact that the model represents a magnetic field purely in the $z$ direction.
Fig. 10. (a) Model magnetic field image as in figure 6.

Visualisations of the x-component of the calculated supercurrent density, along with line profiles across the lower boundary, with (b) no filtering, and a Hanning filter with (c) at the optimal value of 80 and (d).

Bright regions in the $J_x$ images indicate a current to the left, dark regions indicate current to the right, grey indicates no current. The line profiles model calculated current vs. distance into a sample and immediately outside the sample edge.
4.3 Indicator Film Characterisation

The current profiling technique was extended to include Laviano's in-plane field correction discussed in section 2.4. Before this could be achieved, however, knowledge of the magnetic properties of the indicator films was required. Specifically, the magnetic anisotropy field $B_A$ was to be determined from equation (15). For comparison, a value of $B_A = (80 \pm 5)$ mT was determined by Johansen et al. (1996) by application of an external $B_x$ field and a fitting process.

$$B_A = \frac{E_A}{M_0}$$ (15)

Here $M_0$ is the spontaneous magnetisation, which is the value of the magnetisation in the film with no applied field. $E_A$ is the anisotropy energy, which is the energy required to rotate the film's magnetic moment from its "easy" crystalline axis to its "hard" axis, i.e. from the most preferred direction for magnetic penetration to the least preferred direction (Kittel 8th ed., Buschow 2nd ed.). Both of these properties could be determined by taking characteristic magnetisation curves of the films.

Magnetic characteristics of the MO indicator films were determined using the Magnetic Property Measurement System (MPMS) sample magnetometer. Magnetisation in the film was plotted against applied magnetic field over a range of temperatures. This process was repeated for applied fields both parallel to and perpendicular to the plane of the film, thus demonstrating the anisotropy of the film. The film was assumed to be symmetric in the plane.

The magnetisation curves acquired with MPMS are shown in figure 11. Magnetisation was seen to increase linearly with applied magnetic field, as expected for values less than half the saturation magnetisation (Fratello et al. 2004). The saturation magnetisation was not determined for this film, as this is not crucial for determining the anisotropy field. The magnetisation continued to increase linearly with applied field up to fields of 0.2 Tesla. The indicator films are of course designed for such a linear response, to allow smooth rotation of light polarisation over the necessary magnetic field range. The maximum magnetic field used for MOI is less than 0.15T.

The anisotropy energy of the film is related to the relative gradients of its magnetisation curves in the easy and hard directions, as shown in figure 8a. The parallel direction was seen here to be the film's easy axis, as the gradient is steeper, and the film will therefore reach magnetic saturation faster in the parallel direction. The magnetisation gradient for the film perpendicular to the applied field was $3.33 \times 10^8$ A²/N. The gradient for the film parallel to the applied field was $3.96 \times 10^8$ A²/N.

Each direction should have the same value for saturation magnetisation, though the shape of the curve between zero and saturation will be different for different directions (Kittel 8th ed.).

The linear relation between magnetisation and applied field was extrapolated to zero to give the spontaneous magnetisation of the indicator film. Spontaneous magnetisation was most apparent at room temperature, and the value is seen from figure 8b to be $M_0 = 8 \times 10^{-12}$ A/m.
directed in the plane of the film. The component of spontaneous magnetisation perpendicular to the plane of the film was smaller by two orders of magnitude, so could be neglected.

At liquid helium temperatures, as used for MOI of superconductors, the magnetisation curves were steeper, meaning that the spontaneous magnetisation was less easily seen from the graphs. However, measurements at 4.2K and 10K also gave similar values within uncertainty. The spontaneous magnetisation was therefore seen to be temperature-independent within the tested range.

For a more complete discussion and mathematical analysis of the magnetic properties of indicator films see for example Fratello et al. (2004).
4.4 TESTING OF THE IN-PLANE FIELD CORRECTION

Laviano’s in-plane field correction was developed to remove the distorting effects of the x and y magnetic field components on magneto-optical images and their associated current maps, as discussed in section 2.4. The corrective procedure was tested on several MO images of superconductors. Actual MO images were used rather than the simple model that was utilised for the first-order current calculation. This model could not be used here as it does not aim to represent in-plane fields but only the z component of the magnetic field, as an ideal indicator film would show.

The correction is an iterative procedure, involving a cycle of successive approximations of \( B_x \) and \( B_y \) from \( J_x \) and \( J_y \) using equations (7):

\[
B_x(k_x, k_y, h) = \frac{\mu_0}{2} e^{-hk} \sinh \left( \frac{d}{2} k \right) J_y(k_x, k_y)
\]

\[
B_y(k_x, k_y, h) = -\frac{\mu_0}{2} e^{-hk} \sinh \left( \frac{d}{2} k \right) J_x(k_x, k_y)
\]

then \( B_z \) from \( B_x \) and \( B_y \) using equation (11):

\[
B_z^{(n+1)}(n) = \left( 1 + \sqrt{\frac{B_x^2 + B_y^2}{B_A}} \right) B_z^{(n)}
\]

and finally \( J_x \) and \( J_y \) from \( B_z \) using equations (13):

\[
J_x(k_x, k_y) = -\frac{2iB_x(k_x, k_y, h)e^{hk}k_y}{\mu_0 \sinh \left( \frac{d}{2} k \right)}
\]

\[
J_y(k_x, k_y) = \frac{2iB_y(k_x, k_y, h)e^{hk}k_x}{\mu_0 \sinh \left( \frac{d}{2} k \right)}
\]

Then the process may be repeated. This procedure was simplified slightly in this research by substituting equations (13) into equations (7), to give:

\[
\tilde{B}_x = -\frac{i k_x B_z}{k} \quad (16.1)
\]

\[
\tilde{B}_y = -\frac{i k_y B_z}{k} \quad (16.2)
\]

The iteration procedure is then reduced to a two-step process: successive approximations of \( B_x \) and \( B_y \) from \( B_z \) (equations 16), then \( B_z \) from \( B_x \) and \( B_y \) (equation 11). The quantified magneto-optical image data is taken as the first approximation of \( B_z \). The current distribution was calculated separately at each iteration, to allow visual assessment of the current mapping process at each iterative step.
A series of current map images was created with increasing numbers of iterative applications of the correction, for a particular YBCO sample. These were then analysed visually and by taking line profiles across the images. The results are shown in figure 12 for up to ten iterations. According to previous results obtained by Laviano (2002), the linear current profiles should be much flatter in the flux-penetrated area of a superconductor after the in-plane correction and should drop to zero outside the film, more closely resembling the Bean model shown in figure 2.

Similar results were observed in this research, but only when the corrective procedure was applied once or twice to an image. This can be seen for the first two images of figure 12. The in-plane field correction was seen to diverge from the expected physical situation for a large number of iterations. The features of any image would become increasingly distorted as the process was applied a greater number of times, until the image was reduced to a grey monotone.

Such divergence from the physical situation is actually not surprising, considering equation (11):

$$B_z^{(n+1)} = \left(1 + \frac{\sqrt{B_x^2 + B_y^2}}{B_A}\right)B_z^{(n)}$$

and equations (16):

$$\tilde{B}_x = -\frac{i k_x \tilde{B}_z}{k}$$

$$\tilde{B}_y = -\frac{i k_y \tilde{B}_z}{k}$$

For equation (11) to converge to a single value of $B_z$, $B_x$ and $B_y$ must become small for a large number of iterations. Since $B_A$ is fixed, this can be seen from a simple application of the comparison test for series convergence. Because the square root is always positive, it is seen that $B_z^{(n+1)}$ will always be greater than $B_z^{(n)}$ unless $B_x$ and $B_y$ approach zero.

However, equations (16) show that $B_x$ and $B_y$ are proportional to $B_z$ (in Fourier space). Therefore, $B_z$ must also become small as do $B_x$ and $B_y$ for convergence. Hence, it is seen mathematically that $B_z$ will never reach a stable value (unless this value is $B_z = 0$). If the value of $B_z^{(n)}$ has magnitude larger than zero, then $B_z^{(n+1)}$ will be even larger, and successive iterations will continue to give larger values. Thus, no finite value of $B_z$ will ever be determined from this procedure.

There are therefore only two mathematical possibilities for a large number of iterations of this correction: divergence of the data, predicting infinite magnetic fields, or convergence to a flat value of zero field for the whole image. Of these possibilities, divergence of the values of $B_z$ was observed in all applications. In order to avoid such unphysical behaviour, all further current mapping used no more than one application of the in-plane field correction procedure.
Fig. 12. Images of the “corrected” magnetic field, calculated total current density and linear current profiles for different numbers of iterations of the in-plane field correction procedure on a square YBCO sample at 0.12T. The yellow line in the current density image for 1 iteration is the line along which the current profiles were taken. The grey value of the current profiles is proportional to calculated current. The profiles should ideally resemble the Bean model curve in figure 2.
4.5 **Current Profiles**

Current profiles were created for several samples of regular geometries using the method developed in this research, which is described in section 3.3.

Ignoring Laviano’s in-plane field correction, the MO images acquired were taken to be representations of $B_z$ alone. With this simple (and common) assumption, mathematical analysis was applied to the MO images acquired in order to calculate current components $J_x$, $J_y$, the total current $J$ and the magnetisation $M_z$. For each $B_z$ image, images were created of the simulated current components and total current, and a contour plot of the magnetisation. The magnetisation contour lines also indicate the current flow direction, as indicated in section 2.3. Thus, the current flow could be visualised for each superconducting sample analysed.

The process of analysis is identical to that which was applied to the model MO image in section 4.2.

Figures 13 through 16 show the MO images and images of the calculated current distributions for square, rectangular and circular samples. Current flow patterns are well understood for regular geometries such as these, so the results obtained can be verified by comparison to the literature.

The components of current flow in the x and y directions are also shown for the rectangular sample in fig. 13 and for the circular sample in fig. 16. Current flows around a defect are mapped as magnetisation plots in fig. 15. Fig. 16 shows a few sources of error in current mapping such as defects in the indicator film, dirt on the camera and a generally faint image.
Fig. 13. (a) Magneto-optical image of a rectangular sample of zero-field cooled thin film YBa$_2$Cu$_3$O$_{7-\delta}$ (YBCO) under applied magnetic field 0.02T. This is a cropped, greyscale version of figure 5a.

Calculated images of (b) the x current component, (c) the y current component, and (d) the total current.

In (b) and (c), bright regions indicate current in the positive x and y directions respectively, and dark regions indicate current in the negative direction.

In (d), brighter regions indicate higher total current.
Fig. 14. Magneto-optical images of a rectangular sample of YBCO (above) along with calculated total current density images (below). These are the same images as shown in figure 5. (a) 0.02T applied magnetic field. When magnetic field is excluded from the sample, Meissner currents are seen to flow around the edges. (b) 0.09T applied field. Magnetic flux is seen to penetrate the sample, the current image shows uniform current along the penetration depth, dropping to zero in the middle, resembling the Bean model. (c) The residual flux (and residual current) after switching off the 0.09T applied field. Current loops are seen to flow near each edge of the sample, creating the residual flux pattern.

Defects are seen at the top and bottom of the sample, no current is observed in the defects for any of the images.
Fig. 15. MO images of a square sample of YBCO with a prominent defect. Magnetisation contour plots are superimposed in red. These show the flow direction of the current. The applied magnetic fields are (a) 0.02T, (b) 0.05T and (b) 0.09T. (d) shows the residual flux (and residual current) after switching off the 0.09T applied field.

The current flow maps were created analytically using the created current calculator program, then Paint.NET was used to add colour. The current is observed to flow around the sample and intensify toward the edges for images (a) through (c), as expected. It is seen in all images that no current flows across the defect.

Some unphysical current can be seen however, apparently flowing outside the sample, especially in images (a) and (d).
Fig. 16. (a) MO image of a circular YBCO sample with calculated current maps of (b) the x current component, (c) the y component and (d) total current density. Note the unphysical bright regions seen at the top and bottom of the total current density image.

These regions lead to a distorted current flow map (e).

A speck on the camera is visible as a white blob near the top of the images, and several indicator film defects are visible as black spots. These produce unphysical bright and dark spots in the calculated current image.
5 Conclusion

A procedure for magneto-optical imaging and visual analysis of magnetic fields and current flows in superconductors was created, and was applied to test images and samples. The major constituents of this work were: the creation of LabVIEW programs to control the imaging procedure; collecting images of the magnetic field in superconducting samples; and calculation of current flow maps from these images.

The user interface that was created for the imaging process is fairly intuitive and complete, incorporating temperature and magnetic field regulation as well as live MO image feed through the microscope and saving of images. It also allows calibration of images with magnetic field for quantification.

One missing feature is the possibility to adjust brightness and contrast of images as they are captured. This is available in other MO image capture programs and is useful to better see the magnetic features, especially in greyscale images. However, such adjustments of the pixel look-up tables (LUT) lead to increased non-linearity between magnetic field and grey scale value, and are therefore detrimental to quantification. It was deemed more advantageous in this application to keep the LUT linear for quantitative analysis than to allow users to change brightness and contrast for visual appeal.

The quantification and current mapping program created was also successfully implemented. It is able to accurately determine quantitative magnetic field information whenever calibration data is available, though the alternative approximation method could be refined. It uses a linear approximation when calibration data is not available, while it could instead take into account the more complicated dependence in the calibration equation (equation 13). As discussed in that section, this method is not as accurate as the direct empirical calibration, but it has the advantage of being applicable to any image, not just those taken along with the calibration procedure developed here.

The current mapping section of the program was tested with model images and images of samples with regular geometries. Images could be produced of each current component and of the total current density. These appeared accurate for both models and real MO images, and were consistent with current images produced by other groups using a similar procedure. Magnetisation contour maps were also produced, which are useful for showing the current flow patterns in a sample.

The current maps produced were generally accepted as accurate representations of the current in the sample. These were used to map current flows around samples of regular geometries and to show the effects of defects on this current flow. There are seen to be several phenomena leading to inaccurate current maps such as specks on the camera or indicator film defects. These are most pronounced for fainter MO images. This could be minimised by subtraction of a background image of the indicator film from the MO image before current calculation. Some calculations also produced images containing non-physical currents outside the sample. These led to distortion of the current map contours.
The iterative in-plane field correction proposed by Laviano et al. (2002) was designed to correct for such unphysical calculated currents whenever they are caused by in-plane field components. However, this procedure proved ineffective in correcting images. Instead it produced wildly unexpected and unphysical results. It was also shown mathematically that application of many iterations of the corrective procedure will give non-physical non-finite values for the magnetic field. There are three possible causes for this divergent behaviour:

1. There has been an error in the mathematical interpretation used in this research, or the implementation of the procedure.
2. There was an error in the equations used for reference, in the work of Laviano or Roth et al. (1988).
3. The proposed method is ineffective, and is not capable of producing physical results. The calculated magnetic field values will always diverge from the physical current distribution.

It was also seen, by current calculations on a model magnetic field image, that unphysical currents were observed outside the sample even when no in-plane field components were present. This implies that these currents may have some cause other than the neglected field components, which has yet to be identified specifically, but seems to be intrinsic to the current calculations.

The method has proven generally effective in allowing the acquisition of magneto-optical images and calculating current flow from these. The visualisation of current flow in superconductors has many applications which are soon to be explored, though there are still several ways in which the procedure may be refined or improved. The applications and improvements of this technique are discussed in the next section.
6 Future Work

The procedure described here was developed for use in further research - to acquire high-speed, high-resolution magneto-optical images of superconducting samples, to quantify these and to calculate current profiles.

The MOI and current mapping procedure can be utilised in simple tasks such as checking samples for defects, calculating the critical current and observing magnetic penetration in samples. However, the major application would be the analysis of current flow patterns in samples with complex geometries, such as long superconducting wires, Josephson junctions, current-limiting defects, nano-scale bridges and other samples of irregular geometry.

Also important is the observation of characteristically short timescale magnetic events in these irregular samples as well as in regular superconductors. Using high-speed imaging and current calculation, it will be possible to visualise the current associated with such events as vortex motion during flux penetration and flux creep, and the response of vortices to external stimuli such as movement of domain boundaries on shorter timescales than have been previously achieved.

Aside from the many applications of this technique, there are several avenues of future work in which the procedure described here could be improved:

The current mapping program could be given greater functionality, including the ability to subtract a background image from magneto-optical images before quantification. The calculated current flow information could be displayed more clearly. Also, brightness and contrast controls could be added and the calibration procedure improved, as discussed earlier. The program could also be de-bugged made more user-friendly in general, though these are continual issues for any computer program created.

The main extension to the program would be another attempt at application of a procedure to correct for unwanted magnetic field components. Attempts to apply Laviano’s in-plane field correction proved unsuccessful, as discussed in sections 4.4 and 5. Paturi (2005) presents an alternative in-plane field correction which may be a superior technique in terms of computation time, and it may produce better results. Paturi’s correction was not implemented here, simply for lack of time, though it would be very useful to implement it in the future.

A brief attempt at light filtering was made to select only the light for which the indicator film is most sensitive. As discussed in section 3.1, however, this was abandoned early in the research due to the significant reduction in light intensity. To overcome this issue, plans have been made to replace the light source on the microscope with a brighter source. A more serious attempt at filtering may be made after this is installed.

Another reason for replacing the light source is to provide a higher signal-to-noise ratio for the camera. The Photron camera currently used is much preferable to the old Leica camera in terms of its high speed, high resolution and linear response to light intensity; but this camera requires a brighter light source to capture non-noisy images at such high frame rates. It is incapable of acquiring images at a slower shutter speed due to hardware restrictions.
Only static images of magnetic fields in superconductors were used for calculations in this research. However, the apparatus used is also capable of recording magneto-optical video, and the camera can record at up to 120,000 frames per second. Therefore, the procedure and the LabVIEW programs developed in this research can later be utilised for dynamic imaging of magnetic flux behaviour in superconductors. This could possibly be carried out to a higher temporal resolution than has been previously achieved.

It is also possible to calculate the electric field within samples from dynamic magnetic field data. An electric field $E$ is generated by the motion of magnetic vortices during flux penetration (Brandt 1995) and flux creep (Jooss et al. 2003). The electric field is related to the temporal rate of change of the $B$ field through one of the Maxwell equations:

$$\nabla \times E = \frac{\partial B}{\partial t}$$  \hspace{1cm} (17)

The geometry of the $E$ field resembles that of the $B$ field, though the $E$ field lines are piecewise linear and sharp discontinuity lines are present, even in regions where the current flow appears homogeneous. Unlike the magnetic field, the electric field in a superconducting sample is independent of its thickness (Brandt 1995).

The electric field in superconducting samples was not considered in this research, as only static magnetic field images were considered, while the electric field is determined by the time evolution of the magnetic field. If this procedure were to be extended to high-speed analysis of magneto-optical video, it would be possible to calculate the electric field in a superconducting sample during flux penetration.

Overall, the technique researched here may allow high-speed analysis of the behaviour of the magnetic field, (correct) supercurrent distribution and electric field in a superconductor during short timescale events, which may eventually provide new insights into the nature of superconductivity and its applications. By developing a method for high-speed acquisition and quantitative analysis of magneto-optical images, this research paves the way for such endeavours to be undertaken in the near future.
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APPENDIX A

LITERATURE REVIEW ON BIOT-SAVART LAW INVERSION

Magnetic field is related to current by the Biot-Savart law (equation 2), and inversion of this law is a standard procedure for calculation of current from measurements of magnetic field at a distance from the source. It is also commonplace to apply this technique to determination of current profiles from magneto-optical images of superconductors. However, there are several variations on the mathematical analysis applied to invert the Biot-Savart law in the literature and on the final equations obtained, and these are not all consistent with one another.

The final equations used for current calculation in this research were equations (5):

\[
B_z(k_x, k_y, h) = \frac{i \mu_0}{2} \int_x(k_x, k_y) e^{-hk} \sinh \left( \frac{d}{2} k \right)
\]

\[
\bar{J}_y(k_x, k_y) k_y + \bar{J}_x(k_x, k_y) k_x = 0
\]

There are several variations on these equations in the literature. None of these are particularly noteworthy alone, but the sheer volume of possible mistakes in published papers in this area is significant. There is much potential for others entering in the field to accrue misunderstandings when sorting through the literature on the Biot-Savart law inversion.

For example Laviano (2002) has the positions of \(k_x\) and \(k_y\) reversed in equation (5.2); Johansen (1997) uses \(1 - e^{-kd}\) in place of \(\sinh \left( \frac{d}{2} k \right)\); Jooss (1998) is missing the factor of 2 in equation (5.1); while Roussel (2007) has a factor of \(\mu_0\) present on the right hand side of this equation.

Some of these are probably simple typographic errors, while others represent slight differences in the inversion process used and are mathematically similar.

The equations presented here and used in this research were taken to be the most accurate inverse Biot-Savart law and current continuity equations. They were found to be the most consistent with the original work of Roth and with the basic physical laws on which they are based.
APPENDIX B

CURRENT CALCULATION IN LabVIEW

This appendix gives an insight into the internal workings of the supercurrent calculation program created for this research. The actual equations used in the LabVIEW Current Mapper program are applied individually to each element in an array of magnetic field values in Fourier space.

The MO image is first converted into an array of pixel greyscale values, to which a calibration function is applied to obtain an array of magnetic field values. A Fourier transform is then applied to this array. The Fourier array of magnetic field values is then analysed in an element-wise manner, using two embedded self-indexing for loops. The desired quantities are then calculated from the array of B_z values (called B here) along with several parameters. The equations applied to the magnetic field array are applied in a MathScript node as follows.

\[
\begin{align*}
J_x &= -(2iB_{\text{ky}}\exp(hk))/(u_0\sinh(kd/2)) \\
J_y &= (2iB_{\text{kx}}\exp(hk))/(u_0\sinh(kd/2)) \\
M &= (2B\exp(hk))/(u_0\sinh(kd/2)) \\
B_x &= -i\text{kx}B/k \\
B_y &= -i\text{ky}B/k
\end{align*}
\]

These equations are MathScript representations of equations (13), (8) and (10).

This MathScript code is modified when Hanning filtering is applied, using equation (9). For the case of \(k < k_{\text{max}}\), the following additional lines are added to the start:

\[
\begin{align*}
W &= 0.5(1 + \cos(\pi k/k_{\text{max}})) \\
B &= W*B
\end{align*}
\]

For \(k > k_{\text{max}}\), the frequency is cut off, so the code in this case is simply:

\[
\begin{align*}
J_x &= 0 \\
J_y &= 0 \\
M &= 0 \\
B_x &= 0 \\
B_y &= 0
\end{align*}
\]

Therefore, for each \(B_z\) element, there is one \(J_x\) element produced. These elements are combined into an array of the same dimensions as the original \(B_z\) array, and an inverse Fourier transform is applied to obtain a \(J_x\) array in spatial co-ordinates. A similar procedure is applied for the other four quantities calculated in Fourier space.

The \(J_x, J_y\) and total \(J\) arrays are then scaled to a suitable range for representation as a greyscale image; a contour map is created for \(M_z\); and \(B_x\) and \(B_y\) are used for the in-plane correction procedure, if it is applied.