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Abstract

© The Institution of Engineering and Technology 2020. This study investigates the admissibility analysis and stabilisation problems for singular interval type-2 Takagi- Sugeno fuzzy systems with time delay. A generalised integral inequality method is used to obtain the delay-dependent condition. The criteria for admissibility analysis and controller synthesis are given in terms of linear matrix inequalities. In order to reduce the conservatism of the system, some free weighting matrices and advanced integral inequalities are introduced. Finally, two illustrative examples are exhibited to demonstrate the effectiveness of the proposed method.

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Admissibilization of singular interval type-2 Takagi-Sugeno fuzzy systems with time delay

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Abstract: This paper investigates the admissibility analysis and stabilization problems for singular interval type-2 Takagi-Sugeno fuzzy systems with time delay. A generalized integral inequality method is used to obtain the delay-dependent condition. The criteria for admissibility analysis and controller synthesis are given in terms of linear matrix inequalities. In order to reduce the conservatism of system, some free weighting matrices and advanced integral inequalities are introduced. Finally, two illustrative examples are exhibited to demonstrate the effectiveness of the proposed method.

1 Introduction

Singular systems have been extensively studied by many scholars in the last decades due to their great theoretical and practical research value. As is known to all, the regularity and non-impulsiveness or causality make the study of singular systems more difficult and complex than normal systems [1]. Up to now, it's noted that many fruitful results of singular systems have been achieved and a relatively perfect research system has been formed, for example, exponential stability [2], stabilization ([3], [4]), α-dissipativity analysis [5], robust H_∞ control [6], reachable set estimation [7], event-triggered sliding mode control [8] and the references therein. Singular systems play an important role in many practical fields because of their accuracy and naturalness. Some common areas include constrained robot manipulators [9], vibration attenuation model of aircraft transmission system [10], Leontief economic system [11], and so on. Extensive research and application of singular systems make a great progress to the development of society.

Takagi-Sugeno fuzzy model is very effective to realize the modeling and control of complex nonlinear systems, which is usually based on the methods of sector nonlinearity and local approximation [12]. Up to now, a lot of learned researchers have devoted their attentions to T-S fuzzy singular systems ([13]-[20]). The authors in [13] proposed a new stabilization condition for fuzzy singular systems by using the method of delay partitioning to deal with the time-varying delay. In [14], a robust H_∞ controller was designed under the conditions of norm-bounded parametric uncertainties and actuator saturation for a class of discrete fuzzy singular systems. Considering the parameter uncertainties of state matrix and derivative matrix, the authors in [15] designed the robust non-fragile proportional plus derivative state feedback controller to stabilize the closed-loop system. The authors in [16] investigated the robust fault estimation problem of fuzzy singular systems and gave an improved stability criterion in terms of strict LMIs. A fuzzy observer was designed and exponential stability of the closed-loop fuzzy singular systems was studied by using the methods of augmented system and compact form in [17]. In addition, sliding mode control, adaptive sliding mode control and passivity control, and so on, were also investigated in ([18]-[20]).

It should be noted that all the above mentioned researches about fuzzy systems are type-1 fuzzy systems, which don't contain the uncertain information in the fuzzy membership functions. However, we usually need to consider uncertainties and nonlinearities in practical systems, so it's very necessary to study type-2 fuzzy systems due to the great advantages in describing system uncertainties. Stability analysis and controller design of the interval type-2 fuzzy systems were studied for the first time in [21]. The authors in [22] designed a new fuzzy controller by considering the imperfect premise match between the IT2 fuzzy system and the controller. In order to further reduce the conservatism of the system, a very important equation was introduced in [23]. A detailed study about membership function of the IT2 fuzzy system was shown in [24], which was very helpful for the researchers who intended to study the IT2 fuzzy system. Admissibility analysis of the IT2 singular fuzzy system was investigated in [25], in which the state feedback controller and static output feedback controller were designed to stabilize the closed-loop IT2 fuzzy system. Meanwhile, some important research results about IT2 polynomial fuzzy systems have been investigated successfully, such as, controller design [26], stabilization analysis [27], output-feedback tracking control [28], sampled-data output-feedback tracking control [29], etc.

Time delay must be paid enough attention in practical systems, which has a great effect on the performance and stability of systems. How to deal with the time delay problem has always been a research focus in the control field. A very important point to deal with time delay problem is to reduce conservatism and introduce as few slack matrices as possible at the same time, and thus many approaches have been proposed. The common methods include Park's inequality [30], Jensen inequality [31], Wirtinger-based inequality [32], auxiliary function-based integral inequality [33], Bessel-Legendre inequality [34], generalized integral inequality [35], delay partitioning technique [36], etc. According to [35], we know that Jensen inequality, Wirtinger-based inequality, auxiliary function-based integral inequality, Bessel-Legendre inequality can be regarded as special cases of generalized integral inequality with $k = 0$ and different values of l. The study of time delay systems has gone deep into various types of systems, such as, normal time-delay system ([30]-[35], [37]), T-S fuzzy systems [38], T-S fuzzy singular systems ([13], [14], [16], [19], [20], [36]), interval type-2 fuzzy systems ([39], [40]). Delay partitioning technique was adopted to handle the delay term in [36]. The author in [41] investigated the sliding mode observer design problems for uncertain time-varying delay singular systems, in which the idea of delay partition was also used.

The aforementioned studies about IT2 fuzzy systems are nearly about normal state-space systems and there are few results about IT2 fuzzy singular systems at present. At the same time, the application of generalized integral inequality is an effective method to deal with the time delay of system. Motivated by the above reasons, the admissibility analysis and stabilization problems of interval type-2 fuzzy singular system are investigated in this paper. By using the generalized integral inequality method and constructing a proper augmented Lyapunov-Krasovskii function, the sufficient conditions of ensuring the IT2 fuzzy singular system to be admissible are obtained in terms of strict LMIs. Finally, two illustrative examples are shown to prove the effectiveness of the given method.

Notation: Throughout this paper, the notations are standard. $P > 0$ denotes that matrix P is symmetric and positive definite. The superscripts 'T' and '-1' represent the transpose and inverse of a matrix. I and 0 mean identity matrix and zero matrix with appropriate dimensions, respectively. $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote the n-dimensional Euclidean space and the set of $n \times m$ real matrices. The symbol '$*$' stands for the symmetric term of a symmetric matrix and '$\otimes$' denotes Kronecker product. If the dimension of a matrix is not clearly stated, we assume that it has an appropriate dimension.

2 Problem Formulation and Preliminaries

The IT2 T-S fuzzy singular system with time delay is described as follows:

Plant Rule i: **IF** $z_1(x(t))$ is W_{i1} and $\cdots$ and $z_\psi(x(t))$ is $W_{i\psi}$, **THEN**

$$\begin{cases} E\dot{x}(t) = A_i x(t) + A_{hi}x(t-h) + B_i u(t), \\ x(t) = \varphi(t), t \in [-h, 0], i = 1, 2, \ldots, s, \end{cases} \tag{1}$$

where $z_k(x(t))$ and W_{ik} are premise variable and fuzzy set, respectively, $k \in \{1, 2, \ldots, \psi\}$, ψ represents the number of premise variables; s stands for the number of IF-THEN fuzzy rules; $x(t) \in \mathbb{R}^n$ denotes system state and $u(t) \in \mathbb{R}^m$ is control input vector; The matrix E is singular and satisfies rank$(E) = q < n$; A_i, A_{hi} and B_i are system matrices with appropriate dimensions; h denotes the constant delay and $\varphi(t)$ is initial condition of system. The firing strength of the ith fuzzy rule can be derived as follows:

$$\Xi_i(x(t)) = [\underline{\omega}_i(x(t)),\ \bar{\omega}_i(x(t))], \tag{2}$$

where

$$\begin{aligned} &\underline{\omega}_i(x(t)) = \sum_{k=1}^{\psi} \underline{\mu}_{W_{ik}}(z_k(x(t))) \geq 0, \\ &\bar{\omega}_i(x(t)) = \sum_{k=1}^{\psi} \bar{\mu}_{W_{ik}}(z_k(x(t))) \geq 0, \\ &\bar{\mu}_{W_{ik}}(z_k(t)) \geq \underline{\mu}_{W_{ik}}(z_k(x(t))) \geq 0, \\ &\bar{\omega}_i(x(t)) \geq \underline{\omega}_i(x(t)) \geq 0, \end{aligned}$$

in which $\underline{\mu}_{W_{ik}}(z_k(t)) \in [0, 1]$ and $\bar{\mu}_{W_{ik}}(z_k(t)) \in [0, 1]$ represent the lower and upper grade of membership of $z_k(t)$ in $\underline{\mu}_{W_{ik}}$ and $\bar{\mu}_{W_{ik}}$, respectively. Then, the global model of type-2 T-S fuzzy singular system with time delay is obtained:

$$\begin{cases} E\dot{x}(t) = \sum_{i=1}^{s} \omega_i(x(t))[A_i x(t) + A_{hi}x(t-h) + B_i u(t)], \\ x(t) = \varphi(t), t \in [-h, 0], \end{cases} \tag{3}$$

where

$$\begin{aligned} &\omega_i(x(t)) = \underline{\upsilon}_i(x(t))\underline{\omega}_i(x(t)) + \bar{\upsilon}_i(x(t))\bar{\omega}_i(x(t)),\ \forall i \\ &\textstyle\sum_{i=1}^{s} \omega_i(x(t)) = 1,\ 0 \leq \underline{\upsilon}_i(x(t)) \leq 1, \\ &0 \leq \bar{\upsilon}_i(x(t)) \leq 1,\ \underline{\upsilon}_i(x(t)) + \bar{\upsilon}_i(x(t)) = 1, \end{aligned}$$

in which $\underline{\upsilon}_i(x(t))$ and $\bar{\upsilon}_i(x(t))$ are nonlinear functions not necessarily be known but exist.

For the sake of stabilizing the closed-loop type-2 fuzzy singular system, the state feedback controller with s fuzzy rules is designed as follows:

Controller Rule j: **IF** $y_1(x(t))$ is M_{j1}, and $\cdots$ and $y_\psi(x(t))$ is $M_{j\psi}$, **THEN**

$$u(t) = K_j x(t), \tag{4}$$

where $y_k(x(t))$ and M_{jk} represent the premise variable and fuzzy set of the jth fuzzy rule of the controller, $j = 1, 2, \ldots, s$, $k = 1, 2, \ldots, \psi$; s and ψ denote the number of fuzzy rules and premise variables, respectively; K_j is the state feedback gain matrix of the jth fuzzy rule. The firing strength is shown as the following set:

$$\Theta_j(x(t)) = [\underline{\theta}_j(x(t)),\ \bar{\theta}_j(x(t))],\ j = 1, 2, \ldots s \tag{5}$$

where

$$\begin{aligned} &\underline{\theta}_j(x(t)) = \sum_{k=1}^{\psi} \underline{\mu}_{M_{jk}}(y_k(x(t))) \geq 0, \\ &\bar{\theta}_j(x(t)) = \sum_{k=1}^{\psi} \bar{\mu}_{M_{jk}}(y_k(x(t))) \geq 0, \\ &\bar{\mu}_{M_{jk}}(y_k(x(t))) \geq \underline{\mu}_{M_{jk}}(y_k(x(t))) \geq 0, \\ &\bar{\theta}_j(x(t)) \geq \underline{\theta}_j(x(t)) \geq 0, \end{aligned}$$

in which $\underline{\mu}_{M_{jk}}(y_k(t)) \in [0, 1]$ and $\bar{\mu}_{M_{jk}}(y_k(t)) \in [0, 1]$ denote the lower and upper grade of membership of $y_k(t)$ in $\underline{\mu}_{M_{jk}}$ and $\bar{\mu}_{M_{jk}}$, respectively. The overall control input of the system is shown as follows:

$$u(t) = \sum_{j=1}^{s} \theta_j(x(t)) K_j x(t), \tag{6}$$

where

$$\begin{aligned} &\theta_j(x(t)) = \frac{\underline{\gamma}_j(x(t))\underline{\theta}_j(x(t)) + \bar{\gamma}_j(x(t))\bar{\theta}_j(x(t))}{\sum_{j=1}^{s}(\underline{\gamma}_j(x(t))\underline{\theta}_j(x(t)) + \bar{\gamma}_j(x(t))\bar{\theta}_j(x(t)))} \geq 0, \\ &\sum_{j=1}^{s} \theta_j(x(t)) = 1, 0 \leq \underline{\gamma}_j(x(t)) \leq 1, \\ &0 \leq \bar{\gamma}_j(x(t)) \leq 1,\ \underline{\gamma}_j(x(t)) + \bar{\gamma}_j(x(t)) = 1, \end{aligned}$$

in which $\bar{\gamma}_j(x(t))$ and $\underline{\gamma}_j(x(t))$ stand for predefined functions. For brevity, we will use ω_i and θ_j to replace $\omega_i(x(t))$ and $\theta_j(x(t))$ throughout the rest contents of this paper. Combine the IT2 fuzzy singular system (3) with fuzzy controller (6), we have

$$E\dot{x}(t) = \sum_{i=1}^{s}\sum_{j=1}^{s} \omega_i\theta_j[(A_i + B_iK_j)x(t) + A_{hi}x(t-h)]. \tag{7}$$

Remark 1. *In the IT2 fuzzy systems, it is very difficult to design fuzzy controller if the premise variables between the fuzzy model and controller are not perfectly matching. To handle this problem, the authors in [24] and [27] firstly proposed imperfectly premise matching concept and membership-function-dependent techniques.*

In order to get the main results, the following definition and lemmas are shown to prove the sufficient criterions of admissibility analysis and stabilization for IT2 T-S fuzzy singular system with time delay. Consider the following linear singular system with time delay:

$$\begin{cases} E\dot{x}(t) = Ax(t) + A_h x(t-h), \\ x(t) = \varphi(t), t \in [-h, 0], \end{cases} \tag{8}$$

Definition 1. [1]
1) *The singular system in (8) is said to be regular if det$(sE - A)$ is not identically zero.*
2) *The singular system in (8) is said to be impulse-free if deg(det $(sE - A)) = \deg(E)$.*
3) *The singular system in (8) is said to be asymptotically stable, if for any $\varepsilon > 0$ there exists a scalar $\delta(\varepsilon) > 0$ such that for any compatible initial conditions $\varphi(t)$ satisfying $\sup\limits_{-h\leq t\leq 0} \|\varphi(t)\| \leq \delta(\varepsilon)$, the solution $x(t)$ of system in (8) satisfies $\|x(t)\| \leq \varepsilon$ for all $t > 0$. Moreover, $x(t) \to 0,\ t \to \infty$.*
4) *The singular system in (8) is said to be admissible if it is regular, impulse-free and asymptotically stable.*

Lemma 1. [35] *For a matrix $M \in \mathbb{R}^{n\times n} > 0$, scalars $a < b$, matrix $E \in \mathbb{R}^{n\times n}$, non-negative integral k, l and vector $z : [a, b] \to \mathbb{R}^n$, the following inequality holds for $i = 0, 1, \ldots, l$:*

$$\begin{aligned}&\textstyle\int_a^b w_k(s) z^T(s) E^T M E z(s) ds \geq (\int_a^b P_{l,k}(s) \otimes (Ez(s))ds)^T \\ &\times(\Psi_{l,k} \otimes M)(\textstyle\int_a^b P_{l,k}(s) \otimes (Ez(s))ds),\end{aligned} \tag{9}$$

where

$$w_k(s) = (s-a)^k,\quad e_i(s) = (s-a)^i,$$

$$p_{i,k}(s) = e_i(s) - \sum_{j=1}^{i} \psi_{j,k}(e_i(u)) p_{j-1,k}(s),$$

$$\psi_{j,k}(e_i(u)) = \frac{\int_a^b e_i(u) w_k(u) p_{j-1,k}(u) du}{\int_a^b w_k(u) p_{j-1,k}^2(u) du},$$

$$P_{l,k}(s) = w_k(s) col\{p_{0,k}(s), \ldots p_{l,k}(s)\},$$

$$\Psi_{l,k} = diag\left\{\frac{1}{\int_a^b w_k(u) p_{0,k}^2(u) du}, \cdots, \frac{1}{\int_a^b w_k(u) p_{l,k}^2(u) du}\right\}.$$

Remark 2. *Compared with Lemma 1 in [35], the Lemma 1 of this paper introduces the matrix E. The idea of proving is identical to Lemma 1 in [35]. The inequality (9) can be obtained if we will use $Ez(s)$ to replace $z(s)$.*

Remark 3. *Considering the conservatism and computational complexity of the system, $k = 0$ and $l = 2$ will be chosen to study the admissibility and controller design in this paper. By selecting $p_{0,0}(s) = 1$, $p_{1,0}(s) = (s-a) - \frac{(b-a)}{2}$ and $p_{2,0}(s) = (s-a)^2 - (b-a)(s-a) + \frac{(b-a)^2}{6}$ as in [35], we can get the following result:*

$$(b-a)\int_a^b \dot{z}^T(s) E^T M E \dot{z}(s) ds \geq \sum_{k=1}^{3} (2k-1) \Gamma_k^T(z) M \Gamma_k(z), \tag{10}$$

where

$$\Gamma_1(z) = Ez(b) - Ez(a),$$

$$\Gamma_2(z) = Ez(b) + Ez(a) - \frac{2}{b-a}\int_a^b Ez(s) ds,$$

$$\Gamma_3(z) = Ez(b) - Ez(a) + \frac{6}{b-a}\int_a^b Ez(s) ds - \frac{12}{(b-a)^2}\int_a^b \int_s^b Ez(u) du ds.$$

Lemma 2. [42] *Matrix E can be described as $E = E_L E_R^T$ with E_L and E_R be full column rank. Denote P as a symmetric matrix satisfying $E_L^T P E_L > 0$, and matrix X is invertible. U with full row rank is the left null matrix of E and V with full column rank is the right null matrix of E, namely, $UE = 0$ and $EV = 0$. Then $PE + U^T X V^T$ is nonsingular and its inverse can be stated as:*

$$(PE + U^T X V^T)^{-1} = \bar{P} E^T + V \bar{X} U, \tag{11}$$

where $\bar{P}$ is symmetric and $\bar{X}$ is nonsingular such that

$$E_R^T \bar{P} E_R = (E_L^T P E_L)^{-1}, \bar{X} = (V^T V)^{-1} X^{-1} (U U^T)^{-1}.$$

Lemma 3. [43] *For matrices specified by Lemma 2, the following equations hold:*

$$(PE + U^T X V^T)^T E (PE + U^T X V^T)^{-1} = E^T. \tag{12}$$

$$(PE + U^T X V^T)^{-T} E^T (PE + U^T X V^T) = E. \tag{13}$$

3 Main results

In this section, we first exhibit the sufficient criteria of admissibility analysis for the singular type-2 T-S fuzzy systems; then based on parallel distributed compensation method, the state feedback controller is designed.

Theorem 1. *For constant delay h, the unforced singular type-2 fuzzy system in (3) with $u(t) = 0$ is admissible, if there exist symmetric and positive matrices P, Q, R, W_1, W_2 with appropriate dimensions for all $i \in \{1, 2, \cdots, s\}$, such that the following matrix inequality is feasible:*

$$T_i = \begin{bmatrix} T_{11} & T_{12} & T_{13} & T_{14} & hA_i^T R \\ * & T_{22} & T_{23} & T_{24} & hA_{hi}^T R \\ * & * & T_{33} & T_{34} & 0 \\ * & * & * & T_{44} & 0 \\ * & * & * & * & -R \end{bmatrix} < 0, \tag{14}$$

where U, X_{11} and V are defined in Lemma 2.

$$\begin{aligned}
T_{11} &= (P_{11}E + U^T X_{11} V^T)^T A_i + A_i^T (P_{11}E + U^T X_{11} V^T) \\
&\quad + E^T P_{12} E + E^T P_{12}^T E + hE^T P_{13} E + hE^T P_{13}^T E + Q \\
&\quad + h^2 W_1 + \tfrac{h^4}{4} W_2 - 9E^T RE, \\
T_{12} &= (P_{11}E + U^T X_{11} V^T)^T A_{hi} - E^T P_{12} E + 3E^T RE, \\
T_{13} &= hA_i^T P_{12} E + hE^T P_{22} E + h^2 E^T P_{23}^T E - hE^T P_{13} E \\
&\quad - 24E^T RE, \\
T_{14} &= \tfrac{h^2}{2} A_i^T P_{13} E + \tfrac{h^2}{2} E^T P_{23} E + \tfrac{h^3}{2} E^T P_{33} E + 30E^T RE, \\
T_{22} &= -Q - 9E^T RE, \\
T_{23} &= hA_{hi}^T P_{12} E - hE^T P_{22} E + 36E^T RE, \\
T_{24} &= \tfrac{h^2}{2} A_{hi}^T P_{13} E - \tfrac{h^2}{2} E^T P_{23} E - 30E^T RE, \\
T_{33} &= -h^2 E^T P_{23} E - h^2 E^T P_{23}^T E - 192E^T RE - h^2 W_1, \\
T_{34} &= -\tfrac{h^3}{2} E^T P_{33} E + 180E^T RE, \\
T_{44} &= -180E^T RE - \tfrac{h^4}{4} W_2,
\end{aligned}$$

$$P = \begin{bmatrix} P_{11} & P_{12} & P_{13} \\ * & P_{22} & P_{23} \\ * & * & P_{33} \end{bmatrix}.$$

Proof: Firstly, we will certify the regular and impulse-free characteristics of the unforced singular IT2 fuzzy system in (3) with $u(t) = 0$. As a result of rank$(E) = q < n$, it is certain that there exist two invertible matrices M and N that make the following equalities hold:

$$MEN = \begin{bmatrix} I_q & 0 \\ 0 & 0 \end{bmatrix}, \quad (15)$$

$$MA_iN = \begin{bmatrix} A_{i11} & A_{i12} \\ A_{i21} & A_{i22} \end{bmatrix}, \quad (16)$$

$$M^{-T}RM^{-1} = \begin{bmatrix} R_{11} & R_{12} \\ R_{12}^T & R_{22} \end{bmatrix}, \quad (17)$$

$$M^{-T}P_{11}M^{-1} = \begin{bmatrix} P_{1111} & P_{1112} \\ P_{1112}^T & P_{1122} \end{bmatrix}, \quad (18)$$

$$M^{-T}P_{1i}M^{-1} = \begin{bmatrix} P_{1i11} & P_{1i12} \\ P_{1i21} & P_{1i22} \end{bmatrix}, (i = 2, 3) \quad (19)$$

$$M^{-T}U^T = \begin{bmatrix} 0 \\ U_1^T \end{bmatrix}, V^TN = \begin{bmatrix} 0 & V_1^T \end{bmatrix}, \quad (20)$$

where U_1 and V_1 are nonsingular.

From (14), it is easy to know that $T_{11} < 0$. Considering $Q > 0,\ W_1 > 0,\ W_2 > 0$, we can obtain

$$\begin{aligned} &(P_{11}E + U^TX_{11}V^T)^TA_i + A_i^T(P_{11}E + U^TX_{11}V^T) \\ &+E^TP_{12}E + E^TP_{12}^TE + hE^TP_{13}E + hE^TP_{13}^TE \\ &-9E^TRE < 0. \end{aligned} \quad (21)$$

Pre-multiplying and post-multiplying (21) by N^T and N, then based on the equalities (15)-(20), we have

$$\begin{bmatrix} \oplus & \oplus \\ \oplus & A_{i22}^TU_1^TXV_1^T + V_1X^TU_1A_{i22} \end{bmatrix} < 0, \quad (22)$$

where $\oplus$ denotes the elements in a matrix that have no relationship with the following discussions. Then it is easy to get

$$A_{i22}^TU_1^TXV_1^T + V_1X^TU_1A_{i22} < 0, \quad (23)$$

Due to $\omega_i \geq 0$ and $\sum_{i=1}^{s} \omega_i = 1$, from (23), we have

$$\sum_{i=1}^{s} \omega_i(A_{i22}^TU_1^TXV_1^T + V_1X^TU_1A_{i22}) < 0. \quad (24)$$

From (24), we derive that $\sum_{i=1}^{s} \omega_iA_{i22}$ is nonsingular, so the singular IT2 fuzzy system in (3) with $u(t) = 0$ is regular and impulse-free.

Next, the stability of the singular IT2 fuzzy system in (3) is proved. The Lyapunov-krasovskii function is chosen as:

$$V(x_t) = V_1(x_t) + V_2(x_t) + V_3(x_t) + V_4(x_t), \quad (25)$$

where

$$\begin{aligned} V_1(x_t) &= v^T(t)Pv(t), \\ V_2(x_t) &= \int_{t-h}^{t} x^T(s)Qx(s)ds, \\ V_3(x_t) &= h\int_{t-h}^{t}\int_{\theta}^{t} \dot{x}^T(s)E^TRE\dot{x}(s)dsd\theta, \\ V_4(x_t) &= h\int_{t-h}^{t}\int_{\theta}^{t} x^T(s)W_1x(s)dsd\theta + \frac{h^2}{2}\int_{t-h}^{t}\int_{u}^{t}\int_{\theta}^{t} \\ &\quad x^T(s)W_2x(s)dsd\theta du, \\ v(t) &= \left[(Ex(t))^T, \int_{t-h}^{t}(Ex(s))^Tds, \int_{t-h}^{t}\int_{\theta}^{t}(Ex(s))^Tdsd\theta\right]^T. \end{aligned}$$

Defining $\eta(t) = \left[x^T(t), x^T(t-h), \frac{1}{h}\int_{t-h}^{t} x^T(s)ds, \frac{2}{h^2}\int_{t-h}^{t}\int_{\theta}^{t} x^T(s)dsd\theta\right]^T$ and taking the derivative of $V(x_t)$, we have

$$\begin{aligned} \dot{V}_1(x_t) &= 2\dot{v}^T(t)Pv(t) = 2T_1^T \begin{bmatrix} P_{11} & P_{12} & P_{13} \\ * & P_{22} & P_{23} \\ * & * & P_{33} \end{bmatrix} T_2 \\ &= 2T_1^T \begin{bmatrix} P_{11}E + U^TX_{11}V^T & P_{12}E & P_{13}E \\ * & P_{22}E & P_{23}E \\ * & * & P_{33}E \end{bmatrix} T_3, \end{aligned} \quad (26)$$

where

$$T_1 = \begin{bmatrix} E\dot{x}(t) \\ Ex(t) - Ex(t-h) \\ hEx(t) - \int_{t-h}^{t} Ex(s)ds \end{bmatrix},$$

$$T_2 = \begin{bmatrix} Ex(t) \\ \int_{t-h}^{t} Ex(s)ds \\ \int_{t-h}^{t}\int_{\theta}^{t} Ex(s)dsd\theta \end{bmatrix},$$

$$T_3 = \begin{bmatrix} x(t) \\ \int_{t-h}^{t} x(s)ds \\ \int_{t-h}^{t}\int_{\theta}^{t} x(s)dsd\theta \end{bmatrix},$$

$$\dot{V}_2(x_t) = x^T(t)Qx(t) - x^T(t-h)Qx(t-h), \quad (27)$$

$$\dot{V}_3(x_t) = h^2\dot{x}^T(t)E^TRE\dot{x}(t) - h\int_{t-h}^{t} \dot{x}^T(s)E^TRE\dot{x}(s)ds, \quad (28)$$

$$\begin{aligned} \dot{V}_4(x_t) &= h^2x^T(t)W_1x(t) - h\int_{t-h}^{t} x^T(s)W_1x(s)ds + \frac{h^4}{4} \\ &\times x^T(t)W_2x(t) - \frac{h^2}{2}\int_{t-h}^{h}\int_{\theta}^{t} x^T(s)W_2x(s)dsd\theta. \end{aligned} \quad (29)$$

By using Remark 2, from (28), we have

$$-h\int_{t-h}^{t} \dot{x}^T(s)E^TRE\dot{x}(s)ds \leq \eta^T(t)\Sigma\eta(t), \quad (30)$$

where

$$\Sigma = \begin{bmatrix} -9E^TRE & 3E^TRE & -24E^TRE & 30E^TRE \\ * & -9E^TRE & 36E^TRE & -30E^TRE \\ * & * & -192E^TRE & 180E^TRE \\ * & * & * & -180E^TRE \end{bmatrix}.$$

By using Jensen inequality, from (29), we can get the following results:

$$-h\int_{t-h}^{t} x^T(s)W_1x(s)ds \leq -(\int_{t-h}^{t} x(s)ds)^TW_1(\int_{t-h}^{t} x(s)ds), \quad (31)$$

$$\begin{aligned} &-\frac{h^2}{2}\int_{t-h}^{t}\int_{\theta}^{t} x^T(s)W_2x(s)dsd\theta \\ &\leq -(\int_{t-h}^{t}\int_{\theta}^{t} x(s)dsd\theta)^TW_2(\int_{t-h}^{t}\int_{\theta}^{t} x(s)dsd\theta). \end{aligned} \quad (32)$$

Combining (26) - (32), it is easy to obtain

$$\dot{V}(x_t) \leq \sum_{i=1}^{s} \omega_i\eta^T(t)(\bar{\mathrm{T}}_i + h^2\Gamma_i^TR\Gamma_i)\eta(t), \quad (33)$$

where $\bar{\mathrm{T}}_i$ is the first $4n \times 4n$ block of T_i in (14), $\Gamma_i = \begin{bmatrix} A_i & A_{hi} & 0 & 0 \end{bmatrix}$. If $\bar{\mathrm{T}}_i + h^2\Gamma_i^TR\Gamma_i < 0$, then we have

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$\dot{V}(x_t) < 0$. By using Schur complement, we can obtain $\bar{\mathrm{T}}_i + h^2\Gamma_i^T R\Gamma_i < 0$ is equal to (14). This completes the proof. □

Remark 4. *According to [44], when we use the tighter inequalities to reduce the conservatism of the system, the Lyapunov-Krasovskii functional plays an important role. A simple LKFs may not alleviate the conservatism even though we choose a tighter inequality. Therefore, we choose an augmented Lyapunov-Krasovskii functional to reduce the conservatism by introducing the integral term and the double integral term in $V_1(x_t)$ of (25).*

Furthermore, we will design the fuzzy controller to stabilize the singular type-2 T-S fuzzy system with time delay and the results are shown in the following Theorems.

Theorem 2. *For constant delay h, the singular type-2 fuzzy system in (7) is admissible if there exist matrices $P_{11} > 0$, $\begin{bmatrix} P_{22} & P_{23} \\ * & P_{33} \end{bmatrix} > 0$, $Q > 0$, $R > 0$, $W_1 > 0$, $W_2 > 0$ and real matrices $K_j, \Delta_i = \Delta_i^T, W < 0$ with appropriate dimensions for all $i = 1, 2, \cdots, s$, and the membership functions of the fuzzy model and fuzzy controller satisfying the inequalities $\theta_j - \rho_j\omega_j > 0, 0 < \rho_j < 1$ for all $j = 1, 2, \cdots, s$, such that the following matrix inequalities hold:*

$$\Omega_{ij} - \Delta_i < 0, \tag{34}$$

$$\rho_i\Omega_{ii} + (1-\rho_i)\Delta_i < W_{ii}, \tag{35}$$

$$\rho_j\Omega_{ij} + (1-\rho_j)\Delta_i + \rho_i\Omega_{ji} + (1-\rho_i)\Delta_j < W_{ij} + W_{ji}, \quad (i<j) \tag{36}$$

where U, X_{11} and V are defined in Lemma 2.

$$\Omega_{ij} = \begin{bmatrix} \Omega_{11} & \Omega_{12} & \Omega_{13} & \Omega_{14} \\ * & \Omega_{22} & \Omega_{23} & \Omega_{24} \\ * & * & \Omega_{33} & \Omega_{34} \\ * & * & * & \Omega_{44} \end{bmatrix} + h^2\Gamma_{ij}^T R\Gamma_{ij},$$

$$\begin{aligned}
\Omega_{11} &= (P_{11}E + U^T X_{11} V^T)^T (A_i + B_i K_j) + (A_i + B_i K_j)^T \\
&\quad \times (P_{11}E + U^T X_{11} V^T) + Q + h^2 W_1 + \tfrac{h^4}{4} W_2 \\
&\quad - 9E^T RE, \\
\Omega_{12} &= (P_{11}E + U^T X_{11} V^T)^T A_{hi} + 3E^T RE, \\
\Omega_{13} &= hE^T P_{22} E + h^2 E^T P_{23}^T E - 24E^T RE, \\
\Omega_{14} &= \tfrac{h^2}{2} E^T P_{23} E + \tfrac{h^3}{2} E^T P_{33} E + 30E^T RE, \\
\Omega_{23} &= -hE^T P_{22} E + 36E^T RE, \\
\Omega_{24} &= -\tfrac{h^2}{2} E^T P_{23} E - 30E^T RE, \\
\Omega_{22} &= T_{22},\ \Omega_{33} = T_{33},\ \Omega_{34} = T_{34},\ \Omega_{44} = T_{44}, \\
\Gamma_{ij} &= \begin{bmatrix} A_i + B_i K_j & A_{hi} & 0 & 0 \end{bmatrix},
\end{aligned}$$

$$\Delta_i = \begin{bmatrix} \Delta_{i11} & \Delta_{i12} & \Delta_{i13} & \Delta_{i14} \\ * & \Delta_{i22} & \Delta_{i23} & \Delta_{i24} \\ * & * & \Delta_{i33} & \Delta_{i34} \\ * & * & * & \Delta_{i44} \end{bmatrix},$$

$$W_{ii} = \begin{bmatrix} W_{ii11} & W_{ii12} & W_{ii13} & W_{ii14} \\ * & W_{ii22} & W_{ii23} & W_{ii24} \\ * & * & W_{ii33} & W_{ii34} \\ * & * & * & W_{ii44} \end{bmatrix},$$

$$W_{ij} = \begin{bmatrix} W_{ij11} & W_{ij12} & W_{ij13} & W_{ij14} \\ W_{ij21} & W_{ij22} & W_{ij23} & W_{ij24} \\ W_{ij31} & W_{ij32} & W_{ij33} & W_{ij34} \\ W_{ij41} & W_{ij42} & W_{ij43} & W_{ij44} \end{bmatrix}, (i \neq j)$$

$$W = \begin{bmatrix} W_{11} & W_{12} & \cdots & W_{1r} \\ * & W_{22} & \cdots & W_{2r} \\ \vdots & \vdots & \ddots & \vdots \\ * & * & \cdots & W_{rr} \end{bmatrix}.$$

Proof: In order to design controller, we need to simplify the matrix P with $P_{12} = 0$, $P_{13} = 0$. By using $A_i + B_i K_j$ to replace A_i, and we choose the same Lyapunov-krasovskii function as (25), then we can get the following result:

$$\dot{V}(x_t) \leq \sum_{i=1}^{s}\sum_{j=1}^{s} \omega_i\theta_j\eta^T(t)\Omega_{ij}\eta(t). \tag{37}$$

For the sake of reducing the conservatism, we will introduce the following equalities:

$$\sum_{i=1}^{s}\sum_{j=1}^{s} \omega_i(\omega_j - \theta_j)\Delta_i = 0, \tag{38}$$

where $\Delta_i = \Delta_i^T$ is an arbitrary matrix. Based on the conditions (37) and (38), we can obtain

$$\begin{aligned}
\dot{V}(x_t) \leq& \sum_{i=1}^{s}\sum_{j=1}^{s} \omega_i(\theta_j + \rho_j\omega_j - \rho_j\omega_j)\eta^T(t)\Omega_{ij}\eta(t) \\
&+ \sum_{i=1}^{s}\sum_{j=1}^{s} \omega_i(\omega_j - \rho_j\omega_j)\eta^T(t)\Delta_i\eta(t) \\
&- \sum_{i=1}^{s}\sum_{j=1}^{s} \omega_i(\theta_j - \rho_j\omega_j)\eta^T(t)\Delta_i\eta(t) \\
=& \sum_{i=1}^{s}\sum_{j=1}^{s} \omega_i\omega_j\eta^T(t)(\rho_j\Omega_{ij} + (1-\rho_j)\Delta_i)\eta(t) \\
&+ \sum_{i=1}^{s}\sum_{j=1}^{s} \omega_i(\theta_j - \rho_j\omega_j)\eta^T(t)(\Omega_{ij} - \Delta_i)\eta(t).
\end{aligned} \tag{39}$$

Due to the condition $\theta_j - \rho_j\omega_j > 0$ and the inequality (34), we have

$$\begin{aligned}
\dot{V}(x_t) \leq& \sum_{i=1}^{s} \omega_i^2\eta^T(t)(\rho_i\Omega_{ii} + (1-\rho_i)\Delta_i)\eta(t) \\
&+ \sum_{i=1}^{s}\sum_{i<j}^{s} \omega_i\omega_j\eta^T(t)(\rho_j\Omega_{ij} + (1-\rho_j)\Delta_i \\
&+ \rho_i\Omega_{ji} + (1-\rho_i)\Delta_j)\eta(t).
\end{aligned} \tag{40}$$

On the basis of (35) and (36), from (40), we know

$$\dot{V}(x_t) \leq \begin{bmatrix} \omega_1\eta(t) \\ \omega_2\eta(t) \\ \vdots \\ \omega_r\eta(t) \end{bmatrix}^T W \begin{bmatrix} \omega_1\eta(t) \\ \omega_2\eta(t) \\ \vdots \\ \omega_r\eta(t) \end{bmatrix}. \tag{41}$$

Considering $W < 0$, then we can obtain $\dot{V}(x_t) < 0$. This completes the proof. □

Remark 5. *The condition $P > 0$ in Theorem 1 is difficult to handle in the controller design of Theorem 2. In order to obtain controller gain matrices, we need to simplify the representation of matrix P, so we assume that $P_{12} = 0$ and $P_{13} = 0$.*

Remark 6. *The research ideas of (38) and (39) of this paper are firstly proposed in [23], which can alleviate the conservativeness of the system by introducing some slack matrices under the imperfect premise matching. To some degree, it shows that this method has become the main research idea for studying the IT2 fuzzy systems.*

It is easy to know that the conditions in Theorem 2 are not in terms of strict LMIs. In order to gain the controller gain K_j, the following Theorem is shown in the form of LMIs.

Theorem 3. *For constant delay h, the singular type-2 fuzzy system in (7) is admissible if there exist matrices $\bar{P}_{11} > 0$, $\begin{bmatrix} \bar{P}_{22} & \bar{P}_{23} \\ * & \bar{P}_{33} \end{bmatrix} > 0$, $\bar{Q} > 0$, $\bar{R} > 0$, $\bar{W}_1 > 0$, $\bar{W}_2 > 0$ and* *real matrices H_j, L_j, $\bar{X}_{11}$, $\bar{\Delta}_i = \bar{\Delta}_i^T$, $\bar{W} < 0$ with appropriate dimensions for $i, j = 1, 2, \cdots, s$, and the membership functions of the fuzzy model and fuzzy controller satisfying the inequalities $\theta_j - \rho_j \omega_j > 0, 0 < \rho_j < 1$, such that the following matrix inequalities hold:*

$$\begin{bmatrix} \bar{\Omega}_{ij} - \bar{\Delta}_i & h\Theta_{1ij} \\ * & \Theta_2 \end{bmatrix} < 0, \tag{42}$$

$$\begin{bmatrix} \rho_i \bar{\Omega}_{ii} + (1-\rho_i)\bar{\Delta}_i - \bar{W}_{ii} & h_i \Theta_{1ii} \\ * & \Theta_2 \end{bmatrix} < 0, \tag{43}$$

$$\begin{bmatrix} \rho_j \bar{\Omega}_{ij} + (1-\rho_j)\bar{\Delta}_i - \bar{W}_{ij} \\ + \rho_i \bar{\Omega}_{ji} + (1-\rho_i)\bar{\Delta}_j - \bar{W}_{ji} & h_j \Theta_{1ij} & h_i \Theta_{1ji} \\ * & \Theta_2 & 0 \\ * & * & \Theta_2 \end{bmatrix} < 0, \quad (i < j) \tag{44}$$

where U, X_{11} and V are defined in Lemma 2.

$$\bar{\Omega}_{ij} = \begin{bmatrix} \bar{\Omega}_{11} & \bar{\Omega}_{12} & \bar{\Omega}_{13} & \bar{\Omega}_{14} \\ * & \bar{\Omega}_{22} & \bar{\Omega}_{23} & \bar{\Omega}_{24} \\ * & * & \bar{\Omega}_{33} & \bar{\Omega}_{34} \\ * & * & * & \bar{\Omega}_{44} \end{bmatrix},$$

$$\begin{aligned}
\bar{\Omega}_{11} &= A_i(\bar{P}_{11}E^T + V\bar{X}_{11}U) + (\bar{P}_{11}E^T + V\bar{X}_{11}U)^T A_i^T \\
&\quad + B_i(H_jE^T + L_jU) + (H_jE^T + L_jU)^T B_i^T + \bar{Q} \\
&\quad + h^2\bar{W}_1 + \tfrac{h^4}{4}\bar{W}_2 - 9E\bar{R}E^T, \\
\bar{\Omega}_{12} &= A_{hi}(\bar{P}_{11}E^T + V\bar{X}_{11}U) + 3E\bar{R}E^T, \\
\bar{\Omega}_{13} &= hE\bar{P}_{22}E^T + h^2E\bar{P}_{23}^TE^T - 24E\bar{R}E^T, \\
\bar{\Omega}_{14} &= \tfrac{h^2}{2}E\bar{P}_{23}E^T + \tfrac{h^3}{2}E\bar{P}_{33}E^T + 30E\bar{R}E^T, \\
\bar{\Omega}_{22} &= -\bar{Q} - 9E\bar{R}E^T, \\
\bar{\Omega}_{23} &= -hE\bar{P}_{22}E^T + 36E\bar{R}E^T, \\
\bar{\Omega}_{24} &= -\tfrac{h^2}{2}E\bar{P}_{23}E^T - 30E\bar{R}E^T, \\
\bar{\Omega}_{33} &= -h^2E\bar{P}_{23}E^T - h^2E\bar{P}_{23}^TE^T - 192E\bar{R}E^T - h^2\bar{W}_1, \\
\bar{\Omega}_{34} &= -\tfrac{h^3}{2}E\bar{P}_{33}E^T + 180E\bar{R}E^T \\
\bar{\Omega}_{44} &= -180E\bar{R}E^T - \tfrac{h^4}{4}\bar{W}_2, \; h_i = \sqrt{\rho_i}h, \\
\Theta_2 &= -(\bar{P}_{11}E^T + V\bar{X}_{11}U) - (\bar{P}_{11}E^T + V\bar{X}_{11}U)^T + \bar{R},
\end{aligned}$$

$$\bar{\Delta}_i = \begin{bmatrix} \bar{\Delta}_{i11} & \bar{\Delta}_{i12} & \bar{\Delta}_{i13} & \bar{\Delta}_{i14} \\ * & \bar{\Delta}_{i22} & \bar{\Delta}_{i23} & \bar{\Delta}_{i24} \\ * & * & \bar{\Delta}_{i33} & \bar{\Delta}_{i34} \\ * & * & * & \bar{\Delta}_{i44} \end{bmatrix},$$

$$\bar{W}_{ii} = \begin{bmatrix} \bar{W}_{ii11} & \bar{W}_{ii12} & \bar{W}_{ii13} & \bar{W}_{ii14} \\ * & \bar{W}_{ii22} & \bar{W}_{ii23} & \bar{W}_{ii24} \\ * & * & \bar{W}_{ii33} & \bar{W}_{ii34} \\ * & * & * & \bar{W}_{ii44} \end{bmatrix},$$

$$\bar{W}_{ij} = \begin{bmatrix} \bar{W}_{ij11} & \bar{W}_{ij12} & \bar{W}_{ij13} & \bar{W}_{ij14} \\ \bar{W}_{ij21} & \bar{W}_{ij22} & \bar{W}_{ij23} & \bar{W}_{ij24} \\ \bar{W}_{ij31} & \bar{W}_{ij32} & \bar{W}_{ij33} & \bar{W}_{ij34} \\ \bar{W}_{ij41} & \bar{W}_{ij42} & \bar{W}_{ij43} & \bar{W}_{ij44} \end{bmatrix}, (i \neq j)$$

$$\bar{W} = \begin{bmatrix} \bar{W}_{11} & \bar{W}_{12} & \cdots & \bar{W}_{1r} \\ * & \bar{W}_{22} & \cdots & \bar{W}_{2r} \\ \vdots & \vdots & \ddots & \vdots \\ * & * & \cdots & \bar{W}_{rr} \end{bmatrix},$$

$$\Theta_{1ij} = \begin{bmatrix} (\bar{P}_{11}E^T + V\bar{X}_{11}U)^T A_i^T + (H_jE^T + L_jU)^T B_i^T \\ (\bar{P}_{11}E^T + V\bar{X}_{11}U)^T A_{hi}^T \\ 0 \\ 0 \end{bmatrix}.$$

then, the controller gain matrix can be obtained by $K_j = (H_jE^T + L_jU)(\bar{P}_{11}E^T + V\bar{X}_{11}U)^{-1}$.

Proof: By using Schur complement to inequality (34), we have

$$\begin{bmatrix} \Omega_{1ij} - \Delta_i & h\Xi_{ij}^T \\ * & -R \end{bmatrix} < 0, \tag{45}$$

where Ω_{1ij} is the first term of Ω_{ij},

$$\Xi_{ij} = \begin{bmatrix} R(A_i + B_iK_j) & RA_{hi} & 0 & 0 \end{bmatrix}.$$

Pre- and post-multiplying inequality (45) by diag$\{(P_{11}E + U^TX_{11} \times V^T)^{-T}, (P_{11}E + U^TX_{11}V^T)^{-T}, (P_{11}E + U^TX_{11}V^T)^{-T}, (P_{11}E + U^TX_{11}V^T)^{-T}, R^{-1}\}$ and its transpose, and denoting the following transformations:

$$\begin{aligned}
\bar{R} &= (P_{11}E + U^TX_{11}V^T)^{-1}R(P_{11}E + U^TX_{11}V^T)^{-T}, \\
\bar{Q} &= (P_{11}E + U^TX_{11}V^T)^{-T}Q(P_{11}E + U^TX_{11}V^T)^{-1}, \\
\bar{W}_1 &= (P_{11}E + U^TX_{11}V^T)^{-T}W_1(P_{11}E + U^TX_{11}V^T)^{-1}, \\
\bar{W}_2 &= (P_{11}E + U^TX_{11}V^T)^{-T}W_2(P_{11}E + U^TX_{11}V^T)^{-1}, \\
\bar{P}_{ik} &= (P_{11}E + U^TX_{11}V^T)^{-T}P_{ik}(P_{11}E + U^TX_{11}V^T)^{-1}, \\
&\qquad (i, k = 2, 3) \\
\bar{\Delta}_{ikl} &= (P_{11}E + U^TX_{11}V^T)^{-T}\Delta_{ikl}(P_{11}E + U^TX_{11}V^T)^{-1}, \\
&\qquad (k, l = 1, 2, 3, 4) \\
H_j &= K_j\bar{P}_{11}, \; L_j = K_jV\bar{X}_{11}.
\end{aligned}$$

Because $-W^TR^{-1}W \leq -W^T - W + R \; (R > 0)$, then we have

$$\begin{aligned}
&-(P_{11}E + U^TX_{11}V^T)^{-T}\bar{R}^{-1}(P_{11}E + U^TX_{11}V^T)^{-1} \\
&\leq -(P_{11}E + U^TX_{11}V^T)^{-T} - (P_{11}E + U^TX_{11}V^T)^{-1} + \bar{R}.
\end{aligned} \tag{46}$$

According to Lemma 2 and Lemma 3, it's easy to derive the inequality (42) from the inequality (45).

Denote

$$\begin{aligned}
\bar{W}_{iikl} &= (P_{11}E + U^TX_{11}V^T)^{-T}W_{iikl}(P_{11}E + U^T \\
&\quad \times X_{11}V^T)^{-1}, (k, l = 1, 2, 3, 4) \\
\bar{W}_{ijkl} &= (P_{11}E + U^TX_{11}V^T)^{-T}W_{ijkl}(P_{11}E + U^T \\
&\quad \times X_{11}V^T)^{-1}, (i < j, k, l = 1, 2, 3, 4)
\end{aligned}$$

Then, by using the same congruent transformation to (35) and (36) as the process of (34), we have (43) and (44). This completes the proof. □

4 Numerical Examples

In this section, two illustrative examples are provided to show the effectiveness of the proposed method.

Example 1: Consider the singular IT2 fuzzy system with time delay in (1) and matrix parameters are represented as follows:

$$E = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, A_1 = \begin{bmatrix} 1 & 0.2 \\ 0.6 & -0.4 \end{bmatrix}, A_2 = \begin{bmatrix} -1.2 & 0.1 \\ 0.4 & -0.2 \end{bmatrix}$$

$$A_{h1} = \begin{bmatrix} -0.5 & 0.2 \\ 0 & -0.2 \end{bmatrix}, A_{h2} = \begin{bmatrix} -0.1 & 0 \\ 0 & -0.18 \end{bmatrix},$$

$$U^T = V = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

Our purpose is to look for the maximum delay $h_{\max}$ to ensure that the system in (1) with $u(t) = 0$ is admissible. By solving the inequalities of Theorem 1 and the methods given in [45] and [46], the comparison results are shown in Table 1.

Table 1 Comparison of maximum allowed delay $h_{\max}$

Methods	Theorem 3.1 [45]	Corollary 3.2 [46]	Theorem 1
$h_{\max}$	1.4476	1.7076	1.7423

From Table 1, it is easy to see that Theorem 1 of this paper is less conservative than those in [45] and [46], which shows the generalized integral inequality method is effective. But it should be noticed that computational complexity is also a significant factor, so we'd better balance the relationship between conservatism and computational complexity.

Example 2: In order to validate the effectiveness of the controller design, an inverted pendulum example is shown, which is borrowed from [47].

$$\begin{cases} \dot{x}_1(t) = x_2(t), \\ \quad [(M+m)(J+ml^2) - m^2l^2\cos^2(x_1(t))]\dot{x}_2(t) \\ \quad = (M+m)mgx_3(t) - m^2lx_2^2(t)x_3(t)\cos(x_1(t)) \\ \qquad -ml\cos(x_1(t))u(t), \\ 0 = l\sin(x_1(t)) - x_3(t). \end{cases} \tag{47}$$

where $x_1(t) \in [-\pi/2, \pi/2]$ is the angle of the pendulum, $x_2(t)$ is the angular velocity, $x_3(t)$ is the relative horizontal distance, $M = 1.3283kg$ and $m = 0.22kg$ denote the mass of the cart and the pendulum, respectively, $g = 9.8m/s^2$ represents the gravity constant, $l = 0.304m$ is the pendulum length, $J = ml^2/3$ denotes the moment of inertia. $u(t)$ is control input which represents the force applied to the cart.

Based on the local approximation technique, the original pendulum system can be modelled by using two fuzzy rules.

Plant Rule 1: **IF** $x_1(t)$ is about 0, **THEN**

$$E\dot{x}(t) = A_1x(t) + B_1u(t).$$

Plant Rule 2: **IF** $x_1(t)$ is about $\pm\pi/2$ ($|x_1(t)| < \pi/2$), **THEN**

$$E\dot{x}(t) = A_2x(t) + B_2u(t).$$

where

$$E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \ x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix},$$
$$A_1 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & \frac{g}{4l^2/3 - aml^2} \\ l & 0 & -1 \end{bmatrix},$$
$$A_2 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & \frac{g}{4l^2/3 - aml^2\cos^2\beta} \\ l\sin\beta/\beta & 0 & -1 \end{bmatrix},$$
$$B_1 = \begin{bmatrix} 0 & \frac{-a}{4l/3 - aml} & 0 \end{bmatrix}^T,$$
$$B_2 = \begin{bmatrix} 0 & \frac{-a\cos\beta}{4l/3 - aml\cos^2\beta} & 0 \end{bmatrix}^T,$$
$$a = 1/(M+m), \ \beta = 88^\circ.$$

Assume that time delay exists in the system (47), which can be described by the system (1), and the time delay term parameters are assumed as follows:

$$A_{h1} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, A_{h2} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

For the sake of designing the controller, the parameters are set as $\rho_1 = 0.1, \rho_2 = 0.4$. By solving the inequalities of Theorem 3, we can obtain that the maximum delay $h_{\max} = 0.1488$. In the following simulations, time delay $h = 0.05$ is chosen. By solving (42)-(44), we can get some feasible solution parameter matrices and controller gain matrices:

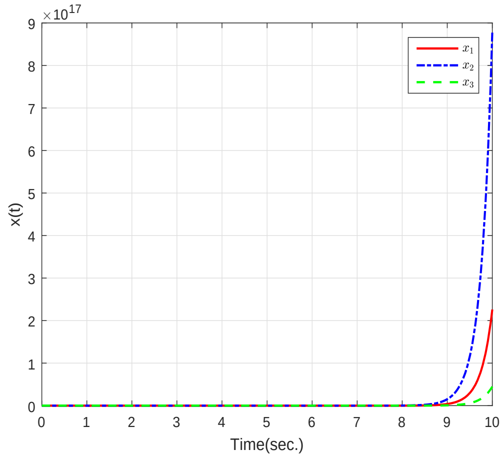

Fig. 1: State trajectories of the open-loop system

$$\bar{P}_{11} = \begin{bmatrix} 0.0048 & -0.0143 & 0.0006 \\ -0.0143 & 0.0630 & -0.0032 \\ 0.0006 & -0.0032 & 0.2904 \end{bmatrix},$$
$$\bar{X}_{11} = 9.1296 \times 10^{-5},$$
$$K_1 = \begin{bmatrix} 497.6235 & 172.9149 & 147.8377 \end{bmatrix},$$
$$K_2 = \begin{bmatrix} 511.6877 & 177.8971 & 154.5409 \end{bmatrix}.$$

The lower and upper membership functions of the fuzzy system are shown by the following format:

$$\underline{\omega}_1(x) = 1 - \frac{1}{1+e^{-(x_1+5)}}, \ \bar{\omega}_1(x) = 1 - \frac{1}{1+e^{-(x_1+3)}},$$
$$\underline{\omega}_2(x) = 1 - \bar{\omega}_1(x), \ \bar{w}_2(x) = 1 - \underline{w}_1(x).$$

Owing to the mismatched membership functions between the system and the controller, we can choose lower and upper membership functions of the controller:

$$\underline{\theta}_1(x) = 1 - \frac{1}{1+e^{\frac{-(x_1-1)}{5}}}, \ \bar{\theta}_1(x) = 1 - \frac{1}{1+e^{\frac{-(x_1-3)}{5}}},$$
$$\underline{\theta}_2(x) = 1 - \bar{\theta}_1(x), \ \bar{\theta}_2(x) = 1 - \underline{\theta}_1(x).$$

From the simulation results, we know that the state trajectories of the open-loop system can't converge to 0 with initial condition $x(0) = \begin{bmatrix} \pi/3 & 0 & 0.2082 \end{bmatrix}^T$ in Fig. 1, which implies the system is not stable. By using the same initial condition, Fig. 2 and Fig. 3 show the state trajectories of the closed-loop system and the trajectory of control input $u(t)$ under the designed fuzzy controller. It is easy to see that the closed-loop system is stable. To further verify the effectiveness of the controller, we choose another set of initial value $x(0) = \begin{bmatrix} -\pi/4 & 0 & -0.1601 \end{bmatrix}^T$, we can get satisfactory results from the Fig. 4 and Fig. 5.

5 Conclusion

The admissibility analysis and stabilization problems of the singular interval type-2 T-S fuzzy system with time delay are investigated in this paper. General integral inequality is used to handle the integral term of the augmented Lyapunov-Krasovskii function. Sufficient conditions of ensuring the closed-loop system's admissibility are

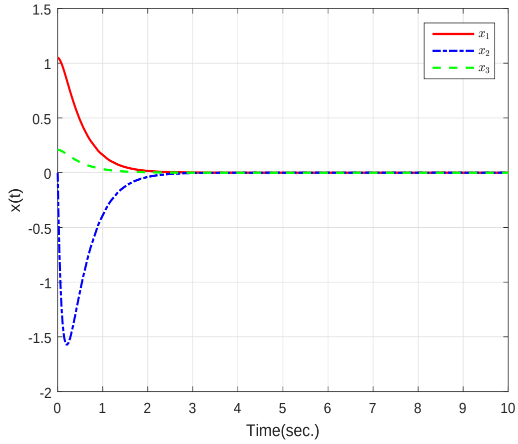

Fig. 2: State trajectories of the closed-loop system with $x_1(t) = \frac{\pi}{3}$

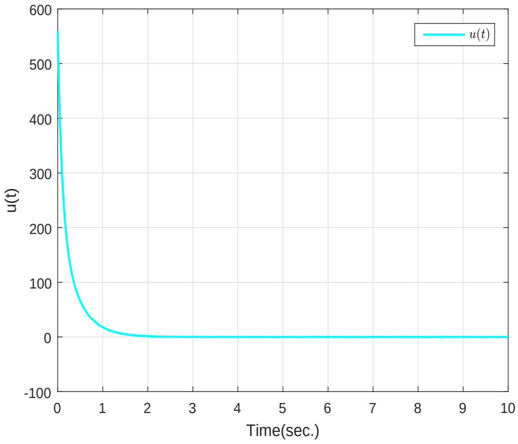

Fig. 3: Control input $u(t)$ with $x_1(t) = \frac{\pi}{3}$

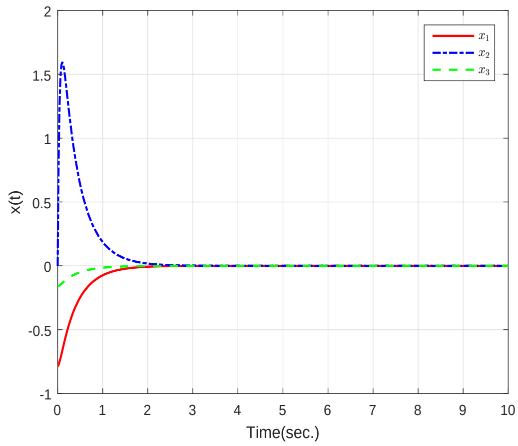

Fig. 4: State trajectories of the closed-loop system with $x_1(t) = -\frac{\pi}{4}$

given in terms of strict linear matrix inequalities. Two examples are given to illustrate the effectiveness of the proposed method. In future works, the reliable dissipative problem and filter design will be considered in the singular interval type-2 fuzzy systems with time-varying delay, which will further enrich the research of singular interval type-2 fuzzy systems.

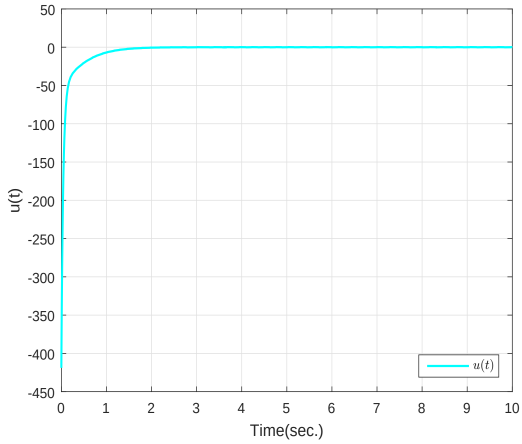

Fig. 5: Control input $u(t)$ with $x_1(t) = -\frac{\pi}{4}$

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