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Mechanistic-empirical permanent deformation models: Laboratory testing, modelling and ranking

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Abstract

© 2020 Elsevier Ltd Geomaterials exhibit elastoplastic behaviour during dynamic and repeated loading conditions. These loads are induced by the passage of a train or vehicle which then generates recoverable (resilient) deformation and/or permanent (plastic) deformation. Modelling this behaviour is still a challenge for geotechnical engineers as it implies the understanding of the complex deformation mechanism and application of advanced constitutive models. This paper reviews on the major causes of permanent deformation and the factors that influence the long-term performance of materials. It will also present the fundamental concepts of permanent deformation as well as the models and approaches used to characterise this behaviour, including: elastoplastic models, shakedown theory and mechanistic-empirical permanent deformation models. This paper will focus on the mechanistic-empirical approach and highlight the evolution of the models, and the main similarities and differences between them. A comparison between several empirical models as well as the materials used to develop the models is also discussed. These materials are compared by considering the reference conditions on the type of material and its physical state. This approach allows for an understanding of which properties can influence the performance of railway subgrade and pavement structures, as well as the main variables used to characterise this particular behaviour. An innovative ranking of geomaterials that relate to the expected permanent deformation and classification (UIC and ASTM) of soil is also discussed because it can be used as an important tool for the design process.

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Mechanistic-empirical permanent deformation models: laboratory testing, modelling and ranking

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Abstract:

Geomaterials exhibit elastoplastic behaviour during dynamic and repeated loading conditions. These loads are induced by the passage of a train or vehicle which then generates recoverable (resilient) deformation and/or permanent (plastic) deformation. Modelling this behaviour is still a challenge for geotechnical engineers as it implies the understanding of the complex deformation mechanism and application of advanced constitutive models. This paper reviews on the major causes of permanent deformation and the factors that influence the long-term performance of materials. It will also present the fundamental concepts of permanent deformation as well as the models and approaches used to characterise this behaviour, including: elastoplastic models, shakedown theory and mechanistic-empirical permanent deformation models. This paper will focus on the mechanistic-empirical approach and highlight the evolution of the models, and the main similarities and differences between them. A comparison between several empirical models as well as the materials used to develop the models is also discussed. These materials are compared by considering the reference conditions on the type of material and its physical state. This approach allows for an understanding of which properties can influence the performance of railway subgrade and pavement structures, as well as the main variables used to characterise this particular behaviour. An innovative ranking of geomaterials that relate to the expected permanent deformation and classification (UIC and ASTM) of soil is also discussed because it can be used as an important tool for the design process.

36 Keywords – mechanistic-empirical permanent deformation models, permanent
37 deformation, laboratory tests, ranking

38

39 Notation list:

40 The most common variables and acronyms are presented:

41 *UIC* - International Union of Railways

42 *ASTM* - American Society for Testing and Materials

43 *SS* - single-stage

44 *MS* - multi-stage

45 *RLT* – repeated load tests

46 M_r - resilient modulus

47 N - load cycles

48 ε_p - permanent strains

49 p - mean stress

50 q - deviatoric stress

51 q_f - deviatoric stress at failure

52 p_{am} - amplitude of the mean stress for train loadings

53 q_{am} - amplitude of the deviator stress for train loadings

54 p_{ini} - mean stress in the initial state of the material

55 q_{ini} - deviator stress in the initial state of the material

56 σ_1 or σ_d - cyclic vertical stress or cyclic deviator stress

57 σ_3 or σ_3 - Horizontal/confining stress

58 $\sigma_1/\sigma_{1,f}$ - failure ratio of the major principal stresses

59 σ_{oct} - octahedral normal stress

- 60 τ_{oct} - octahedral shear stresses
- 61 m and s - defined by the Mohr-Coulomb yielding criterion $q=s + mp$; m is the slope
- 62 W_L - liquid limit
- 63 W_P – plastic limit
- 64 IP – index plasticity
- 65 C_u - coefficient of uniformity
- 66 C_c - coefficient of gradation

1 Introduction

Pavements and railway structures are designed for a certain lifespan and they usually require reconstruction and/or major maintenance work because the materials may experience fatigue, cracking or be under an excessive amount of cumulative permanent deformation (or rutting in the case of pavements) due to the passage of traffic. Plastic or permanent deformation usually occurs in the geomaterials (base, sub-base or subgrade soils) which are responsible for the surface rutting and that can lead to significant passenger discomfort (Puppala et al., 2009). This is why knowing and understanding the deformation and failure mechanisms of geomaterials under dynamic and cyclic loading are so important when designing and planning the maintenance of pavements and railway structures (Li and Selig, 1996). Indeed, an accurate estimation or prediction of the amount of cumulative settlement will help pavements and /or railway structures avoid a mediocre performance (Barksdale, 1972, Monismith et al., 1975, Puppala et al., 1999). This is why the permanent deformation of geomaterials should be included in the design since otherwise it can lead to higher annual rehabilitation costs (Puppala et al., 1999).

Over the past decades, researchers have been so concerned about permanent deformation and they continually search for the most accurate methods and models that will measure and predict these values (Monismith et al., 1975, Lentz and Baladi, 1980, Ullidtz, 1993, Gidel et al., 2001), among others. To achieve these objectives, laboratory investigations using cyclic triaxial tests, simple and cyclic shear tests, resonant column and hollow cylinder tests, among others have been carried out. These tests were often used to determine shear stress-strain behaviour, resilient modulus (M_r) of subgrade geomaterials considering reversible and irreversible deformation under cyclic loads (Gomes Correia, 2004, Correia, 2008). The reversible (elastic) is usually described by non-linear elastic models, but since permanent deformation is more complex, it depends on the accumulation of N loading cycles.

One of the main objectives of this paper is to review on the main causes of permanent deformation on pavements and railway structures and the factors that can increase this phenomenon. It is noted that while the materials should be able to resist permanent deformation, this resistance will depend on the number of load cycles and stress levels (Lekarp and Dawson, 1998), the thickness of the layer, and the granulometry of the material. This also includes other external factors such as the physical state of the soil,

which is often difficult to control because it depends on other environmental aspects such as the moisture content, and degree of saturation, etc.

The behaviour of geomaterials under cyclic loads can be characterised by either using complex elastoplastic models (recoverable and permanent deformation are both considered) or by shakedown theory and mechanistic-empirical models (Hornych and Abd, 2004). The elastoplastic models, however, despite their ability to accurately predict permanent deformation (the loading history is considered because the equation is solved based on incremental steps), they are difficult to implement, time consuming, and complex (Ling et al., 2017). Most of these models only consider a low number of load cycles, which is not in accordance with the *in situ* conditions where the number of loads is up to million cycles. Indeed, these models are very demanding computationally because they require the simulation of repeated load applications in pavements/railway structures. The development of formulations based on cyclic constitutive laws may be expressed through conventional concepts such as the yield condition, hardening and flow rules. The main problem with the numerical implementation is that the increment of permanent deformation per cycle becomes very small quickly, and this leads to problems with the computational accuracy of the results (Abdelkrim et al., 2003). The focus of this work will be on mechanistic-empirical models. These models are based on extensive laboratory testing results, so they can correctly simulate the response of materials; they are easy to implement, and they depend on fewer parameters than conventional elastoplastic models.

In order to select the best testing approach and the most suitable model, it is important to understand the conditions needed for its development such as the properties of the materials tested, degree of compaction, moisture content, etc., as well as the main variables/factors that can influence the response of the material. This paper reviews on the existing methods used to estimate the irreversible deformation of geomaterials, followed by a parametric study that includes comparisons among some selected models on different materials with different classifications (*UIC* and *ASTM*), properties, granulometry, and physical states. This comparison allows to estimate the permanent deformation and to rank materials according to the predicted deformation data and soil classification, which is a helpful tool in the design of the pavement and railway structures. It is also noted that this ranking should be interpreted as a reference value because it depends on several properties and soil conditions.

2 Causes of permanent deformation

Permanent deformation occurs in pavements and railway lines due to repeated traffic loading; and if the volume of traffic is high enough it may lead to permanent deformation and/or structural failure. Therefore, it is crucial to identify the main causes of permanent deformation and adopt appropriate measures, even during the design process. Furthermore, transition zones are also important areas due to the possible development of differential settlement due to differences in stiffness between two contiguous structures such as between a ballasted and slab track, or a bridge and a plain track (Figure 1). In fact, the degradation in these zones often accelerates because the differential settlement increases the dynamic effects (Indraratna, 2019).

Another important cause of permanent deformation is related to the complex stress conditions as geomaterials are subjected to vertical, horizontal and shear stresses during the passage of vehicles as well as the effects of moving loads; all of which implies the rotation of principal stresses.

Flexible road pavements often consist of an upper layer of asphalt over unbound granular layers (base, subbase), which have been compacted over a proper soil subgrade. These unbound granular layers provide a structural role, but in some cases when subgrade has an adequate bearing capacity, rutting still occurs in the granular layers causing fatigue cracking in the bituminous layers. This means the bituminous layers and the subgrade will lead to permanent deformation. In rigid pavements, only the subgrade leads to the development of permanent deformation.

Post settlement in railway structures is often caused by the self-weight of the embankment and traffic loading. Permanent deformation includes the settlement of the roadbed beneath the track and the subgrade. Several studies show that the main influencing factors are the level of stress and stress history; in fact the experimental results and field measurements show that the dynamic and cyclic loads from train passage make a significant contribution. It is noted that although both structures are affected by permanent deformation, pavements and railway structures have different dynamic effects on the structure where the dynamic effects are more severe on railway structures.

Permanent deformation has a significant effect on the performance of structures because it leads to an increase in maintenance operations and costs, and reduces ride quality.

Despite this is long-term behaviour, it must be predicted during the design stage because although the accumulation is very small during each cycle, it may still lead to

the ultimate collapse of the structure (excessive rutting) due to the accumulation of millions of cycles.

Figure 2 & Figure 3 show that the permanent deformation can be defined and characterised by the accumulation of small increments of deformation during N loading cycles. Several studies show that permanent strains depend directly on the mean and deviator (p and q , respectively), levels of stress (Lekarp et al., 2000). The mean stress is dependent on the sum of the principal stresses while the deviator stress is dependent on the sum of squares of the differences of the principal stresses:

$$p = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3} \quad (1)$$

$$q = \sqrt{\frac{1}{2} \times \left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 \right]} \quad (2)$$

2.1 Factors influencing the permanent deformation

Permanent deformation is a complex process that directly depends on the number of load cycles (N) and stress levels. While these factors can be divided into load-related factors and material properties, the load factors will include applied stress levels, number of load applications, the strength of material, as well as the loading history and the effect of principal stress rotation. However, other factors can also influence the plastic strain:

moisture content (degree of saturation), matric suction, fine content, density (degree of compaction), aggregate type, particle size distribution (gradation), and the amount and type of fines (plastic or non-plastic) (Lekarp, 1999, Xiao et al., 2015). Gidel et al. (2001) found that for unbound granular materials, the mineralogical nature of material (including aggregate mineralogy and particle morphology) also influences permanent deformation (Coronado et al., 2011). This factor has important effects on particle shape, the quality of the fines content, and sensitivity to water and surface roughness. However, the stress level and number of load cycles emerge as the most important factors. In fact those models that only consider the value N should not be used to predict permanent deformation because it is too simple and lacks accuracy.

In specific cases of pavements and railway structures, the stress conditions are important because the track foundations are subjected to complex vertical, horizontal and shear stresses. Furthermore, railways and pavements present other complex problems related to the effects that moving loads have on track foundation; for instance, the rotation of principal stresses affects plastic strain. Chan (1990) showed an increase in permanent deformation when the rotation of principal stresses were included. These tests took place in a hollow cylindrical apparatus which allows for simulation in the laboratory and similar conditions to be verified *in situ*.

3 Laboratory testing

There are a number of laboratory tests currently used to evaluate the permanent deformation of geomaterials; they attempt to reproduce *in situ* stress conditions in pavements and railway structures. The cyclic triaxial test is the most widely used to study of geomaterials subjected to cyclic loads. However, in these tests, the principal stresses are always horizontal or vertical, which may not always correspond to *in situ* conditions where the materials are subjected to moving loads and rotations of principal stresses. The cyclic vertical stress, designated as σ_1 , is applied, and horizontal stress, designated as the confinement stress - σ_3 is also applied. This test begins when deviator stress is applied, and then the applied vertical progressively increases until it reaches an allowable displacement of the apparatus.

Another test used to evaluate permanent deformation is a cyclic torsional test, which also includes a hollow cylinder test. This particular test enables the magnitude and the direction of principal stresses to be controlled. In fact, this test device has been developed

to study permanent deformation and resilient behaviour by considering the rotation of principal stresses, and then applying the laboratory results to *in situ* conditions. In this case, the specimen is cylindrical and hollow. The main limitations of this test are the dimensions of the sample and the stress path can only be simulated on the symmetrical plane of the track. However, the results show that the permanent deformations are often higher in this test than the convention cyclic triaxial tests due to the simulation of the rotation of principal stresses.

Cyclic shear tests can also be used to study the dynamic behaviour of soil. In this test the sample is cylindrical and a horizontal shear force is applied to the bases of the samples. This test can also be used to evaluate stability under seismic events, to quantify the degradation of shear stress in cohesive soils under cyclic loads, and to evaluate the liquefaction parameters of non-cohesive soils under cyclic loading.

There are also other approaches for studying the permanent deformation of geomaterials; for example, while expensive, physical models (1-to-1 prototype model test) can be used to evaluate the performance of geomaterials and the stability of structures subjected to dynamic and repeated loading. These models will provide actual field data to help understand the behaviour of railways and pavements under moving loads, and enable the study of permanent deformation and the stresses induced in soil through proper measurement devices.

While small scale models of railways were introduced in the past (Momoya Y. et al., 2005, Al Shaer A. et al., 2008), several models focusing on the development of permanent deformation of railways under cyclic loadings or moving loads at low speed (Bian et al., 2014) are currently available. However, the field measurements and numerically calibrated results show that increased train speed has a huge influence on the long-term performance of a structure (i.e., higher dynamic stresses and higher permanent deformation). A test facility should be able to simulate the actual speed of a moving train under very large number of load cycles. This means that full-scale models are better because the measured results from a test can be considered directly in the track design and maintenance operations as this approach reduces any uncertainties in the measurements (Bian et al., 2014).

4 Modelling approaches

Permanent deformation can be predicted either by numerical simulations using elastoplastic models utilising the shakedown theory or mechanistic-empirical deformation models based on laboratory tests such as cyclic triaxial tests or hollow cylinder apparatus. In addition, the accumulated permanent deformation can be measured by the repeated load triaxial (RLT) tests developed in the laboratory.

Mechanistic-empirical permanent deformation models have become much more complex due to the inclusion of other variables, apart from the number of load cycles and stresses. More recently, a study to correlate the resilient modulus (M_r) with the permanent strains (ε_p) has been developed. Studies that relate the two parameters (M_r and ε_p) have shown that for subgrade soils, M_r has a significant effect on the ε_p or pavement rutting since soils with a higher M_r present less permanent strain (Orobio and Zaniewski, 2011, Rahman and Gassman, 2019). However, mixed soils such as silty sand or sandy silts still show significant rutting despite having higher resilient characteristics (Ullidtz, 1993, Puppala et al., 2009).

Existing permanent deformation models can be divided into two main categories: rutting models based purely on mechanics, and mechanistic-empirical models. Rutting models are based on elastoplastic theory (Desai, 1980, Desai and Faruque, 1984, Vermeer, 1982, Uzan, 1999, Chazallon, 2000, Chazallon et al., 2002, Chazallon et al., 2006). While they can consider how the stress levels and stress paths can affect permanent deformation, they are very complex and difficult to use because: (i) the increment of permanent strain per cycle is very small; and (ii) the response of the track in each cycle is a challenge for cyclic constitutive models (Chazallon et al., 2006) and numerical implementation (Abdelkrim et al., 2003) due to the extensive calculation time and cumulative errors.

Mechanistic-empirical models often describe a relationship between the number of load cycles and the accumulated permanent deformation. They are very simple to use, the numerical results can be obtained very quickly, and they also predict permanent deformations very well. However, one of the particularities of these models is that their derivation from triaxial tests; which means the conventional and well-known heart-shaped stress path (stress rotation) induced by the passage of trains cannot be reproduced in this type of laboratory test.

Mechanistic-empirical models can be divided into single-stage models and multi-stage models. A single-stage implies that the repetitive load tests are carried out at one stress level in one test; in this instance, multiple specimens are tested at different stress levels. Multi-stage models can test multiple levels of stress in one test on one specimen. This approach enables the effects that the stress level and stress history has on permanent deformation to be considered (Grégoire, 2011).

Shakedown theory

Shakedown theory is another approach used to characterise load-deformation responses of geomaterials. This theory was developed initially based on the behaviour of metals subjected to repeated loading. Werkmeister et al. (2002) further developed to apply for unbound granular materials. The cyclic loads may not lead to instantaneous collapse of the structure since they can induce plastic strain in the material in every load cyclic. Indeed, if the load level is lower than a critical limit, the material will show permanent deformation in the first load cycles. However, after a certain number of N , the material will respond elastically to the subsequent load cycles. This phenomenon is defined as shakedown and this critical limit is called shakedown limit.

Here, the material is divided into three categories (by considering its stress dependency): range A, range B, and range C, as shown in Figure 4.

Werkmeister (2003) defined the shakedown limits based on the repeated load triaxial test (RLT); these values are also defined by EN13286-7 (2004a):

- Range A: $(\varepsilon_p^{5000} - \varepsilon_p^{3000}) < 0.045 \times 10^{-3}$
- Range B: $0.045 \times 10^{-3} < (\varepsilon_p^{5000} - \varepsilon_p^{3000}) < 0.4 \times 10^{-3}$
- Range C: $(\varepsilon_p^{5000} - \varepsilon_p^{3000}) > 0.4 \times 10^{-3}$

These limits represent the deformation that accumulates between the 3000th and 5000th number of load cycles.

Figure 4 shows that *Range A* is designated as plastic shakedown where the permanent deformation stabilises after a finite number of load cycles and the material becomes completely resilient. In fact, after a certain point, the cyclic stress does not cause any damage to the structure, which means that the permanent deformation does not increase and the failure does not occur. *Range B* is designated as the intermediate-range where

accumulated deformation increases without complete stabilisation. In this case, permanent deformation develops at a very slow rate and is almost linear. *Range B* occurs when the repeated load cycles overcome plastic shakedown and failure can occur after a large number of load cycles. *Range C* is defined as incremental collapse because the permanent strain accumulated per cycle increases until the failure occurs. This is caused by shear failure associated with the reorientation and rearrangement of particles, breakage, and the slip and loss of friction between the particles (Sun, 2019). Indeed, failure can occur after a low number of loading cycles. This type of behaviour should never be accepted, unlike materials in the range *A* or even range *B*. It is reported in recent studies that the shakedown limits were used for the pavement analysis (Qian et al., 2018, Qian et al., 2019b) and in railway structures (Alves Costa et al., 2018).

The shakedown analysis is used to find the shakedown limit of structure under cyclic load. In the case of railway structure, the shakedown is used to predict whether the settlement will keep increasing or reach a stable status. Indeed, there are studies (Werkmeister S, 2005, Brown et al., 2012) about laboratory studies based on different types of materials (granular and soils) where the possibility of shakedown under traffic loads was evaluated (Liu and Wang, 2019). Indeed, the shakedown approach has becoming very popular in pavement engineer, where some progress had been reported (Collins and Boulbibane, 2000, Chazallon et al., 2009a, Chazallon et al., 2009b, Chazallon et al., 2012, Brown et al., 2012). Keep this phenomenon in mind, in order to prevent excessive permanent strain in a certain structure it is important to guarantee that the loading level is below the elastic shakedown limit. Indeed, the assessment of this limit is not easy since demands a significant computational effort mainly in 3D modeling since the loading time-history needs to be considered.

Considering the classical limit analysis theory, two main theorems were formulated to define the limits of shakedown load; the lower bound theorem (conservative solution) and the upper bound theorem (unconservative solution). The application of the lower bound theorem demands an accurate evaluation of the elastic response of the track's foundation due to the traffic. A numerical model can be used to assess the elastic dynamic response of the system (Alves Costa et al., 2018). During the shakedown phenomenon, the structure is subjected to a load that is lower than this limit but higher than the elastic limit and, after the accumulation of permanent deformation in the beginning (finite number of cycles), the material start to show elastic response and after this point, there is no accumulation

of permanent deformation. This behavior is explained by the total stress field that is given by the sum of the residual stress field with origin in the previous cycles. These conclusions were also obtained by Wang and Yu (2013). The key point of this problem is the assesement of the oad that is compatible with shakedown state. The lower bound theorem states that an elastic perfectly plastic material will shakedown if it is possible to fund a time-independent, self-equilibrated, residual stress field thatcombined with cyclic elastic stresses gives rise to a stress field that no violates the yielding criterion. The shakedown will occur if the following condition is respected (any time and any location):

$$f(\lambda\sigma_{ij}^e + \sigma_{ij}^o + \sigma_{ij}^r) \leq 0 \quad (3)$$

where f represents the yielding criterion, λ is a load factor of stress field, σ_{ij}^o is the rest stress state, $\lambda\sigma_{ij}^e$ are the elastic stresses induced by the cyclic loads and the residual stress field is given by σ_{ij}^r . The maximum value of λ , corresponds to the lower value of the shakedown multiplier. The kinematic shakedown theorem states that if any kinematic acceptable mechanism of plastic deformation can be found, the shakedown phenomenon can not occur since the structure will fail due to fatigue (Alves Costa et al., 2018).

Having realised the importance of elastoplastic models and shakedown theory as approaches to predict permanent deformation, the mechanistic-empirical models are also an essential approach and will be presented in the next section. These models are simple and they present an elegant formulation with good results, particularly when compared to the laboratory tests and *in situ* measurements.

5 Mechanistic-empirical permanent deformation models

The mechanistic-empirical model is often derived from laboratory test results such as the triaxial cyclic tests, direct shear tests or large-scale cubical tests. There have been a number of predictive models used to study permanent deformation range from purely empirical to mechanistic and plasticity theory-based models. However, some of these models are only applicable to specific stress states or testing conditions so they have never been evaluated for a wide range of stress states, or types of materials and their physical conditions (Xiao et al., 2015).

It is noted that most of the developed models considered total stress conditions, while ignoring the importance of suction. With regards to the resilient modulus (M_r), there have been several studies about the influence that suction has on the M_r even though this

influence is not widely used in the study of plastic strains (Coronado et al., 2016). Salour and Erlingsson (2016) used a triaxial testing system to control the pore-water pressures of a specimen, so the tests for permanent deformation took place with the control matric suction of the soil samples, and an effective stress approach has been used for modelling.

The mechanistic-empirical permanent deformation models can be divided by considering the materials used in the tests, the approach used during the tests, and the complexity (variables included in the model), as illustrated in Figure 5 and described in the following sections.

5.1 Materials

Empirical permanent deformation models can be divided by classifying materials tested as cohesive and granular materials. In the Annex, Table 1 and Table 2 present summaries of the empirical models that characterise the permanent deformation of cohesive materials, fine-grained soils–clays and grained soils–silts, respectively. In addition, Table 3 and Table 4 present key features of empirical models that can be used to obtain the irreversible deformation of granular materials classified as sands and sandy gravels, respectively.

5.1.1 Cohesive materials

The plastic strain that accumulates in cohesive materials is a result of the plastic deformation of shear strain, the accumulation of strain due to compaction, consolidation, and associated residual accumulation of excess water pressure. Models for cohesive materials should consider the stress state, the type of soil, and the physical state of the materials, and variables related to the moisture content and dry density. These types of materials are also influenced by environmental conditions and traffic loading, so regardless of the physical state of the material, its influence is considered in most models, and indirectly in the material parameters/constants, as summarised in the Table 1 and Table 2.

The shakedown behaviour of cohesive soils is characterised by the stress-strain loop, and when the load increases, the progressive transition of material from plastic shakedown through plastic creep to incremental collapse can be identified (Yang and Huang, 2007). Yang et al. (2008) indicated that it is not possible to identify the pattern for the cohesive materials shown in Figure 4. Under lower levels of stress, the subgrade (of a pavement or railway structure) will reach an equilibrium strain state after a certain number of load

cycles. In these cases, plastic strain accumulates very slowly as the number of load cycles increases. As expected, the rutting of pavements or even the failure of subgrade can be prevented by controlling the accumulation of excessive plastic strain by keeping repetitive load levels below the critical stress. Tests results have shown that subgrade soils can easily accumulate excessive plastic strain at high water content, but this will vary depending on the environmental conditions (Yang and Huang, 2007).

5.1.2 Granular materials

There have been substantial studies on the use of permanent deformation models for granular materials because of their common use in pavement/railway structures (Indraratna, 2013). The study of granular materials subjected to repetitive loads is fostered by the gradual accumulation of multiple increments of plastic deformation (Figure 3), that could lead to pavement failure due to excessive rutting (Lekarp et al., 2000). In order to justify the use of these materials in pavements some researchers tried to correlate repetitive loading with simple static loading tests (Lentz and Baladi, 1980), but there was no consensus with this approach because granular materials respond differently under static and cyclic loads (Sweere, 1990). Other studies also tried to correlate resilient and plastic deformation through a mathematical expression.

With regard to resilient behaviour, researchers choose a different approaches. Jouve et al. (1987) proposed decomposing axial and horizontal stresses and strains into volumetric and shear components, even though this expression depends on the permanent volumetric and shear strain ($N > 100$) with the mean and deviatoric stresses. Other studies have expressed permanent strain as a function of the number of load cycles (Barksdale, 1972, Sweere, 1990, Wolff and Visser, 1994, Khedr, 1985, Paute et al., 1988, Paute et al., 1994), and some researchers related permanent strain with the stress level and stress ratio (Brown and Hyde, 1975, Barksdale, 1972, Pappin, 1979). Lekarp et al. (2000) found a dependency between permanent strain, and the maximum stress ratio and length of the stress path in p - q space. In fact, applied stress is one of the primary factors that can influence the permanent deformation of aggregates (Xiao et al., 2015). To simulate the correct response of granular materials subjected to cyclic loads (similar to *in situ* conditions) in a laboratory, variable confining pressure triaxial tests are a practical approach because the effect of horizontal and vertical stresses can be combined.

The behaviour of granular materials under cyclic loads is complex due to the gradual accumulation of permanent strain with the number of load cycles. When a cyclic load is applied to a material there is a gradual accumulation of plastic deformation, a reduction in the number of voids, and an increase in stiffness (Erlingsson and Rahman, 2013, Ba, 2018). As with cohesive soils, the shakedown theory can be used to classify the response of the material in terms of permanent deformation. Furthermore, physical characteristics such as granulometry (poor or well graded) can also influence the response of granular materials, as well as their physical state (Gomes Correia, 2000). However, in most empirical models these variables are not directly considered.

5.2 Approach

Based on approach and procedures for a cyclic load test, mechanistic-empirical models can be categorised as either single-stage (SS) or multi-stage (MS) models. Single-stage models mean that repeated load test (RLT) are performed at one stress level in one test, while multi-stage models mean that RLT tests are performed at multiple stress levels in one test on one specimen. This approach is a relatively recent which considers how the stress level and the stress history will affect permanent deformation (Erlingsson and Rahman, 2013). Actually, a multi-stage RLT better represents reality and the real conditions of soils in the field that are subjected to cyclic loads. With the MS approach, the influence of different stress paths with different magnitudes (representing the influence of the effects of stress history) can be analysed. This approach is referred to in EN13286-7 (2004a) and it allows for the application of a certain number of consecutive stress paths to the same specimen, thus reducing the time spent in the laboratory, but it also increases the complexity compared to the single-stage RLT. Gidel et al. (2001) introduced this concept into their work, as did Erlingsson and Rahman (2013) noting that their work was based on the time-hardening concept introduced earlier by Lytton et al. (1993) and Gidel et al. (2001). In this method the permanent deformation formulation is modified (Erlingsson and Rahman, 2013):

$$\varepsilon_p = f_1(N)f_2(p, q, \varepsilon_r) \quad (2)$$

to

$$\varepsilon_p = f_1(N - N_{i-1} + N_i^{eq})f_2(p_i, q_i, \varepsilon_{ri}) \quad (3)$$

where, the subscript i refers to the i th stress path and N_i^{eq} is determined by considering the following expression:

$$N_i^{eq} = f_3(\hat{\varepsilon}_{pi-1}, p_i, q_i, \varepsilon_{ri}) \quad (4)$$

where, $\hat{\varepsilon}_{pi-1}$ is the accumulated permanent strain at the end of the $(i-1)$ th stress path.

Despite the reduction in time, most of the models presented in the bibliography are developed under the single-stage RLT like the Korkiala-Tanttu (2005) model.

5.3 Complexity

Empirical permanent deformation models have been changing over time with an increase in their complexity (variables included in the model). Most models establish a relationship between permanent deformation with the number of load cycles and the levels of stress. Some initial studies on resilient and permanent strains only considered the number of load cycles (Barksdale (1972)). Despite their simplicity, these models have some historical significance and enable us to understand why stresses are an important factor when determining permanent deformation. In fact, some of these models did consider the stress conditions indirectly in their constants, as presented in Table 5. Some studies showed the importance of the stress state and its influence was considered in the permanent deformation models, as shown in Table 6. It is noted that most of the empirical permanent deformation models the stress levels are defined through the deviatoric (q) and mean (p) stresses.

The most recently well-known models are an improvement on the others. Indeed, the first model only depended on the number of load cycles (N), whereas later models depended on the stress levels and some included the influence of stress indirectly (through their constants); these models are still simple as those developed by Monismith et al. (1975), Sweere (1990) and Huurman (1997).

Most models depend directly on the mean (p) and deviatoric (q) stresses and evaluate the length and slope of the stress path through the relationship established between q and p , as does the model developed by Hyde (1974), Lekarp and Dawson (1998) and Rahman and Erlingsson (2015). This model is dependent on the factor S_f :

$$\varepsilon_p(N) = \alpha N^{b_{S_f}} S_f$$

$$S_f = \frac{\left(\frac{q}{p_a}\right)}{\left(\frac{p}{p_a}\right)^\alpha} \quad (5)$$

Nevertheless, these models have considered the influence of stress dependency by including the stress ratio; for example Huurman (1997) included the effect of the stress

ratio on one of his model constants by considering the failure ratio of the major principal stresses $\sigma_1/\sigma_{1:f}$. This formulation means the deformations are larger when the failure ratio is close to failure. The model define the ratio as the relationship between deviatoric stress and deviatoric stress at failure (q/q_f) because the deviatoric stress is supposed to be the most dominating stress component for permanent strains (Li and Selig, 1996, Chai and Miura, 2002, Korkiala-Tanttu, 2005, Chow et al., 2014, Xiao et al., 2015, Yesufa and Hoffa, 2015, Gu et al., 2016). The model developed by Gidel et al. (2001) and further extended by Chen et al. (2014) is more complex because it depends on the failure criterion by including the strength parameters m and s of the *Mohr-Coulomb* yielding criterion.

Subsequently, there have been several models including the influence of initial stress indirectly and explicitly (Chai and Miura, 2002, Chen et al., 2014, Ling et al., 2017, Wei et al., 2017) that can explain the apparent decrease of permanent deformation as the initial mean stress increases. Put simply, a higher initial stress leads to a greater distance to the failure criterion which then leads to a lower value of permanent deformation. Some other recent models based on the power-law, $\varepsilon_p = A \cdot N^b$ include, among other factors, the octahedral normal and shear stresses (σ_{oct} and τ_{oct}) instead of the p and q in the formulation of the model (Puppala et al., 1999, Puppala et al., 2009, Cai et al., 2015). There are more complex models that also include the influence of the physical state of the material, as for example, the model by Xiao et al. (2015), but this model depends on several parameters that can be very difficult to obtain.

To simulate and predict permanent deformation, the most suitable model for each analysis/situation must be chosen, therefore the model must be selected according to the material to be tested and its physical condition. The model must then be evaluated according to its complexity in terms of modelling to the number of variables required by the formulation, and to its efficiency and accuracy with regards to expected results. A simpler model could easily represent *in-situ* conditions rather than complex models that depend on several variables that may be needed to carry out certain laboratory tests.

6 A comparison of permanent deformation models – parametric study

This section attempts to compare different permanent deformation models available for different types of soils considering the model developed by Chen et al. (2014). Whereas this comparison depends on the soil classification, the results go beyond the soil type.

Indeed, while two soils can be integrated into the same classification (UIC or ASTM), the laboratory conditions may differ greatly and therefore lead to different results.

It is noted that Chen et al. (2014)'s model considers the stress level (the model includes the amplitude of diagram p - q), the number of load cycles, the initial stress state, and it quantifies the proximity of the peak stress point (p_{max} , q_{max}) to the *yielding criterion*. This particular model, therefore, includes several important conditions in its analysis of permanent deformation.

For all the experimental data, calibration was performed to find the best fit for the experimental data through the parameters $\varepsilon_{p,o}$, α and B (correspond to material properties), presented in the following equation:

$$\varepsilon_1^p(N) = \varepsilon_1^{p0} [1 - e^{-BN}] \left(\frac{\sqrt{p_{am}^2 + q_{am}^2}}{p_a} \right)^a \cdot \frac{1}{m \left(1 + \frac{p_{ini}}{p_{am}} \right) + \frac{s}{p_{am}} - \frac{(q_{ini} + q_{am})}{p_{am}}} \quad (6)$$

where, p_{am} and q_{am} are the amplitude of the mean stress and deviator stress for train loadings, m and s are defined by the yielding criterion $q=s + mp$; and p_{ini} and q_{ini} are the mean and deviator stress in the initial state of the material.

These selected materials are representative of different types of materials (silt and sand), the percentage of fines, and the granulometry (poor and well graded sands), as depicted in Table 7. The models are calibrated considering the same stress paths for all materials during the calibration process. The stress path described by Chen et al. (2014) is carried out by a cyclic deviator stress of 24 kPa and a constant confining stress of 60 kPa. During the cyclic tests the stress ratio (σ_d/σ_c) is constant at 0.4, which is a representative ratio in the subgrade of a full scale model test. It is noted that other confining stresses from 60 kPa to 210 kPa were also tested. The stress path and the *Mohr-Coulomb yielding criterion* of each material are shown in Figure 6 showing the stress path applied on the cyclic triaxial test (with a stress ratio equal to 0.4); and the failure envelopes of different materials used in the laboratory tests. The failure envelopes are defined through the *Mohr-Coulomb yielding criterion* from the expression $q = s + mp$.

It is seen that the distance between the stress path and the failure envelopes influence the development of permanent deformation. In order to understand which main factors influence cumulative settlement, and compare the long term response of the materials selected, three main models were selected and calibrated as introduced by Huurman (1997), Chen et al. (2014) and Rahman and Erlingsson (2015); these models have been formulated and developed under different conditions in terms of materials and applied stress ratios, as described below.

With the Huurman (1997) model, the samples are tested according to their optimum moisture content and with a confining pressure equal to 12 kPa, whereas with the Chen et al. model, the specimens are saturated and the boundary is the drainage condition. With the Rahman and Erlingsson (2015) model, the influence that the moisture content has on permanent deformation, and the degree of saturation, is analysed but this analysis only considers the optimum moisture content for each type of soil. Confining pressure, σ_3 is tested as 27.6, 41.4 and 55.2 kPa, and cyclic deviator stress varies according to the stress path. Thus, the development of these models includes laboratory conditions such as soils under different state conditions and different stress conditions.

Whereas Huurman (1997) considers seven poorly graded sands and one well-graded sand (designated as crusher), only *C. Bruynweg* and *Crusher* corresponds to the poor and well-graded sands are analysed (Table 8). In addition, the *C. Bruynweg* was selected as the stress ratio is similar to the one carried out by Chen et al. (2014). The remaining sands were tested for stress conditions close to failure ($\sigma_1/\sigma_{1,f}$ close or equal to one), which is outside the representative stress conditions of this study. In this work, the parameters of Huurman's model are adjusted so that the parameters ε_{p0} , a and B can be obtained; this adjustment took place through the formulation *lscurvefit* in *Matlab*.

In Rahman and Erlingsson (2015), two silty sands are analyzed and compared with the parameters of Chen's model. The study also shows a correlation between the ε_{p0} and α parameters and the moisture content. In the work developed by Chen et al. (2014), a coarse sand and a silt are analysed and then the properties of these materials are divided according to their granulometry (Table 8), their state conditions (Table 9), and their failure parameters (Table 10).

It is important to first consider Huurman (1997) work in order to understand the influence that granulometry (well and poorly graded soil) has on permanent deformation. The

analysis utilises the following stress ratios: 0.695, 0.803, 0.911. Figure 7 shows that well-graded sand has larger permanent deformation values for each stress ratio, which means that granulometry has an important influence on permanent deformation, as expected.

Table 11 presents the calibrations of the models for each selected soil where a regression analysis was performed. It is seen that there is some correlation (exponential and polynomial) between the three constant variables - ε_{p0} , α and β (superior to 0.7) - but correlation between the constant variables and UIC classification (QS1, QS2, and QS3) is only residual.

The comparison between the permanent deformation curves presented in Figure 8. It is seen that the results of silty sand and coarse sand (43% and 27% of fines) are very close, whereas the well and poorly-graded sand and silt are at opposite sides (the highest and smallest value of permanent deformation). This result is related to QS3-SW sand because a lower value was expected in comparison to QS2-SP. Another important fact is related to the influence of the stress ratio; in this instance, the permanent deformation results are not 100% aligned with those presented in Figure 6 because higher permanent deformation is expected when the stress path is close to the *yielding criterion* that depends on the values s and m of the *Mohr-Coulomb yielding criterion*. This means that in addition to the strength properties, the soil physical properties (represented by the constants in the model) are also important in permanent deformation.

7 Ranking the materials

The materials and calibration process described in the previous section enable the results to be interpreted and the materials can be ranked according to their resistance to permanent deformation (Coronado et al., 2011). This information can then be used to predict permanent deformation during the design process. The ranking process provides reference values according to the type of material as well as the moisture content close to optimum conditions. In reality, Figure 9 is a re-interpretation of Figure 8. This innovative procedure enables the expected values of permanent deformation to be compared according to the type of material through the UIC and ASTM soil classification; this ranking also includes different types of soils (clays and sands) and different granulometry. Figure 9 shows that the materials classified as CL-ML have a higher permanent deformation than the sands (SM, SP, and SW), while the sands with fines have a higher deformation than the SP and SW sands; therefore this parameter influences the permanent

deformation and ranking of the geomaterials. During the design process the long-term behaviour of the materials can be estimated, even though the permanent deformation will be influenced by their state conditions such as the moisture content; therefore the influence of this parameter in this analysis must be traduced.

Among other factors, moisture content also influences permanent deformation. Puppala et al. (2009) developed a model which depends on the octahedral normal and shear stresses (Table 1 and Table 3). Different materials, clay, silt, and sand with moisture contents ranging from dry of optimum, optimum and wet of optimum were tested. Three different confining pressures were applied to the clay and silt (0, 21 and 42 kPa), and confining pressures of 21, 48 and 97 kPa were applied to the sand. The deviatoric stress was determined by considering a percentage of the maximum deviatoric stress of each soil sample obtained in the unconsolidated undrained triaxial tests, so the deviatoric stress varies from $0.2\sigma_{d,f}$, $0.4\sigma_{d,f}$ and $0.6\sigma_{d,f}$.

To evaluate the influence that the moisture content has on permanent deformation, the results from Puppala et al. (2009) are used, but this analysis only focuses on the behaviour of clay (QS1 – CL) because it was not contemplated in previous studies. This exercise shows how important this parameter is, and the consequences in terms of permanent deformation when the moisture content moves away from the optimum conditions. Material properties are presented in Table 12, and as with the previous models and materials, the curve of permanent deformation has been adjusted to obtain the model parameters.

Figure 10 shows that the differences between dry, optimum and wet materials in terms of permanent deformation after 30 000 cycles is significant; the optimum material is 4-8 times higher than the dry material, and the wet material is 1-2 times higher than the optimum conditions. Therefore, when the moisture content is far away from optimum conditions, the variations of permanent deformation in the clays are substantial. In dry conditions, this value decreases significantly and in wet conditions, it increases which means that maximum permanent deformation occurs when the moisture content is beyond optimum (wet) due to an increase in the residual accumulation of excess water pressure.

8 Conclusions

The ability to determine the permanent deformation of geomaterials is very important when modelling and evaluating the performance of pavements and railway structures; it

is also a key factor when estimating future maintenance operations and the respective costs. This review paper has aimed to frame the main reasons why permanent deformation models were developed, particularly in pavements and railway structures. However, some issues related to traffic-induced permanent deformation weren't evaluated in this paper, such as the estimation of the dynamic stresses and strategies to simulate the effect of rotation of principal stresses. Indeed, these topics will be evaluated in further works related with the numerical modelling of a railway structure, including the influence of the train's speed in the stress results and permanent deformation. It is expected that the train's speed will increase the stress amplitude (the stress path is close to the yielding criterion) and also the permanent strain.

In this paper, it was found that permanent deformation can be evaluated and predicted through laboratory tests, and most mechanistic-empirical models were based on cyclic triaxial tests.

Main concepts of permanent deformation and the laboratory tests used to predict them were reviewed, the approaches used to simulate permanent deformation such as elastoplastic models, shakedown theory, and mechanistic-empirical models were discussed. The definition of the shakedown is a very popular approaches nowadays since allows to prevent plastic deformation that can be critical after some years of exploration. This method became popular in pavement engineering with inclusion of 3D modelling and generalization of the method for non-isotropic materials. However, the application of the shakedown analysis in railway engineering is limited. The shakedown approach however, demands the accurate assessment of the response of the track foundation due to railway traffic, which also demands powerful numerical modelling. It was found that the mechanistic-empirical model reduced the time needed for calculations, it has simple and elegant formulations, good predictive results, and is easy to implement in a numerical scheme. These mechanistic-empirical models were defined and categorised according to the materials tested, the approach used in the laboratory (single-stage or multi-stage), the complexity of the formulation, and the variables included in the mathematical expression. It was concluded that the permanent deformation models should also include the influence of the main stress variables (p and q), the initial state of the materials, the number of load cycles, and the distance to the rupture line; moreover the constant parameters should also reflect the influence of factors such as the state conditions of the material. However, those

complex mechanistic-empirical models that depend on several variables can also imply a need for tests other than a standard triaxial cyclic test, in order to obtain more parameters.

The mechanistic-empirical models summarised in the Annex were divided according to the type of material (clay, silt, sand and gravel) to better understand the model's formulation and the condition of the material tested in terms of its granulometry (C_u and C_c) and plasticity properties (W_L , W_P , and IP). The Tables also include some observations regarding the laboratory tests and classification of the material (UIC and ASTM). This information (including the classification) can be very helpful when modelling sub-structure and predicting its performance.

Considering the available mechanistic-empirical models as well as understanding the main variables and factors that can affect the long-term response of geomaterials, it is important to evaluate the robustness of the models and their sensitivity. Thus, some models and materials were selected to perform a parametric study. The choice depends on the data available for each material and the diversity and variability of the geomaterials (type of soils) in terms of its granulometry, percentage of fines, moisture content, and plasticity properties, etc.

In the comparison analysis: silt, silty sand, and sand were divided according to properties such as the percentage of fines (27% and 43%) and granulometry (poor and well-graded materials). Selected experimental results found in the literature were then adapted to Chen's model. The calibration process was performed to compare the long-term response of these geomaterials, after which they were defined and ranked based on the results of permanent deformation. This ranking tool can be used during the design process for estimating the permanent deformation of a certain type of material after N loading cycles, while considering the *ASTM* and *UIC* classification. As expected, the comparison showed that the values of permanent deformation decreased according to the UIC classification, so well-classified materials showed a reduction in permanent deformation. Furthermore, the deformation behaviour of these materials also depended on their granulometry, the percentage of fines, and the moisture content. Based on the proposed ranking, a brief analysis has been carried out to understand and quantify the influence of the moisture content. The results showed that for a material classified as QS1 (CL), designated as clay, the moisture content exerts a significant influence on permanent deformation.

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Annex

An extensive list of the permanent deformation based on laboratory tests are presented in this Annex.

Although these tables are constructed in a uniform manner, especially the nomenclature of the variables and the parameters and characteristics of the materials, the authors still maintained the original symbols and nomenclature describe in the original paper.

Some models were developed using certain SI units, and where possible these units are described in the column “*variables and empirical constants*”.

The permanent deformation is an adimensional parameter that can also be designated as permanent strain and its nomenclature varies, as mentioned above. The permanent strain is defined as change in the length of the material and expressed as a function of the length being changed. The plastic/permanent deformation is defined as not recoverable. The materials that are part of the pavements and railway structures accumulate some amount of permanent deformation due to the traffic cyclic load. The stress parameters (induced by the cyclic loads) are one of the main inputs of these models. The inclusion of this parameter in the permanent deformation model varies as well as the nomenclature. Indeed, there are models dependent on the p and q (mean and deviatoric) stresses or on the octahedral stresses, for example.

In the permanent deformation tables the deviator stress is represented by q_d , q or σ_d , the variable for the atmospheric pressure/reference stress is represented by σ_{atm} , p_0 , p_a or p_{ref} , while the equation models were developed under different conditions, so permanent deformation is represented in the following tables by the nomenclatures: ε_p , $\varepsilon_{p1,N}$, ε_z^p , $\varepsilon_{q,r}$ (axial residual strain), ε_1^p , $\varepsilon_p(N)$, $\varepsilon_1^p(N)$. The constants of the models used to fit the model to the available data, are represented by different letters and symbols which are defined as material parameters, model constants, and model parameters and soil parameters, as listed in the permanent deformation tables.

Table 1. Summaries of permanent deformation models for fine-grained soils - clays

UIC	ASTM classification	Author's model	Equation Model	Variables and Empirical Constants	Parameters and characteristics	Observations
-	-	Monismith et al. (1975)	$\varepsilon^p = AN^b$	<ul style="list-style-type: none"> - ε_p is the plastic deformation (%); - N is the number of load cycles; - A and b are two parameters that represent the influence of other factors. 	-	<ul style="list-style-type: none"> - This model has a limited practical used since there are other factors, besides N, that can influence the permanent deformation; - b is an independent parameter on the deviatoric stress; - The coefficient A corresponds to the plastic deformation after the first cyclic load.
-	-	Chai and Miura (2002)	$\varepsilon^p = a \left(\frac{q_d}{q_f} \right)^m \left(1 + \frac{q_s}{q_f} \right)^n N^b$	<ul style="list-style-type: none"> - q_s is the initial static deviatoric stress; - $n = \text{constant}$. 	-	<ul style="list-style-type: none"> - This model includes the effects of initial static stress, the magnitude and number of applied loads associated to the passage of the vehicle, and the properties of the subsoil in terms of compression and strength.
-	-	Yesufa and Hoffa (2015)*	$\varepsilon_{p1,N} = N^{m(1-(\sigma_d/q_s))} \varepsilon_{p1,0}$	<ul style="list-style-type: none"> - $\varepsilon_{p1,N}$ is the axial permanent deformation for the cyclic load N; - $\varepsilon_{p1,0}$ is the cumulative axial permanent deformation after the first cyclic load; 	-	<ul style="list-style-type: none"> - The parameters of these materials were found through triaxial tests: the static deviatoric stress comes from the difference between the axial and confining stress in failure; - $\varepsilon_{p1,0}$ is from the equivalent static load in the first cyclic

				<ul style="list-style-type: none"> - σ_d is the deviatoric stress induced by the passage of the vehicle; - q_s is the static stress measured on the soil; - m is a constant determined to fit the laboratory test's results. 		load (based on the plasticity theory).
-	-	Cai et al. (2015)	$\varepsilon_z^p = \alpha_1 N^{\alpha_2} \left(\frac{\sigma_{oct}}{\sigma_{atm}} \right)^{\alpha_3} \left(\frac{\tau_{oct}}{\sigma_{atm}} \right)^{\alpha_4} \left(\sqrt{1 + 4\eta^2} \right)^{\alpha_5}$	<ul style="list-style-type: none"> - $\alpha_1, \alpha_2, \alpha_3, \alpha_4$ and α_5 are constants of the model and determined through laboratory tests; - σ_{oct} and τ_{oct} are the normal and shear octahedral stresses, respectively; - σ_{atm} is the reference stress; - η is the cyclic torsional stress ratio. 	-	<ul style="list-style-type: none"> - This model is based on a series of tests using the hollow cylinder apparatus. The characteristics of the permanent deformation of a sand were then studied in drained conditions by considering different confining stress levels, ratio of the cyclic vertical stress and ratio of the cyclic torsional stress; - The model was developed to include the rotation of principal stresses.
QS0 High plasticity	CH Fat clay	Li and Selig (1996)	$\varepsilon^p = a \left(\frac{q_d}{q_f} \right)^m N^b$	<ul style="list-style-type: none"> - q_d is the traffic-load-induced dynamic deviator stress; - q_f is the static failure deviator stress of soil; 	<ul style="list-style-type: none"> - a=0.64; - b=0.10; - m=1.7. 	

				a, b and m are soil parameters and are related to the plasticity index of the subsoil.		
QS0 Heavy clay	CH Fat Clay	Puppala et al. (1999)	$\log \varepsilon^p$ $= \log A$ $+ \alpha \log N + \beta \log \left(\frac{\sigma_{oct}}{\sigma_{atm}} \right)$	A, β and α are soil parameters; σ_{oct} and τ_{oct} are the octahedral normal and shear stresses, respectively.	$W_L=93\%$; $W_P=29\%$; $I_P=64\%$ (high plasticity); Sand content=0%; Silt content=5%; Clay content=95%.	The model was tested based on triaxial tests; During this study, materials such as well graded sand, silty clay and clay were considered.
QS0 $W_L=60\%$ High plasticity	(without enough data to carry out the ASTM classification)	Wei et al. (2017) – based on Guo (2013) - (Wenzhou structural clay)	$\varepsilon_{q,r} = AN^b$; $A = \varepsilon_{q,r} = \alpha K_q K_T K_\alpha K_\sigma$	<ul style="list-style-type: none"> $\varepsilon_{q,r}$ is the axial residual strain; A is the residual strain generated in the first cycle; b is a material parameter determining the cumulating rate of the residual strain with loading cycles and is equal to the gradient of the residual strain curve in the $\ln \varepsilon_{q,r} - \ln N$ coordinate; K_q and K_T are functions to reflect the effect of wheel load-induced stress; K_α and K_σ are functions to reflect the effect of initial static 	$W_n=60-62\%$; $G_s=2.71$; $S_t=5.9$; M (critical stress ratio) = 1.31.	<ul style="list-style-type: none"> This model is based on the power function developed by Monismith et al. (1975); The parameter A depends on the initial stress state and the imposed dynamic stress on non-remoulded Wenzhou clay samples (Guo, 2013). The samples were collected from a depth of 6-7 m; The parameters are based on the work by Xiao et al. (2014) who carried out a series of tests based on the hollow cylinder apparatus.

				stress state and dynamic stress combinations.		
QS1 (low plasticity)	CL Lean clay	Hyde (1974)	$\varepsilon_1^p = a \frac{q}{\sigma_3}$	<ul style="list-style-type: none"> a is a constant; σ_3 is the confining stress; q is deviator stress. 	<ul style="list-style-type: none"> $W_L=32\%$; $W_P=18\%$; $I_P=14\%$; $G_s=2.74$. 	<ul style="list-style-type: none"> The tests included a constant and variable confining stress; During the analysis, the behaviour of the marl was studied; The author states that these results would be the same if medium confining stress was applied.
QS1 Low to medium plasticity	CL Lean clay	Li and Selig (1996)	$\varepsilon^p = a \left(\frac{q_d}{q_f} \right)^m N^b$	<ul style="list-style-type: none"> q_d is the traffic-load-induced dynamic deviator stress; q_f is the static failure deviator stress of soil; a, b and m are soil parameters and are related to the plasticity index of the subsoil. 	<ul style="list-style-type: none"> $a=0.84$; $b=0.13$; $m=2.0$. 	
QS1 Silty clay	CL Lean clay	Puppala et al. (1999)	$\log \varepsilon^p = \log A$ $+ \alpha \log N + \beta \log \left(\frac{\sigma_{oct}}{\sigma_{atm}} \right)$	<ul style="list-style-type: none"> A, β and α are soil parameters; σ_{oct} and τ_{oct} are the octahedral normal and shear stresses, respectively. 	<ul style="list-style-type: none"> $W_L=44\%$; $W_P=15\%$; $I_P=29\%$ (medium plasticity); Sand content=10%; Silt content=70%; Clay content=20%. 	<ul style="list-style-type: none"> The model was tested based on triaxial tests; During this study, well-graded sand, silty clay and clay were tested.
QS1 Silty clay	CL Lean clay	Puppala et al. (2009)	$\varepsilon^p = \alpha_1 N^{\alpha_2} \left(\frac{\sigma_{oct}}{P_{ref}} \right)^{\alpha_3} \left(\frac{\tau_{oct}}{P_{ref}} \right)^{\alpha_4}$	<ul style="list-style-type: none"> $\alpha_1, \alpha_2, \alpha_3$ and α_4 are the model constants 	<ul style="list-style-type: none"> $W_L=28.19\%$; $I_P=12.55\%$ (low plasticity); $G_s=2.63$; 	<ul style="list-style-type: none"> The model is based on 10,000 cycles; It includes the effects of mean and shear stresses;

				<p>determined from laboratory tests; σ_{oct} and τ_{oct} are the octahedral normal and shear stresses, respectively; P_{ref} is the reference stress.</p>	<p>Passing #200 =80%; Maximum dry unit weight = 17.10 kN/m³; W_{opt}=17.11%; c=60 kPa; ϕ=18°.</p>	<p>The model was tested based on the cyclic triaxial tests; The permanent deformation of clay, silt and sand were measured from samples of compacted soil with different water content.</p>
QS1 Silty clayed sand	CL Lean Clay	Puppala et al. (2009)	$\varepsilon^p = \alpha_1 N^{\alpha_2} \left(\frac{\sigma_{oct}}{P_{ref}} \right)^{\alpha_3} \left(\frac{\tau_{oct}}{P_{ref}} \right)^{\alpha_4}$	<p>$\alpha_1, \alpha_2, \alpha_3$ and α_4 are the model constants determined from laboratory tests; σ_{oct} and τ_{oct} are the octahedral normal and shear stresses, respectively; P_{ref} is the reference stress.</p>	<p>W_L=16.70%; I_P=7.50% (low plasticity); G_s=2.70; Passing #200=38%; Maximum dry unit weight =16.9 kN/m³; W_{opt}=19.3%; c=103 kPa; ϕ=35°.</p>	<p>The model is based on 10,000 cycles; It includes the effects of the mean and shear stresses; The model was tested based on the cyclic triaxial tests; The permanent deformations of clay, silt and sand were measured from samples of c soil compacted with different water contents.</p>
QS1	CL Lean clay	Wei et al. (2017) – based on Xiao et al. (2014) - Clay (Shanghai structural soft clay)	$\varepsilon_{q,r} = AN^b;$ $A = \varepsilon_{q,r} = \alpha K_q K_T K_\alpha K_\sigma$	<p>$\varepsilon_{q,r}$ is the axial residual strain; A is the residual strain generated in the first cycle; b is a material parameter determining the cumulating rate of the residual strain with loading cycles and is equal to the gradient of the residual strain curve</p>	<p>I_P=21.8%; I_L= 1.35 (medium plasticity); G_s=2.74; W_n=51.8%; e₀=1.402; M (critical stress ratio) = 1.28.</p>	<p>The model is based on the power function developed by Monismith et al. (1975); The parameter A depends on the initial stress state and the imposed dynamic stress on samples of non-remoulded Wenzhou clay (Guo, 2013). The samples were collected from a depth of 6-7 m; The parameters were determined based on the</p>

				<p>in the $\ln \varepsilon_{q,r} - \ln N$ coordinate;</p> <p>- K_q and K_T are functions to reflect the effect of wheel load-induced stress;</p> <p>- K_α and K_σ are functions to reflect the effect of initial static stress state and dynamic stress combinations.</p>		<p>work by Xiao et al. (2014) who performed a series of tests based on the hollow cylinder apparatus.</p>
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Table 2. Summarises of permanent deformation models for fine-grained soils - silts

UIC	ASTM classification	Author's model	Equation model	Variables and Empirical Constants	Parameters and characteristics	Observations
QS1 Low to medium plasticity	ML Silt	Li and Selig (1996)	$\varepsilon^p = a \left(\frac{q_d}{q_f} \right)^m N^b$	q_d is the traffic-load-induced dynamic deviator stress; q_f is the static failure deviator stress of soil; a , b and m are soil parameters and are related to the plasticity index of the subsoil.	$a=1.2$; $b=0.18$; $m=2.4$.	
QS1	ML-CL Silt - Lean clay	Chen et al. (2014)	$\varepsilon_1^p(N) = \varepsilon_1^{p0} \left[1 - e^{-BN} \right] \left(\frac{\sqrt{p_{am}^2 + q_{am}^2}}{p_a} \right)^a$ $\frac{1}{m \left(1 + \frac{p_{ini}}{p_{am}} \right) + \frac{s}{p_{am}} - \frac{(q_{ini})}{p_{am}}}$	$\varepsilon_1^p(N)$ is the strain settlement; ε_1^{p0} , B , a , s and m are material parameters; $p_a=100$ kPa; p_{ini} and q_{ini} are the mean and deviator stress in the initial state of the layer; p_{am} and q_{am} are the amplitude of mean stress and deviator stress for train loadings, respectively; s is the intersection of the <i>Mohr-Coulomb yielding criterion</i> in the q axle in the p - q space;	$W_L=35$ %; $W_P=24$ %; $I_P=11$ % (low to medium plasticity); $G_s=2.67$; $C_u=2.51$; $C_c=1.32$; Maximum dry density =15.89 kN/m ³ ; Hydraulic permeability = 5.3×10^{-7} m/s; $c'=11.7$ kPa; $\phi'=16.4^\circ$.	The model is based on Gidel's model (Gidel et al., 2001); This model takes into account the direct influence of the initial stress state.

				- m is the slope of the yielding criterion.		
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Table 3. Permanent deformation models for granular soils (sands)

UIC	ASTM classification	Author's model	Equation model	Variables and Empirical Constants	Parameters and characteristics	Observations
-	-	MEPDG model (Ara and Division, 2004)	$\varepsilon_p = \beta_s \left(\frac{\varepsilon_0}{\varepsilon_r} \right) e^{-(\rho/N)^\beta} \varepsilon_v$	<ul style="list-style-type: none"> - β_s is the global calibration coefficient; - ε_r is the resilient strain imposed in the laboratory test; - ε_v is the average vertical resilient strain (in the base layer of the flexible pavements, for example). 	-	<ul style="list-style-type: none"> - This model considers the effect of the stress on permanent deformation by linearly projecting the plastic deformation obtained from the laboratory tests through vertical strains. This projection is an assumption without any experimental or mechanical justification; it is therefore inaccurate due to the nonlinear effect that stress has on the permanent deformation of unbound granular materials - UGM (Gu et al., 2016).
-	-	Korkiala-Tanttu (2005)	$\varepsilon_p(N) = CN^b \frac{R}{A - R}$	<ul style="list-style-type: none"> - C is a parameter dependent on the stress and also compaction degree and water content; - b is a parameter dependent on the stress level, ratio of failure, compaction degree and water content; 	-	<ul style="list-style-type: none"> - This model is based on the Sweere's model (Sweere, 1990). - It uses the deviatoric stress ratio to capture the nonlinear effect of the stress state. However, plastic deformation is infinity when the load cycles go to infinity, so

				<ul style="list-style-type: none"> - A corresponds to the maximum value of the ratio (the author suggests a value close to 1.05); - $R = \frac{q}{q_f}$, q is the deviatoric stress and q_f corresponds to the failure deviatoric stress; 		the model cannot demonstrate the hardening and softening behavior of UGM.
-	-	Chow et al. (2014)	$\varepsilon^p = AN^B \sigma_d^C \left(\frac{\tau_f}{\tau_{max}} \right)^D$	<ul style="list-style-type: none"> - σ_d is the deviatoric shear stress; - τ_f is the shear stress; - τ_{max} is the shear strength; - A, B, C and D are the regression coefficients. 	-	<ul style="list-style-type: none"> - The model includes the power functions of deviatoric shear stress and the shear strength ratio; - The author developed 16 types of materials with one confining pressure (=34.5 kPa) and three deviatoric stresses. The RLT tests show that this model has higher R^2 values but when the number of load cycles (N) goes to infinity, the accumulated plastic deformation also goes to infinity at one confining pressure. Moreover, the deviatoric shear stress term affects the shear ratio in the model and both parameters represent the softening behavior of the material

						without including the hardening phenomenon.
-	-	Xiao et al. (2015)	$\varepsilon_p = a \left(\frac{\theta}{p_0} \right)^b \left(\frac{\tau_f}{\tau_{max}} \right)^c N^d$ $\cdot EXP \left[e \right.$ $\cdot (S - S_{opt})$ $+ f \cdot wPI$ $+ g \cdot wPI$ $\cdot \frac{\tau_f}{\tau_{max}} + h$ $\cdot (S - S_{opt})$ $\cdot \frac{\theta}{p_0} \left. \right]$	<ul style="list-style-type: none"> - θ is the sum of the principal stresses; - τ_f/τ_{max} is the ratio related to the shear stress; - S corresponds to the saturation level; - S_{opt} corresponds to the optimum saturation level; - wPI corresponds to a weighted plasticity index; - a, b, c, d, e, f, g and h are the parameters determined based on regression analysis. 	-	<ul style="list-style-type: none"> - Triaxial cyclic tests took place on the limestone, dolomite and gravel samples (materials used in base, sub-base and subgrade layers treated in Illinois); - The cohesion and friction angle were determined for three confining stresses.
QS1 Silty sand with high fine content	SM Silty sand	Rahman and Erlingsson (2015) and Salour and Erlingsson (2016)	$\varepsilon_p(N) = \alpha N^{bS_f} S_f$ $S_f = \frac{\left(\frac{q}{p_a} \right)}{\left(\frac{p}{p_a} \right)^\alpha}$	<ul style="list-style-type: none"> - q is deviatoric stress; - p is mean stress; - p_a atmospheric pressure; - a, b and α are the regression parameters. 	<ul style="list-style-type: none"> - $C_u \approx 28$; $C_c \approx 0.54$; - $G_s = 2.68$; - Fine content = 42.2%; - Maximum dry density = 19.6 kN/m³; - $W_{opt} = 10.1\%$. 	<ul style="list-style-type: none"> - The triaxial tests were carried out by considering the constant confining stress; - They considered high stress levels and reduced stress levels.
QS1 Silty sand with high fine content	SM Silty sand	Rahman and Erlingsson (2015) and	$\varepsilon_p(N) = \alpha N^{bS_f} S_f$ $S_f = \frac{\left(\frac{q}{p_a} \right)}{\left(\frac{p}{p_a} \right)^\alpha}$	<ul style="list-style-type: none"> - q is deviatoric stress; - p is mean stress; - p_a atmospheric pressure; 	<ul style="list-style-type: none"> - $C_u \approx 33$; $C_c \approx 0.75$; - $G_s = 2.67$; - Fine content = 27.4%; 	

		Salour and Erlingsson (2016)		a, b and α are the regression parameters.	Maximum dry density = 20.3 kN/m ³ ; W _{opt} = 7.6%.	
QS2	SP Poorly graded sand	Huurman (1997)	$\varepsilon_1^p(N) = A \left[\left(\frac{N}{1000} \right)^B + C \cdot \left(e^{D \cdot \frac{N}{1000}} - 1 \right) \right]$	A, B, C, D are parameters function of stress level and stress ratio ($\sigma_1/\sigma_{1,f}$).	C _u =1.69; C _c =1.39; Maximum dry Density = 16.68 kN/m ³ ; W _{opt} =12.5%; c=4.08 kPa; $\phi=43.9^\circ$.	Cyclic triaxial tests were carried out on the sands used in the base layers; The samples were subjected to a confining stress equal to 12 kPa (corresponds to a depth equal to 0.60m); During the test, a million cycles were applied (frequency equal to 5 Hz).
QS2	SP Poorly graded sand	Huurman (1997)	$\varepsilon_1^p(N) = A \left[\left(\frac{N}{1000} \right)^B + C \cdot \left(e^{D \cdot \frac{N}{1000}} - 1 \right) \right]$	A, B, C, D are parameters function of stress level and stress ratio ($\sigma_1/\sigma_{1,f}$).	C _u =1.69; C _c =0.998; Maximum dry Density = 16.73 kN/m ³ ; W _{opt} =13.0%; c=6.76 kPa; $\phi=43.0^\circ$.	
QS2	SP Poorly graded sand	Huurman (1997)	$\varepsilon_1^p(N) = A \left[\left(\frac{N}{1000} \right)^B + C \cdot \left(e^{D \cdot \frac{N}{1000}} - 1 \right) \right]$	A, B, C, D are parameters function of stress level and stress ratio ($\sigma_1/\sigma_{1,f}$).	C _u =1.88; C _c =1.04; Maximum dry Density = 16.56 kN/m ³ ; W _{opt} =14.0%; c=6.34 kPa; $\phi=41.8^\circ$;	
QS2	SP Poorly graded sand	Huurman (1997)	$\varepsilon_1^p(N) = A \left[\left(\frac{N}{1000} \right)^B + C \cdot \left(e^{D \cdot \frac{N}{1000}} - 1 \right) \right]$	A, B, C, D are parameters function of stress level and stress ratio ($\sigma_1/\sigma_{1,f}$).	C _u =2.10; C _c =1.05; Maximum dry Density = 16.90 kN/m ³ ; W _{opt} =12.5%;	

					c=5.60 kPa; φ=48.2°;	
QS2	SP Poorly graded sand	Huurman (1997)	$\varepsilon_1^p(N) = A \left[\left(\frac{N}{1000} \right)^B + C \cdot \left(e^{D \cdot \frac{N}{1000}} - 1 \right) \right]$	A, B, C, D are parameters function of stress level and stress ratio ($\sigma_1/\sigma_{1,f}$)	C _u =1.70; C _c =1.10; Maximum dry Density = 15.61 kN/m ³ ; W _{opt} =14.50%. c=7.19 kPa; φ=42.8°;	
QS2	SP Poorly graded sand	Huurman (1997)	$\varepsilon_1^p(N) = A \left[\left(\frac{N}{1000} \right)^B + C \cdot \left(e^{D \cdot \frac{N}{1000}} - 1 \right) \right]$	A, B, C, D are parameters function of stress level and stress ratio ($\sigma_1/\sigma_{1,f}$).	C _u =2.35; C _c =1.14; Maximum dry Density = 16.16 kN/m ³ ; W _{opt} =14.5%; c=7.99 kPa; φ=39.7°.	Cyclic triaxial tests took place on the sands used in the base layers; The samples were subjected to a confining stress equal to 12 kPa (corresponds to a depth equal to 0.60m); During the test, a million cycles were applied (frequency equal to 5 Hz).
QS2	SP Poorly graded sand	Huurman (1997)	$\varepsilon_1^p(N) = A \left[\left(\frac{N}{1000} \right)^B + C \cdot \left(e^{D \cdot \frac{N}{1000}} - 1 \right) \right]$	A, B, C, D are parameters function of stress level and stress ratio ($\sigma_1/\sigma_{1,f}$).	C _u =3.76; C _c =1.37; Maximum dry Density = 16.16 kN/m ³ ; W _{opt} =13.5%; c=7.48 kPa; φ=42.9°;	
QS2	SP Poorly graded sand	Sweere (1990)	$\log(\varepsilon_1^p(N)) = a + b \log(N)$	a, b – material parameters for each stress level, separate permanent strain parameters a and b need to be determined from the results of the cyclic load triaxial test.	Density = 16.79 kN/m ³ ; C _u = 1.68; C _c = 1.10.	Application of 10 ⁶ cyclic loads during the triaxial cyclic tests; The samples consisted of granular materials used in the base layers, and sands; The UGM samples were subjected to triaxial cyclic tests.

				**The Sweetes's formula is valid for granular materials (Gidel et al., 2001).		
QS2 Sa (>0.063 to 2)	SP Poorly graded sand	Puppala et al. (2009)	$\varepsilon^p = \alpha_1 N^{\alpha_2} \left(\frac{\sigma_{oct}}{P_{ref}} \right)^{\alpha_3} \left(\frac{\tau_{oct}}{P_{ref}} \right)^{\alpha_4}$	$\alpha_1, \alpha_2, \alpha_3$ and α_4 are the model constants determined from laboratory tests; σ_{oct} and τ_{oct} are the octahedral normal and shear stresses, respectively; P_{ref} is the reference stress.	$W_L=26.40\%$; $G_s=2.71\%$; Passing #200=0.70%; $C_u=1.79$; $C_c=0.89$; Maximum dry unit weight =15.70 kN/m ³ ; $W_{opt}=13.70\%$; $c=20$ kPa; $\phi=42^\circ$;	The model is based on 10,000 cycles, and includes the effects of mean and shear stresses; The model was tested based on the cyclic triaxial tests; The permanent deformation of clay, silt and sand were measured using the soil samples compacted with different water content.
QS2 Gravelly sand (SaGr)	SP Sand poorly graded	Chen et al. (2014)	$\varepsilon_1^p(N) = \varepsilon_1^{p0} \left[1 - e^{-BN} \right] \left(\frac{\sqrt{p_{am}^2 + q_{am}^2}}{p_a} \right)^a$ $\frac{1}{m \left(1 + \frac{p_{ini}}{p_{am}} \right) + \frac{s}{p_{am}} - \frac{(q_{ini})}{p_{am}}}$	$\varepsilon_1^p(N)$ is the strain settlement; ε_1^{p0} , B , a , s and m are material parameters; $p_a=100$ kPa; p_{ini} and q_{ini} are the mean and deviator stress in the initial state of the layer; p_{am} and q_{am} are the amplitude of mean stress and deviator stress for train loadings, respectively.	$G_s=2.66$; $C_u=4.8$; $C_c=0.62$; Maximum dry density =20.69 kN/m ³ ; Minimum dry density =15.89 kN/m ³ ; Hydraulic permeability = 3.2×10^{-5} m/s; $c'=0$ kPa; $\phi'=33^\circ$.	The model is based on Gidel's model (Gidel et al., 2001); It considered the direct influence of the initial stress state.

<p>QS2 Sa (>0.063 to 2) Fine sand</p>	<p>SP Poorly graded sand</p>	<p>Cai et al. (2015)</p>	$\varepsilon_z^p = \alpha_1 N^{\alpha_2} \left(\frac{\sigma_{oct}}{\sigma_{atm}} \right)^{\alpha_3} \left(\frac{\tau_{oct}}{\sigma_{atm}} \right)^{\alpha_4} \left(\sqrt{1 + 4\eta^2} \right)^{\alpha_5}$	<p>$\alpha_1, \alpha_2, \alpha_3, \alpha_4$ and α_5 are constants of the model and determined through laboratory tests; σ_{oct} and τ_{oct} are the normal and shear octahedral stresses, respectively; σ_{atm} is the reference stress η is the cyclic torsional stress ratio.</p>	<p>$G_s=2.70$; $C_u \approx 1.63$; $C_c \approx 0.89$; $D_{50}=0.17$ mm; $e_{max}=1.142$; $e_{min}=0.628$.</p>	<p>The model is based on a series of tests using the hollow cylinder apparatus. The permanent deformation of a sand was studied in drained conditions by considering different levels of confining stress, and the ratios of cyclic vertical stress and cyclic torsional stress; This model was developed to incorporate the rotation of principal stresses.</p>
<p>QS2</p>	<p>SP Poorly-graded sand</p>	<p>Lekarp and Dawson (1998)</p>	$\frac{\varepsilon_1^p(N_{ref})}{(L/p_0)} = a \cdot \left(\frac{q}{p} \right)_{max}^b$	<p>a and b: N_{ref}, $L = \sqrt{q^2 + p^2}$; $p_0 = 100$ kPa (reference mean stress).</p>	<p>Dry density=14.91 kN/cm³; $C_u \approx 1.5$; $C_c \approx 1.0$.</p>	<p>Triaxial cyclic tests and hollow cylinder tests were carried out on limestone, sand and gravel.</p>
<p>QS2</p>	<p>SP Poorly-graded sand</p>	<p>Qian et al. (2019a)</p>	$\varepsilon_N^{acc} = (\alpha_1 \Delta\gamma + \alpha_2) + (\alpha_3 \Delta\gamma + \alpha_4) \ln \frac{N}{100}$	<p>ε_N^{acc} is the Nth vertical accumulative strain in percentage; N is the number of load cycles; $\Delta\gamma$ is the calculated shear strain amplitude in percentage $\alpha_1 - \alpha_4$ are model parameters</p>	<p>$G_s=2.54$; $C_u \approx 1.65$; $C_c \approx 1$; $D_{50}=0.16$ mm $e_{max} \approx 0.931$; $e_{min} \approx 0.611$.</p>	<p>Drained cyclic triaxial tests with different levels of initial mean effective stress (p_s), relative density (D_r), initial static stress ratio (n_s) and cyclic stress ratio were performed on saturated Toyoura sand. Indeed, the effects of cyclic stress ratio and initial mean effective</p>

						<p>stress on the permanent deformation;</p> <p>The estimated permanent strains show a good agreement with the measured data. The deviation between the estimation and the measurement is less than 20%.</p>
QS3	SP Poorly graded sand	Hyde (1974)	$\varepsilon_1^p = a \frac{q}{\sigma_3}$	<p>a is a constant;</p> <p>σ_3 is the confining stress;</p> <p>q is deviator stress.</p>	<p>G_s=2.72;</p> <p>C_u≈16;</p> <p>C_c≈0.69;</p> <p>Maximum dry density=21.76 kN/m³;</p> <p>W_{opt}=7.5±0.5%;</p>	<p>The tests were carried out under a constant and variable confining stress;</p> <p>During the analysis, the behaviour of the marl was studied;</p> <p>The author states that the results would be the same if medium confining stress was applied.</p>
QS3	SP Poorly-graded sand	Lekarp and Dawson (1998)	$\frac{\varepsilon_1^p(N_{ref})}{(L/p_0)} = a \cdot \left(\frac{q}{p}\right)_{max}^b$	<p>a and b:</p> <p>N_{ref},</p> <p>$L = \sqrt{q^2 + p^2}$;</p> <p>p₀ = 100 kPa (reference mean stress).</p>	<p>Dry density=20.10 kN/m³;</p> <p>C_u≈32;</p> <p>C_c≈0.2.</p>	<p>Triaxial cyclic tests and hollow cylinder tests were carried out on limestone, sand and gravel.</p>
QS3 Sa(>0.063 to 2)	SW Well-graded sand	Puppala et al. (1999)	<p>log ε^p</p> <p>= log A</p> <p>+ $\alpha \log N + \beta \log \left(\frac{\sigma_{oct}}{\sigma_{atm}}\right)$</p>	<p>A, β and α are soil parameters;</p> <p>σ_{oct} and τ_{oct} are the octahedral normal and shear stresses, respectively.</p>	<p>% sand =100.</p>	<p>The model was tested based on triaxial tests;</p> <p>During his study, well-graded sand, silty clay and clay were considered.</p>

QS3	SW Well-graded sand	Huurman (1997)	$\varepsilon_1^p(N) = A \left[\left(\frac{N}{1000} \right)^B + C \right] \cdot \left(e^{D \cdot \frac{N}{1000}} - 1 \right)$	<p>A, B, C, D are parameters function of stress level and stress ratio ($\sigma_l/\sigma_{l,f}$)</p>	<p> $C_u=10.5$; $C_c=1.25$; Maximum dry Density = 17.21 kN/m³; $W_{opt}=10.5\%$; $c=8.68$ kPa; $\phi=50.2^\circ$. </p>	<p> Cyclic triaxial tests were carried out on the sands used in the base layers; The samples were subjected to a confining stress equal to 12 kPa (corresponds to a depth equal to 0.60m); During the test, a million cycles were applied (frequency equal to 5 Hz). </p>
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Table 4. Permanent deformation model for granular soils (gravels and sandy gravels)

UIC	ASTM classification	Author's model	Equation model	Variables and Empirical Constants	Parameters and characteristics	Observations
QS2 Sandy gravel (GrSa) $C_u \approx 200$; $C_c \approx 0.8$	GP-GM/GC Poorly-graded sandy gravel (no information about the fines)	Rahman and Erlingsson (2015)	$\varepsilon_p(N) = \alpha N^{bS_f} S_f$ $S_f = \frac{\left(\frac{q}{p_a}\right)}{\left(\frac{p}{p_a}\right)^\alpha}$	<ul style="list-style-type: none"> - q is deviatoric stress; - p is mean stress; - p_a atmospheric pressure; - a, b and α are the regression parameters. 	<ul style="list-style-type: none"> - $G_s=2.54$; - Maximum dry density =21.67 kN/m³; - Fines Content = 10.2%; - $W_{opt}=6\%$. 	The triaxial tests took place by considering constant confining stress; The tests considered high stress levels and reduced stress levels.
QS2 Sandy gravel (GrSa) $C_u \approx 73$; $C_c \approx 1.5$	GW-GM/GC Well-graded sandy gravel (no information about the fines)	Rahman and Erlingsson (2015)	$\varepsilon_p(N) = \alpha N^{bS_f} S_f$ $S_f = \frac{\left(\frac{q}{p_a}\right)}{\left(\frac{p}{p_a}\right)^\alpha}$	<ul style="list-style-type: none"> - q is deviatoric stress; - p is mean stress; - p_a atmospheric pressure; - a, b and α are the regression parameters. 	<ul style="list-style-type: none"> - $G_s=2.68$; - Fines Content = 8.6%; - Maximum dry density = 23.05 kN/m³; - $W_{opt}=6.9\%$. 	
QS2 Sandy gravel (GrSa) $C_u \approx 98$; $C_c \approx 2.8$	GW-GM/GC Well-graded sandy gravel (no information about the fines)	Rahman and Erlingsson (2015)	$\varepsilon_p(N) = \alpha N^{bS_f} S_f$ $S_f = \frac{\left(\frac{q}{p_a}\right)}{\left(\frac{p}{p_a}\right)^\alpha}$	<ul style="list-style-type: none"> - q is deviatoric stress; - p is mean stress; - p_a atmospheric pressure; - a, b and α are the regression parameters. 	<ul style="list-style-type: none"> - $G_s=2.64$; - Fines Content = 6.5%; - Maximum dry density = 22.16 kN/m³; - $W_{opt}=6\%$. 	
QS2 Sandy gravel (GrSa)	GW-GM/GC Well-graded sandy gravel (no	Rahman and Erlingsson (2015)	$\varepsilon_p(N) = \alpha N^{bS_f} S_f$	<ul style="list-style-type: none"> - q is deviatoric stress; - p is mean stress; - p_a atmospheric pressure; 	<ul style="list-style-type: none"> - $G_s=2.64$; - Fines Content = 12%; 	

$C_u \approx 117$; $C_c \approx 2.2$	information about the fines)		$S_f = \frac{\left(\frac{q}{p_a}\right)}{\left(\frac{p}{p_a}\right)^\alpha}$	- a , b and α are the regression parameters.	- Maximum dry density = 21.77 kN/m ³ ; - $W_{opt} = 6.5$ %.	
QS2-QS3 (average to good soils) Classification based on the LA and MDE results.	-	Gidel et al. (2001)	$\begin{aligned} \varepsilon_p(N) &= \varepsilon^0 \left[1 - \left(\frac{N}{100} \right)^{-B} \right] \\ &\times \left(\frac{L_{max}}{p_a} \right)^u \left(m + \frac{s}{p_{max}} - \frac{q_{max}}{p_{max}} \right)^{-1} \end{aligned}$	- ε^0 , B and u are material parameters - p_{max} and q_{max} are the maxima applied hydrostatic stress and the deviator stress, respectively. - s is the intercept of the Mohr-Coulomb failure line in the p - q space - m is the slope of this failure line - p_a correspond to the reference stress (100 kPa). - The value L_{max} is calculated considering the following expression: - $L_{max} = \sqrt{p_{max}^2 + q_{max}^2}$	- 0/20 mm (limestone unbound granular material); - LA=22; - MDE=15; - $p_{dSPO} = 2450$ kg/m ³ ; - $W_{SPO} = 4.4$ %; - Fines content=10%; - D=160 mm; - H=320 mm.	- This model is based on cyclic triaxial tests on two samples where the particles had different dimensions: hard limestone (0/20 mm) and microgranite (0/10 mm); - The tests included several load levels (40) at almost 20,000 cycles.

<p>QS2-QS3 (average to good soils) Classification of the subgrade (based on the LA and MDE results):</p>	-	<p>Gidel et al. (2001)</p>	$\varepsilon_p(N) = \varepsilon^0 \left[1 - \left(\frac{N}{100} \right)^{-B} \right] \times \left(\frac{L_{\max}}{p_a} \right)^u \left(m + \frac{s}{p_{\max}} - \frac{q_{\max}}{p_{\max}} \right)^{-1}$	<p>ε^0, B and u are material parameters</p> <p>$p_{\max}$ and $q_{\max}$ are the maxima applied hydrostatic stress and the deviator stress, respectively.</p> <p>s is the intercept of the Mohr-Coulomb failure line in the p-q space</p> <p>m is the slope of this failure line</p> <p>p_a correspond to the reference stress (100 kPa).</p> <p>The value $L_{\max}$ is calculated considering the following expression:</p> <p>$L_{\max} = \sqrt{p_{\max}^2 + q_{\max}^2}$</p>	<p>0/10 mm (granular material obtained from microgranite); LA=20; MDE=13; p_dSPO=2200 kg/m³; W_{SPO}=6.3% Fines content=7.8%; D=76.2 mm.</p>	
<p>QS2 – QS3 Sandy gravel (GrSa)</p>	<p>GW Well-graded gravel</p>	<p>Ling et al. (2017)</p>	$\varepsilon_{acc}(N) = f_{ampl} f_n f_p f_N$	<p>f_{ampl} describes the range intensity of PD with the increase of the cyclic stress amplitude;</p> <p>f_p represents the influence of the initial mean stress;</p> <p>f_n represents the influence of the initial stress ratio;</p>	<p>$C_u \approx 60$; $C_c \approx 1.3$; Sand content=40%; Gravel content =60%;</p>	<p>The study focused on granular materials.</p>

				<ul style="list-style-type: none"> - f_N characterizes the trend of the variation's deformation with the number of cyclic loads. 		
QS3	GP Poorly-graded gravel	Lekarp and Dawson (1998)	$\frac{\varepsilon_1^p(N_{ref})}{(L/p_0)} = a \cdot \left(\frac{q}{p}\right)_{max}^b$	<ul style="list-style-type: none"> - a and b ; - N_{ref}; - $L = \sqrt{q^2 + p^2}$; - $p_0 = 100$ kPa (reference mean stress). 	<ul style="list-style-type: none"> - Dry density=21.18 kN/m³; - $C_u \approx 92$; - $C_c \approx 6.4$. 	<ul style="list-style-type: none"> - Triaxial cyclic tests and hollow cylinder tests took place on limestone, sand and gravel.
QS3 Sandy gravel (GrSa) $C_u \approx 26$; $C_c \approx 0.9$	GP Poorly-graded sandy gravel	Rahman and Erlingsson (2015)	$\varepsilon_p(N) = \alpha N^{bS_f} S_f$ $S_f = \frac{\left(\frac{q}{p_a}\right)}{\left(\frac{p}{p_a}\right)^\alpha}$	<ul style="list-style-type: none"> - q is deviatoric stress; - p is mean stress; - p_a atmospheric pressure; - a, b and α are the regression parameters. 	<ul style="list-style-type: none"> - $G_s = 2.49$; - Fines Content (<0.075 mm) = 3.8%; - $W_{opt} = 7.5\%$; - Maximum dry density = 20.89 kN/m³. 	<ul style="list-style-type: none"> - The triaxial tests considered constant confining stress; - The tests also considered high stress levels and reduced stress levels.
QS3 Sandy gravel (GrSa) $C_u \approx 35$; $C_c \approx 4.6$	GP Poorly-graded sandy gravel	Rahman and Erlingsson (2015)	$\varepsilon_p(N) = \alpha N^{bS_f} S_f$ $S_f = \frac{\left(\frac{q}{p_a}\right)}{\left(\frac{p}{p_a}\right)^\alpha}$	<ul style="list-style-type: none"> - q is deviatoric stress; - p is mean stress; - p_a atmospheric pressure; - a, b and α are the regression parameters. 	<ul style="list-style-type: none"> - Fines Content (<0.075 mm) = 1.4%; - $W_{opt} = 5\%$; - $G_s = 2.61$; - Maximum dry density = 21.18 kN/m³. 	<ul style="list-style-type: none"> - The triaxial tests considered constant confining stress; - The tests also considered high stress levels and reduced stress levels.
QS3	GW Well-graded Gravel	Lekarp and Dawson (1998)	$\frac{\varepsilon_1^p(N_{ref})}{(L/p_0)} = a \cdot \left(\frac{q}{p}\right)_{max}^b$	<ul style="list-style-type: none"> - a and b; - N_{ref}; - $L = \sqrt{q^2 + p^2}$; 	<ul style="list-style-type: none"> - Dry density=20.50 kN/m³; - $C_u \approx 75$; - $C_c \approx 1.3$. 	<ul style="list-style-type: none"> - Triaxial cyclic tests and hollow cylinder tests took place on limestone, sand and gravel.

				$p_0 = 100 \text{ kPa}$ (reference mean stress);		
QS3	GW Well-graded Gravel	Lekarp and Dawson (1998)	$\frac{\varepsilon_1^p(N_{ref})}{(L/p_0)} = a \cdot \left(\frac{q}{p}\right)_{max}^b$	a and b ; N_{ref} $L = \sqrt{q^2 + p^2}$; $p_0 = 100 \text{ kPa}$ (reference mean stress).	Dry density=22.16 kN/m^3 ; $C_u \approx 55$; $C_c \approx 1.3$.	
QS3 Sandy gravel (GrSa) $C_u \approx 16$; $C_c \approx 1.6$	GW Well-graded sandy gravel	Rahman and Erlingsson (2015)	$\varepsilon_p(N) = \alpha N^{bS_f} S_f$ $S_f = \frac{\left(\frac{q}{p_a}\right)}{\left(\frac{p}{p_a}\right)^\alpha}$	q is deviatoric stress; p is mean stress; p_a atmospheric pressure; a, b and α are the regression parameters.	$G_s = 2.64$; Fines Content= 2.4%; Maximum dry density = 20.59 kN/m^3 ; $W_{opt} = 5.5 \%$.	The triaxial tests considered constant confining stress; The tests also considered high stress levels and reduced stress levels.
QS3 Sandy gravel (GrSa) $C_u \approx 20$; $C_c \approx 1.5$	GW Well-graded sandy gravel	Rahman and Erlingsson (2015)	$\varepsilon_p(N) = \alpha N^{bS_f} S_f$ $S_f = \frac{\left(\frac{q}{p_a}\right)}{\left(\frac{p}{p_a}\right)^\alpha}$	q is deviatoric stress; p is mean stress; p_a atmospheric pressure; a, b and α are the regression parameters.	$G_s = 2.63$; Fines Content= 2.2%; Maximum dry density = 20.35 kN/m^3 ; $W_{opt} = 5.5 \%$.	
QS3	GW Well-graded Grave (granite aggregate)	Gu et al. (2016)	$\varepsilon^p = \varepsilon_0 e^{-\left(\frac{p}{N}\right)} (\sqrt{J_2})^m (\alpha I_1 + K)^n$ $\alpha = \frac{2 \sin \phi}{\sqrt{3}(3 - \sin \phi)}$ $K = \frac{c \cdot 6 \cos \phi}{\sqrt{3}(3 - \sin \phi)}$	J_2 is the second invariant of the deviatoric stress tensor; I_1 is the first invariant of the stress tensor; c and ϕ are the cohesion and friction angle, respectively;	$\gamma_d = 2.162 \text{ kg/m}^3$; $\omega = 6.7\%$; $PI = 4$; $LL = 25$; $\phi = 51.3^\circ$; $c = 20.2$ kPa ; $C_u \approx 25$; $C_c \approx 2.8$.	This model used the shear strength ratio and stress terms in the Drucker-Prager model to construct a stress- dependent model; The stress term $\sqrt{J_2}$ represents the softing

				- $\varepsilon_0, \rho, \beta, m$ and n are model coefficients.		effects that deviatoric shear stress has on the material, and a larger value yields larger permanent deformation; The power m is always positive and the term $\alpha I_1 + K$ indicates the hardening/strengthening effect that hydrostatic stress has on UGM; it is highly affected by the strength parameter (cohesion and friction angle);
QS3	GW Well-graded Gravel (limestone conglomerate aggregate)	Gu et al. (2016)	$\varepsilon^p = \varepsilon_0 e^{-\left(\frac{\rho}{N}\right)} (\sqrt{J_2})^m (\alpha I_1 + K)^n$ $\alpha = \frac{2 \sin \phi}{\sqrt{3}(3 - \sin \phi)}$ $K = \frac{c \cdot 6 \cos \phi}{\sqrt{3}(3 - \sin \phi)}$	<ul style="list-style-type: none"> - J_2 is the second invariant of the deviatoric stress tensor; - I_1 is the first invariant of the stress tensor; - c and ϕ are the cohesion and friction angle, respectively; - $\varepsilon_0, \rho, \beta, m$ and n are model coefficients. 	<ul style="list-style-type: none"> - $\gamma_d=1.934 \text{ kg/m}^3$; - $\omega= 13.5\%$; - PI=non-plastic; - LL=not applicable; - $\phi=54.9^\circ$; $c=66.2 \text{ kPa}$; - $C_u \approx 45$; $C_c \approx 1.25$. 	<ul style="list-style-type: none"> - A higher value of $\alpha I_1 + K$ leads to smaller plastic deformation. The power n is always a negative number. The authors carried out a compressive strength test to determine the cohesion, the friction angle, and also α and K by plotting the diagram $\sqrt{J_2} - I_1$ and Repeated Load Triaxial Tests ($N=10,000$ cycles).

Table 5. Empirical models dependent on the number of load cycles

Authors	Permanent Strains	Parameters
Barksdale (1972)	$\varepsilon_1^p = a + b \log (N)$	a and b are constants
Khedr (1985)	$\frac{\varepsilon_1^p}{N} = A \cdot N^{-b}$	b is a material parameter A is a material and stress-strain parameter given as a function of shear stress ratio and resilient modulus
Paute (1988)	$\varepsilon_1^p = A_0 \frac{\sqrt{N}}{\sqrt{N} + D} + \varepsilon_1^p (100)$	A_0 – parameter function of the stress level, D is a regression parameter $\varepsilon_1^p (100)$ – permanent deformation after the first 100 cycles
Tseng and Lytton (1989b)	$\varepsilon^p = \varepsilon_0^p e^{-\left(\frac{\rho}{N}\right)}$	ε_p is the permanent strain ε_0^p is the maximum permanent strain N is the number of load cycles ρ is the scale factor β is the shape factor
Sweere (1990)	$\log \left(\varepsilon_1^p (N) \right) = a + b \log (N)$	a, b – material parameters * for each stress level, separate permanent strain parameters a and b need to be determined from the results of cyclic load triaxial test

		** <i>The Sweeners's formula is valid for granular materials (Gidel et al., 2001)</i>
Hornych (1993)	$\varepsilon_1^p(N) = A \left[1 - \left(\frac{N}{100} \right)^{-B} \right] + \varepsilon_1^p(100)$	- A and B - $\varepsilon_1^p(100)$ - permanent deformation after the first 100 cycles
Vuong (1994)	$\varepsilon_1^p = \varepsilon_1^r \left(\frac{a}{b} \right) N^c$	- a, b and c - ε_1^r is the resilient axial strain
Wolff and Visser (1994)	$\varepsilon_1^p = (cN + a)(1 - e^{-bN})$	- a, b and c are regression parameters
Huurman (1997)	$\varepsilon_1^p(N) = A \left[\left(\frac{N}{1000} \right)^B \right] + C \cdot (e^{D \cdot \frac{N}{1000}} - 1)$	- A, B, C, D are parameters function of stress level

Table 6. Empirical permanent deformation models dependent on the number of load cycles and stress levels

Authors	Permanent Strains	Parameters
Barksdale (1972)	$\varepsilon_1^p = \frac{q/a \cdot \sigma_3^b}{1 - \left[\frac{R_f \cdot q \cdot (1 - \sin \phi)}{2(C \cdot \cos \phi + \sigma_3 \sin \phi)} \right]}$	<ul style="list-style-type: none"> $a \cdot \sigma_3^b$ is a relationship defining the initial tangent modulus as a function of confining pressure (σ_3); a and b are constants R_f is the ratio of the applied deviator stress at failure q_f ϕ is the friction angle and C is the cohesion
Hyde (1974)	$\varepsilon_1^p = a \frac{q}{\sigma_3}$	<ul style="list-style-type: none"> a is a constant; σ_3 is the confining stress; q is deviator stress.
Shenton (1974)	$\varepsilon_1^p = K \left(\frac{q_{max}}{\sigma_3} \right)^\alpha$	<ul style="list-style-type: none"> K and α q_{max} is the maximum deviatoric stress applied
Pappin (1979)	$\varepsilon_1^p = f_n(N) \cdot L \cdot \left(\frac{q^0}{p^0} \right)^{2.8}$	<ul style="list-style-type: none"> $f_n(N)$ – depends on the number of cycles (shape factor); q^0 is the modified deviator stress ($\sqrt{2/3} \cdot q$) p^0 is the modified mean normal stress ($\sqrt{3} \cdot p$) $L = \sqrt{p^2 + q^2}$ (length of stress path)
Lentz and Baladi (1980)	$\varepsilon_1^p = \varepsilon_{0.95S} \ln \left(1 - \frac{q}{S} \right)^{-0.15} + \left[\frac{n(q/S)}{1-m(q/S)} \right] \ln(N)$	<ul style="list-style-type: none"> $\varepsilon_{0.95S}$ is the axial strain at 95% of the deviatoric stress at failure m is the slope of the failure line S is the deviatoric stress at failure

Tseng and Lytton (1989a)	$\varepsilon_1^p = \varepsilon_0^p e^{-(\rho/N)^\beta}$	<p>ε^p is the permanent strain of the granular material;</p> <p>ε_0^p is the maximum permanent strain;</p> <p>N is the number of load cycles;</p> <p>ρ is the scale factor;</p> <p>β is the shape factor.</p>
Paute et al. (1994)	$\varepsilon_1^{p*} = f(N) \cdot A$ $= f(N) \cdot \frac{\frac{q}{(p+p^*)}}{b \left(m - \frac{q}{(p+p^*)} \right)}$	<p>ε_1^{p*} is the permanent axial strain after the first 100 cycles</p> <p>b – regression parameter</p> <p>p^* is a stress parameter defined by the intersection of the static failure line and p-axis in p-q space</p> <p>m is the slope of the failure line in p-q space</p> <p>$f(N)$ function of the number of cycles N</p>
Nishi (1994)	$\varepsilon_{1,ult}^p = k \frac{q^a}{p^b}$	<p>a and b</p> <p>$\varepsilon_{1,ult}^p$ is the ultimate permanent axial strain</p>
Lekarp and Dawson (1998)	$\frac{\varepsilon_1^p(N_{ref})}{(L/p_0)} = a \cdot \left(\frac{q}{p} \right)_{max}^b$	<p>a and b</p> <p>N_{ref}</p> <p>$L = \sqrt{q^2 + p^2}$,</p> <p>$p_0 = 100$ kPa (reference mean stress)</p>

Table 7. Materials selected for the preliminary analysis of the permanent deformation

Author	Soils	ASTM Classification	UIC classification	Observations
Chen et al. (2014)	Sand	SP	QS2	Compacted at field conditions and saturated
	Silt	CL-ML	QS1	Compacted at field conditions and saturated
Salour and Erlingsson (2016) and Erlingsson et al. (2017)	Silty Sand (42.2% fines)	SM	QS1	Compacted at optimum compaction – standard Proctor
	Silty Sand (27.4% fines)	SM	QS1	Compacted at optimum compaction - standard Proctor
Huurman (1997)	Well-graded Sand	SP	QS2	Compacted at optimum compaction - standard Proctor
	Poor-graded Sand	SW	QS3	Compacted at optimum compaction - standard Proctor

Table 8. Physical properties of the materials - granulometry

Authors	Huurman (1997)		Salour and Erlingsson (2016) and Erlingsson et al. (2017)		Chen et al. (2014)	
Material/ Properties	C. Bruynweg	Crusher	Silty Clay (42.2% fines content)	Silty Clay (27.4% fines content)	Silt	Coarse sand
C _u	2.1	10.5	28	33	2.51	4.68
C _c	1.05	1.25	0.54	0.75	1.32	0.62
D ₁₀	0.148	0.217	-	-	-	-
D ₃₀	0.219	0.784	-	-	-	-
D ₅₀	0.280	1.722	-	-	-	-
D ₆₀	0.31	2.28	-	-	-	-
CBR (%)	22	15.7	-	-	-	-

Table 9. State conditions of the materials

Authors	Huurman (1997)		Salour and Erlingsson (2016) and Erlingsson et al. (2017)		Chen et al. (2014)	
Material/ Properties	C. Bruynweg	Crusher	Silty Clay (42.2% fines content)	Silty Clay (27.4% fines content)	Silt	Coarse sand
$\gamma_{\text{dry,max}}$ (kg/m ³)	1723	1755	1998	2070	1620	2110
W_{opt} (%)	12.5	10.5	10.1	7.6	-	-
γ_{wet} (kg/m ³)	1942	1937	-	-	-	-
Liquid limit (%)	Non- plastic	Non- plastic	Non- plastic	Non- plastic	35	Non- plastic
Plastic limit	Non- plastic	Non- plastic	Non- plastic	Non- plastic	24	Non- plastic

Table 10. Strength properties of the materials

Authors	Huurman (1997)		Salour and Erlingsson (2016) and Erlingsson et al. (2017)		Chen et al. (2014)	
Material/ Properties	C. Bruynweg	Crusher	Silty Clay (42.2% fines content)	Silty Clay (27.4% fines content)	Silt	Coarse sand
ϕ (°)	48.2	50.20	36.18	45.66	11.7*	0*
c (kPa)	5.60	8.68	15.82	10.43	16.4*	33*
s (kPa)**	9.90	14.90	31.80	19.10	24.8	0
m **	1.98	2.07	1.47	1.88	0.62	1.33

* In this case, the author described the saturated samples (silt and coarse sand) in ϕ' and c'

** s and m are the parameters used to define the failure envelope: $q=s+mp$

Table 11. Parameters of Chen's model and its relationship with the ASTM and UIC classification

	ε_{p0}	α	β	ASTM classification	UIC classification
Silt (Chen et al., 2014)	0.0029	0.2680	0.3390	CL	QS1
Silty sand (43% fines) (Erlingsson and Rahman, 2013)	0.0067	0.6500	0.2000	SM	QS1
Silty sand (27% fines) (Erlingsson and Rahman, 2013)	0.0069	0.6500	0.2000	SM	QS1
Coarse sand (Chen et al., 2014)	0.0018	0.3340	0.3390	SP	QS2
Poor-graded sand (Huurman, 1997)	0.0505	0.0104	0.0018	SP	QS2
Well-graded sand (Huurman, 1997)	0.0486	0.0111	0.0569	SW	QS3

Table 12. Properties of the clay (in optimum conditions) tested in the Puppala et al. (2009)

Properties and parameters	Clay
ϕ (°)	18
c (kPa)	60
Liquid limit (%)	28.19
Plasticity index (%)	12.55
Maximum dry unit weight (kN/m ³)	17.10
W _{opt} (%)	17.11
α_1	0.24
α_2	0.31
α_3	-1.77
α_4	2.85

Figure 1. Example of a rail track at a transition zone

Figure 2. Illustration of the accumulation of permanent deformation under cyclic loads (after two cycles)

Figure 3. Accumulation of permanent strain (adapted from Erlingsson et al. (2017))

Figure 4. Illustration of shakedown theory (Erlingsson et al., 2017)

Figure 5. Division of the mechanistic-empirical permanent deformation models

Figure 6. Total stress path selected and failure envelopes of Mohr-Coulomb yielding criteria

Figure 7. Comparison between the permanent deformation of a poorly graded (C. Bruynweg) and well-graded (crusher) sand under different stress ratios

Figure 8. Permanent deformation of the materials applying the Chen et al. (2014) model

Figure 9. Permanent deformation predicted by Chen et al. (2014)'s model for different soils (N=30,000 cycles) under a stress ratio $\sigma_1/\sigma_{1:f} = 0.4$

Figure 10. Influence of the moisture content in the permanent deformation of a clay (Puppala's model): a) permanent deformation results applying the Chen et al. (2014) model; b) Ranking of permanent deformation for clay