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Development of adaptive structures working with magnetorheological elastomers and magnetic force

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Department of Mechanical, Materials & Mechatronic

Development of Adaptive Structures Working with Magnetorheological Elastomers and Magnetic force

Miao Guo

This thesis is presented as part of the requirements for the Award of the Degree of Doctor of Philosophy of Engineering

University of Wollongong

April 2013
The work presented in this thesis consists of two major parts: (a) An investigation of the dynamic response of magnetorheological elastomers (MREs) sandwich beam under non-homogeneous magnetic field strength; (b) The development of a positive-negative-stiffness vibration isolator based on MREs and magnetic force.

(I) The sandwich structure theory is based on the Mead & Markus model and was extended such that a vibration differential equation of an MRE sandwich beam was deduced. The coverage area and strength of the external magnetic field on the dynamic characteristics of a sandwich structure was also studied. It focused on an investigation of MRE cantilever sandwich beam and a clamped-clamped sandwich beam.

MREs based on silicone rubber and carbonyl iron particles were fabricated and their dynamic performance under magnetic fields of different strengths were tested using a rheometer.

An MRE sandwich beam was fabricated by placing an MRE between two thin layers of aluminium. An experimental test rig was set up to investigate the vibration of the MRE sandwich beam under non-homogeneous magnetic fields. Both experimental results showed that the first natural frequency of the MRE sandwich beam decreased as the magnetic field that applied on to the beam was moved from the clamped end of the beam to the free end of the beam. The MRE sandwich beam also had the capability of shifting the first natural frequency left when the magnetic field in the activated regions was increased.

(II) A novel positive-negative-stiffness vibration isolator was developed using MRE and a magnetic force to vary the stiffness. This vibration isolator can work over a relatively wide range of excited frequencies. A mathematical model of the isolator was derived and a prototype of the positive-negative stiffness was fabricated. The test rig was setup and the transmissibility of the system under different current intensities was tested. The dynamic characteristics of the isolator under different
current intensities were simulated with Matlab. Both the experimental and simulated results show that the system’s natural frequency increased when a positive current was applied and decreased when a negative current was applied. The simulation results also demonstrated that the positive-negative isolator can efficiently suppress vibration after the current in the coils has been tuned.
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LIST OF SYMBOLS

b width of beam
B magnetic flux density
E Young’s modulus
E* equivalent Young’s modulus
G shear modulus
h_{1,3} height of face layer of sandwich beam
h_2 height of MRE layer of sandwich beam
\rho density of face layer or MRE layer
J_p average particle polarisation
w transverse displacement of the beam
Y geometric parameter
g shear parameter
E_i Young’s modulus of the i\textsuperscript{th} layer
H_i thickness of the i\textsuperscript{th} layer
D central distance between face layers
f(x,t) external force
\gamma core loss factor
\mu mass per unit length of beam
f natural frequency
\alpha fractional unsaturated area of particle
\mu_0 magnetic permeability of a vacuum (4\pi \times 10^{-7} \text{ H/m})
\mu_1 relative permeability
\phi volume fraction of particles
\( \zeta \) non-dimentional length co-ordinate
\( \tau \) shear stress
\( \gamma \) shear strain
\( G^* \) complex shear modulus
\( G' \) storage modulus
\( G'' \) loss modulus
\( l \) length of the beam
\( l_1 \) length of the MRE part under a magnetic field
\( l_2 \) length of the whole MRE layer
\( y_1 \) distance between magnets on top layer and sanwich beam
\( y_2 \) distance between magnets on bottom layer and sanwich beam
\( M \) magnetisation of the particles
\( d_p \) the diameter of the particles
\( m \) mass of steel plate
\( EM1 \) upper electromagnet 1
\( EM2 \) lower electromagnet 2
\( H \) magnetic field
\( V \) volume of the air gap between the magnetic poles and the middle mass
\( S \) surface area of the air gap
\( g \) initial size of each gap
\( N_1 \) winding number of electromagnet 1
\( N_2 \) winding number of electromagnet 2
\( N_3 \) winding number of electromagnet 3
$F_1$  magnetic force from electromagnet 1
$F_2$  magnetic force from electromagnet 2
$A$  section area of MRE in vibration isolator
$h$  thickness of MRE in vibration isolator
$M_1$  mass of middle mass
$K_1$  stiffness of MRE
$C_1$  damping coefficient of MRE
$M_2$  top mass of vibration isolator
$K_2$  stiffness of spring
$x_f$  input of vibration isolator
$x_1$  displacements of $m_1$
$x_2$  displacement of $m_2$
$X_f$  Laplace transform of displacement $x_f$
$X_2$  Laplace transform of displacement $x_2$
$I^+$  positive current
$I^-$  positive current
(\)  derivative with respect to time
(\(\))  second derivative with respect to time
CHAPTER 1
INTRODUCTION

1.1 Research Background and Motivation

In industry, there are many different kinds of unwanted vibrations and noises, so a variety of methods have been developed to suppress undesirable vibrations. Adaptive structures such as sandwich structures, tuned vibration absorbers and vibration isolators are normally used to suppress the vibration.

In recent years, there has been an increasing trend towards developing Magnetorheological (MR) adaptive structures to control vibration because MR materials have rheological properties that can be changed continuously, rapidly, and reversibly by applying a magnetic field. Those studies have shown that MR adaptive structures vary in their responses to vibration, such as the amplitude of vibration, natural frequencies, and loss factor, when subjected to a magnetic field.

Some work has been done in investigating the capabilities of MR materials to suppress vibration in adaptive structures. Sandwich beams are known to have an extremely high strength to weight ratio compared to a single homogeneous beam. The other advantage of a sandwich structure is its adaptability, which means its application is very popular in the construction of airplanes, guided missiles and space ships, where weight is a big factor in the design of such structures.

Homogeneous three-layered adaptive beams with MR materials sandwiched between two elastic layers were studied and the vibration suppression capabilities of MR adaptive beams were observed in the form of shifts in their natural frequency, variations in loss factors, and amplitudes of vibration. Despite this there has been little research on MR Elastomer sandwich beams under a non-homogeneous magnetic field, so one main part of this research focuses on investigating the
dynamic response of an MR elastomer sandwich structure under different magnetic fields and boundary conditions.

Conventional vibration absorbers can only effectively suppress a single frequency excitation of the vibrating systems. If for example, the sources of vibration have a time-varying frequency or wide frequencies, a conventional absorber using a traditional elastomer, whose stiffness and damping can not change, will lose its effect; indeed, it will aggravate the vibration. It is therefore necessary to implant variable elements to tune the properties of the vibration absorbers. In practice changeable elements can be employed such as piezoceramics, magnetorheological fluids (MRFs), electrorheological fluids (ERF) and MR elastomers can be used.

Several groups have made use of MR materials to develop novel adaptive tuned vibration absorbers because of their easy implementation, very fast response, good maintenance, high stability and effective control.

Some researchers have investigated MRE-based vibration absorbers where MREs are used as field-dependent springs. All of these studies showed a frequency shift capability. The stiffness of an MRE-based vibration absorber increases when the external magnetic field strength or current intensity increases, which is called a positive-stiffness vibration absorber.

Another way to attenuate vibrations is using vibration isolators. The methods of vibration isolation can be classified as: passive, active, and semi-active. The classical approach to isolating vibration uses a passive system of springs which resist the movement of vibration by exerting an opposing force proportional to its displacement, and a damper which removes kinetic energy and dissipates it as heat. The dynamic stiffness of an isolator spring should be as low as possible in order to increase the region of isolating vibration. However, if a linear spring is used, a low stiffness isolator can cause unacceptably large static deflection. So there have been some investigations into developing a vibration isolator whose stiffness can be decreased through mechanism or active control strategy in order to broaden the working frequency range.
CHAPTER 1. INTRODUCTION

This research makes good use of variable stiffness, which includes positive and negative stiffness, and combines them together to propose a new kind of vibration isolator, which is capable of working in a relatively wide range of frequencies. A totally novel vibration isolator-Positive-Negative-stiffness vibration isolator has been designed and manufactured. The experimental rig is set up to test the dynamic characteristics of the proposed vibration isolator.

1.2 Research Objective

The overall aim of this thesis is to develop adaptive structures for attenuating vibration by working with MREs and magnetic force.

The specific objectives of the thesis are:

1) The fabrication and characterisation of MR elastomers;
2) Derivation of the equations of an MRE sandwich beam under different boundary conditions and localised magnetic field using the M&M method.
3) An experimental study of the dynamic response of an MRE sandwich beam under different magnetic field strengths and magnetic field locations;
4) The design and manufacture of a novel positive-negative stiffness vibration isolator based on MRE and magnetic force;
5) An experimental study of the proposed novel positive-negative stiffness vibration isolator under different current intensities;
6) The dynamic response of the proposed positive-negative vibration isolator are investigated by simulation.

1.3 Outline

This thesis begins with an introduction to the research background, motivation and objective of this project.
Chapter 2 gives a literature review of MRE fabrication, properties, modelling and application of MRE.

Chapter 3 extends the Mead & Markus model and the deduced vibration differential equation of an MRE sandwich beam and studies the coverage and strength of an external magnetic field on its dynamic characteristics. It focuses on the investigation of an MRE sandwich beam under different boundary conditions.

In Chapter 4, a magnetorheological elastomer (MRE) is manufactured and its dynamic properties are tested, and an MRE sandwich beam is also fabricated. The experimental test rig is set up to investigate the vibration of an MRE sandwich beam under a non-homogeneous magnetic field. The MRF sandwich beam is also fabricated by placing MRF between two thin layers of aluminum to compare the vibration of the MRF sandwich beam with an MRE sandwich beam under a non-homogeneous magnetic field.

Chapter 5 proposes a totally novel positive-negative stiffness vibration isolator which used MREs and a magnetic force to achieve a variable stiffness. A mathematical model of this new vibration isolator is introduced and a positive-negative stiffness isolator is designed and manufactured. The experiment is set up and the dynamic response of the system is tested and simulated with Matlab.

Chapter 6 concludes the main findings and presents ideas for future research work.
2.1 MRE materials

Magnetorheological (MR) materials are smart materials which have MR effects and many unique properties under magnetic fields. Since the MR effects were discovered by Rabinow in 1948 [1], MR materials have developed into a family containing MR fluids, MR foams and MR elastomers [2].

MRFs and MREs have similar field response properties in that there are some distinct differences in the way these two classes of materials are typically intended to operate. The most significant difference is that MREs operate within a pre-yield regime [3-4] while MRFs typically operate in a post yield continuous shear or flow regime [5-6]. In addition, the strength of MRFs is characterised by yield stress while MREs are characterised by field-dependent modulus. With regard to application, MRE devices are used to tune the natural frequency of a structure, which is dominated by its stiffness [7-9]. While MRF devices provide a damping function, which is the process of dissipating energy [10-12].
Conventional MREs consist of three components, namely polarised particles, the matrix, and additives [13]. A photo of an MR elastomer is shown in Figure 2.1.

(1) Polarised particles

The particles used in MR materials are generally magnetised ferromagnetic, low coercivity, finely divided particles of iron, nickel, cobalt, iron-nickel alloys, iron-cobalt alloys, iron-silicon alloys and the like, which are spherical or nearly spherical in shape, and have a diameter of about 1 to 100 micrometre. A preferred material is the iron microspheres particulate known as carbonyl iron. Since these particles are used in non-colloidal suspensions, it is better if the particles are at the small end of the suitable range, preferably a nominal diameter of 1 to 10 micrometer or a particle size that can avoid sedimentation, although MR materials with larger size particles generally have a larger MR effect [14]. But for MR elastomers, sedimentation has been overcome so the particle size in MREs can be chosen from a larger range. There were many patents for improved particles for MR materials. For example, in US20050064191, hydrophobic metal particles that were used as the
magnetised particles in MREs were provided [15]. Hydrophilic metal particles, such as carbonyl iron particles, are made hydrophobic by reacting with a surface hydroxyl on a solid metal particle with a reactive surfactant. The particles are coated with a reactive surfactant which covers at least 90% of their surfaces and are then washed with a low viscosity synthetic hydrocarbon to remove any excess surfactant. The particles are stabilised against oxidation and irreversible coagulation during further processing, and formulation and are then used in magnetorheological materials.

(2) Matrix

The matrix, or host material, is the basic component of an MRE and is the only structural difference between an MRE and MR fluid. MRE has a cross linking host material which avoids the disadvantages that can occur in a fluid hosted MR material (such as liquid leakage and possible corrosion of the container). Although the selection of host material will not affect the rheological property of an MRE directly, it can become a crucial factor in certain applications especially when we are to consider the long term stability of the MR composites [16-17].

If the particles are distributed homogeneously inside the matrix rubber during the curing process under a zero magnetic field, the properties of the composite will be isotropic. But if an external magnetic field is applied to the polymer composite during cross linking, the particles will align themselves parallel to the magnetic field and become locked in place during the final cure. These kinds of aligned composites are strongly anisotropic in their mechanical, electrical, magnetic, and thermal properties [18]. Natural rubber is one of the commonly used matrix materials.

The matrix of MR elastomers also has lots of choices, such as thermoplastic rubber, silicone rubber, plastic, natural and synthetic rubber, and so on. Among them, natural rubber has very good mechanical properties, flexibility, and processing performance and is very suitable for a practical MR elastomer [19-23]. Butyl rubber also has excellent chemical stability, insulation, and a high damping factor, so it is
suitable for an MR elastomer based shock absorber. In addition, silicone rubber and thermoplastic rubber have been widely used to prepare an MR elastomer in lab because they are easy to process [19-21, 23-29]. The other matrices include acrylonitrile rubber [30-31] vinyl PVA glutaraldehyde synthetic polymers [32], polyurethane oxygen fluorine polymer [28], polyurethane [33] and formaldehyde plastic [34]. All these examples show that the choice of matrix for an MR elastomer is very flexible.

(3) Additives

Additives are used to adjust the mechanical properties or electrical performance of MREs. In MREs, silicone oil is usually used as an additive. When the molecules of silicone oil enter into the matrix, the gaps between the matrix molecules are increased and the conglutination of molecules is decreased. Apart from increasing the plasticity and fluidity of the matrix, the additives can average the distribution of internal stress in the materials, which makes them ideal for fabricating MRE materials [35]. Graphite powder is a kind of additive which can adjust the conductivity of MREs [36-37]. By introducing graphite micro-particles into the elastic matrix, MREs become electro-conductive. This property of MREs can be used for achieving magneto-resistors, magnetic field sensors, transducers of mechanical distortions, and strains, etc [38]. Chen et al. demonstrated that carbon black can improve the mechanical performance and reduce the damping ratio of MR elastomers [39].

2.2 Fabrication of MRE

There are two categories of MREs: isotropic MREs and anisotropic MREs. Figure 2.1 shows the major steps in fabricating these two MREs [30, 33-38, 40].
The difference between the fabrication procedures of isotropic and anisotropic MREs is that whether the mixture of polarised particles, matrix and additives is curing under a magnetic field.

The phenomenon of magnetic particle displacement under the action of a magnetic field has been observed in [41] in a thin film of composite using a metallographic optical microscope. A schematic representation of this phenomenon is shown in Figure 2.3 (a, b) while in Figure 2.3 (c, d) the photos were obtained by optical microscopy. In the absence of a field, the particles in isotropic composites are distributed randomly in Figure 2.3 (b) and (d); under the influence of a field they move to new positions in Figure 2.3 (a) and (c), and their initial positions are shown by open circles in Figure 2.3 (a). This displacement is completely reversible, i.e. when the magnetic field is switched off the particles return to their initial positions. It should be noted that structuring the magnetic particles within the composites under the influence of a magnetic field can also be confirmed by the material surface structuring [42].
Figure 2.3: A schematic representation of magnetic particle displacements under an external magnetic field: (a) in the presence of a magnetic field; (b) without a field. Photo of an optical microscope under the action of an external magnetic field: (c) aligned MRE particles under a magnetic field; (d) randomly dispersed MRE particles without a magnetic field [41].

The general procedure for fabricating an anisotropic MR elastomer with natural rubber is similar to conventional rubber. Normally, the ingredients are natural rubber, zinc oxide, stearic acid, sulfur, and iron particles. After all the ingredients are evenly
mixed in a mixing machine at a temperature as high as 120º, the mixture is packed to a mould and then cured under an electric-magnetic field for a certain time. The samples are then left at room temperature for more than 24 hours prior to testing. The chain formation results from the anisotropic magnetic forces among the particles. MREs fabricated by this method are called anisotropic MREs [33], whereas for isotropic MREs, carbonyl iron particles are first immersed in silicone oil and then mixed with silicone rubber. All the ingredients in the beaker were mixed with a stirrer bar for about 5 min at room temperature. After all the ingredients were evenly mixed, the mixture was put under a vacuum to remove air bubbles, and then cured for 24 hours at room temperature in an open sheet mould without an external magnetic field [40]. According to current research, anisotropic MREs have a larger MR effect than isotropic MREs [43]. Gong et al. have done research on the determining factor in the formation of ordered microstructures in anisotropic MREs and the relationship between their microstructures and viscoelastic properties. Their results show that the viscoelastic properties of field-dependent MREs increased with magnetic flux densities applied during testing. MREs prepared under high magnetic fields have a large field-induced shear modulus and high MR effect [3].

2.3 Mechanical property of MRE

MREs contain viscoelastic properties and magnetorheology [44]. The magnetorheology of MREs is described as a reversible change in modulus in an applied magnetic field. Aligned MREs have mostly been characterised at relatively low frequencies (1 to 20 Hz) to measure changes in the dynamic shear modulus induced by the external magnetic field. The controllable behaviour of MREs is typically characterised by their field dependent modulus. Many features of the particles such as the size, shape, distribution in the matrix, and percentage volume of the ferromagnetic particles in the elastomer matrix can have an effect on the overall behaviour of MRE.
Experiments on double lap shear specimens of MREs were reported in many references [13, 18, 30-31, 33-34, 43, 45-48]. The change in modulus increased monotonically with an increasing volume percentage of iron, but the maximum change in modulus increased to nearly 0.6 MPa as the iron volume concentration increased to 30%. The researchers also observed a pronounced drop off in the MR effect and a corresponding increase in field dependent energy dissipation at strains above 1±2%. This strain dependency was attributed to the onset of a magnetic yielding of the particle chains. Ginder et al. [22, 44] found a substantial magnetorheology over the entire frequency range studied. The increase in the shear modulus varied initially with the strength of the magnetic field but became saturated at higher strength fields. When the magnetic field was increased from 0 to 0.56 Tesla the consequent increase in shear modulus was nearly 2 MPa and the frequency of the resonance was shifted upward by over 20% [22]. Zhou et al. [44] stated that the changes of dynamic shear storage modulus can be over 50%, while Gong et al. [40] said it can be over 100%. Lokander et al. [30-31] studied the dynamic shear modulus for isotropic MR elastomers with different filler particles and matrix materials. They measured the magnetorheology as a function of the amplitude of strain and found that it decreased rapidly with increasing strain within the measured range, and is not dependent on the frequency of testing. The fact is that the absolute magnetorheological effect of isotropic MR rubber materials with large irregular iron particles is independent of the matrix material. The relative MR effect can be increased by the addition of plasticisers which means that softer matrix materials will show a greater relative magnetorheology [30-31]. Chen et al. [49] optimized the fabrication method and fabricated good natural rubber based MREs with high modulus by investigating the influences of a variety of fabrication conditions on the MREs performances, such as the type of matrix, external magnetic flux density, and temperature, plasticiser and iron particles. The results show that the content of iron particles plays a significant role in improving MRE performance.

Stepanov et al. [41] studied the field dependence of viscoelastic properties of both isotropic and structured MREs under three typical loading modes, including elongation, static shear, and dynamic shear. The experimental results show that MRE
samples exhibited a giant increase in the field dependence of elastic modulus. Also, the pronounced effect of pseudo-plasticity has been observed, which is manifested by a considerable increase in the shear loss modulus of the composites.

Li et al. [4] presented both experimental and modelling studies of the viscoelastic properties of MR elastomers under harmonic loading. Various sinusoidal loadings with different strain amplitude and frequencies were applied to study the stress responses. Figure 2.4 shows the stress-strain relationships of the MRE sample at a constant strain amplitude of 10% but under magnetic fields that varied from 0 to 440 mT. It can be seen that all stresses and strains form nice elliptical shapes, the areas of which increased steadily with the increment of the magnetic fields. These results demonstrate that MRE materials have controllable mechanical properties. The slope of the main axis of the elliptical loops varied with the magnetic field, which means that the modulus of MREs varied with the magnetic field and therefore MREs exhibit variable stiffness and damping properties. Lerner and Cunefare [50] developed and tested an MRE absorber operating in shear, longitudinal, and squeeze modes. The natural frequency changes for these three modes was 183%, 473%, and 510%, respectively. These results also indicate that the MREs had a considerable increase in the field dependence modulus.
2.4 Model of MR Elastomer

In ER and MR fluids research, a number of parametric and non-parametric models were proposed to predict their dynamic properties. At sufficiently small strain amplitudes, ER and MR fluids behave as linear viscoelastic properties, but if these materials are subjected to large strain amplitude excitations, which are beyond the yield strain, they behave like nonlinear viscoelastic properties [5-6, 51-53]. The Fourier transform method was used to analyse the nonlinear viscoelastic properties [6, 52]. These studies have provided both quantitative and qualitative characterisations of the dependence on the dynamic properties of ERF and MRF of...
key influencing factors, such as the external field strength, strain amplitude, and frequency. On the other hand, parametric and phenomenological models [5, 53] were developed to describe the linear and non-linear viscoelastic properties of ER and MR fluids and devices. These parametric models were widely used for analysis and control of practical MR fluid devices.

A number of models were developed to describe the performance of MREs since accurate models can predict the performance of MR elastomers under differently controlled magnetic fields. The mechanical properties of MREs can be divided into two distinctive regimes: composite properties (with different filler fractions) without a magnetic field applied, and the composite properties with a magnetic field applied. Jolly et al. [13] presented a point-dipole model, where the MR effect was studied as a function of particle magnetisation. The dipolar model was augmented with a proposed mechanism of how magnetic flux density distributes itself with the particle network. This mechanism takes into account the magnetic non-linearities associated with the gradual saturation of permeable particles as the applied field increases. The schematic is shown in Figure 2.5, where it is assumed that uniform saturation occurs in each particle in the region (s). Furthermore, the distances decreases from r to zero as the average composite flux density increases from zero to some large level.

Figure 2.5: particle saturation model [13].
The Einstein-Guth-Gold equation [48] is widely used to approximate the shear modulus of MREs in zero field. In this model, MREs are considered to be an elastomer filled with randomly distributed, spherical rigid particles. For the field-responsive behaviour of MREs, Jolly et al. [54] proposed a quasi-static model to explain the increase in the modulus by calculating the magnetic interaction between the adjacent particles. This model takes into account the gradual saturation of the permeable particles as the applied field increases. The pre-yield modulus $G$ of the particle network is simply stress divided by strain,

$$G \approx \frac{\phi J_p^2}{2\mu_1\mu_0 h^3}, \quad \varepsilon < 0.1 \quad (2.1)$$

Where $J_p$ is the average particle polarisation and this is also the dipole moment magnitude per unit particle volume:

$$J_p \approx \frac{3/2\alpha^3 B + (1 - \alpha^3)J_s}{1 + 3/2\phi/3} \quad (2.2)$$

Ginder et al. [45] and Davis [48] used finite element methods to analyse the modulus increase under varied magnetic fields. Dorfmann et al. [55] derived a model that represented Maxwell’s equation by using the mechanical and thermodynamic balance law. Shen et al. [33] developed a model that takes into account the magnetic interactions of dipoles in the same chain and the nonlinear properties of the host composites. Most of models are based on the basis of a dipole model for particle energy interaction, with the assumption that the particles are the same size and shape. These models are based on previous studies of MR fluids and the structure of particle chains. Their results showed that the field-dependent modulus of MREs varies with the square of the saturated magnetisation of the particles.

An effective permeability model was proposed to predict the field-induced modulus of MREs. Based on the effective permeability rule and taking into account
the particle’s saturation, the model was proposed to predict the mechanical performances of MREs with complex structures and components [56]. Unlike conventional methods, the iron particles are coated with magnetisable soft shell’s consisting of nano-size ferrite powder and polymer gel. As shown in Figure 2.6 (a), in order to fabricate this kind of magnetisable soft shell, nano-size ferrite, and polymer gel are pre-requisites. The structural comparison between conventional MREs and the newly proposed MREs is shown in Figure 2.6 (b) and (c), where the nano-particles additives around the micro-particles are zoomed out.

**Figure 2.6**: Newly constructed MREs (a) Fabrication process (b) construction of traditional MR elastomers, and (c) MR elastomers with nano-size particles additive [56].
Zhang et al. [57] presented theoretical and experimental studies of the mechanical performance and magnetorheological effects of magnetorheological elastomers (MREs) fabricated with mixtures of large and small particles. In the model, the saturation of MREs has also been taken into account. Their theoretical and experimental results indicated that the MREs fabricated with different particle sizes can provide larger field dependent modulus. Chen et al. [3] proposed a finite column to calculate the field induced shear modulus from the observation of microstructures.

In MRE, the iron particles can be idealised as chains of particles locked in the elastomer. Shen et al. indicated the arrangement of the iron particles in a magnetic field before and after deformation by a shear force [33]. The shear modulus that increased under the magnetic field is determined by equation 2.3

\[ \Delta G \approx \frac{9}{8} \frac{\Phi C m^2 (4 - \gamma^2)}{\gamma_0 \pi^2 \alpha^3 \mu_0 \mu_r (1 + \gamma^2)^2} \]  

(2.3)

It shows that the shear modulus change was quadratically proportional to the value of the dipole moment. The average distance between two adjacent particles in a chain has an important effect on the value of the dipole magnetic moment, and therefore, the change in the MRE’s modulus was greatly affected by the ratio of the mean distance between two adjacent particles to the mean radius of the particles. In order to obtain a large change in the modulus, the MRE should have a relatively high particle volume fraction, and the iron particles should have a highly saturated flux density [33]. Li et al. [4] proposed a four parameter viscoelastic model to describe the performance of MR elastomers, as shown in Figure 2.7, by extending the classical three parameter standard solid model. In this model, a spring element which is parallel with the standard model for representing the field dependence of the modulus was introduced.
2.5 MRE Adaptive Structures

MR elastomes with a magnetic field responsive to rheology hold promise in enabling simple variable and controllable stiffness devices in that they have many prospective applications such as mechanical actuators (artificial muscle [58]), sensors [37], controlled vibration dampers, adaptive structures and variable impedance surfaces. The Ford Motor Company has patented an automotive bushing that uses an MRE, but their manufacture is not yet widespread enough to have standards for production. A new method and apparatus for varying the stiffness of a suspension bushing with an MRE has been presented which improves customer satisfaction by reducing any dissatisfaction associated with braking events which can result in noise, vibration and harshness [59]. Ginder et al. [60] proposed that MR elastomer bushings may be used to reduce rough braking because they can be stiffened transiently during braking events, thus shifting the frequency and / or lowering the amplitude of relevant suspension vibrations.
An adaptive structure is one whose geometric and inherent structural characteristics can be changed beneficially in response to external stimulation by either remote commands or automatic means.

### 2.5.1 Sandwich structure

Sandwich beams have been in use for many years in various industries due to their high stiffness-mass ratio [61]. A typical construction of a sandwich beam consists of two stiff layers and a soft core. A kind of typical sandwich structure with an MRE core is shown in Figure 2.8.

![Figure 2.8: A kind of typical sandwich structure with an MRE core](image)

Lots of papers can be found in literature covering de-bonding problems [62], bucking problems [61], and dynamic mechanical problems of curved [63] or non-curved [64] beams. Many works proved that the transverse flexibility of the core affects the overall behaviour of a sandwich beam, such as stresses and displacement [61, 64]. The field dependant dynamic property of MRE based sandwich beams, composed of conductive or non-conductive layers, is addressed theoretically through a high order model [65].
Ginder et al. reported that the field induced increase in moduli of MR elastomers is effective at frequencies well above 1 kHz. Vianney et al. did an experimental study investigating the controllability of the vibration characteristics of magnetorheological fluid cantilever sandwich beams. Diverse excitation methods were considered as well as a range of magnetic field strengths and configurations [66]. The controllability of the beam’s response to vibration was observed in the form of variations in the amplitude of vibration and shifts in natural frequency. Zhou [67] indicated that the sandwich configuration is an alternative to developing smart structures because it takes advantage of the field-controllable shear modulus of MRE and enhances the bulk flexural rigidity through the layers.

MREs hold promise in enabling the manufacture and application of simple variable stiffness devices. The pioneering application of MREs was presented in the vehicle industry. In US5816587 [60], a novel method and apparatus for varying the stiffness of a suspension bushing containing an MRE was presented. This allows for improved customer satisfaction by reducing the dissatisfaction associated with braking events, such as shudder, which can result in increased noise, vibration, and harshness [68].

2.5.2 Vibration Absorber

A vibration absorber is used to reduce vibration to some sensitive equipment [69-71]. These devices look similar but are totally different in the way they reduce vibration. Two variations on the traditional two-degrees-of-freedom, lumped mass absorber system have been developed. The first was the 1909 design by Frahm [72], which is most effective when the primary system is excited near its resonant frequency. This system consists of an undamped, single-degree-of-freedom absorber. The second variation on Frahm’s work, which includes damping in the absorber, was refined by Den Hartog [73]. This absorber system has the advantage of decreasing the amplitude of vibration over a wider frequency range, but it is not as effective as
the undamped system when the excitation frequency and fundamental resonance are closely matched. Fully active systems which reduce vibration by both dissipating and supplying energy have been investigated by Hrovat et al. [74], Karnopp et al. [75], Burdess and Metcalfe [76] and Guntur and Sankar [77]. Applications of active vibration control have been developed by Tanaka and Kikushima [78-79], Mecki and Seering [80], Zimmerman and Cudney [81] and Gerhold and Rocha [82].

Conventional passive elastomeric materials are widely used in components for mounting, isolating vibration, and sealing on an automobile, and in many other commercial products and processes. Elastomers that possess tunable mechanical properties could dramatically enhance the functionality of these components. The application of an electrical current to the coil generates a magnetic field in the elastomer and generally increases the spring rate or stiffness and damping of these components. Such tunable components could be used to alter the ride and handling, or control the noise, vibration, and harshness on vehicles [60].

The US20036623364 [83] provided a damper assembly to reduce the vibration of a driveline. The driveline assembly was adapted to transmit rotational power from an engine system to a plurality of drive wheels. MREs can also be used as the smart spring in a dynamic vibration absorber. Albanese [26] and Holdhusen [68] presented MRE absorbers working on a compress mode and Deng [84] presented an MREs absorber working on a shear mode. Their results indicated that the frequency of a vibration absorber can be tuned in a wide frequency range, and the controlled frequency band was expanded too. Hoang et al. [85] presented a conceptual adaptive tunable vibration absorber (ATVA) with soft MREs to reduce the vibration of vehicle powertrain systems. A schematic diagram is shown in Figure 2.12, in which the inner cylinder with lugs is fixed on the rotating shaft. The MRE material operates like a torsional spring and is put into the gap between the inner and outer cylinders. Like the inner cylinder, there are lugs on the outer cylinder that cause tangential elastic forces as well as elastic torque to form between these cylinders. Therefore, the outer cylinder can vibrate with the inner cylinder. There are also electromagnetic coils, which are supplied by a DC current to make a magnetic field through the MRE layer. Numerical results show that the ATVA can effectively work with the MRE
material in a wide frequency range, from around 7 to 70 Hz. Lerner and Cunefare [50] used MREs as field dependent springs within three vibration absorber configurations, and to determine their vibration absorption characteristics.

Figure 2.9: A schematic diagram of the ATVA. 1: inner cylinder 2: rotating shaft; 3: lug; 4: electromagnetic coils; 5: outer cylinder; 6: MRE material [85].

Tuned Dynamic Vibration Absorbers (TDVA) have been used effectively to suppress the vibration of machines and structures. TDVA technology has found wide applications because it offers high reduction of vibration, good stability, low cost, low power, and simplicity of implementation. Examples of such systems include machines, automobiles, aircrafts, generators, engines and motors, and building structures [86-87]. However, the effectiveness of a conventional TDVA is always limited due to its narrow frequency ranges. In many practical applications, off-tuning of a TDVA occurs because of structural changes or varying usage patterns and
loading conditions. To overcome these shortcomings, adaptive tuned dynamic vibration absorbers (ATDVAs) have been studied extensively. An ATDVA is similar to a conventional TDVA but with adaptive elements that can be used to change the tuned condition. Commonly, adaptive stiffness elements are employed to vary the natural frequency of the device such that an ATDVA may be tuned to track uncertain or time-varying excitation frequencies.

The use of MREs to develop ATDVAs are expected to have many advantages: very fast response (less than a few milliseconds), simple structure, easy implementation, good maintenance, high stability, and effective control. Ginder, et al. [45] have done the research that utilised MREs as variable-spring-rate elements to develop an ADTVA. Their results indicate that a natural frequency can range from 580 Hz to 710 Hz at a magnetic field 0.56 Tesla. However, the natural frequency variation was only 22% from its central frequency. Deng and Gong [88] developed an adaptive tuned vibration absorber (ATVA) based on the unique characteristics of magnetorheological elastomers (MREs), whose modulus can be controlled by an applied magnetic field. This ATVA works in shear mode and consists of a dynamic mass, a static mass, and smart spring elements with MREs. The schematic diagram of the proposed ATVA is shown in Figure 2.10. The working principle of the system is as below. The magnetic field is created by two coils in the electromagnets and the field strength is controlled by the coil current, provided by an external DC power. As the shear modulus of MREs depends on the field strength, the equivalent stiffness of the ATVA changes with the field strength as well as the coil current. Consequently, the natural frequency of the ATVA can be controlled by the coil current, so its natural frequency can be changed by tuning the coil current to trace the external excitation frequency. When the tuned ATVA frequency matches the excitation frequency, the vibration can be attenuated significantly.

Lerner and Cunefare [50] use MREs as field dependent springs within three vibration absorber configurations, and to determine their vibration absorption characteristics. They showed that if a vibration absorber has flexible design constraints, a squeeze mode device will yield the largest frequency shift for active, semiactive, and hybrid vibration absorber design configurations. MREs comprised of
35% iron content by volume yielded the largest frequency shifts when incorporated in a squeeze mode device.

![Figure 2.10](image.png)

Figure 2.10: A schematic diagram of the developed ATVA: 1. Cover; 2. Guiderod; 3. Linear bearing; 4 magnetic conductor; 5 shear plate; 6. MREs; 7. Base; 8. Electromagnet; 9 mounting shell [88].

Xu et al. recently made an attempt to attach a voice coil motor to an MRE ATVA to counteract the damping force [89]. Zhang and Li [9] developed a new adaptive tuned dynamic vibration absorber (ATDVA) working with magnetorheological elastomers (MREs) which work in a shear mode and where the magnetic field is generated by a magnetic circuit. A real time control logic was proposed to evaluate the control effect. The simulation results indicate that the control effect of MRE ATDVA can be improved significantly. The device consisted of four basic components: an absorber mass, two MREs, an iron core, and a coil of magnetic wire. The magnetic field is controlled by the intensity of the electrical
current in the coil and an induced magnetic field is imposed in the direction of the particle chains in MREs, and therefore it works in the shear mode.

### 2.5.3 Vibration Isolator

Researchers have applied MREs to tuned vibration absorbers, and vibration isolators using MREs have also been studied recently. Some attempts have been made to use smart materials, which allow the stiffness or damping to vary during operation. Time varying damping elements include magnetorheological fluids (MRFs), electrorheological fluids (ERFs) [90] and piezoelectrics [91-92], whose damping characteristics can be changed if an electric field is applied for electrorheological fluids or a magnetic field for magnetorheological fluids. A high yield stress and an increase in damping for MR or ER dampers can occur by increasing the field strength to the point where the chains solidify. The fluid can return to its original state in milliseconds if the external electric field or magnetic field is removed [54, 93]. The MR fluid can be controlled with a low voltage, current driven power supply outputting only 1-2 amps. So there are a number of researchers who have recently undertaken their study after recognising the significant potential of devices based on MR fluids [94-114].

The rubber element of a vibration isolator may be operated in shear, compression, torsion, or a combination of them. The composition of the rubber determines its material properties such as the resilience, normal modulus, hardness, creep, damping, compression relaxation and temperature dependence. Difference types of commercially available rubbers are compared in [115-117].

Traditionally, MRE materials are used in both compression and shear modes. Studies have been conducted which demonstrate the benefits of using MRE materials as components of vibration isolation systems in compression and shear [4, 23, 44-45, 50, 89, 118-126]. MRE materials have been used for tunable automotive bushings to increase vehicle handling, and to reduce vibration and noise [23, 124-126].
Kavlicoglu et al. [127] developed a novel MRE mount using 0.5-inch thick MRE layers and built-in electromagnets, which provided a wide controllable compression static stiffness range for protecting a system with variable payloads from external shocks and vibration. A new type of MR fluid–elastomer vibration isolator, which has the capacity for controllable damping and dynamic stiffness over a wide range of frequencies and displacement amplitudes, was proposed in [128]. This new vibration isolator has potential in applications where tuning the vibration characteristics is desired. An investigation into MRE isolators in reducing the vibration of structural systems has been carried out in [4, 129-130], and a prototype of an MRE isolator has been manufactured and experimentally tested in [131], where the cost of manufacturing the MRE isolator hardware is comparable to an inexpensive audio speaker.

![Schematic of the proposed MRE mount][1]

**Figure 2.11**: Schematic of the proposed MRE mount [127].

Du and Li [132] proposed a concept design for an MRE isolator and its behaviour was experimentally evaluated. Then, an integrated seat suspension system which includes a quarter-car suspension and a seat suspension together with a driver body model has been proposed. Moreover a continuously variable stiffness semi-
active control strategy has been developed for the MRE isolator so that it can largely approximate the active isolator when it is in an energy dissipation state. A schematic diagram of the proposed MRE seat isolator using the fabricated MRE samples is shown in Figure 2.12. It consists of a core and base, coil, non-magnetic rings and MRE samples. The coils, core, and base are from an electromagnet which is used to generate varying magnetic fields. The non-magnetic rings partially support the weight of the coil so that it does not press on to the MRE too much, and also prevent the magnetic field lines from directly passing through the MRE without crossing the core. If the rings do not exist, the magnetic field lines pass through the MRE along its surface without crossing it, which would not have any MR effect. In the design, the MRE functions like a variable spring. The intensity of the current in the coils can be controlled by electrical power to adjust the intensity of the magnetic field. This means the modulus of the MRE can be controlled in real time, i.e. the stiffness of the spring can be adjusted to fit the particular need.

Li et al. [133] designed, tested and evaluated an MRE isolator and a simulation analysis on the effectiveness of its vibration control. By altering the system’s stiffness based on the nature of excitation through an appropriate stiffness control algorithm, the vehicle’s vibration energy input to the seat can be reduced to avoid resonance. The isolator’s stiffness is actively controlled to establish a nonresonant state against base excitations, thus suppressing the seat’s response.
Figure 2.12: A schematic diagram of the MRE seat isolator [132].

Some researchers focused on the feasibility of semi-active control devices to reduce the seismic response of base isolated structures which can be both adaptable and stable, while maintaining low external power requirements [134]. A smart structural control system employing semi-active control devices has been proposed and studied by [135] and [136]. Recently, a new type of semi-active base-isolation system has been proposed to incorporate MREs in base-isolation systems. For civil engineering applications of MREs, Hwang et al. [137] carried out a conceptual study on the application of MREs to base isolation systems for building structures.

Yu et al. [138-140] provided some views regarding the ideal engine mount system that should isolate vibration caused by engine disturbance in various speed ranges and prevent engine bounce from shock excitation.
An application in audible frequency ranges is shown in [141]. The MR elastomer vibration isolator on an elastic base consists of an MRE bushing attached to an iron shell. It is shown that the transmitted energy flow into the base is significantly reduced. These concepts can help reduce the noise radiated from the submerged facilities. Since the change of stiffness of MRE by the application of a magnetic field is a rapid and reversible process, the conclusion is drawn that an MRE vibration isolator will be a very interesting choice for conventional rubber vibration isolators in reducing vibration and noise. An MRE vibration isolator is in fact a semi-active structure that can be used to dissipate vibrational energy. The method used for a semi-active device is an on-off control which can switch a control signal between the maximum and minimum values based on some switching conditions provided by skyhook or groundhook control, etc [142]. York et al. [128] proposed a new design concept utilising an MR composite material through encapsulating MR fluids into an elastomer [143-144]. This type of material is similar to other MR elastomer systems. However, the magnetisable particles are contained within a void in the elastomer casing and do not need alignment during the curing process before the matrices solidify, unlike typical MR elastomers [18, 22].

2.6 Conclusions

In this chapter a review of the fabrication, properties and modelling of magnetorheological elastomers (MREs) and different applications of MREs has been presented with the aim of developing the adaptive structures where the MREs are incorporated to reduce vibration.

Overall, the review presented in this chapter has clearly outlined the lack of use of MREs for sandwich structure under non-homogeneous magnetic field strength and for two–degrees of freedom of variable stiffness vibration isolator application. New works should therefore to be carried out in order to design novel MR adaptive structures which can work in a relatively wide frequency range.
CHAPTER 3
NUMERICAL ANALYSIS OF AN MR ELASTOMER SANDWICH BEAM
UNDER NON-HOMOGENEOUS MAGNETIC FIELDS

This chapter extends the Mead & Markus [145-146] model and derives a vibration differential equation of an MRE sandwich beam to study the effect of the coverage and strength of an external magnetic field and its dynamic characteristics. It focuses on investigating an MRE cantilever sandwich beam and a clamped-clamped sandwich beam.

3.1 MRE sandwich beam modelling based on the MM approach

![Diagram of MRE sandwich beam under localised magnetic field]

**Figure 3.1**: A schematic of an MRE Cantilever Sandwich Beam under a Localised Magnetic Field
The MM model was extended and the core layer was divided into $N$ independent parts, where the magnetic field was applied respectively. This means there are different properties on every part which are analysed independently and then the whole sandwich beam model can be built through the continuous boundary conditions of each part. Figure 3.1 is a schematic of an MRE cantilever sandwich beam under localised magnetic field.

The following assumptions were made in the analysis:

1) The surface layers are purely elastic and suffer no shear deformation normal to the layer faces.
2) The surface layers should not affect the distribution and strength of the magnetic field.
3) The middle layer materials are only subjected to the shear deformation associated with axial displacements and the longitudinal direct stresses are negligible.

The shear modulus of the core was assumed to be a distributing function along the longitudinal direction because the MRE part of the core is affected by the magnetic field while the non-MRE parts are not [147].

1) Transverse normal strains in both core and skins were neglected so that the transverse displacements of all points on a cross section are equal.
2) There was perfect bonding between layers of the sandwich beam and hence there was no slippage or delamination between the layers during deformation.

The Mead and Markus model can be expressed as a six-order differential equation such that the sandwich beam is divided into $N$ parts and the transverse vibration differential equations of each part can be expressed as follows, which is based on Mead and Markus [145-146].

$$\frac{\partial^6 w}{\partial x^6} - g(1+Y)\frac{\partial^4 w}{\partial x^4} + \frac{\mu}{D_t} \left( \frac{\partial^4 w}{\partial x^2 \partial t^2} - g\frac{\partial^2 w}{\partial t^2} \right) = -\frac{gf(x,t)}{D_t} \quad (3.1)$$
CHAPTER 3. NUMERICAL ANALYSIS OF MR ELASTOMER SANDWICH BEAM UNDER NON-HOMOGENEOUS MAGNETIC FIELDS

\[ Y = \frac{d^2 b}{D_f} \left( \frac{E_i H_i E_3 H_3}{E_i H_i + E_3 H_3} \right) \]

\[ g = g' (1 + i \beta) = \frac{1}{H} \left( \frac{1}{E_i H_i + E_3 H_3} \right) G (1 + i \gamma) \]

where \( w \) is transverse displacement of the beam

\( Y \) is geometric parameter

\( g \) is shear parameter

\( E_i \) is Young’s modulus of the \( i \)th layer

\( H_i \) is thickness of the \( i \)th layer

\( D \) is central distance between face layers

\( f(x,t) \) is external force

\( \gamma \) is core loss factor

\( \mu \) is mass per unit length of beam

When the beam is under free vibration, the solution can be expressed in the form

\[ w(x,t) = W(x, \omega)e^{i\omega t} \quad (3.2) \]

Assume \( f \) is 0 and substitute equation (3.2) into equation (3.1), then:

\[ W_{(x)}^{iv} - g(1 + Y)W_{(x)}^{iv} - \omega_n^2 (1 + i \beta_n) \left( \frac{D}{D_f} \right) (W_{(x)}^{ii} - gW_{(x)}) = 0 \quad (3.3) \]
CHAPTER 3. NUMERICAL ANALYSIS OF MR ELASTOMER SANDWICH BEAM UNDER NON-HOMOGENEOUS MAGNETIC FIELDS

Set:  \[ x = \xi l \]

\[ \Omega_n^2 = \omega_n^2 (1 + i \beta_n) \frac{mt}{D}, \]

\[ g^* = gt^2 \]

Where \( \xi \) is the non-dimensional length co-ordinate, \( l \) is the length of the beam, \( \omega_n, \beta_n \) are the \( n \)th resonant frequency and \( n \)th is the modal loss factor, so equation (3.3) can be simplified as:

\[ W(\xi)^{vi} - g^*(1 + Y)W(\xi)^{iv} - \Omega_n^2 (W(\xi)^{ii} - g^7W(\xi)) = 0 \]  \( (3.4) \)

Since equation (3.4) is of the sixth order, the functions \( W(\xi) \) may be expressed in the form:

\[ W(\xi) = A_1 e^{i\lambda_1 \xi} + A_2 e^{-i\lambda_2 \xi} + A_3 e^{i\lambda_3 \xi} + A_4 e^{-i\lambda_4 \xi} + A_5 e^{i\lambda_5 \xi} + A_6 e^{-i\lambda_6 \xi} \]  \( (3.5) \)

So, the mode shape function of every part can be expressed as:

\[ W_k(\xi) = A_{k1} e^{i\alpha_{k1} \xi} + A_{k2} e^{-i\alpha_{k2} \xi} + A_{k3} e^{i\alpha_{k3} \xi} + A_{k4} e^{-i\alpha_{k4} \xi} + A_{k5} e^{i\alpha_{k5} \xi} + A_{k6} e^{-i\alpha_{k6} \xi} \]  \( (3.6) \)

Where \( W_k \) is the mode shape function of the \( k \)th part;

\( A_{k1}, \ldots, A_{k6} \) are the constant of each mode shape

\( a_{k1}, a_{k2}, a_{k3} \) are the six complex roots of the complex characteristic equation

\[ a_k^6 - g_k^* (1 + Y) a_k^4 - \Omega_n^2 (a_k^2 - g_k^*) = 0 \]  \( (3.7) \)

\[ \delta_k^3 - g_k^* (1 + Y) \delta_k^2 - \Omega_n^2 (\delta_k - g_k^*) = 0 \]  \( (3.8) \)

It is convenient to introduce the symbols

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CHAPTER 3. NUMERICAL ANALYSIS OF MR ELASTOMER SANDWICH BEAM UNDER NON-HOMOGENEOUS MAGNETIC FIELDS

\[ Q_k = -\frac{1}{3} \Omega \frac{n^2}{g_k} \left(1 + Y\right)^2 \]

and

\[ R_k = \frac{1}{6} \Omega \frac{n^2}{g_k} \left(1 + Y\right) - \frac{1}{2} \Omega \frac{n^2}{g_k} \left(1 + Y\right)^3 + \frac{1}{27} g_k^3 \left(1 + Y\right)^3 \]

\[ D_k = Q_k^3 + R_k^3 \]

If \( D_k \leq 0 \), then:

\[ \delta_{1k} = 2\sqrt{-Q_k} \cos\left(\frac{\alpha_k}{3}\right) + \frac{1}{3} g_k \left(1 + Y\right) \]

\[ \delta_{2k} = 2\sqrt{-Q_k} \cos\left(\frac{2\pi}{3}\right) + \frac{1}{3} g_k \left(1 + Y\right) \]

\[ \delta_{3k} = 2\sqrt{-Q_k} \cos\left(\frac{4\pi}{3}\right) + \frac{1}{3} g_k \left(1 + Y\right) \]

where,

\[ \alpha_k = \cos^{-1} \left(\frac{R_k}{\sqrt{-Q_k^3}}\right) \]

If \( D_k > 0 \), then

\[ \delta_{1k} = S_k + T_k + \frac{1}{3} g_k \left(1 + Y\right) \]

\[ \delta_{2k} = -\frac{1}{2} (S_k + T_k) + \frac{i}{2} \sqrt{3} (S_k - T_k) + \frac{1}{3} g_k \left(1 + Y\right) \]

\[ \delta_{3k} = -\frac{1}{2} (S_k + T_k) - \frac{i}{2} \sqrt{3} (S_k - T_k) + \frac{1}{3} g_k \left(1 + Y\right) \]

where,

\[ S_k = \sqrt{R_k + \sqrt{Q_k^3 + R_k^2}} \]

\[ T_k = \sqrt{R_k - \sqrt{Q_k^3 + R_k^2}} \]

3.2 Boundary conditions
The boundary conditions and continuous conditions of each part of the sandwich beam are as follows [148-149]:

1) Transverse displacement prevented at the clamped end:
   \[ W = 0 \]

2) Transverse displacements of adjacent parts are equal:
   \[ W_i = W_j \quad j = i + 1 \]

3) Rotation prevented at the clamped end:
   \[ W^' = 0 \]

4) Rotation of adjacent part is equal:
   \[ W_i^' = W_j^' \]

5) Bending moment at the clamped end:
   \[ M = W^{iv} - g^*(1 + Y)W'' - \Omega_n^2W = 0 \]

6) Bending moments of adjacent parts are equal:
   \[ M_i = M_j \]

7) Shear force at the clamped end:
   \[ S = W^v - g^*(1 + Y)W''' - \Omega_n^2W' = 0 \]

8) Shear forces of adjacent parts are equal:
   \[ S_i = S_j \]

9) The longitudinal displacement of the neutral surface of the face plate are equal:
   \[ (U_{1,3,i}) = W_i^v - g^*YW_i''' - \Omega_n^2W_i' \]

10) The axial forces of adjacent part are equal:
3.3 Solutions

Based on the boundary conditions and continuous conditions shown in section 3.2, the whole model can be obtained. This chapter gives the solution of a cantilever sandwich beam and clamped-clamped sandwich beam.

The face layers are AL and middle layer is MRE, which can be seen in Figure 3.1. The left domain of core indicated by shading means that the MRE is under a magnetic field but the right part is without an external magnetic field. So the boundary conditions and continuous conditions of every part in this model are as follows:

Clamped end:

1) Transverse displacement $W = 0$;

2) Rotation $W' = 0$;

3) The longitudinal displacement of face plate

$$(U_{1,3})_i = W_i^y - \gamma W'^{'''} - \Omega_n^2 W_i = 0;$$

Free end:

1) Bending moment $M = W^y - \gamma (1 + Y) W' - \Omega_n^2 W = 0$;

2) Shear force $S = W^y - \gamma (1 + Y) W'^{'} - \Omega_n^2 W' = 0$;

3) The longitudinal displacement of the neutral surface of the face plate

$$(U_{1,3})_i = W_i^y - \gamma W''^y - \Omega_n^2 W_i = 0;$$

\[ P_i = W_i^y - \gamma W_i^{'''} - \Omega_n^2 W_i \]  

(3.9)
Adjacent parts are continuous:

1) Transverse displacements $W$ are equal;
2) Rotation of adjacent parts $W'$ are equal;
3) Bending moments $M$ are equal;
4) Shear forces $S$ are equal;
5) The longitudinal displacements of neutral surface of face plate are equal $U_1=U_3$;
6) The axial forces of adjacent part $P$ are equal.

So there are twelve constraint conditions and twelve unknown coefficients, the equations can be expressed in term of the matrix:

$$
\begin{bmatrix}
[B_{11}]_{6\times6} & [B_{12}]_{6\times6} \\
[B_{11}]_{6\times6} & [B_{12}]_{6\times6} \\
[B_{21}]_{3\times6} & [A_{21}] \\
[B_{21}]_{3\times6} & [A_{26}]
\end{bmatrix}
= 0
$$

(3.10)

where,

$$
[B_{11}] = 
\begin{bmatrix}
1 & 1 & 1 & 1 & 1 & 1 \\
1 & a_{11} & -a_{11} & a_{12} & -a_{12} & a_{13} & -a_{13} \\
1 & -D_{11} & D_{11} & -D_{12} & D_{13} & -D_{13}
\end{bmatrix}
$$
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\[ [B_{11}] = \begin{bmatrix}
  e^{a_{ij} \theta_1} & e^{-a_{ij} \theta_1} & e^{a_{ij} \theta_1} & e^{-a_{ij} \theta_1} & e^{a_{ij} \theta_1} & e^{-a_{ij} \theta_1} \\
  a_{11} e^{a_{ij} \theta_1} & -a_{12} e^{-a_{ij} \theta_1} & a_{12} e^{a_{ij} \theta_1} & -a_{13} e^{-a_{ij} \theta_1} & a_{13} e^{a_{ij} \theta_1} & -a_{13} e^{-a_{ij} \theta_1} \\
  B_{11} e^{a_{ij} \theta_1} & B_{11} e^{a_{ij} \theta_1} & B_{12} e^{a_{ij} \theta_1} & B_{12} e^{a_{ij} \theta_1} & B_{13} e^{a_{ij} \theta_1} & B_{13} e^{a_{ij} \theta_1} \\
  E_{11} e^{a_{ij} \theta_1} & E_{12} e^{a_{ij} \theta_1} & E_{12} e^{a_{ij} \theta_1} & E_{13} e^{a_{ij} \theta_1} & E_{13} e^{a_{ij} \theta_1} & E_{13} e^{a_{ij} \theta_1} \\
  C_{11} e^{a_{ij} \theta_1} & -C_{11} e^{-a_{ij} \theta_1} & C_{12} e^{a_{ij} \theta_1} & -C_{12} e^{-a_{ij} \theta_1} & C_{13} e^{a_{ij} \theta_1} & -C_{13} e^{-a_{ij} \theta_1} \\
  D_{11} e^{a_{ij} \theta_1} & -D_{11} e^{-a_{ij} \theta_1} & D_{12} e^{a_{ij} \theta_1} & -D_{12} e^{-a_{ij} \theta_1} & D_{13} e^{a_{ij} \theta_1} & -D_{13} e^{-a_{ij} \theta_1}
\end{bmatrix} \]

\[ [B_{12}] = \begin{bmatrix}
  e^{a_{ij} \theta_1} & e^{-a_{ij} \theta_1} & e^{a_{ij} \theta_1} & e^{-a_{ij} \theta_1} & e^{a_{ij} \theta_1} & e^{-a_{ij} \theta_1} \\
  a_{21} e^{a_{ij} \theta_1} & -a_{21} e^{-a_{ij} \theta_1} & a_{22} e^{a_{ij} \theta_1} & -a_{22} e^{-a_{ij} \theta_1} & a_{23} e^{a_{ij} \theta_1} & -a_{23} e^{-a_{ij} \theta_1} \\
  B_{21} e^{a_{ij} \theta_1} & B_{21} e^{a_{ij} \theta_1} & B_{22} e^{a_{ij} \theta_1} & B_{22} e^{a_{ij} \theta_1} & B_{23} e^{a_{ij} \theta_1} & B_{23} e^{a_{ij} \theta_1} \\
  E_{21} e^{a_{ij} \theta_1} & E_{21} e^{a_{ij} \theta_1} & E_{22} e^{a_{ij} \theta_1} & E_{22} e^{a_{ij} \theta_1} & E_{23} e^{a_{ij} \theta_1} & E_{23} e^{a_{ij} \theta_1} \\
  C_{21} e^{a_{ij} \theta_1} & -C_{21} e^{-a_{ij} \theta_1} & C_{22} e^{a_{ij} \theta_1} & -C_{22} e^{-a_{ij} \theta_1} & C_{23} e^{a_{ij} \theta_1} & -C_{23} e^{-a_{ij} \theta_1} \\
  D_{21} e^{a_{ij} \theta_1} & -D_{21} e^{-a_{ij} \theta_1} & D_{22} e^{a_{ij} \theta_1} & -D_{22} e^{-a_{ij} \theta_1} & D_{23} e^{a_{ij} \theta_1} & -D_{23} e^{-a_{ij} \theta_1}
\end{bmatrix} \]

\[ [B_{2}] = \begin{bmatrix}
  B_{21} e^{a_{ij} \theta_2} & B_{21} e^{a_{ij} \theta_2} & B_{22} e^{a_{ij} \theta_2} & B_{22} e^{a_{ij} \theta_2} & B_{23} e^{a_{ij} \theta_2} & B_{23} e^{a_{ij} \theta_2} \\
  C_{21} e^{a_{ij} \theta_2} & -C_{21} e^{-a_{ij} \theta_2} & C_{22} e^{a_{ij} \theta_2} & -C_{22} e^{-a_{ij} \theta_2} & C_{23} e^{a_{ij} \theta_2} & -C_{23} e^{-a_{ij} \theta_2} \\
  D_{21} e^{a_{ij} \theta_2} & -D_{21} e^{-a_{ij} \theta_2} & D_{22} e^{a_{ij} \theta_2} & -D_{22} e^{-a_{ij} \theta_2} & D_{23} e^{a_{ij} \theta_2} & -D_{23} e^{-a_{ij} \theta_2}
\end{bmatrix} \]

Where,

\[ B_{ij} = a_{ij}^4 - g_k^{*} \left( 1 + Y \right) a_{ij}^2 - \Omega_n^2 \]

\[ C_{ij} = a_{ij}^5 - g_k^{*} \left( 1 + Y \right) a_{ij}^3 - \Omega_n^2 a_{ij} \]

\[ D_{ij} = a_{ij}^5 - g_k^{*} Y a_{ij}^3 - \Omega_n^2 a_{ij} \]

\[ E_{ij} = a_{ij}^4 - g_k^{*} Y a_{ij}^2 - \Omega_n^2 \]

where, \( i=1,2 \) \( j=1,2, \) \( q_i = \frac{l_i}{l}, \) \( i=1,2 \)

\( l \) is the length of the beam, \( l_i \) is the length of the MRE part under a magnetic field, and \( l_2 \) is the length of the whole MRE layer.
In equation (3.10), not all $A_{ij}$ are zero, so the determinant of the coefficient matrix must be zero. After the natural frequency is calculated, the mode shape coefficient $A_{i1}, \ldots, A_{i6}$ vibration characteristic equation can be determined.

Figure 3.2 is a schematic of an MRE Clamped-Clamped Sandwich beam under localised magnetic field.

The face layers are aluminium and middle layer is MRE. The central domain of the core indicated by shading means that the MRE is under a magnetic field but the left and right part is the MRE without an external magnetic field. So the boundary conditions and continuous conditions of every part in this model are as follows:

**Figure 3.2**: A Schematic of MRE Clamped-Clamped Sandwich Beam under a Localised Magnetic Field

Clamped end:

1) Transverse displacement $W = 0$;

2) Rotation $W' = 0$;
3) The longitudinal displacement of neutral surface of face plate

\[
(U_{1,3})_i = W_i^{y'} - g^{n} W_i^{y'''} - \Omega^{n}_{n} W_i^{'2} = 0;
\]

Adjacent parts are continuous:

1) Transverse displacement \( W \) is equal;

2) Rotation of adjacent part \( W' \) is equal;

3) Bending moment \( M \) is equal;

4) Shear force \( S \) is equal;

5) The longitudinal displacement of the neutral surface of the face plate is equal to

\[
U_1 = U_3;
\]

6) The axial forces of adjacent part \( P \) are equal.

There are eighteen constraint conditions and eighteen unknown coefficients, so the equations can be expressed in terms of the matrix:

\[
\begin{bmatrix}
[B_1]_{3 \times 6} & [B_{12}]_{6 \times 6} & [B_{21}]_{6 \times 6} & [B_{22}]_{6 \times 6} & [B_3]_{3 \times 6}
\end{bmatrix}
\begin{bmatrix}
A_{11} \\
\vdots \\
A_{16} \\
A_{21} \\
\vdots \\
A_{26} \\
A_{31} \\
\vdots \\
A_{36}
\end{bmatrix} = 0 \quad (3.11)
\]
In equation (3.10),

\[
\begin{bmatrix}
B_1
\end{bmatrix} =
\begin{bmatrix}
1 & 1 & 1 & 1 & 1 \\
\alpha_1 & -\alpha_1 & \alpha_2 & -\alpha_2 & \alpha_3 & -\alpha_3 \\
D_1 & -D_1 & D_2 & -D_2 & D_3 & -D_3
\end{bmatrix}
\]

\[
\begin{bmatrix}
B_{11}
\end{bmatrix} =
\begin{bmatrix}
e^{\alpha_1 q_1} & e^{-\alpha_1 q_1} & e^{\alpha_2 q_1} & e^{-\alpha_2 q_1} & e^{\alpha_3 q_1} & e^{-\alpha_3 q_1} \\
a_1 e^{\alpha_1 q_1} & -a_1 e^{-\alpha_1 q_1} & a_2 e^{\alpha_2 q_1} & -a_2 e^{-\alpha_2 q_1} & a_3 e^{\alpha_3 q_1} & -a_3 e^{-\alpha_3 q_1} \\
B_1 e^{\alpha_1 q_1} & B_1 e^{-\alpha_1 q_1} & B_2 e^{\alpha_2 q_1} & B_2 e^{-\alpha_2 q_1} & B_3 e^{\alpha_3 q_1} & B_3 e^{-\alpha_3 q_1} \\
E_1 e^{\alpha_1 q_1} & E_1 e^{-\alpha_1 q_1} & E_2 e^{\alpha_2 q_1} & E_2 e^{-\alpha_2 q_1} & E_3 e^{\alpha_3 q_1} & E_3 e^{-\alpha_3 q_1} \\
C_1 e^{\alpha_1 q_1} & -C_1 e^{-\alpha_1 q_1} & C_2 e^{\alpha_2 q_1} & -C_2 e^{-\alpha_2 q_1} & C_3 e^{\alpha_3 q_1} & -C_3 e^{-\alpha_3 q_1} \\
D_1 e^{\alpha_1 q_1} & -D_1 e^{-\alpha_1 q_1} & D_2 e^{\alpha_2 q_1} & -D_2 e^{-\alpha_2 q_1} & D_3 e^{\alpha_3 q_1} & -D_3 e^{-\alpha_3 q_1}
\end{bmatrix}
\]

\[
\begin{bmatrix}
B_{12}
\end{bmatrix} =
\begin{bmatrix}
e^{\alpha_1 q_2} & e^{-\alpha_1 q_2} & e^{\alpha_2 q_2} & e^{-\alpha_2 q_2} & e^{\alpha_3 q_2} & e^{-\alpha_3 q_2} \\
a_2 e^{\alpha_1 q_2} & -a_2 e^{-\alpha_1 q_2} & a_2 e^{\alpha_2 q_2} & -a_2 e^{-\alpha_2 q_2} & a_3 e^{\alpha_3 q_2} & -a_3 e^{-\alpha_3 q_2} \\
B_1 e^{\alpha_1 q_2} & B_1 e^{-\alpha_1 q_2} & B_2 e^{\alpha_2 q_2} & B_2 e^{-\alpha_2 q_2} & B_3 e^{\alpha_3 q_2} & B_3 e^{-\alpha_3 q_2} \\
E_2 e^{\alpha_1 q_2} & E_2 e^{-\alpha_1 q_2} & E_2 e^{\alpha_2 q_2} & E_2 e^{-\alpha_2 q_2} & E_3 e^{\alpha_3 q_2} & E_3 e^{-\alpha_3 q_2} \\
C_1 e^{\alpha_1 q_2} & -C_1 e^{-\alpha_1 q_2} & C_2 e^{\alpha_2 q_2} & -C_2 e^{-\alpha_2 q_2} & C_3 e^{\alpha_3 q_2} & -C_3 e^{-\alpha_3 q_2} \\
D_2 e^{\alpha_1 q_2} & -D_2 e^{-\alpha_1 q_2} & D_2 e^{\alpha_2 q_2} & -D_2 e^{-\alpha_2 q_2} & D_3 e^{\alpha_3 q_2} & -D_3 e^{-\alpha_3 q_2}
\end{bmatrix}
\]

\[
\begin{bmatrix}
B_{21}
\end{bmatrix} =
\begin{bmatrix}
e^{\alpha_1 q_2} & e^{-\alpha_1 q_2} & e^{\alpha_2 q_2} & e^{-\alpha_2 q_2} & e^{\alpha_3 q_2} & e^{-\alpha_3 q_2} \\
a_2 e^{\alpha_1 q_2} & -a_2 e^{-\alpha_1 q_2} & a_2 e^{\alpha_2 q_2} & -a_2 e^{-\alpha_2 q_2} & a_3 e^{\alpha_3 q_2} & -a_3 e^{-\alpha_3 q_2} \\
B_2 e^{\alpha_1 q_2} & B_2 e^{-\alpha_1 q_2} & B_2 e^{\alpha_2 q_2} & B_2 e^{-\alpha_2 q_2} & B_3 e^{\alpha_3 q_2} & B_3 e^{-\alpha_3 q_2} \\
E_2 e^{\alpha_1 q_2} & E_2 e^{-\alpha_1 q_2} & E_2 e^{\alpha_2 q_2} & E_2 e^{-\alpha_2 q_2} & E_3 e^{\alpha_3 q_2} & E_3 e^{-\alpha_3 q_2} \\
C_2 e^{\alpha_1 q_2} & -C_2 e^{-\alpha_1 q_2} & C_2 e^{\alpha_2 q_2} & -C_2 e^{-\alpha_2 q_2} & C_3 e^{\alpha_3 q_2} & -C_3 e^{-\alpha_3 q_2} \\
D_2 e^{\alpha_1 q_2} & -D_2 e^{-\alpha_1 q_2} & D_2 e^{\alpha_2 q_2} & -D_2 e^{-\alpha_2 q_2} & D_3 e^{\alpha_3 q_2} & -D_3 e^{-\alpha_3 q_2}
\end{bmatrix}
\]
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\[ [B_{22}] = \begin{bmatrix}
    e^{a_{ij}q_1} & e^{-a_{ij}q_1} & e^{a_{ij}q_2} & e^{-a_{ij}q_2} & e^{a_{ij}q_3} & e^{-a_{ij}q_3} \\
    a_{31} e^{a_{ij}q_1} & -a_{31} e^{-a_{ij}q_1} & a_{32} e^{a_{ij}q_2} & -a_{32} e^{-a_{ij}q_2} & a_{33} e^{a_{ij}q_3} & -a_{33} e^{-a_{ij}q_3} \\
    B_{31} e^{a_{ij}q_1} & B_{31} e^{-a_{ij}q_1} & B_{32} e^{a_{ij}q_2} & B_{32} e^{-a_{ij}q_2} & B_{33} e^{a_{ij}q_3} & B_{33} e^{-a_{ij}q_3} \\
    E_{31} e^{a_{ij}q_1} & E_{31} e^{-a_{ij}q_1} & E_{32} e^{a_{ij}q_2} & E_{32} e^{-a_{ij}q_2} & E_{33} e^{a_{ij}q_3} & E_{33} e^{-a_{ij}q_3} \\
    C_{31} e^{a_{ij}q_1} & -C_{31} e^{-a_{ij}q_1} & C_{32} e^{a_{ij}q_2} & -C_{32} e^{-a_{ij}q_2} & C_{33} e^{a_{ij}q_3} & -C_{33} e^{-a_{ij}q_3} \\
    D_{31} e^{a_{ij}q_1} & -D_{31} e^{-a_{ij}q_1} & D_{32} e^{a_{ij}q_2} & -D_{32} e^{-a_{ij}q_2} & D_{33} e^{a_{ij}q_3} & -D_{33} e^{-a_{ij}q_3} 
\end{bmatrix} \]

\[ [B_{3}] = \begin{bmatrix}
    e^{a_{ij}q_1} & e^{-a_{ij}q_1} & e^{a_{ij}q_2} & e^{-a_{ij}q_2} & e^{a_{ij}q_3} & e^{-a_{ij}q_3} \\
    a_{31} e^{a_{ij}q_1} & -a_{31} e^{-a_{ij}q_1} & a_{32} e^{a_{ij}q_2} & -a_{32} e^{-a_{ij}q_2} & a_{33} e^{a_{ij}q_3} & -a_{33} e^{-a_{ij}q_3} \\
    D_{31} e^{a_{ij}q_1} & -D_{31} e^{-a_{ij}q_1} & D_{32} e^{a_{ij}q_2} & -D_{32} e^{-a_{ij}q_2} & D_{33} e^{a_{ij}q_3} & -D_{33} e^{-a_{ij}q_3} 
\end{bmatrix} \]

where:

\[ B_{ij} = a_{ij}^4 - g_k^*(1 + Y) a_{ij}^2 - \Omega_n^2; \]

\[ C_{ij} = a_{ij}^5 - g_k^*(1 + Y) a_{ij}^3 - \Omega_n^2 a_{ij}; \]

\[ D_{ij} = a_{ij}^5 - g_k^* Y a_{ij}^3 - \Omega_n^2 a_{ij}; \]

\[ E_{ij} = a_{ij}^4 - g_k^* Y a_{ij}^2 - \Omega_n^2; \]

where \( i=1,2 \) and \( j=1,2,3 \);

\[ q_i = \frac{l_i}{l}, \quad i=1,2,3 \]

\( l \) is the length of the beam, \( l_1, l_2, l_3 \) are the lengths of different parts of the MRE layer shown in Figure 3.2.

In equation (3.11), not all the \( A_{ij} \) are zero, so the determinant of the coefficient matrix must be zero. After the natural frequency is calculated, the mode shape coefficient \( A_{ij}, \ldots, A_{i} \) and vibration characteristic equation can be determined.
3.4 Numerical analysis of MRE sandwich beam under a non-homogeneous magnetic field

The numerical analysis of an MRE cantilever sandwich beam and MRE clamped-clamped sandwich beam is based on the analytic method according to Section 3.3.

The dynamic characteristics of a sandwich beam were obtained under different boundary conditions and non-homogeneous magnetic fields. The MRE parameters in this numerical analysis are based on the research results of Nayak [150]. This section focused on the cantilever sandwich beam and clamped-clamped sandwich beam where the strength of the magnetic field was set at 500 mT. Table 3.1 and Table 3.2 are the dimensions and material parameters of the sandwich beam. The configuration of the MRE cantilever sandwich beam under different localised magnetic fields is shown in Figure 3.3.

**Table 3.1**: The dimensions and material parameters of the face layer

<table>
<thead>
<tr>
<th>Dimension /m</th>
<th>material</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length /L</td>
<td>Width /b</td>
</tr>
<tr>
<td>0.36</td>
<td>0.04</td>
</tr>
</tbody>
</table>
Table 3.2: The dimensions and material parameters of the MRE layer

<table>
<thead>
<tr>
<th>Dimension /m</th>
<th>Material</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length /L</td>
<td>Width b</td>
</tr>
<tr>
<td>0.36</td>
<td>0.04</td>
</tr>
<tr>
<td>Height (h₂)</td>
<td>0.00188</td>
</tr>
<tr>
<td>Shear modulus G/MPa</td>
<td>calculated according to Nayak [150]</td>
</tr>
<tr>
<td>Density ρ (kg/m³)</td>
<td>3400</td>
</tr>
</tbody>
</table>

Figure 3.3: A schematic of the MRE Cantilever Sandwich Beam under Different Local Magnetic Fields
Table 3.3 is the first natural frequency of an MRE cantilever sandwich beam under a non-homogeneous magnetic field according to the configuration of Figure 3.3.

**Table 3.3**: The first natural frequencies of an MRE cantilever sandwich beam under a non-homogeneous magnetic field

<table>
<thead>
<tr>
<th>Configuration</th>
<th>Fundamental Natural frequency $f/\text{Hz}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>10.672</td>
</tr>
<tr>
<td>(2)</td>
<td>11.543</td>
</tr>
<tr>
<td>(3)</td>
<td>12.790</td>
</tr>
<tr>
<td>(4)</td>
<td>10.593</td>
</tr>
<tr>
<td>(5)</td>
<td>10.487</td>
</tr>
<tr>
<td>(6)</td>
<td>10.303</td>
</tr>
<tr>
<td>(7)</td>
<td>10.094</td>
</tr>
<tr>
<td>(8)</td>
<td>11.861</td>
</tr>
<tr>
<td>(9)</td>
<td>14.010</td>
</tr>
</tbody>
</table>

Table 3.3 shows that the difference in the first natural frequency of an MRE cantilever sandwich beam under different localised magnetic field strengths is very obvious. When the strength of the external magnetic field (500mT) was fully applied on to the MRE layer, the first natural frequency was 14.010 Hz which is the largest...
in all the configurations. For experimental considerations, the strength of magnetic field applied for simulation should not be very strong.

If the MRE core is divided into 4 parts and the smallest natural frequency is obtained when a localised magnetic field applied close to the free end of sandwich beam is 10.303Hz, which is 69.5% of fully covered by the magnetic field. Figure 3.4 is a schematic of the MRE Clamped-Clamped Sandwich Beam under Different Local Magnetic Fields.

**Figure 3.4**: A schematic of the MRE Clamped-Clamped Sandwich Beam under Different Local Magnetic Fields
Table 3.4 is the first natural frequency of the MRE Clamped-Clamped sandwich beam under a non-homogeneous magnetic field, according to the configuration in Figure 3.4.

Table 3.4: The first natural frequency of MRE Clamped-Clamped sandwich beam under non-homogeneous magnetic field

<table>
<thead>
<tr>
<th>Configuration</th>
<th>First natural frequency $f/Hz$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>24.950</td>
</tr>
<tr>
<td>(2)</td>
<td>25.781</td>
</tr>
<tr>
<td>(3)</td>
<td>25.781</td>
</tr>
<tr>
<td>(4)</td>
<td>24.950</td>
</tr>
<tr>
<td>(5)</td>
<td>29.541</td>
</tr>
<tr>
<td>(6)</td>
<td>29.541</td>
</tr>
<tr>
<td>(7)</td>
<td>21.392</td>
</tr>
<tr>
<td>(8)</td>
<td>33.375</td>
</tr>
<tr>
<td>(9)</td>
<td>37.580</td>
</tr>
</tbody>
</table>
As is shown in Table 3.4, the smallest first natural frequency is gained when a magnetic field was applied onto the middle of half an MRE layer, which is only 57% compared with fully covered by the magnetic field. In addition, the same results were obtained from configuration (2) and (3) because the boundary conditions are the same for clamped-clamped sandwich beam. The same results were noticed for configurations (5) and (6). Moreover it is shown in configurations (5), (6), and (7), when the strength and coverage of magnetic field are certain, the smallest first natural frequency was obtained if the coverage of magnetic field was in the middle of the MRE.

Figure 3.5: Theoretical First Natural Frequency of MRE Cantilever Sandwich Beam at different covered regions of the Magnetic Field Region
Figure 3.5 is the theoretical first natural frequency of an MRE cantilever sandwich beam when the region of applied magnetic field increased under magnetic field strength 500 mT. From figure above, the natural frequency increased 30% if the area covered by external magnetic field changes from 20% (configuration 1 in Figure 3.3) to 100% (configuration 9 in Figure 3.3)

The first natural frequency of the MRE cantilevered sandwich beam when the magnetic field moved from the clamped end to the free end is shown in Figure 3.6. The covered area by applied magnetic field is 25% of whole length and the strength is 500 mT. The x axis is the distance between the clamped end of the beam to the right side of the applied magnetic field.
Figure 3.6: Theoretical first Natural Frequency of MRE Cantilever Sandwich Beam when Magnetic Field moves from clamped end to free end

Figure 3.6 shows that the first natural frequency decreases slightly (when the strength of the external magnetic field moved from the clamped end to the free end).

Figure 3.7 is the effect of the coverage of the MRE under a magnetic field on the first natural frequency of the MRE clamped-clamped sandwich beam when the strength of the magnetic field was 500 mT.
Figure 3.7 shows that the first natural frequency of the clamped-clamped sandwich beam increased by 50.6% when the area covered by magnetic field increased from 25% to 100%.

3.5 Conclusions

In this chapter, the model based on M&M was extended, the differential equations of the MRE sandwich beam under a non-homogeneous magnetic field were derived, and the dynamic characteristics of the sandwich beam with different boundary conditions and under different magnetic field strength were calculated.
Moreover the effect of applied magnetic fields on dynamic response of MRE sandwich beam was analysed. The results are as follows:

(1) The first natural frequency of MRE cantilever sandwich beam increased about 30% when the area of localised magnetic field strength gradually increased from the clamped end to full coverage of the beam. While, the first natural frequency could decrease when the localized magnetic field moved from the clamped end to the free end of the sandwich beam.

(2) The first natural frequency of MRE clamped-clamped sandwich beam increased 50.6% when the area of localised magnetic field increased from one end to full coverage of the beam. In addition the first natural frequency could decrease when the external magnetic field moved from the clamped end to the middle of the beam.
CHAPTER 4 EXPERIMENTAL INVESTIGATION ON VIBRATION CHARACTERISTICS OF MAGNETORHEOLOGICAL ELASTOMER SANDWICH BEAM UNDER NON-HOMOGENEOUS MAGNETIC FIELD

4.1 Introduction

The recent development of an MRE based sandwich structure was initiated. The sandwich structures apply metal skins to enhance their bulk zero-field flexural rigidity and utilise MRE cores to magnetically alter their bulk flexural rigidity due to the field dependent transverse shear modulus of the cores. Cai et al. [151] presented an analytical approach for analysing the vibration of a cantilever or simply supported base beam with passive constrained layer damping (PCLD) patch. The governing equation of motion of the beam was derived on the basis of an energy approach and the Lagrange equation. Kovac et al. [152], Hyer et al. [153], and Daya et al. [154] performed analytical studies of the nonlinear vibration of a sandwich beam, while Iu et al. [155] proposed a numerical model for the nonlinear vibration of multilayer sandwich beams. In their approach, the structure was discretised in space by the finite element method, and the periodic solutions of the resulting set of ordinary differential equations were obtained by using the harmonic balance method.

Yalcintas and Dai [156] presented a detailed analysis of the vibration control capabilities of adaptive structures based on MR and ER materials, and also compared their minimum vibration rates, time response, and energy consumption rates. Their studies showed the vibration minimisation capabilities of MR and ER adaptive beams at different rates and environmental conditions.
They also investigated the vibration suppression capabilities of magnetorheological materials in adaptive structures. Homogeneous three-layered adaptive beams with MR materials sandwiched between two elastic layers were considered. The results of their investigation showed that MR material applications in adaptive structures have effective vibration control capabilities [157].

This attempt may lead to stiffness-controllable structures with high initial stiffness and wide field-controllable ranges of stiffness. Zhou, Lin, and Wang [148] investigated the effect of structural parameters on the field-dependent rigidities of single-layer sandwich beams with MRE cores through finite element analysis, where the dynamic stiffness of these sandwich beams could be modelled very well with a spring-mass model in the low frequency range. Furthermore, a multilayer sandwich configuration was introduced to provide the structures with higher bulk field-dependent rigidities. The structural design rules for multilayer sandwich beams for achieving the desired zero-field rigidities and relative change ranges of the field-dependent rigidities were also presented. Zhou and Wang [67, 147, 158] also carried out a theoretical study on MRE embedded smart sandwich beams with non-conductive skins and conductive skins based on higher order sandwich beam theory respectively. Wei and co-workers [159] did a preliminary experiment on the vibration characteristics of an MRE sandwich beam, and the results showed that the natural frequencies increased and the vibration amplitude of each mode decreased when the magnetic field was applied to the MR elastomers beam. Dwivedy, Mahendra, and Sahu [160] used higher order theory to derive the governing equations of motion of a soft-cored symmetric sandwich beam with magnetorheological elastomer subjected to periodic axial load. The regions of parametric instability for simple and combined resonances was also investigated for simply supported, clamped-pinned, clamped-guided, and clamped-free end conditions by modified Hsu’s method. The instability regions of the system with and without an MRE patch, and with different magnetic field strengths and permeability of skin materials has also been studied. Navak et al. [161] investigated the dynamic analysis of a three-layered symmetric sandwich beam with MRE embedded
viscoelastic core and conductive skins subjected to a periodic axial load. The governing equation of motion was derived by extending Hamilton’s principle along with Galarkin’s generalized method. The effects of the magnetic field, length of MRE patch, core thickness, percentage of iron particles, and carbon blacks on the regions of parametric instability for first three modes of vibration have been studied. Ying and Ni [162] studied the micro-vibration response of a clamped–free sandwich beam with an MR elastomer core and a supplemental mass under stochastic support micro-motion excitation. The dynamic behaviour of an MR elastomer was described and the sixth-order partial differential equation of motion was also derived. The numerical results illustrated how the core parameters of the MR elastomer influenced the root-mean-square velocity response and the high response reduction capacity sandwich beam. Choi et al. [163] investigated the dynamic behaviour of MRE cored sandwich beams with steel skins. Modelling the dynamic behaviour was carried out by adopting a higher order sandwich beam theory. The experimental responses were generated from a specially designed test rig to study dynamic behaviour, damping effects, localised magnetic field effects and energy dissipation with varying topology.

However, those research works for an MRE sandwich beam in the current literature were mostly only carried out under homogeneous magnetic fields, not considering the non-homogenous magnetic field effects on the beam. The aim of this study therefore is to address this limitation and to investigate the vibration response of the MRE sandwich beam under non-homogeneous magnetic field.

In this chapter, the MRE was manufactured and tested, and the MRE sandwich beam was also fabricated by placing the MRE between two thin layers of aluminium. The experimental test rig was set up to investigate the vibration of the MRE sandwich beam under non-homogeneous magnetic field. The experimental results showed that the first natural frequency of the MRE sandwich beam decreased as the magnetic field applied onto the beam was moved from the clamped end to the free end of the beam. It is also noted that the MRE sandwich beam had the capabilities of
left shifting the first natural frequency when the magnetic field was increased in the activated regions.

4.2 Sandwich structure

The structure of sandwich plates generally consists of two relatively thin external materials called the facings separated by and bonded to a relatively thick internal structure called the core. The facings are usually of a material which has high strength and stiffness compared to the core which is normally of a lighter density and relatively low strength and stiffness. Due to the nature of their construction, sandwich plates are known to have an extremely high strength-weight ratio compared to a single homogeneous plate.

Some advantages of sandwich construction are:

(1) The cross sections of the sandwich are composite. They usually consist of a low to moderate stiffness core which is connected to two stiff exterior facing sheets. The composite has a considerably higher shear stiffness to weight ratio than an equivalent beam made of only the core material or the face-sheet material. The composite also has a high tensile strength to weight ratio.

(2) The high stiffness of the face sheet leads to a high bending stiffness to weight ratio for the composite.

(3) Deformation due to bending moments or bending deformation, and

(4) Deformation due to transverse forces is also called shear deformation.

Open and closed cell structured foams like polyvinylchloride, polyurethane, polyethylene or polystyrene foams, balsa wood, syntactic foams and honeycombs are commonly used core materials.

Laminates of glass or carbon fibre reinforced thermoplastics or mainly thermoset polymers (unsaturated polyesters, epoxies) are widely used as layer
materials, although sheet metal is also used as layer materials in some cases. The core is bonded to the layers with an adhesive.

Metal composite material (MCM) is a type of sandwich formed from two thin layers of metal bonded to a plastic core in a continuous process under controlled pressure, heat, and tension [164].

The strength of the composite material is largely dependent on two factors:

(1) The outer layers: If the sandwich is supported on both sides and then stressed by a force in the middle of the beam, then the bending moment will introduce shear forces into the material. The shear forces result in the bottom layer being in tension and the top layer being in compression. The core material spaces keep these two layers apart, fundamentally the thicker the core material, the stronger the composite

(2) The interface between the core and the layer: Because the shear stresses in the composite material changes rapidly between the core and the layer, the adhesive layer also sees some degree of shear force. If the adhesive bond between the two layers is too weak, the most probable result will be delamination.

4.3 Fabrication of MRE sandwich beam

4.3.1 Fabrication of MRE material

In this study silicone sealant (SELLEYS PTY.LIMITED, Australia) and the Poly(dimethylsiloxane) fluid (SIGMA-ALDRICH Company, USA) were chosen as the matrix and the dispersed particles are carbonyl iron, which has a diameter of 4.5-5.2 um (SIGMA Company, USA), and is shown in Figure 4.1.
The mass fraction ratio of iron, silicone oil and silicone rubber in the mixture is 7:1:2. The carbonyl iron particles were placed into a container and then Poly fluid was poured into it and mixed them with the silicone sealant. All the materials mixed thoroughly. The mixture was poured into a mould after the air bubbles were removed. After 24 hours curing under room temperature, an MRE sample was fabricated with an iron particles mass fraction of 70%. The process is shown in Figure 4.2.
4.3.2 MRE material rheological properties

In the three-layered MRE sandwich beam configuration, the MRE materials experience shear stress and shear strain that is confined in the pre-yield regime. The stress-strain relationship based on linear viscoelastic theory is given by the following equation:

\[ \tau = G^* \gamma \]  \hspace{1cm} (4.1)

In equation (4.1), \( \tau \) is shear stress, \( \gamma \) is shear strain, and \( G^* \) is the complex shear modulus represented in the form

\[ G^* = G' + G''i \]  \hspace{1cm} (4.2)
where $G'$ is the storage modulus and $G''$ is the loss modulus. The storage modulus is proportional to the average energy stored during a cycle of deformation per unit volume of the MRE material. The loss modulus is proportional to the energy dissipated per unit volume of the MRE material over a cycle.

In this study, the relationship between the storage modulus, loss modulus and the magnetic field applied is expressed by the experiment done on a parallel plate rheometer (Physica MCR 301, the Anton Paar Company, Germany) and a Magneto-Rheological Device (MRD 180, Anton Paar Companies, Germany) that was used to measure the MREs' properties, and which can be seen in Figure 4.3. The MagnetoRheolgical Device was equipped with an electromagnetic kit which can generate a magnetic field perpendicular to the direction of the shear flow. Specifically, a 20mm diameter parallel plate measuring system with a 1 mm gap was used. The samples
were sandwiched in parallel between a rotary disk and a base. The stress and strain signals were output from two ports detected through the DAQ board (Type: LabViEW PCI-6221, National Instruments Corporation.U.S.A) and transferred to a computer where the data are processed.

The MRE sample is about Φ20 mm×1.15 mm thick. At a constant angular frequency $\omega=5$ rad/s, the relationship was under current intensities of 0, 0.25, 0.5, 0.75, 1, 1.25, 1.5, 1.75 and 2A, respectively. The magnetic flux density was applied through the tuning current. Figure 4.4 shows the relationships between the storage modulus of MRE and the strain under different magnetic flux density (current intensity), and Figure 4.5 shows the relationship between the loss modulus of MRE and the strain under a different magnetic flux density (current intensity). From the two figures, we noted that the storage modulus and the loss modulus increased if the magnetic flux density (current intensity) was increased, although the storage modulus and loss modulus increased very slowly when the intensity of the magnetic field was below 1000 Gs.

Figure 4.6 shows the storage modulus and loss modulus as a function of the magnetic flux density when the shear strain was below 1%.

The results shows that MRE dynamic properties can be changed when the external magnetic flux density is tuned, therefore, it is possible to apply MRE into sandwich beam to adjust dynamic properties of the whole structure through changing applied magnetic flux density.
Figure 4.4: The storage modulus of MRE under different magnetic fields
Figure 4.5: The loss modulus of MRE under different magnetic fields
Figure 4.6: Storage modulus and loss modulus as a function of the magnetic flux density

4.3.3 Fabrication of sandwich beam with MRE core

Figure 4.7 is the profile of the MRE sandwich beam that consist of three layers: the top and bottom layers are aluminium and the core is MRE material. Thin aluminium plates were chosen for elastic surface plates due to their low damping properties and relatively high stiffness compared to the MREs. Moreover, aluminium’s relative magnetic permeability is equal to one, which indicates that it does not affect the distribution and strength of the magnetic field. The beam is L=360 mm long and
b=40 mm wide, and the upper and lower layers of aluminium are \( h_1 = 1.61 \) mm thick, and the layer of MREs is \( h_2 = 2.98 \) mm thick.

133g of iron, 38g of silicone sealant and 19g of Poly fluid were placed into a container and stirred thoroughly. A sheet of aluminium was placed into 500 mm×40 mm×3 mm steel plate slot, and then the MRE mixture was packed evenly onto the surface of the aluminium sheet. To prevent the MRE from dissipating under the magnetic field, a rectangular aluminium frame, \( L_1 = 2 \) mm wide by \( h_2 = 2.98 \) mm thick was made to bond onto the bottom layer of aluminium. Then a sheet of aluminium was placed onto the MRE and then pressed down to ensure that the upper surfaces of the aluminium and the steel plate are parallel. A heavy steel plate was placed onto the sandwich beam and it was allowed to cure for about 24 hours at room temperature.
CHAPTER 4 EXPERIMENTAL INVESTIGATION ON VIBRATION CHARACTERISTICS OF MAGNETORHEOLOGICAL ELASTOMER SANDWICH BEAM UNDER NON-HOMOGENEOUS MAGNETIC FIELD

**Figure 4.7**: A schematic of the MRE sandwich beam: (a) Schematic of three layers; (b) Photograph of top and bottom layer of aluminium and (c) Photograph of the MRE sandwich beam

### 4.4 Experimental set-up

In this experimental test, the MRE sandwich beam was clamped onto a fixed platform in a cantilever configuration. **Figure 4.8** presents a schematic configuration and photograph of the experimental set-up, which is integrated with magnets, sensor, shaker, and signal analysis equipment. The instruments used in the experiment include LabVIEW programming, PCI multi-function Data Acquisition (DAQ) board, laser sensor, shaker, and power amplifier.

The DAQ board produced by National Instruments Inc. with model of PCI 6221 was connected to a Connector Block produced by National Instruments Inc. with a model of SCB-68, both of which were used to generate and acquire the signals. The laser sensor produced by MICRO-EPSILON Company with a model of the opto NCDT 1700 was used to measure the vibration at a single location. This sensor could measure displacements from 0 to 20 mm. The shaker produced by Sinocera Piezotronics Inc. with model of JZK-5 was driven by the voltage signal from the power amplifier with a model of YE5871A, also produced by Sinocera Piezotronics Inc. This voltage signal is generated by DAQ output through LabVIEW programming. The shaker could generate a nominal force up to 50 N with a displacement of 7.5 mm and a bandwidth from 0 to 5 kHz. The LabVIEW programming was used to obtain and analyse the analogue signals from the laser sensor.

Permanent magnets (Block 50x5x50 mm N40SH, Frenergy Magnets) were used to generate a magnetic field over the test beam. A magnetic field was applied in a vertical direction to beam surface. According to the parameters of permanents magnets, the maximum magnetic field strength that can be generated is
approximately 1300 Gs. Variations in the level of the magnetic field were obtained by changing the distance $y_1$ and $y_2$ between the permanent magnets.

A rod external to the shaker was connected to the beam at the actuation location, 110 mm from the fixed end of the beam. The point of the laser was located 20 mm away from the free end of the beam. The experimental procedure was carried out as follows: the signal output from the DAQ was sent to the shaker through the amplifier. The shaker provides the external vibration over the test beam. Data from the vibration of the test beam is acquired by the laser sensor and sent to the computer through DAQ. The LabVIEW programming in the computer processes the input signal. Then, the vibration in the frequency domain, and natural frequencies and amplitudes of the vibration are presented in the output of the analysis results.
**Figure 4.8**: Experimental setup for MRE sandwich beam: (a) Schematic; (b) Photograph
4.5 Dynamic Response of MRE Sandwich Beam under Different Magnetic Field Strengths

In the test, the LabVIEW program was set using a swept sine actuation at a range of 0-30 Hz with 0.1 Hz increment, and the amplitude was set at \( v = 200 \) mV. The magnetic field was provided by a single group magnet, which included three magnets in each group.

**Figure 4.9** shows the vibration of MRE sandwich beam at different activated regions. In the test, the distance between the magnetic poles was 70 mm (\( y_1 = 20 \) mm, \( y_2 = 50 \) mm), and the magnetic flux density in the centre of the activated region was 950 Gs. The magnets were moved from the clamped end of the beam to the free end of the beam along the x direction shown in **Figure 4.8**.

**Figure 4.10** displays the experimental first natural frequency at different regions when the magnetic field was activated. From these two figures, the first natural frequency of the MRE sandwich beam decreased quickly as the magnets moved towards the free end. Compared to the frequency under zero magnetic fields, the first natural frequency decreased by 13.9%. Stiffening the MRE in regions away from where the beams were clamped resulted in a decrease in the natural frequency of the beam compared to the natural frequency without a field. This behaviour agrees with the experimental results carried out by Lara-Prieto and his co-workers [165], where the first natural frequency of a cantilever PET MR beam decreased when the magnet moved towards the free end. This was also similar to the theoretical prediction made by Yalcintas and Coulter [66], where the natural frequency of a simply supported ER beam decreased when the beam was activated only in the central regions, away from the clamped ends.

**Figure 4.11** shows the vibration of an MRE sandwich beam under different intensity of magnetic field. During the experiments, the intensity of the magnetic field was changed by adjusting \( y_1 \), and \( y_2 \) was set to be 50 mm, as shown in Table 4.1. A magnetic field was applied onto the particular region of \( x = 23 \) cm. The
The experimental first natural frequency of an MRE sandwich beam under different intensity of magnetic fields is shown on Figure 4.12. It is also noted that the first natural frequency of the MRE sandwich beam decreased as the magnetic field increased in intensity, which showed the frequency left shift trend. In comparison to the frequency under zero magnetic fields, the first natural frequency decreased by 11.5%.

Figure 4.9: Vibration of MRE sandwich beam at different magnetic field regions
Figure 4.10: Experimental natural frequency at different activated magnetic field region

Table 4.1: the relationship of magnetic field intensity $B$ and distance $y_1$ and $y_2$

<table>
<thead>
<tr>
<th></th>
<th>$y_1$</th>
<th>$y_2$</th>
<th>$B$(mT)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>70</td>
<td>50</td>
<td>37</td>
</tr>
<tr>
<td>2</td>
<td>45</td>
<td>50</td>
<td>48</td>
</tr>
<tr>
<td>3</td>
<td>20</td>
<td>50</td>
<td>95</td>
</tr>
</tbody>
</table>
This interesting phenomenon is a little different from our common sense, in that the frequency increased as the intensity of the magnetic field increased. However, as shown in Figure 4.4, Figure 4.5, and Figure 4.6, the storage modulus $G'$ and loss modulus $G''$ changed very slowly when the intensity of the magnetic field was less than 1000 Gs. At the same time, the equivalent Yong’s modulus $E^*$ of the MRE will decrease when the magnetic field increases in intensity, as certified by Zhou [44]. As the influence of $G^*$ slowly increased, the quicker $E^*$ decreased, the first natural frequency decreased as the magnetic field increased.

![Figure 4.11: Vibration of MRE sandwich beam under different magnetic field intensity](image-url)
MREs are viscoelastic materials. The constitutive equation for this kind of material is significantly nonlinear and it is often described in terms of the principal extension ratios through the Ogden strain potential. This means that the shear modulus of an MRE is dominated by the stress condition, or strain energy. When a magnetic field is applied, an MRE will deform along the direction of the magnetic field. If we adopt a single chain model, similar to that used in [18, 48] to analyse the shear stress induced by inter-particle magnetic forces, the total magnetic energy density (energy per unit volume) is given by [166].

Figure 4.12: Experimental first natural frequency under different magnetic field intensity
where $M$ is the magnetisation of the particles, $d_p$ is the diameter of the particles, $\mu_0$ is the permeability of free space, $\phi$ is the volume fraction of the MRE and $r_0$, $\gamma = \tan \theta$, $\varepsilon = (r-r_0)/r_0$ represent the relative position of two adjacent particles in a chain after deformation, respectively (see Figure 4.13).

The complex shear modulus $G^*$ is assumed to be the sum of the shear modulus $G_0$ with no field applied and the shear modulus $\Delta G(B)$ induced by inter-particle magnetic forces. Let $\varepsilon = 0$ (no normal strain occurs); the shear stress induced by the application of a magnetic field can be deduced by taking the derivative of the total magnetic energy density in equation (4.3)
Figure 4.13: Magnetic interaction between two dipoles

With respect to $\gamma$:

$$
\tau_s = \frac{\partial U}{\partial \gamma} = \frac{\phi (4 - \gamma') M' d' \mu_i}{8c' (1 + \gamma')^{3/2}}
$$

(4.4)

For the small deformation of $\gamma$, equation (4.4) can be expressed as

$$
\Delta G(B) = \frac{\phi \mu_s M^2 d_s}{2 r_s^3}
$$

(4.5)
CHAPTER 4 EXPERIMENTAL INVESTIGATION ON VIBRATION CHARACTERISTICS OF MAGNETORHEOLOGICAL ELASTOMER SANDWICH BEAM UNDER NON-HOMOGENEOUS MAGNETIC FIELD

When the magnetic field was applied along the z direction, let $\gamma=0$ (no shear strain occurs); the stress induced by the application of a magnetic field can also be computed by taking the derivative of the total magnetic energy density with respect to $\varepsilon$:

$$\sigma_\varepsilon = \frac{\varphi M^2 d_0^\gamma \mu_n}{4 r_0^\gamma} - \frac{\varphi M^2 d_0^\gamma \mu_n}{r_0^\gamma} \varepsilon$$  \hspace{1cm} (4.6)

So, Yong’s modulus was induced by interparticle magnetic forces and can be given as

$$\Delta E(B) = -\frac{\varphi M^2 d_0^\gamma \mu_n}{r_0^\gamma}$$  \hspace{1cm} (4.7)

Equation (4.7) reveals a negative field induced modulus caused by exchange between the magnetic energy and the strain energy.

The complex shear modulus $G^*$ and the equivalent Young’s modulus $E^*$ can be expressed as

$$G^* = G_o + \Delta G(B)$$  \hspace{1cm} (4.8)

$$E^* = E_o + \Delta E(B)$$  \hspace{1cm} (4.9)

As shown in equations (4.4), (4.7), (4.8) and (4.9), the complex shear modulus $G^*$ increased, while the equivalent Young’s modulus $E^*$ will become smaller when the magnetic field increases. Furthermore, the decreased $\Delta E (B)$ was twice the increased $\Delta G (B)$. At the same time, as shown in Figures 4.4-4.6, the storage
modulus $G'$ and loss modulus $G''$ increased very slowly when the intensity of the magnetic field was 100 mT, which means the complex shear modulus $G^*$ showed little change. Under the influence of the slowly increasing $G^*$ and the more quickly decreasing $E^*$, the first natural frequency decreased as the magnetic field increased, as shown in Figures 4.11 and 4.12.

4.6 Comparison of MRF sandwich beam and MRE sandwich beam

MR materials have developed into a family with MR fluids (MRFs), and MR foams and MR elastomers (MREs) [2]. The most common MR material is MRFs. The general criterion to estimate the MR effect of MRFs is the capability of dynamic yield stresses to vary within a post-yield regime under an externally applied magnetic field. A lot of applications based on MRFs benefit from the properties of the dynamic yield stress which can be continuously, rapidly, and reversibly controlled by the applied magnetic field. But MRFs also have shortcomings, such as deposition, environmental contamination and sealing problems. Also, particle residue can degrade the performance of MR devices, which hinders their wide application [33]. MREs, the structural solid analogs of MRFs, may be a good solution to overcome these disadvantages.

Yalcintas and Dai investigated the capabilities of magnetorheological (MR) materials to suppress vibration in adaptive structures. The adaptive structures based on MR materials were formed by embedding MR materials between two elastic layers, and by applying different magnetic field levels over the three-layered composite configuration. Variations in stiffness and damping properties of the structures were achieved in response to different levels of magnetic fields applied levels [157]. Ginder et al. reported that the field induced increase in the moduli of the MR elastomers was effective at frequencies well above 1 kHz. Vianney et al. did an experimental study investigating the controllability of the vibration of
magnetorheological fluid cantilever sandwich beams. Diverse excitation methods were considered as well as a range of magnetic field strengths and configurations [66]. The controllability of the beam’s vibration was observed in the form of variations in amplitudes of vibration and shifts in natural frequency. However, there are few studies on the application of MRFs to control the vibration of flexible beams, especially under a non-homogeneous magnetic field.

4.6.1 Fabrication of a sandwich Beam with an MRF core

The Commercial MR fluid selected for this study was a hydrocarbon-based fluid called MRF-132AD manufactured by Lord Corporation. It contains carbonyl iron particles, which are widely used for MR fluids because of their high magnetic permeability and low coercivity, making the fluid suitable for reversible systems. MR fluid always works in the pre-yield region in this application of sandwich beam, so it is considered to be a linear viscoelastic material.

The MRF sandwich beam consists of three 1.61 mm thick layers of aluminium plates which were machined to the dimensions shown in Figure 4.14. The layers were glued together and sealed to avoid any leakage. Next, to be able to fill the cavity of the sandwich beam, four 3 mm diameter holes were drilled through the top Al layer of the beam. One hole was drilled at the free end of the beam and the other one in the opposite side, very close to the clamping part. Then, the MR fluid was injected into the beam using a hypodermic syringe. Figure 4.14 shows a schematic of the central layer of the sandwich beam and Figure 4.15 shows the design of the top AL layer with the location of the drilled holes.

Holes 1 and 3 were used to pour in the MR fluid while holes 2 and 4 were used to release the air. This way of filling the sandwich beam worked well without any air bubbles being trapped inside. Finally, the four holes were sealed. Figure 4.16 shows structure of the MRF sandwich beam.
Figure 4.14: The central layer of the sandwich beam with an MRF core

Figure 4.15: A schematic of the top Al layer
4.6.2 Dynamic characteristic of MRF sandwich beam under different strength magnetic fields

In this experiment, the MRF sandwich beam was clamped onto a fixed platform using a cantilever configuration. Figure 4.17 presents a schematic configuration of the experimental set up integrated with magnets, sensor, shaker, and signal analysis equipment. The instruments and measuring methods used in the experiment are the same as mentioned before.
4.6.3 Results analysis and discussion

In the test, the LabVIEW program was set using a swept sine actuation at a range of 0-30 Hz with 0.1 Hz increment, and the amplitude was set at $v=150$ mV. The magnetic field was provided by a single group magnet, which including three magnets in each group.

Figure 4.18 shows the vibration of cantilevered MRF sandwich beam at different activated regions. In test (a), the strength of the magnetic field in the centre of the activated region was 450 Gs. The magnets were moved from the clamped end to the free end of the beam, as shown in Figure 4.17.
Figure 4.18: Vibration of MRF sandwich beam at different activated regions under different strength magnetic fields: (a) B=450Gs  (b) B=1000Gs
Figure 4.18 (b) also shows the vibration of the cantilevered MRE sandwich beam at different activated regions where the strength of the magnetic field in the centre of the activated region was 1000 Gs. It is obvious that the first natural frequency of the MRE sandwich beam decreased as the magnets moved towards the free end. Compared to the first natural frequency in the absence of magnetic fields, the first natural frequency decreased 19.2% when the magnet was moved away from the clamped end from the beam, under a magnetic field of 1000 Gs.

This behaviour agrees well with the theoretical predication of Yalcintas and Coulter [66], where the natural frequency of a simply supported ER beam decreased when the beam was activated only in the central regions, away from the clamped ends. Stiffening the fluid in the regions away from the clamps of the beam results in a decrease in the natural frequency of the beam compared with the natural frequency in the absence of the field. Major changes in frequency are observed as the magnet moves towards the free end [165].

Figure 4.19 shows that the vibration of the cantilevered MRF sandwich beam under non-homogeneous field. The magnetic fields were provided by three groups of magnets with three magnets in each group. In this test, the distance between every two adjacent group magnets was 45 mm and x=35 mm. It is obvious that the first natural frequency of the MRF sandwich beam decreased by 23.6% as the strength of the magnetic field increased. It also shows the left frequency shift trend.

Figure 4.20 shows that the natural frequency of the cantilevered sandwich beam decreased when the magnets were moved away from clamped end to free end of beam and the strength of magnetic field increased at a specifically activated region. The first natural frequency changed dramatically when the external magnetic field was closed to the free end of the sandwich beam.
Figure 4.19: Vibration of the MRF cantilevered sandwich beam under different magnetic flux density
4.6.4 Conclusions

A number of researches done on ER and MR fluid sandwich structures showed that the natural frequency shifts to higher frequency when the strength of an external magnetic field increased. This chapter revealed the possibility of shifting the natural frequency to a lower frequency by applying a non-homogenous magnetic field, which can be achieved by partially activating a region of the sandwich beam.

It was noticed that the storage modulus and the loss modulus of MRE sample increased if external magnetic flux density (current intensity) was increased. And it
was also observed that the first natural frequency of the MRE sandwich beam decreased by 13.9\% when the single group magnet moved from the clamped end to the free end, and the first natural frequency decreased by 11.5\% when the applied magnetic field increased from 0 Gs to 950 Gs in the specific location of x=23 cm.

It was also observed that the first natural frequency of the MRF sandwich beam decreased by 6.4\%, 19.2\%, and 21.8\% respectively when the single group magnet moved from the clamped end to the free end when the magnetic field increased from 460 to 1000 Gs. It also showed that the first natural frequency of the MRF sandwich beam decreased by 23.6\% as the strength of the magnetic field increased from 0 to 900 Gs under non-homogeneous.

Both of the experimental results agreed well with the numerical analysis in Chapter 4 where the natural frequency of the cantilevered sandwich decreased when the magnetic field moved from the clamped end to the free end, as shown in Table 3.3.
CHAPTER 5
DESIGN AND DEVELOPMENT OF POSITIVE-NEGATIVE STIFFNESS VIBRATION ISOLATOR

5.1 Introduction

Many instruments are very sensitive to mechanical vibration and acoustic noise, which can come from different sources. For this reason vibration is becoming prevalent because machine operating speeds are increasing and machine tools are cutting more deeply.

A number of groups use vibration isolators over a wide range of applications which can be divided into four major categories: machinery isolation [167], building or seismic isolation [168], automotive isolation [169], and isolation of aerospace structures [170]. The criteria for isolation is how much unwanted force or oscillating disturbance can be mitigated. This can be the result of reducing forces or vibrations coming from a foundation or a piece of equipment, or it can be the result of reducing forces or vibration coming from the equipment to the foundations. The ideal vibration isolator has high static stiffness to support the isolated structure and low dynamic stiffness to ensure low transmission of dynamic forces [171]. The classical approach to vibration isolation uses a passive system of springs, which resist the movement of the vibration because it exerts an opposing force proportional to its displacement, and a damper which removes kinetic energy and dissipates it as heat. The dynamic stiffness of an isolator spring should be as low as possible in order to increase the region of vibration isolation. However, if a linear spring is used a low stiffness isolator can cause unacceptably large static deflection. An elastic support vibration isolator [78-79] effectively reduces the transmission of vibrations from the machine to the ground, but it also has drawbacks. First, the machine itself still vibrates, thus hindering accuracy and reducing its life [78]. Second, isolator itself
tends to induce vibrations of large amplitude [79], and finally, the elastic support may exert an exciting force at the resonant frequency of the system [78]. Both passive absorbers and isolators have been optimised for use in a given frequency range. Previous studies have investigated various objective functions and determined optimal values for stiffness and damping in a vibration reduction system [72, 172-174]. In a conventional mass-spring system, static deflection increases as the stiffness of the support is reduced, and a lower limit on the stiffness is imposed on the allowable displacement by constraints. An example of a system with local regions of zero stiffness is a parallel connection of vertical and inclined springs [175-178].

There are three types of potential vibration isolators: passive, semi-active and active. Passive isolators featuring elastic materials and hydraulic oils provide design simplicity and cost effectiveness but their performance limitations are inevitable. On the other hand, active isolators may provide high control performance over a wide frequency range but they require high power sources and complex configurations. Recently, in an effort to solve the disadvantages of active isolators, semi-active isolators have been introduced.

An isolator designed in one field can usually be applied in any of the other fields. In the majority of cases, the base is flexible and vibrates with an unpredictable waveform. The model of a single degree of freedom is often used to present vibration isolation, which consists of a linear stiffness spring in parallel to a damper [179-180]. Vibration attenuation is obtained when the excited frequencies are greater than $\sqrt{2}$ times the natural frequency of the isolation system. For excited frequencies below $\sqrt{2}$ times the natural frequency, especially those close to the natural frequency, the vibration level of the isolated equipment is actually amplified compared to that of the base. So it is very evident that isolating vibration over a wider frequency range requires the system to have small stiffness to get a lower natural frequency. A conventional isolation system, which consists of spring and damper, that incorporates a passive, negative stiffness mechanism can achieve a quasi zero stiffness
characteristic while still keeping a high static stiffness [181]. Carrella et al. [182] proposed a passive high static low dynamic stiffness isolator that combined mechanical and magnetic springs where the negative stiffness is obtained by arranging the magnets in an attracting configuration.

Mizuno [183] proposed a new vibration isolation system which uses zero-power magnetic suspension. Since a zero-power system behaves as if it has a negative stiffness, combining it with a normal spring can generate infinite stiffness against disturbances on the isolation table. It enables the system to perform well in reducing vibration transmitted from ground and in suppressing direct vibration. The principles of the proposed vibration system are as follows, it is assumed there is a serial spring consisting of a normal spring and a spring whose stiffness is negative. If the absolute value of the stiffness of the normal spring is equal to the negative spring, the total stiffness of the serial spring becomes infinite and therefore, even if direct disturbance acts on the isolation table, the table has no steady-state displacement. It seems that this suspension system has infinite stiffness. Moreover, if the absolute value of the stiffness of each spring is low enough, the isolation table is isolated from ground vibration very well. That enables the system to have good characteristics in isolation from ground vibration and suppression of the effects of direct disturbance. Mizuno et al. [184] proposed a new vibration isolation system using negative stiffness realized by active control technique.

Carrella explored the design of an high-static-low-dynamic stiffness (HSLDS) isolator where positive stiffness from linear springs is counteracted by negative stiffness from attracting magnets. The main advantage of the HSLDS system is its load bearing capability [182].

Choi et al. [185] presented an experimental and theoretical analysis of a vibration isolation system using an MR fluid based semi-active isolator. A nonlinear hysteresis model with simplicity in form is proposed to describe the hysteresis force characteristics of the MR isolator. Liao et al. [186] developed a tuneable stiffness and damping vibration isolator based on magnetorheological elastomers. In this
isolator, four MRE elements are used as tuneable springs, whose stiffness can be controlled by varying the magnetic field. A voice coil motor, which is controlled by the relative velocity feedback of the payload, is used as the tuneable damper of the isolator. The experimental results indicate that the responses of the payload are suppressed significantly in comparison to a passive system. Zhou [187] developed a novel vibration isolator that possessed the characteristics of high-static-low-dynamic stiffness (HSLDS) and can act passively or semi-actively. The HSLDS property is obtained by connecting a mechanical spring, in parallel with a magnetic spring that is constructed by a pair of electromagnets and a permanent magnet.

Platus et al. presented a negative stiffness mechanism (NSM) [188-189]. It consists of two bars which have a length L. The bars are hinged at the centre and supported at their outer ends on pivots which are free to move horizontally. They are loaded in compression by an opposing force P, and the motions are constrained in the plane of the paper. The bars are aligned and in a state of unstable equilibrium where the centre hinge is displaced downwards an amount s and is held in equilibrium by a force $F_N$ which opposes the displacement, the angle of the bar to the horizon is $\theta$.

A typical vertical motion isolator uses a conventional spring to support the weight of the payload and a negative-stiffness mechanism (NSM) to cancel some or all of the stiffness of the spring, thereby producing low or zero net vertical stiffness. It has been used successfully to simulate zero gravity while testing large space structures [190]. A unique characteristic of the zero-power control system is that it behaves as if it has a negative stiffness. When an external force is applied to the mass in a mass-spring system, the mass moves to the direction of the applied force. In a zero-power controlled system, the suspended object moves to a new equilibrium position located in the direction opposite to the applied force [183]. Platus [188] exploited the beams under axial load in a specific configuration to achieve a negative stiffness in combination with a positive stiffness, and hence low-dynamic stiffness. This chapter presents the development of a positive-negative stiffness isolator. The isolator uses an MR elastomer and magnetic force to achieve a variable stiffness. The variable stiffness isolator is able to work over a relatively wide range of excited
frequencies. A mathematical model of the isolator was derived, and a prototype of the positive-negative stiffness has been fabricated. The test rig was setup and the transmissibility of the system under different current intensities was tested. The dynamic characteristics of this isolator under different current intensities were simulated with Matlab. Both the experimental and simulated results show that the system’s natural frequency increases when a positive current is applied, and decreases when a negative current is applied. The simulation results also demonstrated that the positive-negative isolator can efficiently suppress vibration after tuning the current of coils.

5.2 Fabrication and dynamic properties of MRE samples

Other MRE samples with iron particles mass fraction 80% were fabricated. The mass fraction ratio of iron, silicone oil, and silicon rubber in the mixture was 8:1:1. The dynamic properties were also tested and compared to the MRE with the 70% iron particles mass fraction which was fabricated in Chapter 4. Table 5.1 shows the mass fraction of the ingredients of the MRE samples.

<table>
<thead>
<tr>
<th></th>
<th>iron</th>
<th>Silicone oil</th>
<th>Silicone sealant</th>
</tr>
</thead>
<tbody>
<tr>
<td>MRE samples</td>
<td>80 %</td>
<td>10 %</td>
<td>10 %</td>
</tr>
</tbody>
</table>

Table 5.1: The mass fraction of the ingredients of the MRE samples

Similar testing was applied to the MRE sample2. Figure 5.1 shows the relationship between $G'$, $G''$ and the strength of the magnetic field.
Figure 5.1: The storage modulus and loss modulus of the MRE samples

The storage modulus increased from 0.223×10^5 Pa to 1.27×10^5 Pa 4.695 when the magnetic field increased from 0 to 220 mT. The loss modulus increased from 0.657×10^4 to 2.93×10^4 at the same time.
In the same way, the complex modulus increased from $0.233 \times 10^5$ Pa to $1.30 \times 10^5$ Pa and was plotted in Figure 5.2.
Figure 5.3 shows that the damping factor of MRE samples increased steadily when the strength of magnetic field increased.

Figure 5.2 shows that the complex modulus |G*| of the MRE samples increased dramatically when the strength of the magnetic field increased from 0~220 mT because this 80% iron MRE sample has a higher mass fraction ratio iron than the 70% iron MRE sample fabricated in Chapter 4.

When the strength of the magnetic field was less than 120 mT, it showed that the 80% MRE had a lower complex modulus than the 70% iron MRE, which means 80% has a relatively low stiffness. For a vibration isolator, it is good to have a
relatively low natural frequency to mitigate vibration over a wide frequency, which is why the 80% MRE is better for vibration isolator.

5.3 The working principle of magnetic negative stiffness

A schematic of magnetic negative stiffness is shown in Figure 5.4. The plate with a mass of \( m \) is made of steel. The two electromagnets are arranged to generate an attracting force on the mass acting as a negative stiffness spring. The upper one is denoted as electromagnet 1 (EM1) and the lower one is denoted as electromagnet (EM2)

![Schematic of a magnetic negative stiffness system](image)

**Figure 5.4**: Schematic of a magnetic negative stiffness system
The attracting force of the mass is varied by tuning the current intensity, which therefore generates a negative stiffness in the isolator. If the finite reluctance of the iron is neglected, the flux density is controlled by the current in the coil through

\[ B = \frac{\mu_0 \times N \times I}{2 \times g} \]  

(5.1)

When a magnetic field is applied to the middle mass, the energy stored in the air gaps between the iron core and mass plate is:

\[ W = \frac{1}{2} B \times H \times V = \frac{1}{2} B \times H \times (2 \times S \times g) \]  

(5.2)

Where \( B \) is the magnetic flux density, \( H \) is the magnetic field, \( V \) is the volume of the air gap between the magnetic poles and the middle mass, \( S \) is the surface area of the air gap and \( g \) is the initial size of each gap. The magnetic force applied onto the middle mass is [191]:

\[ F_0 = \frac{dW}{dg} = B \times H \times S = \frac{B^2 \times S}{\mu_0} = \frac{\mu_0 \times N^2 \times I^2 \times S}{4 \times g^2} \]  

(5.3)

Where \( \mu_0 = 4\pi \times 10^{-7} \) N·A\(^2\)\ is the magnetic field constant of vacuum, \( N \) is the winding number and \( I \) is the current in the coil. When the middle mass moves relative to electromagnet 1 or electromagnet 2, assuming that the relative displacement is \( x \), the magnetic force from electromagnet 1 is:

\[ F_1 = \frac{\mu_0 \times N^2 \times I^2 \times S}{4 \times (g - x)^2} = \frac{F_0 \times g^2}{(g - x)^2} \]  

(5.4)
when $x \ll g$, the equation above can be simplified as:

$$F_1 = F_0(1 + \frac{2}{g}x + \frac{3}{g^2}x^2)$$

(5.5)

The magnetic force from electromagnet 2 can also be expressed as:

$$F_2 = F_0(1 - \frac{2}{g}x + \frac{3}{g^2}x')$$

(5.6)

Thus, the net magnetic force applied on the middle mass with the displacement of $x$ is simplified as

$$\Delta F = F_2 - F_1 = -\frac{4}{g}F_0x$$

(5.7)

The equation above shows that the net magnetic force $\Delta F$ can be considered as a negative spring with a negative stiffness of $-4F_0/g$

### 5.4 The positive-negative stiffness vibration isolator

#### 5.4.1 The design of positive-negative stiffness vibration isolator
Figure 5.5: A schematic of the developed positive-negative stiffness vibration isolator

The schematic of the positive-negative stiffness vibration isolator is shown in Figure 5.5 and the 3D structure is shown in Figure 5.6. The vibration isolator consists of a top mass, a middle mass, a base, three electromagnets, two MREs, two connecting plates, eight ball bearings, and four guide pins. The base plate, which is made of aluminium, is connected to a shaker. The magnetic field applied to the MRE is generated by electromagnet (EM) 3, which the stiffness can increase. The middle mass is connected to the top mass through a connecting plate. Four guide pins are used to make sure that the top mass and middle mass move smoothly in a vertical direction.
Two springs are arranged between the top plate and middle plate so that the whole stiffness is $2.0 \times 10^3$ N/m and two pieces of 80% MRE were chosen with dimension 25 mm×25 mm×5 mm (Length×Width×Thickness).

The stiffness of the isolator depends partly on the MREs field-dependent modulus:

$$k = GA/h$$  \hspace{1cm} (5.8)
G is the MRE shear modulus, and A and h are the section area and thickness of MREs, respectively. The positive-stiffness of the isolator can be adjusted through different magnetic flux intensities. According to the MRE dynamic properties in Figure 5.2 and equation (5.8), we can get an equivalent stiffness of the MRE under different strength magnetic fields, as shown in Table 5.2.

<table>
<thead>
<tr>
<th>Magnetic field intensity (mT)</th>
<th>0</th>
<th>125</th>
<th>200</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equivalent stiffness (N/m)</td>
<td>1.35E3</td>
<td>3E3</td>
<td>6.75E3</td>
</tr>
</tbody>
</table>

Table 5.3: The parameters of the vibration isolator for test

<table>
<thead>
<tr>
<th>Top mass (kg)</th>
<th>M₂</th>
<th>1.7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stiffness (N/m)</td>
<td>K₂</td>
<td>2.0E3</td>
</tr>
<tr>
<td>Damping Coefficient (N s/m)</td>
<td>C₂</td>
<td>0</td>
</tr>
<tr>
<td>Middle mass (kg)</td>
<td>M₁</td>
<td>0.8</td>
</tr>
<tr>
<td>Stiffness of MRE (N/m)</td>
<td>K₁</td>
<td>1.35E3</td>
</tr>
<tr>
<td>Damping Coefficient (N s/m)</td>
<td>C₁</td>
<td>30</td>
</tr>
<tr>
<td>Magnetic force (N)</td>
<td>F₀</td>
<td>0</td>
</tr>
<tr>
<td>Winding Turns of EM 1</td>
<td>N₁</td>
<td>330</td>
</tr>
<tr>
<td>Winding Turns of EM 2</td>
<td>N₂</td>
<td>180</td>
</tr>
</tbody>
</table>
CHAPTER 5. DESIGN AND DEVELOPMENT OF POSITIVE-NEGATIVE STIFFNESS VIBRATION ISOLATOR

<table>
<thead>
<tr>
<th>Winding Turns of EM 3</th>
<th>$N_3$</th>
<th>180</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gaps of air (mm)</td>
<td>$g$</td>
<td>4</td>
</tr>
</tbody>
</table>

Table 5.3 lists the initial parameters of the proposed vibration isolator. When the winding turns of EM 1 N is 330 and the g is 5 mm, the relationship between the magnetic field strength and current intensity can be obtained, shown in Figure 5.7. It was measured by Bell-5180 Hall Effect Gauss / Tesla Meters.

![Magnetic field strength B (mT) vs Current I (A) graph]

**Figure 5.7**: The relationship between the strength of the magnetic field and current intensity
5.4.2 Mathematical model

The designed positive-negative stiffness isolator was modelled as two mass-spring-damper system, and the physical model of the isolator is shown in Figure 5.8. In this model $k_1$ and $k_2$ are the stiffness of spring and magnetorheological (MR) elastomer respectively, $c_1$ is the structure damping factor and $c_2$ is the damping coefficient of the MR elastomer. The motion equation of the isolator is represented by:

\[
\Delta F = F_2 - F_1 = -\frac{4}{g} F_0 x = -\frac{4}{g} F_0 (x_i - x_f)
\]

**Figure 5.8**: Mathematical model of the positive-negative stiffness isolator
\[
\begin{align*}
\begin{cases}
m_1\dddot{x}_1 + c_2(\dddot{x}_1 - \ddot{x}_2) + k_2(x_1 - x_2) + c_1(\dddot{x}_1 - \ddot{x}_j) + k_1(x_1 - x_j) = -\Delta F = \frac{4}{g} F_0(x_1 - x_j) \\
m_2\dddot{x}_2 + c_2(\dddot{x}_2 - \ddot{x}_1) + k_2(x_2 - x_1) = -F_1 = -F_0 \left[ 1 + \frac{2}{g} (x_2 - x_1) \right]
\end{cases}
\end{align*}
\]

Where \( x_j \) is the input and \( x_1 \) and \( x_2 \) are the displacements of \( m_1 \) and \( m_2 \). In this design, assume \( F_0 \) is 0. Thus the equations (1) and (2) can be simplified as:

\[
\begin{align*}
\begin{cases}
m_1\dddot{x}_1 + c_2(\dddot{x}_1 - \ddot{x}_2) + k_2(x_1 - x_2) + c_1(\dddot{x}_1 - \ddot{x}_j) + k_1(x_1 - x_j) = 0 \\
m_2\dddot{x}_2 + c_2(\dddot{x}_2 - \ddot{x}_1) + k_2(x_2 - x_1) = -F_1 = 0
\end{cases}
\end{align*}
\]

and the transmissibility of input \( x_j \) and output \( x_2 \)

\[
\eta_2 = \frac{X_2}{X_f}
\]

\[
X_2 = \frac{(s c_1 + k_1)(s c_2 + k_2)}{(m_2 s^2 + c_2 s + k_2)(m_1 s^2 + c_2 s + k_2 + c_1 s + k_1) - (s c_2 + k_2)^2}
\]

= 

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\[
c, c_s s^2 + (k, c_2 + k, c_i) s + k, k_2
\]
\[
m, m_2 s^2 + (m, c_2 + m, c_i + m, c_2) s^2 + (m, k_2 + m, k_i + c, c_2 + m, k_i) s^2 + (k, c_2 + k, c_i) s + k, k_2
\]

\[
= \frac{b_2 s^2 + b_1 s + b_0}{a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0}
\]

(5.13)

Where \( X_f \) and \( X_2 \) are the Laplace transform of displacement \( x_f \) and \( x_2 \)

\[
\begin{align*}
  b_2 &= c, c_2 \\
  b_1 &= k, c_2 + k, c_i \\
  b_0 &= k, k_2 \\
  a_4 &= m, m_2 \\
  a_3 &= m_2 (c_2 + c_1) + m, c_2 \\
  a_2 &= m_2 (k_2 + k_1) + m, k_2 + c, c_2 \\
  a_1 &= k_1 c_2 + k, c_i \\
  a_0 &= k, k_2
\end{align*}
\]

\[
\begin{align*}
  \left| \frac{X_2}{X_f} \right| &= \left| \frac{(sc_1 + k_1)(sc_2 + k_2)}{(m_2 s^2 + c_2 s + k_2)(m_1 s^2 + c_2 s + k_2 + c_1 s + k_1) - (sc_2 + k_2)^2}_{s=j\omega} \right| \\
  &= \left| \frac{b_2 s^2 + b_1 s + b_0}{a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0} \right|_{s=j\omega}
\end{align*}
\]

105
\[
\begin{align*}
\omega & = \frac{b_0 - b_2 \omega^2 + b_1 \omega j}{a_4 \omega^4 - a_2 \omega^2 + a_0 + (a_1 \omega - a_3 \omega^3) j} \\
& = \sqrt{\frac{(b_0 - b_2 \omega^2)^2 + (b_1 \omega)^2}{(a_4 \omega^4 - a_2 \omega^2 + a_0)^2 + (a_1 \omega - a_3 \omega^3)^2}} \\
& \quad \text{(5.14)}
\end{align*}
\]

### 5.5 Numerical simulation results of positive-negative stiffness isolator

In order to get the simulation results, a Simulink model was built according to the equation of motion of the isolator. The chirp signal was set as the vibration input of the base with an initial frequency of 0Hz, a target time of 20s, and the frequency at target time was 20Hz. After the Fast Fourier Transform of the input of the base and output of the top mass, the transmissibility of the isolator was obtained at different frequencies. Different parameters were set when a positive current I+ and negative current I- were applied.

Using a control method, the smallest level of vibration can be obtained. When the positive current intensity I+ is applied, the transmissibility peak occurs at a high frequency which is denoted as \( f_h \). When a negative current intensity I- is applied, the transmissibility peak occurs at a relatively low frequency denoted as \( f_l \). When the excited frequency is greater than \( f_h \), the current intensity I should be 0. When the excited frequency is less than \( f_l \), the positive current intensity I+ should be applied. When the excited frequency is between \( f_l \) and \( f_h \), the vibration level of isolator should be tuned by the EM based on the transmissibility curve.

The parameters for simulation are set as Table 5.4. The Matlab program was developed to simulate the vibration isolator. The Chirp signal is used as input whose initial frequency was 0 Hz, Target time was 20 seconds, and the frequency at target
time was 20 Hz. Three sets of parameters were set to test the dynamic response of the vibration isolator. The yellow line in Figure 5.9, Figure 5.11, and Figure 5.13 is the input chirp signal whose amplitude is 1 and the cyan line is the response of the top mass.

The transmissibility of the positive-negative stiffness vibration isolator under different currents is shown in Figure 5.10, Figure 5.12, and Figure 5.14. When the current was 0, which means no power was applied to the vibration isolator, the response of the top mass gradually increased firstly, but then decreased after 5.5 seconds compared to the input signal, which can be seen in Figure 5.9. The maximum response happened at 4.4 seconds when the chirp signal frequency was 4.4 Hz, and where the peak transmissibility of the positive-negative stiffness occurred, as shown in Figure 5.10. Similarly, when the positive current $I_+ = 3$A, a magnetic field was applied onto the MR elastomer, the response of the top mass also increased, but then decreased after 7.5 seconds compared to the input signal. It is noted from Figure 5.11 that the maximum response happened at 5.8 seconds when the chirp signal frequency was 5.8 Hz, and where the transmissibility of the positive-negative stiffness curve was at its maximum, as shown in Figure 5.12.
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Table 5.4: Parameters of simulated vibration isolator

<table>
<thead>
<tr>
<th></th>
<th>Top mass kg</th>
<th>Stiffness N/m</th>
<th>Damping Coefficient N s/m</th>
<th>Middle mass /kg</th>
<th>Stiffness of MRE N/m</th>
<th>Damping Coefficient N s/m</th>
<th>Magnetic force N</th>
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<td>M₂</td>
<td>1.7</td>
<td>2.0e3</td>
<td>0</td>
<td>0.8</td>
<td>1.35e3 (0mT)</td>
<td>30</td>
<td>0</td>
</tr>
<tr>
<td>(1)</td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>(2)</td>
<td></td>
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<tr>
<td>(3)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1.35e3</td>
<td>30</td>
<td>1.5</td>
</tr>
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For data from set (1)

Figure 5.9: The dynamic response of positive-negative stiffness vibration isolator when I=0A

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Figure 5.10: The transmissibility of positive-negative stiffness vibration isolator when $I=0A$

For data from set (2):
Figure 5.11: The dynamic response of positive-negative stiffness vibration isolator when \( I^+ = 3A \)

Figure 5.12: The transmissibility of positive-negative stiffness vibration isolator when \( I^+ = 3A \)
For data from set (3),

**Figure 5.13:** The dynamic response of the positive-negative stiffness vibration isolator when $I=2A$
When a negative current $I_{-}=2A$ was applied on electromagnet 1 and electromagnet 2, the magnetic force applied onto the middle mass works like a negative spring, the dynamic response of the top mass was maximum until 2.2 seconds then kept decreasing for 5 seconds. It remained stable between 5 and 10 seconds, as shown in Figure 5.13. At 2.2 seconds the response of top mass peaked when the chirp signal frequency was 2.2 Hz and where the transmissibility of the positive-negative stiffness curve was maximum, as illustrated in Figure 5.14.

The simulated transmissibility of the isolator based on the equation of motion under different current intensities is shown in Figure 5.15. As can be seen, the natural frequency increased from 4.4 Hz to 5.8Hz when the current intensity in
electromagnet 3 changed from 0 A to 3 A, and the current intensity in both EM 1 and 2 was 0 A. However, the natural frequency decreased from 4.4 Hz to 2.2 Hz when the current intensity in both EM 1 and 2 was 2 A, which in EM 3 was 0 A.

**Figure 5.15**: Simulated results of transmissibility of the developed vibration isolator
Figure 5.16: Simulated results of transmissibility of the developed vibration isolator with control method

Figure 5.16 shows the simulated transmissibility of the isolator after tuning the current intensity. It can be seen that the current tuning guarantees that the isolator maintains its best performance.

5.6 Experimental Investigation of Positive-Negative Stiffness Vibration Isolator
5.6.1 Experimental setup

The schematic of the experimental setup is shown in Figure 5.17. The base of the isolator is connected to a shaker (Vibration Test System, AURORA, Model No.: VG 100-8), which is driven by a signal source from the power amplifier (Crown D-150A) whose signal is provided by the Data Acquisition (DAQ) board (Type: LabVIEW PCI-6221, National Instruments Corporation U.S.A) and a computer. Two accelerometers (CA-YD-106 SINOCERA Piezotronics, Inc.) are used to measure the vibration of the base and top aluminium plate. The signals are amplified by Charge amplifier (YE5851A from SINOCERA Piezotronics, Inc) and processed by DAQ board and computer. Two DC power supplies (GW INSTEK GPC-3030D, GW GPR-3030D) were used to tune the currents of the coils and adjust the strength of the magnetic field of the electromagnets and stiffness of the MRE, and therefore the dynamic response of the isolator can be changed.

The testing program was designed in LabVIEW. The essential part of this system is the vibration package, which was used to generate the swept sine signals and display and record the test results for the analysis of the isolator. The program measures the frequency response of the device with a swept sine. The start frequency and stop frequency determine the frequency range of the measurement.

In this experiment, the start frequency was 0.2 Hz and the stop frequency was 20 Hz. The number of steps was set as 100, which determines the total number of test frequencies. The current intensity in the coil 1 is ranged from 0 to 3A, while in coils 2 and 3 it changed from 0 to 2A.

The negative stiffness of the isolator depends on the pair of electromagnets, which work like a negative stiffness spring when the magnetic poles are opposite. The different current intensity in coils 2 and 3 can adjust the negative-stiffness of the isolator.
5.6.2 Dynamic characteristics of positive-negative stiffness vibration isolator and comparison with simulated results

![Schematic of the experiment setup](image)

**Figure 5.17**: Schematic of the experiment setup

Figure 5.18 shows the experimental and simulated transmissibility of the positive-negative stiffness vibration isolator at various current intensities. From these figures, the natural frequency of the isolator changed from 2.6 Hz, when the current intensity in coil 2 and 3 was 2 A respectively, to 5.2 Hz when the current intensity was 3 A in coil 1 (apply magnetic field strength to MRE). The relative change in frequency of this isolator was about 100% when the stiffness changed from a
negative state to a positive state. This shows that the experimental results agreed with the simulated results.

![Graph](image)

**Figure 5.18**: The experimental and simulated results of the transmissibility of isolator under different currents

The peak transmissibility of the isolator, when there was no external current intensity applied between the experimental and simulated results were slightly different because the vibration isolator proposed is a two level model (two degrees of freedom model) so the transmissibility is related to many parameters, which can be
seen from the equations of motion of the vibration isolator. During the experiment, there were probably some factors that affected the results. For example, It was hard to enable get the vibration isolator to move in a vertical direction, and moreover, the amplitude of force from the shaker may have been influenced and friction between the different parts of the vibration isolator probably affected the test results as well.

5.7 Conclusions

In this chapter, MR elastomers with iron particles mass fraction of 80% were fabricated and the storage modulus and loss modulus were tested under different strength magnetic fields. A positive-negative stiffness vibration isolator was designed and manufactured, which can be controlled by adjusting the current intensities of the MR elastomers and the pair of electromagnets A mathematical model of the system was developed based on the isolator. The experiment was set up and the dynamic performance of the isolator under positive current intensity I+ and negative current intensities I− was studied experimentally. The results showed that the natural frequency of the isolator changed from 2.6 Hz, when the current intensity in EM coil 2 and EM coil 3 were 2 A respectively, to 5.2 Hz when the current intensity is 3A in EM coil 1. The relative change in frequency of the isolator was about 100% when the stiffness changed from a negative state to a positive state. In addition, the isolator was investigated numerically and the results agreed well with the experimental data, which demonstrated that the natural frequency of the isolator can be changed over a relatively wide frequency range. Furthermore, a simple tuning method was proposed to adjust the current intensity of the isolator to ensure the best performance when the excitation frequency varied.
6.1 Conclusions

In summary, the following conclusions can be drawn from this thesis:

Two kinds of MREs based on silicone rubber and carbonyl iron particles were fabricated and the dynamic performances of these MREs under different strength of magnetic fields were tested using a Rheometer. It is noted that the storage modulus and loss modulus increased if the magnetic field (current intensity) increased, but the storage modulus and loss modulus only increased very slowly when the intensity of the magnetic field was below 1000 Gs. This showed that the complex modulus |G*| of MRE 2, whose mass fraction ratio of iron was 80%, increased dramatically compared to MRE 1, whose mass fraction of iron was 70%, when the magnetic field strength increased from 0~220 mT.

The theoretical analysis of a sandwich structure was extended and derived the differential equations of an MRE sandwich beam based on the M&M method and the dynamic response of the sandwich structure was based on different boundary conditions and different localised magnetic fields. The numerical results show that different MR performances can be obtained depending on the location of the magnetic fields applied. The first natural frequency of the MRE cantilever sandwich beam increased when the area of localised magnetic field gradually increased from the clamped end to full coverage of the beam. The first natural frequency of the MRE clamped-clamped sandwich beam increased when the area of localised magnetic field increased from one end to full coverage of the beam, although the first natural frequency could decrease when the external magnetic field moves from the clamped end to the middle of the beam.

The dynamic response of a cantilevered sandwich beam with an MRE core and an MRF core under a non-homogeneous magnetic field were investigated
experimentally. The results reveal the possibility of shifting the natural frequency to a lower frequency by applying a small non-homogeneous magnetic field by partially activating a region of the sandwich beam. It has been observed that the first natural frequency of both the MR Elastomer and MR Fluid sandwich beam decreased when a single group magnet moved from the clamped end to the free end, and also the first natural frequency decreased when the magnetic field increased in a particular location.

Finally, a Positive-Negative stiffness vibration isolator based on MRE and a magnetic force was designed and manufactured. The MREs implanted into the proposed vibration isolator worked in shear mode and the strength of the magnetic field was controlled by a coil and DC power supplies. Two electromagnets were arranged to generate a magnetic force, which worked like a negative spring. This magnetic force was controlled by a pair of coils and a DC power supply. The equations of motion based on the positive-negative vibration isolator were derived and the transmissibility of the vibration isolator was calculated under different frequencies. Numerical simulations were used to validate the performance of the proposed Positive-Negative Stiffness Vibration Isolator, and also the transmissibility of the vibration isolator was obtained under different current intensities.

### 6.2 Future Research Work

There are some areas for further experimental and theoretical investigations that need to be addressed, such as:

(a). A controller to control the Positive-Negative stiffness vibration isolator needs to be designed;

(b). The parameter of the Positive-Negative stiffness vibration isolator needs to be optimised according to the actual requirements in order to get the best performance.
(c). An experimental set up for a positive-negative vibration isolator needs to be conducted, including the control algorithm.

(d). The frequency dependent magnetic field preload effect on magnetorheological elastomer core of sandwich beam should be carried out based on M&M theory.

(e). The dynamic characterisation of the magnetic circuit of two electromagnets under different exciting forces needs to be simulated.
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REFERENCES


APPENDIX A

THE PARAMETERS OF THE PERMANENT MAGNETS

Neodymium Block Magnets

Length: 50mm (1.968" in.)
Width: 50mm (1.968" in.)
Height: 5mm (0.196" in.)
Grade: N40SH
Coating: Nickel (Ni)
Magnetised Direction: Through height – 5mm
Theoretical Holding Force: ~ 16.965kgs
Surface Gauss: 6356.64 Gauss