2013

Implications of PVD and vacuum preloading on viscoplastic behaviour of soft soils

Kourosh Kianfar

University of Wollongong, kk975@uowmail.edu.au

Recommended Citation

UNIVERSITY OF WOLLONGONG

COPYRIGHT WARNING

You may print or download ONE copy of this document for the purpose of your own research or study. The University does not authorise you to copy, communicate or otherwise make available electronically to any other person any copyright material contained on this site. You are reminded of the following:

Copyright owners are entitled to take legal action against persons who infringe their copyright. A reproduction of material that is protected by copyright may be a copyright infringement. A court may impose penalties and award damages in relation to offences and infringements relating to copyright material. Higher penalties may apply, and higher damages may be awarded, for offences and infringements involving the conversion of material into digital or electronic form.
Department of Civil, Mining and Environmental Engineering

Implications of PVD and Vacuum Preloading on Viscoplastic Behaviour of Soft Soils

by:

Kourosh Kianfar, BSc, Msc

This thesis is presented as part of the requirement for the award of the degree of Doctor of Philosophy from University of Wollongong, NSW Australia

May 2013
CERTIFICATION

I, Kourosh Kianfar, declare that this thesis, submitted in fulfillment of the requirement for the award of doctor of Philosophy in the Department of Civil Engineering at the University of Wollongong, is wholly my own work unless otherwise referenced or acknowledged. The document has not been submitted for qualifications at any other academic institution.

Kourosh Kianfar
May 2013
LIST OF PUBLICATION


Kianfar, K., Indraratna, B., Rujikiatkamjorn, C., “Radial Consolidation Model Incorporating the Effects of Vacuum Preloading and Non-Darcian Flow”, Submitted to Geotechnique and accepted for publication.

ABSTRACT

Vacuum assisted radial consolidation has been increasingly used to stabilise soft clays worldwide. The advantages of the combined vacuum-surcharge pressure over the conventional surcharge alone such as reduction of the embankment height, increasing the rate of consolidation, and increasing the factor of safety of the embankments against failure are appreciated however they are not precisely addressed in the literature. In this research a comprehensive laboratory study was conducted to capture the effects of combined vacuum-surcharge pressure during radial consolidation. The modification of a Rowe cell was initiated at the University of Wollongong to monitor lateral effects of vacuum pressure on excess pore water pressure and associated effective stresses. The new apparatus was then employed to develop a radial consolidation model incorporating the effects of vacuum pressure and non-linear flow relationship via flow velocity-hydraulic gradient relationship during radial consolidation.

Based on the laboratory observation, degree of consolidation and back calculated coefficient of consolidation increase with increasing vacuum pressure-total surcharge ratio (VSR). Coefficients of consolidation back calculated from excess pore water pressure dissipation showed lower values than those back calculated from settlement data. Empirical formulations were then developed to calculate both these coefficients for varying VSR based on settlement determined from oedometer testing.

Decrease in excess pore water pressure across the soil due to the application of vacuum pressure inside the vertical drains is not immediate and its non-linear variation depends on the radial distance from the drain and elapse time after the application of vacuum. During laboratory studies in this research, it was shown that
the application of a combined vacuum-surcharge pressure results in a higher rate of excess pore pressure dissipation and settlement than the application of conventional surcharge alone. Higher gain in effective stresses and higher overconsolidation ratio were consequently observed after the removal of vacuum. Considering these effects, a design methodology was proposed incorporating VSR to minimise the removal of embankments which in turn reduces both the time and the cost of the projects and plays as a more environmentally friendly technique by reducing the extent of earthworks that is typically associated with surcharge-only embankments.

Using modified Rowe cell, new non-linear flow relationship was proposed during vacuum assisted radial consolidation eliminating the shortcomings of the conventional methods. The relationship was then used to develop a radial consolidation model incorporating the effects of vacuum pressure. It was shown that the proposed model can effectively be used for both laboratory and field conditions and its predictions provided better agreement with the measured data than the existing models.
ACKNOWLEDGEMENTS

I would like to express my gratitude to my supervisors Professor Buddhima Indraratna and Dr. Cholachat Rujikiatkamjorn. Throughout this research they have been a continuous source of guidance and inspiration, and a deep fountain of knowledge and wisdom upon which to draw. They have also been unfailing in their encouragement and belief in my ultimate success. Please accept my grateful and sincere thanks.

I would also like to thank Mr. Alan Grant, the senior technical officer at the University of Wollongong, and all the technical officers in the Civil Engineering Department. Through this research period, particularly the testing program and laboratory investigation they have been a constant source of friendly advice, and have provided me with very high standard of technical support. Thank you all.

I would also like to acknowledge the Australia Research Council for providing the scholarship for this research, and also the support received from Coffey Geotechnics, Douglas Partners, the Transport and Maritime Services (NSW), and the Queensland Department of Transport and Main Roads.

Heartfelt thanks are also offered to my mother and late father. Throughout my life they have been a constant source of encouragement, support, and sacrifice, without whom I would not have been able to extend my education this far. From my parents I learned the value and importance of knowledge, from them I learned to explore the unknown and reach towards a better, more fulfilling life.

Finally, and most importantly, I wish to express my deepest thanks and gratitude to my wife, Mahvash Kashani. Her unfailing love, support, encouragement and sacrifice, have proven to be my primary source of strength throughout these studies. I wholeheartedly dedicate this work to my beloved wife, my partner, the one who has always been beside me during the ups and downs of my life.
TABLE OF CONTENTS

CERTIFICATION ........................................................................................................ ii
LIST OF PUBLICATION ........................................................................................... iii
ABSTRACT ................................................................................................................ iv
ACKNOWLEDGEMENTS ............................................................................................. vi
TABLE OF CONTENTS ............................................................................................ vii
LIST OF FIGURES ..................................................................................................... x
LIST OF TABLES ....................................................................................................... xvi
LIST OF NOTATIONS ............................................................................................... xvii

1 INTRODUCTION ........................................................................................................ 1
  1.1 Objectives of the Research .................................................................................. 1
  1.2 Background ......................................................................................................... 2
  1.3 Significance of the Research .............................................................................. 3
  1.4 Organisation of the Thesis .................................................................................. 5

2 LITERATURE REVIEW ............................................................................................ 8
  2.1 General ................................................................................................................. 8
  2.2 Deformation of Clay .......................................................................................... 10
    2.2.1 Consolidation Settlement ............................................................................. 10
    2.2.2 Time-Dependent Behavior of Clays .......................................................... 14
  2.3 Wick Drains and Horizontal Consolidation ...................................................... 25
    2.3.1 Governing Equations for Radial Consolidation ........................................ 25
    2.3.2 Prefabricated Band Drains .......................................................................... 29
    2.3.3 Smear Zone, Horizontal Permeability, and Radial Consolidation ........... 34
    2.3.4 Discharge Capacity of the Drains ............................................................... 40
    2.3.5 Settlement due to the Radial Consolidation .............................................. 43
    2.3.6 Excess Pore Water Pressure ..................................................................... 44
    2.3.7 Lateral Displacement .................................................................................. 46
    2.3.8 Effect of Vacuum Preloading .................................................................... 47
  2.4 Reliability of the Soil Parameters Used in the Existing Consolidation Models and Flow Relationship .......................................................... 52
  2.5 Implication of Non-Darcian Flow .................................................................... 53
  2.6 Summary ............................................................................................................. 57

3 EXPERIMENTAL PROCEDURE ............................................................................ 59
<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.1</td>
<td>General</td>
<td>59</td>
</tr>
<tr>
<td>3.2</td>
<td>Standard Rowe Cells</td>
<td>60</td>
</tr>
<tr>
<td>3.2.1</td>
<td>History and Advancement of Rowe Cells</td>
<td>60</td>
</tr>
<tr>
<td>3.2.2</td>
<td>Horizontal Drainage During Consolidation</td>
<td>64</td>
</tr>
<tr>
<td>3.2.3</td>
<td>Salient Features and Advantages of the Rowe Cell</td>
<td>65</td>
</tr>
<tr>
<td>3.2.4</td>
<td>Types of Tests</td>
<td>67</td>
</tr>
<tr>
<td>3.3</td>
<td>Effects of the Drainage Material</td>
<td>70</td>
</tr>
<tr>
<td>3.4</td>
<td>Modified 150 mm Diameter Rowe Cell</td>
<td>75</td>
</tr>
<tr>
<td>3.4.1</td>
<td>General Features of the Modified Cell</td>
<td>75</td>
</tr>
<tr>
<td>3.4.2</td>
<td>Capabilities of the Modified Cell</td>
<td>77</td>
</tr>
<tr>
<td>3.5</td>
<td>Hardware Used in the Rowe Consolidation Tests</td>
<td>79</td>
</tr>
<tr>
<td>3.5.1</td>
<td>GDS Pressure/Volume Controllers</td>
<td>80</td>
</tr>
<tr>
<td>3.5.2</td>
<td>GDS Data Logger</td>
<td>81</td>
</tr>
<tr>
<td>3.5.3</td>
<td>Pore Pressure and Displacement Transducers</td>
<td>82</td>
</tr>
<tr>
<td>3.5.4</td>
<td>Vacuum Pump, Gauges and Regulators</td>
<td>83</td>
</tr>
<tr>
<td>3.6</td>
<td>Software Used with the Experimental Investigations</td>
<td>84</td>
</tr>
<tr>
<td>3.7</td>
<td>Summary</td>
<td>87</td>
</tr>
<tr>
<td>4.1</td>
<td>General</td>
<td>88</td>
</tr>
<tr>
<td>4.2</td>
<td>Background</td>
<td>89</td>
</tr>
<tr>
<td>4.3</td>
<td>Location of Average Pore Water Pressure</td>
<td>91</td>
</tr>
<tr>
<td>4.4</td>
<td>Determination of Degree of Consolidation (DOC)</td>
<td>98</td>
</tr>
<tr>
<td>4.5</td>
<td>Determination of the Coefficient of Radial Consolidation (ch)</td>
<td>105</td>
</tr>
<tr>
<td>4.6</td>
<td>Model Validation</td>
<td>114</td>
</tr>
<tr>
<td>4.7</td>
<td>Practical Implications</td>
<td>115</td>
</tr>
<tr>
<td>4.8</td>
<td>Summary</td>
<td>118</td>
</tr>
<tr>
<td>5.1</td>
<td>General</td>
<td>120</td>
</tr>
<tr>
<td>5.2</td>
<td>Test Set Up and Procedure</td>
<td>122</td>
</tr>
<tr>
<td>5.3</td>
<td>Excess Pore Water Pressure Response</td>
<td>123</td>
</tr>
<tr>
<td>5.4</td>
<td>Changes in Effective Stress</td>
<td>137</td>
</tr>
</tbody>
</table>
LIST OF FIGURES

Figure 2.1: Consolidation settlement of a soil layer .................................................. 12
Figure 2.2: Creep test performed at a low stress level: (a) Stress-strain relationship; (b) Stress history; and (c) Strain history (after Augustesen et al., 2004) .......... 15
Figure 2.3: Definition of creep stages considering a creep test at constant stress performed in a triaxial apparatus: (a) Strain versus time and (b) log strain versus log time (after Augustesen et al., 2004) .................................................. 15
Figure 2.4: Definition of primary, secondary, and tertiary compression: (a) Strain versus log time and (b) log strain rate versus log time (after Augustesen et al., 2004) ................................................................. 16
Figure 2.5: Definition of secondary compression ..................................................... 18
Figure 2.6: Development of quasi-preconsolidation pressure due to secondary compression (after Bjerrum, 1967). ................................................................. 20
Figure 2.7: Illustration of instant time line, reference time line, limit time line, and time lines for positive and negative equivalent times ......................... 22
Figure 2.8: Relaxation test (A → B): (a) Stress-strain relationship; (b) strain history; (c) stress history ........................................................................................................... 24
Figure 2.9: Plan of Drain Well Pattern and Fundamental parameters of Each Well 28
Figure 2.10: Some different band drains - (a), (b), (c), and (d) drains with central core surrounded by filter sleeves, (e) and (f) drains without filter sleeve (after Moseley and Kirsch, 2004) ......................................................................................... 31
Figure 2.11: Plan of drain patterns and influence zone of each drain ..................... 32
Figure 2.12: Typical Mandrels for Prefabricated Vertical Drains (after Holtz et al., 1991) ..................................................................................................................... 33
Figure 2.13: Typical ratio of kh/kv versus radial distance from a centrally located prefabricated vertical Flodrain (after Indraratna and Redana, 1998) ........... 36
Figure 2.14: Variation of soil permeability around vertical drain (Onoue et al. 1991) ......................................................................................................................... 37
Figure 2.15: Vacuum-assisted preloading systems, (a) Membrane system, (b) Membrane-less system ......................................................................................... 49
Figure 2.16: Schematic diagram of vacuum preloading system, (a) Combined vacuum and surcharge load (b) Surcharge load (c) Vacuum load (after Mohamedelhassan and Shang, 2002)) ......................................................................................... 50
Figure 3.1: One of the first Rowe cells developed by Rowe and Barden (1966) ....... 61
Figure 3.2: Main features of the Rowe hydraulic consolidation cell ................. 62
Figure 3.3: Loading and drainage conditions for consolidation in Rowe cell(a), (c),
(e), and (g) “Free strain” loading,(b), (d), (f), and (h) “Equal strain” loading 69
Figure 3.4: Permeability tests in Rowe cell with different flow conditions,(a), (b)
vertical flow conditions,(c), (d) radial flow conditions ......................... 70
Figure 3.5: (a) Vylon discs, and (b) Sintered Bronze discs used as drainage material
at the base of Rowe cell .................................................................... 72
Figure 3.6: (a) 75 mm standard Rowe cell, (b) Location of pore pressure measurement
at the base of 75 mm Rowe cell, and (c) Hardware setup for consolidation tests
with 75 mm Rowe cell ...................................................................... 74
Figure 3.7: Pore pressure response of the transducers when Vylon or sintered Bronze
discs are inserted into the drainage hole at the base plate of the 75 mm Rowe cell
................................................................................................. 75
Figure 3.8: (a) Modified 150 mm Rowe cell at University of Wollongong, (b)
Assembled developed and standard 150 mm Rowe cells,(c) Comparison of the
cell body height of the developed and standard cells,(d) and (e) Location of pore
pressure measurement at the base of developed 150 mm Rowe cell ........ 78
Figure 3.9: Hardware used in Rowe consolidation tests (Soils and Marine Laboratory,
SMART Building, UOW) .................................................................. 80
Figure 3.10: GDS Enterprise Level Pressure/Volume Controller ...................... 81
Figure 3.11: 8-Channel GDS Data Logger .................................................. 82
Figure 3.12: (a) Pore Pressure Transducer, (b) Displacement Transducer ......... 83
Figure 3.13: Vector LD-5 Vacuum Pump .................................................... 84
Figure 3.14: Vacuum Gauge and Vacuum Regulator ................................... 84
Figure 3.15: (a) Different Test Types can be run by GDSLAV software, and (b) An
Example of a Live Graph and data Shown on Screen ............................ 86
Figure 4.1: Theoretical location of the average excess pore water pressure in a unit
cell based on Equation 4.3 .................................................................. 93
Figure 4.2: Modified 150 mm Rowe cell .................................................... 94
Figure 4.3: Lateral distributions of the excess pore pressure using the modified Rowe
cell, Test No. 1 ................................................................................ 97
Figure 4.4: Lateral distributions of the excess pore pressure using the modified Rowe
cell, Test No. 2 ................................................................................ 97
Figure 4.5: Lateral distributions of the excess pore pressure using the modified Rowe cell, Test No. 3

Figure 4.6: Comparison of the DOC based on measured settlements and pore pressures, Test No. 4, (b) Test No. 5

Figure 4.7: Comparison of the DOC based on measured settlements and pore pressures, Test No. 6, (b) Test No. 7

Figure 4.8: Comparison of the DOC based on measured settlements and pore pressures, Test No. 8, (b) Test No. 9

Figure 4.9: Degree of consolidation vs. time for different vacuum pressure-total surcharge ratios, (a) based on the settlement data, (b) based on the pore pressure data.

Figure 4.10: Predicted and measured DOC based on excess pore pressure dissipation, Test No. 4, (b) Test No. 5

Figure 4.11: Predicted and measured DOC based on excess pore pressure dissipation, Test No. 6, (b) Test No. 7

Figure 4.12: Predicted and measured DOC based on excess pore pressure dissipation, Test No. 8, (b) Test No. 9

Figure 4.13: Predicted and measured DOC based on settlement, Test No. 4, (b) Test No. 5

Figure 4.14: Predicted and measured DOC based on settlement, Test No. 6, (b) Test No. 7

Figure 4.15: Predicted and measured DOC based on settlement, Test No. 8, (b) Test No. 9

Figure 4.16: (a) chp and chs values, and (b) chs/chp for different VSR

Figure 4.17: Measured and predicted excess pore water pressure vs. time at r = 0.52R, (a) Test No. 10, and (b) Test No. 11

Figure 4.18: Measured and predicted degree of consolidation based on settlement data, (a) Test No. 10, and (b) Test No. 11

Figure 5.1: Excess pore water pressure distribution with time at different radii: (a) Test 1: removal of 50 kPa surcharge after 8.0 hours, (b) Test 2: removal of 50 kPa surcharge after 10.0 hours, (c) Test 3: removal of 50 kPa surcharge after 12.0 hours.
Figure 5.2: Excess pore water pressure distribution with time at different radii – first 20 hours: (a) Test 1: removal of 50 kPa surcharge after 8.0 hours, (b) Test 2: removal of 50 kPa surcharge after 10.0 hours, (c) Test 3: removal of 50 kPa surcharge after 12.0 hours.

Figure 5.3: Excess pore water pressure distribution with time at different radii: (a) Test 4: removal of 50 kPa vacuum after 8.0 hours, (b) Test 5: removal of 50 kPa vacuum after 10.0 hours, (c) Test 6: removal of 50 kPa vacuum after 12.0 hours.

Figure 5.4: Excess pore water pressure distribution with time at different radii – first 20 hours: (a) Test 4: removal of 50 kPa vacuum after 8.0 hours, (b) Test 5: removal of 50 kPa vacuum after 10.0 hours, (c) Test 6: removal of 50 kPa vacuum after 12.0 hours.

RR = (r-rw)/(R-rw)

Figure 5.5: Drop in excess pore water pressures at different locations for Tests 1-3 due to the removal of surcharge. Test 1: removal of 50 kPa surcharge after 8.0 hours, Test 2: removal of 50 kPa surcharge after 10.0 hours, Test 3: removal of 50 kPa surcharge after 12.0 hours.

Figure 5.6: Increase in excess pore water pressures at different locations for Tests 4-6 due to the removal of vacuum. Test 4: removal of 50 kPa vacuum after 8.0 hours, Test 5: removal of 50 kPa vacuum after 10.0 hours, Test 6: removal of 50 kPa vacuum after 12.0 hours.

Figure 5.7: Comparison of the average excess pore water pressures of the tests conducted with surcharge alone, with the tests conducted with a combination of surcharge and vacuum for: (a) surcharge/vacuum removal = 8.0 hours, (b) surcharge/vacuum removal = 10.0 hours, (c) surcharge/vacuum removal = 12.0 hours.

Figure 5.8: Average increase in effective stress for: (a) tests with surcharge alone (Tests 1, 2, and 3), (b) tests with a combined surcharge-vacuum pressure (Tests 4, 5, and 6).

Figure 5.9: Comparison of the increase in average effective stress of the tests conducted with surcharge alone, and the tests conducted with a combination of surcharge and vacuum for: (a) surcharge/vacuum removal = 8.0 hours, (b) surcharge/vacuum removal = 10.0 hours, (c) surcharge/vacuum removal = 12.0 hours.
Figure 5.10: Decrease of the average effective stress with the time of the surcharge/vacuum removal. ................................................................. 142
Figure 5.11: OCR versus the removal time of surcharge/vacuum ................. 142
Figure 5.12: Axial strain for: (a) tests with surcharge alone (Tests 1, 2, and 3), (b) tests with a combined surcharge-vacuum pressure (Tests 4, 5, and 6) .......... 144
Figure 5.13: Axial strain for: (a) removal of surcharge/vacuum after 8.0 hours, (b) removal of surcharge/vacuum after 10.0 hours, (c) removal of surcharge/vacuum after 12.0 hours, ................................................................. 145
Figure 5.14: Relationship between time, load, and degree of consolidation ....... 147
Figure 5.15: Relationship between VP/SP and average excess pore pressure, AEPP = Average excess pore pressure; SP = Surcharge pressure; VP = Vacuum pressure ........................................................................................................... 148
Figure 6.1: (a) Modified 150 mm Rowe Cell, (b) Base Plate of the Rowe Cell showing the locations of pore pressure measurement. Dimensions are in mm. ............ 156
Figure 6.2: Excess pore water pressure distributions at different radii in 150 mm Rowe cell. ........................................................................................................ 158
Figure 6.3: Lateral distribution of excess pore water pressure in 150 mm Rowe cell .............................................................................................................. 159
Figure 6.4: Flow velocity – hydraulic gradient relationship ......................... 160
Figure 6.5: Unit cell; r_s = is the radius of smear zone and r_w is the radius of drain well ........................................................................................................... 162
Figure 6.6: Average excess pore water pressure dissipation in 75 mm Rowe cell (a) comparison with Darcian-based model, (b) comparison with Hansbo’s non-Darcian based model ................................................................. 169
Figure 6.7: Axial strain in 75 mm Rowe cell (a) comparison with Darcian-based model, (b) comparison with Hansbo’s non-Darcian based model .............. 170
Figure 6.8: Subdivisions of reclaimed area at the Port of Brisbane ............... 173
Figure 6.9: Soil properties and profile at the reclamation site, Port of Brisbane. (after Indraratna et al. 2011) ................................................................. 175
Figure 6.10: (a) Embankment height, (b) Settlement, and (c) Excess pore water pressure for Sections WD2 and WD4, Port of Brisbane, Australia ............ 177
Figure 6.11: Flow velocity – hydraulic gradient relationship for areas WD2 and WD4 in Port of Brisbane ........................................................................ 178
Figure 6.12: (a) Embankment height, (b) Settlement, and (c) Excess pore water pressure for area VC1 in Port of Brisbane.
LIST OF TABLES

Table 3-1: Dimensions of Rowe cells (Head, 1986) ................................................................. 64
Table 3-2: Dimension information of the drainage discs .......................................................... 71
Table 3-3: Comparison of the modified 150 mm Rowe cell with the Standard cell... 77
Table 4-1: Properties of Kaolin ................................................................................................ 95
Table 4-2: Information of the tests run with the modified Rowe cell .................. 96
Table 4-3: Information of the test series using 75 mm Rowe cell ................................. 99
Table 4-4: Information of the radial consolidation test with the 75 mm Rowe cell ..115
Table 5-1: Information from the test conducted using a 150 mm Rowe Cell ..........123
Table 5-2: Comparison of the excess pore water pressure before and after
surcharge/vacuum removal for Tests 1-6..........................................................133
Table 5-3: Developed preconsolidation pressures at the end of the tests...........139
Table 6-1: Properties of Kaolin ...........................................................................................154
Table 6-2: Tests conducted with the 150 mm modified Rowe Cell......................155
Table 6-3: ch values calculated based on different approaches............................167
Table 6-4: Radial consolidation test using 75 mm Rowe cell .................................168
Table 6-5- PVD characteristics and improvement scheme in area VC1 at Port of
Brisbane ....................................................................................................................174
Table 6-6- Soil profiles for individual sections at the Port of Brisbane............175
LIST OF NOTATIONS

c_c  compression coefficient

c_h  coefficient of consolidation in the radial direction (m²/year)

c_v  coefficient of consolidation in the vertical direction (m²/year)

D_f  equivalent diameter of influence zone in field (m)

D_l  diameter of influence zone in laboratory (m)

i  hydraulic gradient

i_t  threshold hydraulic gradient in Hansbo's model

k  coefficient of permeability based on linear flow (m/sec)

k_h  coefficient of permeability in the horizontal direction in undisturbed soil (m/sec)

k_h  coefficient of permeability in the horizontal direction in smear zone (m/sec)

l  soil thickness in a unit cell (m)

m  constant in Hansbo's flow relationship

m_v  coefficient of the soil volume compressibility (m²/kN)

n  ratio of the unit cell radius to the radius of drain = \[ \frac{R}{r_w} \]

P_0  magnitude of the vacuum pressure at drain – soil interface (kPa)

Q  volume of flow (m³)

r  radial distance measured from the centre of drain (m)

r_s  radius of the smear zone (m)

r_w  drain radius (m)

R  effective radius or radius of consolidation cell (m)

s_u  undrained shear strength (kPa)
$t$  time (hour)

$u$  excess pore water pressure at a given time outside the smear zone (kPa)

$u'$  excess pore water pressure at a given time within the smear zone (kPa)

$\bar{u}$  average excess pore water pressure in unit cell at a given time (kPa)

$\bar{u}_0$  initial average excess pore water pressure in unit cell (kPa)

$v$  flow velocity (m/sec)

$LL$  liquid limit (%)

$PL$  plastic limit (%)

$SP$  surcharge pressure (kPa)

$VP$  vacuum pressure (kPa)

$VSR$  vacuum – total surcharge ratio = $VP/(SP + VP)$

$\alpha_c, \alpha'_c$  constants in the proposed flow relationship (m/sec)

$\beta$  constant in the proposed flow relationship

$\gamma_w$  unit weight of water (kN/m$^3$)

$\kappa$  coefficient of permeability based on non-linear flow in Hansbo's model (m/sec)

$\varepsilon$  axial strain at a given time

$\eta, \eta_s, \text{and } \eta_n$  coefficients in the proposed consolidation model (see Appendix A)
1 INTRODUCTION

1.1 Objectives of the Research

This research provides an original contribution to the field of Geotechnical Engineering through exclusive laboratory testing and evaluating the corresponding soil parameters, as well as analytical modelling pertaining to vacuum assisted radial consolidation. The objectives of this research study are outlined as follows:

- Summarising the theoretical and practical works in the field of soft soil improvement by applying surcharge loading with vertical drains and vacuum preloading, and then outlining various issues most relevant to this area of research.

- Development/modification of the equipment for measuring the behaviour of soil during radial consolidation when a surcharge and/or vacuum pressure is applied.

- Characterisation of the most important soil parameters used in consolidation theories, including identifying how and when they should be used, with particular reference to the coefficient of consolidation and rate of consolidation.

- Identifying the differences between the surcharge and vacuum pressures and then explaining how the soil responds to the application and removal of these loads during radial consolidation.

- Identifying the issues surrounding the use of conventional soil parameters in existing consolidation models.
• Development of an analytical model for vacuum assisted radial consolidation using a new approach to determine the related soil and the model parameters.

1.2 Background
One of the main concerns of geotechnical engineers is how to improve the soil properties to ensure that it performs properly under the expected environment and design superstructures. Most deposits of natural soft soil usually need improvement to some extent, which means that as populations increase and land becomes limited, construction on soft soils cannot begin until the soil has been improved adequately to withstand the designed loads. This problem has been widely addressed in those countries where most of the population expects to live in coastal areas on land that usually consists of highly compressible soft, marine soils. Soft soils are categorised as those with a relatively low shearing strength (usually around 10-30 kPa) and high compressibility. These soils mainly consist of clay and silt particles and have a low bearing capacity and exhibit large deformation under the load of superstructures.

Different soft soil improvement techniques are now widely available; but the decision on which technique should be used generally depends on factors such as future land use, the importance and sensitivity of the superstructures, period of construction, the area needing improvement, the cost of it being improved, and other post construction criteria. One of the most commonly used techniques to improve soft soil is to construct embankments on top of them as a surcharge preload which is usually higher than the future design load. This will preconsolidate the soil enough to achieve the required effective stresses and enable it to carry the design loads within the required criteria and minimise post construction settlement.
However, this technique alone is only effective for thin layers of soft soils where the drainage path is relatively short. For thick marine clays this method often requires a long period of consolidation which is not ideal in many land reclamation projects.

A combination of vertical drains with conventional surcharge loading to consolidate the thick layers of soft clays would significantly shorten consolidation by creating shorter radial drainage paths. A wide range of the prefabricated drains are now available on the market, which makes it even easier to use this technique. Indeed, this technique is now one of the most commonly used and economical methods of ground improvement in land reclamation projects worldwide. The embankment dimension required can be defined based on the effective stresses needed in the soil, although this sometimes exceeds the practical criteria. Construction issues, preparing the material for the embankment, cost of constructing the embankment, and a limited corridor for construction are some of the criteria defining the maximum dimension of the embankments.

One of the techniques developed to minimise the construction of high embankments is to use vacuum pressure in conjunction with surcharge preloading and vertical drains. This method is now widely known and used, so the focus of this research is on this combination. With this technique, part of the embankment (surcharge) can be substituted by vacuum pressure and therefore the height of the embankment can be reduced. However, this method is now used worldwide but there are several aspects which are not completely understood and must be addressed.

1.3 Significance of the Research
The application of a surcharge pressure alone almost immediately increases the pore
water pressure at any distance from the drains, whereas the propagation of vacuum pressure from the drains across the soil and how it affects the pore water pressure at different distances from the drain have not yet been completely understood. The average effective stresses in the soil also depend on the lateral distribution of excess pore water pressure, which in turn relates to the propagation of vacuum. The aim of this research is to identify how vacuum pressure propagates laterally across the soil and how it affects the radial distribution of excess pore water pressures. The results of this research will provide an understanding of how the effective stresses in soil can increase its shear strength and stability.

One of the issues which always arise for practical engineers is whether the rate of settlement or the rate of excess pore water pressure dissipation should be used to calculate the degree of consolidation. It is already understood that these two rates are different, but how this difference should be adopted in design and analysis has not been studied in detail. This study will address this fundamental issue and clearly show when and how settlement and/or pore pressure data should be analysed and used to calculate the degree of consolidation and gain in shear strength.

It is still not clear what the differences are between the use of conventional surcharge loading and vacuum pressure. It is already known from various recent case studies that vacuum can be cheaper than the cost of constructing surcharge embankments, but the advantage of vacuum over conventional surcharge during consolidation is not completely understood and is still not documented in the literature. In this research the effects of surcharge and vacuum pressures on the change of excess pore water pressure, effective stresses, and axial strain will be
categorised and the difference between surcharge and vacuum pressures will be identified, including how to combine them in order to optimise their performance and reduce the cost and duration of consolidation.

The condition of flow during vacuum assisted radial consolidation has not been addressed properly, so in this study laboratory testing will be conducted to examine and characterise flow-deformation aspects during vacuum assisted radial consolidation. Moreover, from this study a comprehensive analytical model which incorporates the actual flow rule and the effects of vertical drains, smear zone, and vacuum preloading, will be presented.

1.4 Organisation of the Thesis
This thesis is presented in different chapters to highlight the various aspects of the study. Chapter 2 presents a comprehensive literature review related to the deformation of clay that includes the effective stresses, time, vertical drains, and vacuum pressure while it also summarises the analytical, numerical, and experimental works conducted by the writer. The most important parameters related to radial consolidation are also highlighted and comprehensively discussed in this chapter.

Chapter 3 discusses the various aspects and advantages of Rowe Cells over the conventional oedometer apparatus, and explains how different types of tests, including vertical and radial consolidation with different boundary conditions and drainage paths, can be conducted using these cells. Then, a modified 150 mm Rowe Cell is described and its advantages over commonly used Rowe Cells are explained and discussed in detail, followed by a description of the hardware and software used
in laboratory testing.

Chapter 4 presents the outcomes of the writer’s laboratory program to distinguish the effects of changing the vacuum pressure-total surcharge ratio on the rate of consolidation. Then the degrees of consolidation based on analyses of settlement and excess pore pressure are introduced, and the differences between them are highlighted. The back calculated coefficients of radial consolidation based on the settlement and pore pressure data are then discussed, and the relationship between these two, as well as their relationship with the vacuum-surchage ratio, are then produced.

Chapter 5 presents a complete picture of the effects of the application and removal of surcharge and vacuum pressures on the excess pore water pressure, effective stresses, and axial strain during radial consolidation. It also shows these effects and compares them at different distances from the drain, including the effects of vacuum on the rate of consolidation and its effects on the gain in effective stress. At the end of this chapter a conceptual design approach which explains how a combination of surcharge and vacuum can be used to consolidate soft soils quicker and thus reduce the costs and duration of soil reclamation projects is shown. The optimum time to remove the vacuum and surcharge according to the proposed design approach is then discussed and calculated.

Chapter 6 shows how the flow velocity—hydraulic gradient relationship during vacuum assisted radial consolidation can be captured using the modified Rowe Cell. The actual flow type which is captured during consolidation and includes all of the phenomena occurring in soil (such as migration of grains, change in cross
section, and change in porosity) during consolidation, is then used to develop a comprehensive analytical model that incorporates the effects of vertical drains, smear zone, vacuum pressure, and non-Darcian flow. The advantages of this proposed model over existing models are highlighted in this chapter.

Finally, Chapter 7 presents a general conclusion to the research conducted in this study, including a summary of the outcomes from each preceding chapter.
2 LITERATURE REVIEW

2.1 General
Soft soils generally consist of clay and silt with a moisture content that is close to or beyond their liquid limits. Their shear strength and bearing capacity are low, and they will yield significant settlement under structural and non-structural loads. Both short term and long term deformations are important and must be taken into account in projects involving soft soils. Soft soils also have low permeability and their consolidation upon loading takes a notable time. Due to their low shear strength, soft soils need to be improved before a design load can be applied. Improving thick layers of soft soils is among the most time consuming and costly projects, and therefore any technique which reduces the time and the cost of the projects is highly appreciated. To this end, numerous analytical, numerical, and empirical studies have been done to produce different models and techniques, and to predict the behavior and the strength gain of soils during the improvement process.

Various methods have been used to stabilise soft soils, each of which has some benefits and limitations. The most prominent methods for improving soft soil can generally be categorised as:

- Thermal-based Techniques,
- Explosion-based Methods,
- Electro-kinetic Techniques,
- Physical Modification Methods,
Of these, preloading is one of the most popular methods for increasing the bearing capacity and reducing the post construction settlement of soft soils. Improving the shear strength of soft soils can be achieved by reducing the water content, which in turn reduces the void volume in the soil and brings soil grains closer together. Therefore, any technique which reduces the water content quicker while taking into account the stability of the soil during treatment would be more acceptable.

Placing earth fills as surcharges on top of saturated soils produces excess pore water pressure which makes a hydraulic gradient between the points inside the soil mass, and the points on the drainage boundary. The drainage boundary is usually at the ground surface, and therefore the hydraulic gradient is usually in the vertical direction. The hydraulic gradient causes water to move towards the drainage boundary which gradually reduces the water content of soil and dissipates the excess pore water pressure. The amount and rate at which the water is reduced depend on several factors, including the total surcharge pressure, and the thickness and permeability of the soil. The total surcharge pressure is chosen based on the design loads and it controls the reduction in the water content (void ratio). The thickness of the soil and its permeability, dictate the rate of consolidation.

Utilising vertical drains in conjunction with earth fills is a more effective method of improving soft soils. Vertical drains play two main important roles. First, they shorten the drainage path by making a horizontal hydraulic gradient in addition to the vertical one, and second, because the horizontal permeability of the soils is
usually higher than the vertical permeability, they result in a higher rate of consolidation.

Various types of drains have frequently and effectively been used: sand drains, sand compaction piles (SCP), sandwick drains, drains made from wood and cardboard, and also prefabricated geosynthetic vertical drains (PVD). Usually, the same method can be used to design and predict the behaviour of soils treated with vertical drains irrespective of the type of drain, although minor considerations or adjustments might be needed.

2.2 Deformation of Clay
Experimental investigations have shown that clay exhibits an elasto-viscoplastic behaviour under loading. Elastic deformation is a time-independent behaviour. In some of the elasto-viscoplastic models, primary consolidation, which occurs with time, is also considered to be a time-independent deformation. In these models, time and time dependency are assumed to be related to viscous effects in the soil skeleton such as creep, stress relaxation, and the strain-rate effects. Therefore the process of consolidation, which is a reduction in pore water, is not regarded as a true time effect. However, consolidation is physically connected with time and time is one of the important parameters concerning the consolidation of soils.

2.2.1 Consolidation Settlement
Soft saturated soils experience prominent settlement under increasing external loads, with most deformation being caused by consolidation. Increasing the total stress caused by a surcharge introduces immediate excess pore water pressure into the soil mass, whereas consolidation continues until the excess pore water pressure
completely dissipates and the effective stresses increase. Most of the studies regarding consolidation are based on Terzaghi’s well-known theory of one-dimensional consolidation. Equation 2.1 is the equation to calculate the consolidation settlement of a layer of soil with a total thickness $H$. In this equation $s_c$ is consolidation settlement, $m_v$ is coefficient of volume compressibility (Eq. 2.2), and $\Delta \sigma'$ is the increase in effective stress of a thin layer of thickness $dz$ at depth $z$.

$$s_c = \int_0^H m_v \Delta \sigma' dz$$  \hspace{1cm} 2.1

$$m_v = \frac{1}{1+e_0} \cdot \frac{e_0 - e_1}{\sigma_1' - \sigma_0'}$$  \hspace{1cm} 2.2

Where, $\sigma_0'$ = initial effective stress;

$\sigma_1'$ = final effective stress due to an increase in total stress;

$e_0$ = initial void ratio;

$e_1$ = void ratio after an increase in effective stress from $\sigma_0'$ to $\sigma_1'$.

The degree of consolidation at any depth $z$ can be calculated using Eq. 2.3, where $U_z$ is the degree of consolidation at depth $z$, $u_0$ and $u$ are the initial excess pore water pressure and excess pore water pressure at a given time, respectively.

$$U_z = \frac{u_0 - u}{u_0} = 1 - \frac{u}{u_0}$$  \hspace{1cm} 2.3

Based on Terzaghi’s theory of one-dimensional consolidation, $u$ can be found
by using the partial differential Equation 2.4 where \( t \) is time, \( z \) is depth, and \( c_v \) is the coefficient of consolidation (Eq. 2.5).

\[
\frac{\partial u}{\partial t} = c_v \frac{\partial^2 u}{\partial z^2} \tag{2.4}
\]

\[
c_v = \frac{k}{m_\omega \gamma_w} \tag{2.5}
\]

Where, \( k \) is soil permeability and \( \gamma_w \) is unit weight of water.

The solution of Eq. 2.5 for the excess pore water pressure at depth \( z \) at a given time \( t \) is:

\[
u = \sum_{n=1}^{n=\infty} \left( \frac{1}{d} \int_0^{2d} u_0 \sin \frac{n\pi z}{2d} \, dz \right) \left( \sin \frac{n\pi z}{2d} \right) \exp \left( -\frac{n^2 \pi^2 c_v t}{4d^2} \right) \tag{2.6}
\]

where, \( d \) = length of the longest drainage path.
For a particular case in which \( u_0 \) is constant throughout the layer of clay:

\[
\frac{u}{u_0} = \sum_{n=1}^{\infty} \frac{2u_0}{n\pi} \left( 1 - \cos(n\pi) \right) \left( \sin \frac{n\pi z}{2d} \right) \exp \left( -\frac{n^2\pi^2 c_d t}{4d^2} \right)
\]

2.7

Because only odd values of \( n \) are relevant in Eq. 2.7, it is more convenient to make the following substitutions:

\[
n = 2m + 1
\]

2.8

and

\[
M = \frac{\pi}{2} (2m + 1)
\]

2.9

and

\[
T_v = \frac{c_d t}{d^2}
\]

2.10

\( T_v \) is the time factor which is a dimensionless number.

Substituting Eqs. 2.8, 2.9, and 2.10 into Eq. 2.7 results:

\[
\frac{u}{u_0} = \sum_{m=0}^{\infty} \frac{2u_0}{M} \left( \sin \frac{Mz}{d} \right) \exp(-M^2 T_v)
\]

2.11

Combining Eqs. 2.3 and 2.11, the degree of consolidation at depth \( z \) and time \( t \) can
be obtained:

\[ U_z = 1 - \sum_{m=0}^{\infty} \frac{2}{M} \left( \sin \frac{Mz}{d} \right) \exp(-M^2T_v) \]  

2.12

The average degree of consolidation at time \( t \) for constant \( u_0 \) is then given by:

\[ U = 1 - \frac{1}{2d} \int_0^{2d} u dz \frac{1}{u_0} = 1 - \sum_{m=0}^{\infty} \frac{2}{M^2} \exp(-M^2T_v) \]  

2.13

2.2.2 Time-Dependent Behavior of Clays

The time-dependent viscous behaviour of clays includes creep, stress relaxation, and the strain rate effect. Based on these phenomena, there are three standard tests utilised to identify the time-dependent response of the soil: creep tests, stress relaxation tests, and constant rate of strain tests (CRS tests).

2.2.2.1 Creep and its Stages

Consider a soil which is sheared to point A (Figure 2.2a) in the stress-strain space.

The creep phenomenon is then initiated at this point under the effect of constant stress over time (Figure 2.2b), and although the stress is constant afterwards, the strain gradually increases and the strain state moves towards B (Figure 2.2c). Therefore, it can be said that creep is a gradual increase in strain, while the stress is kept constant.

By considering the creep process at a constant stress while utilising a triaxial test and plotting the strain-time diagram (Figure 2.3a), the process can be divided into three parts: (1) Primary or transient creep, (2) secondary or stationary creep, and
(3) tertiary or accelerating creep. The primary, secondary, and tertiary phases can be characterised by a decreasing, constant, and an increasing strain rate, respectively (Figure 2.3a). This can be seen more clearly in Figure 2.3b where the logarithm of the strain rate is plotted against the logarithm of time. It should be noted that tertiary creep eventually leads to failure of the soil.

Figure 2.2: Creep test performed at a low stress level: (a) Stress-strain relationship; (b) Stress history; and (c) Strain history (after Augustesen et al., 2004)

Figure 2.3: Definition of creep stages considering a creep test at constant stress performed in a triaxial apparatus: (a) Strain versus time and (b) log strain versus log time (after Augustesen et al., 2004)
Evaluating the settlement of soil with an oedometer test, introduces three distinct phases that must be distinguished from the creep stages in a triaxial test. Primary, secondary, and tertiary compressions are three phases that can be defined by plotting strains versus the logarithm of time (Figure 2.4a) with step loading tests performed in an oedometer apparatus. The first one is primary consolidation where excess pore water pressure dissipates. In an oedometer test, secondary and tertiary compressions correspond to pure creep which can be characterised by considering the relationship between log (time) and strain (Figure 2.4a). This relationship is linear during secondary compression and non-linear through a tertiary one. Figure 2.4b shows a plot of strain rate versus time both in the logarithmic scale for a single load increment in an oedometer test. In this test the strain rate always decreases over time, which means only primary creep can be observed in an oedometer test and secondary and tertiary creeps cannot be captured in this type of test. In an oedometer test, only volumetric creep occurs since the soil cannot have shear failure and does not experience deviatoric creep. Therefore, tertiary creep cannot occur in oedometer test.
The strain rate decreases during primary creep, it is constant during secondary creep, and increases during tertiary creep, while it always decreases over time in the primary, secondary, and tertiary compressions.

One of the most important and contentious issues regarding consolidation and creep is defining the end of primary consolidation (EOP). It is defined as the time that excess pore water pressure completely dissipates and transfers to the effective stresses, and based on one hypothesis, it is when creep starts. There are two well known approaches which have been adopted to estimate creep:

*Hypothesis A*: It is assumed that the location of EOP and the value of pre-consolidation pressure are not affected by the thickness of the soil, and therefore the values of strain at the EOP are unique. This means that the soil does not produce any time-dependent creep behaviour during the dissipation of pore water pressure. Based on this hypothesis, the secondary compression (creep) occurs only after primary consolidation has been completed (Ladd et al. 1977; Ladd et al. 1977; Mesri and Choi, 1985a; Mesri and Choi, 1985b).

*Hypothesis B*: Based on this hypothesis, time-dependent strains (creep) take place during primary consolidation as well, which means that the strain at the EOP is not unique. (Suklje, 1957; Bjerrum, 1967; Barden, 1969; Leroueil et al. 1985; and Yin and Graham, 1989b).

Some researchers (e.g. Aboshi, 1973) have confirmed that real soil behaviour is somewhere between these two hypotheses, which correspond to two extreme cases. However, a general agreement has not been made on whether there is a combination of primary and secondary compression during the dissipation of pore
water pressure.

2.2.2.2 One-Dimensional Creep Tests

Clays exhibit creep behaviour under constant effective stresses, especially normally consolidated clays. Various researchers have comprehensively investigated the creep (secondary compression) phenomenon under one-dimensional conditions. Secondary compression is usually identified when an approximately linear relationship between the vertical strain ($\varepsilon_z$) or void ratio ($e$) and the logarithm of time ($t$) is observed. The coefficient of secondary compression represents this linear relationship (Figure 2.5). This coefficient can be represented based on the void ratio or vertical strain, with the most commonly used definitions given by:

$$c_{ae} = \frac{\Delta e}{\Delta \log(t)} ; \quad c_{ae} = \frac{\Delta e}{(1+e_0)\Delta \log(t)} = \frac{e_z}{\Delta \log(t)} = \frac{c_{ae}}{1+e_0}$$  \hspace{1cm} (2.14)$$

where, $e =$ void ratio; $\varepsilon_z =$ vertical strain; $t =$ time; $e_0 =$ initial void ratio; and $c_{ae}$ and $c_{ae}$ are coefficients of secondary compression with respect to $e$ and $\varepsilon$. 
Many researchers (e.g. Walker and Raymond, 1968; Mesri and Godlewski, 1977) believe that the coefficient of secondary compression has a linear relationship with the coefficient of primary consolidation $c_{ce}$ ($c_{ce} = \Delta \epsilon / \Delta \log \sigma'_z$). Also, it is established (e.g. Tavenas et al., 1978; and Graham et al., 1983) that coefficients of primary and secondary compressions are not constant and depend on the applied effective stresses. Moreover, with regards to the creep phenomenon, the linear strain-time behaviour with respect to a $\epsilon_z - \log(t)$ diagram (Figure 2.5) has not normally been observed, so a general non-linear behaviour is usually proposed (Leonards and Girault, 1961; Bjerrum, 1967; Leroueil et al. 1985; and Yin, 1999).

Increased apparent preconsolidation pressure will be recorded with secondary compression. Figure 2.6 shows how the void ratio and strain rate progressively decrease when a soil is subjected to a constant effective stress for a long period of time. The strain rate will increase when soil is reloaded, while the resulting compression curve moves to the curve of constant strain rate corresponding to the new strain rate, and then a preconsolidation pressure $\sigma'_{z,pc1}$ associated with this new
strain rate will result. Leonnards and Ramiah (1960) were the first who reported this “quasi-preconsolidation pressure” when it was later explained in detail by Bjerrum (1967).

![Diagram](image1)

Figure 2.6: Development of quasi-preconsolidation pressure due to secondary compression (after Bjerrum, 1967).

The time-dependency of the compressibility of clay due to secondary compression was first formulated by Buissman (1936). Afterwards, Taylor (1942) considered the one dimensional compression of clay and argued that a single stress-strain curve cannot represent the compression of clay, including the effects of secondary compression. There is instead a family of representative curves called “time lines,” each of which corresponds to a different duration of the applied load. Moreover, a feature of this approach is that each time line represents a different
preconsolidation pressure \( (\sigma'_{z,pc}) \). Taylor’s observation was confirmed by Bjerrum, (1967) when he proposed the concept of the parallel lines in the \( e - \log \sigma'_{z} \) space for delayed compression. The parallel lines are actually a series of equilibrium relationships after different time periods of sustained loading (Figure 2.6).

The one-dimensional time dependent behaviour of clays was studied by Yin and Graham (1989a, b, 1994) by developing a series of models. Additional studies were conducted by Yin and Graham (1996), Yin and Zhu (1999), Yin and Graham (1999) and Yin et al. (2002). The Yin and Graham models are actually further developments of Bjerrum’s work and the strain rate approach. The concept of “equivalent time” is a new aspect in Yin and Graham’s work that was utilised to model the creep behaviour of normally consolidated and over-consolidated clays as a function of \( \sigma'_{z}, \dot{\sigma}'_{z}, \varepsilon_{z} \), and \( \dot{\varepsilon}_{z} \). The behaviour of clay under a variety of test conditions (e.g., relaxation tests, tests with constant rate of strain, or constant rate of stress) can be predicted by taking advantage of this concept. Compared to Bjerrum’s model and the strain-rate approach, the important improvements in Yin and Graham’s models are the ability to distinguish normally consolidated and over consolidated clays, as well as modelling the relaxation behaviour. Important concepts in connection with Yin and Graham’s model are: (1) equivalent time, (2) reference time line, (3) instant time line, and (4) limit time line (Figure 2.7).

**Equivalent time**: Unique time lines for the constant duration of loading will not be resulted from Bjerrum’s model in all cases (Yin and Graham, 1989b). The time lines having equal values of equivalent time \( (t_{e}) \) but not necessarily equal real time durations were introduced by Yin and Graham. The time needed to creep from a
reference time line is defined as equivalent time \((t_e)\). It is equal to zero for the current value of the vertical strain \(\varepsilon_z\) under a constant vertical effective stress. Under a conventional multi-stage loading test for a normally consolidated range with a constant load increment ratio and constant load durations, the equivalent time \((t_e)\) and the duration of the load increments are equal. However, they are quite different in the over-consolidation range, depending on the OCR. The equivalent time represents a unique creep strain rate and a larger equivalent time shows smaller creep strain rates.

Reference time line: To calculate the equivalent time, a reference is needed, which is called the reference time line. It is defined as a line where \(t_e\) equals zero (Yin and Graham (1994). Below the reference time line the equivalent times are positive and above it they are negative in the range of \(0 < t_e < \infty\) and \(-t_0 < t_e < 0\),
respectively. \( t_0 \) is a material parameter. Figure 2.7 represents the location of the reference time line in the \( \varepsilon_z - \sigma'_z \) space.

*Instant time line:* Instantaneous strains are defined by using the instant time lines. In contrast to the elastic-plastic instant time line defined by Bjerrum (1967), these lines are assumed to be purely elastic. The instant elastic response of a soil skeleton due to changes in the effective stress is actually captured by the instant time line (Yin and Graham, 1989, 1994). The location of the instant time line in \( \varepsilon_z - \sigma'_z \) space can be observed in Figure 2.7.

*Limit time line:* A unique limit time line exists on \( \varepsilon_z - \sigma'_z \) space beyond which soil represents a time-independent behavior (Yin and Graham, 1994). It is defined as a line with an equivalent time \( t_e = \infty \) and a corresponding creep rate equal to zero (see Figure 2.7). Two different types of general elastic-viscous-plastic models were presented by Yin and Graham. One of them is formulated by means of logarithmic functions (Eq. 2.15) and the other by means of power functions (Eq. 2.16).

\[
\varepsilon_z = \frac{\kappa}{\nu \sigma_z} \sigma'_z + \frac{\psi}{v t_0} \exp \left[ -\left( \varepsilon_z - \varepsilon_{z0} \right) \frac{\nu}{\psi} \left( \frac{\sigma'_z}{\sigma'_{z0}} \right)^{\lambda/\psi} \right]
\]

where, \( \varepsilon_{z0} = \) initial strain corresponding to the initial effective stress \( \sigma'_{z0} \); \( \nu = \) specific volume; \( \kappa = \) material parameter which describes the elastic stiffness of the soil; \( \lambda = \) elastic-plastic material parameter; \( \psi = \) creep parameter that is constant for a given soil; and \( t_0 = \) intrinsic time parameter.
\[ \dot{\varepsilon}_z = a_2 n_1 \left( \frac{\sigma'_u - \sigma'_{20e}}{\sigma'_u} \right)^{n_1-1} \sigma'_u + \left( f^\text{ep}_\infty - f^\text{ep}_0 \right) \frac{n_3}{t_0} \times \left( 1 - \frac{\varepsilon_z - f^\text{ep}_0}{f^\text{ep}_\infty - f^\text{ep}_0} \right) \left( n_3 + 1 \right)/n_3 \]  

2.16

In Eq. 2.16, the first part on the right hand side is the elastic or instant component of the total strain rate, and the second term is the viscous creep component of the total strain rate. \( f^\text{ep}_0 \) indicates the function of the reference time line; \( f^\text{ep}_\infty \) indicates the function of the limit time line; \( \sigma'_u \) = unit stress; \( a, n_1, n_3, \sigma'_{20e} \) = model parameters; and \( t_0 \) = intrinsic time parameter.

Both Eqs. 2.15 and 2.16 can be solved to obtain solutions for creep, relaxation, the constant rate of strain, and the constant rate of stress. Generally, a good agreement between the predictions and the experimental results of the one-dimensional viscous behavior of soft soils is observed. The power function approach results in the best agreements because the options for calibrating the model have been improved, compared to the logarithmic model (Yin and Graham, 1989a). To this end, it is worth mentioning that 11 parameters need to be determined when using

![Figure 2.8: Relaxation test (A → B): (a) Stress-strain relationship; (b) strain history; (c) stress history](image-url)
the power function, whereas the logarithmic model only involves 5 parameters.

2.2.2.3 Stress Relaxation

Figure 2.8 demonstrates a stress relaxation test. The stress-strain state at point A represents a sheared soil. Beyond this point, a stress relaxation process is initiated. The total strain remains constant over time during the stress relaxation process (time duration from A to B). Stress relaxes during this process which means it gradually decreases over time. Therefore, the characterisation of the stress relaxation test can be represented by constant total strain and decreasing stress.

2.2.2.4 Constant Rate of Strain

In the constant rate of strain (CRS) test, a constant total strain rate \( \dot{\varepsilon} = \frac{d\varepsilon}{dt} \) is applied to the sample throughout the experiment. To obtain the stress-strain relationship, the stress needs to be measured.

2.3 Wick Drains and Horizontal Consolidation

2.3.1 Governing Equations for Radial Consolidation

Utilising wick drains to improve soft soils changes the condition of one-dimensional flow to the more sophisticated three-dimensional flow towards the drains. An analytical approach to the effects of the wick drain on the consolidation of soils was originally proposed by Barron (1948). The effects of radial and vertical drainages are actually combined in his model. Barron considered the horizontal drainage under two different hypotheses, equal strain and free strain. He showed that the average consolidation obtained is almost the same under these two assumptions. In the laboratory when a rigid loading platform is usually used, or in the field if the clayey soil is relatively thick, the equal strain hypothesis can be applied. Barron’s work was
originally based on Terzaghi’s (1925) one-dimensional consolidation theory. The following basic assumptions were made in Barron’s theory:

- All vertical loads are initially carried by excess pore water pressure $u$.
- All the compressive strains within the soil occur in the vertical direction.
- Each well has its own influence zone and it does not have any effect on the rate of consolidation of points outside this zone.
- The zone of influence of each well is a circle.
- The load distribution is uniform over the zone of influence of each well.

Based on the second assumption, the basic partial differential equation for three-dimensional consolidation is:

$$
\frac{k_h}{\gamma_w} \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) + \frac{k_v}{\gamma_w} \left( \frac{\partial^2 u}{\partial z^2} \right) = \frac{a_v}{1+e} \left( \frac{\partial u}{\partial t} \right)
$$

2.17

where, $k_h = \text{coefficient of soil permeability in horizontal direction}$,

$k_v = \text{coefficient of soil permeability in vertical direction}$,

$u = \text{excess pore water pressure}$,

$\gamma_w = \text{water unit weight}$,

$e = \text{void ratio}$,

$a_v = \text{coefficient of soil compressibility} = -\frac{de}{dp}$, and

$p = \text{effective pressure}$.

For symmetrical conditions and flow towards the drain well, Eq. 2.17 becomes:
where, \( r \) and \( z \) are cylindrical coordinates.

Barron divided the soil into unit cells where each one surrounds a drain well symmetrically. Each drain well has its own influence zone and an axisymmetric condition exists inside each unit cell (Figure 2.9). The installation of prefabricated drains creates a smear zone around each well, and the smear zone has lower soil permeability and higher compressibility near the drains. In addition, the permeability of the drain itself can cause well resistance. The effects of the smear zone and well resistance were incorporated into an explicit analytical solution by Hansbo (1981). Hansbo’s solution yields Eq. 2.19a for the average excess pore water pressure \( \bar{u} \), and Eq. 2.19b for the average degree of consolidation, \( \bar{U}_h \) at a specific depth, \( z \).

\[
\bar{u} = \bar{u}_0 \exp \left( -\frac{8T_h}{\mu} \right) \tag{2.19a}
\]

\[
\bar{U}_h = 1 - \frac{\bar{u}}{\bar{u}_0} = 1 - \exp \left( -\frac{8T_h}{\mu} \right) \tag{2.19b}
\]

\[
\mu = \frac{n^2}{n^2 - 1} \left( \ln \frac{n}{s} + \frac{k_h}{k_w} \ln s - \frac{3}{4} \right) + \frac{s^2}{n^2 - 1} \left( 1 - \frac{s^2}{4n^2} \right) + \frac{k_h}{k_w} \frac{1}{n^2 - 1} \left( \frac{s^{n-1}}{4n^2} - s^2 + 1 \right) + \pi z (2l - z) \frac{k_h}{q_w} \left( 1 - \frac{1}{n^2} \right) \tag{2.19c}
\]
where, $\bar{u}_0$ is the initial average excess pore water pressure, $T_h = \frac{c_h t}{D_e^2} = \text{time factor in radial consolidation}$, $c_h$ is the coefficient of consolidation in radial consolidation, and $t$ is time.
time, \( D_e \) is the diameter of unit cell, \( n = \frac{D_e}{d_w} \), \( d_w \) is the equivalent diameter of the drain, \( s = \frac{d_s}{d_w} \), \( d_s \) is the diameter of the smear zone, \( k_h \) is the permeability of soil in the horizontal direction in the undisturbed zone, \( k'_h \) is the permeability of soil in the horizontal direction in the smear zone, \( l \) is the maximum vertical drainage distance in a unit cell, and \( q_w \) is the vertical discharge capacity of the drain at a hydraulic gradient in the drain \( i_w = 1 \).

2.3.2 Prefabricated Band Drains
Due to the cost of manufacturing and improvements in the installation equipment and techniques, prefabricated band drains have replaced other types of drains and using PVDs (prefabricated vertical drains) with surcharge is now considered to be an established and suitable method to improve soft soil. Band drains usually consist of plastic cores surrounded by geotextile filters, although drains without filters can be found on the market as well. Figure 2.10 shows some of the band drains. They feature plastic cores that are usually like vertical pipes woven together with small cross sectional areas to allow the water to drain to the surface. The core is also suitable for applying a vacuum pressure propagating along the drain. More than a hundred different types of prefabricated wick drains can now be found on the market, some of which are: Geodrain (Sweden), Mebradrain (Netherlands) and Alidrain (England). They are made with different cross sectional shapes, discharge capacity, and types of filters.

Vertical drains are commonly installed in square or triangular (Figure 2.11) patterns. The influence zone of each drain \((D_e)\) is defined as the area covering the
pore water flowing towards a drain, and is a function of drain spacing ($S$). The shape of the actual influence zone for drains installed in a square pattern is square, and it is hexagon for the triangular pattern (Figure 2.11, dash lines). For analytical purposes, there is a need to convert these shapes into an equivalent circular shape. The diameter of a circle ($D_e$) with an area equal to each shape (rectangle or hexagon) is then calculated. For the square pattern, it results in $D_e = 1.13S$, and for the triangular one $D_e = 1.05S$.

The triangular pattern is usually preferred due to a more uniform consolidation between the drains, but it might be easier to lay out and control the installation of drains on a field using the square pattern.
Figure 2.10: Some different band drains - (a), (b), (c), and (d) drains with central core surrounded by filter sleeves, (e) and (f) drains without filter sleeve (after Moseley and Kirsch, 2004)
PVDs are installed into the soil using installation equipment and a mandrel. The mandrel is driven into the soil by either dynamic or static methods. A vibrating hammer or conventional drop hammer is used in the dynamic method whereas the mandrel is driven into the soil with a static load in the static method. In real projects mandrels have different cross sectional shapes (e.g. Figure 2.12). Driving a mandrel into the soil produces shear strain and excess pore water pressure around it, to some extent. The degree of disturbance in this area depends on factors such as the mandrel size and shape, soil macrofabric and installation procedure. Due to this disturbance, the ratio of $k_h/k_v$ reduces significantly in the disturbed (smear) zone ($k_h$ and $k_v$ are the coefficients of soil permeability in horizontal and vertical directions respectively).

Analytical solutions for radial consolidation problems are mostly based on the circular cross sectional shape of drains. Prefabricated drains are usually made in

Square pattern: $D_e = 1.13S$

Triangular pattern: $D_e = 1.05S$

Figure 2.11: Plan of drain patterns and influence zone of each drain
two different shapes: *band drains* and *circular drains*. To solve the equations for the band drains, the rectangular cross section is converted into an equivalent circular one by calculating and using an equivalent diameter of the band drains. Different researchers have come up with different conversion formulae (e.g. Eqs. 2.20 to 2.24), where $a$ and $b$ are the actual width and thickness of the band drain respectively, and $d_w$ is the equivalent diameter.

\[
d_w = \frac{2(a+b)}{\pi} \quad \text{(Hansbo, 1981)}
\]

\[
d_w = \frac{a+b}{2} \quad \text{(Atkinson and Eldred, 1981)}
\]

\[
d_w = \left(\frac{4ab}{\pi}\right)^{0.5} \quad \text{(Fellenius and Castonguay, 1985)}
\]
\[ d_w = 0.5a + 0.7b \]  \hspace{1cm} \text{(Long and Covo, 1994)} \hspace{1cm} 2.23

\[ d_w = d_e - 2\sqrt{(s^2)} + b \]  \hspace{1cm} \text{(Pradhan et al., 1993)} \hspace{1cm} 2.24

where,

\[ s^2 = \frac{1}{4} d_e^2 + \frac{1}{12} a^2 - \frac{2a}{\pi^2} d_e \]

and, \( d_e \) is equivalent diameter of unit cell.

Equation 2.20 is based on the equivalent perimeter of a band drain and circular drain, and Eq. 2.22 is based on the equivalent area of them. Equation 2.21 was proposed to account for the throttle that occurs near the drain. Based on an electrical analogy, Long and Covo (1994) developed Eq. 2.23 to determine the equivalent diameter of the drain. By considering the flow net of a circular area draining to a central rectangular drain, Pradhan et al. (1993) proposed Eq. 2.24. Although, different researchers might suggest one of the above equations as the best (e.g. Rixner et al. 1986), some studies show that there is minimal difference in the consolidation rate calculated by using any of the above equations (e.g. Indraratna and Redana, 2000; Welker et al. 2000).

2.3.3 **Smear Zone, Horizontal Permeability, and Radial Consolidation**

Prefabricated vertical drains are installed and driven in soft soils using mandrels. Other types of drains, like sand drains, are usually installed in pre-bored holes made by using augers/drills. During installation, the soil is disturbed and remolded near the drains to some extent, and this area is called the smear zone. The permeability of the soil in the smear zone decreases and its compressibility increases, which decreases
the rate of consolidation and increases the consolidation time. This implies that the extent of the smear should be kept to a minimum, and that depends on parameters such as the size and shape of the mandrel and the properties of the soil (e.g. Rowe and Barden, 1966; Lo, 1998; Eriksson et al. 2000). The greatest smear effects occur in the highly sensitive clays with a prominent macro fabric. Smear zone has an approximately elliptical shape around prefabricated band drains and circular shape around circular drains such as sand drains (Indraratna and Redana, 1998b; Welker et al. 2000).

Many researchers believe that the disturbance in the smear zone increases closer to the drain, and therefore, the permeability of the soil also reduces more towards the drain (Bergado et al. 1991; Madhav et al. 1993; Indraratna and Redana, 1998b; Chai and Miura, 1999; Hird and Moseley, 2000; Sharma and Xiao, 2000; Hawlader et al. 2002). Due to drain installation, the permeability of the soil near the drain can be reduced by one order of magnitude (Bo et al. 2003). Also, measurements have shown that the horizontal permeability of the soil close to the drain can be the same as its vertical permeability (Hansbo, 1981; Indraratna and Redana, 1998b). It is reported that for different undisturbed soils $k_h/k_v$ varies between 1.36-2.0 (Tavenas et al. 1983; Bergado et al. 1991; Shogaki et al. 1995).

Indraratna and Redana (1998) conducted laboratory investigations and showed that $k_h/k_v$ in the smear zone varied between 0.9-1.3, with an average of 1.15, whereas $k_h/k_v$ varied between 1.4-1.9, with an average of 1.63 in the undisturbed zone (Figure 2.13).
To estimate radial consolidation, one of the most important parameters which must be determined is the coefficient of radial consolidation $c_r$. It can be determined by using Eq. 2.25, where $c_v$ can easily be determined by utilising ordinary oedometer tests.

$$c_h = \left( \frac{k_h}{k_v} \right) c_v$$  \hspace{1cm} 2.25

Onoue (1988) used a modified oedometer apparatus to determine the coefficient of horizontal consolidation and permeability of the soil. He found that for undisturbed soils, the $c_h/c_v$ ratio is from 2.7 to 6.3 and $k_h/k_v$ is between 2.6 and 4.4. By back calculating the field measured data, the ratio of $c_h/c_v$ and hence $k_h/k_v$ was found to be equal to unity for the smear zone (disturbed soil).
Onoue et al. (1991) studied the horizontal permeability of soil using a especial consolidometer (305 mm diameter by 140 mm thick) and a central drain. Sedimented Boston blue clay was used in the tests. The results showed that the permeability of soil varied with the radius and three different smear zones could be distinguished (Figure 2.14). In Figure 2.14, zone I is the undisturbed region and zones II and III are the intermediate and remolded smear regions, respectively. In this figure \(k_h\) is the horizontal permeability of the soil in the disturbed zone, \(k_{h0}\) is the horizontal permeability of the soil in the undisturbed region, and \(r_w\) is the radius of the well drain. Figure 2.14 shows that the horizontal permeability of soil in the smear zone reduced from 60% to 20%, with respect to the permeability of the undisturbed soil.

Bergado et al. (1991) used a large scale consolidometer and studied the smear effects due to the installation of vertical drains. They used a consolidometer with a
diameter of 45.5 cm and a height of 92 cm. Reconstituted samples of Bangkok soft clay were used in the research. They also used standard oedometer tests to measure the properties of samples taken from various distances from drain. Terzaghi’s method and Asaoka’s (1978) method were used to back calculate the coefficient of horizontal consolidation \( c_h \). It was found that the \( k_h/k_v \) ratio is close to unity in the smear zone, but between 1.5 and 2.0 in the undisturbed zone. Moreover, they used the laboratory model to predict the behaviour of a full scale embankment and found that the predicted and measured total settlements were in good agreement. It was also verified that smaller mandrels create less smear.

However, observations have shown that the horizontal permeability of soil reduces in the smear zone towards the drain, but the most common analytical methods for radial consolidation still use a constant reduced horizontal permeability across the smear zone (e.g. Hansbo, 1981). Ambiguity would arise with this when considering the “size” of the smear zone. Typically, the outer radius of the smear zone \( r_s \) is used to define the extent of smearing. There are generally two ways to define \( r_s \). One defines the point where horizontal permeability begins to drop below the permeability in the undisturbed zone, while the other defines the point where a smear zone with a reduced constant permeability would exhibit equivalent effects compared to the actual distribution of permeability, the latter representing the smaller values for \( r_s \). This indicates that permeability close to the drain has a greater effect on the rate of consolidation compared to permeability away from the drain (Bergado et al. 1991; Chai and Miura, 1999; Sharma and Xiao, 2000; Hird and Moseley, 2000; Hawlader et al. 2002).
Another important common issue related to permeability is the correlation of field permeability to the one determined in laboratory. To evaluate this, the performance of two full-scale test embankments constructed on soft clay deposits in the eastern coastal region of China was studied by Shen et al. (2005). One embankment was constructed on natural subsoil and the other was constructed on subsoil improved with prefabricated vertical drains. The field performance of the two embankments was analysed using the finite element method, while the hydraulic conductivity of the subsoil and discharge capacity of the PVDs were investigated numerically. Modified Cam-Clay model was used in this investigation. The results of a back analysis for the embankment on natural subsoil showed that the hydraulic conductivity ratio \( \frac{C_f}{C_l} \) of the field to the laboratory values is about 6. Also, to analyse the PVD improved subsoil, a simple approach using the equivalent vertical hydraulic conductivity of PVD improved subsoil (Chai et al. 2001) was used. They found that PVDs increase the bulk vertical hydraulic conductivity of soft soil about 30 times compared to the original non-treated subsoil, if the PVDs are installed at a spacing of 1.5 m. Well-resistance and smear effect were taken into account in this study.

In a conventional consideration of PVD improved soft soils, it is assumed that the coefficient of volume compressibility \( m_v \) and the coefficient of horizontal permeability \( k_h \) of the soil are constant, but during consolidation, \( m_v \) varies over a wide range of applied pressure and \( k_h \) changes with the change of void ratio. Indraratna et al. (2005b) took the effects of variable \( m_v \) into account, by replacing it with the compressibility indices \( c_c \) and \( c_r \) which define the slopes of \( e - \log \sigma' \)
curve. Moreover, by considering the $e^{-\log k_h}$ relationship where the slope is $C_k$, they took into account the effect of a variable $k_h$. It was assumed, based on previous work of Tavenas et al. (1983), that there is a linear relationship between the average void ratio and logarithm of horizontal permeability ($\bar{e} = e_0 + C_k \log (k_h/k_{hi})$). The permeability index ($C_k$) was generally considered to be independent of the stress history ($p'_c$) (Nagaraj et al. 1994). Indraratna et al. (2005b) proposed a new model incorporating the aforementioned effects and compared the results with the conventional solution of Hansbo, (1981). They found that $C_c/C_k$ and the load increment ratio ($\Delta p/\sigma_i$) have significant effects on the improved soft soil performance. They argued that when $C_c/C_k < 1.0$, the actual rate of consolidation is higher than that determined by using the conventional solution of Hansbo (1981), and the rate of consolidation decreases with the reduction of the load increment ratio ($\Delta p/\sigma_i$). Also, when $C_c/C_k > 1.0$, the consolidation process takes place at a slower rate compared to the conventional solution where the rate of consolidation increases with a decreasing load increment ratio ($\Delta p/\sigma_i$).

2.3.4 Discharge Capacity of the Drains
One of the steps taken to improve soft soil is the design and selection of drains. Different shapes and types of drains are available on the market, but their discharge capacity must be considered in advance to find what effect it will have on the consolidation rate. The discharge capacity of a drain ($q_{w}$) is the rate of water flow per unit hydraulic gradient, and in the laboratory it is usually measured at 20°C. Numerous studies have been conducted to discover the minimum required discharge
capacity of drains which do not delay consolidation process (e.g. Den Hoedt, 1981; Kremer et al. 1982; Mesri and Lo, 1991; Bo et al. 2000).

The average discharge capacity of a drain can be determined from Eq. 2.26:

\[ q_w = \frac{Q}{i} = Q \frac{dl}{dh} \]  

where, \( Q \) is the average discharged water per unit time, \( i \) is the hydraulic gradient, \( l \) is the flow length, and \( h \) is the head of water.

The minimum discharge capacity of drain should be somehow related to the compressibility and permeability of the soil because the drain should be capable of freely draining the whole quantity of water squeezing out of the soil, which in turn depends on the compressibility and permeability of the soil. For a 30 m long by 100 mm wide drain, based on 40 mm/day allowable settlement, Den Hoedt (1981) stated that the required discharge capacity should be at least \( 3 \times 10^{-6} \text{ m}^3/\text{s} \).

Based on back analysis of data from three major projects for improving soft soil, Mesri and Lo (1991) proposed Eq. 2.27 for the discharge factor \( (D) \) and argued that the required discharge capacity should be 5 times the discharge factor \( D \). Therefore, \( q_w = 5k_h l_m^2 \) where \( k_h \) is soil permeability in the horizontal direction and \( l_m \) is the maximum drainage length.

\[ D = \frac{q_w}{k_h l_m^2} \]  

Bo (2004) proposed Eq. 2.28 for the required average discharge capacity,
which connects it to the volumetric strain of soil. Due to the different rate of settlement during consolidation, flow rate is faster during the early stages of consolidation compared to the latter stages. Therefore the discharge capacity required for the early stages of the consolidation may be greater than the one estimated from Eq. 2.28.

\[
q_w = \frac{(QL)\gamma_w}{\Delta\delta'} = \frac{H\varepsilon_v(D_{e}/2)^2\pi L\gamma_w}{\Delta\delta't} \tag{2.28}
\]

where, \(Q\) is the average flow rate, \(L\) is the prefabricated vertical drain length, \(\gamma_w\) is unit weight of water, \(\Delta\delta'\) the additional effective stress, \(H\) is soil thickness, \(\varepsilon_v\) is the soil volumetric strain, \(D_e\) is the effective diameter of the unit cell around the drain, and \(t\) is the time to complete the primary consolidation.

The variation of settlement with time, in a case of PVD improved soft soil for a full scale test embankment, was monitored and the data compared with the calculated results by Shen et al. (2005). Different discharge capacities \((q_w)\) were analysed and taken into account, and showed that if \(q_w\) is almost more than \(79 - 100 \, \text{m}^3/\text{year}\), it would not have much effect on the rate of settlement.

Table 2-1 summarises the discharge capacity specified at a number of soft soil improvement projects in different countries. The table indicates that for a straight condition the specified discharge capacity ranges from \(5 \, \text{to} \, 100 \times 10^{-6} \, \text{m}^3/s\) and for a kinked PVD, from \(6.3 \, \text{to} \, 32.5 \times 10^{-6} \, \text{m}^3/s\).
Table 2-1. Summary of discharge capacity specified in different projects (after Bo, 2004)

<table>
<thead>
<tr>
<th>Country</th>
<th>Straight Condition</th>
<th>Buckled Condition</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>DC</td>
<td>Test condition</td>
</tr>
<tr>
<td>Netherlands</td>
<td>&lt;10 m thick</td>
<td>&gt;10</td>
</tr>
<tr>
<td></td>
<td>&gt;10 m thick</td>
<td>&gt;50</td>
</tr>
<tr>
<td>Singapore</td>
<td>&gt;25</td>
<td>&gt;10</td>
</tr>
<tr>
<td>Thailand</td>
<td>&gt;16</td>
<td>300 kPa, 28 days, ( i = 1 )</td>
</tr>
<tr>
<td>Hong Kong</td>
<td>&gt;5.0</td>
<td>200 kPa</td>
</tr>
<tr>
<td>Malaysia</td>
<td>&gt;6.3</td>
<td>400 kPa, ( i = 1 )</td>
</tr>
<tr>
<td>Australia</td>
<td>&gt;100</td>
<td>300 kPa</td>
</tr>
<tr>
<td>Finland</td>
<td>&gt;10</td>
<td>100 kPa</td>
</tr>
<tr>
<td>Greece</td>
<td>&gt;10</td>
<td>100 kPa</td>
</tr>
</tbody>
</table>

Note: DC is the Discharge Capacity in \( m^3/s \times 10^{-6} \)

2.3.5 Settlement due to the Radial Consolidation

Reducing post construction settlement is one of the main objectives of radial consolidation projects, and an accurate prediction of soft soil deformation is a definite requirement. Using the equivalent plane strain models to analyse multiple drains beneath embankments is becoming a popular method in this area. Indraratna and Redana (1997) introduced an equivalent plane strain model for PVD improved soils incorporating the effects of the smear zone. They utilised the modified Cam-clay theory and CRISP92 finite-element code in their study. To validate the mathematical model and numerical results, they compared the outcomes with the performance of an embankment stabilised with vertical drains on soft Muar clay in Malaysia (Indraratna et al. 1994). They compared the field measurements with three different finite element analyses: one for perfect drains (no smear), one with smear, and one considering smear with increased horizontal permeability. With regards to settlement at the centre line of the embankment, the last finite element analysis
showed the best results. Following this, Indraratna and Redana (2000) introduced a numerical model that included the effects of smear and well resistance for soft soils improved with vertical drains. In this study they compared the numerical results with the measured data from different embankments, and once again they achieved reasonable results for the predicted settlement.

Shen et al. (2005) studied the performance of PVD improved sub-soils by monitoring two full scale embankments and comparing the data with the FEM results using Cam-Clay model. They found that for the PVD improved case, primary consolidation was finished in almost 600 days, but beyond this period FEM underestimates the settlement, so the inability of the Modified Cam-Clay to model secondary compression was suggested to be the cause of this issue.

Generally speaking, regarding settlement and the rate of settlement, there are some discrepancies in all of the studies between the analytical results and field measured data. Although between the settlement, excess pore water pressures and lateral displacements, the best prediction is usually achieved for settlements. However, when the long term performance of the improved soft soils is considered, there is more difference between the field measurements and predicted data.

2.3.6 Excess Pore Water Pressure
Excess pore water pressure resulting from improving soft soils is one of the most important parameters that must be accurately deal with because it controls the effective stresses inside the soil which specify its shear strength. The stability of embankments strongly depends on the shear strength of the soil, and the effective stress and excess pore pressure. Therefore, by connecting with the stability issues,
excess pore water pressure must be precisely predicted, measured, and analysed. Shen et al. (2005) compared the measured and calculated data for two full scale embankments through the changes in pore water pressure measured over time, and at different depths, for both natural and PVD improved embankments. They found noticeable differences between the measured and calculated data for both cases, although it was mentioned that the reason was not clearly known.

Yin and Zhu (1999) used an elastic-viscoplastic model in a finite element program to study why the excess pore water pressure in the clay underneath the Tarsiut caisson increased after construction had been completed. They found that the creep compression nature of the clay is the main internal factor causing this increase. The volumetric strain (change) produced due to the dissipation of excess pore water pressure was described as an external factor which caused a decrease in effective stress and an increase in excess pore water pressure. It was also argued that for this increase in excess pore water pressure, the change in volumetric strain caused by the dissipation of excess pore water must be smaller than that caused by creep compression.

Many researchers have been working on the prediction of excess pore water pressure and its lateral and vertical distribution in PVD improved soft soils. Some of the outcomes are significant, although there are still discrepancies between the analytical results and the data measured from the field and lab. Also, an unanticipated increase in excess pore water pressure occurred in several cases at the end of primary consolidation for reasons that are still not clear.
2.3.7 Lateral Displacement

The lateral displacement of soft soils improved with prefabricated vertical drains was also studied by Shen et al. (2005). The calculated and measured data were compared with each other and there were significant differences between them. Also, the lateral displacement for the PVD improved case was much larger at the end of construction than for the unimproved case. However, the calculated lateral displacements were not so different. For PVD improved subsoil, the measured results showed that the place where maximum lateral displacement occurred was about 4 m deeper than for the unimproved case, although the calculated values were the same for unimproved and improved soils, and it was almost 6 m below the ground. It was mentioned that the exact reason was not completely clear but could probably be due to spatial variations in the sub-soil condition.

Indraratna et al. (2005b) investigated PVD improved soft soils incorporating the effects of vacuum preloading and found the results of the numerical lateral displacement did not match very well with the measured data, and this discrepancy was more noticeable in the weathered crust. The sources of this discrepancy were thought to be the anisotropy of soil, 2D plane strain modelling and inability of Cam-Clay theory to model the performance of the crust.

Further research is needed to obtain better results for analytical and numerical studies regarding the lateral displacement and excess pore water pressure to establish enhanced matching between the predicted results and measured data.
2.3.8 Effect of Vacuum Preloading

Kjellman (1952) was the first who proposed the idea of applying vacuum preloading to land reclamation. Afterwards, this technique has been successfully used in different projects worldwide (Holtz, 1975; Chen and Bao, 1983; Bergado et al. 1998; Chu et al. 2000; Indraratna et al. 2005a, c). A nominal 80 kPa of vacuum pressure was usually used in this design. In land reclamation where more applied load is needed, a combination of vacuum preloading and surcharge can be used, but in every case where a surcharge is applied, using vacuum preloading reduces the required height of the embankment and the cost of the project (Chu et al. 2000).

Two types of vacuum preloading systems are currently used commercially: vacuum preloading system with membrane (Figure 2.15a) and the membrane-less vacuum preloading system (Figure 2.15b). With the first one, a membrane is placed over the sand blanket to ensure an airtight region above the PVDs. In this system, a vacuum pressure is applied through the sand blanket to the surface of the soil and along the drains. The advantage of this system is that it accelerates the dissipation of excess pore water pressure radially towards the PVDs and vertically towards the surface. In a membrane-less system, vacuum pressure is applied directly along the drains and propagates laterally across the soil, but it is not applied to the ground surface. Each of these systems has its own advantages, but their effectiveness depends on the properties of the soil, the depth of the clay, drain spacing, the type and geometry of drains, and so on. Selecting which vacuum system to use is more or less based on empirical assessments and design experience.

A combined load system for vacuum and surcharge pressures was described
by Mohamedelhassan and Shang (2002). They adopted the one-dimensional Terzaghi’s consolidation theory. Figure 2.16 shows how the law of superposition has been used to analyse the mechanism of combined vacuum-surcharge loading. After which the average degree of consolidation due to the combined vacuum-surcharge loading can be expressed by Eq. 2.29.

\[ U_{ave} = 1 - \sum_{n=0}^{\infty} \frac{8}{(2n+1)^2 \pi^2} e^{\frac{-(2n+1)^2}{4} \pi^2 \frac{c_{vc} t}{H^2}} \]  

where, \( c_{vc} \) is the coefficient of consolidation representing the effects of vacuum – surcharge combined loading, and \( t \) is time.
The effects of vacuum preloading for a single vertical drain were studied analytically by Indraratna et al. (2005a), who verified their analytical approach by numerically using ABAQUS software. Indraratna et al. (2005a) also proposed two analytical models for axi-symmetric and equivalent plane strain conditions. By evaluating the excess pore pressure, consolidation settlement and time analyses, they considered the effects of the magnitude and distribution of vacuum pressure.

Figure 2.15: Vacuum-assisted preloading systems, (a) Membrane system, (b) Membrane-less system
Vacuum preloading was modelled by a uniformly constant negative pore pressure at the top of the soil and a linearly varying negative pore pressure along the drain. It was argued that a higher vacuum preloading ratio results in faster dissipation of pore pressure and a consequently higher rate of consolidation. Moreover, it was shown that the length of the drain has a reverse relationship with the magnitude of vacuum pressure propagated at the bottom of the drain.

Figure 2.16: Schematic diagram of vacuum preloading system, (a) Combined vacuum and surcharge load (b) Surcharge load (c) Vacuum load (after Mohamedelhassan and Shang, (2002))
Indraratna et al. (2005c) proposed an analytical and numerical model to analyse PVD-improved soft soils incorporating vacuum preloading. This model was also proposed for both axi-symmetric and equivalent plane strain conditions where the distribution of vacuum preloading along the drain and laterally across the soil was taken into account. Also, the effects of the smear zone and well resistance were incorporated. They used a numerical model to analyse the performance of an embankment and compared the results with the measured data. They also used the Modified Cam-Clay theory and ABAQUS finite element code in this study. They discovered that using vacuum preloading with vertical drains helps soft soils consolidate quicker providing that using suitable vacuum pressure ratio (usually more than 1.0). For short drains (less than 10 metres long), they argued that the distribution of vacuum pressure along the drains is nearly trapezoidal and tends to be triangular for long drains, where the maximum magnitude is at the top of the drain and zero at bottom. Moreover, they argued that considering the constant lateral distribution of the vacuum in the unit cells produces better results (but this is based on the drain spacing and is true for small spacing).

Vacuum pressure directly affects the pore water pressure and hence the effective stresses, while the surcharge pressure due to the embankment load increases the effective stress by dissipating the pore water pressure. Moreover, the surcharge pressure creates lateral outward movement of the subsoil near the toe of the embankment, while the vacuum pressure moved the subsoil laterally inward in this area. As a result, and due to the low shear strength of the soft subsoil, the surcharge pressure can create soil instability close to the toe of the embankment, whereas
vacuum pressure increases the stability of the soil.

2.4 Reliability of the Soil Parameters Used in the Existing Consolidation Models and Flow Relationship

Terzaghi (1925) developed the conventional theory of one-dimensional consolidation based on the assumptions that soil volume change is small compared to its initial volume (small strain theory), and coefficient of permeability, compressibility, and external load are constant. Schiffman (1958) further developed the solution by incorporating the influence of permeability variation and time-dependency of the loading during consolidation. Barron (1948) developed analytical solutions for radial consolidation for the cases of free strain and equal axial strain, which were further modified by various researchers including Kjellmam (1952), Yoshikuni and Nakanodo (1974), Onoue (1988), and Zeng and Xie (1989). Hansbo (1960) developed a radial consolidation model based on a non-linear flow relationship. The effects of a varying coefficient of horizontal permeability and coefficient of compressibility during radial consolidation of clay were incorporated into a Darcian based analytical model by Indraratna et al. (2005a). Sathananthan and Indraratna (2006) developed a plane-strain lateral consolidation model which is applicable to both Darcian and non-Darcian flows. The results from the proposed model provide a more realistic lateral distribution of the excess pore water pressure than those predicted under Darcian flow conditions. Most of the existing analytical models for radial consolidation are based on the Darcy’s law for permeability which might not be the case for the consolidation of fine-grained soils under low hydraulic gradients (Hansbo, 1960; 2001). All of the existing Darcian-based and non-Darcian based
models rely on the accurate determination of the coefficient of permeability and coefficient of consolidation. Different techniques for the measurement of coefficient of permeability results different values (e.g. Olsen, 1965; 1966). The value of the coefficient of consolidation also depends on the adopted calculation method (e.g. Vinod, et al. 2010).

Both linear and non-linear relationships between flow velocity and hydraulic gradient have been observed in fine-grained clays (e.g. Hansbo, 1960; Olsen, 1966). In general, there is no unique relationship between the flow rate and hydraulic gradient due to the factors such as the applied hydraulic gradient and seepage induced consolidation (Olsen, 1966). During radial consolidation, phenomena such as the migration of grains, seepage induced consolidation, clogging of the flow channels, reorientation of the grains, change in the hydraulic gradient, possible change in temperature, and change in the average viscosity of pore water due to the change in pores, might occur simultaneously. This may produce a relationship between the flow rate and hydraulic gradient during radial consolidation which is completely different from those that were determined based on conventional permeability or oedometer tests. Therefore, to capture more realistic flow characteristics, the actual flow relationship must be obtained during consolidation.

2.5 Implication of Non-Darcian Flow

It has been shown experimentally that Darcy’s law for permeability is not always valid for a low porosity soil (Winterkorn, 1954; and Schmid, 1957) and for seepage flow under low hydraulic gradients (Buissong, 1953; and Kezdi, 1958). Hansbo (1960) also showed that the relationship between flow velocity and hydraulic
gradient of normally consolidated plastic clays under low hydraulic gradients deviates from Darcy’s law. Based on the laboratory observations, a relationship between the flow velocity and hydraulic gradient was proposed to capture a non-linear correlation under low hydraulic gradient, followed by a linear relationship at a higher hydraulic gradient (Fig. 2.17). The relationships between the flow velocity and hydraulic gradient are expressed by (Hansbo, 1960):

\[ v = \kappa i^m \quad \text{when} \quad i \leq i_t \]  

\[ v = k(i - i_0) \quad \text{when} \quad i \geq i_t \]  

\[ i_0 = i_t (m - 1)/m \]
where, $v$ is the flow velocity, $i$ is the hydraulic gradient, $m$ is a constant which depends on the soil type, void ratio, and temperature, $\kappa$ is the coefficient of permeability based on non-linear flow and $k$ is the coefficient of permeability based on linear flow for a given soil.

The relationship between $\kappa$ and $k$ is given by:

$$k = m\kappa i^m$$  \hspace{1cm} (2.33)

It is shown that the best agreement between the analytical and field data could be obtained if $m$ equals 1.5 (Hansbo, 1960; 2001).

According to Hansbo (1960), $i_l$ is the hydraulic gradient needed to overcome the maximum binding energy of mobile pore water and it can vary between 4 and 10; however, Dubin and Moulin (1986) reported that $i_l$ values can be in the range of 8 to 35. Based on the above equations, Hansbo (1960) developed a radial consolidation model that captured non-Darcian flow and explained that the hydraulic gradient in the field is much lower than the hydraulic gradient in a small scale laboratory test on radial consolidation. The relationship between the field and laboratory conditions based on hydraulic gradient can then be determined by:

$$\left( \frac{\partial u}{\partial r} \right)_t = \frac{D_f}{D_l} \left( \frac{\partial u}{\partial r} \right)_f$$  \hspace{1cm} (2.34)
where, \( u \) is the excess pore water pressure, \( D \) is the influence zone diameter, \( r \) is the radius measured from the centre of the drain, and subscripts \( f \) and \( l \) represent the field and laboratory conditions, respectively.

Equation 2.34 shows that the pore water pressure gradients that occur in the small scale laboratory tests are much higher than those in the field. Hansbo (1960) emphasised that an inaccurate prediction can result from this inconsistency, if the proposed limit (\( \ell_L \)) between the non-linear and linear relationships of the pore water flow and hydraulic gradient (Fig. 2.17) exceeds in the laboratory tests but not in the field. Therefore this non-linear model is only applicable to field conditions.

Several previous test results of saturated clays have also shown a deviation from Darcy’s law (e.g. Lutz and Kemper, 1959; Miller and Low, 1963; Zou 1996). However, Olsen (1965, 1966) showed that those deviations can be attributed to possible errors in the conventional measuring techniques for obtaining the permeability of fine-grained soils. The errors can be due to atmospheric contamination, the long time intervals needed to obtain a measurable flow rate, the large hydraulic gradients used in the laboratory tests, and the partially confined or unconfined clay fractions within the rigid skeletons of coarser particles in the laboratory tests. Olsen (1966) stated that resolving these errors can result in relationships between the flow rate and hydraulic gradient that imply validation of Darcy’s law. Probable exceptions can be observed in extremely fine-grained clays where large hydraulic gradients exist in shallow unconfined sediments (Olsen, 1966). Mitchell and Younger (1967) also emphasised that the hydraulic gradient in the laboratory during consolidation tests was much larger than in field conditions.
However, in contrast to Olsen’s (1966) opinion, they argued that at low hydraulic gradients the deviations from Darcy’s law are very significant and any analysis of field consolidation based on the laboratory results requires scrutiny. Pane et al. (1983) argued that very small hydraulic gradient conditions are required to reliably measure the permeability of soft clays, while a higher applied hydraulic gradient (i.e. >1.4) can induce some consolidation during a permeability test. Several studies have reported that possibly attributed to this, the hydraulic conductivity decreases as the hydraulic gradient increases (e.g. Mitchell and Younger, 1967; Gairon and Swartzendruber, 1975; Foreman and Daniel, 1984; Acar et al. 1985; Edil and Erickson, 1985; Carpenter and Stephenson, 1986; Kodikara and Rahman, 2002).

2.6 Summary
Consolidating soft soils via the radial drainage of pore water and using different types of vertical drains for this purpose, has been extensively carried out. The design and prediction of soil behaviour during radial consolidation alone, or a combination of radial and vertical consolidation is much more sophisticated than vertical consolidation alone. Different parameters are involved in this sophisticated process, some of which are:

- The 3D nature of radial consolidation compared to the 1D nature of vertical consolidation,

- The process of installing drains which disturbs the soil and changes the parameters of the soil near the drains,

- Difficulties with measuring the radial consolidation parameters of the soil in the laboratory,
Sophisticated analytical and numerical methods which must be used for radial consolidation, and

Unclear nature of vacuum preloading consolidation when used in conjunction with vertical drains.

However, numerous analytical models have been developed and effectively used for radial consolidation, but there is still a need to use numerical assessment in some cases, due to the inefficiency of the existing analytical models or sophisticated nature of the problems. Converting the actual 3D radial consolidation problems to the equivalent 2D plane strain ones for analysis and design would simplify the analytical procedure and reduce the complexity of most cases.

For vacuum assisted cases, especially when the vacuum is combined with the surcharge load, extensive research still needs to be conducted to clarify the nature of vacuum propagation inside the soil and to find the effects of vacuum compared to the surcharge pressure. Parameters such as (a) amount of vacuum required, (b) optimum time to remove the vacuum, (c) most effective vacuum-surcharge ratio, and (d) effects of vacuum on the rate of consolidation and dissipation of pore pressure, are some important and practical issues which are not completely understood. In this research, an attempt has been made to clarify the above mentioned effects of vacuum pressure on the consolidation of soft soils, in the following chapters.
3 EXPERIMENTAL PROCEDURE

3.1 General
The oedometer apparatus has been used to determine the consolidation parameters of low permeable soils since it was first introduced by Terzaghi (1925). Oedometer tests are among the most commercially used tests by practitioners and researchers to find the compressibility and coefficient of consolidation of fine grained soils. The compressibility of soil is expressed in terms of the modulus of volume change or the coefficient of volume compressibility. It is a parameter that represents a change in the volume of soil when it is loaded, and it is used to calculate the total consolidation settlement of soil under any specific load. The time-dependent behaviour of soil during consolidation is represented by the coefficient of consolidation (Eq. 2.5) which is a measure of the rate of settlement.

Although, conventional oedometers have been extensively utilized, they do have the following limitations:

- The samples are usually small,
- The drainage path is usually in the vertical direction and therefore only one dimensional drainage is possible,
- Pore pressures cannot be measured in conventional oedometers,
- Soil deformation is only in the vertical direction and therefore only 1D consolidation happens in the oedometers,
- The loading disc is rigid, which means that oedometers can only model equal
strain conditions, and

- Only step loadings can be applied to the samples of soil in the oedometer tests.

Some researchers have overcome a number of these limitations by applying further modifications to the oedometers for specific studies. Although, under more sophisticated conditions such as radial drainage including vertical drains and vacuum preloading, conventional oedometers are inadequate. One piece of equipment that does not suffer from the same limitations as oedometers, and can be used to study the effects of radial consolidation and vacuum preloading on soil, is the Rowe cell. With this device, different loading systems, assorted drainage paths, various drainage boundary conditions, and diverse strain conditions can be studied. Moreover, Rowe cells can be further modified to capture the lateral distribution of vacuum pressure.

In this chapter, the advantages of the standard Rowe cells are discussed and one of them is further modified to capture additional consolidation parameters such as the lateral distribution of pore and/or vacuum pressures in soft compressible soils. Also, the problem of the pervious material that commercially is used in the cells is discussed and a substitution is suggested. The complete hardware and software used during experimental studies are then explained in detail.

## 3.2 Standard Rowe Cells

### 3.2.1 History and Advancement of Rowe Cells

The standard Rowe cells which are commercially used nowadays were first developed at Manchester University (Rowe and Barden, 1966). The main reason for developing these cells was to overcome most of the disadvantages of conventional
oedometer apparatus. Figure 3.1 shows one of the first cells developed by Rowe and Barden, and Figure 3.2 represents a schematic drawing of the Rowe hydraulic consolidation cell, including its main features. Unlike oedometer where samples of soil are loaded mechanically through a lever system, a hydraulic loading system is utilised in the Rowe cells where water pressure is applied onto a convoluted flexible diaphragm. Large diameter samples of soil can be tested and large settlement deformations are allowed and measureable with this loading arrangement.

Rowe (1954) introduced the principle of a diaphragm loading system for confined compression tests on sand. Air pressure was applied onto a flat flexible membrane in contact with a 10” diameter sample of soil. In 1966 and 1967 at Manchester University, four different size cells (3 in, 6 in, 10 in, and 20 in diameter) were built and became commercially available. To measure pore water pressures
during the tests, all the cells were fitted with pore pressure transducers. Bishop and co-workers at Imperial College independently designed two different sizes of hydraulic oedometers fitted with the flexible membranes (Simons and Beng, 1969). The samples which could be tested in these two oedometers were 1” high by 4” in diameter, and ¾” high by 3” in diameter, and the pore water pressure could also be measured during the tests.

To minimise the entrapped air in the soft, convoluted membrane, and also to reduce the possibility of the air diffusing, Barden (1974) used a modified 10” Rowe cell fitted with a flat diaphragm made from butyl rubber. Barden did this in order to examine the consolidation of unsaturated clays. Measuring pore water pressure in consolidation tests goes back to the research experiments in the USA. The

---

Figure 3.2: Main features of the Rowe hydraulic consolidation cell
consolidation of soil in triaxial equipment while measuring the pore water pressure had been established in Britain by Bishop and Henkel (1957). Leonards and Girault (1961) used a manually controlled mercury/water pore pressure device and measured the pore water pressures in a 112 mm diameter conventional oedometer cell. A major advancement in the measurement of pore water pressure was made when an electrical pressure transducer was utilised by Whitman et al. (1969). Afterwards, pore pressure transducers and related monitoring devices attached to soil testing equipment, enabled Rowe and Barden to use them in their cells from the beginning.

Nowadays, Rowe cells are produced commercially, in three different sizes, 75 mm, 150 mm, and 250 mm in diameter. These sizes are actually the nominal inner diameter of the cells. The dimensions of the Rowe cells are listed in Table 3-1. 75 mm and 150 mm Rowe cells are usually fitted with two points at the base of the cells to measure the pore water pressures. At these points, small, porous discs are inserted at the top of the two channels connected to the pore pressure transducers. One of these points is at the centre of the base plate and the other is 0.55 times the radius of the cell R away. In 250 mm cells, there are four points where the pore water pressures may be measured, and again, one of them is at the centre and the others are 0.55R, 0.1R, and 0.9R away.
Table 3-1: Dimensions of Rowe cells (Head, 1986)

<table>
<thead>
<tr>
<th>Nominal diameter</th>
<th>3 in (75 mm)</th>
<th>6 in (150 mm)</th>
<th>10 in (250 mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Sample diameter:</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Exact equivalent</td>
<td>( mm ) 76.2</td>
<td>152.4</td>
<td>254</td>
</tr>
<tr>
<td>New series</td>
<td>( mm ) 75.7</td>
<td>151.4</td>
<td>252.3</td>
</tr>
<tr>
<td><strong>Sample area</strong></td>
<td>( mm^2 ) 4560</td>
<td>4500</td>
<td>18241</td>
</tr>
<tr>
<td><strong>Recommended sample height</strong></td>
<td>( mm ) 30</td>
<td>50</td>
<td>90</td>
</tr>
<tr>
<td><strong>Sample volume (based on recommended height)</strong></td>
<td>( cm^3 ) 136.8</td>
<td>135</td>
<td>912.2</td>
</tr>
</tbody>
</table>

### 3.2.2 Horizontal Drainage During Consolidation

Bishop and Henkel (1957) utilised filter papers as peripheral drains to carry out drainage in the radial (horizontal) direction for the first time during consolidation. Subsequently Rowe (1959) showed that the side filter papers in the triaxial cells cannot form an effective drain for lacustrine clay. This inefficiency was concluded by Escario and Uriel (1961) when they satisfactorily used a surrounding layer of 5 mm thick micaceous sand instead. In a standard oedometer apparatus, McKinlay (1961) utilised a porous stainless steel ring instead of the conventional one to carry out radial consolidation tests. A peripheral drain made from Vylon porous plastic (normally 1.5 mm thick), which was used by Rowe and Barden in their cells, has proven that it can play the role of an efficient drain and provide free drainage.

Rowe (1959) carried out radial consolidation tests using conventional oedometer equipment, with an inwards flow towards a central sand drain. For the radial consolidation tests of layered soils, Rowe and Shields (1965) used a central vertical sand drain with different sized samples and modelled the horizontal drainage
layers using porous ceramic discs.

### 3.2.3 Salient Features and Advantages of the Rowe Cell

Several advantages of the Rowe cell can be distinguished compared to conventional oedometer apparatus. The hydraulic loading system, the control facilities, the ability to measure pore water pressure, and testing large diameter samples are the main characteristics responsible for these gains. The advantages of the Rowe cell due to its hydraulic loading system can be summarised as:

- Applying axial pressure hydraulically via air or water pressure onto a flexible membrane places less vibration onto the samples of soil, compared to the traditional lever loading system of oedometers.
- High pressure, usually up to 1000 kPa, can be easily applied even to the large samples.

Except for the very stiff samples, deformation of the loading system can be ignored and no correction is required.

The control facilities available in the Rowe cells are as follows:

- Individual and combined vertical and radial drainage can be imposed onto the samples.
- Undrained conditions to apply a load can be modelled by closing the drainage valves. This enables full development of excess pore water pressure before consolidation begins. Moreover, the initial instant deformation due to opening the drainage line can be measured independently from the consolidation settlement.
- Immediate response of the pore water pressure to loading/deformation
conditions can be easily tracked, enabling the beginning and end of the primary consolidation phase to be established.

- In addition to surface settlement, changes in the volume of soil can be measured using the volume of water draining out from the samples.

- Incremental back pressure can be utilised to saturate the sample until the required B value or effective stress has been obtained before starting the consolidation stage.

- An elevated back pressure can be used to ensure full saturation of the sample during the test. This gives a rapid response to the pore water pressure and reliable time relationships (Lowe et al., 1964).

- Both rigid and flexible plates can be fitted on top of the sample to model equal strain and free strain conditions, respectively.

- Precise loading control, including initial loading at low pressure, can easily be carried out.

- Vacuum pressure can be applied to the surface and/or base of the sample, or into the vertical central drain, or to the peripheral area of the sample.

The following points related to the size of the samples in the Rowe cell may be summarised:

- Data that is more reliable can generally be gathered from the larger samples. In comparison with the size of the samples in the traditional oedometer tests, it is reported that higher and more reliable values of the coefficient of consolidation \( (c_v) \) can be measured with samples that are 150 mm diameter by 50 mm thick, or even larger (McGown et al., 1974). With the larger
samples, Lo et al. (1976) reported better agreement between the predicted and measured values of rate of displacement and the magnitude of settlement. This might be attributed to the fact that the structural viscosity has less effect in larger samples.

- In layered deposits, the fabric of the soil has critical effects on the drainage pattern where it can be captured using larger samples, thus enabling a more realistic estimation of the rate of consolidation (Rowe, 1968; 1972).

- Preparing the smaller samples creates more disturbances in the soil, which affects the \( e - \log p' \) plot and may also obscure the stress history. Consequently, the pre-consolidation pressure and over-consolidation ratio can be underestimated. Moreover, at lower stresses a much lower coefficient of volume compressibility \( (m_v) \) can be obtained. Utilising high quality, large diameter samples minimises these deficiencies because the microfabric of the soil is not as disturbed as much.

- More reliable measurements of the vertical and horizontal permeabilities of soil can be attained using larger samples which produce the fabric of the soil much better.

- The effects of a central drain can be more accurately modelled and analysed by using larger Rowe cells.

### 3.2.4 Types of Tests

Consolidation tests under four different conditions of drainage and subjected to two distinct types of loading (“free strain” or “equal strain”) can be simulated using a Rowe cell. Figure 3.3 represents all the different drainage and loading conditions for
consolidation tests that are applicable for a Rowe cell. Figure 3.3a and b, show consolidation with one way vertical drainage for “free strain” and “equal strain” conditions, respectively. Similar cases for two way drainage conditions are shown in Figure 3.3c and d. Figure 3.3e and f correspond to consolidation with outward radial drainage for cases of “free strain” and “equal strain”, respectively. Similarly, inward radial drainage towards a central vertical drain is represented in Figure 3.3g and h.

In addition of these capabilities, a Rowe cell can be utilised to run the permeability tests to find the permeability of soil in both vertical and horizontal directions with a different setup in the cell. Figure 3.4 illustrates four diverse types of permeability tests using a Rowe cell. Two back pressures with different values are needed to run the permeability tests with a Rowe cell. These tests can be conducted independently or during the consolidation tests. This enables the permeability of the soil to be measured continuously under changing effective stresses and void ratios during consolidation deformation. Figure 3.4a represents the permeability test with a vertical flow of water upwards and Figure 3.4b shows the same, but downwards. Permeability tests with radial drainage of water outwards to a pervious peripheral lining and inwards to a central drain are shown in Figure 3.4c and d, respectively.
Figure 3.3: Loading and drainage conditions for consolidation in Rowe cell (a), (c), (e), and (g) “Free strain” loading, (b), (d), (f), and (h) “Equal strain” loading.
3.3 Effects of the Drainage Material

As previously explained, there are small holes (channels) at the bottom of the Rowe cell base plate for drainage or to measure the pore water pressure. Small pervious discs must be inserted on top of these holes to prevent soil from clogging the pore pressure measurement system. The type of pervious material and the size of their apertures is very important because the apertures must be small enough to prevent grains of soil from draining out of the cell while being large enough to be more permeable than the soil to allow the water to drain out freely as the soil consolidates.

Figure 3.4: *Permeability tests in Rowe cell with different flow conditions, (a), (b) vertical flow conditions, (c), (d) radial flow conditions*
Some manufacturers producing Rowe cells insert small Vylon discs into the top of the drainage channels. These discs are usually 2.5-5.0 mm thick, and it has been reported that Vylon provides reasonably free drainage when it is used as a pervious peripheral lining sheet or a flexible pervious sheet (disc) at the top of the sample (see Figure 3.3). However, Vylon is not a suitable material to use as the pervious discs inserted into the drainage channels of the Rowe cell. To examine this disadvantage of Vylon, several tests have been done with two different drainage materials.

Four different tests have been done to discover the best size and type of material to use for the pervious discs used as draining elements at the base of Rowe cells. A standard 75 mm diameter Rowe cell was used for these tests. Two different materials, Vylon and sintered Bronze (Grade 30B) were used for the small pervious discs. Table 3-2 includes a summary of the dimensions of discs used in these tests and Figure 3.5 shows the discs.

Table 3-2: Dimension information of the drainage discs

<table>
<thead>
<tr>
<th>Disc type</th>
<th>Vylon</th>
<th>Sintered Bronze</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>At central location</td>
<td>At distance</td>
<td>At central location</td>
</tr>
<tr>
<td>diameter</td>
<td>thickness</td>
<td>diameter</td>
<td>thickness</td>
</tr>
<tr>
<td>12.0</td>
<td>2.7</td>
<td>8.5</td>
<td>2.7</td>
</tr>
</tbody>
</table>
Figure 3.6 shows a 75 mm Rowe cell, the location where pore pressure measurements are taken at the base of cell, and the hardware used for the tests. The following is information from the four tests:

Test No. 1: Vylon discs were inserted into the holes and the cell was filled with de-aired water. Then under undrained conditions, 50 kPa axial pressure was applied to the sample (water) and the response of the pore pressure transducers at two locations (in the centre and at a distance $r = 0.55R$) was recorded.

Test No. 2: Same as Test No. 1, but instead of water, a Kaolin (liquid limit=50%) slurry with 70% water content was prepared and used as the soil sample.

Test No. 3: Same as Test No. 1, but instead of Vylon, sintered Bronze discs inserted into the drainage holes.
Test No. 4: Same as Test No. 3, but instead of water the same Kaolin slurry used in Test No. 2 was used in this test.

In addition to the type and permeability of the drainage discs, the viscosity and stiffness of the sample in the cell also has an effect on the time period during which the applied axial pressure is completely captured by the pore pressure transducers. Therefore, the water used in the cell in Tests No. 1 and 3 was to eliminate the effects of viscosity and stiffness in the samples. This was actually conducted to first examine the workability of the draining discs, regardless of the properties of the samples of soil.

Figure 3.7 shows the results of these tests, from which the following points are notable:

- Eliminating the viscosity and stiffness of the sample by using water in the cell (Tests No. 1 and 3, Figure 3.7) shows that the type of material used for the discs at the drainage holes, and their size, have no effect on the transformation of the applied surcharge to the transducers. This is easily visible from the figure where all of the relevant graphs coincide with each other, regardless of the type and size of the discs.

- The results of Test No. 4 show that where sintered Bronze was used in the drainage holes with Kaolin slurry as the soil sample, both of the discs responded quickly and in exactly the same way (in around 100 seconds the transducers showed that the B value exceeded from 0.95).

For the Vylon discs with Kaolin slurry (Test No. 2), even after 200 seconds, the transducers showed that the B value still did not reach 0.95, and showed constant
values thereafter. Also, the smaller disc showed a lower rate of increase in pore pressure which was completely different from the larger one.

Figure 3.6: (a) 75 mm standard Rowe cell, (b) Location of pore pressure measurement at the base of 75 mm Rowe cell, and (c) Hardware setup for consolidation tests with 75 mm Rowe cell
The difference between the sintered Bronze and Vylon discs (and also between the two sizes of Vylon discs) is greater when a stiffer sample or a soil with a higher viscosity is used in the tests. From that, it can be concluded that sintered Bronze discs (Grade 30B) can effectively be used as the drainage material inserted into the holes at the base of the Rowe cells, and provide acceptable free drainage conditions.

3.4 Modified 150 mm Diameter Rowe Cell

3.4.1 General Features of the Modified Cell

It was decided to use the 75 mm and 150 mm Rowe cells for different purposes...
during this research. Moreover, it was decided to use remolded samples instead of undisturbed ones since the isotropic homogenous samples would eliminate any other effect which can alter the results of the tests. Furthermore, preparing slurries and pre-consolidating them inside the Rowe cell was the best way to eliminate any possible disturbance to the sample when transferring it from the preconsolidation cell to the Rowe cell. Pore water pressure at different radii needed to be monitored during radial consolidation for some of the tests. Therefore a suitable consolidation cell with several locations for pore water pressure measurement should be used.

A 250 mm standard Rowe cell could be the ideal size to fulfill the requirements of the study since there are four positions at the base of the cell at different radii where pore pressure measurement can be taken. Also, the cell is high enough to allow the slurries to be preconsolidated internally. However, due to the size of the cell and the time required to pre-consolidate and then consolidate samples of this size, and the time needed to run the required numbers of tests, it was decided to modify a 150 mm Rowe cell.

In a standard 150 mm Rowe cell there is only one (excluding the central one) location where the pore water pressure can be measured and the cell might not be high enough to have a sample with reasonable thickness after pre-consolidating the slurry in the cell, so a new, modified 150 mm Rowe cell was designed and fabricated at the University of Wollongong. In this cell the height of the main body of the cell was increased and four locations (including the central one) were provided for pore water pressure measurement at different radii. Figure 3.8 shows the modified 150 mm Rowe cell compared with the standard cell, and also the locations at the base of
the cell where pore pressures can be measured. Information regarding the dimensions is summarised in Table 3-3 as well.

<table>
<thead>
<tr>
<th>Sample size</th>
<th>Number of locations for pore pressure measurement</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard Rowe cell</td>
<td>Developed Rowe cell</td>
</tr>
<tr>
<td>Sample height (mm)</td>
<td>Diameter (mm)</td>
</tr>
<tr>
<td>50</td>
<td>150</td>
</tr>
<tr>
<td>80</td>
<td>150</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
</tr>
</tbody>
</table>

### 3.4.2 Capabilities of the Modified Cell

When disturbed samples in the form of a slurry are used in a Rowe cell, the thickness of the samples after pre-consolidation, depending on the properties of the soil and initial water content, would be approximately half its initial thickness. This means that with a standard 150 mm Rowe cell, the thickness of the sample after pre-consolidation might reduce to 25 mm, which is slightly more than the thickness of a sample from a conventional oedometer. This size would not represent the microfabric of the soil as would be expected from large Rowe cells, and also, the effects of the boundary conditions might be prominent because the ratio of height to diameter of the sample may be around 1:6. By increasing the body height of the Rowe cell in the modified cell, a sample almost twice as thick as one obtained in a standard cell, is achievable after the slurry has pre-consolidated. In essence, the sample after pre-consolidation in the modified Rowe cell is as thick as the initial thickness of the sample (slurry) in a standard Rowe cell.
Figure 3.8: (a) Modified 150 mm Rowe cell at University of Wollongong, (b) Assembled developed and standard 150 mm Rowe cells, (c) Comparison of the cell body height of the developed and standard cells, (d) and (e) Location of pore pressure measurement at the base of developed 150 mm Rowe cell.
The other advantage of the modified 150 mm Rowe cell is its ability to capture the lateral distribution of pore water pressure during radial consolidation tests. Moreover, when a vacuum pressure is applied to the vertical drain inside the sample, the lateral propagation of the vacuum across the soil can be monitored. Moreover, a simultaneous measurement of pore water pressure with time and radius gives a 3D pattern of the excess pore water pressure dissipation, which clarifies the time dependency of the consolidation of any specific element of soil in three dimensional space. All of these advantages are realised in the following chapters.

3.5 Hardware Used in the Rowe Consolidation Tests

The hardware used in the Rowe consolidation tests were:

- A 75 mm standard Rowe cell and a modified 150 mm Rowe cell,
- A set of GDS pressure/volume controllers,
- An 8-channel GDS data logger,
- Pore pressure transducers,
- Displacement transducers,
- Vacuum pump, and
- Vacuum gauges and regulators.

Figure 3.9 shows the main hardware used in the experimental investigations. The Rowe cells and their features have already been described in detail. What follows now is a description of the other above mentioned equipment.
3.5.1 **GDS Pressure/Volume Controllers**

Figure 3.10 shows one of the GDS Enterprise Pressure/Volume Controllers used for the consolidation tests. This pressure controller can be used to apply cell (surcharge) pressure or back pressure. It uses de-aired water to apply pressure or to control the volume. This device can be controlled directly from a computer using compatible software, or as a completely stand-alone device with the Smart Keypad (Figure 3.10). In stand-alone mode it works as a constant pressure source which can replace traditional laboratory pressure sources such as mercury column, compressed air, pumped oil, and dead weight equipment. It also can be used in this mode as a gauge.
to measure the volumetric change of the sample. With PC based compatible software, this device can be used to run any consolidation tests under different conditions of stress and strain control.

![GDS Enterprise Level Pressure/Volume Controller](image)

**Figure 3.10: GDS Enterprise Level Pressure/Volume Controller**

The technical specification of this pressure controller are as follows: its maximum pressure range is 1000 kPa, the nominal volumetric capacity is 200cc, its resolution of measurement for pressure is 1 kPa and for volume is 1 mm$^3$. Its accuracy for measuring pressure is less than 0.25% of the full range and is less than 0.4% of the measured value for volume with less than +/-50 mm$^3$ backlash.

### 3.5.2 GDS Data Logger

Figure 3.11 shows an 8-channel GDS data logger used in the testing. It can simultaneously acquire data from eight different sources, such as displacement or pore pressure transducers.
3.5.3 Pore Pressure and Displacement Transducers

The pressure transducers used in the experimental investigations could accurately measure both positive and negative (vacuum) pore pressures. Negative pore pressures needed to be measured due to the application of vacuum pressure, and positive pore pressures needed to be measured due to the application of surcharge. The pressure range of these transducers was 2 bar (200 kPa) and their accuracy was 0.04% of the full scale. The accuracy of the pore pressure transducers usually decreases when their full range increases. Hence, it was decided to use the transducers with lower full scale. Figure 3.12a represents one of these transducers. Displacement transducers with a range of +/-12.5 mm were used in the tests. Their accuracy was 0.5% of the full range. Figure 3.12b shows one of them with an electrical connection. The reason for choosing transducers with a longer travelling distance was to be able to monitor higher displacements during the consolidation of soft soil, especially during the pre-consolidation stages of the tests.
3.5.4 Vacuum Pump, Gauges and Regulators

A Vector LD-5 vacuum pump was used to apply vacuum pressure to the samples during vacuum assisted consolidation tests. The high performance of these pumps by providing constant long lasting vacuum pressure can be mentioned as their main advantages for the required testing. This vacuum pump can produce a maximum 95 kPa for weeks or even months without any drop in the amount of vacuum. However, a maximum of 80 kPa vacuum pressure was used for the tests in this research. To control the amount of vacuum and to apply different vacuum pressures to the different tests, vacuum gauges and vacuum regulators were used between the vacuum pump and Rowe cells. Figure 3.14 shows the assembly of a vacuum regulator with a vacuum gauge used in the system.

Figure 3.12: (a) Pore Pressure Transducer, (b) Displacement Transducer
3.6 Software Used with the Experimental Investigations

The GADSLAB software was used in connection with the pressure controllers and data logger to automatically control the pressures applied to the samples in the Rowe cells and to collect the measured parameters from the data logger. GDSLAB software has the following key features:

Figure 3.13: Vector LD-5 Vacuum Pump

Figure 3.14: Vacuum Gauge and Vacuum Regulator
- Multiple tests can be run using a single computer. This enables having single hardware (i.e. data loggers) connected to the same computer but they are shared between different test stations.

- Users can set up multiple test stages before commencing testing or at any specific time during testing.

- It is possible to pause, stop, or resume the test stages at any point in time. Test stages can also be set to automatically stop on a number of user defined criteria (e.g. maximum surcharge pressure).

- Several parameters relevant to the measured values can be calculated and displayed on the monitor.

- Data is saved to a data file and the user can specify the time interval on a linear, square root, or log scale.

- All the measured and calculated data can be displayed graphically on the monitor in real time, and the data and graphs displayed can be changed at any time before starting or during testing.

- Different test stations can be independently configured, started, run, or stopped.
Figure 3.15: (a) Different Test Types can be run by GDSLAB software, and (b) An Example of a Live Graph and data Shown on Screen
Different types of tests, such as “Step Loading”, “Constant Rate of Strain”, “Constant Rate of Load”, and “Constant Stress” can be run with GDSLAB software (Figure 3.15a). Figure 3.15b gives an example of a live graph and data in GDSLAB software shown on the computer screen.

### 3.7 Summary

Due to the advantages of Rowe cell, it was decided to use this type of the equipment for radial consolidation tests conducted with and without vacuum pressure. The hydraulic system used to apply the load, with capability of simulating different types of vertical and radial consolidation tests, and the ability to apply vacuum pressure in conjunction with surcharge pressure can be mentioned as some of the main features of the Rowe cell.

A modified 150 mm Rowe cell was designed and used to preconsolidate the slurries inside the cell itself, as well as to capture the lateral propagation of pore water pressure and vacuum pressure during consolidation tests.

A complete and compatible set of hardware and software were used to run the consolidation tests. With this set, vacuum assisted radial consolidation tests and normal consolidation tests with surcharge pressure alone could be run, and required parameters could be measured, saved, and even analysed during the tests.

The capabilities of the systems used in this research will be realised more in the following chapters when the results of the tests are presented.
4 LABORATORY EVALUATION OF COEFFICIENT OF CONSOLIDATION DUE TO THE APPLICATION OF VACUUM PRESSURE

4.1 General
The best way to examine whether a parameter has a noticeable effect on the process of consolidation is to analyse its effects on the degree of consolidation (DOC). Therefore, to determine whether vacuum pressure works exactly the same as surcharge pressure or whether it affects the soil differently, one of the best ways is to consider the degree of consolidation. That means if vacuum affects the process of consolidation, substituting some part of the surcharge pressure with vacuum pressure would result in a different degree of consolidation.

It is already argued that vacuum pressure in conjunction with vertical drains affects consolidation and accelerates the rate of consolidation (e.g. Indraratna et al., 2005a). In that case, when a combined surcharge and vacuum pressure are applied to the soil, not only must the degree of consolidation be affected, but also the back calculated coefficient of consolidation should differ from the conventionally calculated one. The coefficient of consolidation is normally calculated based on the settlement curves that resulted from oedometer or other similar testing, without the application of vacuum.

Another interesting issue is whether the degree of consolidation should be calculated based on settlement or the dissipation of excess pore water pressure. This question usually arises for any kind of consolidation problem regardless of the water
drainage path and application of surcharge and/or vacuum pressures. Therefore, related to vacuum assisted problems, this question should be considered as well.

To this end, the main aim of this chapter is to answer the following questions:

- Is there any difference between DOC calculated based on settlement and pore pressure dissipation when vacuum assisted consolidation problems are considered?
- What is the effect of vacuum pressure on the degree of consolidation?
- What is the effect of vacuum pressure on the coefficient of radial consolidation?

To answer these questions in this chapter of research, two different size Rowe cells (a 75mm standard cell and a 150 mm modified one) were used to simulate vacuum assisted radial consolidation with different vacuum pressure-total surcharge ratio (VSR). VSR is defined as VP/TP = VP/(SP+VP), where VP is the magnitude of the applied vacuum pressure, SP is the surcharge pressure, and TP = SP+VP is the total pressure. The degree of consolidation for different values of VSR was determined based on settlement and excess pore pressure data. The coefficient of horizontal consolidation \( c_h \) was then back calculated and the variation of \( c_h \) based on the two techniques was compared. At the end, a semi-empirical model for predicting the coefficient of horizontal consolidation is proposed based on the variation of VSR.

4.2 Background

Gibson et al. (1967) derived the one-dimensional consolidation equations in a non-
linear form for a thin layer of saturated clay layer for the large strain condition, while considering the variable compressibility and permeability of the soil. Based on their proposed assumptions, the DOC calculated from the strain and excess pore water pressure can be distinctly differentiated. Later, Gibson et al. (1981), based on a finite non-linear consolidation theory for thick homogeneous layers, showed that the conventional consolidation theory seriously underestimates the excess pore water pressure ($u_w$), which results in an over estimation of the effective stress.

Lee et al. (1992) developed an explicit analytical solution for the one-dimensional consolidation of layered soils. They also showed that the overall average degree of consolidation corresponding to settlement is different from that governed by $u_w$. More recently, Xie and Leo (2004) developed an explicit analytical solution for 1D large strain consolidation where they showed that unlike classical small strain theory, the average DOC as separately defined by the pore pressure and strain, are also different theoretically.

To further investigate the consolidation of soils in 1D and 2D spaces with variable properties, Huang et al. (2010) combined the coupled Biot (Biot, 1941) consolidation theory with the finite element method and demonstrated that the independent analyses of excess pore pressure and settlement cannot provide the same DOC.

In the field, the time of vacuum pressure and surcharge load removal is critical to achieve the desired DOC. Settlement data are commonly used to calculate the DOC and to back calculate the coefficient of lateral consolidation (e.g. Sridharan et al., 1996; Vinod et al. 2010). In vacuum consolidation, the change in $u_w$ reflects the
efficiency of the vacuum system, so it is imperative to assess the degree of consolidation based on excess pore water pressure dissipation (Chu and Yan, 2005).

Surcharge pressure and vacuum pressure affect the soil in two different ways. When a surcharge load is applied to a saturated fine grain soil, it transfers immediately and completely to the pore water pressure. A hydraulic gradient is then created between the elements inside the soil mass and the elements on the boundary, which in turn results in the movement of pore water towards the drainage boundary. This drainage of water out of the soil dissipates excess pore pressures and transfers it to the effective stresses.

While, a vacuum pressure does not have any immediate effect on the pore water pressures of the elements inside the soil mass, it immediately reduces the water pressure at the boundary to negative values, which again creates a hydraulic gradient. The pore water is then sucked out of the soil due to this hydraulic gradient creating negative pressures inside the pores. Suction inside the soil brings the particles closer together, while increasing the effective stresses. Due to this difference of surcharge and vacuum pressures, their effects on the consolidation rate would be differentiated as well.

In the rest of this chapter, a systematic approach has been adopted to study the possible effects of vacuum pressure on the DOC and coefficient of radial consolidation, which both show the effects on the rate of consolidation.

### 4.3 Location of Average Pore Water Pressure

The degree of consolidation and the coefficient of radial consolidation can be calculated based on the trends of average excess pore water pressure ($\bar{u}_w$) or
settlement. For the former, excess pore water pressure \((u_w)\) needs to be simultaneously measured at different radii to enable \(\bar{u}_w\) to be calculated, or it needs to be measured at a location that represents \(\bar{u}_w\). Under the Rowe cell condition where the smear zone can be ignored, the magnitude of \(\bar{u}_w\) can be determined from Eq. 4.1 (Barron, 1948):

\[
\bar{u}_w = \bar{u}_{w0}\exp \left( -\frac{\beta T_h}{\mu} \right)
\]

where, \(\bar{u}_w\) is the average excess pore water pressure in the unit cell, \(\bar{u}_{w0}\) is the initial average excess pore pressure, \(T_h = \frac{c h t}{D_e^2}\) is the time factor for radial consolidation, \(t\) is time, \(D_e = 2R\) is the diameter of the unit cell, and 
\[
\mu = \left[ \frac{n^2}{n^2 - 1} \ln(n) - \frac{(3n^2 - 1)}{4n^2} \right],
\]

where \(n = \frac{D_e}{D_w}\), and \(D_w\) is the diameter of the drain. Also, the excess pore pressure at any time as a function of radius in a unit cell can be represented by Eq. 4.2 (Indraratna et al. 2005b):

\[
u_w(r) = \frac{\bar{u}_{w0}}{\mu R^2} \exp \left( -\frac{\beta T_h}{\mu} \right) \left[ R^2 \ln \left( \frac{r}{R_w} \right) - \frac{(r^2 - r_w^2)}{2} \right]
\]

where, \(u_w(r)\) is the excess pore water pressure due to the surcharge at radius \(r\) measured from the centre of the unit cell. Combining Eqs. 4.1 and 4.2 results Eq. 4.3:
\[
\ln \left( \frac{nR}{r} \right) - 0.5 \left( \frac{r^2}{R^2} - \frac{1}{n^2} \right) - \mu = 0
\] 4.3

Equation 4.3 gives the theoretical loci of the \(\bar{u}_w\) in a unit cell. Based on this equation, the location of the equivalent average excess pore pressure \(\left( \frac{r^2}{R^2} \right)\) versus \(n\) can be plotted (Figure 4.1). With this plot it is easy to position the location of the equivalent average pore water pressure inside a unit cell for any given \(n\) value. Then, with a single direct measurement, the average pore pressure can be captured without any calculation.

![Figure 4.1: Theoretical location of the average excess pore water pressure in a unit cell based on Equation 4.3](image)

For the standard Rowe cell (75mm diameter), the DOC based on pore pressure can be directly determined using Eq. 4.1 if the \(\bar{u}_w\) can be determined from
the measured data. For radial consolidation under surcharge loading only, Shields (1963) showed that the location of the $u_w$ in a standard Rowe cell with $n = 20$ is located at a distance of $0.55R$ from the centre of the cell. To find the location of $u_w$ in a unit cell under the effects of combined surcharge and vacuum pressures, the modified 150 mm Rowe cell was used (Fig. 3.8). Figure 4.2 shows a schematic drawing of the cell, including the locations of the pore water pressure measurements, which can be measured at four different points at the base of the cell to capture its actual radial distribution. The central hole can be used either for drainage or to measure the pore water pressure. Kaolin was used as the model for soft soil, and its properties are summarised in Table 4-1.

![Figure 4.2: Modified 150 mm Rowe cell](image)
Three different test series were conducted using the modified Rowe cell, as summarised in Table 4-2. Kaolin slurry was prepared with a moisture content of at least 1.5 times its liquid limit, and then transferred to the cell and subjected to a preconsolidation pressure of 30 kPa. To ensure that the sample was saturated, Skempton’s B parameter of at least 0.99 was obtained. A circular central sand drain with a diameter of 14.5 mm was then installed in the sample via a pre-bored hole to minimize any smear effect. Subsequently, the application of surcharge pressure on top of the sample and/or application of the vacuum pressure at the central drain was simulated to represent the membraneless technique in the field. The excess pore pressure was monitored and measured during the tests at four different locations (Figure 4.2).

Table 4-1: Properties of Kaolin

<table>
<thead>
<tr>
<th>Property</th>
<th>Standard</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Index Properties:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Liquid Limit (%)</td>
<td>ASTM D4318</td>
<td>55</td>
</tr>
<tr>
<td>Plastic Limit (%)</td>
<td>ASTM D4318</td>
<td>27</td>
</tr>
<tr>
<td>Specific Gravity</td>
<td>ASTM D4318</td>
<td>2.7</td>
</tr>
<tr>
<td>Percent Sand (%)</td>
<td>ASTM D6913</td>
<td>12</td>
</tr>
<tr>
<td>Percent Silt Size (%)</td>
<td>ASTM D422</td>
<td>26</td>
</tr>
<tr>
<td>Percent Clay Size (%)</td>
<td>ASTM D422</td>
<td>62</td>
</tr>
<tr>
<td><strong>Engineering Properties:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Slope of consolidation line, ( \lambda ), in ( v - \ln p' ) plot</td>
<td>ASTM D2435</td>
<td>0.17</td>
</tr>
<tr>
<td>Slope of swelling line, ( \kappa ), in ( v - \ln p' ) plot</td>
<td>ASTM D2435</td>
<td>0.03</td>
</tr>
<tr>
<td>Specific volume at ( p' = 1 , kPa ) on the 1D consolidation line ( (N_c) )</td>
<td>ASTM D2435</td>
<td>2.85</td>
</tr>
<tr>
<td>Angle of internal friction ( (\phi) )</td>
<td>ASTM D4767</td>
<td>27°</td>
</tr>
<tr>
<td>Slope of critical state line in ( q - p' ) plot</td>
<td>ASTM D4767</td>
<td>1.07</td>
</tr>
</tbody>
</table>
Table 4-2: Information of the tests run with the modified Rowe cell

<table>
<thead>
<tr>
<th>Test No.</th>
<th>Sample Diameter (mm)</th>
<th>Drain Diameter (mm)</th>
<th>n</th>
<th>SP (kPa)</th>
<th>VP (kPa)</th>
<th>VSR = ( \frac{VP}{SP + VP} )</th>
<th>Average pore pressure location ( \frac{R_{ave}}{R} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>151</td>
<td>14.5</td>
<td>10.41</td>
<td>80</td>
<td>20</td>
<td>0.2</td>
<td>0.545</td>
</tr>
<tr>
<td>2</td>
<td>151</td>
<td>14.5</td>
<td>10.41</td>
<td>20</td>
<td>80</td>
<td>0.8</td>
<td>0.510</td>
</tr>
<tr>
<td>3</td>
<td>151</td>
<td>14.5</td>
<td>10.41</td>
<td>0</td>
<td>80</td>
<td>1.0</td>
<td>0.502</td>
</tr>
</tbody>
</table>

Figure 4.3 show the lateral distributions of \( u_w \) for different combinations of surcharge-vacuum pressures. Circles represent the measured data and the star symbols show the calculated loci of the \( \bar{u}_w \). For the surcharge application alone where \( n = 10.4 \), the theoretical location of \( \bar{u}_w \) is at a distance \( \frac{R}{R} = 0.565 \) (c.f. Figure 4.1). It can be seen that for the vacuum assisted consolidation tests, the location of \( \bar{u}_w \) also depends on the vacuum pressure-total surcharge ratio (VSR = VP/(SP + VP)). Higher values of VSR shift the location of the \( \bar{u}_w \) towards the centre of the sample. A significant shift in the location of \( \bar{u}_w \) can be observed at the early stages of consolidation. For VSR = 0.2, 0.8, and 1.0, radial locations \( \frac{R}{R} \) of the average excess pore pressure varied between 0.48-0.59, 0.48-0.53 and 0.45-0.54, respectively. Although the locations of \( \bar{u}_w \) are very similar, the effect of vacuum pressure on the location of \( \bar{u}_w \) can be significant when the drain spacing in the field becomes larger.

The location of the pore pressure measurement is important to calculate the corresponding DOC where only a single location is considered. To conveniently calculate the DOC based on the excess pore pressure dissipation, the point of measurement should coincide with the location that accurately represents \( \bar{u}_w \).
Figure 4.3: Lateral distributions of the excess pore pressure using the modified Rowe cell, Test No. 1

Figure 4.4: Lateral distributions of the excess pore pressure using the modified Rowe cell, Test No. 2
Determination of Degree of Consolidation (DOC)

This section of the chapter focuses on the determination of the degree of consolidation (DOC) based on both pore water pressure and settlement for the vacuum assisted radial consolidation tests. Several tests were conducted using a standard Rowe cell with an internal diameter of 75.5 mm. Kaolin samples were prepared according to the procedures discussed earlier, and their test details are summarised in Table 4-3. Pore water pressure ($u_w$) was measured at two locations at the base of the cell. One of the measurement points was located in the centre of the cell to monitor the vacuum pressure (VP) in the drain. Another measurement point was ideally located to coincide with the location of the average excess pore water pressure.

Figure 4.5: Lateral distributions of the excess pore pressure using the modified Rowe cell, Test No. 3
pressure $\bar{u}_w$. The location of $\bar{u}_w$ was chosen as $\frac{r_{ave}}{R} = 0.52$. Based on the results presented in the previous section, this location can accurately represent the average pore water pressure $(\bar{u}_w)$ in the selected cell with the $n$ value equal to 10.07 for different vacuum pressure-total surcharge ratio (VSR).

Table 4-3: Information of the test series using 75 mm Rowe cell

<table>
<thead>
<tr>
<th>Test No.</th>
<th>Sample Diameter (mm)</th>
<th>Drain Diameter (mm)</th>
<th>$n$</th>
<th>$SP$ (kPa)</th>
<th>$VP$ (kPa)</th>
<th>$SP + VP$ (kPa)</th>
<th>VSR = $\frac{VP}{SP + VP}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>75.5</td>
<td>7.5</td>
<td>10.07</td>
<td>100</td>
<td>0</td>
<td>100</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>70</td>
<td>30</td>
<td>0.3</td>
<td>70</td>
<td>30</td>
<td>100</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>60</td>
<td>40</td>
<td>0.4</td>
<td>60</td>
<td>40</td>
<td>100</td>
<td>0</td>
</tr>
<tr>
<td>7</td>
<td>50</td>
<td>50</td>
<td>0.5</td>
<td>50</td>
<td>50</td>
<td>100</td>
<td>0</td>
</tr>
<tr>
<td>8</td>
<td>40</td>
<td>60</td>
<td>0.6</td>
<td>40</td>
<td>60</td>
<td>100</td>
<td>0</td>
</tr>
<tr>
<td>9</td>
<td>30</td>
<td>70</td>
<td>0.7</td>
<td>30</td>
<td>70</td>
<td>100</td>
<td>0</td>
</tr>
</tbody>
</table>

The degree of consolidation based on settlement is determined by:

$$U_s = \frac{s}{s_\infty}$$

where, $s$ is settlement at a given time, and $s_\infty$ is the ultimate settlement.

The DOC based on the dissipation of pore pressure can be determined by:

$$U_p = \frac{\bar{u}_{w0} - \bar{u}_w}{\bar{u}_{w0} + P_0}$$

where, $\bar{u}_{w0}$ is the initial average excess pore pressure, $\bar{u}_w$ is the average pore pressure at a given time, and $P_0$ is the magnitude of the vacuum pressure at the drain-soil interface.
Figures 4.6 to 4.8 compare the degree of consolidation calculated based on settlement (Eq. 4.4), with the one resulted from excess pore water pressure dissipation (Eq. 4.5) for six different tests (Table 4-3). The consolidation rates based on the analyses of settlement are higher than those based on the dissipation of pore pressure. For all the tests the total surcharge pressure (SP+VP) was equal to 100 kPa, but the VSR was different for each individual test (Table 4-3).

Figure 4.9 reflects the effect of VSR on the rate of consolidation by comparing the curves of the degree of consolidation of these tests based on both methods. For clarity, only four curves are shown in this figure. The effect of the VSR on the rate of consolidation is more distinguishable when the DOC was calculated based on the pore water pressure (Figure 4.9b). This result is expected due to vacuum assisted consolidation; suction directly affects the dissipation of pore pressure (Chu and Yan, 2005).
Figure 4.6: Comparison of the DOC based on measured settlements and pore pressures, Test No. 4, (b) Test No. 5
Figure 4.7: Comparison of the DOC based on measured settlements and pore pressures, Test No. 6, (b) Test No. 7

SP = 60 kPa, 
VP = 40 kPa, 
VSR = 0.4

SP = 50 kPa, 
VP = 50 kPa, 
VSR = 0.5

VSR = VP / TP = VP / (SP + VP)
Figure 4.8: Comparison of the DOC based on measured settlements and pore pressures, Test No. 8, (b) Test No. 9
Figure 4.9: Degree of consolidation vs. time for different vacuum pressure-total surcharge ratios, (a) based on the settlement data, (b) based on the pore pressure data.

\[ VSR = \frac{VP}{TP} = \frac{VP}{SP+VP} \]
4.5 Determination of the Coefficient of Radial Consolidation ($c_h$)

Excess pore water pressure ($u_w$) at radius $r$ under the effect of both surcharge and vacuum pressures can be calculated using the modified formulation provided by Indraratna et al. (2005):

\[
\begin{align*}
\frac{u'_w}{k_h} &= \frac{1}{k_h \mu R^2} (u_{w0} + P_0) \exp \left( -\frac{8T_h}{\mu} \right) \left[ R^2 \ln \left( \frac{r}{r_w} \right) - \frac{(r^2 - r_s^2)}{2} \right] - P_0 \\
\frac{u_w}{k_h} &= \frac{1}{k_h \mu R^2} (u_{w0} + P_0) \exp \left( -\frac{8T_h}{\mu} \right) \left[ R^2 \ln \left( \frac{r}{r_w} \right) - \frac{(r^2 - r_s^2)}{2} \right] \\
&\quad + \frac{k_h}{k_h} \left[ R^2 \ln \left( \frac{r_s}{r_w} \right) - \frac{(r^2 - r_s^2)}{2} \right] - P_0
\end{align*}
\]

\[
\mu = \frac{n^2}{n^2 - 1} \left[ \ln \left( \frac{n}{s} \right) + \frac{k_h}{k_h'} \ln (s) - \frac{3}{4} \right] + \frac{s^2}{n^2 - 1} \left( 1 - \frac{s^2}{4n^2} \right) \\
&\quad + \frac{k_h}{k_h' n^2 - 1} \left( \frac{s^4 - 1}{4n^2 - s^2 + 1} \right)
\]

where, $u'_w$ is the excess pore pressure within the smear zone, $u_w$ is the excess pore pressure outside the smear zone, $k_h$ is the soil permeability outside the smear zone, $k_h'$ is the soil permeability within the smear zone, $r_s$ is the radius of the smear zone, and $s = \frac{r_s}{r_w}$.

Coefficients of consolidation based on pore water pressure dissipation ($c_{hp}$) and settlement ($c_{hs}$) can be determined by:
where, $U_p$ and $U_s$ are the DOCs based on the dissipation of pore water pressure and settlement, respectively.

In Eq. 4.6, a constant value of $c_h$ is usually calculated from the 1D oedometer tests (settlement based) or from a radial consolidation test with surcharge alone. To examine whether Eq. 4.6 can accurately predict the consolidation behaviour for any given VSR, Eqs. 4.7 and 4.8 were used to back calculate the coefficients of radial consolidation ($c_{hp}$ and $c_{hs}$) and the least squares method was adopted to best fit the predicted DOC with the measured one. Figures 4.10 to 4.12 illustrate the comparison between the predicted and measured DOC in relation to pore pressures. The same is represented in Figs. 4.13 to 4.15 related to settlement. The back calculated values of $c_{hp}$ and $c_{hs}$ from these figures are then plotted in Figure 4.16a. The values of both $c_{hp}$ and $c_{hs}$ increase with the VSR. Figure 4.16a shows that the unique value of $c_h$ obtained from conventional surcharge testing is unable to represent the degree of consolidation and pore pressure dissipation for different values of VSR. With this conclusion, the effect of VSR should somehow be considered in the calculations. One way to do this is to correlate the value of $c_h$ to the values of $c_{hp}$ and $c_{hs}$. 

\[
c_{hp} = -\frac{\mu D_p^2}{8t} \ln \left(1 - U_p \right) \tag{4.7}
\]

\[
c_{hs} = -\frac{\mu D_s^2}{8t} \ln \left(1 - U_s \right) \tag{4.8}
\]
Figure 4.10: Predicted and measured DOC based on excess pore pressure dissipation, Test No. 4, (b) Test No. 5

\[ SP = 100 \text{ kPa}, \quad VP = 0.0, \quad VSR = 0.0 \]

\[ SP = 70 \text{ kPa}, \quad VP = 30 \text{ kPa}, \quad VSR = 0.3 \]
Figure 4.11: Predicted and measured DOC based on excess pore pressure dissipation, Test No. 6, (b) Test No. 7
Figure 4.12: Predicted and measured DOC based on excess pore pressure dissipation, Test No. 8, (b) Test No. 9.

\[ VSR = \frac{VP}{TP} = \frac{VP}{SP+VP} \]
Figure 4.13: Predicted and measured DOC based on settlement, Test No. 4, (b) Test No. 5
Figure 4.14: Predicted and measured DOC based on settlement, Test No. 6, (b) Test No. 7
Figure 4.15: Predicted and measured DOC based on settlement, Test No. 8, (b) Test No. 9

\[ VSR = \frac{VP}{TP} = \frac{VP}{(SP+VP)} \]
Therefore, from Figure 4.16a, Eqs. 4.9 and 4.10 are proposed for $c_{hp}$ and $c_{hs}$ values, respectively.

\[ c_{hp} = c_h [\alpha_1 + \alpha_2 \exp(\alpha_3 * VSR)] \]  
\[ c_{hs} = c_h [\beta_1 + \beta_2 \exp(\beta_3 * VSR)] \]

Figure 4.16: (a) $c_{hp}$ and $c_{hs}$ values, and (b) $c_{hs}/c_{hp}$ for different VSR
In the above equations, \( c_h \) is the coefficient of radial consolidation for the case of surcharge load alone \((VSR = 0.0)\) based on the settlement analysis. From Fig. 4.16a, the empirical coefficients in Eqs. 4.9 and 4.10 can be determined as follows:

\[
\alpha_1 = 0.45 \quad , \quad \alpha_2 = 0.29 \quad , \quad \alpha_3 = 1.4
\]

and,

\[
\beta_1 = -0.17 \quad , \quad \beta_2 = 1.18 \quad , \quad \beta_3 = 0.65
\]

Figure 4.16b shows the ratio of \( \frac{c_{hs}}{c_{hp}} \) for different VSR. Interestingly, the value of \( \frac{c_{hs}}{c_{hp}} \) is almost constant at about 1.4, suggesting that while the values of both \( c_{hp} \) and \( c_{hs} \) depend on VSR, it does not have any effect on \( \frac{c_{hs}}{c_{hp}} \). This makes the calculation even easier because by knowing the ratio of \( \frac{c_{hs}}{c_{hp}} \) and using either Eqs. 4.9 or 4.10, both coefficients of radial consolidation corresponding to pore pressure and settlement can be obtained.

**4.6 Model Validation**

To validate the proposed model, two additional tests (Test No. 10 and 11) were conducted using the standard 75 mm Rowe Cell (Table 4-4). Excess pore water pressure \((u_w)\) data for these tests are plotted in Figure 4.17. The degree of consolidation is calculated based on settlement for these tests and the results are presented in Figure 4.18. Using Eq. 4.9, the \( c_{hp} \) values can be calculated for Test No.
10 and 11 to be 2.59 and 2.80 m²/year, respectively. Using Eq. 4.10, the $c_{hs}$ values for these tests are 3.67 and 3.94 m²/year, respectively. An acceptable agreement between the proposed solutions and the measured data are shown in Figure 4.17 and Figure 4.18.

### Table 4-4: Information of the radial consolidation test with the 75 mm Rowe cell

<table>
<thead>
<tr>
<th>Test No.</th>
<th>Sample Diameter (mm)</th>
<th>Drain Diameter (mm)</th>
<th>$n$</th>
<th>SP (kPa)</th>
<th>VP (kPa)</th>
<th>$SP + VP$ (kPa)</th>
<th>VSR = $\frac{VP}{SP + VP}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>75.5</td>
<td>7.5</td>
<td>10.07</td>
<td>55</td>
<td>45</td>
<td>100</td>
<td>0.45</td>
</tr>
<tr>
<td>11</td>
<td>75.5</td>
<td>7.5</td>
<td>10.07</td>
<td>45</td>
<td>55</td>
<td>100</td>
<td>0.55</td>
</tr>
</tbody>
</table>

#### 4.7 Practical Implications

In order to assess the effectiveness of consolidation, the key question that is raised is whether the DOC should be calculated based on settlement or the dissipation of excess pore water pressure. From a practical point of view, there are two main factors governing the benefits of soft soil improvement, (a) the increased shear strength, and (b) the reduced post-construction settlement. The rate of increase in shear strength depends on the rate of excess pore pressure dissipation (i.e. change in effective stress), hence the long term shear strength is influenced by any residual (undissipated) excess pore water pressure. Therefore, if the governing design criteria is based on the shear strength of the treated foundation (e.g. reclamation), then the analysis must adopt $c_{hp}$ and $U_p$. On the other hand, if the settlement control is the governing criteria (e.g. railway), then the analysis must capture the role of $c_{hs}$ and $U_s$. 
Figure 4.17: Measured and predicted excess pore water pressure vs. time at $r = 0.52R$, 
(a) Test No. 10, and (b) Test No. 11
Figure 4.18: Measured and predicted degree of consolidation based on settlement data, (a) Test No. 10, and (b) Test No. 11
For instance, when an embankment cannot be constructed to its full height in one single stage due to the low shear strength of subsoil, construction is usually carried out in several stages. At each stage of construction a portion of the embankment is raised and then allowed to rest until a desired DOC is achieved before proceeding to the next stage of construction. In these situations, the coefficient of consolidation ($c_{hp}$) and degree of consolidation ($U_p$) corresponding to the dissipation of excess pore water pressure must be used to evaluate the increase in shear strength. In contrast, using $c_{hs}$ and $U_s$ instead would result in overestimating the shear strength of the subsoil. When settlement at a given time, or total settlement is of interest, the calculations must be based on $c_{hs}$ and $U_s$ because using $c_{hp}$ and $U_p$ will result in an underestimation of settlement for a given load.

4.8 Summary
Vacuum assisted radial consolidation tests were conducted using two different sized Rowe cells. Application of vacuum pressure in radial consolidation alters the location of the average pore water pressure. The coefficient of consolidation based on excess pore pressure dissipation ($c_{hp}$) is shown to be less than that based on settlement ($c_{hs}$), and they are both affected by the vacuum-surcharge ratio ($VSR$). Higher VSR results in an increase in the rate of consolidation. For the same equivalent total surcharge, using a constant coefficient of consolidation determined from oedometer testing cannot accurately predict the rate of consolidation under different VSR. However, knowing the value of $c_h$ determined from the oedometer time-settlement curve, the values of both $c_{hs}$ and $c_{hp}$ can easily be calculated using
Eqs. 4.9 and 4.10. Depending on VSR, a semi-empirical model is proposed to determine the coefficients $c_{hp}$ and $c_{hs}$. The predictions from the proposed equations agree well with the results of the consolidation tests using Rowe cell. Irrespective of VSR, the ratio of $\frac{c_{hs}}{c_{hp}}$ remains almost constant at about 1.4.

Due to the difference in DOC based on settlement and excess pore water pressure dissipation, one must analyse both the measured settlement and excess pore pressure to evaluate the performance of a soft soil foundation subjected to radial consolidation with surcharge and vacuum application. It is suggested that to evaluate the gain in shear strength during consolidation, the DOC obtained based on $c_{hp}$ or excess pore pressure measured in the field, should be used to avoid overestimating the shear strength.
5 RESPONSE OF EXCESS PORE WATER PRESSURE, EFFECTIVE STRESS AND SETTLEMENT TO THE APPLICATION AND REMOVAL OF SURCHARGE AND/OR VACUUM PRESSURES

5.1 General
The drainage path and soil hydraulic conductivity are among the factors which control the distribution and dissipation of excess pore water pressure and the rate of settlement during consolidation. Consolidation of soil also depends on the amount, duration, and rate of applied external load (i.e. surcharge and/or vacuum pressures). To assess the stability of soil in relation to effective stresses gained during consolidation, the time-dependent responses of excess pore water pressure to the applied load must be examined carefully. To determine the time required to apply an external load to achieve a certain degree of consolidation, it is imperative to know the rate and amount of settlement for a given time.

It is already understood that surcharge and vacuum pressures work differently, but their effects on the excess pore water pressure, effective stress and settlement have yet to be thoroughly investigated. The application of surcharge pressure in the field is usually achieved by gradually constructing embankments which represent a gradual increase in the external load and excess pore water pressure in the soil mass, while vacuum pressure is usually applied instantaneously
by switching the vacuum pumps on. The process of removing the surcharge and vacuum pressure in the field is also quite different in that the former can be removed gradually, but the latter can be removed immediately. In the laboratory, surcharge and vacuum pressures can be applied in whichever way it is required, but step loading and unloading are commonly used for both of them, so it represents their instantaneous application and removal.

Applying an instantaneous surcharge increases the excess pore water pressure almost immediately by the same amount in the entire fully saturated soil mass, whereas vacuum pressure requires a certain period to propagate across the soil mass (see Chapter 4). Furthermore, the ways that surcharge and vacuum pressures change the effective stresses are quite different; surcharge loading generates excess pore water pressure through increased total stress while the dissipation of excess pore pressure increases the effective stresses, but vacuum pressure through constant total stress decreases the pore water pressure and thereby increases the effective stresses. These differences, with their instantaneous effects and effects during the dissipation of excess pore water pressure on soil consolidation, needs more detailed study.

In combined surcharge and vacuum consolidation projects, vacuum pressure may not need to be applied for the entire duration of consolidation. Applying vacuum pressure in a certain project can be uneconomical over time. Therefore, determining the optimum removal time for the vacuum is important to ensure that the desired degree of consolidation is achieved while simultaneously minimising its application time. This optimum time is often determined based on the experience of engineers so more research needs to be conducted to establish the optimum time for vacuum
 removal.

In the previous Chapter the general effects of the vacuum pressure and vacuum-surcharge ratio on the degree of consolidation and coefficient of consolidation were discussed in detail. In this Chapter, based on laboratory measurements, the effects of the application and removal of surcharge and vacuum pressures on excess pore water pressure, effective stress, and settlement during radial consolidation were studied and the difference between surcharge and vacuum is highlighted. Also, the optimum time for removing vacuum pressure is discussed, and how to determine this optimum time based on the pore water pressure measurements is shown.

5.2 Test Set Up and Procedure
A 150 mm diameter Rowe cell (Figs. 4.2) was used to study the effects of surcharge and vacuum pressures on the radial consolidation of clays. The pore water pressure was measured at the base of the cell at four different locations, and the surface axial settlement was measured at the centre of the top of the cell. Kaolin was used as a soil sample and its properties are represented in Table 4.1. The preparation and pre-consolidation of the samples, specifications and installation of the drains, and application of the surcharge and vacuum were exactly same as previously explained in Section 4.3. Six different tests were conducted and the details are summarised in Table 5-1.
Table 5-1: Information from the test conducted using a 150 mm Rowe Cell

<table>
<thead>
<tr>
<th>Test No.</th>
<th>Sample Diameter (mm)</th>
<th>Drain Diameter (mm)</th>
<th>SP (kPa)</th>
<th>VP (kPa)</th>
<th>SP + VP (kPa)</th>
<th>VSR = VP / (SP + VP)</th>
<th>Time of Surcharge or Vacuum removal (hr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>151 14.5 10.41</td>
<td></td>
<td>100</td>
<td>0</td>
<td>100</td>
<td>0</td>
<td>8</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td>100</td>
<td>0</td>
<td>100</td>
<td>0</td>
<td>10</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td>100</td>
<td>0</td>
<td>100</td>
<td>0</td>
<td>12</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td>50 50</td>
<td>0.5</td>
<td>50</td>
<td>5</td>
<td>8</td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
<td>50 50</td>
<td>0.5</td>
<td>50</td>
<td>5</td>
<td>10</td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
<td>50 50</td>
<td>0.5</td>
<td>50</td>
<td>5</td>
<td>12</td>
</tr>
</tbody>
</table>

The total time for each test was 144 hours. For the Tests 1-3, 100 kPa surcharge pressure was applied onto the surface of the samples at the start of the tests (time = 0). Half of the surcharge (50 kPa) was removed after 8, 10, and 12 hours, respectively, the remaining surcharge (50 kPa) for all of them was removed after 72 hours, and then the tests continued for a further 72 hours. For Tests 4-6, a 50 kPa surcharge pressure was applied onto the surface of the samples, and 50 kPa of vacuum pressure was simultaneously applied at the central drains at the start of the tests (time = 0). The vacuum pressure was removed after 8, 10, and 12 hours for Tests 4-6, respectively, the surcharge pressure was removed after 72 hours, and then all the tests continued for further 72 hours. When the first part of the surcharge for Tests 1-3 was removed, and when the vacuum for Tests 4-6 was removed, the drainage path was connected to a stand pipe piezometer to simulate the effect of the water table in the field. This would also allow a reverse movement of water into the soil due to dilation when the load was removed, and it also prevents air from entering the soil samples.
5.3 Excess Pore Water Pressure Response

During the tests the pore water pressure was measured at the base of the cell at four different radii. To normalise the radial distance from the centre of the cell, a Radius Ratio (RR) was defined as:

\[ RR = \frac{(r - r_w)}{(R - r_w)} \]  

where, \( r \) is the radius from the centre, \( r_w \) is the radius of the vertical drain, and \( R \) is the radius of the cell.

The locations where the pore water pressure was measured were at \( RR = 0.0, 0.19, 0.48, \) and \( 0.77 \). Figure 5.1 illustrates the distribution of excess pore water pressure at different \( RR \) for Tests 1-3, and it also shows that there are three distinct steps in the application and removal of the surcharge that should be carefully considered. Step 1 is the time of the application of surcharge, Step 2 is the time when the first half of the surcharge was removed, and Step 3 is when the second half of the surcharge was removed. To identify the responses of the pore water pressure during the first two steps more clearly, the first 20 hours of the tests are re-plotted in Fig. 5.2.

When the surcharge pressure was applied under undrained conditions, several minutes (usually 10-15 minutes for the soil tested) were required for the excess pore water pressure to reach its maximum value (equal to the applied surcharge). Therefore, in all the tests described in this Chapter, the drainage valve was first closed until the excess pore water pressure attained its maximum value, and then it
was opened to allow for consolidation to occur. In the vacuum assisted tests (Tests 4-6), vacuum pressure was applied inside the drain at the same time as the drainage valve was opened.

Figure 5.2 shows that when the drainage valve was opened (time = 0.0), the excess pore water pressure first increased (more than the surcharge applied) further away from the drain and decreased closer to the drain. This phenomenon can be attributed to the boundary effects in laboratory testing where the wall of the cell acts like a rigid boundary and where the drainage boundary (drain-soil interface) begins to operate as soon as the drainage valve is opened. The trends and rates of excess pore water pressure during this time when stresses are being redistributed do not represent the general response of the soil during consolidation, and therefore should not be used to calculate parameters such as the hydraulic gradient. Figure 5.2 shows that after approximately half an hour or less, all the excess pore water pressure curves have a similar trend and thereafter the soil behaviour is systematic at different distances from the drain.
Figure 5.1: Excess pore water pressure distribution with time at different radii: (a) Test 1: removal of 50 kPa surcharge after 8.0 hours, (b) Test 2: removal of 50 kPa surcharge after 10.0 hours, (c) Test 3: removal of 50 kPa surcharge after 12.0 hours.
Figure 5.2: Excess pore water pressure distribution with time at different radii – first 20 hours:
(a) Test 1: removal of 50 kPa surcharge after 8.0 hours, (b) Test 2: removal of 50 kPa surcharge after 10.0 hours, (c) Test 3: removal of 50 kPa surcharge after 12.0 hours.
Figure 5.2 shows that when the first half of the surcharge was removed, the drop in excess pore water pressure was different at different radii. Also, the difference in the excess pore water pressure at different distances from the drain in each test was reduced immediately after the surcharge was removed. This can be seen by comparing the differences in excess pore water pressure immediately before and after the surcharge removal (Fig. 5.2), which shows that the stresses were redistributed again at this stage of the experiment. Figure 5.2 shows that depending on where the measurement points are located and when the surcharge is removed, excess pore water pressures continued to drop immediately after the surcharge was removed, or first dropped to certain values and then increased quickly, followed by a continuous decrease. Once again the excess pore pressure responses during this time (stress redistribution), do not represent the general behaviour of the soil during consolidation.

When the excess pore water pressures dissipated completely (after 72 hours), the second half of the surcharge (50 kPa) was removed (Fig. 5.1). It is interesting that the pore water pressures at this stage dropped below atmospheric pressure and showed suction (negative values) in the soil. It took at least 15 hours for the pore pressures to return to zero. When the remaining surcharge was removed at this stage, water moved inwards from the drain into the soil, and the negative pore pressures could be attributed to a tendency of the soil to expand at a higher volume rate than the inward movement of water. When the effective stresses are plotted (in the following sections) the effects of this behaviour can be seen and compared with the vacuum assisted tests.
Similar graphs for the vacuum assisted tests (Tests 4-6) are presented in Figs. 5.3 and 5.4. Again, when the surcharge was applied, and when the vacuum and surcharge were removed, the behavior was similar. But, unlike removing the surcharge pressure, removing the vacuum pressure increases the excess pore water pressure in the soil mass (Fig. 5.4). Figure 5.4 shows that when the vacuum was removed from the first two tests (Tests 4 and 5), the excess pore water pressure increased and then began to dissipate again, whereas in Test 6 (Fig. 5.4c) the excess pore water pressure increased to zero and remained constant thereafter. In Tests 4 and 5 (Figs. 5.4a and b), the excess pore water pressure was negative near the drain just before the vacuum was removed and positive further away, but in Test 6 (Fig. 5.4c) the excess pore water pressure was negative in all locations just before the vacuum was removed. The reason why the excess pore water pressure in Test 6 increased to zero when the vacuum was removed and then remained constant thereafter was because, before the vacuum was removed the excess pore water pressure was negative in the entire soil mass.
Figure 5.3: Excess pore water pressure distribution with time at different radii: (a) Test 4: removal of 50 kPa vacuum after 8.0 hours, (b) Test 5: removal of 50 kPa vacuum after 10.0 hours, (c) Test 6: removal of 50 kPa vacuum after 12.0 hours.
Figure 5.4: Excess pore water pressure distribution with time at different radii – first 20 hours: (a) Test 4: removal of 50 kPa vacuum after 8.0 hours, (b) Test 5: removal of 50 kPa vacuum after 10.0 hours, (c) Test 6: removal of 50 kPa vacuum after 12.0 hours.
To further explore the effects of the time of the surcharge/vacuum removal on changes in the excess pore water pressures at different distances from the drain, the excess pore water pressures at different locations were compared before and after surcharge/vacuum had been removed. Table 5-2 summarises the recorded values for the excess pore water pressures at these times. From Table 5-2 the change (drop) in pore water pressures at different locations and times due to the removal of half (50 kPa) of the surcharge can be calculated and plotted (Fig. 5.5) for Tests 1-3. Figure 5.5 shows that the effect of the removal of surcharge on the drop in excess pore water pressure was less closer to the drain, and also when the surcharge was removed later. This was expected but it is imperative to know how this distribution is related to the distance from the drain and the time of the surcharge removal when the degree of consolidation and gain in soil shear strength are of interest. From Table 5-2 the change (increase) in pore water pressures at different locations and times due to the removal of vacuum can be calculated and plotted (Fig. 5.6) for Tests 4-6. Figure 5.6 shows that the effect of the removal of vacuum on the increase in excess pore water pressure was more closer to drain, and also when the vacuum was removed later. This is exactly opposite to the effects of surcharge removal. Therefore, the comparison of Figs. 5.5 and 5.6 shows that removing the surcharge influences more on excess pore water pressure further away from the drain, while removing the vacuum creates more changes in excess pore water pressure closer to the drain. Moreover, removing the surcharge later has less effect on the excess pore water pressure, while removing the vacuum later imparts a greater influence on the excess pore water pressure.
Table 5-2: Comparison of the excess pore water pressure before and after surcharge/vacuum removal for Tests 1-6.

<table>
<thead>
<tr>
<th>Test No.</th>
<th>Excess pore pressure at RR = 0.19</th>
<th>Excess pore pressure at RR = 0.48</th>
<th>Excess pore pressure at RR = 0.77</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Before surcharge removal</td>
<td>After surcharge removal</td>
<td>Before surcharge removal</td>
</tr>
<tr>
<td>1</td>
<td>47.48</td>
<td>13.19</td>
<td>63.18</td>
</tr>
<tr>
<td>2</td>
<td>36.22</td>
<td>11.64</td>
<td>52.52</td>
</tr>
<tr>
<td>3</td>
<td>28.70</td>
<td>5.45</td>
<td>41.47</td>
</tr>
<tr>
<td>4</td>
<td>-12.43</td>
<td>6.16</td>
<td>2.12</td>
</tr>
<tr>
<td>5</td>
<td>-13.50</td>
<td>7.16</td>
<td>0.32</td>
</tr>
<tr>
<td>6</td>
<td>-22.53</td>
<td>0.55</td>
<td>-12.74</td>
</tr>
</tbody>
</table>

Figure 5.5: Drop in excess pore water pressures at different locations for Tests 1-3 due to the removal of surcharge. Test 1: removal of 50 kPa surcharge after 8.0 hours, Test 2: removal of 50 kPa surcharge after 10.0 hours, Test 3: removal of 50 kPa surcharge after 12.0 hours, \[ \phi = \left( r - r_w \right) / \left( R - r_w \right) \]
The practical implication of the above comparison is that in the field, pore water pressure piezometers are usually installed half way between the drains, which means considering the unit cells, whereas piezometers are installed closer to the boundary, which is actually the furthest location from the drain. Therefore, piezometers can show the maximum change in the pore water pressures in the unit cells when the surcharge is removed, while they show the minimum when the vacuum is removed.

Figure 5.6: Increase in excess pore water pressures at different locations for Tests 4-6 due to the removal of vacuum. Test 4: removal of 50 kPa vacuum after 8.0 hours, Test 5: removal of 50 kPa vacuum after 10.0 hours, Test 6: removal of 50 kPa vacuum after 12.0 hours, \( RR = \frac{(r - r_w)}{(R - r_w)} \)
Figure 5.7 shows the average excess pore water pressures between the tests conducted with surcharge alone, and those tests conducted with a combination of surcharge and vacuum when the surcharge and vacuum were removed at different times. In Chapter 4 it was shown how the vacuum pressure and vacuum-surcharge ratio (VSR) affected the degree of consolidation and the back calculated coefficients of consolidation. With increasing VSR, the degree of consolidation for a given time increased with a corresponding increase in the coefficient of radial consolidation. Here again, the advantage of vacuum pressure over conventional surcharge can be observed from Fig. 5.7 where the former generally represents a higher rate of average excess pore water pressure dissipation. However, while all of these trends can be observed they do not show that much difference between the surcharge and the combination of the surcharge and vacuum here. The reason for this is that only one vacuum-surcharge ratio was used for all of the tests, and by increasing this ratio (as already shown in Chapter 4) the difference would be more visible.
Figure 5.7: Comparison of the average excess pore water pressures of the tests conducted with surcharge alone, with the tests conducted with a combination of surcharge and vacuum for: (a) surcharge/vacuum removal = 8.0 hours, (b) surcharge/vacuum removal = 10.0 hours, (c) surcharge/vacuum removal = 12.0 hours.
5.4 Changes in Effective Stress

The changes in average effective stress with time for all of the tests are shown in Fig. 5.8. Figure 5.8a shows the changes for Tests 1-3 that only a surcharge was used, while Fig. 5.8b represents the same for Tests 4-6 where a combination of surcharge and vacuum was used. Both parts of Fig. 5.8 show that removing the surcharge/vacuum later resulted in a larger decrease in the average effective stress gain in the samples. This was actually expected because leaving the surcharge/vacuum for longer times results in greater dissipation of the excess pore water pressures, and therefore gain in effective stresses, which in turn results in a larger decrease in the effective stresses by removing the external loads (surcharge or vacuum).

To better distinguish the effects of surcharge and vacuum, and the duration of the application of these loads on the average effective stresses in the samples and discuss about the differences, a comparison was made between the tests conducted with surcharge alone and the tests conducted with a combination of surcharge and vacuum by plotting them in the same graph for every duration (8, 10, and 12 hours after starting the tests) of surcharge/vacuum (Fig. 5.9). Figure 5.9 shows two interesting aspects: firstly, when the first half (50 kPa) of the surcharge or vacuum was removed, all the tests showed a higher gain in the effective stresses in the tests conducted with a combination of vacuum and surcharge compared to those tests conducted with a surcharge alone. Secondly, after the removal of the surcharge/vacuum, the decrease in the effective stresses was more in the tests conducted with surcharge alone, which resulted in even more difference between
those cases with a combination of vacuum and surcharge, and those cases with surcharge alone, after the external loads were removed.

To better analyse the difference between surcharge and vacuum pressure, the decrease in the average effective stresses of all of the tests were calculated and plotted in Fig. 5.10. This figure compares the gain in effective stresses before and after the external loads were removed with the time the surcharge/vacuum was removed. Figure 5.10 shows that the gain in average effective stresses before the external loads (surcharge or vacuum) were removed for tests conducted with surcharge and vacuum are on average, 23% more than those in similar tests conducted with a surcharge alone. Also, it shows that this difference after the external loads were removed increased to an average of 39%. These values with Fig. 5.10 clearly show the advantages of vacuum pressure over conventional surcharge alone, especially when the stability of the embankments is of interest. These differences also imply significant practical implications between the two approaches.

Preconsolidation pressure \( p'_{c} \) for each test after the total load was removed can be calculated by adding the initial effective stress \( p'_0 \), which was the same (30 kPa) for all the tests, to the total gain in effective stress (which can be calculated from Figure 5.8). Table 5-3 shows the calculated and developed preconsolidation pressures at the end of the tests. In this Table, time represents the removal time of the first half of the surcharge for the tests conducted with surcharge alone, and also represents the time of the vacuum was removed for the vacuum assisted tests.
Table 5-3: Developed preconsolidation pressures at the end of the tests

<table>
<thead>
<tr>
<th>Time (hr)</th>
<th>Preconsolidation Pressure (kPa) For the test conducted with</th>
<th>Preconsolidation Pressure (kPa) For the tests conducted with surcharge + vacuum</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>surcharge alone</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>84.50</td>
<td>85.50</td>
</tr>
<tr>
<td>10</td>
<td>88.70</td>
<td>89.50</td>
</tr>
<tr>
<td>12</td>
<td>90.17</td>
<td>92.25</td>
</tr>
</tbody>
</table>

Calculation of the OCR (overconsolidation ratio) can now be conducted by knowing the preconsolidation pressure developed for each test. Figure 5.11 shows the associated OCR versus the removal time of the surcharge/vacuum. Again this figure shows the advantage of using a combination of vacuum and surcharge pressure rather than using surcharge alone by the resulting higher OCR for the vacuum assisted tests, which in turn results in less post-construction settlement for the improved clay.
Figure 5.8: Average increase in effective stress for: (a) tests with surcharge alone (Tests 1, 2, and 3), (b) tests with a combined surcharge-vacuum pressure (Tests 4, 5, and 6)
Figure 5.9: Comparison of the increase in average effective stress of the tests conducted with surcharge alone, and the tests conducted with a combination of surcharge and vacuum for: (a) surcharge/vacuum removal = 8.0 hours, (b) surcharge/vacuum removal = 10.0 hours, (c) surcharge/vacuum removal = 12.0 hours
Figure 5.10: Decrease of the average effective stress with the time of the surcharge/vacuum removal.

Figure 5.11: OCR versus the removal time of surcharge/vacuum removal.
5.5 Settlement Response
The axial strains for the tests conducted with only a conventional surcharge and a combination of surcharge and vacuum pressure are presented in Fig. 5.11. A comparison between the axial strains of similar tests with and without vacuum pressure is made in Fig. 5.12. Figures 5.12a, b, and c represent the tests where the surcharge and vacuum pressures were removed after 8.0, 10.0, and 12.0 hours, respectively. It can be seen that when the first part of the external loads (surcharge/vacuum) were removed the vacuum assisted tests generally produce a higher axial strain for a given time than those tests conducted with surcharge alone. This clearly proves that the rate of settlement for the same period of time was higher in the vacuum assisted tests, and therefore primary consolidation finishes in a shorter time when vacuum is used in conjunction with a conventional surcharge. This is also consistent with the results provided in Chapter 4 where the back calculated coefficients of radial consolidation for the vacuum assisted tests were higher compared to those tests conducted with surcharge alone.
Figure 5.12: Axial strain for: (a) tests with surcharge alone (Tests 1, 2, and 3), (b) tests with a combined surcharge-vacuum pressure (Tests 4, 5, and 6)
Figure 5.13: Axial strain for: (a) removal of surcharge/vacuum after 8.0 hours, (b) removal of surcharge/vacuum after 10.0 hours, (c) removal of surcharge/vacuum after 12.0 hours.
5.6 Optimum Design Procedure for a Surcharge-Vacuum Combined Consolidation

In the previous sections of this chapter the key advantages of vacuum pressure in conjunction with surcharge pressure to consolidate soft soils more effectively were highlighted. In this section a procedure is described to show how vacuum pressure can be used to apply a higher load (compared to the design load) to consolidate the soil quicker and also minimise the cost of removing the surcharge. Also, in this procedure the optimum time for removing the vacuum is explained and calculated.

The settlement \( S_{c1} \) of a layer of soft soil due to the design load for the soil improvement \( \Delta \sigma_1' \) can be calculated from Eq. 5.2:

\[
S_{c1} = m_v H_0 \Delta \sigma_1'
\]  \hspace{1cm} \text{5.2}

where, \( m_v \) is the coefficient of the soil volume change, and \( H_0 \) is the initial thickness of the soft soil layer.

If instead of \( \Delta \sigma_1' \) a higher load of \( \Delta \sigma_2' \) applies to the soil where \( \Delta \sigma_1' < \Delta \sigma_2' \), then settlement of the soil would be \( S_{c2} \) and that can be calculated from Eq. 5.3:

\[
S_{c2} = m_v H_0 \Delta \sigma_2'
\]  \hspace{1cm} \text{5.3}

Then, the required degree of consolidation \( (U_{req.}) \) under the higher load \( (\Delta \sigma_2') \) and the required time for this degree of consolidation \( (t_{U_{req.}}) \) can be calculated from Eqs.
5.4 and 5.5. Figure 5.14 graphically shows the relationships presented in these equations.

\[ U_{\text{req.}} = \frac{S_{c1}}{S_{c2}} = \frac{\Delta \sigma'_1}{\Delta \sigma'_2} \]  

\[ \frac{t_{U_{\text{req.}}}}{t_{95\%}} = \frac{\ln(1-U_{\text{req.}})}{\ln(0.05)} \]  

Figure 5.14: Relationship between time, load, and degree of consolidation

The relationship between the average excess pore water pressure, vacuum and surcharge loads, and degree of consolidation can be represented by Eq. 5.6 and can be plotted as shown in Figure 5.15. Equations 5.5 and Fig. 5.14 are presented for cases where a combination of surcharge and vacuum pressure is adopted to
consolidate the soil, not for cases where only the vacuum pressure (without any surcharge) is considered.

\[
\frac{\bar{u}}{SP} = 1 - \left(1 + \frac{VP}{SP}\right)U
\]

where, \(\bar{u}\) is the average excess pore water pressure, \(SP\) is surcharge pressure, \(VP\) is vacuum pressure, and \(U\) is degree of consolidation.

By using Figure 5.14 and Figure 5.15, the following procedure can be adopted to design a combination of surcharge and vacuum pressures for accelerating the consolidation...
scheme:

1- Calculate the required time for 95% of consolidation (t95%).

2- Specify the desired time (t) to achieve the desired settlement under the improvement scheme.

3- Calculate the ratio of t/t95%.

4- From Figure 5.14 by using the curve for the relationship between the time and degree of consolidation, determine the required degree of consolidation.

5- Based on the degree of consolidation defined in step 4, use Figure 5.14 and define the load ratio (Load/Design Load). Where, the denominator is the load originally designed for the consolidation scheme, and the numerator is a higher value which will produce the required consolidation of the soil in a shorter time.

6- By increasing the design load using the load ratio defined in step 5, the time that can be saved for the consolidation of soil is t-t95%.

7- To eliminate the need to remove the surcharge after improving the soil, select \( SP = S_{c1} \times \gamma_{emb.} \), where \( \gamma_{emb.} = \text{Unit weight of embankment} \), and select \( VP = \Delta \sigma'_{z} - SP \), then use the degree of consolidation defined in step 4, the average excess pore water pressure at the required degree of consolidation can be calculated from Figure 5.15.

8- Excess pore water pressure at a given radius from the drain can then be calculated from Eq. 5.7. In the field the piezometers are usually installed half way between the drains, therefore it is the only location where pore water pressure is usually recorded and that is why Eq. 5.6 needs to be used to
calculate the excess pore water pressure at the location of piezometers.

\[
 u = \frac{(\bar{u} + P_0)}{\mu R^2} \left\{ \left[ R^2 \ln \left( \frac{r}{r_s} \right) - \frac{(r^2 - r_s^2)}{2} \right] + \frac{k_h}{k_h} \left[ R^2 \ln \left( \frac{r_s}{r_w} \right) - \frac{(r_s^2 - r_w^2)}{2} \right] \right\} - P_0 \quad 5.7
\]

where, \( r = (\text{Drain Spacing/2}) \).

9- When the excess pore water pressure halfway between the drains reaches the value calculated from Eq. (6), the vacuum pressure can be removed and there will be no need to remove the surcharge.

10- If the calculated VP in Step 7 exceeds the maximum practical applicable vacuum pressure (almost 90 kPa), then (VP-90) needs to be added to the surcharge and an equivalent part of the surcharge needs to be removed after consolidation has been completed.

5.7 Summary
Six different radial consolidation tests were conducted where the excess pore water pressure was measured at different radii, and the axial settlement was also measured during each test. Three tests were conducted using a conventional surcharge alone and three with a combination of surcharge and vacuum pressure. Surcharge and vacuum pressures were applied to the samples at the beginning of each test. To study the effects of surcharge/vacuum removal on the excess pore water pressure, axial strain, and effective stresses, half of the total stress in the tests conducted with the application of surcharge alone, and the vacuum pressure in the vacuum assisted tests
were removed at different times. To simulate the complete removal of embankments in the field, the remaining surcharge was removed when the excess pore water pressures were completely dissipated.

The vacuum assisted tests generally showed a higher rate of excess pore water pressure dissipation and settlement than the tests conducted with a conventional surcharge alone. It is shown that the change in excess pore water pressure due to removal of the surcharge was less closer to the drain, and when surcharge was removed later. However, it was the opposite for the vacuum whereby the change in excess pore water pressure due to the vacuum being removed was more closer to the drain, and the effect was greater if vacuum was removed later. This further proves that VP is a direct negative pore pressure and not a total stress as previously considered by many.

It was also shown that the average effective stresses just before and after the removal of the first part of the external loads (surcharge/vacuum) in the vacuum assisted tests were an average of 23% and 39%, respectively, higher than those in similar tests conducted with surcharge alone. This clearly shows the advantage of vacuum pressure over conventional surcharge pressure, especially when the stability of the embankments is important. Moreover, it was shown that the decrease in effective stresses due to the external loads (surcharge/vacuum) being removed was less in the tests conducted with a combination of vacuum and surcharge, compared to the tests conducted with a surcharge alone. This again is an advantage related to the stability issues. Moreover, it was shown that the generated OCR for the vacuum assisted consolidation is usually higher than the consolidation of soils under
Finally, considering the above mentioned advantage of vacuum pressure over conventional surcharge alone, a design procedure was developed and presented to accelerate the consolidation process by applying a combination of surcharge and vacuum pressure. This combination is higher than the design load for the soil improvement scheme and it was shown how to design the amount of surcharge and vacuum to minimise the need to remove the surcharge after completing consolidation. It was also shown that the optimum time for removing the vacuum can be defined by observing the excess pore water pressure half way between the drains.
6 RADIAL CONSOLIDATION MODEL INCORPORATING THE EFFECTS OF VACUUM PRELOADING AND NON-DARCIAN FLOW GENERAL

6.1 General
Existing radial consolidation model mostly employ coefficient of consolidation and coefficient of permeability in the formulations. Defining these two parameters is not an easy task and the methods used for defining them affect the resulted values. Therefore the consolidation models will be affected by the errors encountered in the determination of these parameters. More detail about the issues related to determining coefficient of consolidation and coefficient of soil permeability is presented in Chapter 2.

Moreover, there is not an agreement about the flow velocity-hydraulic gradient relationship in the soil whether it follows a Darcian or non-Darcian relationship (ref. Chapter 2). This issue becomes more sophisticated for flow relationship during consolidation especially when vacuum assisted consolidation is applied. There is not enough information on this issue in the literature. To overcome these issues, a modified 150 mm Rowe cell equipped with pore water pressure measurement was used to capture the flow relationship during vacuum assisted radial consolidation. Based on the measured data, a radial consolidation model incorporating the effects of vacuum preloading is proposed based on a non-linear relationship between the flow velocity and hydraulic gradient. Then the predictions of the proposed consolidation model were then compared with the predictions based
on the Hansbo’s Darcian and non-Darcian models. The agreement between the proposed model and the measured data is shown and the advantages of the proposed model compared to the existing models are discussed. An embankment case history taken from the reclamation project at the Port of Brisbane, Australia, was then analysed based on the current solution, and compared with the field measurements.

6.2 Laboratory Determination of the Radial Flow Characteristics During Consolidation Tests

A modified 150 mm Rowe cell (Figure 6.1) was utilised to capture the flow relationship during radial consolidation tests. Figure 6.1 shows a schematic illustration of the Rowe cell and the locations of the pore water pressure measurements (A, B, C, and D). Kaolin was used as the soil, and its properties are summarised in Table 6-1. Three tests were conducted with different combinations of surcharge (SP) and vacuum (VP) pressure to capture the flow relationship, and the details are summarised in Table 6-2. The clay slurry was prepared by mixing Kaolin with de-aired water to obtain a water content of at least 1.5 times its liquid limit and was then kept in a sealed container for a few days before transferring it to the cell. To ensure that the sample was fully saturated, Skempton’s B parameter was checked to achieve at least $B = 0.99$, and then it was preconsolidated under 30 kPa. A 14.5mm diameter sand drain was installed in the centre of the sample via a pre-bored hole to minimise the smear effect. A surcharge pressure of 30 kPa was re-applied until strain rate was less than 0.015 %/hr. An additional vertical load was then imposed in tandem with the vacuum pressure applied to the central drain to mimic the membraneless technique in the field. The excess pore water pressure was monitored
during the tests and measured at four different locations (Figure 6.1b).

Table 6-1: Properties of Kaolin

<table>
<thead>
<tr>
<th>Property</th>
<th>Standards</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Liquid Limit (%)</td>
<td>ASTM D4318</td>
<td>55</td>
</tr>
<tr>
<td>Plastic Limit (%)</td>
<td>ASTM D4318</td>
<td>27</td>
</tr>
<tr>
<td>Specific Gravity</td>
<td>ASTM D4318</td>
<td>2.7</td>
</tr>
<tr>
<td>Percent Sand (%)</td>
<td>ASTM D6913</td>
<td>12</td>
</tr>
<tr>
<td>Percent Silt Size (%)</td>
<td>ASTM D422</td>
<td>26</td>
</tr>
<tr>
<td>Percent Clay Size (%)</td>
<td>ASTM D422</td>
<td>62</td>
</tr>
<tr>
<td>Engineering Properties:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Slope of consolidation line, $\lambda$, in $v - ln p$ plot</td>
<td>ASTM D2435</td>
<td>0.17</td>
</tr>
<tr>
<td>Slope of swelling line, $\kappa$, in $v - ln p$ plot</td>
<td>ASTM D2435</td>
<td>0.03</td>
</tr>
<tr>
<td>Specific volume at $p = 1$ kPa on the 1D consolidation line</td>
<td>ASTM D2435</td>
<td>2.85</td>
</tr>
<tr>
<td>Friction angle ($\phi$)</td>
<td>ASTM D4767</td>
<td>27°</td>
</tr>
<tr>
<td>Slope of critical state line in $q - p'$ plot</td>
<td>ASTM D4767</td>
<td>1.07</td>
</tr>
</tbody>
</table>

Table 6-2: Tests conducted with the 150 mm modified Rowe Cell

<table>
<thead>
<tr>
<th>Test No.</th>
<th>Sample Diameter (mm)</th>
<th>Drain Diameter (mm)</th>
<th>$n$</th>
<th>$SP$ (kPa)</th>
<th>$VP$ (kPa)</th>
<th>$VSR = \frac{VP}{SP+VP}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>151</td>
<td>14.5</td>
<td>10.41</td>
<td>60</td>
<td>40</td>
<td>0.4</td>
</tr>
<tr>
<td>2</td>
<td>151</td>
<td>14.5</td>
<td>10.41</td>
<td>50</td>
<td>50</td>
<td>0.5</td>
</tr>
<tr>
<td>3</td>
<td>151</td>
<td>14.5</td>
<td>10.41</td>
<td>40</td>
<td>60</td>
<td>0.6</td>
</tr>
</tbody>
</table>

Note: $n = \frac{R}{r_w}$, where $R$ is the radius of the cell, and $r_w$ is the radius of the drain.
Figure 6.1: (a) Modified 150 mm Rowe Cell, (b) Base Plate of the Rowe Cell showing the locations of pore pressure measurement. Dimensions are in mm.
The calculation of the hydraulic gradient in the radial direction was based on the measurements at two points (i.e. Points B and C in Fig. 2b). The middle locations were chosen to calculate the hydraulic gradient, because, they were sufficiently far enough from both boundaries (sand drain and the cell wall) to minimise any boundary effects. Figure 6.2 represents the distribution of excess pore water pressure with time at different radii for Tests 1-3. Lateral distributions of the excess pore water pressure at different times for these tests are also shown in Figure 6.3. Figures 6.2 and 6.3 provide a complete pattern of the dissipation of excess pore water pressure with time and radius, and faster dissipation of excess pore water pressure in the soil elements closer to the drain can be observed. The hydraulic gradient between locations B and C can now be calculated by:

\[ i_{BC} = \frac{1}{\gamma_w \left( R^2 - r_{BC}^2 \right)} \frac{u_C - u_B}{r_C - r_B} \]  

6.1

In this equation, \( i_{BC} \) is the hydraulic gradient between points B and C, \( \gamma_w \) is the unit weight of water, \( u_B \) and \( u_C \) are the excess pore water pressures at a given time at locations B and C, respectively, and \( r_B \) and \( r_C \) are the radii of Points B and C, respectively, measured from the centre of the cell (drain).

The flow velocity (\( v_{BC} \)) was calculated half way between points B and C based on the rate of the volume change of the soil sample with the internal and external radii \( r_{BC} \) and \( R \), respectively, hence:

\[ v_{BC} = \frac{\partial}{\partial t} \left( \frac{R^2 - r_{BC}^2}{2 r_{BC}} \right) \]  

6.2

where, \( \varepsilon \) is the axial strain.
Figure 6.2: Excess pore water pressure distributions at different radii in 150 mm Rowe cell.
Figure 6.3: Lateral distribution of excess pore water pressure in 150 mm Rowe cell
Figure 6.4 shows the relationship of flow velocity with hydraulic gradient for all tests. The equation based on the measured data can be represented by:

\[ v = \alpha_c i^\beta \]  \hspace{1cm} 6.3

where, \( v \) is the flow velocity, \( i \) is the hydraulic gradient, and \( \alpha_c \) and \( \beta \) are constants which depend on the type of soil.

From Figure 6.4, \( \alpha_c \) and \( \beta \) are defined to be \( 3.2 \times 10^{-10} \) m/sec, and 1.3, respectively. These values represent the best fit to the average measured data. The flow velocity-hydraulic gradient curve is obtained here neglecting the horizontal displacements of the soil. The average measurement data (dash line in Figure 6.4) is
used to calculate $\alpha_c$ and $\beta$. It should be mentioned that the use of average values can somehow smooth the differences between Darcian and non-Darcian flow, blurring the threshold hydraulic gradient in Hansbo’s (1960) model and lowering the value of the power law; however, that is the best way to minimise the errors in testing.

### 6.3 Radial Consolidation Model Incorporating Non-Darcian Flow and Vacuum Application

Hansbo’s (1960, 2001) model relies on an accurate measurement and calculation of the coefficient of permeability ($k_h$) and the coefficient of radial consolidation ($c_h$). Moreover, the threshold hydraulic gradient ($i_t$) in the model (ref. Chapter 2 for detail) needs to be accurately defined. In this section, the proposed exponential flow rule (Eq. 6.3), which is independent of any threshold hydraulic gradient is used to develop an analytical model for radial consolidation that incorporates the effects of vacuum preloading. Apart from $\alpha_c$ and $\beta$, $m_v$ is the only soil parameter required in this model. Based on Figure 6.4, the non-linear relationship of the flow velocity-hydraulic gradient is incorporated in the formulation of radial consolidation. By considering a unit cell (Figure 6.5), the hydraulic gradient ($i$) in the radial direction at distance $r$ from the centre of the cell can be calculated from Eq. 6.4. It should be noted that in the following approach, gravity and flow in the vertical direction have not been considered.

$$ i = \frac{\partial h}{\partial r} = \frac{1}{\gamma_w} \frac{\partial u}{\partial r} \quad 6.4 $$

where, $h$ is the head of water.
The flow rate \( \frac{\partial Q}{\partial t} \) can then be calculated based on:

\[
\frac{\partial Q}{\partial t} = vA = \alpha_c l^\beta A = \alpha_c \left( \frac{1}{\gamma} \frac{\partial u}{\partial r} \right)^\beta (2\pi rl) \tag{6.5}
\]

\[
\frac{\partial Q}{\partial t} = \frac{\partial e}{\partial t} \pi (R^2 - r^2)l \tag{6.6}
\]

Equating Eqs. 6.5 and 6.6 results in:

\[
\frac{\partial u}{\partial r} = \gamma \left( \frac{1}{2\alpha_c \frac{\partial e}{\partial t}} \right)^\frac{1}{\beta} \left( \frac{R^2-r^2}{r} \right)^\frac{1}{\beta} \tag{6.7}
\]
Solving Eq. 6.7 (see Appendix A for the complete solution) leads to:

\[ \bar{u} = \left\{ (1 - \beta) \left( -\frac{2\alpha_c}{m_v} \right) \left[ \frac{n^2 - 1}{2n^2} \frac{R^2}{\eta_w} \right]^{\beta} t + (\bar{u}_0 + P_0)^{(1-\beta)} \right\}^{1/(1-\beta)} - P_0 \]  \hspace{1cm} 6.8

and,

\[ u' = \eta_s \left( \frac{\alpha_c}{\alpha_{c'}} \right)^{\frac{1}{\beta}} \left( \frac{n^2 - 1}{2n^2} \frac{R^2}{\eta} \right) \left\{ (1 - \beta) \left( -\frac{2\alpha_c}{m_v} \right) \left[ \frac{n^2 - 1}{2n^2} \frac{R^2}{\eta_w} \right]^{\beta} t \right\}^{1/(1-\beta)} - P_0 \]  \hspace{1cm} 6.9

for \( r_w \leq r \leq r_s \)

and,

\[ u = \eta_r \left( \frac{n^2 - 1}{2n^2} \frac{R^2}{\eta} \right) \left\{ (1 - \beta) \left( -\frac{2\alpha_c}{m_v} \right) \left[ \frac{n^2 - 1}{2n^2} \frac{R^2}{\eta_w} \right]^{\beta} t \right\}^{1/(1-\beta)} + (\bar{u}_0 + P_0)^{(1-\beta)} \]  \hspace{1cm} 6.10

for \( r_s \leq r \leq R \)

and,

\[ \varepsilon = -m_v \left\{ (1 - \beta) \left( -\frac{2\alpha_c}{m_v} \right) \left[ \frac{n^2 - 1}{2n^2} \frac{R^2}{\eta_w} \right]^{\beta} t + (\bar{u}_0 + P_0)^{(1-\beta)} \right\}^{1/(1-\beta)} - (\bar{u}_0 + P_0) \]  \hspace{1cm} 6.11

where, \( \bar{u} \) is the average excess pore water pressure in the unit cell, \( u' \) and \( u \) are the excess pore water pressures within and outside the smear zone, respectively, for a given time, \( m_v \) is the coefficient of the soil volume compressibility, \( \bar{u}_0 \) is the initial average excess pore water pressure in the unit cell, \( P_0 \) is the magnitude of the vacuum pressure at drain-soil interface, \( \alpha_{c'} \) is the substitute of \( \alpha_c \) in Eq. 6.3 for the
flow relationship in the smear zone, \( r_w \) and \( r_s \) are the radii of the drain and smear zone, respectively, \( n = \frac{R}{r_w} \), and \( \eta, \eta_s, \) and \( \eta_n \) are the coefficients which are defined in Appendix A.

### 6.4 Validation of the Non-Darcian Model

To validate the proposed radial consolidation model, the model predictions were compared with the measured data, predictions made by radial consolidation model based on Darcy’s flow (Hansbo, 1981), and radial consolidation model based on non-linear flow proposed by Hansbo (1960, 2001). Based on the Darcian flow, Eqs. 6.12 and 6.13 can be used to calculate the excess pore water pressures in the smear and undisturbed zones, respectively (Indraratna et al. 2005b).

\[
\begin{align*}
\eta' &= \frac{k_h}{k_h'} \left( \frac{\bar{u}_0 + P_0}{\mu R^2} \right) \exp \left( \frac{-2 \eta c_h}{\mu R^2} \right) \left[ R^2 \ln \left( \frac{r}{r_w} \right) - \frac{(r^2 - r_w^2)}{2} \right] - P_0 \\
\quad \text{for } r_w \leq r \leq r_s
\end{align*}
\]

\[
\begin{align*}
\eta &= \frac{(\bar{u}_0 + P_0)}{\mu R^2} \exp \left( \frac{-2 \eta c_h}{\mu R^2} \right) \left[ \left[ R^2 \ln \left( \frac{r}{r_s} \right) - \frac{(r^2 - r_s^2)}{2} \right] + \frac{k_h}{k_h'} \left[ R^2 \ln \left( \frac{r_s}{r_w} \right) - \frac{(r^2 - r_w^2)}{2} \right] \right] - P_0 \\
\quad \text{for } r_s \leq r \leq R
\end{align*}
\]

where,

\[
\eta = \frac{n^2}{n^2 - 1} \left[ \ln \left( \frac{n}{\sqrt{\eta_s}} \right) + \frac{k_h}{k_h'} \ln (s) - \frac{3}{4} \right] + \frac{s^2}{n^2 - 1} \left( 1 - \frac{s^2}{4n^2} \right) + \frac{k_h}{k_h'} \frac{1}{n^2 - 1} \left( \frac{s^4 - 1}{4n^2} - s^2 + 1 \right)
\] 6.14

\( k_h \) and \( k_h' \) are the coefficients of permeability in the horizontal direction within the
smear and undisturbed zones, respectively, and \( c_R \) is the coefficient of radial consolidation.

Average excess pore water pressure and axial strain based on Darcy flow at any given time in the unit cell can be calculated by Eqs. 6.15 and 6.16, respectively (Indraratna et al. 2005b):

\[
\tilde{u} = (\tilde{u}_0 + P_0) \exp \left( -\frac{2c_R t}{\mu R^2} \right) - P_0 \tag{6.15}
\]

\[
\varepsilon = -m_v (\tilde{u}_0 + P_0) \left[ \exp \left( -\frac{2c_R t}{\mu R^2} \right) - 1 \right] \tag{6.16}
\]

Based on Hansbo’s (1960) non-linear flow relationship (Eq. 1), Eqs. 6.17-6.20 can be formulated for the excess pore water pressures within and outside the smear zone, the average excess pore water pressure, and the axial strain, respectively. Note that the original Hansbo’s (1960) equations have now been revised to include vacuum pressure \( (P_0) \). To formulate Eqs. 6.17-6.20, same procedure as explained in Appendix A can be followed and \( \beta \) and \( \frac{a_c}{\alpha_c} \) are substituted by \( m \) and \( \frac{k_h}{k_h^*} \), respectively.

\[
u' = \eta_s \left( \frac{k_h}{k_h^*} \right)^{\frac{1}{m}} \left( \frac{n^2 - 1}{2n^2} \frac{R^2}{\eta} \right) \left( 1 - m \right) \left( -\frac{2c_R \gamma_w}{m \frac{n^2 - 1}{m - 1} \frac{R^2}{\eta \gamma_w} t} \right) \left( \frac{1}{(1-m)} \right) - P_0 \tag{6.17}
\]

\[\text{for } r_w \leq r \leq r_s\]
To determine the soil parameters \( m_v \) and \( c_h \), a radial consolidation test (SP=100 kPa and VP=0.0) with a standard 75 mm Rowe cell was conducted and the value of \( m_v \) was calculated to be 0.97 \( m^2/MN \). Based on the settlement data, \( c_h \) was calculated according to three different methods: (A) Steepest Tangent Method (Vinod et al, 2010), (B) Square Root Method (Sridharan et al, 1996), (C) Log-Log Method (Robinson, 2009). In addition, by assuming \( c_h = c_v \) a vertical consolidation test (SP=100 kPa) with the 75 mm Rowe cell was conducted and Logarithm-of-Time Method was employed to determine \( c_h \) (Approach D). Due to employing remolded soil, the effect of anisotropy is considered to be minimal in this study. The related information for the above four approaches is summarised in Table 6-3. To use Hansbo’s non-linear model, the values of \( i_0 \) and \( i_t \) based on Figure 6.4 were determined to be 15 and 45, respectively, and for the value of \( m \) the general value of
1.5 recommended by Hansbo (2001) is used.

Table 6-3: $c_h$ values calculated based on different approaches

<table>
<thead>
<tr>
<th>Test No.</th>
<th>Sample Diameter (mm)</th>
<th>Drain Diameter (mm)</th>
<th>$n$</th>
<th>$SP$ (kPa)</th>
<th>$VP$ (kPa)</th>
<th>$c_h$ ($m^2/year$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Row cell (Radial consolidation)</td>
<td>Row cell (Vertical consolidation)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(A)</td>
<td>(B)</td>
<td>(C)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Steepest Tangent Method</td>
<td>Square Root Method</td>
<td>Log-Log Method</td>
</tr>
<tr>
<td>4</td>
<td>75.5</td>
<td>7.5</td>
<td>10.07</td>
<td>100</td>
<td>0</td>
<td>4.90</td>
</tr>
</tbody>
</table>

To validate the proposed model, measured data from a test with a combination of surcharge and vacuum pressure were also obtained using a 75 mm Rowe cell, as shown in Table 6-4. The average excess pore water pressure and axial settlement were compared with the predictions based on the proposed model, Darcian flow model, and the model based on Hansbo’s non-Darcian flow. Figure 6.6a compares the measured average excess pore water pressure with the predictions from the proposed model (Eqs. 6.9 and 6.10) and the Darcian-based model (Eqs. 6.12 and 6.13) considering the four different approaches (A, B, C, and D) for the calculation of $c_h$. Figure 6.6b shows the comparisons between the laboratory results and the predictions based on Hansbo’s non-linear flow rule (Eqs. 6.17 and 6.18). The time-dependent axial strains based on Eqs. 6.11, 6.16, and 6.20 are presented in Figure 6.7; comparing the predictions based on proposed model, Darcian based model and Hansbo’s non-Darcian based model. Predictions of Hansbo’s non-Darcian based model show faster dissipation of excess pore water pressure and higher
settlement for all values of $c_r$ at a given time, compared to the proposed model and Darcian based model. This is because, under higher hydraulic gradients, Hansbo’s non-linear flow rule yields higher water flow velocity than the conventional Darcy’s law and proposed model. A good agreement between the predictions of the proposed model with the measured data can be observed as well as a sensitivity of the existing models to the method of $c_r$ calculation where the calculated $c_r$ can vary more than 250% (see Table 6-3) adopting different approaches (A, B, C, and D).

Table 6-4: Radial consolidation test using 75 mm Rowe cell

<table>
<thead>
<tr>
<th>Test No.</th>
<th>Sample Diameter (mm)</th>
<th>Drain Diameter (mm)</th>
<th>$n$</th>
<th>$SP$ (kPa)</th>
<th>$VP$ (kPa)</th>
<th>$SP + VP$ (kPa)</th>
<th>$VSR = \frac{VP}{SP + VP}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>75.5</td>
<td>7.5</td>
<td>10.07</td>
<td>40</td>
<td>60</td>
<td>100</td>
<td>0.6</td>
</tr>
</tbody>
</table>
Figure 6.6: Average excess pore water pressure dissipation in 75 mm Rowe cell (a) comparison with Darcian-based model, (b) comparison with Hansbo’s non-Darcian based model.
Figure 6.7: Axial strain in 75 mm Rowe cell (a) comparison with Darcian-based model, (b) comparison with Hansbo’s non-Darcian based model.
6.5 Advantages of the Model and Practical Implications

During the conceptual design when the observational field data are unavailable, parameters such as the coefficient of soil volume compressibility \( (m_v) \), the coefficient of consolidation \( (c_h) \), and the coefficient of permeability \( (k_h) \) need to be evaluated first. Independent tests are usually used to calculate \( c_h \) and \( k_h \) and often result in different values of \( c_h \) and \( k_h \). Among the above three mentioned parameters, \( m_v \) is considered to be more reliable and does not depend on the test procedures or the calculation method and using this parameter instead of \( c_h \) eliminates the uncertainties related to the calculation of \( c_h \).

Capturing flow relationship by conducting the above technique not only eliminates errors and uncertainties that could probably occur while using conventional procedures to calculate permeability, it also provides a more realistic flow relationship during consolidation. With the proposed technique, \( \alpha_c \) and \( \beta \) define the seepage flow behaviour under the entire range of hydraulic gradient under both field and laboratory conditions. Another advantage of the proposed consolidation model is that it only needs \( m_v \) to be determined, which can easily be calculated based on settlement and applied pressure. Moreover, in comparison to Hansbo’s (2001) model, the proposed flow relationship and the consolidation model have the following advantages:

1. Although, Hansbo (2001) stated that the non-linear part of the flow relationship can be accurately used for the entire range from low to high hydraulic gradients, the threshold hydraulic gradient \( (i_t) \) must still be accurately defined to calculate \( i_0 \) and \( m \). Determining this gradient is not an
easy task and may not be accurate unless the flow relationship is available. The wide range of the reported values for the $i_t$ (e.g. Hansbo, 1960; Dubin and Moulin, 1986) shows that the $i_t$ changes considerably in different soil, and inaccuracy in defining $i_t$ might significantly decrease the accuracy of the predictions. In contrast, in the proposed Author’s model, there is no threshold and a non-linear flow relationship can be used to describe the entire range of the hydraulic gradients.

(2) The non-linear part of Hansbo’s model is only applicable to the field conditions (Hansbo, 1960) when the hydraulic gradient is small. For cases where the hydraulic gradient is large, such as in the projects where a high surcharge preloading is required, or when the behaviour of the soil close to the drains is of interest, Hansbo’s non-linear model may deviate from the measured data while the proposed model is able to capture the entire range of hydraulic gradients applicable for both laboratory and field conditions.

6.6 Application to a Case Study

To demonstrate the accuracy of the prediction of the proposed model under field conditions, the measured data obtained from the reclamation area at the Port of Brisbane (Australia) were analysed. The site was divided into 8 areas (Figure 6.8) and a combination of conventional surcharge and vertical drains (PVDs) was used for the soil improvement scheme. In two of the sub-divisions (VC1 and VC2, Figure 6.8), vacuum pressure was used in conjunction with surcharge fill and vertical drains. In this section, the measured data in non-vacuum areas (WD2 and WD4) were
employed to determine flow relationship, whereas the measured data from vacuum area (VC1) was compared with the predictions based on the proposed and conventional models.

The characteristics of ground improvement for the areas WD2, WD4, and VC1 are summarised in Table 6-5 and the properties and general profile of the soil for the entire reclamation area reported by Indraratna et al. (2011) are presented in Figure 6.8:

Figure 6.8: Subdivisions of reclaimed area at the Port of Brisbane.

The characteristics of ground improvement for the areas WD2, WD4, and VC1 are summarised in Table 6-5 and the properties and general profile of the soil for the entire reclamation area reported by Indraratna et al. (2011) are presented in Figure 6.8:
6.9. The profile and thickness of the soil layers for sections WD2, WD4, and VC1 are shown in Table 6-6. Embankment height, measured surface settlement, and measured excess pore water pressure for areas WD2 and WD4 are shown in Figure 6.10. Surface settlement and pore water pressure at a depth of 9.2 and 14 m were measured at the centre of sub-divisions WD2 and WD4, respectively. From Figure 6.10, the data after the excess pore water pressures reaching their maximum values were used to determine the seepage flow relationship. The average hydraulic gradient in each area was calculated based on the excess pore water pressures at the drain-soil interface and at the drain influence zone. The seepage velocity was calculated at the half way between these two locations, based on 6.2. Figure 6.11 represents the associated $v$-$i$ plots for areas WD2 and WD4. Accordingly, the values of $\alpha_c$ and $\beta$ were calculated to be $1.1 \times 10^{-10} \text{ m/sec}$ and 1.35, respectively.

<table>
<thead>
<tr>
<th>Section</th>
<th>Drain type</th>
<th>Drain length (m)</th>
<th>Drain spacing in square pattern (m)</th>
<th>Clay thickness (m)</th>
<th>Total fill height (m)</th>
<th>Treatment scheme</th>
</tr>
</thead>
<tbody>
<tr>
<td>WD2</td>
<td>Circular drains with 34 mm diameter</td>
<td>22.5-27.5</td>
<td>1.3</td>
<td>20.0-23.5</td>
<td>7-7.2</td>
<td>Surcharge</td>
</tr>
<tr>
<td>WD4</td>
<td>Band drains (100x4 mm²)</td>
<td>27.0-28.7</td>
<td>1.3</td>
<td>22.5-24.5</td>
<td>6.1</td>
<td>Surcharge</td>
</tr>
<tr>
<td>VC1</td>
<td>Circular drains with 34 mm diameter</td>
<td>14.0-26.5</td>
<td>1.2</td>
<td>9.0-21.0</td>
<td>3.2</td>
<td>Surcharge + 65 kPa vacuum</td>
</tr>
</tbody>
</table>
Table 6-6: Soil profiles for individual sections at the Port of Brisbane

<table>
<thead>
<tr>
<th>Area</th>
<th>Dredged Mud</th>
<th>Upper Holocene Sand</th>
<th>Upper Holocene Clay</th>
<th>Lower Holocene Clay</th>
</tr>
</thead>
<tbody>
<tr>
<td>WD2</td>
<td>1-2.5</td>
<td>1-3</td>
<td>2-5</td>
<td>18-20</td>
</tr>
<tr>
<td>WD4</td>
<td>1.5-2.2</td>
<td>1-2</td>
<td>1.5-3.5</td>
<td>18-23</td>
</tr>
<tr>
<td>VC1</td>
<td>2-3</td>
<td>2-3</td>
<td>2-3</td>
<td>5-18</td>
</tr>
</tbody>
</table>

Surface settlement and pore water pressure at a depth of 10.1 m were measured at the centre of sub-division VC1. In the analysis, the ratio of $\alpha_c/\alpha'_c$ and $s$ were adopted as 2 and 3, respectively. Based on the settlement curves for the areas WD2 and WD4, the coefficient of volume compressibility ($m_v$) and $c_h$ values for clay were back calculated to be 1.0 m$^2$/MN and 2.0 m$^2$/year, respectively. For Hansbo’s
model, the general value of \( m = 1.5 \) recommended by Hansbo (2001) is used and from Figure 6.11, \( i_0 \) and \( i_1 \) were determined to be 4 and 12, respectively.

Figure 6.12 shows the height of the embankment, the associated settlement and pore pressure data at the centre line of Section VC1, in comparison with the predictions from the conventional Darcian-based model, and Hansbo’s non-Darcian based model during the period when the maximum height of the embankment and vacuum pressure were applied to the soil. Both the Darcian-based and Hansbo’s non-Darcian based models overestimate the settlement and underestimate the excess pore water pressure, in comparison with the proposed non-Darcian model. This can be attributed to flow characteristics on the rate of consolidation. Figure 6.12 shows that the proposed model can accurately predict settlement whereas under-estimate the excess pore water pressure.
Figure 6.10: (a) Embankment height, (b) Settlement, and (c) Excess pore water pressure for Sections WD2 and WD4, Port of Brisbane, Australia.
Figure 6.11: Flow velocity – hydraulic gradient relationship for areas WD2 and WD4 in Port of Brisbane
Figure 6.12: (a) Embankment height, (b) Settlement, and (c) Excess pore water pressure for area VC1 in Port of Brisbane
6.7 Summary

Laboratory radial consolidation tests subjected to vacuum and surcharge loading using modified Rowe cell were conducted to determine seepage flow characteristics during radial consolidation. Based on the excess pore water pressure measurement, a non-linear relationship between flow velocity and hydraulic gradient was proposed for the entire consolidation process. The advantages of the proposed flow relationship can be summarised as follows: (a) it provides more realistic flow behaviour during consolidation and (b) in contrast to relationship proposed by Hansbo (1960), threshold hydraulic gradient is not required to differentiate between the linear and non-linear flow relationships.

Classical radial consolidation models often rely on the accurate calculation of $c_h$. Different methods of the determination of $c_h$ provide different values. In this study, it is shown that the value of $c_h$ can vary more than 250% resulting in uncertainties in the prediction of excess pore pressure and settlement. In contrast, an analytical model has been developed based on the observed flow relationship to predict the radial consolidation behaviour of soft soil under both surcharge and vacuum preloading. In the proposed model, instead of $c_h$, $m_v$ is used and that can be determined readily from settlement-effective stress curve. The predictions from the proposed model agree well with the laboratory results based on Rowe cell testing whereas the predictions of the other models vary due to the adopted method of the calculation of $c_h$. The proposed solution gives a greater accuracy of the settlement and excess pore water pressure prediction, when applied to a selected case history (Port of Brisbane Australia). In this analysis, the smear effect (due to mandrel driven
prefabricated drains) was included and the flow relationship was determined based on the measurement in the adjacent areas. For a given drain pattern, the findings of this study confirm that the flow relationship is a major factor influencing the embankment settlement and excess pore water dissipation.
7 CONCLUSION AND RECOMMENDATIONS

7.1 General Summary
A comprehensive Introduction and a Literature Review on the behaviour and improvement of soft soils via vertical drains and vacuum preloading were presented in Chapters 1 and 2. These chapters summarised the advancements in the analytical, numerical, and experimental methods used to improve soft soil and highlighted the areas that still require research efforts specifically focused on the application of combined surcharge-vacuum loading. Chapter 3 presented a discussion on the Rowe consolidation cell and its advantages and described a new 150 mm Rowe cell developed in-house. Chapter 4 emphasised the difference between the consolidation parameters back-calculated from the trends of settlement and excess pore water pressure dissipation, and demonstrated how these parameters change with the vacuum-surcharge ratio. Chapter 5 presented a comprehensive laboratory study elaborating the different roles of surcharge and vacuum on the changes to excess pore water pressure, settlement, and effective stresses. Finally, Chapter 6 introduced an innovative analytical non-Darcian radial consolidation model incorporating the effects of vertical drains and vacuum preloading. Specific outcomes and salient features of this research study are described below.

7.2 Novelty of the Experimental Procedure
Modification of the Rowe cell used in the experimental studies and the adopted test procedure indicated the following advantages and novelties over the existing
equipment and test procedures:

- The modified 150 mm Rowe cell was equipped with a higher cell body than a standard 150 mm Rowe cell that enables the preconsolidation of slurries to a desired soil thickness before installing the drain;

- Measuring pore water pressure at four locations at different radial distances at the base of the cell enables the lateral distribution of excess pore water pressure to be measured and to quantify the radial propagation of vacuum pressure. The actual effects of surcharge and/or vacuum pressure on the soil elements at any radial distance from the drain at a given time can then be studied, while making a more accurate comparison between the effects of surcharge and vacuum on the consolidation of soft soils;

- Convenience of determining the degree of consolidation (DOC) based on excess pore water pressure dissipation and a comparison of the back-calculated consolidation parameters from the excess pore pressure and settlement analyses;

- Measurement of the excess pore water pressure at four different radii during vacuum assisted radial consolidation tests revealed that when the vacuum pressure was applied to the drain, it did not propagate immediately across the soil. The propagation of vacuum pressure across the soil tends to follow an exponential trend with the maximum value at the drain-soil interface and the minimum value at the cell body-soil interface. The maximum value was equal to the applied vacuum and remained constant. The minimum value increased with time as the vacuum pressure propagated across the soil, and reached its
maximum (i.e. applied vacuum) at the end of consolidation. These observations clearly suggest that the effect of vacuum pressure on the excess pore water pressure depends on the distance from the drain, whereas when a surcharge is applied to the soil, excess pore water pressure increases evenly across the soil.

7.3 Analytical Method and its Salient Features
The proposed practical procedure to capture the flow relationship and the corresponding analytical model formulated for the vacuum assisted radial consolidation incorporate several salient features over the existing solutions. These characteristics can be summarized as follow:

- Unlike Hansbo’s non-linear flow relationship, the proposed non-linear $v-i$ relationship does not have any threshold hydraulic gradient, and therefore it eliminates the difficulties and uncertainties surrounding the threshold hydraulic gradient;

- The proposed flow relationship covers a wide range of hydraulic gradients (0 to 100) by a single non-linear formulation, which can be used for both laboratory and field conditions, whereas Hansbo’s non-Linear flow relationship can be only used for field conditions of relatively low hydraulic gradients;

- The procedure proposed to capture the flow relationship is directly associated with the radial consolidation tests and automatically capture the salient aspects such as the changes to the applied hydraulic gradient, migration of soil grains, changes in the soil cross section, etc. Therefore, the
typical errors encountered in conventional methods of determining flow relationships are reduced;

- The proposed consolidation model can incorporate the effects of the smear zone, the type of vertical drains, the variation of vacuum pressure, and the non-linearity of the flow relationship;
- The proposed model uses the coefficient of soil volume compressibility \( m_v \) instead of the coefficient of permeability \( k_h \) and coefficient of consolidation \( c_h \), which are often affected by the method of calculation and the method of measurement. Therefore, the uncertainties in determining \( c_h \) and \( k_h \) are minimised in the proposed model.

### 7.4 Benefits of Combined Vacuum-Surcharge Pressure Over Surcharge Alone

In this study, it is shown that the application of the combined vacuum-surcharge pressure has several advantages over the conventional surcharge alone in the consolidation of soft soils. These benefits include both technical and environmental aspects and they can be summarized as follow:

- A higher rate of the excess pore water pressure dissipation and a higher rate of settlement occur in the vacuum assisted consolidation for the same magnitude of total stress. This results in an earlier release of land for construction activities, plus reducing project costs;
- The rate of increase in effective stresses corresponding to vacuum assisted consolidation (both vertical and horizontal) results in a higher shear strength of the improved soil;
• A greater OCR for the vacuum assisted consolidation can be attained compared to surcharge alone. Therefore, the post construction settlement is expected to be less;
• Vacuum pressure decreases the lateral displacements, and therefore reduces the risk of embankment failure (i.e. increased stability);
• Increased vacuum-total surcharge ratio (VSR) eliminates the need for fill removal after the desired DOC is attained;
• Reduction of the embankment height in the vacuum assisted consolidation generates savings and contributes to an earlier completion of construction;
• Vacuum assisted consolidation is a more environmentally friendly technique by reducing the extent of earthworks that is typically associated with surcharge-only embankments.

7.5 Recommendations for Future Research
This area of research can be extended further by conducting the following studies:
• In the entire laboratory tests conducted in this research, Kaolin was used as the soil sample. It would be worthwhile if a few other natural clays can be further calibrate the proposed model;
• All soil samples used in the current laboratory studies were disturbed. This offered the simplicity and advantage of eliminating the uncertainties regarding soil homogeneity. Undisturbed samples should be used to study the effects of soil anisotropy and the structure;
• As discussed in Chapter 5, only one value of vacuum-surcharge ratio
(VSR=0.5) was used for all the tests. It is recommended to conduct tests using different VSR, for examining the model validity for a broader range of test conditions;

- This research was based on the laboratory and analytical studies and numerical tools have not been employed for the assessment. It is suggested for future research to numerically model the Rowe cell consolidation and compare the results with the provided laboratory and analytical outcomes.
REFERENCES


REFERENCES


REFERENCES

Prefabricated vertical drains-design and performance. CIRIA.


REFERENCES


REFERENCES

International Conference on Soil Mechanics and Foundation Engineering, University of Toronto Press, 1 (Paper 2/44).
REFERENCES


APPENDIX A: ANALYTICAL SOLUTION OF THE RADIAL CONSOLIDATION MODEL

The complete solution of the radial consolidation equation (Eq. 6.7) is presented here in detail. To start the solution, Eq. 6.7 is represented here again by:

$$
\frac{\partial u}{\partial r} = \gamma_w \left( \frac{1}{2\alpha_c \alpha_t} \right)^{\frac{1}{\beta}} \left( \frac{R^2-r^2}{r^3} \right)^{\frac{1}{\beta}}
$$  \hspace{1cm} (A1)

Using binomial series, the last term in the above equation can be expanded to:

$$
\left( \frac{R^2-r^2}{r} \right)^{\frac{1}{\beta}} = \left( \frac{R^2}{r} - r \right)^{\frac{1}{\beta}} = \left[ \left( R^2 \right)^{\frac{(1-\beta)}{\beta}} r^{\left( \frac{1-\beta}{\beta} \right)} - \frac{1}{\beta} \left( R^2 \right)^{(1-\beta)} r^{\left( \frac{2-\beta}{\beta} \right)} + \frac{1-\beta}{2(1-\beta)} \left( R^2 \right)^{(1-2\beta)} r^{\left( \frac{4-\beta}{\beta} \right)} - \frac{(1-\beta)(1-2\beta)}{3(1-3\beta)} \left( R^2 \right)^{(1-3\beta)} r^{\left( \frac{6-\beta}{\beta} \right)} + \ldots \right]
$$  \hspace{1cm} (A2)

Substitution of Eq. A2 into Eq. A1 gives:

$$
\frac{\partial u}{\partial r} = \gamma_w \left( \frac{1}{2\alpha_c \alpha_t} \right)^{\frac{1}{\beta}} \left[ \left( R^2 \right)^{\frac{(1-\beta)}{\beta}} r^{\left( \frac{1-\beta}{\beta} \right)} - \frac{1}{\beta} \left( R^2 \right)^{(1-\beta)} r^{\left( \frac{2-\beta}{\beta} \right)} + \frac{1-\beta}{2(1-\beta)} \left( R^2 \right)^{(1-2\beta)} r^{\left( \frac{4-\beta}{\beta} \right)} - \frac{(1-\beta)(1-2\beta)}{3(1-3\beta)} \left( R^2 \right)^{(1-3\beta)} r^{\left( \frac{6-\beta}{\beta} \right)} + \ldots \right]
$$  \hspace{1cm} (A3)

Integrating Eq. A3 in the $r$ direction with the boundary condition $u = -P_0$ at $r = r_w$ yields:
where; 

\[ u' = \eta_s Y_w \left( \frac{1}{2\alpha_c} \frac{\partial e}{\partial t} \right)^{\frac{1}{\beta}} - P_0 \quad \text{for} \quad r_w \leq r \leq r_s \quad (A4) \]

\[
\eta_s = \begin{cases} 
\left( \frac{\beta}{\beta-1} \right) (R^2) \left( \frac{1}{\beta} \right) \left[ r^{(1-1/\beta)} - r_w^{(1-1/\beta)} \right] \\
- \left( \frac{1}{3\beta-1} \right) (R^2) \left( \frac{1-\beta}{\beta} \right) \left[ r^{(3-1/\beta)} - r_w^{(3-1/\beta)} \right] \\
+ \left( \frac{1-\beta}{2\beta(5\beta-1)} \right) (R^2) \left( \frac{1-2\beta}{\beta} \right) \left[ r^{(5-1/\beta)} - r_w^{(5-1/\beta)} \right] \\
- \left( \frac{1-\beta}{3\beta^2(7\beta-1)} \right) (R^2) \left( \frac{1-3\beta}{\beta} \right) \left[ r^{(7-1/\beta)} - r_w^{(7-1/\beta)} \right] \\
+ \ldots 
\end{cases} 
\quad (A5) 
\]

In the above, \( u' \) is the excess pore water pressure in the smear zone, \( P_0 \) is the magnitude of vacuum pressure at the drain-soil interface, \( r_s \) is the radius of smear zone, and \( \alpha_c' \) is the substitute of \( \alpha_c \) in Eq. 8 for the flow relationship in the smear zone (flow relationship in the smear zone can be represented by \( v = \alpha_c'i^\beta \) ).

Integrating Eq. A3 in the \( r \) direction outside the smear zone with the boundary condition \( u = u' \) at \( r = r_s \) gives:

\[ u = \eta_n Y_w \left( \frac{1}{2\alpha_c} \frac{\partial e}{\partial t} \right)^{\frac{1}{\beta}} - P_0 \quad \text{for} \quad r_s \leq r \leq R \quad (A6) \]

where;
Appendix A

ANALYTICAL SOLUTION OF THE RADIAL CONSOLIDATION MODEL

\[ \eta_n = \left\{ \begin{aligned} &\left( \frac{\beta}{\beta-1} \right) (R^2)^{\frac{1}{2\beta}} \left[ r^{(1-\frac{1}{\beta})} + \left( \frac{a_c}{\alpha_c^2} \right)^{\frac{1}{2\beta}} - 1 \right] r_s^{(1-\frac{1}{\beta})} - \left( \frac{a_c}{\alpha_c^2} \right)^{\frac{1}{2\beta}} r_w^{(1-\frac{1}{\beta})} \\ &- \left( \frac{1}{3\beta-1} \right) (R^2)^{\frac{1-2\beta}{2\beta}} \left[ r^{(1-\frac{1}{\beta})} + \left( \frac{a_c}{\alpha_c^2} \right)^{\frac{1}{2\beta}} - 1 \right] r_s^{(1-\frac{1}{\beta})} - \left( \frac{a_c}{\alpha_c^2} \right)^{\frac{1}{2\beta}} r_w^{(1-\frac{1}{\beta})} \\ &+ \left( \frac{1-\beta}{2\beta(3\beta-1)} \right) (R^2)^{\frac{1-2\beta}{2\beta}} \left[ r^{(1-\frac{1}{\beta})} + \left( \frac{a_c}{\alpha_c^2} \right)^{\frac{1}{2\beta}} - 1 \right] r_s^{(1-\frac{1}{\beta})} - \left( \frac{a_c}{\alpha_c^2} \right)^{\frac{1}{2\beta}} r_w^{(1-\frac{1}{\beta})} \\ &- \left( \frac{1-\beta}{3\beta^2(7\beta-1)} \right) (R^2)^{\frac{1-2\beta}{2\beta}} \left[ r^{(1-\frac{1}{\beta})} + \left( \frac{a_c}{\alpha_c^2} \right)^{\frac{1}{2\beta}} - 1 \right] r_s^{(1-\frac{1}{\beta})} - \left( \frac{a_c}{\alpha_c^2} \right)^{\frac{1}{2\beta}} r_w^{(1-\frac{1}{\beta})} \\ &+ \ldots \end{aligned} \right\} \quad (A7) \]

The average excess pore water pressure in the unit cell can then be calculated from:

\[ \bar{u} = \frac{\int_{r_w}^{r_s} 2\pi r u'dr + \int_{r_s}^{R} 2\pi r udr}{\pi (R^2 - r_w^2) l} = \frac{2}{(R^2 - r_w^2)} \left[ \int_{r_w}^{r_s} r u'dr + \int_{r_s}^{R} r udr \right] \quad (A8) \]

Substituting Eqs. A4 and A6 into Eq. A8 and taking the integrals gives:

\[ \bar{u} = \frac{2n^2}{n^2 - 1} \frac{2\pi}{R^2} \frac{\eta_{gw}}{R^2} \left( \frac{1}{2\alpha_c} \frac{\partial \epsilon}{\partial t} \right)^{\frac{1}{2}} - P_0 \quad (A9) \]

where,
Then, the well known relationship between the strain rate and excess pore pressure dissipation rate (Eq. A11) can be substituted in Eq. A9 which results in Eq. A12 representing an alternative expression for $\bar{u}$.

$$\frac{\partial \varepsilon}{\partial t} = -m_v \frac{\partial \bar{u}}{\partial t}$$  \hspace{1cm} (A11)$$

where, $m_v$ is the coefficient of volume compressibility.

$$\bar{u} = \frac{2n^2 \eta \gamma_{\nu}}{n^2 - 1} \frac{1}{R^2} \left( - \frac{m_v}{2\alpha_\nu} \frac{\partial \bar{u}}{\partial t} \right)^\frac{1}{\beta} - P_0$$  \hspace{1cm} (A12)$$

Rearranging the above equation gives:

$$\frac{\partial \bar{u}}{\partial t} = \left( - \frac{2\alpha_\nu}{m_v} \right) \left[ \frac{n^2 - 1}{2n^2} \frac{R^2}{\eta \gamma_{\nu}} \left( \bar{u} + P_0 \right) \right]^{\beta}$$  \hspace{1cm} (A13)$$

and,
\[(\bar{u} + P_0)^{-\beta} \partial \bar{u} = \left(\frac{-2\alpha_c}{m_w}\right) \left[\frac{n^2-1}{2n^2} \frac{R^2}{\eta \gamma_w}\right]^\beta \partial t \] (A14)

Integration of Eq. A14 gives:

\[\bar{u} = \left\{ (1 - \beta) \left(\frac{-2\alpha_c}{m_w}\right) \left[\frac{n^2-1}{2n^2} \frac{R^2}{\eta \gamma_w}\right]^\beta t + \left(\bar{u}_0 + P_0\right)^{(1-\beta)} \right\}^{\frac{1}{1-\beta}} - P_0 \] (A15)

Differentiating Eq. A16 with respect to \(t\) results in:

\[\frac{\partial \bar{u}}{\partial t} = \left(\frac{-2\alpha_c}{m_w}\right) \left[\frac{n^2-1}{2n^2} \frac{R^2}{\eta \gamma_w}\right]^\beta \left\{ (1 - \beta) \left(\frac{-2\alpha_c}{m_w}\right) \left[\frac{n^2-1}{2n^2} \frac{R^2}{\eta \gamma_w}\right]^\beta t \right\}^{\frac{\beta}{(1-\beta)}} \] (A16)

Substituting Eq. A16 into Eq. A11 yields:

\[\frac{\partial \varepsilon}{\partial t} = \left(\frac{2\alpha_c}{m_w}\right) \left[\frac{n^2-1}{2n^2} \frac{R^2}{\eta \gamma_w}\right]^\beta \left\{ (1 - \beta) \left(\frac{-2\alpha_c}{m_w}\right) \left[\frac{n^2-1}{2n^2} \frac{R^2}{\eta \gamma_w}\right]^\beta t \right\}^{\frac{\beta}{(1-\beta)}} \] (A17)

Combining Eqs. A4 and A6 with Eq. A17 leads to Eqs. A18 and A19 for the excess pore water pressures inside and outside the smear zone, respectively, thus:

\[u' = \eta_w \left(\frac{\alpha_c}{\alpha_c'}\right)^{\frac{1}{\beta}} \left(\frac{n^2-1}{2n^2} \frac{R^2}{\eta}\right) \left\{ (1 - \beta) \left(\frac{-2\alpha_c}{m_w}\right) \left[\frac{n^2-1}{2n^2} \frac{R^2}{\eta \gamma_w}\right]^\beta t \right\}^{\frac{1}{(1-\beta)}} - P_0 \] (A18)

for \(r = r_w \leq r \leq r_s\)

and,
Integrating Eq. A17 results in the following equation for the axial strain at a given time:

\[ \varepsilon = -m_v \left\{ \left(1 - \beta\right) \left( -\frac{2a_c}{\eta m_v} \left[ \frac{n^2-1}{2n^2 \eta \gamma_w} \right]^\beta \right) t + \left( \bar{u}_0 + P_0 \right)^{(1-\beta)} \right\}^{-\frac{1}{1-\beta}} - \left( \bar{u}_0 + P_0 \right) \]  

(A20)