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Predicting the Behaviour of Coal Wash and Steel Slag Mixtures under Triaxial Conditions

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Abstract: The effective reuse and recycling of granular waste materials, namely coal wash (CW) and Basic Oxygen Steel slag (BOS), is economically beneficial and environmentally sustainable. Nevertheless, due to the heterogeneity of these granular waste materials, their ultimate adoption as structural fills must be supported by constitutive relationships capable of accurately describing the stress-strain behaviour under representative field loading conditions. In this paper, a critical-state-surface generalised-plasticity model is presented and its predictive capability using an explicit set of soil parameters is demonstrated for drained triaxial compression tests.

Key words: generalised plasticity, critical state surface, blended waste material, triaxial conditions

1. INTRODUCTION

Coal wash (CW) and Basic Oxygen Steel slag (BOS) are wastes from the coal mining and steel making industries. Currently, in the Wollongong region (NSW, Australia), these granular wastes are produced at an annual rate exceeding two million tons. Their effective reuse and recycling through large-scale geotechnical projects, such as land and port reclamation, will minimize land usage for stockpiling, reduce disposal costs and preserve natural resources (Indraratna et al., 2012; Rujikiatkamjorn et al., 2013; Chiaro et al., 2014). For the Wollongong's Port Kembla Outer Harbour expansion project, the Centre for Geomechanics and Railway Engineering of the University of Wollongong has been requested to explore the use of CW-BOS blends as potential reclamation fills. As described in Chiaro et al. (2014), comprehensive laboratory and field investigations were carried out on CW, BOS and a variety of CW-BOS blends to evaluate their geotechnical properties, such as compaction characteristics, shear strength and bearing capacity, permeability, particle breakage and swelling (i.e. volumetric expansion). Nevertheless, due to the heterogeneity of these granular waste materials, their ultimate adoption as structural fills must be supported by constitutive relations accurately describing the stress-strain behaviour under representative field loading conditions.

To address this issue, in this paper, the results of a series of monotonic drained triaxial shear tests is reported. The stress-strain and volumetric behaviour and CS characteristics of the three investigated waste material mixtures is described for specimens compacted at a 90% Standard Proctor effort and consolidated at three different confining pressures (30, 120 and 220 kPa), as representative of field conditions. Note that, since the coal wash and steel slag are very coarse materials, it is expected that their behaviour will be prevalently drained in the field, therefore this paper concentrates merely on predicting the behaviour of such waste materials under drained conditions.

By taking advantage of CS characteristics, experimentally observed for waste material mixtures, the concept of Critical State Surface (CSS), which may invoke a parallelism of CSLs (Rahman, 2009; Rahman et al., 2011; Rahman and Lo, 2014), is introduced in this study. Consequently, a set of simple empirical expressions is defined to capture the overall triaxial drained behaviour of different waste materials under the

CS framework. More importantly, a model based on generalised plasticity and critical state concept, having a similar structure of those presented by Ling and Yang (2006) and Manzanal et al. (2011), is proposed. The main features of the proposed model are: (i) a critical state surface (CSS) is introduced to account for coupled dependency of stress-strain to density and pressure level for blended CW and BOS mixes; (ii) the concept of state parameter (ψ^*) is used to identify the position of current density state from the CSS in the $p'-e$ plot; (iii) the stress-dilatancy, plastic potential and hardening rules are related to ψ^* and CSS; and (iv) the stress-strain-dilatancy behaviour of various blended wastes consolidated at different void ratio and confining pressure levels (Table 1) can be described using an explicit set of 11 soil parameters. Note that, the term “explicit set of material parameters”, as used in this study, would identify a set of parameters that do not need to be changed if the behaviour of different mixtures is evaluate by the proposed model, rather than indicating a unique set of soil parameters valid for all tested materials.

The main advantage of generalised plasticity models lies in their versatility and ability to describe the proper stress-strain behaviour of numerous soil types under both monotonic and cyclic loadings without the need to define explicitly yield surface, plastic potential and hardening rules (Pastor et al., 1990; Ling and Liu, 2003). In critical state constitutive models, it is postulated that soils sheared to a state of large strain would continue to deform under constant void ratio and pressure (Roscoe et al., 1958; Scholfield and Wroth; 1968; Vesic and Clough; 1968). An important advantage of such models, it is their ability to predict soil behaviour over a wide range of void ratios and confining pressures, using a single set of soil parameters for elasticity, critical state, dilatancy etc. (e.g. Jefferies, 1993; Imam et al., 2005; Modoni et al., 2011).

2. CRITICAL STATE CHARACTERISTICS FOR GRANULAR WASTE MATERIALS

2.1 Critical State Locus for a given waste material

Similarly to granular soils (e.g. sands, gravels etc.), the stress-strain behaviour of granular waste materials under drained shear conditions is void ratio (e) and stress level (p') dependent. Typical stress-strain relationships and volumetric strain responses observed for compacted CW specimens ($e_0 = 0.403-0.429$; measured at the end of isotropic consolidation) are shown in Figs. 1(a) and 1(b). It can be observed that under a relatively low confining pressure of $p_0' = 30$ kPa, compacted CW specimens show a dilative response (i.e. increase in volume) and the peak deviator stress is followed by strain softening. Alternatively, at higher p_0' of 120 and 220 kPa, compacted CW exhibits contractive behaviour (i.e. decrease in volume) besides strain hardening. Nevertheless, despite the different dilative/compressive behaviour, it can be seen that at shear strain (ε_q) of approximately 20%, a state of continuous shear deformation with a constant volume under constant applied deviator stress was achieved for all specimens. This ultimate condition is referred to the critical state locus (CSL; Roscoe et al., 1958; Scholfield and Wroth, 1968; Vesic and Clough, 1968), and its projection in the $p'-q$ and $p'-e$ plots are shown in Figs. 1(c) and 1(d), respectively. Specifically, CSL sets the boundary between dilatancy/strain softening and contraction/strain hardening of CW specimens for a given value of p' . As pointed out in Fig 1(d), the type of behaviour observed for CW depends on the difference between the current void ratio (e) and the critical void ratio (e_{css}^*), which is generally defined by the state parameter ψ^* (Been and Jefferies, 1985) as follows:

$$[1] \quad \psi^* = e - e_{css}^*$$

If the current state lies above the CSL, ψ^* is positive and soil behaviour is expected to be contractive. Alternatively, if the current state lies below the CSL, ψ^* is negative and the soil behaviour is then expected to be dilative. At the CSL, ψ^* is zero and dilatancy is suppressed (i.e. volume remains constant).

2.2 Critical State Surface for blended waste materials

In the case of blended CW-BOS materials, experimental evidence suggests that the CSL is not unique. For instance, as shown in Fig. 2(a), the addition of BOS to CW produces a change of CSL in the $p'-e$ plane along the vertical direction, where for a given pressure level (p') the void ratio at critical state (e_{css}^*) increases with the BOS content (f_{BOS}). Simultaneously, CSL slightly rotates (anti-clockwise) in the $p'-q$ plot (Fig. 2(b)) leading to a small increase in critical stress ratio (M_{css}^*) with the increasing BOS content (f_{BOS}). Significantly, for any given CW-BOS mixture a well-definite CSL can be defined in the $p'-q-e$ space, indicating the existence of a *Critical State Surface* (CSS) as presented in Fig. 3, which can be described using the following expressions:

$$[2] \quad e_{\text{css}}^*(f_{\text{BOS}}) = e_{\Gamma}(f_{\text{BOS}}) - \lambda(f_{\text{BOS}}) \ln p'$$

where $e_{\Gamma}(f_{\text{BOS}}) = 0.695 + 0.229f_{\text{BOS}}$ and $\lambda(f_{\text{BOS}}) = 0.061$ (= constant);

$$[3] \quad q_{\text{css}}^*(f_{\text{BOS}}) = M_{\text{css}}^*(f_{\text{BOS}}) p'$$

in which $M_{\text{css}}^*(f_{\text{BOS}}) = 1.44 + 0.12f_{\text{BOS}}$

In the above, $e_{\text{css}}^*(f_{\text{BOS}})$, $e_{\Gamma}(f_{\text{BOS}})$ and $\lambda(f_{\text{BOS}})$ are the critical void ratio; critical void ratio at reference pressure of 1kPa and slope of CSS in $p'-e$ plot, respectively; $q_{\text{css}}^*(f_{\text{BOS}})$ and $M_{\text{css}}^*(f_{\text{BOS}})$ are the critical deviator stress and slope of CSS in $p'-q$ plot, respectively; p' is the current effective mean stress.

3. GOVERNING EQUATION OF PROPOSED MODEL

The proposed model is formulated for the case of axisymmetric triaxial stress conditions ($\sigma_2' = \sigma_3'$) and ($\varepsilon_2 = \varepsilon_3$), wherein the stress and strain invariants are defined as:

$$[4] \quad \boldsymbol{\sigma} = \begin{Bmatrix} q \\ p' \end{Bmatrix} = \begin{Bmatrix} \sigma_1' - \sigma_3' \\ (\sigma_1' + 2\sigma_3')/3 \end{Bmatrix}$$

$$[5] \quad \boldsymbol{\varepsilon} = \begin{Bmatrix} \varepsilon_q \\ \varepsilon_v \end{Bmatrix} = \begin{Bmatrix} 2(\varepsilon_1 - \varepsilon_3)/3 \\ \varepsilon_1 + 2\varepsilon_3 \end{Bmatrix}$$

where, q is the deviator stress; p' is the effective mean stress; ε_q is the shear strain; ε_v is the volumetric strain; σ_1' and σ_3' are the effective major and minor principal stresses, respectively; and ε_1 and ε_3 are the major and minor principal strains, respectively.

By specifying the relationship between the stress increment ($d\sigma$) and the strain increment ($d\varepsilon$), and referring to the deviator stress space, the elasto-plastic behaviour is described as follows:

$$[6a] \quad \{d\sigma\} = \mathbf{M}^{ep} \{d\varepsilon\}$$

$$[6b] \quad \begin{Bmatrix} dq \\ dp' \end{Bmatrix} = \begin{Bmatrix} \mathbf{M}^e - \frac{\mathbf{M}^e \mathbf{m}_g \mathbf{n}_f^T \mathbf{M}^e}{H + \mathbf{n}_f^T \mathbf{M}^e \mathbf{m}_g} \end{Bmatrix} \begin{Bmatrix} d\varepsilon_q \\ d\varepsilon_v \end{Bmatrix}$$

where, $\mathbf{M}^e$ is the elastic stiffness matrix; $\mathbf{M}^{ep}$ is the elasto-plastic stiffness matrix (Mzor and Zienkiewicz, 1984); $\mathbf{m}_g$ is the plastic flow direction vector; $\mathbf{n}_f$ is the loading direction vector and H is the plastic modulus.

In this paper, for the specific case of strain-controlled drained triaxial compression loading, where $d\varepsilon_q$ is known and $dp' = dq/3$, Equation (6b) yields to the two following governing equations:

$$[7] \quad dq = \left(\frac{3GH + 3GKm_q n_v}{H + 3Gm_q n_q + Km_v n_v} \right) d\varepsilon_q - \left(\frac{3GKm_q n_v}{H + 3Gm_q n_q + Km_v n_v} \right) d\varepsilon_v$$

$$[8] \quad d\varepsilon_v = \frac{G}{K} \left(\frac{H + Km_v n_v - 3Km_q n_q}{H + 3Gm_q n_q + Km_q n_v} \right) d\varepsilon_q$$

where, G and K are the shear and bulk moduli, respectively; m_q, m_v are the two component of the plastic flow vector ($\mathbf{m}_g$); and n_q, n_v are the two component of the loading direction vector ($\mathbf{n}_f$).

3.1 Elastic and plastic strains

Noteworthy is the fact that, the proposed model assumes that for any given shear stress increment both elastic and plastic deformations do always occur, so that a purely elastic region does not exist, i.e. soil continuously yields from the very small strains. The plastic strain increment ($d\varepsilon^p$) is given by

the difference between the total strain increment ($d\epsilon$) and the elastic increment component ($d\epsilon^e$), which is computed using the well-established theory of elasticity (Poulus and Davis, 1974):

$$[9] \quad d\epsilon_q^p = d\epsilon_q - d\epsilon_q^e = d\epsilon_q - dq / 3G$$

$$[10] \quad d\epsilon_v^p = d\epsilon_v - d\epsilon_v^e = d\epsilon_v - dp' / K$$

where,

$$[11] \quad G = \frac{3(1-2\nu)}{2(1+\nu)} K$$

$$[12] \quad K = \frac{(1+e)p'}{\kappa}$$

in the above ν is the Poisson's ratio; p' is the current effective mean stress; κ is the swelling-recompression index; and e is the current void ratio.

3.2 Stress-dilatancy relationship (flow rule)

Change of volumetric behaviour in different stages of drained shearing can be described by the stress-dilatancy relationships (e.g. Rowe, 1962; Nova and Wood, 1979; Bolton, 1986; Pradhan and Tatsuoka, 1989; Chiaro et al., 2013), which relate the ratio of plastic strain increments ($d_g = d\epsilon_v^p / d\epsilon_q^p$) to the stress ratio ($\eta = q / p'$). Similar to other granular material, also in the case of granular waste mixtures, dilatancy is zero not only when reaching the CSS ($d_g = 0$ for $\eta = M_{css}^*$), but also before (i.e. $d_g = 0$ for $\eta \neq M_{css}^*$). This latter condition, which marks the change of soil behaviour from contractive to dilative, is referred to as the phase transformation (PT) state (Ishihara et al., 1975). To account for the combined effects of void ratio and stress level on stress-dilatancy behaviour of waste materials, the following exponential dilatancy relationship proposed by Li and Dafalias (2000) is employed:

$$[13] \quad d_g = \xi_g \left[M_{css}^* \exp(\mu_g \psi^*) - \eta \right]$$

where, ξ_g and μ_g are dilatancy material constants. Note that, Equation (13) is an extension of the linear stress-dilatancy proposed by Manzari and Dafalias (1997) and satisfies the condition of CSS

and PT where the volume change is equal to zero for $\psi^* = 0$ and $\eta = M_{\text{css}}^* \exp(\mu_g \psi^*)$, respectively. For contractive soil, $M_{\text{css}}^* \exp(\mu_g \psi^*) > M_{\text{css}}^*$ and thus PT state is never reached during the shearing process. Alternatively, for dilative soil, the PT state is achieved when $\eta = M_{\text{css}}^* \exp(\mu_g \psi_{\text{pt}}^*)$, where ψ_{pt}^* is the value of state parameter at PT state.

3.3 Plastic flow (loading direction)

In generalised plasticity, the stress-strain behaviour of soils can be described without the need to define explicitly the yield and plastic potential surfaces (Pastor et al., 1990; Ling and Liu, 2003 etc.). Instead, the plastic flow direction and loading direction vectors can be used. In the triaxial space, plastic flow direction vector ($\mathbf{m}_g$) is given as:

$$[14] \quad \mathbf{m}_g = \begin{cases} m_q = 1 / \sqrt{1 + d_g^2} \\ m_v = d_g / \sqrt{1 + d_g^2} \end{cases}$$

The non-associate flow rule was adopted and the loading direction vector ($\mathbf{n}_f$) was defined as:

$$[15] \quad \mathbf{n}_f = \begin{cases} n_q = 1 / \sqrt{1 + d_f^2} \\ n_v = d_f / \sqrt{1 + d_f^2} \end{cases}$$

where, d_f is the loading direction component:

$$[16] \quad d_f = \xi_f [\eta_f \exp(\mu_f \psi^*) - \eta]$$

in which ξ_f , η_f and μ_f are material parameters describing the plastic potential.

3.4 Plastic modulus

In the proposed model, the expression for plastic modulus (H) proposed by Li and Dafalias (2000) and modified by Ling and Yang (2006) was used, and the dependency of H on void ratio and confining pressure level was accounted by the state parameter (ψ^*):

$$[17] \quad H = h_0 \left(\frac{\eta_{\text{pk}}^*}{\eta} - 1 \right) \left(1 - \frac{\xi_f}{(\xi_f - 1)} \frac{\eta}{\eta_f} \right)^4 \sqrt{\frac{p'}{p_{\text{atm}}}}$$

where according to Li and Dafalias (2000)

$$[18] \quad \eta_{pk}^* = M_{css}^* \exp(-\mu_{pk} \psi^*)$$

in the above h_0 and μ_{pk} are hardening material constants. It is noteworthy, H depends on the difference between the current stress ratio η and the virtual peak stress ratio η_{pk}^* : H may be positive (hardening) for $\eta_{pk}^* > \eta$, negative (softening) for $\eta_{pk}^* < \eta$ or zero (peak failure) for $\eta_{pk}^* = \eta$.

4. EVALUATION OF MODEL PARAMETERS

The model is calibrated against experimental results, in order to obtain an explicit set of 11 model parameters (Table 2) to be used to simulate the stress-strain behaviour of CW-BOS blends compacted at different void ratios ($e_0 = 0.429-0.519$) and confining pressures ($p_0' = 30-220$ kPa).

In this study, an acceptable estimation of elastic properties for monotonic shearing was obtained by fitting the initial stage of isotropic consolidation tests on compacted CW-BOS specimens:

$\kappa(f_{BOS}) = 0.002 - 0.0003 f_{BOS}$. The Poisson's ratio was assumed to be constant and equal to $\nu = 0.25$.

Evaluation of CSS parameters for the CW-BOS blended mix was presented in Fig. 3, where: the critical void ratio at reference pressure of 1kPa is $e_r(f_{BOS}) = 0.695 + 0.229 f_{BOS}$; the slope of CSS in $p'-e$ plot is $\lambda(f_{BOS}) = 0.061$ (i.e. constant); and the slope of CSS in $p'-q$ plot is $M_{css}^*(f_{BOS}) = 1.44 + 0.12 f_{BOS}$.

A typical calibration of dilatancy parameters is presented in Fig 4, in which $\xi_g = 1.45$. Alternatively, $\mu_g = 3$ was determined by evaluating Equation (13) at the PT state, thus:

$$[19] \quad d_g = 0 \Rightarrow M_{css}^* \exp(\mu_g \psi^*) - \eta = 0 \Rightarrow \mu_g = \frac{1}{\psi_{pt}^*} \ln \left(\frac{\eta_{pt}}{M_{css}^*} \right)$$

where, η_{pt} and ψ_{pt}^* are the values of η and ψ^* at phase transformation state, respectively.

In the basic generalised plasticity models, η_f is independent from confining pressure and constant for a given material. In addition, the ratio between η_f and M_{css}^* is similar to the relative density of the

soil. Here, for compacted CW-BOS blend, the following relationship was used, which allows calculating η_f once M_{css}^* is known:

$$[20] \quad \eta_f = \frac{D_c}{100} M_{css}^*$$

where, D_c (%) is the degree of compaction.

Furthermore, $\mu_f = 5$ was determined by matching the shape of the ε_v - ε_q relationship and, as suggested by Manzanal et al. (2011), $\zeta_f = \zeta_g$ was selected. Finally, $h_0 = 100$ MPa was obtained by fitting both the ε_q - q curves, while $\mu_{pk} = 14$ was determined by evaluating Equation (18) at the deviator peak stress state where:

$$[21] \quad H = 0 \Rightarrow M_{css}^* \exp(m_{pk} \psi^*) - \eta = 0 \Rightarrow m_{pk} = \frac{1}{\psi_{pk}^*} \ln \left(\frac{M_{css}^*}{\eta_{pk}} \right)$$

in the above, η_{pk} and ψ_{pk}^* are the values of η and ψ^* at deviator peak stress state, respectively.

5. COMPARISON BETWEEN EXPERIMENTAL RESULTS AND MODEL SIMULATIONS

Performance of the proposed model to simulate the observed behaviour of CW-BOS blend under monotonic drained triaxial shear loadings was investigated in Fig. 5 by comparing the numerical simulations (lines) with the experimental data (symbols). It can be seen that, despite the change in void ratio and confining pressure, the monotonic drained response of blended wastes can be satisfactorily captured by the proposed model in terms of both the stress-strain relationship and the volumetric change response. In particular, for all CW-BOS blends examined, the dilative behaviour followed by strain softening was observed for specimens consolidated at low confining pressure. Also, the contractive response besides strain hardening observed for specimens undergoing higher confining pressures was well depicted by the model using an explicit set of soil parameters.

6. CONCLUSIONS

The effective reuse and recycling of granular waste materials, such as coal wash (CW) and Basic Oxygen Steel slag (BOS), through large-scale geotechnical project is advantageous and sustainable,

as proven by a series of laboratory and field investigations. Nevertheless, due to the heterogeneity of these granular by-products, their ultimate adoption as structural fills must be reinforced by robust constitutive relations for accurately describing their complex stress-strain behaviour under representative field loading conditions. Hence, in this paper a critical-state-surface generalised-plasticity model for describing the stress-strain behaviour and volumetric change response of coal wash and BOS slag mixtures over a wide range of stress conditions was presented. It was demonstrated that, the proposed model was able to adequately simulate the monotonic drained triaxial response of compacted CW-BOS blended wastes. In particular, using an explicit set of soil parameters, the dilative behaviour with strain softening of specimens consolidated at low confining pressure as well as the contractive response besides strain hardening of those specimens consolidated at higher confining pressure was well predicted by the model.

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LIST OF SYMBOLS

e	current void ratio
e_{css}^*	void ratio at critical state
e_{Γ}	critical void ratio at a reference pressure of 1kPa
f_{BOS}	BOS slag content
M_{css}^*	slope of critical state surface in the $p'-q$ plot
p', q	current effective mean stress and deviator stress
p_{atm}	atmospheric pressure (=100 kPa)
q_{css}^*	critical deviator stress
η_f, μ_f, ξ_f	model parameters for plastic potential
μ_g, ξ_g	model parameters for dilatancy
h_0, μ_{pk}	model parameters for hardening
$\varepsilon_1, \varepsilon_3$	major and minor principal strain
$\varepsilon_q, \varepsilon_q^e, \varepsilon_q^p$	total, elastic and plastic shear strain
$\varepsilon_v, \varepsilon_v^e, \varepsilon_v^p$	total, elastic and plastic volumetric strain

λ	slope of critical state surface in the p' - e plot
σ'_1, σ'_3	effective major and minor principal stress
$\epsilon, \epsilon^e, \epsilon^p$	total elastic and plastic strain invariant
σ	stress invariant
$d\epsilon, d\sigma$	strain and stress increment
$\mathbf{M}^e, \mathbf{M}^{ep}$	elastic and elasto-plastic stiffness matrix
$\mathbf{m}_g, \mathbf{n}_f$	plastic flow direction and loading direction vector
m_q, m_v	components of plastic flow direction vector
n_q, n_v	components of loading direction vector
dp', dq	effective mean stress and deviator stress increment
$d\epsilon_q, d\epsilon_v$	shear strain and volumetric strain increments
G, K, H	shear, bulk and plastic modulus
ν	Poisson's ratio
κ	swelling-recompression index
d_f, d_g	loading direction component and stress dilatancy ratio
$\eta, \eta_{pk}, \eta_{pt}$	current stress ratio, stress ratio at deviator peak state and stress ratio at phase transformation state
ψ^*	state parameter

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Table 1. Consolidated drained triaxial compression tests

Material	Test	Isotropic consolidation		Critical state		
		p_0' (kPa)	e_0	q_{css} (kPa)	p'_{css} (kPa)	e_{css}
Coal wash (CW) $f_{\text{BOS}} = 0.0$	CDT1	28.2	0.429	82	59	0.447
	CDT2	123.9	0.417	248	374	0.359
	CDT3	221.9	0.403	425	609	0.326
CW-BOS 75/25 $f_{\text{BOS}} = 0.25$	CDT4	28.1	0.485	78	53	0.514
	CDT5	119.3	0.476	355	241	0.422
	CDT6	217.2	0.457	631	429	0.386
CW-BOS 50/50 $f_{\text{BOS}} = 0.50$	CDT7	29.1	0.519	656	438	0.556
	CDT8	118.6	0.504	374	243	0.467
	CDT9	219.2	0.495	81	56	0.431

Note: For all specimens, e_0 corresponds to a degree of compaction $D_c \approx 90\%$ of standard Proctor compaction tests

Table 2. Parameter of proposed model

Soil parameters		Values
Elastic	κ	$0.002 - 0.0003 f_{\text{BOS}}$
	ν	0.25
Critical state surface	e_{r}	$0.695 + 0.229 f_{\text{BOS}}$
	λ	0.061
	M_{css}	$1.44 + 0.12 f_{\text{BOS}}$
Dilatancy	ξ_{g}	1.45
	μ_{g}	3
Loading direction	ξ_{f}	1.45
	μ_{f}	5
Plastic modulus	h_0 (kPa)	100000
	μ_{pk}	14

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