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© 2020 Elsevier Ltd The transformation of energy is an intrinsic process that is needed to trigger the internal erosion of soils subjected to a fluid flow, but how to capture this process is not understood very well. This is why this study aims to address these complex processes through a numerical fluid-particle coupling simulation. The computational fluid dynamics (CFD) is used to model fluid flows which is coupled with the discrete element method (DEM) employed to simulate soil particles. Detailed migration of particles and fluid variables are recorded to enable their kinetic energy to be computed. Successful experiments are used to demonstrate how the numerical method can be used to model the internal erosion associated with energy computation. This study shows a good agreement between the numerical and experimental results in terms of the hydraulic conductivity and erosion rate of soils subjected to upward flows. A significant loss in energy is also found as fluid flows through the soil whereas only a small amount of kinetic energy is needed to make particles migrate at a considerable degree. The influence that the porosity and uniformity of soils has on the transformation of energy is also discussed in the paper.

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THE ENERGY TRANSFORMATION OF INTERNAL EROSION
BASED ON FLUID-PARTICLE COUPLING

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Abstract

The transformation of energy is an intrinsic process that is needed to trigger the internal erosion of soils subjected to a fluid flow, but how to capture this process is not understood very well. This is why this study aims to address these complex processes through a numerical fluid-particle coupling simulation. The computational fluid dynamics (CFD) is used to model fluid flows which is coupled with the discrete element method (DEM) employed to simulate soil particles. Detailed migration of particles and fluid variables are recorded to enable their kinetic energy to be computed. Successful experiments are used to demonstrate how the numerical method can be used to model the internal erosion associated with energy computation. This study shows a good agreement between the numerical and experimental results in terms of the hydraulic conductivity and erosion rate of soils subjected to upward flows. A significant loss in energy is also found as fluid flows through the soil whereas only a small amount of kinetic energy is needed to make particles migrate at a considerable degree. The influence that the porosity and uniformity of soils has on the transformation of energy is also discussed in the paper.

Keywords: fluid-particle coupling, internal erosion, energy transformation, mud pumping, particle migration
1. Introduction

The loss of soil mass and bearing capacity followed by substantial deformation is inerrable when soil foundations are subjected to internal erosion, as reported in many previous studies on dams, roads and railways [1-4]. A typical example is the softening of subgrade due to heavy haul rail loads that cause the fluidised subgrade soil (i.e., mud pumping) to migrate upwards. The major cause of this problem is the increased excess pore water pressure due to railway cyclic loads that reduce the effective stress and force the subgrade fines to migrate, which results in the loss of soil mass and increasing void ratio, i.e., internal erosion in the subgrade. Although this issue has received considerable attention over the past year [5-7], it is still one of the most critical issues causing substantial maintenance costs for railways [8-10].

Studies on energy transformation in geomechanics are generally limited due to the lack of practical methods by which it can be measured and validated. Despite this limitation, much effort has still been made to capture the energy that evolves in this geomechanical phenomena [11]. For example, deflections and vibrations of the tracks induced by the passage of trains can be used to estimate the energy transferred through the ground [12, 13], however the amount of energy loss has not been estimated well. Furthermore, how this energy is accumulated and transferred to the kinetic energy that causes subgrade particles to migrate upwards has not been understood in detail. This context indicates that while the transformation of energy is theoretically acknowledged as an inherent aspect during the passage of trains, the solutions to capture this process has also not been determined well.

Using the discrete element method (DEM) to model soils has significant advantages in tracking the information of soil elements in detail, which enables the micro-characteristics of the soils to be better understood. This is why this method has successfully been adopted to analyse the energy transformation in soils [14-18]. For example, the kinetic energy induced
while soil is being deformed can be computed directly by accumulating individual components of discrete particles. Moreover, combining DEM with computational fluid dynamics (CFD) provides a wider application of DEM where the energy and momentum interaction between the fluid and solid phases especially in filtration and internal erosion issues can be well represented [19-21]. Based on these advantages, recent studies have successfully captured the mutual transformation of energy between fluid and solid phases, such as the effects of debris flow on water reservoirs [15] and internal erosion [21]. Despite this certain success, there are still limitations such as using a 2D model which results in certain inaccuracies of particle behaviour, especially in relation to the projected (from 3D to 2D) granular assembly and the resulting implications on particle migration under the applied hydraulic forces. In addition, previous studies [15, 21] use relatively uniform soil particles that can be quite different from an actual granular soil fabric (e.g., $C_u$ often exceeding 3). Therefore, more effort is still required to examine the CFD-DEM coupling approach in analysing the energy evolution of fluid-soil systems.

This current study uses a fluid-particle coupling approach to investigate how energy is transformed while soil is subjected to an upward flow under an increasing hydraulic gradient. The DEM is used to model granular soils with different soil properties while the OpenFOAM-based CFD [22] is used to describe the fluid variables [23]. The potential energy induced by fluid applying onto the system is calculated from the input hydraulic gradient whereas the components of kinetic energy of fluid and particles are computed as their responses to the given potential energy through internal erosion process.

2. Theoretical background

2.1 Discrete Element Method

The discrete element method (DEM) [24] is applied in the current study to simulate granular
soils, however, the total hydraulic force $F_f$ is incorporated into the conventional governing
equations of particles to consider the influence of fluid flows as follows:

$$m_i \frac{dU_{pi}}{dt} = \sum_{j=1}^{n_i^c} F_{ci,j} + F_{gi} + F_{fi}$$

$$I_i \frac{d\omega_{pi}}{dt} = \sum_{j=1}^{n_i^c} (M_{c,ij} + M_{r,ij})$$

where $U_{pi}$ and $\omega_{pi}$ are the translational and angular velocities of particle $i$, respectively while
$m_i$ is the solid particle mass; $F_{c,ij}$ and $M_{c,ij}$ are the contact force and torque acting on particle
$i$ by particle $j$ (or walls) with $n_i^c$ denoting the number of total contacts of particle $i$; $t$ denotes
the time while $F_{g,i}$ is the gravitational force and $I_i$ is the inertia moment of particle $i$; $M_{r,ij}$ is
the rolling friction torque. The subscripts $p$ and $f$ denote particle and fluid, respectively.

The nonlinear Hertz-Mindlin contact model is adopted to simulate particle contacts as
this model has shown reasonable results when describing the contact behaviour of granular
materials [25-27]. In particular two components, i.e., the normal $F_{cn}$ and tangential $F_{ct}$ of the
contact force $F_c$ are determined based on the corresponding contact stiffness, i.e., $k_{cn}$ and
$k_{ct}$ which is computed by [25]:

$$k_{cn} = \frac{4}{3} E^* \sqrt{R^* \delta_n}$$

$$k_{ct} = 8G^* \sqrt{R^* \delta_n}$$

where $\delta_n$ is the overlap of two particles $i$ and $j$ in contact; $R^*$, $E^*$ and $G^*$ are the effective
radius, Young and shear modulus, respectively. The subscripts $cn$ and $ct$ represent the normal
and tangential components of the contact. How to calculate these parameters can be found in
previous studies [23, 28].
In this study, the rolling resistance is simulated based on the directional constant torque model [16, 17]:

\[ M_r = \frac{\omega_{rel}}{|\omega_{rel}|} \mu_r R^* F_{cn} \]  

where \( M_r \) and \( \mu_r \) are the rolling torque and friction coefficients while \( \omega_{rel} \) represents the relative angular velocity between two particles in contact. As this study aims to model soils having low effective stress (i.e., no compression load) and subjected to small level of hydraulic gradient \( i \) (i.e., \( i < 1.5 \)), a small rolling resistance such as \( \mu_r = 0.1 \) is used. In fact, Goniva et al. [29] based on experimental validation indicate that a \( \mu_r \geq 0.1 \) would result in a reasonable rolling resistance of granular materials.

### 2.2 Computational fluid dynamics based on Navier-Stokes theories

The current study used the modified Navier-Stokes (NS) equations where the effect of solid particles on the fluid flow is considered as follows:

\[
\frac{\partial n_f}{\partial t} + \nabla \cdot (n_f U_f) = 0 \tag{6}
\]

\[
\frac{\partial (\rho_f n_f U_f)}{\partial t} + \nabla \cdot (\rho_f n_f U_f U_f) = -n_f \nabla p I + \nabla \cdot (n_f \tau_f) + \rho_f n_f g - f_p \tag{7}
\]

where \( n_f \) is the porosity of a fluid cell with \( n \) denoting the overall porosity in the computed domain while \( \rho_f, \tau_f, U_f \) and \( p \) are the density, viscous stress, velocity and pressure of fluid, respectively; \( I \) is the identity tensor and \( g \) is the gravitational acceleration vector. The term \( f_p \) denotes the mean volumetric particle-fluid interaction force which represents the effect of solid particles on the fluid phase.

The finite volume method (FVM) is used to solve the NS equations in this study, so that the fluid domain is discretised into a finite number of cells where the governing equations are solved locally using the averaged fluid velocity and pressure. The averaged
fluid variables at each fluid cell are used to estimate fluid forces acting on particles in the
cell, so that it is a cost effective method in computation. With respect to this approach, the
mean volumetric particle-fluid interaction force $f_p$ in a fluid cell is calculated by:

$$f_p = \sum_{i=1}^{n_p} \frac{\sigma_i}{V_c} \frac{F_{fi}}{V_c}$$ [8]

where $n_p$ is the number of particles in the cell with a volume $V_c$, and $F_{fi}$ is the total hydraulic
force induced by fluid acting on particle $i$. Because not all particles locate fully in the fluid
cell, the contribution of each particle to $f_p$ is weighted by the factor $\sigma_i$ which is defined as
the ratio between the internal volume of particle $i$ in the fluid cell to its total volume. To
estimate partial volume of a particle, divided void fraction technique which is able to divide
the particle into a number of smaller parts is adopted in this study [29].

2.3 Hydraulic forces acting on particles due to fluid flowing

The total fluid-particle interaction force $F_f$ can include the drag force, the pressure gradient
force, the viscous force, and other unsteady forces such as the virtual mass, the Basset and the
lift forces [30]. Considering a small hydraulic gradient, i.e., $i < 1.5$ as used in the current
simulation, the drag, pressure gradient and viscous forces are usually predominant over other
unsteady forces. Therefore only major components of $F_f$ are considered as follows:

$$F_f = F_d + F_{vp} + F_{v,r}$$ [9]

where $F_d$ is the drag force, $F_{vp}$ is the pressure gradient force and $F_{v,r}$ is the viscous force.
The drag force $F_d$ is induced by the variation of the point stress tensor in the fluid flowing
through the particle while $F_{vp}$ and $F_{v,r}$ are generated due to the macroscopic stress variation,
i.e., varying fluid stress tensor over different fluid cells [31].

Basically, there are a number of methods to estimate the drag force such as Ergun
By comparing these models, Kafui et al. indicate that Di Felice can better present a smooth variation of drag force over porosity. This is why it has been used widely in recent studies where similar materials to the current study are used. Di Felice is hence adopted in this study as follows:

\[
F_d = \frac{1}{8} C_d \rho_f \pi D_p^2 n_f^2 (U_f - U_p)|U_f - U_p| n_f^{-x}
\]  \hspace{1cm} \text{[10]}

where \(D_p\) is the diameter of the particle. The drag coefficient \(C_d\) is computed with respect to the particle Reynolds number \(Re_p\) by:

\[
C_d = \left(0.63 + \frac{4.8}{\sqrt{Re_p}}\right)^2
\]  \hspace{1cm} \text{[11]}

where the particle Reynolds number \(Re_p\) is determined as:

\[
Re_p = \frac{n_f \rho_f D_p |U_f - U_p|}{\mu_f}
\]  \hspace{1cm} \text{[12]}

where \(\mu_f\) is the fluid viscosity. In Eq. [12], the porosity function \(n_f^{-x}\) represents the presence of other particles in the cell in relation to the power factor \(x\) which can be estimated by [30]:

\[
\chi = 3.7 - 0.65 \exp \left( - \frac{(1.5 - \log_{10} Re_p)^2}{2} \right)
\]  \hspace{1cm} \text{[13]}

The pressure gradient force which is generated when there is a change in fluid pressure over a surface, i.e., \(\nabla p\) including buoyant and external components [30]. The computation of this force acting on a particle having a volume \(V_p\) can be written as:

\[
F_{vp} = -(\nabla p \mathbf{I})V_p
\]  \hspace{1cm} \text{[14]}

The viscous force is normally induced when there is a difference in deviator (shear) stress over flowing space. This force is calculated by [30, 31]:
2.4 Energy conservation of fluid flowing through soils

Given a fluid flowing through a porous soil medium under a hydraulic gradient $i$, the conservation of energy in the fluid-soil system can generally be written as [35]:

$$\sum E_{in} = \sum E_{out} + \sum E_{work} + \sum E_{dis}$$  \hspace{1cm} [16]

where $E_{in}$ and $E_{out}$ are the total input energy and the total energy maintained at the outlet of the system, respectively. $E_{work}$ is the total energy to get work done in the system (e.g., moving soil particles) and $E_{dis}$ represents the dissipation or loss of energy while fluid is flowing through the soil.

In this study, the internal energy (by molecular and atomic motion) and the thermal aspect are not considered because they are insignificant, whereas the energy lost due to flow friction depends mainly on the fluid viscosity, the coefficients of friction, and the contact surface between fluid and solid elements. The net potential energy $E_{f,net}^p$ (or $E_{in} - E_{out}$) for a unit volume of flow (volumetric potential energy of fluid) is computed by:

$$E_{f,net}^p = \rho_f g i L$$ \hspace{1cm} [17]

where $i$ is the hydraulic gradient over the flow length $L$; $i$ can vary over time, i.e., $i(t)$. By considering the incremental form of $i$, the total input $E_{f,net}^p$ can be rewritten by:

$$\sum E_{f,net}^p = \rho_f g L \sum i_k$$ \hspace{1cm} [18]

where $i_k$ is the value of hydraulic gradient at $k^{th}$ step during $i$ increasing.

The potential energy induced by a fluid flow can be transferred into the following components: (1) the kinetic energy of particles $E_{p}^k$; (2) the kinetic energy of fluid $E_{f}^k$; (3) and the energy dissipation due to friction $E_{dis}$. If the soil assembly deforms and particles displace
due to the fluid flow, the $E_p^k$ would be generated and contribute to the $E_{work}$ shown in Eq. [16]. Clearly, for the same input energy, the larger the friction, the less energy is transferred to the kinetic components, and therefore, there is less internal erosion. The frictional loss of energy depends mainly on the properties of soil such as particle shape and size, and the porous features (e.g., the porosity). Moreover, the viscous properties of fluid that govern how it resists flowing can account for a certain amount of energy loss. If a normal soil foundation such as subgrade and capping layers under a rail track is considered, and the viscosity of the fluid is basically unchanged, the dissipation of energy due to the soil and the foundation properties would receive much more attention.

By considering a fluid flow $Q_f = U_{f,d} \times A_f$ where $U_{f,d}$ is the discharge (or superficial) flow velocity and $A_f$ is the cross-sectional area of the flow during a time $\Delta t$, the total kinetic energy induced by particle migration in a fluid-particle system is computed by:

$$\sum E_p^k = \frac{1}{2} \rho_p \sum_{i=1}^{n} V_{pi} U_{pi}^2$$  \hspace{1cm} [19]

The total kinetic energy $\sum E_p^k$ of the particles can vary significantly over the evolution of flow and particle migration, therefore, the term $\sum E_p^k$ should be considered over time in the whole process, i.e., until the flow is stabilised and there is no more particle migration.

The total kinetic energy of fluid in the entire system can be computed by:

$$\sum E_f^k = \frac{1}{2} \rho_f \sum_{j=1}^{n_{fc}} U_{fj}^2$$  \hspace{1cm} [20]

where $U_{fj}$ is the averaged fluid velocity in cell $j$ and $n_{fc}$ is the total number of fluid cells. In this study, the hydraulic gradient $i$ varies, so the fluid energy is computed with respect to the amount of flow at a certain $i$ during a time $\Delta t$. 
3. Soil properties and numerical parameters

3.1 Soil properties

The internal migration of soil particles while fluid is flowing through is mainly governed by the characteristics of particle size distribution (PSD), as discussed in previous studies [1, 19, 36]. Several parameters are used to represent the characteristics of the PSD of a soil, such as the coefficient of uniformity $C_u$ and the filter ratio $f = D_{15}/d_{85}$ ratio where $D_{15}$ and $d_{85}$ are the particle size at 15 and 85% finer by mass of coarse and fine particles, respectively. In this study, the granular soils with $C_u$ varying from 1.6 to 7 adopted from previous experimental studies [1, 37, 38] (see Table 1) are used to investigate how the internal erosion and corresponding energy transformation can change with different PSD. While soils C and D are sand mixed with gravel to generate different samples having a $C_u$ of 7 and 4.5, respectively, more uniform soils such as Toyoura and Ottawa sand with $C_u$ of 1.6 and 1.9 are also used. Note also that soils C and D have the same coarse fabric (i.e., the same fraction for $D_p > 1.0$ mm) while their fines content varies, i.e., 15 and 10 % with respect to the experimental data [1].

Although these soils have varying PSD with $C_u$ from 1.6 to 7, they are all internally stable for a certain range of hydraulic gradient $i < i_c$ where $i_c$ is the critical hydraulic gradient. For example, the experimental results given by Skempton and Brogan [1] show that the $i_c$ of soil C is about 1.0. Although severe instability such as the suffusion which usually occurs with broadly or gap graded soils is not considered in the current investigation, the soils selected still show a high degree of internal erosion that can be used to demonstrate the energy transformation captured through the current CFD-DEM coupling approach. For example, the experimental results [1] show a considerable amount of fines, i.e., about 9 % by mass being washed out in soil C, thus this data is appropriately taken to validate the current
numerical analysis.

3.2 Numerical modelling

Sampling soil in DEM

Given a real granular soil with a certain PSD, the number of particles at each size in DEM is estimated based on the mass ratio of such a size and the total number of particles in the sample. Fig. 1 shows the current numerical PSD for all the soils used in the present study, which is in a good agreement with previous experimental data. Table 2 compares the porosity and $C_u$ of soil samples in numerical and experimental methods, which indicates a good agreement. The scale of the simulation varies with different soils, however, the size of the particle domain in DEM should be large enough to ensure the uniform distribution of particles and porosity. The size of the numerical soil samples used in this study is summarised in Table 1. While different values of $C_u$ are represented through Cases 1, 5, 6 and 7; Cases 2 to 5 modelling the same PSD (i.e., soil D) are used to examine how varying porosity would affect the energy transformation of internal erosion. The periodic boundary which ensures a continuous particle interaction is used in every case so that the localised behaviour of particles at domain boundaries is minimised. Furthermore, this would help the DEM soil sample be more uniform and representative of real soil, despite the smaller size being used in the numerical approach.

Numerical parameters such as Young’s modulus (E), friction coefficients ($f_r$) and Poisson number ($\nu$) are assumed in this study because of the lack of proper information for determining these parameters in the experimental work. This assumption is made with respect to previous studies where DEM has been used to model granular soils with some success [27, 39, 40]. Specifically, $E = 6.1 \times 10^5$ kPa, $f_r = 0.5$ and $\nu = 0.3$ have been adopted. Some previous investigations [14, 39] show that a reasonable assumption of these parameters would
not considerably affect the results in CFD-DEM coupling, especially the medium granular soils without being compressed as considered in this current study. Note also that varying Young’s modulus in a certain range is reported as not having very much influence on the DEM results, especially for the low effective stress represented in the current simulation [39].

*Modelling fluid flowing through DEM soil*

In the finite volume method (FVM), the fluid cell is considered as macroscopic to soil particles, which means the fluid cells should be bigger than the largest particles in DEM. Details of CFD meshing for different soil samples are summarised in Table 1. Extended paths were also added at the inlet and outlet of the particulate (DEM) domain to ensure computational stability and a fully developed condition of flow. These meshing criteria are used consistently over different samples and hydraulic gradients.

An upward flow is generated at the inlet of the fluid domain (Fig. 2) by controlling the hydraulic pressure. The hydraulic gradient $i$ is estimated based on the drop in pressure and the distance the flow travels through the particulate domain. While a varying pressure is applied at the inlet, the zero-pressure is set at the outlet to mimic the overflow condition in the experiment. This pressure controlled scheme enables a stable flow to be established at a certain value of $i$ over the particulate domain, and the flow velocity to be captured at different elevations of the soil over time. A slip boundary condition that is compatible with the periodic boundary in DEM is used to eliminate flow friction on the cell walls. In this study, $i$ is only increased when there is no more significant migration of particles (fully dissipated flow energy) over the entire sample (see Fig. 3) with respect to the experimental testing [1]. In this simulation, a uniform incremental rate, i.e., 100 Pa/s is applied at each increment of fluid pressure, and normal water with a density of 1000 kg/m$^3$ and dynamic viscosity ($\mu_f$) of $10.04 \times 10^{-4}$ Pa.s is used.
4. Results and discussion

4.1 Evaluating the response of soil to an increasing hydraulic gradient

This section aims to represent the numerical results in terms of particle and fluid behaviour as the tested soil responds to an increasing hydraulic gradient \( i \). Validation is also made by comparing the results with the experimental data during the discussion.

Particle migration and corresponding re-distribution of porosity over the sample height

The migration of soil particles is an essential aspect of assessing internal erosion of soil subjected to a hydraulic flow; this behaviour can be tracked and quantified by using the numerical CFD-DEM coupling method. In essence, the migration characteristics can be represented through the length of migration paths that particles make due to fluid flows (i.e., the travel). As the current study examines the response of granular soils under upward flows, the total displacement of particles located in different layers over the height of the specimen is computed over an increasing \( i \). Here the specimen is divided into 5 equal layers and then the total displacement of all particles in each layer is computed at different values of \( i \). The initial position of particles in different layers is recorded and then updated, thus enabling their travel to be estimated. Fig. 4 shows that the migration of particles develops towards the soil surface (i.e., via piping process) when \( i \) increases, resulting in the concentration of fines on the surface zone (see Fig. 5b), noting that Case 4 (soil D) is used here as a typical example to demonstrate the analysis. Fines (i.e., \( D_p < 1.0 \text{ mm} \)) that locate in the low layers apparently make an overwhelming travel compared to the surface particles which are almost static, especially when \( i \) is still small. For example, where \( i = 0.53 \), the total fines at the bottom, i.e., the 1st layer, travel approximately 90 m while the same size particles in layers 4 and 5 only travel about 14 m and 0.8 m, respectively. While more particles closer to the surface begin to migrate when \( i \) increases as a larger flow pressure travels to the surface zones (see the
pressure gradient force, Eq. [14]), this is still not as significant as the migration by bottom fines. An excessive migration of fines over the entire specimen, especially layers 1 to 4, occurs when $i$ is greater than 1.0.

Corresponding to the migration that fine particles make due to fluid flowing, there is also a re-distribution in porosity over the height of the sample. For example, Fig. 4b shows how the porosity changes when $i$ increases to 0.89 in soil D (Case 4). As fines move upwards and concentrate in the upper layers, the porosity in these regions decreases significantly whereas this process makes soil media looser in the bottom zones. For instance, the average porosity increases from about 0.32 to 0.34 in layer 1 whereas it decreases from 0.33 to 0.32 in layer 5.

Fig. 5 represents soil samples (Case 4, soil D) at initial (Fig. 5a) and $i = 0.89$ (Fig. 5b) stages where there is an apparent concentration of fines on the surface. A larger $i$ would result in an increasing amount of fines being flushed upwards and washed out of the soil sample. Fig. 5c represents the corresponding distribution of fluid variables over the height of the sample at $i = 0.89$. Clearly, the fluid pressure decreases to almost zero at the surface zone but this drop in pressure is not uniform over the height because of the non-uniform distribution in porosity shown in Fig. 4b above. In fact, there is a greater loss in hydraulic pressure in the lower layers at initial stages where more fines concentrate, however this gradually changes when $i$ increases with increasing porosity in the lower regions. Fig. 5d shows how the drop in pressure changes relatively from convex to concave form when $i$ increases to 0.89. In this analysis, the ratio $z/H$ which varies from 0 to 1 represents elevation towards the surface of the sample.

While the previous sections represent the overall evolution of particle migration along the height of soil sample, different size particles will respond differently. For example, Fig. 6 shows the distance that particles with different sizes travel when $i$ increases over time (i.e.,
linear increment), which helps classify the contribution of particles in different sizes to internal erosion. It is obvious that fine soils (i.e., $D_p < 1.0$ mm) are the major source of internal erosion of soil as the finest particles $D_p = 0.4$ mm make predominant migration over other larger particles. For instance, they totally travelled about 245 m which is almost 8 times further than the remaining particles in the soil when $i$ increases to 0.89. It is noteworthy that soils C and D in this paper is made by mixing sand and gravel [1], and as such the gravel functions as the macro fabric of the soil while the fines fill the porous system of this coarse material. For example, the constriction size $C_d$ of gravel G which is used to generate soils C and D is estimated (see Fig. 1) with respect to the analytical method given by Indraratna et al. [36]. Apparently almost 78% of the coarse matrix $C_d$ is larger than the fine particles, i.e., $D_p \leq 0.4$ mm, therefore the finer the particles, the more they travel during fluid flows.

**Evaluation of soil erosion over increasing i**

The amount of fines that migrate and concentrate on the surface of soil samples can be used to indicate how much internal erosion has evolved over time throughout the samples at a certain degree of $i$. Fig. 7 shows the amount of fines counted on the surface of soils C and D over time when $i$ increases. This amount of fines on the surface is normalized over the initial mass of sand in the samples but note that soils C and D have different percentages of sand, i.e., 15 and 10%, respectively. In this analysis where Cases 1 and 5 are modelling the exact experimental values of porosity in soils C and D, they are used to validate the erosive behaviour described in the experimental studies [1]. Apparently, in soil C with a larger $C_u$, a much larger amount of fines (i.e., $D_p \leq 0.6$ mm) have been flushed to the surface when $i$ exceeds 1.0. In fact, the amount of fines concentrating on the surface increases to nearly 18.5 % and 9.1 % as $i$ reaches to about 1.19 in soils C and D, respectively. Compared to the sand in soil C, the amount of fines rising on the surface in soil D actually accounts for only...
about 6.0%.

Interestingly, fines do not concentrate on the surface very much when $i < 0.9$, but they increase rapidly when $i$ increases to 1.0 especially in soil C. This indicates the soil has reached a critical state when the migration (i.e., piping) of fine particles becomes apparent over the entire sample. While further details of how this migration affects the hydraulic behaviour of soil is shown later in this paper, this result indicates a good agreement with experimental observations on the critical state and corresponding hydraulic gradient $i_c$ [1].

For example, most experimental investigations usually consider that soil turns into a critical state under a hydraulic flow when strong general piping occurs over the entire soil specimen. Highly uniform soils such as Ottawa and Toyoura sand (Cases 6 and 7) do not experience significant erosion until they fail due to heave formation; this result corroborates with experimental observations [37, 38].

In addition to fines concentrating on the sample surface, the rate of erosion is usually used to evaluate internal erosion; this parameter is normally defined as the ratio by mass between the particles washed out of the soil to the total amount of original soil. For example, after completing their test, Skempton and Brogan [1] measured the amount of washed out fines compared to the total amount of original sand, which is also used in the current numerical investigation. Fig. 7b represents the erosion rate that develops with increasing $i$ with reference to the experimental data. Where $i < 0.8$, the amount of fines being washed out of the specimen is insignificant but then it increases considerably when $i$ exceeds 0.8. In fact, the erosion rate reaches almost 8.2 % and 2.7 % when $i$ increases to about 1.19 in soils C and D, respectively. Compared to the experimental data, this numerical result agrees quite well, particularly in soil C, however, note that the experimental data for soil D is not given in detail by Skempton and Brogan [1], so it is not included in this discussion. This comparison indicates a reasonable prediction by the current numerical approach with regards to the
internal erosion of granular soils.

*Hydraulic behaviour*

The migration of soil particles results in changes to the porous features of soil associated with varying drops in the hydraulic pressure over height, as discussed in previous sections. This section further addresses the hydraulic properties while increasing $i$. Fig. 8 represents the discharge velocity ($U_{f,d}$) at the soil surface (i.e., superficial velocity) over an increasing hydraulic gradient $i$ in different soils with validation to experimental data. In particular, the numerical results of soils C, D and sandy Ottawa and Toyoura soils (Cases 1, 5, 6 and 7, respectively) are used in this analysis. Note that the PSD and porosity of these numerical samples are similar to the experimental data [1, 37, 38]. While the sandy soils are grouped in Fig. 8a, the gravel and sand mixtures (i.e., soils C and D) which have a much larger hydraulic conductivity $k$ compared to the pure sandy samples, are shown in Fig. 8b.

The numerical and experimental results are in good agreement; for example the numerical prediction matches the experimental data in soil C very well where $i < 1.0$, with an accuracy of 95% over different values of $i$. The results of sandy soils have a certain deviation in $k$ between the numerical and experimental results. For example, the experimental value of $k$ is about $0.5 \times 10^{-3}$ m/s while the numerical prediction gives approximately $0.59 \times 10^{-3}$ m/s for $i < 1.0$. One of possible reasons that cause this deviation is because the current DEM assumes ideally spherical particles whereas particles of real soil can have angular shapes which lead to a difference in the fluid-particle contact area despite using the same PSD. The experimental detail for the hydraulic behaviour of soil D has not been provided by Skempton and Brogan [1], so they are not represented here; however, its hydraulic conductivity is much larger than soil C because soil D is much coarser than soil C, i.e., $C_u = 4.5$ compared to 7.0. In fact, the finest particles in soil C are 0.16 mm in diameter which is much smaller than the finest
particles (i.e., $D_p = 0.4$ mm) in soil D. Previous studies by [41] show that the coarser the particles, the less the fluid-particle contact area and the greater the flow resistance. In addition, the critical hydraulic gradient $i_c$ where $k$ begins to increase is estimated to be almost 1.1 in soil C using the numerical approach, which is reasonably close to the experimental and theoretical (i.e., Tezaghi method) values (i.e., 1.02).

4.2 Energy transformation through internal erosion

Transformation of flow energy

The input potential energy $E_f^p$ of fluid flows is governed by the difference in water heads (i.e., hydraulic gradient $i$) between the bottom (inlet) and the top (outlet) of the soil. An increasing $i$ results in an increasing potential energy of fluid flowing through the particulate medium. Fig. 9 shows how this energy varies over $i$ and how it is transferred to the kinetic energy of fluid $E_f^k$ and particles $E_p^k$. In this analysis, the results of soil D in Case 4 are used as a typical example to demonstrate the transformation process, but the same process also occurs over different soil samples at different magnitudes of energy. Undoubtedly, an increase in the potential energy of fluid results in rising kinetic components, i.e., $E_f^k$ and $E_p^k$, but they only increase rapidly when $i$ exceeds 1.08, which narrows the gap between the potential and kinetic energy in the system. This indicates a critical state where large amounts of soil particles migrate over the entire specimen and increasing the inertial (non-linear) component of the seepage flow. Moreover, the particle migration and heave behaviours, as in an upward growth of the surface, cause the porosity and constriction size to increase, which in turn reduces the flow resistance and increases the kinetic energy.

It is also apparent that most potential flow energy is transferred to the flow kinetic energy of fluid $E_f^k$ while the kinetic energy of particles $E_p^k$ only accounts for a very small part. The kinetic energy of particles is only generated when particles begin to migrate.
Despite the large migration of fine particles (i.e., $D_p < 1.0$ mm) when the hydraulic gradient is large, i.e., $i > 1.0$, their mass is very small whereas the coarse grains with a larger mass are almost static; this means the kinetic energy of particles in a fluid-soil system is very small. To some extent, these current findings agree with previous attempts [21] where the numerical and experimental investigations revealed the very small amount of kinetic energy (i.e., $< 1\%$) that is usually required to cause a large internal erosion of soil. Despite a small contribution to the total kinetic energy of fluid-soil systems, determining $E_p^k$ plays an important role in understanding the energy transformation as this component reflects directly the particle migration and internal erosion characteristics which cannot be captured fully through $E_f^k$. Following sections will further elaborate the behaviour of $E_p^k$ associated with the migration detail of particles.

Fig. 9b also shows that a considerable amount of energy (i.e., 48 to 64\%) is lost when fluid flows through the soil under an $i$ increasing from 0.3 to 1.0. The potential fluid energy is apparently larger than the kinetic energy of fluid and particles especially when $i < 1.0$. For example, the total kinetic energy of fluid $E_f^k$ and the particles $E_p^k$ is almost 0.0145 J which is apparently less than the potential flow energy of (0.038 J), which indicates an approximately 62\% energy loss at $i = 0.8$. It is also interesting that the energy loss increases when $i$ increases, however this loss begins to decrease due to the rapid increase in kinetic components when $i > 1.08$. In this current study where porous cohesionless soils are considered, the energy loss mainly represents the flow friction between the fluid and solid phases such as particles and boundaries. For example, the denser the soil particles, the less the hydraulic conductivity and particle migration, so more energy is needed to make the fluid and particles travel through the soil. Quantitative figures for these aspects will be addressed later in this paper.

How the kinetic energy of particles develops with increasing $i$ is further represented in
Fig. 10; note that in this current study, \( i \) is only increased to a new level when the soil is stabilized. There is a dramatic jump in \( E_p^k \) whenever \( i \) rises to a higher level but this growth of \( E_p^k \) is relatively linear to an increase in \( i \). \( E_p^k \) gradually dissipates over time at each level of \( i \) as the particles migrate and rearrange themselves. A swift change occurs when \( i \) exceeds 1.08; in fact, \( E_p^k \) reaches about \( 910 \times 10^{-9} \) J at \( i = 1.24 \) which is more than double its level at \( i = 1.08 \). Fig. 10b shows how different particles contribute to the entire kinetic energy of the solid phase in soil. In this comparison, the \( E_p^k \) of different size particles is normalised to one of the largest particles; i.e., \( D_p = 10 \) mm which is almost static with lowest kinetic energy. Apparently, the smaller the size, the greater the contribution because it is easier for finer particles to migrate. Interestingly, while the total \( E_p^k \) dissipates immediately after it reaches a peak in a short time at a new level of \( i \) (see Fig. 10a), the detailed analysis shown in Fig. 10b indicates that \( E_p^k \) by finer particles such as the finest \( D_p = 0.4 \) mm takes a longer time to stabilise. When \( i \) reaches 1.08, the deviation in kinetic energy between different particles decreases as coarser particles begin to displace more significantly.

**Influence of porosity and PSD**

Porosity plays a significant role in hydraulic properties and internal erosion behaviour of the soil, thus it imposes a large influence on the energy transformation. For example, for the same type of soil, the more compacted the soil, i.e., the smaller the porosity, the lower the hydraulic conductivity and the lesser the soil migration, which means the smaller the kinetic energy transferred into the system. Fig. 11 shows the maximum kinetic energy \( (E_p^{k,max}) \), see Fig. 10a) that particles can achieve at each level of \( i \) in soil D with varying porosity. Four different levels of porosity, i.e., from 0.29 to 0.36 (Cases 2 to 5, Table 1) are used in this analysis. Obviously, the smaller the porosity, the lower the generated potential energy by fluid considering the same time and \( i \) of the flow, as Fig. 11a shows. This disparity results in
different levels of kinetic energy transferred to solid particles (Fig. 11b); for example, $E_{p,max}^k$ is approximately 4 times larger at $i = 1.0$ when $n$ increases from 0.29 to 0.36. However, by considering the same input $E_f^p$ (i.e., normalize $E_{p,max}^k$ by $E_f^p$), the variation in the $E_{p,max}^k$ in different porosity soils becomes less significant (Fig. 11c). This indicates that reduction in the potential energy of fluid when reducing $n$ at the same $i$ can be a key contributor for the decrease the kinetic energy transferred to particles and corresponding soil erosion.

To understand how varying the uniformity of soil, i.e., $C_u$ can affect the energy transformation, $E_{p,max}^k$ is obtained for soil C (Case 1), soil D (Case 5) and Toyoura sand (Case 7) which have distinct values of $C_u$. Because these soil samples have different soil mass and erosion rates as shown in previous sections, $E_{p,max}^k$ is normalized by the total soil mass and particle travel, i.e., $E_{p,max}^k/(\text{total soil mass} \times \text{total particle travel})$ to represent how much the kinetic energy is needed to make particles travel a unit length per a unit soil mass. The results (Fig. 12) show that soil C having the largest $C_u = 7$ needs much smaller kinetic energy to produce the same migration of particles compared to soils D ($C_u = 4.5$) and Toyoura sand ($C_u = 1.6$). For example, $E_{p,max}^k$ is about $5 \times 10^{-9}$ J in soil C but it increases to $5 \times 10^{-8}$ and $1 \times 10^{-4}$ J in soil D and Toyoura sand, respectively, considering the same input potential energy $i = 1.0$. This explains why widening soil gradation makes the soil more susceptible to internal erosion as usually given through many experimental investigations [1, 3, 4]. Clearly, using the same approach, one is also able to estimate the amount of energy needed to cause a unit erosion rate (i.e., 1%) over different soils, however this aspect is not further detailed in this paper due to the imposed brevity of the article.

**Engineering significance of the energy modelling**

Computing the exact value of kinetic energy transfer in an experimental fluid-soil system is almost impossible, and only a very few studies have ever been attempted in the past,
especially in relation to internal erosion [11, 21]. The current paper demonstrates how the CFD-DEM coupling approach can help to rationalise this process insightfully. For example, traditional laboratory tests such as that conducted on soil C [1] could only account for the amount of particles eroded from the specimen while their detailed time-dependent migration could not be captured properly. However, adopting an approximate approach [21], one is able to estimate the kinetic energy needed for removing the fines fraction from experimental data, for example, soil C based on the rate of erosion (i.e., 9 % by mass) and interstitial velocity (i.e., \( U_f = U_{f,d}/n = 21.3 \times 10^{-3} \) m/s at \( i = 1.0 \)). Assuming that the eroded particles move at the same velocity as the interstitial fluid flow, the estimated kinetic energy of these particles amount to about \( 2.3 \times 10^{-7} \) J for the small soil mass of \( 7.5 \times 10^{-2} \) kg (Table 1), which relatively match the numerical value, i.e., \( E_{p,max}^k = 1.1 \times 10^{-7} \) J at \( i = 0.99 \) (i.e., without normalizing by the total travel path and soil mass, Fig. 12). Obviously, this estimate remains certain limitations, especially compared to the current numerical approach which can offer a more insightful perspective for enhanced understanding of internal erosion of soil particles.

5. Conclusion

This study has shown how energy transformation occurs with internal erosion in granular soils based on the CFD-DEM coupling method. Internal erosion that has successfully been captured and documented in previous experimental studies is re-produced and evaluated using the proposed numerical method. Major findings can be highlighted as follows:

- The numerical simulation could accurately predict the migration of fines that develop over the height and the increasing hydraulic gradient; essentially, the nearer the surface, the less migration there is of particles. When \( i \) reached a critical level such as 1.08 in soil D for example (Case 5), fine particles, i.e., \( D_p < 1.0 \) mm migrated upwards apparently over the entire specimen, and this result could be corroborated by the experimental
observations. The erosion rate found in the numerical simulation was relatively close to the experimental value; for example, 8.2% and 9.0% in soil C by numerical and experimental approaches, respectively.

- Hydraulic variables such as the pressure drop and hydraulic conductivity could be captured reasonably well using the current CFD-DEM coupling. The large amount of fines that migrated when $i$ approached the critical value $i_c$ resulted in significant redistribution of localised porosity over the sample height, leading to a change in the pressure drop of fluid. The hydraulic conductivity predicted by the numerical method agreed well with experimental data; for example the accuracy reached almost 95% in soil C. The numerical $i_c$ captured based on the hydraulic curve in soil C was about 1.1 which was quite close to the experimental value of 1.02.

- The numerical results showed that a large part (e.g., 62% in soil D (Case 4, $i = 0.8$)) of the potential input energy induced by fluid flow was dissipated while a very small percentage of $E^P_f$ was transferred to the kinetic component of soil particles. When $i$ approached $i_c$, there was a rapid increase in the kinetic energy of the fluid-soil system; for example, the energy dissipation decreased to almost 20% at $i = 1.08$ in soil D (Case 4). The kinetic energy by particles $E^P_k$ increased by almost double when $i$ reached 1.24.

- Porosity and $C_u$ play a significant role in determining how much energy was dissipated and how much was transferred to the kinetic component which causes internal erosion. Basically, the larger the porosity and $C_u$, the larger the amount of kinetic energy that soil particles could obtain. As an example, soil D with $n = 0.36$ received almost 4 times more kinetic energy when the soil was compacted to $n = 0.286$ at $i = 1$. Soil C having $C_u = 7$ needed about 10 times smaller kinetic energy to cause the same particle migration in soil D which as $C_u = 4.5$. Although there was a limitation to verifying the actual value of kinetic energy transferred to solid particles, the numerical results generally showed a
reasonable transformation of energy during the internal erosion of soils.

CRediT author statements

Thanh Trung Nguyen: Conceptualization, Writing-Original Draft, Formal Analysis, Methodology, Validation; Buddhima Indraratna: Supervision, Funding acquisition, Writing-Review-Editing; Methodology.

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References


OpenFOAM. OpenFOAM user guide, Version 2.3.0. 2014.


Zhou ZY, Kuang SB, Chu KW, Yu AB. Discrete particle simulation of particle–fluid


### Tables

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<thead>
<tr>
<th>Case No</th>
<th>Soil type</th>
<th>Particle size (mm)</th>
<th>Cu</th>
<th>Initial porosity ($n$)</th>
<th>Size of particle domain (mm)</th>
<th>Soil mass (kg)</th>
<th>Number of particles</th>
<th>Fluid cell size (mm)</th>
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Table 1 Details of different cases in the numerical investigation
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Table 2 Compare numerical to experimental data
Fig. 1 PSD of soils used in the current study
Fig. 2 Typical example of numerical CFD-DEM coupling model
Fig. 3 Incremental scheme of hydraulic gradient
Fig. 4 Response of soil D (case 2) at different depths over increasing \( i \): a) total travel (i.e., total length of the migration paths) of particles; b) corresponding redistribution in porosity.
Fig. 5 Particle migration and corresponding fluid variables in soil D (Case 4): a) initial state; b) fines concentration on the surface at $i = 0.89$; c) space distribution of fluid variables; and d) pressure drop over the height of sample
Fig. 6 Total particle migration with different particle sizes over time
Fig. 7 Development of erosion rate over increasing hydraulic gradient $i$: a) fines concentration on the surface; and b) fines washed out the sample
Fig. 8 Hydraulic conductivity compared with experimental data: a) sandy soils Ottawa and Toyoura; and b) gravel mixed with sand soils C and D
Fig. 9 Energy transformation during internal erosion (soil D, Case 4): a) potential fluid; and b) the energy loss
Fig. 10 Kinetic energy of particles ($E_k^p$) (soil D, case 3): a) energy evolution over time with different levels of $i$; b) energy distribution over different sizes of particles
Fig. 11 Influence of porosity on energy transformation: a) potential energy of fluid $E_f^p$ considering the same $i$ and time; b) corresponding maximum $E_f^k$; c) maximum $E_p^k$ considering the same unit potential energy of fluid.
Fig. 12 Kinetic energy in different $C_u$ soils: a) total travel of particles; b) kinetic energy cost for a unit travel (m) and soil mass