Fringe projection profilometry for the 3D shape measurement of objects with three-dimensional movements

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**Recommended Citation**
Duan, Chengpu; Zhang, Yiwei; Xi, Jiangtao; Tong, Jun; Yu, Yanguang; and Guo, Qinghua, "Fringe projection profilometry for the 3D shape measurement of objects with three-dimensional movements" (2019). *Faculty of Engineering and Information Sciences - Papers: Part B*. 3799.

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Abstract
© 2019 SPIE. Phase shifting profilometry (PSP) is considered as an effective method for 3D shape measurement based on fringe projection. However, PSP is not suitable for dynamic measurement, as it requires that the object be kept still. Movement of the object during the cause of projection of multiple fringe patterns may lead to significant error in the measurement of the 3D shape. A number of approaches were proposed to combat this problem consisting of two steps: Capturing of the movement and then compensation (or correction) of fringe patterns. However, such compensation is only valid for the cases where the object moves or rotates in the way that all points on the object surface change by the same amount. In other words, there is still not a method effective for measuring objects moving in a free 3D space. In this paper, a new method is proposed to combat the problem. Firstly, movement of the object is capturing by means of existing methods, yielding rotation matrix and translation vector, able to characterize arbitrary movement in a 3D space. Secondly, variation of the fringe patterns by the movement is analyzed and formulated, leading to the expressions of phase maps. Based on these expressions, a new method is proposed to compensate the variance on height map, with which PSP can be used to yield improved measurement performance. Computer simulations is carried out to verify the effectiveness of the proposed method.

Disciplines
Engineering | Science and Technology Studies

Publication Details

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This journal article is available at Research Online: https://ro.uow.edu.au/eispapers1/3799
Fringe projection profilometry for the 3D shape measurement of objects with three-dimensional movements

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ABSTRACT

Phase shifting profilometry (PSP) is considered as an effective method for 3D shape measurement based on fringe projection. However, PSP is not suitable for dynamic measurement, as it requires that the object be kept still. Movement of the object during the cause of projection of multiple fringe patterns may lead to significant error in the measurement of the 3D shape. A number of approaches were proposed to combat this problem consisting of two steps: Capturing of the movement and then compensation (or correction) of fringe patterns. However, such compensation is only valid for the cases where the object moves or rotates in the way that all points on the object surface change by the same amount. In other words, there is still not a method effective for measuring objects moving in a free 3D space. In this paper, a new method is proposed to combat the problem. Firstly, movement of the object is capturing by means of existing methods, yielding rotation matrix and translation vector, able to characterize arbitrary movement in a 3D space. Secondly, variation of the fringe patterns by the movement is analyzed and formulated, leading to the expressions of phase maps. Based on these expressions, a new method is proposed to compensate the variance on height map, with which PSP can be used to yield improved measurement performance. Computer simulations is carried out to verify the effectiveness of the proposed method.

Keywords: phase shifting profilometry (PSP), dynamic object measurements; motion compensation

1. INTRODUCTION

Fringe pattern profilometry (FPP) has been extensively studied for three-dimensional (3D) shape measurements due to its non-contact feature [1] and high accuracy [2]. This technique has wide applications, such as inspection, monitoring and manufacturing [3]. Recently, FPP-based 3D shape measurement for dynamic objects whose height distributions vary with time has attracted significant interests [4].

Fourier transform profilometry (FTP) is a popular single-shot FPP technique. Though directly applicable to moving objects, FTP is more sensitive to the influence of the ambient light and reflectivity on the object as compared with multi-shot FPP techniques. Numerous approaches have been studied [5, 6] to improve the performance of FTP, such as the carrier removal techniques [7, 8]. 1-D Fourier transform can also be extended to windowed 2-D Fourier transform to achieve better separation of the height information from noise [9]. The π-shift FTP method [10] uses two line-scan cameras and phase-shifted subpatterns to improve the measurement of objects moving at a constant speed. In addition, improved phase unwrapping methods are proposed for dynamic shape measurements [11]. Even the FTP method has been improved in many different ways, the performance for dynamic objects is still limited due to its single-shot nature.

Phase shifting profilometry (PSP) reconstructs the object shape based on three or more fringe patterns with the noise effect suppressed. The conventional PSP schemes assume static objects and the motion of the objects can significantly degrade the performance due to the loss of the correspondence of the points. In [12], the knowledge of the motion is utilized to improve the performance for dynamic objects with 2D movements. Liu et al. [13] propose an approach that estimates the motion parameters of the object from successively captured 3D frames and then uses the projector’s pinhole model to compensate the motion error. Wang and Han [14] use least squares optimization to iteratively minimize the motion-induced phased error. Another approach [15] uses Hilbert transform to create extra fringe patterns and phase maps and compensates motion-induced error by averaging the original and additional phase maps. All the above mentioned methods are developed for dynamic objects with 2D movements.
In this paper, we propose a novel method based on multiple-shot fringe patterns with phase shifts. We directly retrieve the height information from multiple fringe patterns, utilizing the motion knowledge and the correspondence among the points in different measurements. A pixel-by-pixel model fitting-based method is developed to search for the height. Besides, the method is extended by considering multiple pixels jointly to combat the influence of noise. Examples with three-dimensional movements are demonstrated, which show that the proposed method can effectively improve the measurement accuracy.

2. TRADITIONAL PSP

The FPP system consists of a projector projecting fringe patterns and a camera capturing the patterns deformed by the object, as illustrated in Fig. 1. The distance between the lens of the projector and that of the camera is \( d_0 \) and the distance between the lens of the camera and the reference plane is \( L_0 \). The optical axis \( P_1 - P_2 \) of the projector lens crosses the optical axis \( I_1 - I_2 \) of the camera lens at point \( O \) on the reference plane. A light from point \( P_2 \) to point \( A \) is reflected by the object, producing the deformed fringe pattern. From the image captured by the camera, the phase difference caused by the object as compared to the reference pattern is used for retrieving the height \( h \) of the object.

Consider \( K \)-step PSP. The sinusoidal fringe patterns captured respectively from the reference plane and a static object can be written as:

\[
s_k(x, y) = a(x, y) + b(x, y)\cos \left( \varphi(x, y) + \frac{2\pi(k-1)}{K} \right), \quad k = 1, 2, ..., K, \tag{1}
\]

and

\[
d_k(x, y) = a(x, y) + b(x, y)\cos \left( \varphi(x, y) + \Phi_h(x, y) + \frac{2\pi(k-1)}{K} \right), \quad k = 1, 2, ..., K, \tag{2}
\]

where \( a(x, y) \) is the average intensity and \( b(x, y) \) is the intensity modulation of sinusoidal fringe patterns. Generally, we assume \( a(x, y) \) and \( b(x, y) \) are constant over time. \( \varphi(x, y) \) is the reference phase information and \( \Phi_h(x, y) \) is the phase difference caused by the height of the object. The phase maps for the cases without and with object can be retrieved as

\[
\varphi^s(x, y) = \varphi(x, y) = \tan^{-1} \left( -\frac{\sum_{k=1}^{K} s_k(x, y) \sin(2\pi(k-1)/K)}{\sum_{k=1}^{K} s_k(x, y) \cos(2\pi(k-1)/K)} \right), \tag{3}
\]

and

\[
\varphi^d(x, y) = \varphi(x, y) + \Phi_h(x, y) = \tan^{-1} \left( -\frac{\sum_{k=1}^{K} d_k(x, y) \sin(2\pi(k-1)/K)}{\sum_{k=1}^{K} d_k(x, y) \cos(2\pi(k-1)/K)} \right). \tag{4}
\]

The object-induced phase difference \( \Phi_h(x, y) \) can be found as the difference of (3) and (4), and then unwrapped to obtain the absolute phase value to retrieve the object height as

\[
h(x, y) = \frac{L_0\varphi_h(x, y)}{\Phi_h(x, y) - 2\pi d_0}. \tag{5}
\]

The above PSP approach assumes a static object whose height distribution does not vary with time. For dynamic objects, the height at a pixel \((x, y)\) can vary with time and \( \Phi_h(x, y) \) in the captured patterns (2) may not be constant during the measurement. Consequently, the phase retrieval step in (4) may perform poorly, resulting in poor measurements.
3. HEIGHT RETRIEVAL FOR MOVING OBJECTS

We now introduce a method that directly recovers the height of moving targets from the captured phase-shifted fringe patterns. We assume rigid movements of the object. Assume that \( K \) images are acquired at \( K \) time instants, \( k = 1, 2, \ldots, K \). Let us consider a single point \( P \) on the object surface. Other points can be treated using the same method. Let \((x_k^P, y_k^P, h_k^P)\) be its coordinate in the 3D space at time instant \( k \), where \( h_k^P \) is the height of \( P \). The 3D movement, relative to the time instant \( k = 1 \), can be modelled by:

\[
\begin{bmatrix}
x_k^P \\
y_k^P \\
h_k^P
\end{bmatrix} = R_k \begin{bmatrix} x_1^P \\ y_1^P \\ h_1^P \end{bmatrix} + t_k,
\]

where \( R_k, t_k \) are referred as the rotation matrices and translation vectors:

\[
R_k = \begin{bmatrix}
  r_{11}^k & r_{12}^k & r_{13}^k \\
r_{21}^k & r_{22}^k & r_{23}^k \\
r_{31}^k & r_{32}^k & r_{33}^k
\end{bmatrix},
\quad t_k = \begin{bmatrix} t_1^k \\ t_2^k \\ t_3^k \end{bmatrix}.
\]

From the motion models, we have

\[
x_k^P = f_k(x_1^P, y_1^P, h_1^P) = r_{11}^k \cdot x_1^P + r_{12}^k \cdot y_1^P + r_{13}^k \cdot h_1^P + t_{12}^k,
\]

\[
y_k^P = g_k(x_1^P, y_1^P, h_1^P) = r_{21}^k \cdot x_1^P + r_{22}^k \cdot y_1^P + r_{23}^k \cdot h_1^P + t_{22}^k,
\]

\[
h_k^P = z_k(x_1^P, y_1^P, h_1^P) = r_{31}^k \cdot x_1^P + r_{32}^k \cdot y_1^P + r_{33}^k \cdot h_1^P + t_{32}^k.
\]

If the height and location of \( P(x_1^P, y_1^P, h_1^P) \) are known, then the points corresponding to \( \tilde{P}(x_k^P, y_k^P, h_k^P) \) at different instants can be tracked. In practice, the height \( h_k^P \) is unknown but to be estimated. Meanwhile, if we assume \( h_k^P = h \), then the hypothesized locations of the point in the different images can be found using (8) and (9). Denote the corresponding intensity values in the measurements as

\[
D_1^P(h) = d_1(x_1^P, y_1^P),
\]

\[
D_2^P(h) = d_2(f_k(x_1^P, y_1^P, h), g_k(x_1^P, y_1^P, h)),
\]

\[
\vdots
\]

\[
D_K^P(h) = d_K(f_k(x_1^P, y_1^P, h), g_k(x_1^P, y_1^P, h)).
\]

On the other hand, according to the patterns of PSP, we can model the intensity values at the different locations as

\[
M_1^P(h) = a + b \cos(\Phi_1^P(h) + \frac{2\pi k_0 \phi_0(x, y)}{h(x, y)})
\]

\[
M_2^P(h) = a + b \cos(\Phi_2^P(h) + \frac{2\pi k_0 \phi_0(x, y)}{h(x, y)}) + \Phi_2^P(z_k(x_1^P, y_1^P, h)) + \frac{2\pi (K-1)}{K}
\]

\[
: : : \Phi_K^P(h) = a + b \cos(\Phi_K^P(h) + \frac{2\pi k_0 \phi_0(x, y)}{h(x, y)}) + \Phi_K^P(z_K(x_1^P, y_1^P, h)) + \frac{2\pi (K-1)}{K}
\]

where we have assumed that the average intensity and the intensity modulation are the same for the same point \( P \) in the \( K \) patterns, and

\[
\Phi_k^P(h) = \frac{2\pi k_0 \phi_0(x, y)}{h(x, y)}
\]

is phase difference caused by point \( P \) at time instant \( k \). Consider a system with \( K=3 \). Similarly to the PSP, we can compute the following metrics

\[
M^P(h) = \frac{M^P(h) - M_1^P(h)}{M^P(h) - M_2^P(h)}
\]

\[
= \frac{\cos(\Phi_1^P(h)) - \cos(\Phi_2^P(h)) + \Phi_2^P(z_2(x_1^P, y_1^P, h)) + \Phi_2^P(x_1^P, y_1^P, h)}{\cos(\Phi_1^P(h)) - \cos(\Phi_2^P(h)) + \Phi_2^P(z_2(x_1^P, y_1^P, h)) + \Phi_2^P(x_1^P, y_1^P, h)}
\]

\[
D^P(h) = \frac{D_1^P(h) - D_2^P(h)}{D_1^P(h) - D_2^P(h)}
\]

Define the cost function

\[
J^P(h) = |M^P(h) - D^P(h)|^2.
\]

In the ideal case and ignoring all the ambient light and noise, \( J^P(h) \) is zero if \( h \) is equal to the true height of \( P \). Taking into account the ambient light and noise, we propose to estimate the height \( h_k^P \) as

\[
h_k^P = \arg \min_J J^P(h).
\]

The performance of the proposed method is influenced by the noise. Fig. 2 plotted several examples of the cost function \( J^P(h) \) with different SNRs. It is seen that the minimizer of \( J^P(h) \) can deviate from the true height in practice.
In order to improve performance, we propose an approach based on an idea similar to the local polynomial fitting method (LPF) of [16]. Instead of considering Point \( P \) only while estimating its height, we consider a local region \( \Omega_P \) which includes \( P \) and also its neighboring points \( \{ P' \} \), as indicated in Fig. 3. We assume that \( \Omega_P \) is planar with parameters given by \((\alpha(h), \beta(h), h)\), i.e., the height of a point in \( \Omega_P \) can be modelled as

\[
h_{P'}^p(h) = \alpha(h) \ast (x_{P'} - x_P) + \beta(h) \ast (y_{P'} - y_P) + h, \forall P',
\]

(17)

where \( h \) denotes the hypothesized height at point \( P \) and we assume \( \alpha(h), \beta(h) \) depend on \( h \). In this case, instead of minimizing \( J_P(h) \), we estimate the height for \( P \) as

\[
\hat{h}_P^p = \arg \min_h \sum_{P' \in \Omega_P} J_P^p(h_{P'}^p(h))
\]

(18)

where \( J_P^p(h_{P'}^p(h)) \) denotes the cost function defined in the same way as \( J_P(h) \) in (15).

Ideally we may search over all possibilities of \((\alpha(h), \beta(h), h)\) but this leads to significant complexity. We assume that initial estimates \( \{ \hat{h}_P^p \} \) of the heights in \( \Omega_P \) are available, e.g., by applying FTP to the first image. We then fit the model (17) to these estimates with a hypothesized value of \( h \). The ordinary least squares (OLS) method can be used to compute the model parameters \( \alpha(h), \beta(h) \), and (18) can then be performed.

\[
\begin{array}{c|ccc}
 & p^c & p & p^a \\
\hline
p^c & p^a \\
p & & & \\
p^a & & & \\
\end{array}
\]

Figure 3. Local region \( \Omega_P \)

4. SIMULATION RESULTS

Consider an FPP system with \( l_0 = 4000 \text{mm} \) and \( d_0 = 600 \text{mm} \). The fringe spatial frequency is set to have 20 pixels in one period. As shown in Fig. 4, we consider an example of a hemi-ellipsoidal object placed on the reference plane, which is moving during a three-step measurement process with the following motion parameters (i.e., rotation matrices and translation vectors).
\[ R_2 = \begin{bmatrix} 0.9999 & -0.0083 & -0.0111 \\ 0.0085 & 0.9999 & 0.0111 \\ 0.0110 & -0.0112 & 0.9999 \end{bmatrix}, \quad t_2 = \begin{bmatrix} 4 \\ -2 \\ 1 \end{bmatrix}, \]  
\( R_3 = \begin{bmatrix} 0.9999 & -0.0056 & -0.0139 \\ 0.0057 & 0.9999 & -0.0083 \\ 0.0138 & -0.0084 & 0.9999 \end{bmatrix}, \quad t_3 = \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}. \]  
\( (19) \)  
\( (20) \)

Figure 4. Dynamic object in three successive frames

Figure 5. Reconstructed object by (a) FTP and (b) the proposed method

The reconstruction results at SNR=30 dB are shown in Fig. 5 with different methods. FTP achieves a mean square error (MSE) of 0.0065 in the height estimates. The proposed method effectively reduces the MSE to 0.0008, indicating a higher measurement accuracy is achieved. Fig. 6 shows that the proposed method can effectively improve the performance at different SNRs.

Figure 6. MSE achieved by FTP and the proposed method
5. CONCLUSION

A novel method is developed for measuring the shape of dynamic objects with 3D movement. The proposed method utilizes knowledge of the motion to establish the correspondence between the measurements at different time instants. A cost function inspired by the PSP is formulated, based on which the estimate of the height can be optimized. Higher accuracy as compared with FTP is demonstrated for the proposed method by simulations. Future work includes experimental study of the proposed method with practical motion parameter estimation methods.

6. REFERENCE