Formula for Sediment Transport Subject to Vertical Flows

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Abstract
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Abstract: Sediment transport is a geophysical phenomenon in which sediment particles are driven to move in streamwise and vertical directions by various forces. Almost all existing formulas of sediment transport were derived without considering vertical flows $V$, resulting in a large discrepancy between measured and predicted transport rates, as has been reported in the literature. This paper investigates the effect of vertical motion on sediment transport. It was found that upward fluid velocity increases particles’ mobility, and downward motion increases its stability. Furthermore, the investigation showed that decelerating flows can promote upward flow and vice versa. New equations were developed to express the influence of vertical motion on sediment transport. A reasonably good agreement between measured and predicted sediment transport rates was achieved. DOI: 10.1061/(ASCE)HY.1943-7900.0001592. © 2019 American Society of Civil Engineers.

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Introduction

Sediment transport is a ubiquitous phenomenon in natural waterways. The process of sediment transport is very complex and has attracted intensive research. Generally, sediment transport in well-controlled laboratory conditions can be reasonably predicted by many formulas available in the literature, but this is not true for field data. In these formulas, the basic parameters used are (Yang and Lim 2003) discharge $Q$, mean velocity $U$, channel slope $S$, water depth $h$, channel width $b$, particle size $d$ (or settling velocity $\omega$) and its gradation, sediment density $\rho_s$, and gravitational acceleration $g$. Other fluid parameters include fluid density $\rho$ and fluid viscosity $\nu$. Parameters that reflect vertical motion, such as seepage velocity or vertical pressure force induced by unsteadiness and nonuniformity, are totally neglected in almost all equations for sediment transport, even when the equations are extended to these complex cases.

Sediment transport is a result of driving and resistance forces acting on particles. Some researchers believe that the boundary shear stress $\tau = \rho gh S$ alone can fully express the driving force of sediment transport. Thus, the equations of sediment transport by Einstein (1942), Meyer-Peter and Muller (1948), Yalin (1977), Engelund and Hansen (1972), and Ackers and White (1973) can be written as $\Phi = f(\tau)$, where $\Phi$ is the dimensionless form of sediment discharge $Q$, and $\tau$, is the Shields (1936) shear stress parameter $\tau/[(\rho_s - \rho)g d]$.

Because formulas that use $\tau$, do not always provide good agreement with measured $Q$, some researchers have made attempts to use other parameters to express the driving force, like Velikanov’s (1954) parameter, $U^3/(gh\omega)$. This parameter has been widely used in Russia and China to express sediment transport.

As both the boundary shear stress $\tau$ and mean velocity $U$ sometimes correlate poorly with measured sediment transport rate, another parameter, known as the stream power ($=\tau U$) was proposed by Bagnold (1966), who hypothesized that the work used to transport sediment comes from the stream power. However Yalin (1977) expressed concern that the stream power can be rewritten as $\rho g h Q$, where $q = U h = Q/b$, for a reach where the channel slope $S$ is constant. The concept of stream power implies that the sediment discharge is proportional to the discharge only. This is not correct, because it excludes the influence of other hydraulic parameters such as bedform roughness. In other words, if the flow rate is constant along a river from upstream to downstream, the concept of stream power indicates that the rate of sediment transport only depends on the channel slope $S$. According to Yalin, this is unacceptable.

Like the product of $U$ and $\tau$, the product of $U$ and Sha’s been tested against measured sediment transport. Yang (1973) empirically found that, among the existing hydraulic parameters, the parameter of unit stream power $U S/\omega$ yielded the highest correlation coefficient with measured sediment concentration. This finding significantly advanced the knowledge of sediment transport and improved the accuracy of sediment prediction.

It was probably van Rijn (1984) who was the first one to make attempts to include the influence of bedforms in the formulas for sediment transport. By introducing a new parameter, the shear velocity related grains $u'_s$, he developed equations that depend only on two parameters, that is, $T$ and $d'_s$, which are expressed as

$$T = \frac{u'^2_s - u'^2_{ic}}{u'^2_{ic}} \quad (1)$$

$$d'_s = d \left[ \frac{(\rho_s/\rho - 1)}{\rho^2} \right]^{1/3} \quad (2)$$

where

$$u'_s = \frac{U}{2.5 \ln \frac{h}{2d_s}} \quad (3)$$

In van Rijn’s model, the shear velocity $u'_s$ was introduced to replace the mean velocity $U$. He argued that $u'_s$ is simple and conveniently eliminates the effect of bedform roughness. In his model, the parameter of boundary shear stress was not included, because “the energy gradient $S$ is not an appropriate parameter for morphological computations.”

In the literature, there are three different hydraulic parameters used to express the driving force of sediment transport.
Namely, they are (1) the shear stress $\tau$ or its dimensionless form $\tau_c$; (2) the mean velocity $U$ or its dimensionless form $U'/(gh_{w})$, $US/\omega$, $T$, and; (3) the stream power $\tau U$. It is important to note that these three often give vastly different predictions. For example, in a simple laboratory experiment in which the flume slope, channel width, particle size ($d_{50}$ or $\omega$), and discharge and other parameters are kept constant, only the bed roughness is changed. The formulas using $U$ predict that the small velocity hindered by higher bed roughness will reduce $g_r$. The formulas using $\tau$ predict that, at higher roughness, the higher $\tau$ will yield higher $g_r$, because $\tau = \rho g s \tau$ and higher $h = q/\tau U$ are caused by the lower velocity, or else $g_r$ will be infinite if the roughness is so high that the velocity approaches zero. However, the models of stream power $\tau U$ state that the variation of roughness will not change $g_r$ at all, because $q = \tau U$ constant, as pointed out by Yalin (1977). Therefore, the three models have totally different predictions.

In practice, the concept of sustainable development has been widely accepted worldwide, and many cities, such as Singapore, and Seoul, South Korea, want to replace their existing concrete drainage systems with natural plantation. City planners, water engineers, and citizens are keen to know the consequences of these replacements for sediment transport in a given return-period flood. Obviously, no satisfactory answer can be obtained from the existing models to guide such projects, in which roughness is increased by vegetation. This highlights the fact that further investigation on sediment transport is needed even for steady and uniform flows.

Based on the fact that sediment transport is a near-boundary phenomenon, Yang (2005) modified Bagndöld’s expression by using near-boundary parameters, that is, boundary shear stress $\tau$, and near boundary velocity, which can be represented by $u'_b$; the energy used to support sediment transport is redefined as $E = \tau u'_b$, and the average sediment velocity is assumed to be proportional to $u'_b$, rather than $U$ as in Bagndöld’s expression. The following equation was obtained to express the total load of sediment transport:

$$g_r = \left(\frac{\rho_u}{\rho_s - \rho}\right)ku'_b\left(\frac{E - E_c}{\omega}\right)$$

where $k = 1.1$ is a constant ($=12.2$), which is insensitive to other hydraulic parameters like Froude number, Reynolds number, relative roughness, and Rouse’s number (Yang et al. 2007); and $E_c = (\rho u'_b)^2$ is critical energy needed to transport sediment. The arrows in Eq. (4) indicate that sediment transport is a vector, and its direction follows the near-bed velocity $u'_b$, which is useful especially when the flow directions of upper and lower water layers are different, as in cases like large reservoirs, estuaries, and open seas.

Eq. (4) produces the highest correlation coefficient among the existing parameters, and on average it is 0.8, higher than that of the unit stream power (Yang 2005). All flow parameters used in Eq. (4) represent the characteristics of the boundary region (e.g., boundary shear stress, near bed velocity, and so forth), in which the majority of sediment particles are transported.

This simple review reveals that all models of sediment transport rely only on streamwise parameters, such as $U$, $u'_b$, $\tau$, $E$, and $US$. All equations, including Eq. (4), predict that the higher the streamwise-parameters are, the more particles will be transported (Liu and Chiew 2012). These models developed from unidirectional flows have been widely extended to complex cases, like unsteady, non-uniform flows, and large discrepancies have been observed. Lee et al. (2004) carried out a series of experiments to investigate the validity of this extension. They found that the highest rate of sediment transport never occurs when these streamwise-parameters listed above are the highest; in fact, peak sediment concentration and transport rate always appear after the peak flow rate in a “phase lag.” Even for steady flow Afzalimhr et al. (2007) observed that the very famous Shields diagram is no longer valid in decelerating flows in which the measured critical shear stress is significantly less than the Shields prediction. Many laboratory and field (Hardy et al. 2010; Cellino and Lemmin 2004; Chen et al. 2005) studies have demonstrated that this lag can be caused by tides, waves, and turbulent coherent structures on different spatiotemporal scales. Some researchers have found that upward flow (or ejection) may be responsible for the threshold of particle movement; for example, Wren et al. (2007) found that, along a dune, peak sediment concentration does not appear at the place where the streamwise parameters are highest, but at the location where the upward velocity becomes discernible.

Data from field observations (Hossain et al. 2001) have also confirmed that maximum sediment concentration does not always appear when the streamwise parameters are maximum. Myrhaug (1995) observed sediment transport near a seabed and found that the highest sediment concentration occurs at the lowest velocity ($U \approx 0$), and the highest streamwise velocity always corresponds with lower sediment concentration.

Many attempts have been made to explain these observations. Some useful advances have been made. Francalanci et al. (2008) attributed the phase lag to pressure variation and argued that current theory of sediment transport is based on the assumption of a hydrostatic pressure distribution of the flow field, which is valid only for quasi-steady, quasi-uniform, rectilinear flows; therefore, the existing theory of sediment transport cannot possibly express a large variety of flow conditions, such as erosion induced by a bridge pier. Their analysis showed that the dimensionless Shields number contains the assumption of hydrostatic pressure. Others, like Chiew and Parker (1994) and Cheng and Chiew (1999), have linked the nonhydrostatic pressure with the existence of vertical flow. Systematic observations have been conducted for sediment transport under suction and injection on the interface.

The objective of this paper is to systematically investigate the influence of vertical flows on sediment threshold conditions $\tau_c$, transport load or discharge $g_r$, and suspended concentration $C$. To achieve this, a suitable parameter to represent vertical motion is selected and investigated to see whether the selected parameter can explain the observed phenomena quantitatively and qualitatively.

### Theoretical Considerations

Sediment transport is a joint effect of streamwise and vertical motions of water on particles. It is understandable that all equations become invalid if a vertical motion coexisting in the flow field influences sediment transport. This joint effect was explained clearly by Francalanci et al. (2008), who rewrote the definition of Shields shear stress $\tau_c$, in the following way:

$$\tau_c = 4 \frac{\tau \pi (\frac{\rho}{\rho_s})^2}{3 \pi (\rho_s - \rho)} \omega (\frac{\rho}{\rho_s})^3$$

They argued that the numerator denotes the streamwise friction force and the denominator represents the vertical force. Thus, if pressure is not constant with respect to time and space, then the Shields number must be modified. They used the pressure force to modify the fluid density in the Shields number; high pressure corresponds to a higher value of $\rho_s$ and lower pressure lower $\rho_s$.

As shown in Fig. 1, a simpler model, similar to Francalanci et al.’s (2008) treatment, is considered in this study, in which vertical velocity, rather than the pressure force, represents vertical motion, because velocity is more straightforward and convenient than other parameters like pressure force or hydraulic gradient in the
Fig. 1. Schematic diagram showing interaction of streamwise and vertical motions.

sediment layer. A permeable bed is represented by uniform spherical particles with diameter $d$ where the vertical velocity $V_v$ exists at the interface. The value of the vertical velocity $V_v$ is much less than $U$, but the presence of $V_v$ is very important in controlling sediment transport because it alters the velocity distribution (Schlichting 1979), Reynolds shear stress distribution, flow resistance, and so forth (Yang 2009a, b, c). Its influence on mass transport also needs to be spelled out clearly (Yang 2013; Alfarahli et al. 2014).

For the particles shown in Fig. 1, the settling velocity $\omega$ in still water can be determined by the following force balance equation:

$$C_d\frac{d^2\rho v^2}{2} = \pi \frac{d^3}{6} g(\rho_s - \rho)$$

(6)

where $C_d$ = drag coefficient, which depends on the Reynolds number $R (= \omega d/\nu)$ if $R > 1,000$ and $C_d = 0.45$.

In an environment in which the ambient fluid moves upward with velocity $V_v$, the net settling velocity is reduced to $\omega - V_v$. A reduction in the settling velocity could be treated by altering its density from $\rho_s$ to $\rho_s'$ by assuming that the particle size remains unchanged, and the force balance equation is similar to Eq. (6):

$$C_d\frac{d^2\rho (\omega - V_v)^2}{2} = \pi \frac{d^3}{6} g(\rho_s' - \rho)$$

(7)

From Eqs. (6) and (7), one can derive the following relationship:

$$\frac{\rho_s' - \rho}{\rho_s - \rho} = \alpha \left(1 - \frac{V_v}{\omega}\right)^2$$

(8)

where $\alpha = C_d t/C_d$ and $\alpha = 1$ are assumed in order to simplify the mathematical treatment.

Contrary to Francalanci et al.’s (2008) treatment, in which the fluid density was modified in order to express the pressure influence, Eq. (8) introduces the apparent particle density of $\rho_s'$ and implies that the effects of nonhydrostatic pressure could be equivalently expressed by the variation of sediment density. If an upward velocity ($V_v > 0$) is present, Eq. (8) gives $\rho_s' < \rho_s$, and the sediment can be represented as a lightweight material; a downward velocity ($V_v < 0$) increases $\rho_s'$, implying that the sediment is more difficult to move. Therefore, Eq. (8) demonstrates that an upward velocity $V_v$, promotes the mobility of particles as they become “lighter,” and a downward velocity $V_v$ promotes the stability of particles as the sediment becomes “heavier.”

For unsteady or nonuniform flows, the two-dimensional (2D) continuity equation is

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

(9)

where $u$ and $v$ = streamwise and vertical time-averaged velocities in the $x$- and $y$-directions, respectively, as shown in Fig. 1. The vertical velocity can be determined from Eq. (9) as follows:

$$v = -\int \frac{\partial u}{\partial x} dy$$

(10)

In Eq. (10), $\partial u/\partial x$ is the gradient of streamwise velocity in the $x$-direction. The value is positive if the velocity becomes higher in the streamwise direction (accelerates) and negative if the flow is decelerating. Hence, an accelerating flow yields a negative or downward $v$, and a decelerating flow generates an upward or positive $v$.

At the permeable boundary, the fluid velocity must meet the continuous boundary condition, that is, $V_r'(y = 0) = V_r(y = 0)$, where $V_r'(y = 0)$ is the vertical velocity at the bottom of the fluid layer, and $V_r(y = 0)$ is the velocity at the top of the sediment layer. A downward velocity ($v < 0$) induced by an accelerating flow could generate a downward velocity in the sediment layer, and vice versa for an upward velocity by decelerating flow in upper layer. From Eqs. (8) and (10), it can be inferred that sediment is more easily transported in decelerating flows than in accelerating flows, and this inference may be valid even with a bursting phenomenon or wave conditions in which the accelerating/decelerating phases alternate randomly or regularly. It is necessary to validate this inference using experimental data and field observations.

Critical Shear Stress Subject to $V_s$

Artificial $V_s$

Almost all equations of sediment transport were developed from steady and uniform flows in which the vertical velocity $v$ is equal to 0, as stated in Eq. (10), and then were extended to flows with $v \neq 0$. For example, the Shields diagram has been extended to express sediment movement in nonuniform/unsteady flows, and large discrepancies have been found. Eqs. (8) and (10) may provide a useful tool to improve the Shields diagram’s application when its density is modified:

$$\tau_s' = \frac{\tau_s^c}{(\rho_s' - \rho)gd}$$

(11)

where $\tau_s'$ = critical shear stress with vertical velocity. Eq. (11) predicts that if the Shields number remains unchanged, the observed critical shear stress in a decelerating flow (or phase) should be smaller than the critical shear stress without $V_s$, and an accelerating flow (or phase) needs a higher shear stress to drive particles in motion. This can be seen by inserting Eq. (8) into Eq. (11):

$$\tau_s' = \frac{\tau_s^c}{(\rho_s' - \rho)gd}\left(\frac{\omega}{\omega - V_s}\right)^2$$

(12)

Using Eq. (5), Eq. (12) can be rewritten as

$$\frac{\alpha\tau_s'}{\tau_s} = \left(\frac{\omega}{\omega - V_s}\right)^2$$

(13)

Eqs. (12) and (13) express the relationship between the modified Shields number $\tau_s'$ and the original Shields number $\tau_s$. They predict that the original Shields number may significantly deviate from the Shields curve if there exists a vertical velocity $V_s$.

Eq. (11) includes the influence of the vertical velocity. It demonstrates that an upward velocity reduces the apparent particle density, thereby reducing the critical shear stress, whereas a downward
velocity increases the apparent density, and the required critical shear stress is higher. If the cases with and without vertical velocity are compared, the critical shear stresses \( \tau_c \) and \( \tau'_c \) have the following relationship:

\[
\frac{\tau'_c}{\tau_c} = (1 - Y)^2 \tag{14}
\]

where

\[
Y = \frac{V_s}{\omega} = \beta \frac{V_s}{\omega} \tag{15}
\]

where the correction coefficient \( \beta \) is introduced to express the relationship between the measured \( V \) and the vertical flow in the sediment layer, \( V_s \), as shown in Fig. 1.

A comparison of Eq. (14) with experimental data by Cheng and Chiew (1999), Kavcar and Wright (2009), and Liu and Chiew (2012) is shown in Fig. 2. In Cheng and Chiew’s experiment, the uniform particle size \( d = 1.02 \text{ mm} \) was used, and the velocity \( V_s \) (injection) was measured. Kavcar and Wright (2009) conducted similar experiments with both injection and suction using sediment particles of \( d_{50} = 0.5 \text{ mm} \). Liu and Chiew (2012) observed the critical shear stress for sediment with a median diameter of 0.9 mm in the presence of downward seepage. Fig. 2 shows that the agreement between the measured and predicted critical shear stress is acceptable.

Differing from Eq. (14), Cheng and Chiew (1999) expressed their data using the following empirical method:

\[
\frac{\tau'_c}{\tau_c} = 1 - \left( \frac{V_s}{V_{sc}} \right)^m \tag{16}
\]

where they introduced a new parameter, \( V_{sc} \) for a quick state; and \( m = 1 \sim 2 \) depending on the characteristics of the sediments. The value of \( V_{sc} \) was determined by

\[
V_{sc} = K \left( \frac{\rho_s - \rho}{\rho_s} - 1 \right) (1 - \lambda) \tag{17}
\]

where \( K = \text{hydraulic conductivity}; \) and \( \lambda = \text{bed porosity} \).

By comparing Eqs. (14) and (16), one can see that the equations are functionally similar to each other, but Eq. (14) is simpler without empirical treatment. If \( m = 2 \), Eq. (16) states that upward and downward \( V_s \) have the same effect on \( \tau'_c \), thereby differing from the experimental data.

For steady and nonuniform flows, if there is no seepage from the underground water, then

\[
\frac{d}{dx} \int_0^h \tilde{u} dy = 0 \tag{22}
\]

Otherwise

\[
\frac{d}{dx} \int_0^h \tilde{u} dy = V_s \tag{23}
\]

In natural conditions, groundwater and river water are exchanged. Thus, \( V_s \) is always nonzero; that is, \( Y \neq 0 \), which causes a discrepancy when the Shields curve is compared with field data. Fig. 3 shows the critical shear stress observed by Afzalimhr et al. (2007), Sarker and Hossain (2006), Shvidchenko and Pender (2000), Gaucher et al. (2010), Emadzadeh et al. (2010), Everts (1973), Graf and Suszka (1987), White (1970), Neill (1967), and Carling (1983). The observed critical shear stress largely deviates from the Shields diagram, represented by the solid line \((Y = V_s/\omega = 0)\); the lines in Fig. 3 are results calculated using different values of \( Y \) in Eq. (14). A very small value of \( Y \) can significantly alter the critical shear stress. For example, the median sediment size in the downstream section of the Mississippi River is about 0.37 mm, and its settling velocity is about 4.52 cm/s. If the vertical velocity is 0.4 cm/s or 0.3% of the streamwise velocity, the observed critical shear stress will be 20% higher/lower than the Shields curve’s prediction.

Francalanci et al. (2008) also developed an empirical equation to express the critical shear stress under the influence of vertical velocity:

\[
\frac{\tau'_c}{\tau_c} = \rho_s - \rho (1 + V_s/K) \tag{18}
\]

Comparing Eqs. (14), (16), and (18), one can find that the conditions for \( \tau'_c = 0 \) are, respectively, \( V_s = \omega \), \( V_s = V_{sc} \), and

\[
V_s = K \left( \frac{\rho_s - \rho}{\rho_s} - 1 \right) \tag{19}
\]
theoretical models, because the mobility of particles increases with a channel's slope due to the added gravitational force in the downstream direction, and vice versa. Fig. 3 explains this paradox: if a channel slope is very gentle, then a decelerating flow is likely to occur, and the upward velocity promotes sediment mobility; if a channel slope is very steep, accelerating flows are most likely, and the downward velocity constrains particle mobility.

In Fig. 3, the parallel lines represent Shields curve with different \( Y \) values from Eq. (13), it can be seen that the data points below/above the original Shields curve \( (Y=0) \) can be explained by the nonzero \( Y \), which explains why the measured critical shear stress significantly deviates from Shields' prediction.

### Total Load of Sediment Transport Subject to \( V_s \)

As mentioned previously, sediment transport is a joint effect of streamwise and vertical motions. The near bed streamwise motion can be represented by the parameters \( u'_s \) and \( \tau \), and the vertical motion can be included in the apparent density \( \rho'_s \). Therefore, Eq. (4) can be modified into

\[
\bar{g}_t = \left( \frac{\rho'_s}{\rho'_s - \rho} \right) k u'_s \left( \frac{E - E_s}{\omega - V_s} \right) \tag{24}
\]

For sediment transport in a unidirectional flow, Eq. (24) becomes (Yang 2005)

\[
g_t = k \left( \frac{\rho'_s}{\rho'_s - \rho} \right) \frac{u'_s^2 - u'^s}{\omega} \tag{25}
\]

Eq. (25) may interpret the parameter \( T \) in Eq. (1), which was empirically discovered by van Rijn, but can be derived from Bagnold’s modified theorem.

Inserting Eq. (8) into Eq. (25), one has

\[
g_t(Y) = k \left[ \frac{\rho}{\rho'_s - \rho} \left( 1 - \frac{1}{1 - Y} \right)^2 + 1 \right] \tau \frac{u'^2 - u'^s}{\omega(1 - Y)} \tag{26}
\]

Eq. (26) includes the influence of streamwise and vertical parameters on sediment transport. It shows that for the same \( u'_s, \tau, \) and \( \omega, g_t \) increases with upward velocity \( V_s, (Y > 0) \) but decreases with downward \( V_s \).

To evaluate the influence of \( V_s \) on sediment transport rate, one can compare the sediment transport rate in two cases: with and without the vertical velocity, if other parameters, like sediment size, and streamwise flow conditions remain unchanged. In such cases, sediment discharge in uniform flows can be expressed from Eq. (26) as

\[
g_t(0) = k \frac{\rho}{\rho'_s - \rho} \frac{u'^2 - u'^s}{\omega} \tag{27}
\]

From Eqs. (26) and (27), one has

\[
\frac{g_t(Y)}{g_t(0)} = \frac{\rho}{\rho'_s(1 - Y)} + \frac{\rho'_s - \rho}{\rho(1 - Y)} \tag{28}
\]

A research group at Nanyang Technological University in Singapore carried out a series of experiments to measure the influence of vertical velocity on sediment transport (Cao et al. 2016). The tests in this study were conducted in a rectangular Perspex flume that was 4.8 m long, 0.25 m wide and 0.25 m deep, supported on a steel frame. Fig. 4 presents a schematic drawing of the flume. The sand bed, which was 1 m long and 0.25 m wide, was placed approximately 3 m downstream of the upstream end of the flume. The test section with injection was located in the middle of the sand bed, with a length = 0.1 m and width = 0.25 m. Holes were
drilled in an aluminum plate and placed beneath the sand bed to ensure uniform injection flow distribution. The sediment used in the study was uniformly distributed sand with a median grain size of \( d_{50} = 0.2 \) mm, and a uniformity coefficient \( d_{60}/d_{10} = 1.29 \). The porosity and coefficient of permeability of the sand, which were tested separately, were \( \lambda = 46.6\% \) and \( K = 5.625 \times 10^{-4} \) m/s, respectively. In addition, a liquefaction test was conducted to determine the critical velocity for piping to occur, \( V_{cr} (= 0.0324 \) cm/s).

A sand trap was installed 0.45 m downstream of the seepage zone to collect the sand transported during the experiment (see Fig. 4) and was used to directly measure the bed load transport rate by collecting the sand within a known time period. The trap was connected to a PVC tube with a valve attached to it. The valve was closed during the experiment. After each test, the valve was opened and the collected sand and water drained to a container. The sand was then dried and weighed to directly calculate the bed load transport rate.

Water was stored in a laboratory reservoir. Two different pumps were used to circulate flow in the flume and the injection, subsequently referred to as the main and injection pumps. The flow rates in the flume and injection areas were controlled with two separate valves and monitored using different flow meters. In order to avoid turbulence and ensure uniform flow distribution, small pipes with 20-mm diameter were installed at the entrance to the flume.

The following procedure was adopted in this study to measure the transport rate for a given injection rate:

1. Place the sand on the bed. Level the sand surface in the seepage region to the adjacent Perspex bed level.
2. Open the sand trap valve. Turn on the main pump to a very slow flow rate. Wait until no particles are moving toward the sand trap, then close the valve and let the water slowly fill up the entire flume. The very small amount of sand collected in the sand trap at this time was not included in the determination of the sediment transport rate.
3. Turn off the main pump and turn on the injection pump. Open the valve to the desired injection flow rate.
4. Open the main pump and set the flow rate to 6.775 L/s; this flow rate was used for all tests. When both pumps were opened and set at their respective flow rates, the actual test commenced.
5. Run the test for 30 min. During the test, keep monitoring the flow rates of both pumps to ensure that they remain constant. Videotape the bed load transport behavior in the flume. At the end of the 30-minute duration, turn off both pumps.
6. Collect the wet sand and put it into an oven at 120°C for drying. Weigh the dry sand to calculate the volumetric sediment transport rate.

The measured data (Cao et al. 2016) are presented in Fig. 5, which shows that sediment transport rate can be significantly increased by an upward velocity. If the upward velocity is 80% of the settling velocity \( (Y = 0.8) \), then the predicted sediment transport rate can be increased 50 times \( g_Y(0) \). This explains why scour holes are formed by an upward velocity. Fig. 5 also shows that sediment transport rate is slightly reduced if a downward flow exists. If the downward velocity is equal to the settling velocity, that is, \( Y = -1 \), the sediment transport rate will be reduced to \( 1/3 \) of \( g_Y(0) \), because the particles are “heavier” in this case.

Therefore, it can be inferred from Eq. (28) and Fig. 5 that upward flow dominates the sediment erosion process. In other words, local scour is always associated with upward or decelerating flows if the streamwise parameters are relatively unchanged. This is consistent with experimental results by Oldenziel and Brink (1974), Richardson et al. (1985), and Francalanci (2006). Their experimental results show that injection promotes sediment transport, while suction reduces the rate of sand transport.

![Fig. 5. Comparison of predicted and measured sediment discharge versus vertical motion; \( Y = 0 = \) no vertical velocity; \( Y < 0 = \) accelerating flow in which sediment discharge is reduced; and \( Y > 0 = \) decelerating flow in which sediment discharge is increased significantly.](image-url)
This is particularly supported by the “tearing bottom” phenomenon in the Yellow River in China (Chien and Wan 1998), in which large pieces of the riverbed, as much as several meters long, can be lifted to the free surface like a carpet. Observations show that this phenomenon always occurs when (1) hyperconcentrated flows are formed—that is, the settling velocity of the carpet, \( \omega \), is reduced as the density of hyperconcentrated fluid could be as high as \( 1,400 \text{ kg/m}^3 \) (Chien and Wan 1998); (2) the water level is falling—that is, a decelerating phase generates an upward velocity; and (3) the river flow is spatially decelerating from a narrow channel to a wider channel, which never occurs in channels with constant width or in a transitional reach from a wide channel to a narrow channel. Obviously, these facts imply that a higher and positive \( Y \) is responsible for this phenomenon. Similarly, specially designed artificial floods with high \( Y \) can flush more sediment to the sea, it is more important for the lower reach of Yellow River in order to save floodwater in the arid region.

It should be highlighted that an upward velocity or positive \( Y \) can in some cases trigger large vortices, which can in return lift houses, or ships into pieces; the sizes and fluids of phenomena.

488 Suspended Sediment Concentration Subject to \( V \)

The governing equation for suspended concentration can be derived from the mass continuity equation in the following form (Yang 2007):

\[
\frac{\partial(CV + C'V' - C\omega)}{\partial y} + \frac{\partial(CW + C'W')}{\partial z} = 0 \tag{29}
\]

where \( C' \) = fluctuation of sediment concentration; \( C = \) time-averaged sediment concentration by volume; \( V \) and \( W = \) vertical and lateral time-averaged velocities, respectively; and \( V' \) and \( W' \) are the velocity fluctuation in the \( y \)- and \( z \)-directions, respectively.

If the lateral gradient of Eq. (29) is negligible, the integration of Eq. (29) with respect to \( y \) yields the following equation:

\[
CV + C'V' - C\omega = 0 \tag{30}
\]

where the upper boundary condition at \( y = h \) is applied to determine the integration constant. The importance of \( V \) for suspended sediment was noticed by researchers like Hawksley (1951), Fu et al. (2005), and Steinour (1944). Their research shows that sediment presence induces an upward velocity \( V \), which, together with turbulence, balances sediment settlement.

The second term of Eq. (30) can be expressed by

\[
\varepsilon_s \frac{dC}{dy} = -\omega C(1 - Y) \tag{32}
\]

Rouse assumes that \( \varepsilon_s \) is proportional to the turbulent eddy viscosity, that is

\[
\varepsilon_s = k\nu_s \left( \frac{1 - y}{h} \right) \tag{33}
\]

By inserting Eq. (33) into Eq. (32), one obtains the modified Rouse’s law

\[
\frac{C}{C_0} = \left( \frac{\xi - 1}{\xi} \right)^{Y/2}\tag{34}
\]

\[
Z(Y) = \frac{\omega(1 - Y)}{k\nu_s} \tag{35}
\]

\[
Z(Y) = \frac{1 - Y}{Z(0)} \tag{36}
\]

where \( \xi = \) the reference level at which the sediment concentration is \( C_0 \). Van Rijn (1984) claimed that the measured \( Z(0) \) is different from \( Z(0) = \omega/k\nu_s \) and depends on the flow strength or \( \omega/\nu_s \), but Van de Graaff (1988) concluded that the variation in \( Z \) is not totally clear. Eq. (36) states that the discrepancy in \( Z \) is caused by \( Y \), which was missed in the original Rouse’s number. Eq. (36) indicates that in decelerating flows, the vertical distribution of sediment concentration is more uniform relative to that predicted by the classical Rouse’s law, and the measured Rouse’s number is smaller than the calculated Rouse’s number. An upflow (or \( V > 0 \)) results in the decrease of concentration gradient \( dC/dy \). If \( Y = 1 \), then \( Z(Y) = 0 \).

Nezu and Azuma (2004) observed that the ratio of measured \( Z(Y) \) to calculated \( Z(0) \) was significantly larger than the expected value if the flow experienced downflow. They suggested that Rouse’s diffusion theory cannot be used in practice. This is consistent with Eq. (36), which shows that if \( Y < 0 \), the measured \( Z(Y) \) should be higher than the predicted \( Z(0) \).

Kinoshita (1967) analyzed aerial stereoscopic photos of rivers in floods and discovered a fascinating light-and-dark strip pattern on the water surface. In the light zones, the flows were 10%-20% faster than in the dark zones, and the latter had high concentrations of sediment. Kinoshita deduced that downward (\( V < 0 \)) and upward (\( V > 0 \)) flows existed in these zones. Eq. (36) supports his inference, because the upflow zone functions like a sediment source in which sediment particles become “lighter,” and the downflow zone works like a sink in which particles become “heavier.”

Nezu and Nakagawa (1993) summarized the difference as shown in Table 1, which clearly states that vertical velocity has a significant impact on suspended sediment. Their observation is consistent with Eq. (36). Therefore, it is understandable that experienced river engineers can infer the presence of vertical velocity from the color of the water surface. For example, Vanoni (1946) stated that the periodically spanwise variation of sediment concentration on the surface is the result of interacting secondary currents and sediment transport.
| Table 1. Relationship between vertical velocity and sediment transport |
|---|---|---|
| Type | Upflow region (v > 0) | Downflow region (v < 0) | Region (v = 0) |
| T1:1 | Primary mean velocity | Low (log-wake law) | High (dip-lg-law) | Log law |
| T1:2 | Bed roughness | Low (<5ghS) | High (>5ghS) | Exponential decay law |
| T1:3 | Reynolds shear stress farther from the bed | Higher | Lower | Log law |
| T1:4 | Reynolds shear stress closer to the bed | Higher | Lower | Rouse’s law |
| T1:5 | Suspended load | Higher | Lower | Random small variations |
| T1:6 | Riverbed form | Boil lines, divergence | Foam lines, convergence | Ripples/dunes |
| T1:7 | Bed load | Lower | Higher | 2.5dso |
| T1:8 | Turbulence intensity farther from the bed | Lower | Higher | Rouse’s law |
| T1:9 | Vertical velocity | Lower | Higher | Rouse’s law |

Note: “Higher” and “lower” are in comparison to the “Region” column.
*Observed by Nezu and Nakagawa (1993).

The significance of Eq. (36) is that the net falling velocity of particles in rivers and coastal waters is not a constant but rather is variable. More sediment particles will “float,” that is, higher concentrations appear, in upflows or accelerating flows. Similarly, more particles are deposited on the bed in downflows or accelerating flows; thus, one can infer that the occurrence of maximum sediment concentration always lags behind the appearance of peak flow. This inference could be extended to estuaries and coastal waters in which the highest sediment concentration always occurs when streamwise velocity u = 0, because the highest velocity (shear stress) corresponds with lower sediment concentration. All these phenomena indicate that sediment transport cannot be fully expressed by streamwise parameters alone, and vertical motion should be included.

**Discussion**

This paper considers why the extension of existing sediment transport formulas to nonuniform and unsteady flows is invalid. In addition, the investigation shows that vertical motion is responsible for the invalidity. In the classical theory of sediment transport, the lift force is included as the vertical motion, but it is induced by streamwise motion, and the vertical force is always upward. It becomes zero when the streamwise velocity u = 0. By contrast, this study highlights that vertical motion can be upward or downward and can be independent of the streamwise motion. Hence, sediment transport can be modeled in a more realistic way, especially when sediment particles are transported by waves or are subject to groundwater seepage.

After the introduction of vertical velocity into existing sediment transport theory, many odd phenomena become understandable, as discussed previously. For example, the formation of ripples/dunes can be attributed to instantaneous coherent structures. Its upward velocity or ejection causes severe local scour, but scour holes with the same scour depth cannot be formed by downward velocity during its sweeping period in the bursting phenomenon. In other words, the effect of sediment transport by upward or downward velocity as shown in Fig. 5 is responsible for the bedform formation. It is reasonable to assume that for large particles Y ≈ 0, vertical velocity Vz has little effect as compared to the effect on small particles. This is why ripples can be observed only when a bed is made of fine particles like clay or sand, not of larger particles.

This also is a theoretical basis for riprap protection against local scour using large stones, that is, Y ≈ 0 in torrent flows.

As local scour holes are always related to upward velocity, it can be inferred that a riverbed on the concave side of a curved channel will be very smooth, but scour holes will be found in its convex side due to the action of secondary currents.

This study provides a theoretical framework for sediment transport in unsteady and nonuniform flows and extends existing formulas to complex flows, because the continuity equation, that is, Eq. (9), is universal and can be applied in unsteady flows.

\[ \rho \frac{\partial u}{\partial t} + \rho u \cdot \nabla u = -\nabla p + \rho g z + f_x + f_y + f_z \]

\[ \frac{\partial h}{\partial t} + u \frac{\partial h}{\partial x} + v \frac{\partial h}{\partial y} = 0 \]
For example, by introducing a potential function, one can get the Laplace equation from Eq. (9), and the linear wave theory is obtained from it by using boundary conditions.

It should be highlighted that in Eq. (4), the near bed velocity \( u_i' \) is very difficult to determine in combined wave/current conditions in which Eq. (3) may be invalid. It is suggested that for numerical models, the calculated velocity in the first grid from the boundary be used for \( u_i' \). If this is done, the constant \( k \) in Eq. (4) must be recalibrated and its value depends on the grid size.

Another difficulty in applying this theory is the determination \( V_s \) or \( Y \) in Eqs. (14), (28), and (36). Generally, this \( V_s \) is very small and very hard to measure directly using any existing equipment. Currently empiricism is needed for its determination.

In order to simplify the mathematical treatment, this study assumes that the presence of vertical motion does not alter the streamwise motion significantly, that is, \( V_s \ll U \), and \( u_i' \) and \( \tau \) are not influenced by \( V_s \). This is acceptable if \( V_s \) is very small. If \( V_s \) is very high, these parameters may be influenced by \( V_s \), and the data scatter in Fig. 5 can be improved if this influence is included. In the future, more detailed investigation is needed to clarify the influence of \( V_s \) on \( u_i' \) and \( \tau \).

### Conclusions

This study investigates why the current theory of sediment transport cannot adequately express the initiation, entrainment, transport, and suspension of particles. Among many possible reasons, this study ascribes the invalidity of existing formulas for sediment transport to the missing parameter of vertical motion, which, together with the streamwise parameters, determines sediment transport. With the inclusion of vertical velocity, many phenomena observed in experiments and in the field are well explained. Vertical velocity is ubiquitous in natural and laboratory flows. It can be induced by coherent structures, secondary currents, unsteadiness, nonuniformity, and so on. The role of vertical motion should not be underestimated. Based on this investigation, the present study reaches the following conclusions:

1. **Upward velocity enhances sediment mobility, and downward velocity increases its stability.** Mathematically, the effect of vertical motion on sediment transport can be expressed by the proposed apparent density, which becomes “heavier” when particles experience downward flows. This reduces the sediment transport rate. But particles become “lighter” in “boiling” flows with upward velocity in which sediment discharge is increased significantly.

2. **The critical shear stress for incipient motion of sediment transport is also affected by the vertical velocity.** Upward velocity reduces the critical shear stress, but downward velocity increases the shear stress. After the introduction of apparent density of sediment, the Shields curve can be extended to express the critical shear stress of sediment in unsteady and nonuniform flows.

3. **Upward velocity lightens solid particles, decreasing the gradient of sediment concentration; downward velocity increases particle settling velocity and concentration gradient.** Consequently, vertical velocity causes deviation from the distribution of sediment concentration from Rouse’s law and the Rouse’s number is different from the widely accepted value of \( \omega/\kappa u' \).

4. **Vertical velocity can be induced by a channel’s geometry, nonuniformity, unsteadiness, bursting, groundwater seepage, density stratification, surface waves, and so forth.** Generally, accelerating flows produce downward velocity, and decelerating flows generate upward velocity. The influence of vertical velocity on turbulence structures and sediment transport should not be underestimated.

5. **Upward velocity causes more erosion or local scour.** In contrast to conventional theory, decelerating flows are responsible for bedform formation. An effective way to protect a bed against local scour is to keep \( Y = V_s/\omega \) as small as possible, that is, to increase \( \omega \) (larger and heavier particles) and reduce \( V_s \). This study also ascribes bedform formation to the upward velocity at the sediment transport.

### Notation

The following symbols are used in this paper:

- \( h \) = water depth;
- \( Q \) = discharge;
- \( S \) = energy slope;
- \( U \) = mean velocity;
- \( V \) = vertical velocity;
- \( u \) = time-averaged velocity in the streamwise direction;
- \( u' \) = shear velocity;
- \( \upsilon' \) and \( \mathbf{w}' \) = turbulent velocity fluctuation in the streamwise and wall-normal directions, respectively;
- \( \mathbf{v} \) = Reynolds shear stress;
- \( \mathbf{v}_s = V s/\omega \);
- \( y = \) distance normal to the wall;
- \( Z = \) Rouse’s number;
- \( \alpha = \) factor;
- \( \beta = \) coefficient;
- \( \varepsilon_s = \) sediment diffusion coefficient;
- \( \nu = \) kinematic viscosity;
- \( \xi = y/h \);
- \( \xi_o = \) reference level at which the sediment concentration is \( \text{Ca} \);
- \( \rho = \) fluid density;
- \( \tau = \) Shields number;
- \( \tau = \) boundary shear stress; and
- \( \omega = \) particle fall velocity.

### References


Queries

1. Please provide the ASCE Membership Grades for the authors who are members.

2. Please clarify the intended meaning of “downward motion increases its stability.” In particular, please clarify what “its” refers to.

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4. Please check the hierarchy of section heading levels.

5. Please check the editing of the sentence beginning "Parameters that reflect vertical motion, such as seepage velocity." Is this still your meaning?

6. Please check the editing of the sentence beginning "Like the product of $U$ and $\tau$, the product of $U$ and $S$. " Is this still your meaning?

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18. Please clarify. By “vice versa for an upward velocity by decelerating flow in upper layer” do you mean “a decelerating flow in the upper layer could generate an upward velocity in the sediment layer”?

19. Please check the editing of the sentence beginning "From Eq. (8) and (10), it can be inferred that sediment is more easily transported." Is this still your meaning?

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29. Please check the editing of the sentence beginning "All of these phenomena are initially formed by warm. " Is this still your meaning?

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35. Please check the editing of the sentence beginning "It is reasonable to assume that for large particles $Y \approx 0$, vertical velocity. " Is this still your meaning?

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37. Please check the editing of the sentence beginning "This study investigates why the current theory of sediment transport. " Is this still your meaning?

38. Please check the editing of the sentence beginning “The influence of vertical velocity on turbulence structures and sediment transport. " Is this still your meaning?

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