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Abstract

Rock joints exert an enormous influence on the permeability of a rock mass because they act as interconnecting networks that provide pathways for fluids to permeate and flow within the rock structure. The apertures in rock joints are irregular in nature and induce flows that cannot be described by the parallel-plate theory based on planar joints or the classical cubic flow relationships. In this study, a two-dimensional (2D) hydraulic aperture distribution was considered to develop a mathematical model for fracture flow. In this approach, the three-dimensional Navier-Stokes equation was integrated over the joint aperture and converted to an equivalent 2D flow model. The proposed model was then solved numerically by adopting a well-known algorithm for coupling the pressure and velocity and implementing it in a computer program. The selected program is capable of predicting the deformation of the joint apertures on normal loading, the resulting flow patterns, and the volumetric flow rates associated with permeability tests conducted using a high-pressure triaxial apparatus that was designed and built at the University of Wollongong. The model output for different conditions of confining stresses and hydraulic gradients was computed, and a good agreement with the experimental results was observed.

Disciplines

Engineering | Science and Technology Studies

Publication Details

Indraratna, B., Kumara, C., Zhu, S. & Sloan, S. (2014). Mathematical modeling and experimental verification of fluid flow through deformable rough rock joints. *International Journal of Geomechanics*, 15 (4), 04014065-1-04014065-11.

Mathematical Modelling and Experimental Verification of Fluid Flow through Deformable Rough Rock Joints

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Abstract

Rock joints exert an enormous influence on the permeability of a rock mass because they act as interconnecting networks that provide pathways for fluids to permeate and flow within its structure. The apertures in rock joints are irregular in nature and induce flows that cannot be described by the parallel plate theory based on planar joints or the classical cubic flow relationships. In this study, a two-dimensional hydraulic aperture distribution was considered to develop a mathematical model for fracture flow. In this approach, the three-dimensional Navier-Stokes equation was integrated over the joint aperture and converted to an equivalent two-dimensional flow model. The proposed model was then solved numerically by adopting the SIMPLE (Semi Implicit Method for Pressure Linked Equations) algorithm for coupling the pressure and velocity and implementing it in MATLAB as a computer program. This program is capable of predicting the deformation of the joint apertures upon normal loading, the resulting flow patterns, and the volumetric flow rates associated with permeability tests conducted using the High Pressure Triaxial Apparatus which has been designed and built at the University of Wollongong. The model output for different

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20 conditions of confining stresses and hydraulic gradients was computed, and a good agreement
21 with the experimental results was observed.

22 **Subject Headings:** Rock mechanics, Fluid flow, Mathematical models, Two-dimensional
23 models, Flow simulation

24 **Author Keywords:** Fracture permeability, Fluid flow, Aperture distribution, Triaxial testing

Introduction

The permeability of jointed rock mass plays a major role in many industrial activities such as underground mining, petroleum extraction, ground water extraction, geothermal reservoirs, and underground nuclear repositories. The flow through a real rock fracture is a complex phenomenon due to the irregularity and alteration of its flow domain. The fracture aperture is spatially irregular and, according to the surrounding stress environment, its distribution can be changed. Investigations into the permeability of rock fractures began with the simplified assumption that two parallel plates would represent a planar joint (Lomize 1951; Baker 1955). This led to the development of the cubic formula shown in Eqn. (1) which aims to predict the volumetric fluid flow by coupling the hydraulic and mechanical characteristics using Darcy's law (Snow 1968; Witherspoon *et al.* 1980)

$$Q = \frac{e^3 w}{12\mu} \frac{dP}{dx} \quad (1)$$

where, Q is the flow rate, e is the fracture aperture, w is the fracture width, μ is the dynamic viscosity, and dP/dx is the total pressure gradient. When the flow through fractures is assumed to be Darcian, the intrinsic permeability coefficient of the rock fracture can be taken as $e^2/12$ (Snow 1968). Subsequently, researchers such as Iwai (1976) and Gangi (1978) modified this coefficient of fracture permeability to account for the fracture irregularity and contacts, as well as the pressure distribution. For practical applications, flow through fractures has to be considered in rock fracture networks and the cubic formula, due to its simplicity, is often used for this purpose. When the rock matrix porosity is high so that its permeability is significant, dual porosity models have been developed for permeability calculations of fractured rock strata (Choi *et al.* 1997; Sarkar *et al.* 2004; Crandal *et al.* 2010; Kubeyev 2013). Wang *et al.* (2002) suggested a dual fracture model for calculating the permeability of a fracture network to overcome the complexity of joint sets.

In the parallel plate model the advection and diffusion terms are dropped in favour of one dimensional steady flow, where the pressure gradient is assumed to act in the flow direction only. However, in most cases, the flow does not follow the conventional form of Darcy's law and the reasons for this have been investigated extensively (Witherspoon *et al.* 1980; Zimmerman and Bodvarsson 1996; Zimmerman *et al.* 2004). Barton *et al.* (1985) suggested using an empirical relationship to calculate the hydraulic aperture with the Joint Roughness Coefficient (JRC) of the rock surface, while Neuzil and Tracy (1981) suggested a fractal model to account for the aperture irregularity. Zimmerman *et al.* (1992) demonstrated numerically the permeability variations due to the presence of contacts in different shapes and quantities, and quantified the effective permeability of a fracture with contact areas. Ranjith and Viete (2011) discussed the applicability of the cubic law for non-Darcian flows and suggested the cubic law is still applicable for flows with a Forcheimer number less than 1.3. Singh *et al.* (2014) used an analogue material to simulate the roughness of the fractures subjected to water flow.

Two-dimensional flow models

The effect of an irregular aperture distribution can be studied via two-dimensional (2D) flow models (Fig. 1). First, the flow domain can be viewed from a side parallel to the flow direction (Koyama 2007; Zimmerman and Yeo 2013). Alternatively the flow modelling can be based on the plan view of the fracture (Bear 1993; Kishida *et al.* 2013). In the former, the aperture height is in the direction of the vertical dimension and, therefore, the aperture variation can be incorporated directly into the flow model. However, these models account for the variation of the aperture in one plane only, which is parallel to the flow direction. The variation of aperture lateral to the flow direction is not discussed, and at the same time the lateral velocity component perpendicular to the main flow direction is not included in the flow equations, while the vertical velocity component is included. The latter approach can be

used to define the spatial distribution of flow in a 2D fracture. The velocity in the direction perpendicular to the plane of the aperture is assumed to be negligible, compared to the flow that occurs in the lateral and longitudinal directions of the fracture.

Amadei & Illangasekare (1992) derived a 2D analytical model by assuming one directional flow for steady or transient flows. Here the aperture was not one separate value and the user was able to measure the apertures in preferred intervals and then calculate the associated flow behaviour. Indraratna *et al.* (2002) and Price and Indraratna (2005) suggested the use of Fourier analysis to describe the irregularity of the fracture aperture. Fourier coefficients were calculated using the scanned joint surfaces, and the variations of the fracture aperture were obtained as Fourier functions. Zimmerman and Yeo (2013) used a 2D rock fracture viewed from a side of the fracture and proved that the Navier-Stokes equation can be linearized into the Stokes equation, by neglecting the advection acceleration terms compared to the viscous terms, provided the Reynolds number of the flow is less than about 10. Koyama (2007) modelled flow and particle transport in rock fractures during shear by looking at the fracture from one side using the Reynolds equation. Here the Navier-Stokes equations were solved using commercial software and the simulations were compared with laboratory shear-flow-tracer tests to demonstrate that the Reynolds equation overpredicts the flow rate by roughly 5-10% compared to the Navier-Stokes analysis. Zimmerman *et al.* (2004) conducted laboratory measurements and Navier-Stokes simulations on fracture flow to study the non-linear regimes of fracture flow. They discovered that there is a weak inertia regime within the Reynolds number range 1-10, and the influence is significantly less for the Forchheimer type regime beyond a Reynolds number of 20.

In contrast to 2D models where the fracture is viewed from a side, Bear (1993) modelled the flow in a non-deforming fracture through the plan view by integrating the three-dimensional Navier-Stokes equation in the direction of the aperture height. Kishida *et al.*

(2013) followed the same method and developed a 2D model for non-deforming rock fracture walls, and this model was solved numerically using the highly simplified marker and cell (HSMAC) method.

The main objective of the current study is the prediction of flow rates with changes in the normal stress, and to demonstrate the influence of aperture distribution on the fracture flow. A two-dimensional flow model is proposed which considers the plan view of a rock fracture, and a normal deformation model is coupled with the flow model to predict the flow rates changes with joint normal stress changes.

Mathematical Model Development

A single rock fracture is shown in Fig. 2, where F_u and F_l are the upper and lower joint surface profiles respectively. Since $[X', Y', Z']$ is the global coordinate system, a discontinuity oriented coordinate system $[x, y, z]$ is defined so that the plane (x, y) contains all the mid points of the fracture aperture. The three-dimensional linear momentum conservation of an incompressible fluid flow through a fracture is given by the Navier – Stokes equation below:

$$\frac{\partial(\rho \bar{V})}{\partial t} + \nabla \bullet (\rho \bar{V} \bar{V}) + \nabla p - \rho g \nabla Z' - \nabla \bullet \bar{T} = 0 \quad (2)$$

where $\bar{V}$ is the velocity vector, ρ is the fluid density, p is the fluid pressure, g is the acceleration due to gravity, $\bar{T}$ is the stress tensor given by $\bar{T} = \mu[\nabla \bar{V} + (\nabla \bar{V})^T - 2/3(\nabla \bullet \bar{V})\bar{I}]$, $\bar{I}$ is the unit tensor, and μ is the dynamic viscosity of the fluid. Assuming the fluid is a homogeneous and Newtonian incompressible liquid, Eqn. (2) can be expressed as follows (Bear 1993)

$$\frac{\partial(\rho \bar{V})}{\partial t} + \nabla \bullet (\rho \bar{V} \bar{V}) + \nabla p - \rho g \nabla Z' - \mu \nabla^2 \bar{V} = 0 \quad (3)$$

Bear (1993) modelled water flow through a non-deformable clean rock fracture by integrating Eqn. (3) in the direction of the height of the aperture, or the z direction in Fig. 2, to obtain a 2D flow model where the plan view of the fracture is considered. In order to model water flow through deformable rock fractures, the limits of the integration can be considered to be functions of time, thus yielding a two dimensional depth-averaged momentum conservation equation in the (x,y) space as:

$$\int_{F_l}^{F_u} \left(\frac{\partial(\rho \bar{V})}{\partial t} + \nabla \bullet (\rho \bar{V} \bar{V}) + \nabla p - \rho g \nabla Z' - \mu \nabla^2 \bar{V} \right) dz = 0 \quad (4)$$

Integration of this expression by parts using the Leibnitz integral theorem (Appendix A), leads to:

$$\frac{\partial}{\partial t} (\rho e \bar{V}) = -\dot{\nabla} \bullet (e \rho \bar{V} \bar{V}) + \mu \dot{\nabla}^2 (e \bar{V}) - \frac{12\mu}{e} \bar{V} + \rho g e \dot{\nabla} Z' - e \dot{\nabla} p \quad (5)$$

where

$$\bar{V} = \frac{1}{e} \int_{F_l}^{F_u} \bar{V} dz \quad (6)$$

In the above $\bar{V}$ is the averaged velocity vector in the (x, y) plane, and $\dot{\nabla}$ denotes the corresponding divergence operator. Note that a parabolic distribution of velocities in the vertical direction was assumed in the integration process. The continuity equation for flow is given by Eqn. (7) as follows:

$$\frac{\partial \rho}{\partial t} + \nabla \bullet (\rho \bar{V}) = 0 \quad (7)$$

Integrating Eqn. (7) in the same manner as before gives the depth-averaged continuity equation for an incompressible fluid in two dimensions as:

$$\dot{\nabla} \cdot (\rho e \vec{V}) = 0 \quad (8)$$

140 *Deformation Criteria*

141 The fracture aperture deformation D_n is controlled by the mechanical deformation of
 142 the joint. According to Kulhawy (1975) and Bandis *et al.* (1983), the deformation of the joint
 143 and the effective normal stress have a hyperbolic relationship which is given by Eqn. (9):

$$144 \quad D_{n(t)} = \frac{\sigma'_n D_{nm}}{\sigma'_n + K_i D_{nm}} \quad (9)$$

145 where, σ'_n is the effective normal stress, D_{nm} is the maximum deformation of the fracture
 146 (which is always less than the fracture aperture), K_i is the initial joint normal stiffness when
 147 the applied effective stress is zero, and $D_{n(t)}$ is the total deformation for a particular applied
 148 normal stress. The deformation measured upon initial loading during the test is used to
 149 calculate the initial joint normal stiffness by re-arranging Eqn. (9) into Eqn. (10). A
 150 correction is applied for the deformation of the membrane that added to the fracture aperture
 151 deformation readings (Zhang 2013). Coupling the deformation criteria to the flow model is
 152 described in the next section.

$$153 \quad \frac{1}{D_{n(t)}} = K_i \frac{1}{\sigma'_n} + \frac{1}{D_{nm}} \quad (10)$$

154 For steady state flows, the transient term in Eqn. (5) is removed. Note that Eqn. (5)
 155 cannot be solved analytically for transient or steady state flows because the pressure is linked
 156 with velocity and there is no separate equation to calculate the pressure. Therefore, a
 157 numerical solution method is required to solve Eqns. (5) and (8) , and it can be obtained as
 158 described below.

159

Numerical Solution

The scalar form of Eqn. (5) for steady flow is shown in Eqns. (11) and (12). For relatively low flow velocities through rock fractures, the advection acceleration terms are small compared to the viscous terms (Koyama 2007; Zimmerman and Yeo 2013; Kishida *et al.* 2013). Therefore they are neglected for simplicity and the equations are discretised using the Finite Volume Method (Versteeg and Malalasekara 2007) and solved by the SIMPLE algorithm (Patanker and Spalding 1972). Scalar velocity components (u , v) are depth averaged values and the superscript ‘dot’ is not used as in the vector equation for simplicity.

$$\mu \frac{\partial^2}{\partial x^2}(eu) + \mu \frac{\partial^2}{\partial y^2}(eu) - \frac{12\mu u}{e} + \rho g e \frac{\partial Z'}{\partial x} - e \frac{\partial p}{\partial x} = 0 \quad (11)$$

$$\mu \frac{\partial^2}{\partial x^2}(ev) + \mu \frac{\partial^2}{\partial y^2}(ev) - \frac{12\mu v}{e} + \rho g e \frac{\partial Z'}{\partial y} - e \frac{\partial p}{\partial y} = 0 \quad (12)$$

The flow domain is discretised as a structured mesh that is shown in Fig. 2 as continuous lines. The x -axis is numbered in capital ‘ I ’s and the y -axis in capital ‘ J ’s. The grid nodes are the centres of the control volumes, and the scalar variables of the cell, such as pressure, hydraulic aperture and density, are kept at the cell centre. The two velocity components ‘ u ’ and ‘ v ’ are kept at the faces of the cell, as shown. A staggered grid was created in order to have the grid lines on the centres of the scalar cell faces, which are numbered using ‘ i ’s on the x -axis and ‘ j ’s in the y -axis. The ‘ u ’ and ‘ v ’ velocities of a particular cell, say (I, J) are denoted as $u_{i,j}$ and $v_{I,j}$ respectively. A control volume (cell) for flow calculations and a cell for the ‘ u ’ velocity and a cell for the ‘ v ’ velocity are shown in Fig. 2.

Eqns. (11) and (12) can be integrated over a control volume for u and v respectively, and the discretised equations, adopting the central difference scheme, are as follows:

$$\begin{aligned}
& e_{i,j} \left[-\frac{2\mu}{\Delta x^2} - \frac{2\mu}{\Delta y^2} - \frac{12\mu}{e_{i,j}^2} \right] u_{i,j}^\# + \left[\frac{e_{i-1,j}\mu}{\Delta x^2} \right] u_{i-1,j}^\# + \left[\frac{e_{i+1,j}\mu}{\Delta x^2} \right] u_{i+1,j}^\# \\
& + \left[\frac{e_{i,j-1}\mu}{\Delta y^2} \right] u_{i,j-1}^\# + \left[\frac{e_{i,j+1}\mu}{\Delta y^2} \right] u_{i,j+1}^\# = \varphi_{i,j}
\end{aligned} \tag{13}$$

$$\begin{aligned}
& e_{l,j} \left[-\frac{2\mu}{\Delta x^2} - \frac{2\mu}{\Delta y^2} - \frac{12\mu}{e_{l,j}^2} \right] v_{l,j}^\# + \left[\frac{e_{l-1,j}\mu}{\Delta x^2} \right] v_{l-1,j}^\# + \left[\frac{e_{l+1,j}\mu}{\Delta x^2} \right] v_{l+1,j}^\# \\
& + \left[\frac{e_{l,j-1}\mu}{\Delta y^2} \right] v_{l,j-1}^\# + \left[\frac{e_{l,j+1}\mu}{\Delta y^2} \right] v_{l,j+1}^\# = \varphi_{l,j}
\end{aligned} \tag{14}$$

where

$$\varphi_{i,j} = -(\rho g e)_{i,j} \frac{\Delta Z'}{\Delta x} + \left(e_{i,j} \frac{p_{l+1,j}^\# - p_{l-1,j}^\#}{\Delta x} \right) \text{ and } \varphi_{l,j} = -(\rho g e)_{l,j} \frac{\Delta Z'}{\Delta y} + \left(e_{l,j} \frac{p_{l,j+1}^\# - p_{l,j-1}^\#}{\Delta y} \right)$$

The relations for the gravity and pressure were solved explicitly, together with the pressure, and the quantities $\varphi_{i,j}$, $\varphi_{l,j}$ contain those terms. The terms with the superscript ‘#’ are the solutions of the equations for intermediate velocities and need to be corrected to satisfy the continuity of flow. In the SIMPLE algorithm, these intermediate velocities were treated as the difference between the final velocity and the velocity corrections. The intermediate pressure was also treated as the difference between the final pressure and the pressure correction. Thereafter, a relationship for the ‘ u ’ component velocity corrections (u') and pressure correction (p') was obtained (Appendix B) as shown in Eqn. (15). A similar equation can be obtained for the velocity correction (v') following the same procedure.

$$\begin{aligned}
& e_{i,j} \left[-\frac{2\mu}{\Delta x^2} - \frac{2\mu}{\Delta y^2} - \frac{12\mu}{e_{i,j}^2} \right] u'_{i,j} + \left[\frac{e_{i-1,j}\mu}{\Delta x^2} \right] u'_{i-1,j} + \left[\frac{e_{i+1,j}\mu}{\Delta x^2} \right] u'_{i+1,j} \\
& + \left[\frac{e_{i,j-1}\mu}{\Delta y^2} \right] u'_{i,j-1} + \left[\frac{e_{i,j+1}\mu}{\Delta y^2} \right] u'_{i,j+1} = e_{i,j} \frac{p'_{l+1,j} - p'_{l-1,j}}{\Delta x}
\end{aligned} \tag{15}$$

The general practice of SIMPLE is to omit the neighbour cell velocity corrections in the iteration process. Since these corrections ultimately become negligible, this omission does not affect the solution itself but does influence the speed of convergence. After the omission of neighbour cell velocity corrections, the Eqn. (15) becomes Eqn. (16) and the Eqn. (17) is obtained following the same procedure for the velocity component ‘v’.

$$u'_{i,j} = \frac{1}{\left[-\frac{2\mu}{\Delta x^2} - \frac{2\mu}{\Delta y^2} - \frac{12\mu}{e_{i,j}^2} \right]} \left(\frac{p'_{I+1,j} - p'_{I-1,j}}{\Delta x} \right) \quad (16)$$

$$v'_{i,j} = \frac{1}{\left[-\frac{2\mu}{\Delta x^2} - \frac{2\mu}{\Delta y^2} - \frac{12\mu}{e_{i,j}^2} \right]} \left(\frac{p'_{I,j+1} - p'_{I,j-1}}{\Delta y} \right) \quad (17)$$

In Eqns. (16) and (17), the terms with a dash denote the corrections. Since the intermediate velocities and their corresponding corrections are known, the final velocities can be obtained. If the corrected velocities are substituted into the discretised continuity Eqn. (8), a pressure equation (Poisson type) is obtained (Appendix B), as shown in Eqn. (18).

$$M_{I,j} p'_{I,j} + M_{i,j} p'_{I+1,j} + M_{i-1,j} p'_{I-1,j} + M_{I,j} p'_{I,j+1} + M_{I,j-1} p'_{I,j-1} = \frac{(\rho e u^\#)_{i,j}}{\Delta x} - \frac{(\rho e u^\#)_{i-1,j}}{\Delta x} + \frac{(\rho e v^\#)_{I,j}}{\Delta y} - \frac{(\rho e v^\#)_{I,j-1}}{\Delta y} \quad (18)$$

$$\text{where, } M_{I,j} = -M_{i,j} - M_{i-1,j} - M_{I,j} - M_{I,j-1}, \quad M_{i,j} = -\frac{\rho e_{i,j}}{H_{i,j} \Delta x^2}, \quad M_{i-1,j} = -\frac{\rho e_{i-1,j}}{H_{i-1,j} \Delta x^2},$$

$$M_{I,j} = -\frac{\rho e_{I,j}}{H_{I,j} \Delta y^2}, \quad M_{I,j-1} = -\frac{\rho e_{I,j-1}}{H_{I,j-1} \Delta y^2}, \quad \text{and } H_{i,j} = -\frac{2\mu}{\Delta x^2} - \frac{2\mu}{\Delta y^2} - \frac{12\mu}{e_{i,j}^2}.$$

The intermediate velocities obtained by solving Eqns. (13) and (14) can be substituted in Eqn. (18) to find the pressure corrections, which are then inserted into Eqns. (16) and (17) to

determine the velocity corrections. If the corrections are significant, the new pressure is taken as the intermediate pressure, and the same procedure is repeated until all the corrections become negligible. The model contains the fracture aperture as a variable in 2D space and time which differentiates the model from the usual Navier-Stokes equation. The complete SIMPLE algorithm is shown in Fig. 4.

When compressed, the lower apertures become tight contacts and the flow takes place around the contacts. According to the hyperbolic relationship, an aperture is fully closed when the applied stress is at infinity. However, compared to larger apertures in the domain, the small apertures reach closure earlier. Eqn. (9) was used to calculate the normal deformation of a fracture considering a maximum closure for the domain. Bandis *et al.* (1983) suggested empirical relations to obtain the maximum closure using the usual parameters such as the joint roughness coefficient (JRC), the joint compressive strength (JCS) and some material constants. In this study, the apertures of the domain are treated individually, and therefore, each aperture is given a unique maximum closure. Then Eqn. (9) can be given in the discretized format as follows:

$$D_{n(t)[I,J]} = \frac{\sigma'_{n[I,J]} D_{nm[I,J]}}{\sigma'_{n[I,J]} + K_i D_{nm[I,J]}} \quad (19)$$

The above approach allows the initially smaller apertures to reach closure upon the application of stress while the larger apertures still conduct flow. The initial aperture at zero confining stress was taken as the maximum closure and the initial joint normal stiffness was taken as a constant for all apertures. The coupling of deformation into the flow calculation is shown in Fig. 4.

235 ***Boundary Conditions***

236 Constant pressure boundary conditions were applied at the inlet and outlet, and the
237 inlet pressure was the applied back pressure and the outlet pressure was maintained at zero.
238 No slip velocity boundaries were assumed at the fracture walls in the model derivation and,
239 therefore, the velocities at the wall boundaries are zero. The longitudinal wall boundaries, or
240 the membrane, were assumed to be walls with zero velocity and a zero pressure gradient
241 normal to the wall. The inlet and outlet boundary velocities were calculated along with the
242 domain velocities.

243 **MATLAB Computational Program**

244 Although the model proposed is two-dimensional, standard 2D Navier-Stokes solvers
245 cannot be used to furnish the solution. Thus, it was necessary to implement a new solution
246 scheme using MATLAB. The resulting program consists of graphical user interfaces (GUI)
247 for the user inputs, and solution outputs that are connected to MATLAB functions. The user
248 can select the relaxation factors and the accuracy required for the convergence criteria. The
249 key inputs for the program are a matrix containing the initial fracture aperture distribution
250 over the flow domain and other parameters such as the initial joint normal stiffness and the
251 fluid properties. The method for obtaining the initial fracture aperture is discussed later under
252 the experimental procedure. The fracture dimensions are given and, the domain is discretised
253 by the program according to the user's preferences. The user can activate a confining stress
254 increment and obtain the corresponding flow rate and aperture changes for desired confining
255 stress increment levels. The resulting flow vectors are displayed as arrows with flow
256 directions and relative magnitudes. The background of the plot is either the aperture
257 distribution or a pressure distribution contour map. The outlet flow rate is calculated as the
258 summation of the flow rates of the cell array at the outlet, according to:

$$Q = \sum_{k=1}^m [u_{nk} e_{nk} \Delta y] \quad (20)$$

In the above, m is the number of grids in the y -direction and n is the outlet boundary grid number in the x -direction. The aperture deformation and flow rate are displayed graphically at the end of each calculation.

The solution domain contained 6156 scalar cells with a 1 mm grid spacing. The convergence of the solution was examined for a flow rate with a coarser grid spacing of 2 mm, and up to relatively fine grid spacing of 0.4 mm. A grid spacing of 1 mm was found to converge efficiently and was selected for the analyses. New velocities and pressures were updated at each cycle iteratively, using a relaxation factor of 0.5.

Experimental Procedure

Permeability tests were carried out using the high pressure two-phase triaxial apparatus (HPTPTA) (Indraratna and Haque 1999) to compare with the model predictions. Rock specimen of 54mm diameter with a sub-axial tension crack was used for the tests. The specimen was enclosed in a polyurethane membrane and two cantilever arms were used to measure the deformations of the fracture (Fig. 5). The confining pressure was applied using silicone oil, which does not react with the membrane, and the outlet fluid was collected on an electronic balance to measure the flow rate. A schematic diagram of the test set up is shown in Fig. 6. The sandstone specimen was 114 mm high, and it was tested under 5 confining pressure levels (750kPa, 1MPa, 1.25Mpa, 1.5MPa, and 1.75MPa,) and three levels of inlet pressure (40kPa, 60kPa and 80kPa). Once the flow became steady, the weight of outflow water was recorded in 15 second intervals for each combination of confining pressure and inlet pressure, while the confining pressure, inlet and outlet pressures, and fracture deformation were simultaneously recorded in a data taker.

The fracture aperture wall surface profiles were scanned by a non-contact 3D laser scanner before and after the test. The initial aperture distribution was measured using a silicon rubber solution. The solution was inserted into the fracture, without any pressure being applied to the fracture, and then allowed to set. Once set, the top half of the test specimen was carefully removed and the silicon rubber surface was scanned (Fig. 7). The silicon rubber was then removed from the lower half of the rock specimen and the surface of the lower half was scanned with respect to the same reference point in order to obtain the surface coordinates of both surfaces with the same origin. The difference between the two surface coordinates was then taken as the fracture aperture distribution. The steps of the scanning procedure are illustrated in Fig. 8 (Price and Indraratna 2005).

Results and Discussion

Fig. 9 shows the flow vectors and aperture contours obtained for the fractured sandstone specimen at an inlet pressure of 40 kPa. Fig. 9 shows the velocity and aperture distributions at confining pressures of 750 kPa, 1000 kPa, and 1750 kPa, respectively. The contour maps indicate the aperture deformation at increasing confining pressure in successive plots, while the velocity vectors represent the flow paths for which the size of the arrow is proportional to the magnitude. The smallest apertures with no significant velocity are the contacts of the domain. Fig. 10 shows the flow vectors, as well the pressure contours in the background, for the test subjected to an inlet pressure of 40 kPa and a confining pressure of 1000 kPa. Initially, a uniform pressure distribution (i.e. equal pressure lines are vertical and parallel) was prescribed according to the inlet and outlet pressures. However, for the same aperture distribution shown in Fig. 9(b), when the mathematical model is solved, Fig. 10 verifies that the fluid pressure is not uniformly distributed for irregular apertures. For smaller apertures, the fluid pressure becomes significantly higher relative to the surrounding area

accompanied by an increased flow velocity. In contrast, for larger apertures, the fluid pressure becomes lower with a correspondingly reduced flow velocity.

Fig. 11 shows the model predictions and laboratory test data for the volumetric flow rate against confining stress together with the corresponding Reynolds number. It illustrates the variation of the Reynolds number (Re) with the confining pressure at three different inlet pressures based on Eqn. (21), where the average aperture ($\bar{e}$) can be used as the characteristic linear dimension (Kishida *et al.* 2013; Singh *et al.* 2014), thus:

$$\text{Re} = \frac{\bar{e}U\rho}{\mu} \quad (21)$$

In the above, U is the mean velocity, ρ is the fluid density and μ is the dynamic fluid viscosity. The model predictions are in good agreement with the laboratory data and they validate the proposed approach. Due to the omission of inertia terms in the solution procedure, the relatively high flow rates observed at confining pressures less than 1000 kPa for a corresponding Reynolds number exceeding 10, show a noticeable deviation from the experimental results. Since the model uses the variation of the aperture in space, the effect of the uneven flow path is addressed more explicitly. The model is proposed for underground flow calculations to predict the rock permeability at different depths. Conventional triaxial permeability tests suffer from obvious limitations and practical difficulties. For example, the sample size is considerably smaller and the use of a membrane can introduce boundary effects. Preparing specimens is a challenging task when the rock samples have to be cored with a sub axial fracture. Usually a fracture is induced in a real rock specimen in the laboratory, but identical fracture patterns cannot be guaranteed for conducting multiple tests to improve the reliability of the test data. An aperture distribution can be obtained from a sample with any shape, and a sub-sample of rectangular cross section can be processed and used for the model calculations. The membrane boundary effect can be negated by

considering the side boundaries as symmetrical. In this manner, the permeability in both the longitudinal and lateral directions can be obtained which is often infeasible in conventional triaxial testing.

The evolution of fracture flow during shearing of rock fractures has been researched widely (Yeo *et al.* 1998; Koyama 2007). Rock fracture shearing affects the flow as the fracture walls dilate or compress while shearing. However, actual shearing rates are very small compared to the fluid flow rates, so in the case of predicting flow rates in underground openings or petroleum extraction in the short term, the role of fracture shearing becomes marginal. On the other hand, the fracture shearing is essential for accurately calculating flow rates through fractures over a long period of time. The proposed model is capable of predicting the flow rates given the aperture distribution in local 2D space. The model is coupled with the normal deformation under normal load in order to predict the aperture distribution at different depths, noting that the applied normal stress increases with depth. The model can also be coupled with a shear-dilation model to predict the aperture evolution chronologically and predict the flow rates accordingly.

Conclusion

In this paper, the formulation of a mathematical model and a numerical solution scheme based on MATLAB were presented to determine the velocity and pressure distributions in 2D space for fluid flow through a rough, deformable rock fracture. A MATLAB program incorporating the Authors' model was employed to simulate the triaxial single fracture permeability test, in which the main controlling parameters included the initial aperture distribution, the fracture dimensions, the initial joint normal stiffness, the fluid density, the dynamic fluid viscosity, the number of grids in the 'X' and 'Y' directions of the fracture, and the increments of applied confining pressure. In summary, for a computed

aperture deformation, the program calculates and illustrates graphically the velocity and pressure distributions and the velocity vector plots.

Rock fracture permeability tests were conducted using the High pressure Two-Phase Triaxial Apparatus (HPTPTA) to validate the model. With an acceptable margin of error, the overall model predictions were generally in good agreement with the laboratory results. One of the main reasons for proposing this new model for real rock fracture flows was to demonstrate the changes in the flow due to aperture changes with the applied stress, and to quantify the flow behaviour with increasing confining pressure. Both the model predictions and the experimental results indicate that the relationship between the fracture permeability and the applied normal stress can be represented by a power function.

Unlike most 2D models, a salient feature of the proposed mathematical model is that the joint aperture is not assumed to be constant. In fact, the stress-deformation response of the aperture is fully coupled with the flow equations, thus the magnitude of flow is not only influenced by the effective stress, but also by the changing aperture simultaneously. Subsequently, the overall flow at any time at the outlet is determined by integrating the flow components of the cell array.

The inertia terms of the mathematical model were neglected in the numerical formulation because of the relatively slow fluid flows and for the convenience of eliminating the difficulties attributed to non-linearity. Therefore, the current model calculations are only realistic for slow flow velocities through rock fractures at a Reynolds number less than 10. In this study, the fracture was assumed to be planar with rough fracture walls, thus the model is not expected to be accurate for non-planar discontinuities such as folds. Another limitation of the model is that it considers steady state flows only. Although modification of the model to include transient flows is currently being studied, it is not within the scope of this paper.

In spite of these limitations, the proposed mathematical model provides a rational tool to predict flow through a fracture in lieu of conventional laboratory permeability testing.

Acknowledgement

The authors acknowledge the Australian Research Council (ARC) for funding the research project. Dr. Jan Nemcik of University of Wollongong and Dr. Winton Gale of SCT Pty Ltd are acknowledged for the support given at various times in the project. The authors wish to acknowledge Mr Alan Grant and the Engineering faculty technical staff of the University of Wollongong for their efforts on the laboratory experiments. The second author would like to acknowledge the Endeavour Postgraduate Awards program for its financial support.

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460 **Appendix: A**

461 Integrating the first term in Eqn. (4) by applying Leibniz's rule gives

$$462 \quad \int_{F_b}^{F_t} \frac{\partial(\rho \bar{V})}{\partial t} = \frac{\partial}{\partial t} \int_{F_b}^{F_t} (\rho \bar{V}) dz + \frac{\partial F_b}{\partial t} \rho \bar{V} \Big|_{f_b} - \frac{\partial F_t}{\partial t} \rho \bar{V} \Big|_{f_t} \quad (\text{A1})$$

463 Here $F_b = z_b + f_b$ and $F_t = z_t + f_t$ where z_b and z_t are the z coordinates of the bottom wall and top
 464 wall profile for a particular point in (x, y) space given by f_b and f_t , respectively. Once no-slip
 465 boundary conditions are assumed at the top and bottom surfaces, the last two parts of Eqn.
 466 (A1) vanish. The density of the fluid is constant over the z direction and Eqn. (A1) is reduced
 467 to

$$468 \quad \int_{F_b}^{F_t} \frac{\partial(\rho \bar{V})}{\partial t} = \frac{\partial}{\partial t} (\rho e \bar{V}) \quad (\text{A2})$$

469 where

$$470 \quad \bar{V}(x, y, z, t) = \bar{\dot{V}}(x, y, t) + \widetilde{\bar{V}}(x, y, z, t) \quad (\text{A3})$$

471 The second term of the Eqn. (4) may be integrated using the same method. Assuming no-slip
 472 boundary conditions at the walls and using Eqn. (A3), where $\widetilde{\bar{V}}$ is the difference between the
 473 velocity and the depth-averaged velocity, leads to

$$474 \quad \int_{F_b}^{F_t} \nabla \cdot (\rho \bar{V} \bar{V}) = \dot{\nabla} \cdot (e \rho \bar{\dot{V}} \bar{\dot{V}}) + \dot{\nabla} \cdot (e \rho \widetilde{\bar{V}} \widetilde{\bar{V}}) \quad (\text{A4})$$

475 The last term of the above equation is the dispersive momentum flux and can be neglected
 476 according to Bear (1993). Integrating the fifth term of the Eqn. (4) assuming the same
 477 conditions furnishes

$$478 \quad \int_{F_b}^{F_t} \mu \nabla^2 \bar{V} dz = \mu \dot{\bar{V}}^2 \bullet (e \bar{\dot{V}}) + \mu \left\{ \nabla F_b \bullet \nabla \bar{V} \Big|_{f_b} - \nabla F_t \bullet \nabla \bar{V} \Big|_{f_t} \right\} \quad (A5)$$

479 The last component of this equation can be reduced to $(12\mu\bar{V}/e)$ by assuming non-shearing
 480 walls and a parabolic distribution of velocity inside the fracture at the fully developed stage
 481 of the flow. The remaining pressure and gravity terms in Eqn. (4) can be evaluated simply
 482 and lead to Eqn. (5).

483 **Appendix: B**

484 If the corrected velocity is u , intermediate velocity is $u^\#$ and the correction to be
 485 applied to the intermediate velocity is u' , then

$$486 \quad u = u^\# + u' \quad (B1)$$

487 Similarly, for the pressures

$$488 \quad p = p^\# + p' \quad (B2)$$

489 Substituting Eqns. (B1) and (B2) into Eqn. (13) and subtracting the resulting equation from
 490 Eqn. (13) furnishes Eqn. (15).

491 In order to obtain the pressure equation, Eqn. (16) is substituted into Eqn. (B1) which
 492 gives:

$$493 \quad u = u^\# + \frac{1}{\left[-\frac{2\mu}{\Delta x^2} - \frac{2\mu}{\Delta y^2} - \frac{12\mu}{e_{i,j}^2} \right]} \left(\frac{p'_{I+1,J} - p'_{I-1,J}}{\Delta x} \right) \quad (B3)$$

494 A similar equation can be obtained for the v -velocity by following the same method,
 495 and then those two equations can be used to define the neighbouring velocities of u
 496 ($u_{i,j}, u_{i-1,j}$) and v ($v_{I,j}, v_{I,j-1}$). Thereafter substituting these four equations in the discretised
 497 continuity equation (Eqn. B4) gives the pressure equation (Eqn. 18).

$$498 \quad \frac{[(\rho eu)_{i,j} - (\rho eu)_{i-1,j}]}{\Delta x} + \frac{[(\rho ev)_{I,j} - (\rho ev)_{I,j-1}]}{\Delta x} = 0 \quad (B4)$$

499 **Notation**

500	$\vec{V}$	Velocity vector in 3D (x,y,z)
501	$\bar{\vec{V}}$	Averaged velocity vector in 2D (x,y)
502	e	Aperture
503	ρ	Fluid density
504	Q	Flow rate
505	w	Fracture width
506	μ	Dynamic viscosity of fluid
507	$P = p + \rho g$	Total pressure
508	p	Pressure
509	g	Acceleration of gravity
510	X', Y', Z'	Global coordinate system
511	x, y, z	Discontinuity oriented coordinate system
512	$\bar{\bar{T}}$	Stress tensor
513	D_n, D_m	Normal deformation at time n, maximum deformation
514	u, v	x and y direction components of averaged velocity vector
515	K_i	Initial joint normal stiffness

516	I, J	Main grid indices
517	i, j	Staggered grid indices
518	ϕ, M, H	Variables used to replace expressions
519	F_u, F_l	Upper rock fracture profile and lower rock fracture profile in 3D space

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2D modelling from top view of the fracture

- Bear (1993)
- Kishida et al (2013)

- Zimmerman and Yeo (2013)
- Koyama (2007)
- Indraratna *et al.*, (2002)
- Price & Indraratna (2005)

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