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Abstract

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Keywords

preload, incorporating, columns, slender, effects, tubular, nonlinear, steel, double, concrete-filled, square, analysis

Disciplines

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ABSTRACT

Thin-walled square and circular hollow steel tubes are designed to support the permanent and construction loads of several upper composite floors before filling the concrete into the tubes to form concrete-filled double steel tubular (CFDST) columns. The influences of preloads acting on the steel tubes on the structural responses of slender square CFDST **columns** have not been investigated either experimentally or numerically. This paper presents a fiber-based computational model for the determination of the interaction behavior of local and global buckling in **axially and eccentrically** loaded CFDST thin-walled square slender **columns** including preload effects. The computational modeling method accounts for the influences of the deformations induced by preloads, local-global interaction buckling, geometric imperfections, second-order, and geometric and material nonlinearities. The accuracy of the computational algorithms developed is validated by comparing computations with test data on concrete-filled steel tubular (CFST) columns and finite element results of double-skin CFST (DCFST) columns with preload effects. The computer algorithms are employed to quantify the

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influences of preloads on the local-global interaction buckling responses of CFDST columns with various important parameters. Proposed is a design method for calculating the ultimate loads of concentrically loaded slender square CFDST columns considering preloads. The computational and design models developed are shown to be efficient modeling and design tools for square CFDST slender **columns** taking into account the construction method of high-rise composite buildings.

Keywords: Concrete-filled double steel tubes; preload effects; local-global interaction buckling; nonlinear analysis.

1. Introduction

Concrete-filled double steel tubular (CFDST) columns where the outer tube is square and the inner tube is circular as depicted in Fig. 1(a) offer high ductility, stiffness, strength, and fire-resistance, and have been utilized in tall composite building structures [1, 2]. In the composite construction of a high-rise building, both the circular and square hollow steel tubes are erected together with composite floors several stories before the wet concrete is filled into the hollow tubes to form CFDST composite columns. This construction method greatly speeds up the construction process but induces significant preloads on both the outer and inner tubes. The preloads cause deformations as well as initial stresses in the steel tubes, which may markedly reduce the strength and stiffness of CFDST slender **columns**. The effects of preloads, therefore, must be included in the numerical modeling and structural design of such composite columns for safety. However, no attempts have been made to experimentally and numerically quantify the effects of preloads on the structural responses of slender square thin-walled CFDST columns which may experience the local-global interaction buckling. This paper discusses the

inelastic modeling, behavior and structural design of square CFST slender thin-walled columns including the influences of preloads.

Experimental works on the strengths of CFST columns under preloads as illustrated in Fig. 1(b) have been undertaken by several investigators. Zhang et al. [3] presented experimental results on eccentrically loaded short and slender circular CFST columns where the preload ratios varied from 0.0 to 0.48. The ultimate strength of the columns was observed to reduce with an increase in either the member slenderness, loading eccentricity or the preload ratio. The test results of square CFST columns under preloads provided by Han and Yao [4] indicated that the maximum reduction in the column ultimate strength was up to 20%, but for preload ratio less than 0.3, the strength reduction was only about 5%. Liew and Xiong [5] investigated the influences of preloads on the behavior of rectangular CFST columns and proposed design models based on Eurocode 4 [6]. However, for columns having the preload ratio greater than 0.4, the design method of Liew and Xiong [5] significantly underestimates the ultimate loads of rectangular CFST columns as discussed by Patel et al. [7]. Huang et al. [8] undertook experiments on CFST circular columns with preload ratios varied from 0.0 to 0.54. Test results showed that the small preload ratio had little effect on the ultimate loads of the columns. In addition, it was reported that the preload ratio has a more pronounced influence on the performance of slender columns than that of short ones. Experiments carried out by Li et al. [9] on preloaded concrete-encased CFST columns under long-time sustained loading indicated that the average reduction in the column ultimate load was about 8.7%.

The influences of preloads on the global buckling behavior of CFST composite columns were studied by researchers previously by utilizing numerical approaches [4, 5, 7, 8, 10]. The commercial finite element (FE) software Abaqus was employed by Liew and Xiong [5], Huang

et al. [8], Xiong and Zha [10] for that purpose. Han and Yao [4] developed a numerical technique to study the significance of preloads on the behavior of CFST square columns. However, the local and post-local buckling of the outer box was not incorporated in the analysis which overestimated the strengths of CFST columns. Patel et al. [7] developed an efficient computational method for the interaction buckling simulation of CFST columns loaded biaxially and proposed a design approach supported by numerical results for designing CFST columns subjected to preloads. The behavior of local and post-local buckling of thin-walled sections under biaxial loading was modeled by means of using the local buckling models of Liang et al. [11]. Li et al. [12] investigated the influences of preloads on the strengths of double-skin CFST columns. The preload effects on either outer steel tube or both tubes were studied.

The behavior of CFDST square and rectangular columns without preloads has been ascertained by a number of researchers [13-20]. It was observed that the local buckling of the external steel tube induced the failure of square short CFDST columns while the failure of slender columns was caused by the local-global interaction buckling. The additional confinement exerted by the internal tube of circular section increased the ductility of CFDST columns when compared with CFST columns [1]. Qian et al. [16] examined the influences of high strength concrete on the strengths of square CFDST columns that were loaded eccentrically. Ahmed et al. [20] tested twenty square CFDST short columns loaded either axially or eccentrically. The test parameters included the loading eccentricity and the diameter-to-thickness ratio of the internal steel tube. The inelastic nonlinear analysis of rectangular CFDST columns was performed by Pei [13], Wang et al. [17] and Ahmed et al. [1, 2, 20]. The local buckling of external steel flanges and webs in square CFDST columns was considered in the numerical modeling method proposed by Ahmed et al. [1, 2, 20].

It has been shown that there is a lack of knowledge on the numerical modeling, behavior and design of slender square CFDST columns including the influences of preloads. In this paper, a fiber-based computer modeling method is proposed that calculates the global buckling behavior of loaded square CFDST columns incorporating important features of preloads as well as the local buckling of the outer square steel box. The computational procedure together with efficient solution algorithms are described. The existing experimental results on slender CFST columns and numerical results on double-skin CFST (DCFST) columns subjected to preloads as shown in Fig. 1(c) are used to verify the computer model proposed. The effects of preloads and various design parameters on the interaction behavior of local-global buckling in CFDST columns are examined. A design model is also proposed for determining the ultimate loads of concentrically-loaded CFDST slender columns with preload effects.

2. Computational theory

2.1. Inelastic simulation of cross-sections

The computational modeling technique is formulated on the basis of the theory of fiber analysis, which meshes the cross-section of a square CFDST column by fiber elements as shown in Fig. 2 [1, 21-25]. The sandwiched concrete is discretized by employing a mesh generator developed by Persson and Strang [26]. Because triangular fiber elements are used to discretize the sandwiched concrete area, more elements are employed to mesh the region around the circular boundary. The effect of mesh on the solution is minimized by using the convergence criterion discussed in Section 2.3. The discretization of the core concrete is similar to that of circular CFDST columns [25, 27, 28] where the numbers of steel and concrete layers are prescribed by the user. The strain compatibility is maintained by assuming that the plane section remains

plane under deformation and no longitudinal slippage between the interface of steel and concrete occurs [22, 24]. For the cross-section under uniaxial bending in addition to axial compression illustrated in Fig. 2, the fiber strain is determined from the curvature (ϕ), the depth of the plastic neutral axis (d_n) and the compressive strain at the extreme fiber (ε_t). The stresses of fibers are calculated from the corresponding fiber strains by the nonlinear inelastic constitutive laws of concrete and steel materials. The stresses of the fibers are integrated to calculate the internal moment (M) and axial force (P) as stress resultants [24].

2.2. Local buckling simulation of square steel tubes

The external thin-walled steel section of the square CFDST column under applied loads is susceptible to the outward local buckling. This failure mode markedly reduces the column ultimate loads [11]. Under the combined axial load in addition to uniaxial bending, the top steel flange and webs of the square CFDST column are under either non-uniform or uniform stresses as shown in Fig. 3 [2, 11]. Liang et al. [11] derived a set of equations based on the results of finite element analyses for estimating the initial stresses of plate local-buckling under non-uniform compression. These equations are implemented in the present computational modeling method to accurately predict the onset of local buckling of the outer steel walls of the CFDST square columns with clear width-to-thickness ratios ranging from 30 to 110.

The progressive post-local buckling characteristics of the top steel flange and webs of the square CFDST column are modeled by employing the effective width theory. This theory is implemented in the numerical scheme in which the in-plane stresses within the buckled tube walls are redistributed gradually as illustrated in Fig. 3. The following expressions formulated

by Liang et al. [11] are used to quantify the effective widths b_{e1} and b_{e2} of the steel top flange and webs of the square CFDST column under uniaxial bending shown in Fig. 3:

$$\frac{b_{e1}}{b} = \begin{cases} 0.2777 + 0.01019 \left(\frac{b}{t_o} \right) - 1.972 \times 10^{-4} \left(\frac{b}{t_o} \right)^2 + 9.605 \times 10^{-7} \left(\frac{b}{t_o} \right)^3 & (\alpha_s > 0.0) \\ 0.4186 + 0.002047 \left(\frac{b}{t_o} \right) + 5.355 \times 10^{-5} \left(\frac{b}{t_o} \right)^2 - 4.685 \times 10^{-7} \left(\frac{b}{t_o} \right)^3 & (\alpha_s = 0.0) \end{cases} \quad (1)$$

$$\frac{b_{e2}}{b} = (2 - \alpha_s) \frac{b_{e1}}{b} \quad (2)$$

where b is the clear width of the outer tube wall; t_o is the thickness of the external tube wall; the stress-gradient coefficient α_s defines the ratio of the minimum stress (σ_2) to the maximum stress (σ_1) acting on the tube wall [11]. It should be noted that the effective width formulas defined by Eqs. (1) and (2) are applicable to steel tube walls with clear width-to-thickness ratios ranging from 30 to 110 [11]. This implies that the local buckling of non-compact and slender steel sections as classified in Eurocode 4 [6] is considered in the computational model.

In the numerical modeling, the maximum ineffective width ($b_{ne,max}$) is determined as $(b - b_{e1} - b_{e2})$. The ineffective width (b_{ne}) between zero and $b_{ne,max}$ within the tube wall is determined by means of using the linear interpolation on the basis of the stress level, and is expressed using the following equation given by Liang [24]:

$$b_{ne} = \left(\frac{\sigma_s - \sigma_{cr}}{f_{sy0} - \sigma_{cr}} \right) b_{ne,max} \quad (3)$$

in which σ_s denotes the steel fiber stress, σ_{cr} represents the critical local-buckling stress, and f_{sy0} stands for the yield stress of the external tube material.

2.3. Modeling of column global buckling

The computational modeling method is proposed for pin-ended square slender CFDST beam-columns having preload effects and under the same loading eccentricity (e) at both ends with single curvature bending. The preload ratio is calculated by

$$\beta_a = \frac{P_{pre}}{P_{us}} \quad (4)$$

in which P_{pre} denotes the preload on the hollow steel tubes and P_{us} represents the ultimate axial load of the hollow steel tubes where the local buckling of the external square steel tube is taken into consideration in the numerical analysis.

The part-sine wave displacement function is used to represent the deflected shape of the slender beam-column [29]. As discussed by Liang [29], the curvature (ϕ_m) at the mid-height of the column can be calculated from the displacement function as

$$\phi_m = u_m \left(\frac{\pi}{L} \right)^2 \quad (5)$$

in which u_m represents the deflection that occurs at the column mid-height, and L is the effective length of the column.

The initial geometric imperfection of the steel tube (u_o), and the second-order effects resulting from the interaction between the applied load (P) and the lateral displacement (u_m) are included in the mathematical model. The mid-height lateral displacement (u_{mv}) of the hollow steel tubes under preload is calculated using the load-control approach [7, 30] and is considered as an additional geometric imperfection in the modeling of CFDST columns. The external moment (M_e) at the column mid-length is

$$M_e = P(e + u_m + u_o + u_{mv}) \quad (6)$$

The displacement-control method is used to calculate the interaction of local-global buckling responses of the CFDST slender beam-column with preload effects. The lateral deflection (u_m) occurring at the column mid-height is gradually increased. For each displacement increment, the calculations of the internal axial force (P) as well as the moment (M) are undertaken. The internal axial force computed that satisfies the moment equilibrium condition at the column mid-height is considered as the external axial load applied at the column ends. The residual moment (r_p) generated at each iteration can be expressed in the mathematical form as

$$r_p = M_e - M \quad (7)$$

The moment equilibrium condition is maintained if the residual moment (r_p) satisfies the convergence tolerant $|r_p| < \varepsilon_k = 10^{-4}$. To achieve this condition, the depth of the plastic neutral axis in the cross-section is iteratively calculated by the numerical solution algorithms discussed in Section 2.3.

The computational procedure for determining the axial load-lateral displacement curve of the eccentrically loaded CFDST beam-column considering preloads is described as follows:

- (1) Specify the geometric and material data of the CFDST column.
- (2) Mesh the cross-section of the column into fine fibers.
- (3) Determine the mid-height lateral displacement (u_m) of the hollow steel tubes.
- (4) Assign the geometric imperfection as $u_o + u_{mo}$.
- (5) Initialize the lateral displacement as $u_m = \Delta u_m$.
- (6) Calculate the curvature (ϕ_m) of the column.
- (7) Initialize the neutral axis depths as: $d_{n,1} = D_0$, $d_{n,2} = D_0/2$ and $d_{n,3} = (d_{n,1} + d_{n,2})/2$.
- (8) Determine the axial force P and moment M accounting for local buckling.
- (9) Calculate the values of residual moments $r_{p,1}$, $r_{p,2}$ and $r_{p,3}$.
- (10) Compute the new neutral axis depth ($d_{n,4}$) by the inverse quadratic method.
- (11) Determine fiber stresses from fiber strains by the material stress-strain laws.
- (12) Calculate force P and moment M including local buckling effects.
- (13) Repeat Steps (10)-(12) until $|r_p| < 10^{-4}$.
- (14) Increase the lateral deflection as $u_m = u_m + \Delta u_m$.
- (15) Repeat Steps (6)-(14) until $P < 0.5P_{\max}$ or the displacement limit is exceeded.
- (16) Plot $P-u_m$ curve.

2.3. Numerical solution technique

The plastic neutral axis depth in the cross-section is determined by means of an iterative numerical scheme to obtain the force and moment equilibrium at the column mid-height.

Computer codes that implement the inverse quadratic method have been written to obtain satisfactory solutions to the highly nonlinear, dynamic equilibrium function generated in the iterative-incremental analysis process. Three initial values are required to assign to the neutral axis depth to facilitate the computation. The new neutral axis depth $d_{n,4}$ can be computed by

$$d_{n,j+3} = d_{n,j+1} - r_{p,j+1} \left(\frac{A}{C} \right) \quad (8)$$

$$A = (r_{p,j})^2 (d_{n,j+2} - d_{n,j+1}) + r_{p,j} r_{p,j+1} (d_{n,j+1} - d_{n,j+2}) + (r_{p,j+1} - r_{p,j+2}) r_{p,j+2} (d_{n,j} - d_{n,j+1}) \quad (9)$$

$$C = (r_{p,j+1} - r_{p,j})(r_{p,j+2} - r_{p,j})(r_{p,j+2} - r_{p,j+1}) \quad (10)$$

where j is the iteration number. In the computational analysis, the determination of the true neutral axis depth is continued until the convergence condition is satisfied.

3. Material laws of structural steels

The steel tubes in CFDST columns are subjected to biaxial stresses under axial compression load as discussed by Liang [31]. This effect is considered in the material laws of steel presented in Fig. 4. The equation proposed by Liang [24] is used to calculate the stresses between $0.9\varepsilon_{sy}$ and ε_{st} , where ε_{sy} denotes the yield strain and ε_{st} is the strain at the onset of strain-hardening and is specified as 0.005 [24]. The formula suggested by Mander [32] is employed to compute the stresses in the regime of strain-hardening. The ultimate strain ε_{su} as shown in Fig. 4 is prescribed as 0.2 in the numerical modeling [24].

4. Material laws of confined concrete

4.1. Concrete in compression

Figure 5 shows the idealized two-stage stress-strain curve for confined concrete in compression. The ascending part of the curve is determined by the equations given by Mander et al. [33] as

$$\sigma_c = \frac{f'_{cc}(\varepsilon_c / \varepsilon'_{cc})^\lambda}{(\varepsilon_c / \varepsilon'_{cc})^\lambda + \lambda - 1} \quad (11)$$

where the longitudinal concrete stress and its corresponding strain are expressed as σ_c and ε_c , respectively and λ is calculated as

$$\lambda = \frac{E_c \varepsilon'_{cc}}{E_c \varepsilon'_{cc} - f'_{cc}} \quad (12)$$

$$E_c = 4400 \sqrt{\gamma_c f'_c} \quad (13)$$

where E_c is Young's modulus of concrete, γ_c denotes the reduction factor which considers the column size effect and was proposed by Liang [24] as $1.85 D_c^{0.135}$, in which D_c is computed as $(B_o - 2x_o)$.

The compressive strength (f'_{cc}) and corresponding strain (ε'_{cc}) of confined concrete in Eq. (11) are the function of lateral pressure f_{rp} provided by the steel tube to the confined concrete. The expressions derived by Lim and Ozbakkaloglu [34] to compute f'_{cc} and ε'_{cc} were revised by Ahmed et al. [1] to consider the column size effects (γ_c) and expressed as:

$$f'_c = 5.2 (\gamma_c f'_c)^{0.91} \left(\frac{f_{np}}{\gamma_c f'_c} \right)^a + \gamma_c f'_c \quad \text{in which } a = (\gamma_c f'_c)^{-0.06} \quad (14)$$

$$\varepsilon'_{cc} = \varepsilon'_c + 0.045 \left(\frac{f_{np}}{\gamma_c f'_c} \right)^{1.15} \quad (15)$$

$$\varepsilon'_c = \frac{(\gamma_c f'_c)^{0.225}}{1000} \quad (16)$$

The lateral pressure exerted by the outer steel tube to the sandwiched-concrete is so small that it is taken as zero as discussed by Wang et al. [17]. However, the lateral pressure offered by the internal steel tube f_{pi} is found to increase the ductility as well as the compressive strength of the core concrete. The formulas of Liang and Fragomeni [27] for concrete in circular CFST columns derived based on the studies of Hu et al. [35] and Tang et al. [36] are utilized to calculate f_{pi} , which is given as

$$f_{pi} = \begin{cases} 0.7(v_e - v_s) \left(\frac{2t_i}{D_i - 2t_i} \right) f_{syi} & \text{for } \frac{D_i}{t_i} \leq 47 \\ \left(0.006241 - 0.0000357 \frac{D_i}{t_i} \right) f_{syi} & \text{for } 47 < \frac{D_i}{t_i} \leq 150 \end{cases} \quad (17)$$

where v_e and v_s are the Poisson's ratios of the steel tube with and without concrete infill, respectively, and are given by Tang et al. [36].

The descending part of the stress-strain curve for the confined concrete shown in Fig. 5 is quantified by using the expression provided by Lim and Ozbakkaloglu [34] as

$$\sigma_c = f'_{cc} - \frac{f'_{cc} - f_{cr}}{\left[1 + \left(\frac{\varepsilon_c - \varepsilon'_{cc}}{\varepsilon_{ci} - \varepsilon'_{cc}} \right)^{-2} \right]} \quad (18)$$

where f_{cr} is the residual strength of concrete, and ε_{ci} denotes the strain at the inflection point.

For the core-concrete, f_{cr} and ε_{ci} are calculated by the following expressions [34]

$$f_{cr} = \begin{cases} f'_{cc} & \text{for } \frac{D_i}{t_i} \leq 40 \\ 1.6f'_{cc} \left(\frac{f_{rpi}^{0.24}}{(\gamma_c f'_c)^{0.32}} \right) \text{ and } f_{cr} \leq f'_{cc} - 0.15 (\gamma_c f'_c) & \text{for } 40 < \frac{D_i}{t_i} \leq 150 \end{cases} \quad (19)$$

$$\varepsilon_{ci} = 2.8\varepsilon'_{cc} (\gamma_c f'_c)^{-0.12} \left(\frac{f_{cr}}{f'_{cc}} \right) + 10\varepsilon'_{cc} (\gamma_c f'_c)^{-0.47} \left(1 - \frac{f_{cr}}{f'_{cc}} \right) \quad (20)$$

The residual strength (f_{cr}) of the sandwiched-concrete is determined as $f_{cr} = \beta_c f'_c$. Ahmed et al. [1] derived the strength degradation factor β_c for the sandwiched-concrete based on the experimental work, and is determined as

$$\beta_c = \begin{cases} 1 & \text{for } 0 \leq \frac{B_o}{t_o} \leq 24 \\ 1 - \frac{1}{15} \left(\frac{B_o}{t_o} - 24 \right) & \text{for } 24 < \frac{B_o}{t_o} \leq 33 \\ 0.000062 \left(\frac{B_o}{t_o} \right)^2 - 0.011225 \left(\frac{B_o}{t_o} \right) + 0.705288 & \text{for } 33 < \frac{B_o}{t_o} \leq 100 \end{cases} \quad (21)$$

A value of 0.007 is used for the strain at the inflection point (ε_{ci}) for sandwiched concrete as suggested by Ahmed et al. [1].

4.2. Concrete in tension

The concrete in a slender square CFDST column under axial load combined with bending may be subjected to tensile stresses. The material laws of concrete in tension is presented in Fig. 5 where the stress under tension linearly increases to the tensile strength at which the concrete cracks and reduces linearly to zero. The tensile strength of concrete is determined as $0.6\sqrt{\gamma_c f'_c}$. The ultimate tensile stain is equal to ten times the concrete cracking strain.

5. Verification

In the absence of experimental data, the computational model presented is validated by the test results of **axially and eccentrically loaded** conventional CFST columns having preload effects provided by Han and Yao [4]. The details of the column specimens tested by Han and Yao [4] are tabulated in Table 1. The measured concrete cube strengths have been converted to the cylindrical concrete strengths using a factor of 0.85 as suggested by Oehlers and Bradford [37]. The experimentally measured ultimate loads ($P_{u,exp}$) and the predicted ultimate loads ($P_{u,num}$) of the columns are given in Table 1. **The initial out-of-straightness presented at the column mid-height was taken as $L/1000$.** The agreement between the tests and computations is good. The mean $P_{u,num}/P_{u,exp}$ was calculated as 0.93, giving a standard deviation of 0.04. The experimental and computational load-deflection responses of the columns are presented in Fig. 6. It indicates that the computer model generally simulates well the strength and stiffness of the columns under investigations. The slight difference between experimental and computational results is attributed to the fact that the actual properties of concrete in the tested specimens are unknown.

The computational method is further verified by the finite element results of square DCFST columns with preload effects provided by Li et al. [12]. Table 2 presents the details of the DCFST columns analyzed by Li et al. [12]. In Table 2, $k_{p,FE} = P_{up} / P_{wp}$ defines the strength index, where P_{up} and P_{wp} are the ultimate loads of CFDST columns with and without preload effects. It can be observed that the mathematical method produces results that agree well with the finite element solutions given by Li et al. [12]. The mean $k_{p,num} / k_{p,FE}$ value is 1.02, where $k_{p,num}$ is the strength index obtained using the numerical model developed. The calculated standard deviation value of 0.018 shows an excellent agreement between these two results.

6. Interaction buckling behavior with preload effects

The computational modeling technique formulated was used to determine the influences of the preloads on the interaction responses of local-global buckling in CFDST square columns incorporating various important design parameters. The details of the CFDST columns under consideration are given in Table 3. The local buckling of the outer steel section having a clear width-to-thickness ratio greater than 30 was considered in the analyses. The initial out-of-straightness presented at the column mid-height was taken as $L/1000$.

6.1. Influences of local buckling

Column SC1 in Table 3 was numerically analyzed to study the effects of the local buckling of the steel outer tube on the performance of the slender square CFDST column subjected to a preload ratio of 0.2. Figure 7 presents the axial load-lateral displacement responses predicted by means of considering or ignoring the effect of local plate buckling. It is shown that neglecting the local buckling effects noticeably overestimates the initial bending stiffness and ultimate

load of the CFDST column. The ultimate axial strength is overestimated by 7.4% when local buckling is not included in the calculations. The sensitivities of the column strength curve to the local buckling were examined by changing the length of Column SC1 and the obtained column strength curves are given in Fig. 8. It is shown that local buckling is an influential parameter on the strength of the columns with an L/r ratio less than 40. However, its effect becomes insignificant when the member slenderness greater than 40.

6.2. Influences of preload ratio

The numerical analyses of Columns SC2 to SC5 in Table 3 were undertaken to explore the significance of the ratio of preload on the local-global interaction buckling behavior of CFDST columns. It is seen from Fig. 9 that the preload has a considerable influence on the axial load-displacement curves. Increasing the preload ratio from 0.0 to 0.3, 0.6 and 0.8 decreases the ultimate load of the slender columns by 2.4%, 5.6% and 8.8%, respectively. The load distributions in the components of the columns subjected to different preloads are presented in Fig. 10. The preload ratio has the most prominent effect on the load distribution in the internal steel tube as well as the core-concrete. When increasing the ratio of preload from 0.0 to 0.8, the reduction in the ultimate strength in the external tube, internal tube, sandwiched-concrete and core-concrete was computed as 14.0%, 15.5%, 5.3% and 15.4%, respectively. It can be seen from Fig. 10 that under large deflections, both tubes are under the resultant tensile force regardless of the preload ratio while most of the compression load is carried by the sandwiched-concrete. When the steel tubes are preloaded with a ratio of 0.8, the sandwiched concrete withstands 94.6% of the column ultimate load.

6.3. Influences of preloads and L/r ratio

The interaction local-global buckling behavior of CFDST columns is greatly affected by the member slenderness ratio (L/r). The effects of L/r ratio were examined by analyzing Columns SC6-SC9 in Table 3 where the L/r ratios varied from 20 to 80. The ultimate load and bending stiffness of the columns are significantly reduced by changing the L/r ratio from 20 to 80 as demonstrated in Fig. 11. Increasing the L/r ratio from 20 to 80 causes a decrease in the column ultimate load by 43.7%. As expected, increasing the L/r ratio leads to an increase in the lateral displacement. Figure 12 shows the influences of preloads that have on the column strength curves. The preload effect can be ignored for CFDST short columns that have an L/r ratio less than 20. However, it is seen from Fig. 12 that the influence of preloads on the column strength increases with increasing the L/r ratio. The load distributions in the steel tubes and concrete in CFDST columns with different slenderness ratios are presented in Fig. 13. It is noted that increasing the member slenderness causes a reduction in the load carried by individual components. By increasing the ratio of L/r from 20 to 80, the load shared by the external tube, internal tube, sandwiched-concrete and core-concrete reduce by 47.4%, 65.5%, 20.7% and 72.7%, respectively. The contribution rate of the concrete and steel components to the ultimate loads of CFDST columns is presented in Fig. 14. It is discovered that changing the L/r ratio from 20 to 80 increases the load contribution of the sandwiched concrete from 61.4% to 78.7%, but slightly decreases the contribution of the other components.

6.4. Influences of preloads and e/B_o ratio

Columns SC10-SC14 listed in Table 3 with the e/B_o ratios varied from 0.0 to 0.4 were analyzed by the computational model to ascertain the significance of the e/B_o ratio on the responses of CFDST beam-columns. It is seen from the axial load-deflection responses given in Fig. 15 that the ultimate load and initial bending stiffness of the columns with an L/r ratio of 60 remarkably

decrease with increasing the e/B_o ratio. A strength reduction of 198% is obtained by altering the ratio of e/B_o from 0.0 to 0.4. The effects of e/B_o ratio and preloads are illustrated in Fig. 16, which shows that the load-carrying capacity of the column is significantly reduced by increasing the e/B_o ratio regardless of the preload values. However, for slender CFDST columns having a e/B_o ratio greater than 1.0, the preload has an insignificant influence on the column ultimate load. The load distributions in concrete and steel tubes in the CFDST columns that have different e/B_o ratios are presented in Fig. 17. It is noted that the sandwiched-concrete shares a large portion of the applied load. When columns with large e/B_o ratios are subjected to large deflections, the steel tubes may be in tension. As shown in Fig. 18, increasing the e/B_o ratio causes an increase in the contribution of the sandwiched concrete to the column ultimate load, but decreases the contributions of other components.

6.5. Influences of preloads and B_o/t_o ratio

The influences of preloads and B_o/t_o ratio on the behavior of CFDST columns were ascertained by analyzing Columns SC15-SC18 in Table 3. The thickness of the external steel box was varied to obtain different B_o/t_o ratios. It is confirmed from Fig. 19 that increase the B_o/t_o ratio remarkably reduces the ultimate strength and bending stiffness of the slender CFDST columns. The ultimate strength of the column having an L/r ratio of 45 is reduced by 47.24% by altering the B_o/t_o ratio from 40 to 100. The effect of the B_o/t_o ratio on the column strength curves is shown in Fig. 20. It appears that the B_o/t_o ratio has a marked influence on the column ultimate load regardless of the member slenderness. The effect of preloads and B_o/t_o ratio on the column strengths are illustrated in Fig. 21. Increasing the ratio of preload reduces the strength

of the columns regardless of the ratio of B_o/t_o . When the B_o/t_o ratio is 100, altering the ratio of preload from 0.0 to 0.8 reduces the column ultimate load by 7.8%.

6.6. Influences of preloads and D_i/B_o ratio

Fiber element analyses on Columns SC19-SC22 shown in Table 3 were undertaken to quantify the sensitivities of the load-displacement curves and column strength curves of CFDST columns to preloads and D_i/B_o ratio. The D_i/B_o ratio of the columns varied from 0.33 to 0.66. The load-deflection curves depicted in Fig. 22 demonstrates that increasing the D_i/B_o ratio causes a considerable improvement in the ultimate strength of the columns that have an L/r ratio of 70. The increase in load-carrying capacity is 12.6% when the D_i/B_o ratio is changed from 0.33 to 0.66. The D_i/B_o ratio has also a considerable influence on the strength curves of the columns as illustrated in Fig. 23. However, this effect decreases with an increase in the slenderness of the members. It is seen from Fig. 24 that changing the ratio of preload from 0.0 to 0.8 reduces the column ultimate load by about 12.5%.

6.7. Influences of preloads and D_i/t_i ratio

The computational model was employed to analyze Columns SC23-SC26 with D_i/t_i ratios varying from 20 to 50 listed in Table 3 to ascertain the significance of the preloads and D_i/t_i ratio on the responses of CFDST columns. The computed load-displacement curves and column strength curves are presented in Figs. 25 and 26, respectively. When the D_i/t_i ratio of the column that has the L/r ratio of 50 increases from 20 to 50, the strength of the column decreases by 12.6%. The effect of the preload on the column strength as a function of D_i/t_i ratio is

illustrated in Fig. 27. By increasing the ratio of preload from 0.0 to 0.8, the column ultimate strength is reduced by 7.7%, 7.0%, 6.6% and 6.1% for columns with the D_i/t_i ratios of 20, 30, 40 and 50, respectively.

6.8. Influences of preloads and concrete strength

Columns **SC27-SC30** made of concrete with different strengths varied from 40 MPa to 100 MPa listed in Table 3 were employed to ascertain the effects of preloads and concrete strength on the local-global interaction buckling responses of CFDST columns. It is found from the load-lateral displacement responses of the columns presented in Fig. 28 that using higher strength concrete not only increases the initial flexural stiffness but also remarkably increases the ultimate load of the columns. However, as depicted in Fig. 29, this effect decreases considerably with an increase in the member slenderness. The influence of the preload on the column strengths varied with the concrete strength is illustrated in Fig. 30. Increasing the ratio of preload ratio reduces the ultimate column strength regardless of the concrete strength. It is confirmed that the strength reduction due to preload effects increases with an increase in the strength of concrete. Altering the ratio of preload from 0.0 to 0.8 leads to a 9.3% reduction in the load-carrying capacity of the column with 100 MPa concrete.

6.9. Influences of preloads and steel yield strength

Columns **SC31-SC34** in Table 3 were analyzed by using the computer model to quantify the significance of preloads and steel yield strength on their structural behavior. The simulated load-deflection responses of CFDST columns having different steel yield strengths and an L/r ratio of 60 are given in Fig. 31. The use of higher yield strength steel does not affect the column

initial bending stiffness, but markedly improves their load-carrying capacities. It is seen from Fig. 31 that the increase in the strength of the column is about 24.2% due to the increase of steel strength from 300 MPa to 600 MPa. However, this effect decreases with an increase in the slenderness of the members as shown in Fig. 32. It is interesting to note that the strength reduction due to preload effects increases as the high strength steel is used. As shown in Fig. 33, changing the ratio of preload from 0.0 to 0.8 leads to an 11.8% ultimate load reduction of the column made of steel tubes with 600 MPa yield strength.

7. Proposed design model

The ultimate axial load of CFDST columns incorporating preloads is influenced by the relative slenderness ($\bar{\lambda}$), preloads ratio (β_a) and geometric imperfection of the columns [7]. Although the computational model developed can be used to analyze and design of square CFDST slender columns loaded concentrically and eccentrically including preload effects, a simple design formula is needed for practical design. Based on Eurocode 4 [6], a design model for estimating the ultimate loads of concentrically loaded square CFDST slender columns considering the influence of preloads is developed herein as

$$P_{u,des} = \chi_{pre} P_{uo} \quad (22)$$

where P_{uo} is the column section ultimate strength, which can be calculated as

$$P_{uo} = f_{syo} A_{soe} + f_{syt} A_{sti} + \gamma_{co} f'_{co} A_{co} + \gamma_{ci} f'_{ci} A_{ci} \quad (23)$$

in which A is the cross-sectional area of the components of the CFDST column; subscripts si , co and ci define the inner steel tube, sandwiched concrete and core concrete, respectively; subscript soe is the effective area of the outer tube taking consideration of local buckling effects.

The slenderness reduction factor χ_{pre} of the CFDST column with preload effects is used to take into consideration the influences of preload ratio, relative slenderness and column imperfection and is expressed as

$$\chi_{pre} = \frac{1}{\phi_{pre} + \left[\phi_{pre}^2 - \bar{\lambda}^2 \right]^{0.5}} \leq 1.0 \quad (24)$$

where ϕ_{pre} was given by Patel et al. [7] as

$$\phi_{pre} = \frac{1 + 1.1\alpha_{pre} \left[(1 + \beta_a) \bar{\lambda} - 0.05 \right] + \left[(1 + \zeta) \bar{\lambda} \right]^2}{2} \quad (25)$$

in which α_{pre} is proposed based on the regression analyses on the numerical results of CFDST columns with preload effects as

$$\alpha_{pre} = \frac{1 - \beta_a}{16.5(1.2 - \beta_a) - 5.3(1.2 - \beta_a)^2 - 1.5 + 60\zeta} \quad (26)$$

where ζ is given by Patel et al. [7] as

$$\zeta = \begin{cases} 0 & \text{for } \beta_a \leq 0.4 \\ \frac{\beta_a - 0.4}{4} & \text{for } 0.4 \leq \beta_a \leq 0.8 \end{cases} \quad (27)$$

The relative slenderness ratio $\bar{\lambda}$ of the column is determined as

$$\bar{\lambda} = \sqrt{\frac{P_{uo}}{P_{cr}}} \quad (28)$$

in which P_{cr} is the elastic buckling load of the CFDST column and determined by

$$P_{cr} = \frac{\pi^2 (EI)_{eff}}{L^2} \quad (29)$$

The effective flexural stiffness of the cross-section of CFDST columns $(EI)_{eff}$ is calculated by

$$(EI)_{eff} = E_{s,so} I_{s,so} + 0.6E_{cm,co} I_{c,co} + E_{s,si} I_{s,si} + 0.6E_{cm,ci} I_{c,ci} \quad (30)$$

where E_s and I_s are Young's modulus and second moment of area of the steel tube; E_{cm} and I_c are Young's modulus and second moment of area of concrete; the subscripts *so*, *si*, *co* and *ci* define the outer steel tube, inner steel tube, sandwiched concrete and core concrete, respectively. Young's modulus of concrete can be estimated as

$$E_{cm} = 22000 \left(\frac{f'_c + 8}{10} \right)^{1/3} \quad (31)$$

The comparisons of the fiber analysis results with the calculations by the proposed design model for columns that have the preload ratios of 0.3 and 0.8 are given in Figs. 34 and 35, respectively. It is seen that the calculations by the design model are in good agreement with the numerical results obtained. The design model is further validated against the finite element results of slender square DCFST columns provided by Li et al. [12] in Table 2. The mean value of $k_{p,des} / k_{p,FE}$ is 1.021 with a standard deviation of 0.018. This indicates that the design model yields good ultimate load calculations of CFDST slender columns accounting for preload effects.

8. Conclusions

This paper has described a fiber-based computational method for quantifying the influences of preloads on the local-global interaction buckling behavior of slender square CFDST columns loaded eccentrically. The interaction of local-global buckling, geometric imperfection initially presented, second-order effects, material nonlinearities and concrete confinement have been considered in the mathematical model formulation. The existing test and finite-element results on CFST columns and DCFST columns have been used to validate the computational model developed. The local-global interaction buckling responses of CFDST columns incorporating preloads and important design parameters have been investigated. A design model has been proposed for the design of concentrically loaded slender CFDST columns accounting for preload effects.

It has been shown that the computational method proposed captures well the local-global interaction buckling responses of CFDST columns considering preloads. The local buckling of the external tube considerably reduces the ultimate load of the slender CFDST columns. In

addition, the ultimate strength and load distributions of the components of CFDST columns are reduced by increasing the preload ratio. However, these effects are more pronounced for the preload ratio greater than 0.3. Increasing the slenderness ratio of the column, the eccentricity ratio or the B_o/t_o ratio markedly decreases the stiffness and ultimate strength of CFDST columns under preloads. The D_l/B_o ratio and D_l/t_l ratio have a moderate influence on the ultimate strength of CFDST columns when preloads are applied to the steel tubes. However, these effects become less pronounced as the slenderness of the column increases. Using high strength materials to construct square CFDST slender columns remarkably improves the ultimate load of CFDST columns under preloads.

The proposed design model derived based on Eurocode 4 gives estimations which agree well with numerical results of CFDST columns considering preload effects and can be used by structural engineers in designing CFDST slender columns under axial loading. However, further experimental and numerical works are still required to investigate the effects of preloads on the strength envelopes of slender CFDST columns in order to propose design models for eccentrically loaded slender CFDST columns including preloads.

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