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Abstract

This paper presents a study on the deformation and degradation responses of railway ballast using large-scale laboratory testing and computational modeling approaches. A series of large-scale triaxial tests were carried out to investigate the ballast breakage responses under cyclic train loading subjected to varying frequencies, $f=10-40$ Hz. The role of recycled rubber energy-absorbing mats (REAMs) on reducing ballast breakage was also examined. Laboratory test results show that the ballast experiences significant degradation (breakage) and deformation, while the inclusion of REAMs can reduce the ballast breakage up to about 35%. Numerical modeling using the coupled discrete-continuum approach [coupled discrete-element method-finite-difference method (DEM-FDM)] is introduced to provide insightful understanding on the deformation and breaking of ballast under cyclic loading. Discrete ballast grains were simulated by bonding of many circular elements together at appropriate sizes and locations. Selected cylinders located at corners, surfaces, and sharp edges of the simulated particles were connected by parallel bonds; and when those bonds were broken, they were considered to represent ballast breakage. The subgrade and rubber mat were simulated as a continuum media using FDM. The predicted axial strain ϵ_a and volumetric strain ϵ_v obtained from the coupled DEM-FDM model are in good agreement with those measured in the laboratory. The model was then used to explore micromechanical aspects of ballast aggregates including the evolution of particle breakage, contact force distributions, and orientation of contacts during cyclic loading. These findings are imperative for a more insightful understanding of the breakage behavior of ballast from the perspective of microstructure characteristics of discrete particle assemblies.

Introduction

Keywords

numerical, testing, laboratory, modeling, degradation:, influenced, deformation, performance, ballast

Disciplines

Engineering | Science and Technology Studies

Publication Details

Indraratna, B., Ngo, T. & Rujikiatkamjorn, C. (2020). Performance of Ballast Influenced by Deformation and Degradation: Laboratory Testing and Numerical Modeling. *International Journal of Geomechanics*, 20 (1), 04019138-1-04019138-14.

PERFORMANCE OF BALLAST INFLUENCED BY DEFORMATION AND DEGRADATION: LABORATORY TESTING AND NUMERICAL MODELING

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Abstract: This paper presents a study on the deformation and degradation responses of railway ballast using large-scale laboratory testing and computational modeling approaches. A series of large-scale triaxial tests are carried out to investigate the ballast breakage responses under cyclic train loading subjected to varying frequencies, $f=10-40$ Hz. The role of recycled rubber energy absorbing mats (REAM) on reducing ballast breakage is also examined. Laboratory test results show that the ballast experiences significant degradation (breakage) and deformation, while the inclusion of the REAM can reduce the ballast breakage up to about 35%. Numerical modeling using the coupled discrete-continuum approach (coupled DEM-FDM) is introduced to provide insightful understanding on the deformation and breaking of ballast under cyclic loading. Discrete ballast grains are simulated by bonding of many circular elements together at appropriate sizes and locations. Selected cylinders located at corners, surfaces and sharp edges of the simulated particles are connected by parallel bonds; and when those bonds are broken, they are considered to represent ballast breakage. The subgrade and rubber mat are simulated as a continuum media using the finite difference method (FDM). The predicted axial strain ε_a , volumetric strain ε_v obtained from the coupled DEM-FDM model are in good agreement with those measured from the laboratory. The model is then used to explore micromechanical aspects of ballast aggregates including the evolution of particle breakage, contact force distributions and orientation of contacts during cyclic loading. These findings are imperative for a more insightful understanding of the breakage behavior of ballast from the perspective of microstructure characteristics of discrete particle assemblies.

Keywords: Ballast; Breakage; Laboratory testing; Discrete element modelling; Rubber mat; Micromechanical analysis

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Introduction

With the increasing competition from other means of transport, the railway industry must continually upgrade track systems and apply innovative technologies to minimise the costs of construction and maintenance. The ballast layer plays a crucial part in transmitting and distributing the wheel load from sleepers to the underlying sub-ballast and subgrade at a reduced and acceptable stress level (Selig and Waters 1994, Janardhanam and Desai 1983, Lobo-Guerrero and Vallejo 2005). When subject to heavy freight trains, ballast aggregates experience significant breakage (Zhang *et al.* 2017, Chen *et al.* 2018, Walker and Indraratna 2018, Donohue *et al.* 2009, Indraratna *et al.* 2013). Attributed to excessive degradation of ballast, Australian rail industry spends a mammoth budget on frequent track repair and maintenance (e.g. \$12M annually in NSW alone), plus significant ground improvement prior to track construction, where soft and saturated foundation soils (subgrade) pose considerable challenges.

The breaking of ballast, including particle crushing/splitting, and severe attrition of aggregates significantly influence the shear strength of ballast (Lackenby *et al.* 2007, Lu and McDowell 2010, Xiao *et al.* 2014a, Ishikawa *et al.* 2011, Sun *et al.* 2018). Indraratna *et al.* (2005) conducted large-scale triaxial tests to study the influence of confining pressure (lateral confinement) on the degradation of ballast under cyclic loading. They found that under low confining pressures ($\sigma_3 < 30$ kPa), substantial ballast breakage was observed. Small increases in confining pressure $\sigma_3=30$ -75 kPa resulted in reduced ballast breakage due to increased particle contact area and more uniform inter-granular stresses. However, higher confining pressure ($\sigma_3 > 75$ kPa) resulted in a gradual increase in ballast breakage (mainly across the bodies of grains) due to tighter packing of grains and suppressed dilation. Leng *et al.* (2017) carried out cyclic triaxial test on with coarse-grained granular materials to study the accumulation of permanent deformations under different confining and deviator stress levels. They found two distinct stages of permanent deformation: (i) rapid increase in the accumulative plastic strains until failure; and (ii) a steady state of plastic strain accumulation after a given number of loading cycles. These findings were in agreement with those reported earlier by Lackenby *et al.* (2007).

Guo and Zhu (2017) studied the particle breakage energy and its effect on the stress dilatancy of rockfill aggregates, and they found that the breakage occurred even at low-stress level and it contributed to a considerable reduction of dilatancy of rockfills (Suiker and Borst 2003, Xiao *et al.* 2014b). Ueng and Chen (2000) modified the well-known Rowe's dilatancy law by adding

energy consumption due to particle breakage (E_b) and found that the shear strength of granular aggregates was influenced by the rate of energy consumption related to particle breakage. This could correspond to the rate of increment of the surface areas of grains. They also found that with increased confining pressure, the rate of energy consumption (i.e. due to particle breakage) increased while the peak friction angles decreased, supported by similar findings by Indraratna *et al.* (2011).

A series of large-scale laboratory tests on ballast was carried out by Indraratna *et al.* (2011) at the University of Wollongong, and they found that cyclic loads and impact loads exerted by heavy haul freight train traffic caused significant deformation and degradation leading to poor track geometry. Moreover, the transfer of cyclic stresses to a greater depth might cause differential settlements in the subgrade thus affecting track stability. The complex stress states in a real track environment were influenced by the mid- and high-frequency vibrations stemming from applied train loads (Esvelde 2001, Mishra *et al.* 2014). Damage to sleepers and ballast, and track settlements were attributed to vibrations. Previous studies have suggested that it was imperative to examine the deformation and breakage behavior of ballast under high cyclic loading frequencies (Fu *et al.* 2015; Zhai *et al.* 2015).

There have been numerous research on the use of artificial inclusions (geogrids, geocells, rubber mats, etc.) to improve the performance of railway ballast and to decrease its breakage (Zhai *et al.* 2004, Powrie *et al.* 2007, Biabani *et al.* 2016, Tutumluer *et al.* 2012, Indraratna and Ngo 2018, Leng *et al.* 2017, among others). Field trials conducted on instrumented sections of track at Singleton, NSW showed that the inclusion of rubber mats could reduce vertical and lateral ballast deformation significantly, apart from decreasing the ballast breakage (Indraratna *et al.* 2011). Past laboratory studies using a high capacity impact testing equipment and large-scale track process simulation testing apparatus at University of Wollongong revealed that these rubber mats, depending on their damping properties and energy absorbing capacity could reduce ballast breakage, by attenuating the transmission of impact-induced shock waves effectively (Nimbalkar *et al.* 2012, Navaratnarajah *et al.* 2018).

There has still been limited research on the role of recycled rubber energy absorbing mats (REAM) on the deformation and degradation behavior of ballast in view of micromechanical perspective. This paper presents a study on the testing and modeling the breakage and deformation responses of railway ballast based on large-scale triaxial tests and coupled DEM-FDM modeling, where the

interaction of discrete ballast grains and continuum media (rubber mats, capping and subgrade layers) could be accurately captured.

Large-scale Triaxial Cyclic Testing

A total of six tests were conducted to study the load-deformation behavior of railway ballast and the role of recycled rubber energy absorbing mats (REAM) on decreasing ballast breakage under varied loading frequencies. Details of the testing program is presented in Table 1; 4 tests were carried out for ballast under different frequencies of $f=10$ -40 Hz and two tests were conducted with the inclusion of REAM under frequencies of $f=10$ and 20 Hz. A large-scale triaxial apparatus (Figure 1a) can accommodate specimens of 300 mm in diameter and 600 mm in height (Figure 1b). The main components of the apparatus are: (i) cylindrical triaxial chamber, (ii) axial loading unit, (iii) cell pressure control unit in combination of air and water pressure, (iv) cell pressure and pore pressure measurement system, (v) axial deformation measuring device, and (vi) volumetric change measurement unit (Figure 1c). The volume change of a specimen during loading is measured by a coaxial piston located within a small cylindrical chamber connected to the main cell, in which the piston moves up or down freely depending on the increase or decrease in volume (Indraratna *et al.* 2011).

Properties of Tested Materials

Fresh latite basalt, a quarried igneous aggregate commonly used as railway ballast in New South Wales, Australia was used in this study. This igneous material has been shown to be particularly suitable for use on railway tracks because of its relatively high compressive strength, hardness, and resistance to weathering. The ballast was washed to remove any dust and clay adhering to the surface of the grains and then sorted into the required distribution by passing through a set of standard industrial sieves (aperture size 63-2.36mm). Upon drying (Figure 2a), ballast in different sizes were painted in different colors (Figure 2b) to help distinguish and quantify ballast breakage after the testing. Predetermined weight of each ballast size was mixed thoroughly to ensure homogeneity and to obtain the relevant particle size distribution (Figure 2c). Sandy-soil was used for subgrade and a mixture of sand and crushed basalt was used as sub-ballast (capping) layer (Figure 2d). A layer of rubber mat (10 mm thick) manufactured from recycled tyres was used as REAM (Figure 2e). The particle size distribution of tested materials was selected to abide by the Australian Standards (AS:2758.7: 2015), as shown in Figure 3.

Sample Preparation and Testing Procedure

A cylindrical rubber membrane of 300 mm diameter and 7 mm thick was used to assemble the ballast specimen. The bottom of the cell membrane was filled by a 190mm thick subgrade and compacted to a unit weight of 18.5 kN/m³, followed by a 100 mm-thick compacted capping layer having a unit weight of 20.5 kN/m³. A layer of recycled rubber mats (REAM) was then placed on top of the capping layer. A bottom layer of ballast (100 mm thick) was placed into the cell membrane overlying the REAM and compacted using a hand-held vibrating hammer to attain a unit weight of 15.5 kN/m³. This process was repeated for the remaining two layers of ballast until the final height of the sample reached a height of 600 mm.

After preparing the specimen, the triaxial cell was placed inside the loading frame, and the specimen was filled with water through the base plate. The triaxial chamber was also filled with water and left overnight to ensure saturation. The test specimens were consolidated to a confining pressure of $\sigma_3=20$ kPa, mimicking low confinement in the field (Indraratna *et al.* 2011). Fully drained cyclic loading tests were conducted at a relatively low axial strain rate which allowed excess pore pressure to dissipate completely. Cyclic tests were conducted within the bounds of $q_{cyc,max}=230$ kPa and $q_{cyc,min}=30$ kPa (Figure 4a), subjected to different loading frequencies of $f=10, 20, 30$ and 40 Hz. It is noted that a conditioning loading phase at a frequency of $f=1$ Hz was applied initially to prevent any loss of actuator contact with the specimen during rapid vertical deformation (Figure 4b). Stress-controlled cyclic loading phase then followed and the tests were ceased after $N=500,000$ cycles or when the vertical deformation reached the actuator displacement limit (approx. 30% axial strain). During these tests, the axial strain, ε_a and volumetric change, ε_v were recorded periodically by an automated data logger controlled by a host computer. The ballast specimens were recovered at the end of each test, then dried and sieved, and the changes in particle size were recorded to analyse the extent of breakage.

Laboratory Test Results and Discussion

Axial Strain and Volumetric Strain Responses

Figure 5 shows the measured axial strain, ε_a and volumetric strain, ε_v with the number of load cycle, N under a range of loading frequencies, $f=10-40$ Hz, with and without the REAM. Here, the axial strain increases with an increase in loading cycle and higher the frequency, f applied the greater the axial strain, ε_a measured. There is a distinct trend of increasing axial strain within the

first thousand load cycles, followed by a gradual increase of the ε_a at a decreasing rate (Figure 5a). Indeed, when the ballast aggregates were compressed to a threshold packing arrangement, subsequent loading would initiate volumetric dilation and particle breakage. As expected, the inclusion of the REAM decreases the axial strain of ballast when subjected to frequencies of $f=10$ and 20 Hz. This is because the energy absorbing nature of REAM decreases the energy transferred to the ballast grains that results in less deformation. It is also noted that at the initial loading up to $N=100$ cycle, the REAM causes some increase in the ε_a and this is attributed to the compression of the rubber mat itself that contributes to the total vertical deformation.

Figure 5b shows the volumetric strain, ε_v plotted against the number of load cycles, N . Generally, all specimens exhibited significant volumetric compression at initial loading cycles ($N=1000$) progressing to a threshold compression, followed by a decreasing rate of compression at subsequent loading cycles. The value of ε_v increases with an increase of frequency, f and decreases with the inclusion of REAM. Tests carried under low frequencies of $f=10, 20$ Hz show volumetric compression throughout the tests. Conversely, tests conducted under high frequencies of $f=30$ and 40 Hz exhibit dilation behaviour approximately after $N=10,000$ cycles. Dilatancy can be considered as one of the important tell-tale signs associated with track instability under adverse conditions.

Ballast Breakage

Under cyclic loading, ballast grains deteriorate by the breakage of sharp corners and attrition of asperities, apart from splitting at high confining pressure (Lackenby *et al.* 2007, Qian *et al.* 2018). The volume of ballast broken can be determined by sieving it before and after every test, and then quantifying the differences of particle size gradations using the *BBI* index introduced by Indraratna *et al.* (2005), as illustrated in Figure 6. The amount of ballast breakage subjected to varied frequencies, with and without the inclusion of rubber mat is presented in Figure 7. It is measured that the ballast breakage increases with an increase in frequency, and the granular assemblies having the REAM show a significantly reduced breakage compared to those without rubber mats. The reduction of ballast breakage is primarily reflected by the energy absorbing characteristics of the REAM which could absorb some dynamic loads resulting in less load being transferred to ballast grains; as a result the breakage is decreased. In fact, the inclusion of REAM decreases the *BBI* up to 30.8% and 35.3% for ballast subjected to a frequency of $f=10$ and 20 Hz, respectively.

Resilient Modulus of Ballast

The resilient modulus, M_R can be defined as the ratio of the applied deviator stress ($\Delta q_{cyc} = q_{cyc,max} - q_{cyc,min}$) to the recoverable (resilient) axial strain, $\varepsilon_{a,rec}$ during a loading-unloading cycle.

$$M_R = \frac{\Delta q_{cyc}}{\varepsilon_{a,rec}} \quad (1)$$

During the test, data bursting were initiated at specific cycles to examine the influence of the load and frequency on resilient modulus, M_R of ballast. Typical applied cyclic stress-displacement curves measured from testing are shown in Figure 8a. This data is used to determine the resilient modulus of ballast at a given number of load cycles, N . In this study, the M_R was calculated at $N=1, 50, 100, 500, 1000, 5000, 10000, 50000, 100000, 200000, 300000, 400000, 500000$; and the variations of M_R with and without the inclusion of the REAM under varied f is presented in Figure 8b. It is seen that a considerable increment of the resilient modulus, M_R can be observed up to $N=100,000$ cycles and subsequently, the increment rate becomes marginal. Indeed, rapid densification of ballast during initial load cycles causes an increase in the stiffness of the assembly, and as a result, rapid increase in M_R is observed. After $N=100,000$ cycles, ballast grains attained a more stable condition, and this contributed to the reduction in the rate of increase of M_R . Lackenby *et al.* (2007) studied variations of the resilient modulus of ballast under varied loading conditions and observed similar behavior. It is also noted that the inclusion of the REAM results in a slight increase in the M_R .

Discrete Element Modeling

The discrete element method (DEM) has been widely used to study the micromechanical behavior of railway ballast (McDowell *et al.* 2006, O'Sullivan *et al.* 2008, Huang and Tutumluer 2011, Tutumluer *et al.* 2012, Ngo *et al.* 2014, Jiang *et al.* 2017, among others). Indraratna *et al.* (2010) used DEM to model fresh ballast subjected to 1,000 load cycles to study the shear stress-strain and particle degradation under various load frequencies. Lu and McDowell (2010) introduced a DEM model to simulate ballast subjected to 100 load cycles and found it could still capture the stress-strain behavior of ballast. In these studies, the role of angularity in relation to various shapes and sizes was not captured accurately, and also the interaction of the discrete ballast grains with continuum subgrade was not considered (Lobo-Guerrero and Vallejo 2006, Indraratna *et al.* 2010, Zhang *et al.* 2017). In addition, due to excessive computational time required to carry out cyclic tests in DEM, most previous studies were limited to only a few thousands of loading cycles

(McDowell and Li 2016, Ngo *et al.* 2017). Indraratna *et al.* (2014) simulated a conventional direct shear test in DEM that took nearly 500 hours (using a high performance workstation). Given the excessive cost of solving a practical railway embankment in DEM, and the numerical inefficiency to accurately model a large geometrical problem with different layers of substructure (ballast, capping, subgrade), there is a need to develop a coupled discrete-continuum model that would fully optimize the combined computational effort while providing reasonable accuracy.

Coupled Discrete-Continuum Framework

Ballast aggregates having varying sizes and shapes were simulated in 2D DEM by bonding many cylindrical particles together at predetermined size and locations using parallel bonds (Figure 9a). The breaking of these bonds was considered to represent ballast breakage. Micro-mechanical parameters for ballast grains used in DEM analysis were determined by calibration with laboratory test data, and a set of parameters used in the current DEM analysis is given in Table 2. Figure 9b shows a schematic diagram of a coupled discrete-continuum (DEM-FDM) model, where the layers of subgrade, capping and REAM were modeled by the finite difference method. For simplicity, the REAM was analysed as a linear elastic material. The capping (sub-ballast) was simulated as an elasto-plastic material following the Drucker-Prager yield criterion. The material parameters for the Drucker-Prager model were obtained from triaxial testing carried out on the materials obtained from a quarry in Bombo, New South Wales (Indraratna *et al.* 2011). The subgrade was represented with a conventional elasto-plastic (Mohr Coulomb) model where the model parameters (Young's modulus E , Poisson's ratio ν , cohesion c , friction angle ϕ , and dilatancy angle ψ) were determined in the laboratory, as summarized in Table 3.

The interaction between ballast grains (Zone 1) and capping/subgrade layers (Zone 2) were facilitated by transferring of forces and displacements between the two domains. The schematic diagram of exchanging of contact forces (F_n, F_s) and velocity ($\dot{X}_i^{[E]}$) between the discrete particles and continuum elements at their interfaces is presented in Figure 9c. The superscripts C , E , and P represent contact, element, and particle, respectively.

The contact forces in the interface are determined as:

$$F_i^{[C]} = F_n^{[C]} + F_s^{[C]} \quad (2)$$

257 where, the normal force ($F_n^{[C]}$) and increment of shear force ($\Delta F_s^{[C]}$) are determined by (Itasca
258 2014):

$$259 \quad F_n^{[C]} = K^n U^n n_i ; \quad \text{and} \quad \Delta F_s^{[C]} = -K^s (\Delta X_i^{[C]} - \Delta X_i^{[C]} n_i) \quad (3)$$

260 The resultant force and moment on particles at the interface can be obtained:

$$261 \quad F_i^{[P]} \leftarrow F_i^{[P]} - F_i^{[C]} ; \quad \text{and} \quad M_i^{[P]} \leftarrow M_i^{[P]} - e_{ijk} (X_j^{[C]} - X_j^{[P]}) F_k^{[C]} \quad (4)$$

262 The relative contact velocity at the interface (V_i) is determined by (Itasca 2014):

$$263 \quad V_i = \dot{X}_{i,E}^{[C]} - \dot{X}_{i,E}^{[P]} = \dot{X}_{i,E}^C - [\dot{X}_i^{[P]} + e_{ijk} \omega_j^{[P]} (X_k^{[C]} - X_k^{[P]})] \quad (5)$$

264 where, $\dot{X}_{i,E}^{[C]}$ and $\dot{X}_{i,E}^{[P]}$ are the velocities of element and particle, respectively. $\dot{X}_i^{[P]}$ and $\omega_j^{[P]}$ are the
265 translational and rotational of the particle, and e_{ijk} is the permutation symbol.

266 The velocity of continuum elements at the interface (i.e. displacement) is then determined by:

$$267 \quad \dot{X}_{i,E}^{[C]} = \sum N_j \dot{X}_{i,E}^j \quad (6)$$

268 where, N_j is a shape function, given by: $N_j = (1 + \xi_o)(1 + \eta_o)/4$, $j = 1, 2, 3, 4$; and $\xi_o = \xi_i \xi_j$,
269 $\eta_o = \eta_i \eta_j$; ξ_i and η_i are local coordinates of nodes. At the interface, the shear and the normal
270 contact forces are distributed to the nodal force, $F_i^{[E,j]}$ following to the shape function N_j , given
271 by (Itasca 2014):

$$272 \quad F_i^{[E,j]} = F_i^{[E]} + F_i^{[C]} N_j \quad (7)$$

273 The time step, Δt was selected to ensure numerical stability conditions, given by:

274 $\Delta t_{ct} \leq \min \{\Delta t_{dcr}, \Delta t_{fcr}\}$, in which, $\Delta t_{dcr} = \sqrt{m^d / K^n}$ is the critical time step of DEM region; m^d
275 and K^n are the mass and stiffness of ballast particle; $\Delta t_{fcr} = L_{\min} / C_f$ is the critical time step of
276 FDM region; $L_{\min}$ is minimum length of finite elements; $C_f = \sqrt{E_f(1 - \mu_f) / [\rho_f(1 - \mu_f)(1 - 2\mu_f)]}$
277 is the velocity of elastic wave; E_f , μ_f , and ρ_f are Young's modulus, Poisson's ratio, and
278 density, respectively.

At a given time, Δt , contact forces at interfaces of the discrete DEM domain (Zone 1) were transferred to continuum domain (Zone 2); the displacements of continuum elements were transferred back to the DEM as boundary conditions. The exchange of force-displacement was facilitated by series of interface elements, and a mathematical framework formulated by the authors to distribute forces and moments to nodal points (Figure 9d) is described in the Appendix A. The coupled DEM-FDM model was then be used to perform cyclic tests subjected to varying loading frequencies, $f=10\text{-}40$ Hz and run up to $N=10,000$ cycles.

Predicted Cyclic Stress-Strain Responses

Figure 10 shows typical applied cyclic stresses versus accumulated axial strains obtained from the coupled DEM-FDM model at different load cycles, N , under a given frequency of $f=20$ Hz. It is seen that the predicted axial strains increase considerably up to around 5% within the first 1000 load cycles, followed by gradually increasing axial strains within 5,000 cycles, and then remained relatively stable to the end (10,000 cycles). The area confined by the cyclic (hysteresis) loops becomes increasingly smaller as the number of cycles increases, indicating that the ballast specimen through cyclic densification begins to respond more elastically with time approaching a quasi-shakedown condition. This hysteresis response is in good agreement to that measured earlier in the laboratory by Sun *et al.* (2014).

Comparison of predicted axial strains ε_a obtained from the simulations with those measured experimentally for ballast assemblies subjected to $f=20$ Hz is shown in Figure 11. It is seen that the predicted values of ε_a match reasonably well with laboratory test data, showing that the majority of axial strains occurs within the first $N=1000$ cycles, and the increment of ε_a at subsequent loading cycles occurs at a decreasing rate. This shows that the ballast undergoes significant compression and rearrangement during initial loading cycles, but after achieving a threshold densification, any subsequent loading would resist further compression and thereby promote particle crushing and breakage.

Simulated Particle Breakage

Ballast grains were simulated by bonding of circular elements together and the breaking of these bonds (B_r) was considered to represent particle breakage. Although the number of broken bonds, B_r is different with the BBI , the B_r could still provide a good indication of the amount of ballast breakage. The accumulation of broken bonds with increased load cycle, N under four different frequencies ($f=10-40$ Hz) is presented in Figure 12. It is seen that the B_r increases with an increase in load frequency, f and this trend is similar to the BBI measured for laboratory data (Lackenby *et al.* 2007). Under a given frequency f , the amount of bond breakage increases significantly within the first $N = 5,000$ cycles, and then remains relatively unchanged at subsequent loading cycles. The inclusion of REAM at the interface between the ballast and capping layer results in a decrease in B_r ; this further supports the energy absorbing property of the REAM.

Figure 13 shows snapshots of typical breaking bonds captured from simulations at different load cycles, N at $f=20$ Hz. It shows that the amount of bond breakages increases with increased load cycles, but the inclusion of rubber mats decreases the number of broken bonds. Within the first $N=1000$ cycles, the majority of the broken bonds occurs just below the top loading plate. Zhang *et al.* (2017) investigated the dynamic response of ballast and they found that ballast particles within a depth of 200 mm beneath sleepers would most likely carry a higher stress level, and therefore exhibit more breakage.

Variations of Contact Force Distributions

Figure 14 illustrates the evolutions of contact forces of the ballast assembly together with vertical stress contours in the capping and subgrade layers at different load cycles of $N=100, 1000, 5000$ and 10,000 cycles, for $f=20$ Hz. It is observed that the applied cyclic loads are transmitted to discrete grains in the form of contact force-chains where each contact force is represented at the contact point having a thickness proportional to its intensity. Contact forces in the DEM domain is heterogeneous, where the maximum contact forces change with load cycles. Large contact forces have occurred and concentrated beneath the top-loading plate, and around wall edges, while a significant portion of the load has vertically transmitted to the underlying subgrade. An increase in the number of load cycles results in an increased number of contacts and larger contact force intensities for both cases, i.e. with and without the inclusion of rubber mat. Increased contact force

intensity is synonymous with densification of the sample through rearrangement and breaking of particles. Compared to unreinforced case (Figure 14a), the inclusion of REAM would increase the contact area between ballast and capping and broaden the stress distribution; this in turn eliminates excessive contact forces at the interfaces and help to decrease the deformation and degradation of ballast. Compressive stresses in the capping and subgrade are large around the interface regions but decrease with depth.

Figure 15 shows polar histograms of contact orientations for ballast assemblies subjected to a cyclic loading frequency of $f=20$ Hz, captured at varying load cycles of $N=100$, 1000 and 10000. The polar histogram was plotted by collecting the contact information at a predetermined bin angle of, $\theta = 10^\circ$ and then was normalised by the total number of contacts (O'Sullivan *et al.* 2008, Han *et al.* 2011). The contact orientation can also be described by a Fourier series approximation (Rothenburg and Bathurst 1989):

$$E(\theta) = \frac{1}{2\pi} [1 + a \cos 2(\theta - \theta_n)] \quad (8)$$

where, $E(\theta)$ is the contact density distribution function; a is the magnitudes of directional variations of the contact; and θ_n is major principal direction of contact anisotropy.

It is seen from Figure 15 that at the initial stage of loading ($N=100$ cycles), the majority of contacts are distributed in a vertical direction having the major direction of $\theta_n = 6^\circ$ and 8° for the sample with and without the REAM, respectively. Subjected to subsequent loading cycles, contact force chains develop to resist shear and disperse the loads from the surface into the granular assemblies. Anisotropies of contact orientation grow and rotate considerably during the loading progress and the major direction of contact attains their values of $\theta_n = 13^\circ$ and 16° for rubber mat-reinforced and unreinforced assemblies, respectively (at the $N=1000$ cycles), while the corresponding θ_n values increase up to $\theta_n = 22^\circ$ and 24° at $N=10,000$ cycles. The variations of contact orientations during cyclic loading could be related to the rearrangement of particles (i.e. particles get compacted and form new contacts due to breaking), which results in changes in the major principal contact orientations. This phenomenon clearly explains that the formation of contact force distributions in the ballast assembly during cyclic loading is a dynamic process, significantly influenced by the breakage of the particles.

Conclusions

This paper presents the results of a series of large-scale cyclic triaxial tests and numerical modeling using the coupled discrete-continuum approach to study the deformation and degradation of railway ballast. Six tests were conducted subjected to $q_{cyc,max}=230$ kPa and $q_{cyc,min}=30$ kPa under four different cyclic loading frequencies, i.e. $f=10, 20, 30$ and 40 Hz, and tested up to 500,000 load cycles. The influence of recycled energy absorbing rubber mats (REAM) placed underneath the ballast layer was also examined under two frequencies, $f=10$ and 20 Hz. A coupled DEM-FDM model was developed to simulate cyclic triaxial tests, and the results obtained from these simulations were found to be comparable with laboratory measurements. It is noteworthy that coupling between discrete and continuum domains was implemented using a mathematical approach developed by the Authors in order to transfer forces and displacements. The following conclusions can be drawn:

- The measured test data indicated that axial strain, ε_a increased with an increase in loading cycles; the higher the frequency applied, the greater the measured axial strain. Ballast experienced a considerable axial strain, ε_a within the first $N=1000$ cycles, followed by a gradual increase in ε_a at a decreasing rate towards the end of test. Subjected to frequencies of $f=10$ and 20 Hz, ballast experienced significant volumetric compression throughout the tests, while dilating responses were also measured when subjected to higher frequencies of $f=30$ and 40 Hz.
- The inclusion of recycled rubber energy absorbing mats (REAM) resulted in decreased deformation and degradation of ballast. REAM contributed to less energy being transferred to ballast grains, and thereby reducing particle breakage. The laboratory data showed that BBI decreased up to 30.8% and 35.3% for ballast subjected to frequencies of $f=10$ and 20 Hz, respectively.
- Predicted axial strain (ε_a) and volumetric strain (ε_v) obtained from the coupled DEM-FDM model were in good agreement with the experimental data. The model could effectively capture the evolution of broken bonds of particle clusters and thereby simulating the redistribution and reorientation of contact forces to resist the applied shearing loads.
- This study provided further insight to breakage-induced deformation of ballast as influenced by microstructural characteristics of discrete particle assemblies.

Appendix A: Distributing Forces and Moments at Interface Elements

It is noted that an interface element only receives forces at their nodes (i.e. F_{XA} , F_{YA} , F_{XB} , F_{YB}). Thereby, it is essential to transfer forces and moments (M) from a discrete particle (F_X , F_Y and M) to a continuum element, as derived by the authors (also see Fig.9d) and summarized in this Appendix. Considering the equilibrium of forces in the horizontal and vertical directions, the vertical and horizontal forces can be computed as:

$$\text{Forces on horizontal direction: } F_X = F_{XA} + F_{XB} \quad (9)$$

$$\text{Forces on vertical direction: } F_Y = F_{YA} + F_{YB} \quad (10)$$

Taking the moment equilibrium at the centre (C) of a interface element, given by:

$$M = F_{YA} \times (X_A - X_C) + F_{YB} \times (X_B - X_C) - F_{XA} \times (Y_A - Y_C) - F_{XB} \times (Y_B - Y_C) \quad (11)$$

Equations (9) and (10) can be expressed as:

$$F_X = F_{XA} + F_{XB} = \beta \times F_X + (1 - \beta) \times F_X, \text{ and} \quad (12)$$

$$F_Y = F_{YA} + F_{YB} = \beta \times F_Y + (1 - \beta) \times F_Y \quad (13)$$

in which, the parameter β is given as:

$$F_{XA} = \beta \times F_X \quad \text{and} \quad F_{YA} = \beta \times F_Y \quad (14)$$

$$F_{XB} = (1 - \beta) \times F_X \quad \text{and} \quad F_{YB} = (1 - \beta) \times F_Y \quad (15)$$

Substituting Equations (14) and (15) to Equation (11), results in:

$$M = \beta \times F_Y \times (X_A - X_C) + (1 - \beta) \times F_Y \times (X_B - X_C) - \beta \times F_X \times (Y_A - Y_C) - (1 - \beta) \times F_X \times (Y_B - Y_C) \quad (16)$$

Rearranging Equation (16) gives:

$$\beta = \frac{M - F_Y \times (X_B - X_C) + F_X \times (Y_B - Y_C)}{F_Y \times (X_A - X_B) - F_X \times (Y_A - Y_B)} \quad (17)$$

Equation (17) can be used together with Equations (14) and (15) to transfer forces and moment from the DEM to the FDM as force boundary conditions at nodes. Subroutines (FISH language) were developed to implement above Equations to carry out a coupled DEM-FDM simulation.

Acknowledgements

This research was conducted by the Australian Research Council Industrial Transformation Training Centre for Advanced Technologies in Rail Track Infrastructure (IC170100006) and funded by the Australian Government. Collaboration with Tim Neville (ARTC) for encouraging the use of recycled energy absorbing rubber mats (REAM) in real-life tracks is also appreciated, with subsequent support from organizations including the Australasian Centre for Rail Innovation (ACRI) and RM CRC (Project R2.5.1). The authors are also grateful to Alan Grant, Cameron Neilson, and Duncan Best for their assistance during the laboratory work. The authors also thank Robert Clayton (English editor) for proofreading and professionally editing the manuscript.

References

- AS:2758.7: (2015). "Aggregates and rock for engineering purposes, Part 7." *Railway Ballast. Standard Australia*, Australia.
- Biabani, M.M., Ngo, N.T. and Indraratna, B. (2016). "Performance evaluation of railway subballast stabilised with geocell based on pull-out testing." *Geotextiles and Geomembranes*, 44(4), 579-591.
- Chen, W.-B., Yin, J.-H., Feng, W.-Q., Borana, L. and Chen, R.-P. (2018). "Accumulated permanent axial strain of a subgrade fill under cyclic high-speed railway loading." *International Journal of Geomechanics*, 18(5), 04018018.
- Donohue, S., O'Sullivan, C. and Long, M. (2009). "Particle breakage during cyclic triaxial loading of a carbonate sand." *Géotechnique*, 59(5), 477-482.
- Esveld, C. (2001). *Modern railway track*, MRT Press, The Netherlands.
- Fu, Q., Liu, H., Ding, X. and Zheng, C. (2015). "Numerical investigation of piled raft foundation in mitigating embankment vibrations induced by high-speed trains." *Journal of Central South University*, 22(11), 4434-4444.
- Guo, W.-L. and Zhu, J.-G. (2017). "Particle breakage energy and stress dilatancy in drained shear of rockfills." *Géotechnique Letters*, 7(4), 304-308.
- Han, J., Bhandari, A. and Wang, F. (2011). "DEM analysis of stresses and deformations of geogrid-reinforced embankments over piles." *International Journal of geomechanics, ASCE*, 12(4), 340-350

451 Huang, H. and Tutumluer, E. (2011). "Discrete element modeling for fouled railroad ballast."
 452 *Construction and Building Materials*, 25, 3306-3312.

453 Indraratna, B., Lackenby, J. and Christie, D. (2005). "Effect of confining pressure on the
 454 degradation of ballast under cyclic loading " *Géotechnique*, 55(4), 325–328.

455 Indraratna, B., Ngo, N.T. and Rujikiatkamjorn, C. (2013). "Deformation of coal fouled ballast
 456 stabilized with geogrid under cyclic load." *Journal of Geotechnical and Geoenvironmental*
 457 *Engineering*, 139(8), 1275-1289.

458 Indraratna, B., Ngo, N.T., Rujikiatkamjorn, C. and Vinod, J. (2014). "Behaviour of fresh and
 459 fouled railway ballast subjected to direct shear testing - a discrete element simulation."
 460 *International Journal of Geomechanics, ASCE*, 14(1), 34-44.

461 Indraratna, B., Salim, W. and Rujikiatkamjorn, C. (2011). *Advanced Rail Geotechnology -*
 462 *Ballasted Track*, CRC Press, Taylor & Francis Group, London, UK

463 Indraratna, B., Thakur, P.K. and Vinod, J.S. (2010). "Experimental and numerical study of railway
 464 ballast behavior under cyclic loading." *International Journal of Geomechanics, ASCE*, 10(4),
 465 136-144.

466 Indraratna, B. and Ngo, T. (2018). "Ballast Railroad Design: Smart-Uow Approach", CRC Press,
 467 Taylor & Francis Group.

468 Ishikawa, T., Sekine, E. and Miura, S. (2011). "Cyclic deformation of granular material subjected
 469 to moving-wheel loads." *Canadian Geotechnical Journal*, 48(5), 691-703.

470 Janardhanam, R. and Desai, C.S. (1983). "Three dimensional testing and modeling of ballast."
 471 *Journal of Geotechnical Engineering, ASCE*, 109(6), 783-796.

472 Jiang, M., Li, T., Thornton, C. and Hu, H. (2017). "Wetting-induced collapse behavior of
 473 unsaturated and structural loess under biaxial tests using distinct element method."
 474 *International Journal of Geomechanics*, 17(1), 06016010.

475 Lackenby, J., Indraratna, B., McDowell, G. and Christie, D. (2007). "Effect of confining pressure
 476 on ballast degradation and deformation under cyclic triaxial loading." *Géotechnique*, 57(6),
 477 527-536.

478 Leng, W., Xiao, Y., Nie, R., Zhou, W. and Liu, W. (2017). "Investigating strength and deformation
 479 characteristics of heavy-haul railway embankment materials using large-scale undrained
 480 cyclic triaxial tests." *International Journal of Geomechanics*, 17(9), 04017074.

481 Lu, M. and McDowell, G.R. (2010). "Discrete element modelling of railway ballast under
 482 monotonic and cyclic triaxial loading." *Géotechnique*, 60(6), 459-467.

483 Lobo-Guerrero, S. and Vallejo, L.E. (2005). "Crushing a weak granular material: experimental
 484 numerical analyses." *Géotechnique*, 55(3), 245–249.

485 Lobo-Guerrero, S. and Vallejo, L.E. (2006). "Discrete element method analysis of railtrack ballast
 486 degradation during cyclic loading." *Granular Matter*, 8(3-4), 195-204.

487 McDowell, G.R., Harireche, O., Konietzky, H., Brown, S.F. and Thom, N.H. (2006). "Discrete
 488 element modelling of geogrid-reinforced aggregates." *Proceedings of the ICE - Geotechnical*
 489 *Engineering* 159(1), 35-48.

490 McDowell, G.R. and Li, H. (2016). "Discrete element modelling of scaled railway ballast under
491 triaxial conditions." *Granular Matter*, 18(3), 66.

492 Mishra, D., Qian, Y., Huang, H. and Tutumluer, E. (2014). "An integrated approach to dynamic
493 analysis of railroad track transitions behavior." *Transportation Geotechnics*, 1(4), 188-200.

494 Navaratnarajah, S.K., Indraratna, B. and Ngo, N.T. (2018). "Influence of under sleeper pads on
495 ballast behavior under cyclic loading: experimental and numerical studies." *Journal of*
496 *Geotechnical and Geoenvironmental Engineering*, 144(9), 04018068.

497 Ngo, N.T., Indraratna, B. and Rujikiatkamjorn, C. (2014). "DEM simulation of the behaviour of
498 geogrid stabilised ballast fouled with coal." *Computers and Geotechnics*, 55, 224-231.

499 Ngo, N.T., Indraratna, B. and Rujikiatkamjorn, C. (2017). "Simulation ballasted track behavior:
500 numerical treatment and field application." *ASCE-International Journal of Geomechanics*,
501 17(6), 04016130.

502 O'Sullivan, C., Cui, L. and O'Neill, C. (2008). "Discrete element analysis of the response of
503 granular materials during cyclic loading." *Soils and Foundations*, 48(4), 511-530.

504 Powrie, W., Yang, L.A. and Clayton, C.R.I. (2007). "Stress changes in the ground below ballasted
505 railway track during train passage." *Proceedings of the Institution of Mechanical Engineers:*
506 *Part F: Journal of Rail and Rapid Transit*, 247-261.

507 Qian, Y., Tutumluer, E., Mishra, D. and Kazmee, H. (2018). "Triaxial testing and discrete-element
508 modelling of geogrid-stabilised rail ballast." *Proceedings of the Institution of Civil Engineers*
509 *- Ground Improvement*, 171(4), 223-231.

510 Rothenburg, L. and Bathurst, R.J. (1989). "Analytical study of induced anisotropy in idealized
511 granular materials." *Géotechnique*, 39(4), 601-614.

512 Suiker, A.S.J. and Borst, R. (2003). "A numerical model for the cyclic deterioration of railway
513 tracks " *International Journal for Numerical methods in Engineering*, 57, 441-470.

514 Sun, Q., Indraratna, B. and Ngo, N.T. (2018). "Effect of increase in load and frequency on the
515 resilience of railway ballast." *Géotechnique*, 1-8. DOI: <https://doi.org/10.1680/jgeot.17.P.302>

516 Sun, Q.D., Indraratna, B. and Nimbalkar, S. (2014). "Effect of cyclic loading frequency on the
517 permanent deformation and degradation of railway ballast." *Géotechnique*, 64(9), 746-751.

518 Tutumluer, E., Huang, H. and Bian, X. (2012). "Geogrid-aggregate interlock mechanism
519 investigated through aggregate imaging-based discrete element modeling approach." *International Journal of Geomechanics*, 12(4), 391-398.

520

521 Ueng, T.-S. and Chen, T.-J. (2000). "Energy aspects of particle breakage in drained shear of
522 sands." *Géotechnique*, 50(1), 65-72.

523 Walker, R.T.R. and Indraratna, B. (2018). "Moving Loads on a Viscoelastic Foundation with
524 Special Reference to Railway Transition Zones." *International Journal of Geomechanics*,
525 18(11), 04018145.

526 Xiao, Y., Liu, H., Chen, Y. and Jiang, J. (2014a). "Strength and deformation of rockfill material
527 based on large-scale triaxial compression tests. I: Influences of density and pressure." *Journal*
528 *of Geotechnical and Geoenvironmental Engineering*, 140(12), 04014070.

529 Xiao, Y., Liu, H., Chen, Y. and Jiang, J. (2014b). "Strength and deformation of rockfill material
530 based on large-scale triaxial compression tests. II: Influence of particle breakage." *Journal of*
531 *Geotechnical and Geoenvironmental Engineering*, 140(12), 04014071.

532 Zhai, W.M., Wang, K.Y. and Lin, I.H. (2004). "Modelling and experiment of railwayballast
533 vibrations." *Journal of Sound and Vibration*, 673–683, 673–683.

534 Zhai, W., Wei, K., Song, X. and Shao, M. (2015). "Experimental investigation into ground
535 vibrations induced by very high speed trains on a non-ballasted track." *Soil Dynamics and*
536 *Earthquake Engineering*, 72, 24-36.

537 Zhang, X., Zhao, C. and Zhai, W. (2017). "Dynamic behavior analysis of high-speed railway
538 ballast under moving vehicle loads using discrete element method." *International Journal of*
539 *Geomechanics*, 17(7), 04016157.

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545 *Table 1. Details of cyclic triaxial tests carried out in the laboratory*

Test No.	Use of recycled rubber energy absorbing mats (REAM)	Maximum cyclic loading magnitude, $q_{cyc,max}$ (kPa)	Frequency, f (Hz)
1	No	230	10
2	No	230	20
3	No	230	30
4	No	230	40
5	Yes	230	10
6	Yes	230	20

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547 *Table 2. Micromechanical parameters used for the DEM analysis*

Parameters	Value
Particle unit weight (kN/m^3)	15.5
Radius of particle, r (m)	1.5×10^{-3} to 12×10^{-3}
Friction coefficient, μ	0.82
Contact normal stiffness, k_n (N/m)	2.52×10^8
Contact shear stiffness, k_s (N/m)	1.26×10^8
Contact normal stiffness of wall, $k_{n\text{-}wall}$ (N/m)	3.0×10^8
Shear stiffness of wall of wall, $k_{s\text{-}wall}$ (N/m)	1.5×10^8
Parallel bond normal and shear stiffness, k_p (N/m)	6.84×10^{10}
Parallel bond normal and shear strength, s_p (N/m ²)	4.86×10^6
Parallel bond radius multiplier, r_p	0.50

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561 Table 3. Material properties used for the coupling analysis

<div>Materials</div> <div>Property</div>	Capping	Subgrade	Rubber mat, REAM
Unit weight, γ (kN/m ³)	18.5	20.5	9.80
Elastic modulus, E (MPa)	112	47	12.8
Poisson's ratio, ν	0.3	0.32	0.48
Friction angle, ϕ (degrees)	46	39	-
Angle of dilation, ψ (degrees)	9	4	-
Cohesion, c (kPa)	-	12	-

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