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Fok Hing Chi Tivive

University of Wollongong, tivive@uow.edu.au

Abdesselam Bouzerdoun

University of Wollongong, bouzer@uow.edu.au

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Abstract

1980-2012 IEEE. Through-the-wall radar imaging (TWRI) is a sensing technology that can be used for detecting, locating, and identifying targets inside enclosed building structures. Many of the existing target classification approaches focus on single-target scene. For multi-target classification, the radar signal has to be segregated into different target components. However, target separation in TWRI is a challenging problem since the radar signals consist of both strong wall reflections and weak target echoes. Furthermore, the target signals are attenuated and distorted when propagating through the wall. In this paper, a variational model with low-rank constraint is proposed for decomposing the radar signal into target components and removing the wall returns. Experimental results show that the proposed method can effectively separate the radar signal into different target components.

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Through-the-Wall Target Separation using Low-Rank and Variational Mode Decomposition

Fok Hing Chi Tivive, *Member, IEEE*, and Abdesselam Bouzerdoun, *Senior Member, IEEE*

Abstract—Through-the-wall radar imaging is a sensing technology that can be used for detecting, locating, and identifying targets inside enclosed building structures. Many of the existing target classification approaches focus on single-target scene. For multi-target classification, the radar signal has to be segregated into different target components. However, target separation in through-the-wall radar imaging is a challenging problem since the radar signals consist of both strong wall reflections and weak target echoes. Furthermore, the target signals are attenuated and distorted when propagating through the wall. In this paper, a variational model with low-rank constraint is proposed for decomposing the radar signal into target components and removing the wall returns. Experimental results show that the proposed method can effectively separate the radar signal into different target components.

Index Terms—Target separation, through-the-wall radar imaging, low-rank constraint, variational mode decomposition.

I. INTRODUCTION

Through-the-wall radar imaging (TWRI) is a radar sensing technology with the capability to penetrate through opaque obstacles, such as walls and doors, for detecting, localizing, and identifying stationary or moving targets. It has many potential applications, including search-and-rescue, surveillance, and law enforcement [1]. Over the past two decades, researchers were focused on developing algorithms for image formation [2]–[8], wall clutter mitigation [9]–[11], multipath ghost removal [12]–[14], and target classification [15]–[24]. For target classification problems, several methods were proposed for stationary targets, based on the assumption that the radar data comprises the reflection from a single target [20]–[23]. For multi-target classification problems, however, there is a need to decompose the received radar signal into different individual target components. In [24], the radar signals are firstly converted into an image, which is then segmented into target, clutter and noise regions. Such an approach, however, involves image formation and image segmentation which are time-consuming tasks. Furthermore, the segmented output is dependent on the quality of the formed image. To the best of our knowledge, there is no study that has been devoted to tackling target separation in TWRI. This could be due to the difficulty of the problem since it involves two different

tasks: removing the strong wall reflections and separating the target echoes. Furthermore, due to strong wall attenuations, the target returns are often very weak and distorted, which renders the target separation problem more challenging. Nevertheless, there are source separation approaches that have been developed for other types of radar systems and radar-based applications [25]–[32].

Radar target separation can be regarded as a blind source separation since no prior information about the source signals or the mixing process is available. The objective of blind source separation is to estimate the mixing matrix for determining the source signals. A common approach for blind source separation is the independent component analysis (ICA), which imposes statistical independence and non-Gaussianity constraints on the source signals. ICA was applied to ground penetrating radar to separate the ground bounce from the buried target returns [25]. In [26], a fast-ICA algorithm in conjunction with K-means clustering was developed to separate target signals obtained from passive radar systems. Zhu *et al.* combined eigenvalue decomposition and ICA to separate multi-component signals from a single-channel radar [27]. Several ICA-based approaches were proposed for TWRI not for separating target signals, but for removing wall clutter [28]–[30]. There are other approaches proposed for radar signal separation which are not based on ICA. Wang *et al.* adopted the autoregressive model to separate radar signals of different pulse modulations and applied Bayesian or Akaike information criterion to determine the order of the model [31]. Mehmood and Damarla introduced kernel non-negative matrix factorization (KNMF) for separating seismic signals. The nonnegativity constraint was enforced by first converting the seismic signal into a spectrogram and then applying the KNMF technique to decompose the spectrogram into two non-negative matrices. Nguyen *et al.* exploited the sparsity of the signal representation to extract the radar return of the main rotor blade from the received doppler signal [32]. The objective of the aforementioned approaches is either separating the target signal from the clutter signal or decomposing the radar signal into several target components of similar magnitude.

In this paper, a target separation method is proposed for TWRI, which can estimate the wall reflections and segregate the different target returns. Since the wall reflections are highly correlated across the antennas, it is expected that they lie in a low-rank subspace, as mentioned in [33] and [34]. Moreover, the radar signal, which comprises delayed target echoes, can be modeled as a superposition of several narrow-band signals. Therefore, based on these two observations, a variational model with a low-rank constraint is formulated to capture

F. H. C. Tivive is with the School of Electrical, Computer and Telecommunications Engineering, University of Wollongong, Northfields Avenue, Wollongong NSW 2522, Australia (Email: tivive@uow.edu.au).

A. Bouzerdoun is with the Information and Computing Technology Division, College of Science and Engineering, Hamad Bin Khalifa University, Doha, Qatar, and the School of Electrical, Computer and Telecommunications Engineering, University of Wollongong, Northfields Avenue, Wollongong NSW 2522, Australia.

the wall clutter as a low-rank representation and decompose the radar signal into different modes. Each mode can be considered as an amplitude-modulated frequency-modulated signal carrying a target return. The proposed model is based on the variational mode decomposition [35] and is optimized using the alternating direction method of multipliers (ADMM) technique. Contrary to the traditional ICA-based source separation method, the proposed target separation method does not have the permutation ambiguity because the extracted modes are associated with the initial center frequencies. Thus, when the initial center frequencies are sorted in ascending or descending order, the modes maintain the same order.

The remainder of the paper is organized as follows. Section II presents the signal model for through-the-wall radar imaging and DS beamforming. Section III briefly reviews the variational mode decomposition algorithm for real-valued and complex-valued signals. The proposed variational model for target separation is described in Section IV, followed by experimental results and discussion in Section V. Finally, Section VI gives the conclusion.

II. THROUGH-WALL RADAR SIGNAL AND DELAY-AND-SUM BEAMFORMING

This section presents TWRI signal model and the traditional delay and sum beamforming used for image formation.

A. Through-Wall Radar Signal Model

Consider a mono-static stepped-frequency synthetic aperture radar for imaging targets located behind the wall. Figure 1 depicts the through-the-wall geometry, showing the radar signal propagation through the air-wall-air interfaces. At each scan position, the antenna transmits and receives a wideband stepped-frequency signal comprising M frequencies, equally spaced over the sensing bandwidth. We assume the transceiver is placed at a standoff distance z_{off} and moved horizontally to synthesize an N -element linear antenna array. Moreover, there are P targets behind the wall. Then, the radar signal $y_n(m)$ for the m th frequency at the n th scan position can be modeled as the superposition of the wall return $y_n^w(m)$, target signal $y_n^t(m)$, and noise $e_n(m)$:

$$y_n(m) = y_n^w(m) + y_n^t(m) + e_n(m), \quad (1)$$

for $n = 1, \dots, N$ and $m = 1, \dots, M$. The wall return $y_n^w(m)$ and target signal $y_n^t(m)$ can be expressed as [36]:

$$y_n^w(m) = \sum_{r=1}^R \sigma^w A_r e^{-j2\pi f_m \tau_{n,r}^w}, \quad (2)$$

and

$$y_n^t(m) = \sum_{p=1}^P \int_{\tau_{n,p}^{\min}}^{\tau_{n,p}^{\max}} \sigma_p^t(\tau) e^{-j2\pi f_m \tau} d\tau, \quad (3)$$

where

- σ^w is the complex reflectivity of the wall,
- R is the number of wall reverberations,
- $\tau_{n,1}^w$ is the two-way propagation delay of the direct return from the wall to the n th antenna,

- $\tau_{n,r}^w$ is the two-way propagation delay of the r th wall reverberation for $r > 1$,
- A_r is the path loss factor of the r th wall return,
- $\sigma_p^t(\tau)$ is the complex reflectivity function of the p th target (assuming the target reflectivity is independent of frequency), and
- $\tau_{n,p}^{\min}$ and $\tau_{n,p}^{\max}$ are, respectively, the minimum and maximum two-way propagation delays between the n th antenna and the p th target.

Suppose the stepped-frequency signal has M frequencies over the desired bandwidth $f_M - f_1$,

$$f_m = f_1 + (m-1)\Delta f \quad \text{for } m = 1, \dots, M, \quad (4)$$

where f_1 is the lowest frequency in the desired frequency band and Δf is the frequency step size. The wall and target returns in (2) and (3) can be rewritten as, respectively,

$$y_n^w(m) = \sum_{r=1}^R \vartheta_{n,r}^w e^{-j\omega_{n,r}^w(m-1)} \quad (5)$$

and

$$y_n^t(m) = \sum_{p=1}^P \vartheta_{n,p}^t(m), \quad (6)$$

where $\vartheta_{n,r}^w = \sigma^w A_r e^{-j2\pi f_1 \tau_{n,r}^w}$, $\omega_{n,r}^w = 2\pi \Delta f \tau_{n,r}^w$, $\vartheta_{n,p}^t(m) = \int_{\tau_{n,p}^{\min}}^{\tau_{n,p}^{\max}} \sigma_p^t(\tau) e^{-j2\pi f_1 \tau} e^{-j\omega_{n,p}^t(m-1)} d\tau$, and $\omega_{n,p}^t = 2\pi \Delta f \tau$. From Eq. (5), it is clear that when considering only the direct wall return, the signal associated with the wall resides in a one-dimensional subspace. If the propagation delays of the wall reverberations do not change significantly along the antenna array, the wall contributions are expected to lie in a low-dimensional subspace. On the other hand, the target signal given in (6) can be regarded as an aggregation of P narrow-band signals.

Fig. 1. A schematic diagram of a through-the-wall scene showing the propagation path of the radar signal from the target to the antenna.

B. Delay-and-Sum Beamforming

Delay-and-sum (DS) beamforming is a common image formation technique for TWRI. The scene is firstly divided into a rectangular grid consisting of Q pixels. Then, the magnitude of the q th pixel is computed as follow:

$$I(q) = \left| \frac{1}{NM} \sum_{n=1}^N \sum_{m=1}^M y_n(m) \exp(j2\pi f_m \tau_{n,q}) \right|, \quad (7)$$

where $|\cdot|$ denotes the modulus operator and $\tau_{n,q}$ is the focusing delay between the n th antenna and the q th pixel; for more details on the computation of the focusing delay, the reader is referred to [37].

III. VARIATIONAL MODE DECOMPOSITION AND ITS COMPLEX-VALUED COUNTERPART

Variational mode decomposition (VMD) is a technique that concurrently decomposes a signal into a number of quasi-orthogonal band-limited modes by solving a constrained optimization problem using variational calculus [35]. Each mode is compact at a center frequency and has limited bandwidth in the spectral domain. The estimation of the bandwidth of a mode comprises three steps: (i) apply Hilbert transform to compute the analytic signal, (ii) perform a heterodyne demodulation to shift the frequency of the mode to baseband, and (iii) estimate the bandwidth of the mode through the H^1 -norm (Dirichlet energy) of the demodulated signal, i.e., the square L^2 -norm of the gradient. Let $z(t)$ be a real-valued signal, u_l be the l th mode, ω_l be the l th center frequency, and L be the number of modes. The constrained variational problem can be formulated as

$$\begin{aligned} \min_{\{u_l(t)\}, \{\omega_l\}} & \sum_{l=1}^L \left\| \partial_t \left[\left(\delta(t) + \frac{j}{\pi t} \right) * u_l(t) \right] e^{-j\omega_l t} \right\|_2^2 \\ \text{s.t.} & \sum_{l=1}^L u_l(t) = z(t), \end{aligned} \quad (8)$$

where the square L^2 -norm is understood as $\|\cdot\|_2^2 = \int |\cdot|^2 dt$, $\delta(\cdot)$ denotes Dirac delta function, $j^2 = -1$, ∂_t is the derivative with respect to t , $*$ denotes convolution operator, and $\{u_l(t)\} = \{u_1(t), \dots, u_L(t)\}$ and $\{\omega_l\} = \{\omega_1, \dots, \omega_L\}$ are shorthand notations for the set of all modes and their center frequencies, respectively. By adding a quadratic penalty term and a Lagrange multiplier, Problem (8) can be written as an augmented Lagrangian:

$$\begin{aligned} \mathcal{L}(u_l(t), \omega_l, \lambda(t)) &= \alpha \sum_{l=1}^L \left\| \partial_t \left[\left(\delta(t) + \frac{j}{\pi t} \right) * u_l(t) \right] e^{-j\omega_l t} \right\|_2^2 \\ &+ \left\| z(t) - \sum_{l=1}^L u_l(t) \right\|_2^2 + \left\langle \lambda(t), z(t) - \sum_{l=1}^L u_l(t) \right\rangle, \end{aligned} \quad (9)$$

where $\lambda(t)$ is the Lagrange multiplier, $\langle \cdot \rangle$ is the inner product, and α is a parameter balancing the variational energy term and the fidelity term. The purpose of the Lagrange multiplier is to enforce the constraint, whereas the quadratic penalty is to improve convergence.

The solution to the minimization problem (9) can be defined as the saddle point of the augmented Lagrangian using the alternating direction method of multipliers (ADMM) technique [35]. This optimization technique has global and linear convergence under sufficient conditions [38]. Therefore, ADMM is employed to minimize the objective function (9) by alternately solving the following Subproblems:

$$\begin{aligned} \min_{u_l(t)} & \left\| \partial_t \left[\left(\delta(t) + \frac{j}{\pi t} \right) * u_l(t) \right] e^{-j\omega_l t} \right\|_2^2 \\ & + \left\| z(t) - \sum_{l=1}^L u_l(t) + \frac{\lambda(t)}{2} \right\|_2^2, \end{aligned} \quad (10)$$

and

$$\min_{\omega_l} \left\| \partial_t \left[\left(\delta(t) + \frac{j}{\pi t} \right) * u_l(t) \right] e^{-j\omega_l t} \right\|_2^2 \quad \forall l. \quad (11)$$

Making use of the Parseval/Plancherel Fourier isometry under the L^2 norm, Subproblem (10) can be rewritten in the spectral domain as

$$\begin{aligned} \min_{\hat{u}_l(\omega)} & \left\| (j\omega) [1 + \text{sgn}(\omega + \omega_l) \hat{u}_l(\omega + \omega_l)] \right\|_2^2 \\ & + \left\| \hat{z}(\omega) - \sum_{l=1}^L \hat{u}_l(\omega) + \frac{\hat{\lambda}(\omega)}{2} \right\|_2^2 \quad \forall l, \end{aligned} \quad (12)$$

where $\hat{z}(\omega)$, $\hat{u}_l(\omega)$, and $\hat{\lambda}(\omega)$ are the Fourier transform of $z(t)$, $u_l(t)$, and $\lambda(t)$, respectively. Substituting the variable ω with $\omega - \omega_l$ in the first term of (12), we have

$$\begin{aligned} \min_{\hat{u}_l(\omega)} & \left\| j(\omega - \omega_l) [1 + \text{sgn}(\omega) \hat{u}_l(\omega)] \right\|_2^2 \\ & + \left\| \hat{z}(\omega) - \sum_{l=1}^L \hat{u}_l(\omega) + \frac{\hat{\lambda}(\omega)}{2} \right\|_2^2. \end{aligned} \quad (13)$$

By exploiting the Hermitian symmetry property of a real-valued signal, both terms in (13) can be written as half-space integrals over non-negative frequencies:

$$\begin{aligned} \min_{\hat{u}_l(\omega)} & \int_0^\infty 4\alpha(\omega - \omega_l)^2 |\hat{u}_l(\omega)|^2 + \\ & 2 \left| \hat{z}(\omega) - \sum_{l=1}^L \hat{u}_l(\omega) + \frac{\hat{\lambda}(\omega)}{2} \right|^2 d\omega. \end{aligned} \quad (14)$$

Now, taking the first variation with respect to $\hat{u}_l(\omega)$ and setting it to zero yields

$$8\alpha(\omega - \omega_l)^2 \hat{u}_l(\omega) - 4 \left(\hat{z}(\omega) - \sum_{l=1}^L \hat{u}_l(\omega) + \frac{\hat{\lambda}(\omega)}{2} \right) = 0. \quad (15)$$

The update of the l th mode can be expressed as

$$\hat{u}_l(\omega)^* = \frac{\hat{z}(\omega) - \sum_{i \neq l} \hat{u}_i(\omega) + \frac{\hat{\lambda}(\omega)}{2}}{1 + 2\alpha(\omega - \omega_l)^2}. \quad (16)$$

The solution given in (16) can be regarded as a Wiener filtering of the current residual with signal prior $1/(\omega - \omega_l)^2$, where α controls the width of the Wiener filter. A larger value of α produces a narrower Wiener filter that removes more noise but at the same time reduces the spectral information around the

center frequency of the mode, and vice-versa. Inverse Fourier transform is then applied to the output given in (16) to produce the extracted mode in the time domain.

Similarly, Subproblem (11) is solved in the spectral domain, and it can be rewritten as

$$\min_{\omega_l} \int_0^\infty (\omega - \omega_l)^2 |\hat{u}_l(\omega)|^2 d\omega \quad \forall l. \quad (17)$$

Its solution in the Fourier domain is given by

$$\omega_l^* = \frac{\int_0^\infty \omega |\hat{u}_l(\omega)|^2 d\omega}{\int_0^\infty |\hat{u}_l(\omega)|^2 d\omega} \quad \forall l. \quad (18)$$

The standard gradient ascent technique with a fixed step size $\mu \geq 0$ is used to maximize the Lagrange multiplier

$$\hat{\lambda}(\omega)^* = \hat{\lambda}(\omega) + \mu \left(\hat{z}(\omega) - \sum_{l=1}^L \hat{u}_l^*(\omega) \right). \quad (19)$$

In [39], the variational mode decomposition algorithm was extended to complex-valued signals. Since the spectrum of a complex-valued harmonic is single-sided, the Hilbert transform step is removed. Thus, the constrained variational problem for decomposing a complex-valued signal $g(t)$ can be written as

$$\min_{\{u_l(t)\}, \{\omega_l\}} \sum_{l=1}^L \left\| \partial_t [u_l(t) e^{-j\omega_l t}] \right\|_2^2 \quad \text{s.t.} \quad \sum_{l=1}^L u_l(t) = g(t). \quad (20)$$

Combining the variational energy term with the constraint produces the following augmented Lagrangian:

$$\mathcal{L}(u_l(t), \omega_l, \lambda(t)) = \alpha \sum_{l=1}^L \left\| \partial_t [u_l(t) e^{-j\omega_l t}] \right\|_2^2 + \left\| g(t) - \sum_{l=1}^L u_l(t) + \frac{\lambda(t)}{2} \right\|_2^2. \quad (21)$$

The ADMM algorithm is then applied to minimize (21) by solving the following Subproblems:

$$\min_{u_l(t)} \alpha \left\| \partial_t [u_l(t) e^{-j\omega_l t}] \right\|_2^2 + \left\| g(t) - \sum_{l=1}^L u_l(t) + \frac{\lambda(t)}{2} \right\|_2^2, \quad (22)$$

and

$$\min_{\omega_l} \left\| \partial_t [u_l(t) e^{-j\omega_l t}] \right\|_2^2 \quad \forall l. \quad (23)$$

Their respective solutions in the Fourier domain are given by

$$\hat{u}_l(\omega)^* = \frac{\hat{g}(\omega) - \sum_{i \neq l} \hat{u}_i(\omega) + \frac{\hat{\lambda}(\omega)}{2}}{1 + \alpha(\omega - \omega_l)^2}. \quad (24)$$

and

$$\omega_l^* = \frac{\int_{-\infty}^\infty \omega |\hat{u}_l(\omega)|^2 d\omega}{\int_{-\infty}^\infty |\hat{u}_l(\omega)|^2 d\omega} \quad \forall l, \quad (25)$$

where $\hat{g}(\omega)$ is the Fourier transform of $g(t)$.

IV. PROPOSED TARGET SEPARATION METHOD FOR TWRI

In this section, a variation model with low-rank constraint is proposed for through-the-wall target separation, followed by the Bayesian optimization algorithm for estimating the hyperparameters of the model.

A. Variational Model with Low-Rank Constraint

The signals received across the array aperture contain not only the target reflections, but also the direct wall returns, wall reverberations, and noise. Since the direct wall returns and wall reverberations are stronger than the target reflections, they need to be removed or, at least, significantly suppressed before target separation. Therefore, to remove the wall reflections from the radar signal, the same technique described in [33] and [34] is adopted, i.e., capturing the direct wall returns and the wall reverberations in a low-rank matrix. Let $\mathbf{Y}$ denote a matrix containing radar signals as its columns. Likewise, the wall returns, the target signals, and the noise components are denoted by $\mathbf{Y}^w = [y_n^w(m)]_{MN}$, $\mathbf{Y}^t = [y_n^t(m)]_{MN}$, and $\mathbf{E} = [e_n(m)]_{MN}$, respectively. Equation (1) can be expressed in matrix form as

$$\mathbf{Y} = \mathbf{Y}^w + \mathbf{Y}^t + \mathbf{E}. \quad (26)$$

In [33] and [34], the separation of the wall and target signals was cast as a joint low-rank and sparse constrained optimization problem:

$$\min_{\mathbf{Y}^w, \mathbf{Y}^t} \|\mathbf{Y}^w\|_* + \gamma \|\mathbf{Y}^t\|_1 \quad \text{s.t.} \quad \|\mathbf{Y} - (\mathbf{Y}^w + \mathbf{Y}^t)\|_F^2 \leq \epsilon. \quad (27)$$

where $\|\cdot\|_*$ denotes the nuclear norm of the matrix argument (i.e., the sum of its singular values), $\|\cdot\|_1$ denotes the ℓ_1 -norm (i.e., the sum of the absolute values of matrix entries), $\|\cdot\|_F$ denotes the Frobenius norm, γ is a positive regularization parameter, and ϵ is a noise bound. The low-rank term captures the wall clutter with the assumption that the wall reflections across the antenna array lie in a single low-rank subspace. However, due to the misalignment of the antenna array, the roughness of the wall surface, and the inhomogeneity of the wall, the wall components can span a union of low-rank subspaces. Let $\mathbf{D}$ denote a dictionary of size $M \times Q$, consisting of Q atoms and $\mathbf{Z} = [z_n(i)]_{QN}$ be the low-rank representation obtained from the dictionary $\mathbf{D}$. Therefore, the low-rank constraint is reformulated as

$$\min_{\mathbf{Z}} \|\mathbf{Z}\|_* \quad \text{s.t.} \quad \mathbf{Y}^w = \mathbf{D}\mathbf{Z}. \quad (28)$$

The low-rank model (28), which was introduced by Liu *et al.* [40] for subspace segmentation, was shown to perform better than the robust principal component analysis and could deal with outliers. The proposed variational model with low-rank constraint is formulated as

$$\begin{aligned} \min_{\mathbf{Z}, \{u_{n,l}(m)\}} \quad & \|\mathbf{Z}\|_* + \sum_{n=1}^N \sum_{l=1}^L \left\| \partial_m [u_{n,l}(m) e^{-j\omega_{n,l} m}] \right\|_2^2 \\ \text{s.t.} \quad & \sum_{l=1}^L u_{n,l}(m) = y_n(m) - \sum_{i=1}^Q D(m, i) z_n(i) \quad \forall n. \end{aligned} \quad (29)$$

Similar to the VMD technique, a quadratic penalty term and Lagrange multipliers are added to convert (29) into the following augmented Lagrangian:

$$\begin{aligned} \mathcal{L}(\mathbf{Z}, u_{n,l}(m), \omega_{n,l}, b_n(m)) = & \beta \|\mathbf{Z}\|_* + \\ & \alpha \sum_{n=1}^N \sum_{l=1}^L \left\| \partial_m [u_{n,l}(m) e^{-j\omega_{n,l}m}] \right\|_2^2 + \\ & \left\langle b_n(m), y_n(m) - \sum_{i=1}^Q D(m, i) z_n(i) - \sum_{l=1}^L u_{n,l}(m) \right\rangle + \\ & \sum_{n=1}^N \left\| y_n(m) - \sum_{i=1}^Q D(m, i) z_n(i) - \sum_{l=1}^L u_{n,l}(m) \right\|_2^2, \end{aligned} \quad (30)$$

where β is a penalty parameter controlling the amount of wall clutter to be removed and $b_n(m)$ is the Lagrange multiplier associated with the n th antenna. Using ADMM, the objective function (30) can be decomposed into the following Subproblems:

$$\begin{aligned} & \min_{\mathbf{Z}} \beta \|\mathbf{Z}\|_* + \\ & \sum_{n=1}^N \left\| y_n(m) - \sum_{i=1}^Q D(m, i) z_n(i) - \sum_{l=1}^L u_{n,l}(m) + \frac{b_n(m)}{2} \right\|_2^2, \end{aligned} \quad (31)$$

$$\begin{aligned} & \min_{u_{n,l}(m)} \alpha \left\| \partial_m [u_{n,l}(m) e^{-j\omega_{n,l}m}] \right\|_2^2 + \\ & \left\| y_n(m) - \sum_{i=1}^Q D(m, i) z_n(i) - \sum_{l=1}^L u_{n,l}(m) + \frac{b_n(m)}{2} \right\|_2^2, \end{aligned} \quad (32)$$

and

$$\min_{\omega_{n,l}} \left\| \partial_m [u_{n,l}(m) e^{-j\omega_{n,l}m}] \right\|_2^2 \quad \forall n, l. \quad (33)$$

Subproblem (31) is a least squares problem regularized by a nuclear norm penalty, which can be concisely written as

$$\min_{\mathbf{Z}} \beta \|\mathbf{Z}\|_* + \left\| \mathbf{G} - \mathbf{D}\mathbf{Z} \right\|_F^2, \quad (34)$$

where $\mathbf{G} = [y_n(m) - \sum_{l=1}^L u_{n,l}(m) + 0.5b_n(m)]_{MN}$. This rank minimization problem can be solved using the linearization and proximal technique proposed by Lin *et al.* [42]. First, the quadratic term in (34) can be approximated by the following linearization:

$$\left\| \mathbf{G} - \mathbf{D}\mathbf{Z} \right\|_F^2 \approx \left\| \mathbf{G} - \mathbf{D}\mathbf{Z}^k \right\|_F^2 + \langle \nabla \mathbf{Z}^k, \mathbf{Z} - \mathbf{Z}^k \rangle + \frac{1}{\rho} \left\| \mathbf{Z} - \mathbf{Z}^k \right\|_F^2, \quad (35)$$

where $0 < \rho < 1/\mathcal{S}(\mathbf{D}^T \mathbf{D})$ is a proximal parameter, $\mathcal{S}(\mathbf{D}^T \mathbf{D})$ denotes the spectral radius of $\mathbf{D}^T \mathbf{D}$, and $\nabla \mathbf{Z}^k$ denotes the gradient of $\left\| \mathbf{G} - \mathbf{D}\mathbf{Z}^k \right\|_F^2$ at $\mathbf{Z}^k$ [41]. By plugging (35) in (34) and with simple mathematical manipulations, we obtain the following approximation to (34):

$$\min_{\mathbf{Z}} \beta \|\mathbf{Z}\|_* + \frac{1}{\rho} \left\| \mathbf{Z} - (\mathbf{Z}^k - \rho \nabla \mathbf{Z}^k) \right\|_F^2. \quad (36)$$

Then, Subproblem (36) can be efficiently solved using the singular value thresholding (SVT) technique, which performs two

operations: SVD and soft-thresholding [43]. Let the element-wise soft-thresholding operator be defined by

$$\mathcal{T}(a, b) = \text{sgn}(a) \max(|a| - b, 0), \quad (37)$$

where $\text{sgn}(\cdot)$ denotes the signum function. The minimization of Subproblem (36) can be performed by applying the SVT technique as follows:

$$\text{SVD}(\mathbf{Z}^k - \rho \nabla \mathbf{Z}^k) = \mathbf{U} \mathbf{\Sigma} \mathbf{V}^H, \quad (38)$$

$$\mathbf{Z}^{k+1} = \mathbf{U} \mathcal{T}(\mathbf{\Sigma}, \beta \rho) \mathbf{V}^H, \quad (39)$$

where ^{H} denotes the Hermitian transpose, $\mathbf{U}$ and $\mathbf{V}$ are unitary matrices, and $\mathbf{\Sigma}$ is a diagonal matrix of singular values. Subproblems (32) and (33) are solved in the Fourier domain; their solutions are, respectively, given by

$$\hat{u}_{n,l}(\omega)^{k+1} = \frac{\hat{g}_n(\omega)^{k+1} - \sum_{i \neq l} \hat{u}_{n,i}(\omega) + \frac{\hat{b}_n(\omega)}{2}}{1 + \alpha(\omega - \omega_{n,l})^2} \quad (40)$$

and

$$\omega_{n,l}^{k+1} = \frac{\int_{-\infty}^{\infty} \omega |\hat{u}_{n,l}(\omega)^{k+1}|^2 d\omega}{\int_{-\infty}^{\infty} |\hat{u}_{n,l}(\omega)^{k+1}|^2 d\omega} \quad \forall n, l, \quad (41)$$

where $\hat{g}_n(\omega)^{k+1}$ is the Fourier transform of $g_n(m)^{k+1} = y_n(m) - \sum_{i=1}^Q D(m, i) z_n(i)^{k+1}$. The Lagrange multiplier is updated as

$$\hat{b}_n(\omega)^{k+1} = \hat{b}_n(\omega)^k + \mu \left(\hat{g}_n(\omega)^{k+1} - \sum_{l=1}^L \hat{u}_{n,l}(\omega)^{k+1} \right). \quad (42)$$

The main steps of the proposed variational method with low-rank constraint is described in Algorithm-1, presented in the Appendix. The proposed target separation method has four hyper-parameters: β , α , μ , and L . The first parameter β controls the amount of wall clutter to be removed from the radar signal. The second parameter α is related to the width of the Wiener filter for estimating the modes. The third parameter μ , which is the step size for updating the Lagrange multiplier, is used to enforce the constraint in (29). The last parameter L defines the number of modes to be extracted from the radar signals; overestimating L can lead to mode splitting, whereas underestimating L can cause mode mixing. The following section presents a technique to estimate the hyper-parameters of the proposed method.

B. Hyper-parameters Estimation

If the number of targets and their locations are known, the parameter L is set to the total number of targets and the initial frequency of the p th mode for the n th antenna is defined as

$$f_{n,p} = \Delta f \tau_{n,p}, \quad (43)$$

where $\tau_{n,p}$ is the two-way propagation delay from the n th antenna to the p th target. However, these two parameters are not always available in practical applications. Therefore, Bayesian optimization (BO) with Gaussian process is employed to estimate the hyper-parameters. The rationale for employing the BO technique is that it does not need to evaluate the main objective function for every estimated set of hyper-parameters, and the objective function does not require an

exact functional form. The BO technique has been used for tuning hyper-parameters of machine learning methods, such as convolutional neural networks [44], support vector machines [45], and deep belief networks [46] as it is well-suited for global optimization problems.

Let $\mathcal{M}_\gamma$ be the optimized target separation model using the hyper-parameter vector $\gamma = [\beta, \alpha, \mu, L] \in \Lambda$, where Λ is the bounded hyper-parameter space, and $\mathcal{F}$ denote the cost function that needs to be maximized using the BO technique:

$$\gamma_* = \arg \max_{\gamma \in \Lambda} \mathcal{F}(\mathcal{M}_\gamma(\mathbf{Y})). \quad (44)$$

The BO technique requires a prior $p(\mathcal{F})$ over the function and an acquisition function $\mathcal{A} : \Lambda \rightarrow \mathbb{R}^+$ to determine what point in Λ should be evaluated next using a proxy optimization $\gamma_* = \arg \max_{\gamma} \mathcal{A}(\gamma)$. Let x denote the output of the cost function $\mathcal{F}$. To find the optimal γ vector, the BO technique iterates the following three steps: (i) solve the proxy optimization $\gamma_{v+1} = \arg \max_{\gamma \in \Lambda} \mathcal{A}(\gamma)$, (ii) evaluate the cost function $x_{v+1} \sim \mathcal{F}(\mathcal{M}_{\gamma_{v+1}}(\mathbf{Y})) + \mathcal{N}(0, \sigma^2)$, which can be noisy, and add the resulting data point (γ_{v+1}, x_{v+1}) to the set of observations $\mathcal{D}_{v+1} = \{\gamma_j, x_j\}_{j=1}^{v+1}$, and (iii) update $p(\mathcal{F}|\mathcal{D}_{v+1})$ and $\mathcal{A}(\mathcal{F}|\mathcal{D}_{v+1})$, where v is the iteration index. Gaussian process is a prominent choice for $p(\mathcal{F})$, due to its flexibility and tractability. It is specified by its mean function $m(\gamma)$ and covariance function $c(\gamma_i, \gamma_j)$. Using the property of Gaussian distribution, the prior mean and covariance can be computed in a closed form. The prior mean function can be assumed to be zero in Gaussian process without any loss of generality, and hence the Gaussian process can be fully defined by the covariance function. For hyper-parameters, the ARD Matérn 5/2 kernel is used as the covariance function [47]; it is given by

$$c_{M52}(\gamma_i, \gamma_j) = \theta(1 + \sqrt{5}d(\gamma_i, \gamma_j)) + \frac{5}{3}(d(\gamma_i, \gamma_j))^2 \exp(-\sqrt{5}d(\gamma_i, \gamma_j)), \quad (45)$$

where $d(\gamma_i, \gamma_j)$ is the Mahalanobis distance and θ is the characteristic length scale. The characteristic length scale defines how far apart the input γ can be for the response value to become uncorrelated. The expected improvement is used as the acquisition function [49]. Suppose the current best observation at iteration v is $\gamma_* = \arg \max_{\gamma_i \in \mathcal{Y}_{1:v}} \mathcal{F}(\mathcal{M}_\gamma(\mathbf{Y}))$. The expected improvement EI can be expressed in closed form as [49]:

$$\text{EI}_v(\gamma) = \begin{cases} (m(\gamma) - \mathcal{F}(\mathcal{M}_{\gamma_*}))\eta(z) + s(\gamma)\kappa(z), & \text{if } s(\gamma) > 0 \\ 0, & \text{if } s(\gamma) = 0, \end{cases} \quad (46)$$

where $z = (m(\gamma) - \mathcal{F}(\mathcal{M}_{\gamma_*}))/s(\gamma)$, $m(\gamma)$ and $s(\gamma)$ are the mean and the standard deviation of the posterior Gaussian process at γ_* , $\eta(\cdot)$ and $\kappa(\cdot)$ are the cumulative distribution function and probability density function of a standard normal distribution, respectively.

The output of the cost function $\mathcal{F}$ is defined as the Gini index of the saliency map, which is computed from the

formed image. The Gini index (GI) of the saliency map m_s is computed as

$$\text{GI} = 1 - 2 \sum_{i=1}^G \frac{|\hat{m}_s(i)|}{\|\hat{\mathbf{m}}_s\|_1} \left(\frac{G-i+0.5}{G} \right), \quad (47)$$

where $\hat{\mathbf{m}}_s$ denotes a column vector containing the elements of the saliency map in ascending order, i.e., $|\hat{m}_s(1)| < |\hat{m}_s(2)|, \dots, < |\hat{m}_s(G)|$. The GI lies within the range $[0, 1]$, where a GI of zero means a least sparse map and a GI of one yields the most sparse map.

The saliency map is obtained by converting the extracted modes into an image and applying the two-dimensional Fourier transform. The composite image is computed as follows: the L modes are converted into L mode images using (7) $\{I_1, \dots, I_L\}$, and the L modes are added to produce another image I_p . The composite image $\hat{I} \in \mathbb{R}^G$ is then computed as

$$\hat{I}(i) = \frac{I_p(i)}{L} \sum_{j=1}^L I_j(i) \quad \forall i. \quad (48)$$

Note that each mode image is re-scaled to $[0, 1]$ by dividing by the maximum pixel value. To compute the saliency map, the composite image $\hat{I}$ is rearranged into an $R \times C$ matrix, i.e., $\hat{I} \in \mathbb{R}^G \rightarrow \mathbb{R}^{R \times C}$, and the two-dimensional (2D) Fast Fourier transform is performed on local sliding windows:

$$M_s(i, j) = \max(|\text{FFT2}\{\hat{I}(m, n), (m, n) \in W\}|) \\ i, m = 1, \dots, R, j, n = 1, \dots, C, \quad (49)$$

where FFT2 and W denote, respectively, 2D fast Fourier transform and the 2D local window centered on the pixel (m, n) of the composite image $\hat{I}$. Figures 2(a) and (b) depict, respectively, the images I_p and $\hat{I}$ obtained from the extracted modes. The background clutter in Fig 2(b) is significantly suppressed compared to Fig 2(a). Figure 2(c) illustrates the saliency map using a 5×5 local window.

The estimation of the hyper-parameter vector γ using the BO technique is a two-step procedure. In the first step, the number of targets and their locations are determined. Let $\tau_{Z_{\text{off}}}$ and τ_R denote, respectively, the two-way propagation delays from the antenna to the wall located at the standoff distance Z_{off} and the maximum down-range of the region-of-interest. The frequencies of the modes lie in the interval $[\Delta f \tau_{Z_{\text{off}}}, \Delta f \tau_R]$. Suppose there are L targets in the scene. The radar signal is decomposed into L modes, and the initial frequency of the i th mode is defined as

$$\omega_i = 2\pi \Delta f (\tau_{Z_{\text{off}}} + i d_\tau), \quad i = 1, \dots, L, \quad (50)$$

where Δf denotes the frequency step size and $d_\tau = (\tau_R - \tau_{Z_{\text{off}}})/L$. The target separation method employs the estimated hyper-parameters to find the target signal, which is then used to form a saliency map. A two-dimensional cell-averaged CFAR (CA-CFAR) detector is applied to the saliency map to detect the correct number of targets and estimate their locations. In the second step, with the estimated number of targets, the initial frequencies of the modes are recomputed, and the BO technique is applied again to refine the hyper-parameters β , α , and μ .

Fig. 2. Images formed using the estimated target signals: (a) the beamformed image of the scene, (b) image weighted by the mode images and (c) the saliency map.

V. EXPERIMENTAL RESULTS

In this section, the proposed target separation method for through-the-wall radar is evaluated on simulated and real radar data. The experimental setup for collecting simulated and real radar signals is firstly described, followed by an analysis of performance in terms of the low-rank dictionary type, the target position, and the number of targets, using simulated data. Then, the proposed method and an ICA-based technique are evaluated on real radar data.

A. Experimental Setup and Evaluation Protocol

Two radar datasets are collected for the evaluation of the target separation method. The first dataset is obtained using the full-wave simulation software XFDTD, developed by Remcom¹. An array aperture of 51 elements with an inter-element spacing of 0.028 m is synthesized to interrogate a region-of-interest of size 4 m × 4 m behind the wall. The antenna array is placed at a standoff distance of 1 m in front of the wall, which has a thickness of 0.15 m and the following dielectric properties: $\epsilon_r = 7.5$ and $\sigma = 0.05$ S/m. A modulated Gaussian pulse with a center frequency of 2.8 GHz is used as an excitation signal, where the time domain response is then converted into a stepped-frequency signal with 401 frequencies covering the frequency band [1.8, 3.8] GHz. All simulated radar signals are obtained using the above radar and scene specifications.

The second dataset comprises signals collected at the radar imaging laboratory of the Centre for Signal and Information Processing (CSIP), University of Wollongong, Australia. The radar signals are acquired in a room without any RF absorbers. A network analyzer is used to generate a stepped-frequency waveform covering 1 to 4 GHz frequency band with a step size of 5 MHz. The transceiver is placed at a standoff distance of 1 m in front of a wooden wall of thickness 0.08 m and moved horizontally to synthesize a linear array aperture of length 1.6 m with 51 elements. The region-of-interest behind the wall has the same size as the simulated scene.

Since it is difficult to obtain the individual ground-truth target signals, due to the wall attenuation and the distortion caused by wall clutter mitigation, we adopt target-to-clutter ratio (TCR) of the formed target image as the evaluation

metric. The TCR of an image is defined as the ratio of the average pixel power in the target region to the average pixel power in the clutter region, i.e.,

$$\text{TCR} = \frac{N_c \sum_{j \in A_t} |I(j)|^2}{N_t \sum_{j \in A_c} |I(j)|^2}, \quad (51)$$

where A_t is the selected target region, A_c is the clutter region defined as the entire image excluding the target region, N_c and N_t are, respectively, the number of pixels in the clutter and target regions. Let TCR_B be the TCR of the image of the populated scene without wall clutter mitigation and TCR_j be the TCR of a target image associated with the j th mode. The TCR gain of the j th target image is computed as

$$\text{TCR}_{\text{gain}} = \frac{\text{TCR}_j}{\text{TCR}_B}. \quad (52)$$

The stopping criteria for the proposed variational method are defined as follows. The number of iterations is set to 100, and the average relative difference between two consecutive extracted modes is equal or below the predefined threshold δ ,

$$\frac{\sum_{n=1}^N \sum_{l=1}^L \|\mathbf{u}_{n,l}^{k+1} - \mathbf{u}_{n,l}^k\|_2^2}{\sum_{n=1}^N \sum_{l=1}^L \|\mathbf{u}_{n,l}^k\|_2^2} \leq \delta, \quad (53)$$

where $\delta = 10^{-5}$ and $\mathbf{u}_{n,l}^k = [u_{n,l}^k(1), \dots, u_{n,l}^k(M)]^T$. For the BO technique, the search intervals of the hyper-parameters are set as follows: $0 \leq \eta \leq 1$, $10 \leq \alpha \leq 10^{10}$, and $10^{-10} \leq \mu \leq 0.1$. Note that the relation between the hyper-parameter β and η is given by $\beta = \eta \|\mathbf{D}^\dagger \mathbf{Y}\|_\infty$, where $\mathbf{D}^\dagger$ is the pseudo-inverse of $\mathbf{D}$.

B. Effect of the Low-Rank Dictionary

Different synthesis dictionary can be applied to the proposed method for capturing the wall clutter as a low-rank representation. Three types of synthesis dictionaries are investigated: the first one contains wavelet packet bases, the second one consists of modulated discrete prolate spheroidal sequences (DPSS) [36], and the third one is defined as an identity matrix. Setting the synthesis dictionary to an identity matrix is equivalent to estimating the low-rank matrix directly from the radar data, where the low-rank matrix contains the wall clutter. This low-rank estimation is similar to the model described in [33] and [34]. Wavelet packet dictionary (WPD), on the other hand,

¹Website: www.remcom.com

is employed to promote signal sparsity. Here, Daubechies 5 wavelet was used to create a set of basis functions using three levels of decomposition. In [36], a modulated DPSS dictionary was employed to separate the wall signals from the target signals. The DPSS bases provide a very high degree of approximation accuracy in a mean square error sense for multiband sample vectors [51].

A through-the-wall radar scene based on the specification given in Section V-A is simulated using the EM software with one dihedral behind the wall, similar to the one shown in Fig. 2. The proposed method is applied to separate the target signal from the wall return. The hyper-parameters of the variational model are initially set as follows: $L = 20$, $\mu = 10^{-5}$, and $\alpha = 10^5$. The initial frequencies of the modes are determined using Eq. (50) with $\Delta f = 5 \times 10^6$, $\tau_{Z_{\text{off}}} = 2/c$, and $\tau_R = 10/c$, where c is the speed of light in free space. Since the hyper-parameter β is defined as $\beta = \eta \|\mathbf{D}^\dagger \mathbf{Y}\|_\infty$, the parameter η is varied from 10^{-4} to 10^{-1} . The TCR of the target image is computed for each η value. Figure 3 shows the changes in TCR as a function of the parameter η . An over-complete synthesis dictionary such as WPD or modulated DPSS achieves higher TCR than the identity matrix. Using an over-complete synthesis dictionary allows the wall components to span in a union of low-dimensional subspaces, and hence facilitates the capturing of the wall clutter as a low-rank representation. With an identity matrix, on the other hand, the wall clutter is constrained to reside in a single low-rank subspace. Increasing η enlarges the low-rank subspace that contains not only the wall returns, but also some of the target reflections. Figure 3 shows DPSS provides a higher TCR; therefore, it is used for the remainder of the experiments.

Fig. 3. Target-to-clutter ratio (TCR) of the proposed target detection method for different low-rank synthesis dictionaries as a function of the parameter η .

C. Effect of Target Location

In a through-the-wall radar scene, targets can be located close to each other and also close to the wall, inducing wall-target and target-to-target interactions that can affect the performance of the target separation method. Different through-the-wall radar scenarios are simulated to investigate the effect of the target location on the TCR gain of the proposed technique. The first scene (*H-colocation*) comprises two dipoles, which are located horizontally at the same down-range of 1.5 m from the wall with a distance of 1 m between the two dipoles. The second scene (*V-colocation*)

has two dipoles co-located vertically, where the first dihedral is positioned at a down-range of 1.5 m and the second dihedral at 2.5 m. In the third scene (*D-colocation*), the two dipoles are placed along the diagonal axis with a separation distance of 1 m. The top-view of the three scenes is shown in Fig. 4. In addition to the target location, the distance between the two dipoles is gradually reduced at a step size of 0.2 m for all three scenes. The performance of the proposed method is measured according to (52). The number of targets and the target locations are assumed to be known, and the initial frequency of each mode is computed according to (43). The BO technique is applied once to tune the hyper-parameters η , α , and μ .

Fig. 4. Geometry of the simulated TWRI scene containing two dipoles co-located (a) horizontally, (b) vertically, and (c) diagonally.

Table I presents the average TCR gain values of the proposed target separation method for all three simulated scenes at different target separation distances. When the two dipoles are co-located vertically or diagonally, the changes in the TCR gain of the target image are small, indicating that the proposed method can separate the target signals for such an arrangement. However, for targets co-located horizontally, it is harder to segregate the target returns when the two targets are very close to each other. For example, reducing the inter-target separation from 1 m to 0.8 m decreases the average TCR gain by 1.48 dB. Decreasing the inter-target separation further to 0.4 m results in a drop of 2.19 dB. Figure 5 shows the target images associated with the modes extracted from the three simulated scenes for an inter-target separation of 0.4 m. The target images depicted in Figs. 5(a) and (b) for the H-colocation scene have strong overlapping target reflections, whereas the remaining target images have focused target reflections. This demonstrates that when the targets are co-located horizontally with a small inter-target separation, the proposed method has some difficulty separating the target returns. On the other hand, when the targets are co-located vertically or diagonally, the proposed method can still separate the target returns even when the inter-target separation is small.

D. Effect of the Number of Targets

The next experiment is to evaluate the target separation method on populated scenes. Two simulations are performed with different number of targets behind the wall. Figures 6(a)

TABLE I
TARGET-TO-CLUTTER RATIO GAIN (IN DB) OF THE PROPOSED TARGET
SEPARATION METHOD FOR DIFFERENT INTER-TARGET SEPARATIONS.

Inter-target separation	Simulated through-the-wall scenes		
	H-colocation	V-colocation	D-colocation
1.0 m	11.07	12.09	13.67
0.8 m	9.59	12.90	13.15
0.6 m	9.03	13.37	13.17
0.4 m	8.88	12.57	13.46

Fig. 5. Images obtained from the proposed target separation method associated with (a) mode-1 and (b) mode-2 at an inter-target separation of 0.4 m for the H-colocation scene. Similarly, images (c) and (d) are for the V-colocation scene, and images (e) and (f) are for the D-colocation scene.

and (b) show the top-view of two scenes with 4 and 6 dihedrals, respectively. The targets depicted in Fig. 6(a) are positioned diagonally with an inter-target separation of 0.6 m. Those shown in Fig. 6(b) are co-located vertically and horizontally, where the horizontal distance between T1 and T2 is 1.2 m and the vertical distance between T1 and T3 is 0.75 m. Table II presents the TCR gain of each target image in the two scenes. All the target images in both the four- and six-dihedral scenes have high TCR gain. The gradual drop in TCR gain from T1 to T4 of the four-dihedral scene is due to the signal propagation loss, with T4 positioned farthest from the antenna array. Similar change in TCR gain is observed for the

six-dihedral scene. Figures 7 and 8 depict the individual target images of the populated scenes. The target images show that the proposed method has removed successfully the wall return and segregated the target signals. In the six-dihedral scene, T1 and T2 are co-located horizontally with a 1.2 m inter-target separation. The radar signal received by each antenna contains target returns with different propagation delays. Hence, the proposed method can easily separate the two targets, as shown in Figs. 8(a) and (b). When the targets are positioned far away from the antenna, e.g., T5 and T6, they will have similar two-way propagation delays; therefore, it is more difficult to separate the targets.

Fig. 6. Geometry of the simulated TWRI scene containing (a) four dihedrals and (b) six dihedrals.

TABLE II
TARGET-TO-CLUTTER RATIO GAIN (IN DB) OF THE INDIVIDUAL TARGET
IMAGES OF THE POPULATED SCENES.

Populated scene	T1	T2	T3	T4	T5	T6
4-Dihedral scene	25.15	24.77	22.34	19.94		
6-Dihedral scene	23.40	22.57	20.33	20.36	18.60	18.49

E. Evaluation of Target Separation with Real Data

Two experiments are conducted at the CSIP radar imaging laboratory. In the first experiment, three dihedrals are placed behind the wall (see Fig. 9(a)), and in the second experiment, five objects are placed at different locations, as shown in Fig. 9(b). The five objects are three dihedrals and two trihedrals; their sizes are shown in Table III. A two-step procedure is employed to estimate the regularization parameters of the target separation model. Firstly, the BO technique is employed to determine the hyper-parameters, except the number of modes L which was fixed to 20. The estimated hyper-parameters are then used to form the populated target image, where the 2D-CFAR detector is applied to detect the targets and find their locations. The parameters of the 2D-CFAR detector were set as follows. The number of reference clutter cells was 10, the number of guard cells was 8, and the probability of false alarm rate was 10^{-5} . Based on the target locations, the initial frequencies of the modes are determined according to (43). Then, the BO technique is applied to refine the hyper-parameters β , α , and μ . Since there is no existing

Fig. 7. Individual target images of the 4-dihedral scene: (a) T1, (b) T2, (c) T3, and (d) T4.

target separation method developed for TWRI, we implement the traditional ICA-based technique as the baseline method. Here, the complex-valued joint approximate diagonalization of eigen-matrices (JADE) algorithm is used to decompose the data matrix into a set of independent components. Our preliminary results showed that applying JADE algorithm directly to the received radar signals without wall clutter mitigation performs poorly. Therefore, background subtraction is used to remove the wall and background clutter by subtracting the reference data from the measured data, where the reference data is obtained from the TWRI scene without the targets.

Figures 10 and 11 depict the target images obtained from the proposed target separation method and ICA-based method for the 3-target scene, respectively. Figures 12 and 13 show their corresponding target images for the 5-target scene. Comparing the target images in Figs. 10 and 12 with those in Figs. 11 and 13 shows that the proposed target separation method is more effective than the ICA-based technique. The ICA-based method, which relies on background subtraction for wall clutter mitigation, produces images containing target reflections and some background clutter. In terms of average TCR gain, the proposed method achieves 22.40 dB for 3-target scene and 23.77 dB for 5-target scene, whereas the average TCR gain of the ICA-based technique is below 15 dB, see Table IV.

VI. CONCLUSION

Signals received by through-the-wall radar contain not only multiple target echoes, but also strong wall reflections and background noise. For target classification, there is a need to remove the wall return and separate the individual target signals. This paper presented a variational model with low-rank constraint for through-the-wall target separation. The

Fig. 8. Individual target images of the 6-dihedral scene: (a) T1, (b) T2, (c) T3, (d) T4, (e) T5, and (f) T6.

Fig. 9. Geometry of the real TWRI scene: (a) 3-target scene and (b) 5-target scene.

proposed method employs a low-rank model for removing the wall clutter and a variational model for decomposing the radar signal into a set of modes, where each mode contains an individual target signal. These two tasks, i.e., wall clutter removal and target separation are formulated into a constrained optimization problem, which is solved using the ADMM technique. Experiments using simulated and real radar data

TABLE III
DESCRIPTION OF THE OBJECTS USED FOR THE 3-TARGET AND 5-TARGET
REAL THROUGH-THE-WALL RADAR SCENES.

Target	Description
D1	Dihedral reflector with plane size: 40 cm height $\times$ 22.5 cm wide
D2	Dihedral reflector with plane size: 20 cm height $\times$ 22.5 cm wide
D3	Dihedral reflector with plane size: 22 cm height $\times$ 24.5 cm wide
T1	Triangular trihedral reflector with edge length of 42.5 cm
T2	Triangular trihedral reflector with edge length of 42.5 cm

Fig. 10. Target images obtained from the proposed target separation method for the 3-target scene (a) D1, (b) D2, and (c) D3.

Fig. 11. Target images obtained from ICA-based method for the 3-target scene: (a) D1, (b) D2, and (c) D3.

were conducted to evaluate the effectiveness of the proposed target separation method. The experiment results showed that the proposed method can simultaneously remove the wall clutter and segregate the different target echoes from the received signals; it achieves higher TCR than the ICA-based technique.

TABLE IV
AVERAGE TARGET-TO-CLUTTER RATIO GAIN (IN dB) OF THE TARGET
SEPARATION METHODS, EVALUATED ON REAL RADAR DATA.

Target separation method	Average TCR gain	
	3-target scene	5-target scene
Proposed variational method	22.40	23.77
ICA-based method	14.68	14.77

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VII. APPENDIX

REFERENCES

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Algorithm 1 – Optimization algorithm

Input: $\mathbf{D} = \mathbf{D}_{\text{DPSS}}$, $\mathbf{B}^0 = \mathbf{0}$, $\mathbf{G} = \mathbf{Y}$, $\rho = 1/(4\mathcal{S}(\mathbf{D}^H \mathbf{D}))$, $\delta = 10^{-5}$, L , η , α , μ , $\{\omega_{n,l}^0\}_{l=1,n=1}^{L,N}$, $K = 100$,
Output: $\{\mathbf{u}_{n,l}\}_{l=1,n=1}^{L,N}$, $\mathbf{Z}$

- 1: $\mathbf{Z} = \mathbf{D}^\dagger \mathbf{Y}$;
- 2: $\beta = \eta \|\mathbf{Z}\|_\infty$;
- 3: $\hat{u}_{n,l}(\omega) = 0 \quad \forall n, l$;
- 4: **for** $k = 0$ to K **do**
- 5: $\text{SVD}(\mathbf{Z}^k + 2\rho \mathbf{D}^H (\mathbf{G}^k + 0.5\mathbf{B}^k - \mathbf{D}\mathbf{Z}^k)) = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^H$;
- 6: $\mathbf{Z}^{k+1} = \mathbf{U}\mathbf{T}(\mathbf{\Sigma}, \beta\rho)\mathbf{V}^H$;
- 7: $(\mathbf{Y}^w)^{k+1} = \mathbf{D}\mathbf{Z}^{k+1}$;
- 8: $\hat{\mathbf{x}}_n^{k+1} = \text{FFT}(\mathbf{y}_n - (\mathbf{y}_n^w)^{k+1})$;
- 9: $\hat{u}_{n,l}(\omega)^{k+1} = \frac{\hat{x}_n(\omega)^{k+1} - \sum_{i \neq l} \hat{u}_{n,i}(\omega)^k + \frac{\hat{u}_{n,l}(\omega)}{2}}{1 + \alpha(\omega - \omega_{n,l}^k)^2} \quad \forall n, l$;
- 10: $\omega_{n,l}^{k+1} = \frac{\int_{-\infty}^{\infty} \omega |\hat{u}_{n,l}(\omega)^{k+1}|^2 d\omega}{\int_{-\infty}^{\infty} |\hat{u}_{n,l}(\omega)^{k+1}|^2 d\omega} \quad \forall n, l$;
- 11: $\hat{b}_n(\omega)^{k+1} = \hat{b}_n(\omega)^k + \mu(\hat{x}_n(\omega)^{k+1} - \sum_{l=1}^L \hat{u}_{n,l}(\omega)^{k+1})$;
- 12: $(\mathbf{u}_{n,l}^t)^{k+1} = \text{IFFT}(\hat{\mathbf{u}}_{n,l}^{k+1}) \quad \forall n, l$;
- 13: $(\mathbf{y}_n^t)^{k+1} = \sum_{l=1}^L \mathbf{u}_{n,l}^{k+1} \quad \forall n$;
- 14: $\mathbf{b}_n^{k+1} = \text{IFFT}(\hat{\mathbf{b}}_n^{k+1})$;
- 15: $\mathbf{B}^{k+1} = [\mathbf{b}_1^{k+1}, \dots, \mathbf{b}_N^{k+1}]$;
- 16: $(\mathbf{Y}^t)^{k+1} = [(\mathbf{y}_1^t)^{k+1}, \dots, (\mathbf{y}_N^t)^{k+1}]$;
- 17: $\mathbf{G}^{k+1} = \mathbf{Y} - (\mathbf{Y}^t)^{k+1}$;
- 18: **if** $\frac{\sum_{n=1}^N \sum_{l=1}^L \|\mathbf{u}_{n,l}^{k+1} - \mathbf{u}_{n,l}^k\|_2^2}{\sum_{n=1}^N \sum_{l=1}^L \|\mathbf{u}_{n,l}^k\|_2^2} \leq \delta$ **then break**;
- 19: **end if**
- 20: **end for**

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Fig. 12. Target images obtained from the proposed target separation method for the 5-target scene: (a) D2, (b) D1, (c) T1, (d) D3, and (e) T2.

Fig. 13. Target images obtained from the ICA-based method for the 5-target scene: (a) D2, (b) D1, (c) T1, (d) D3, and (e) T2.

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Fok Hing Chi Tivive (M'06) received the BE(Hons) in Telecommunications from Edith Cowan University and the Ph.D degree in computer engineering from the University of Wollongong, Australia, in 2001 and 2006, respectively. Since 2006, he has been with the School of Electrical Computer and Telecommunications Engineering, University of Wollongong, as a Postdoctoral Research Fellow. His research interests include machine learning, pattern recognition, image processing, ground penetrating radar, and through-the-wall radar imaging.

Abdesslem Bouzerdoum (M'89-SM'03 IEEE) graduated with MSEE and Ph.D. degrees, both in Electrical Engineering, from the University of Washington, Seattle, USA. In 1991, he joined Adelaide University, South Australia. In 1998, he moved to Perth, Western Australia, as an Associate Professor with Edith Cowan University. In September 2004, he became Professor of Computer Engineering and Head of School of Electrical, Computer and Telecommunications Engineering at the University of Wollongong. From 2007 to 2013, he served as Associate Dean (Research) with the Faculty of Informatics, University of Wollongong. He is currently serving as the Head of Information and Computing Technology Division, College of Science and Engineering, Hamad Bin Khalifa University, Qatar, and a Senior Professor of Computer Engineering at the University of Wollongong. Dr. Bouzerdoum held several Visiting Professor Appointments at Institut Galile, Universit Paris-13, LAAS/CNRS, Toulouse, Institut femto-st, Besancon, France, Villanova University, USA, and the Hong Kong University of Science and Technology. From 2009 to 2011, he was a member of the ARC College of Experts and Deputy Chair of the EMI (Engineering, Mathematics and Informatics) panel (2010–2011).

Dr. Bouzerdoum is the recipient of the *Eureka Prize for Outstanding Science in Support of Defence or National Security* (2011), the *Chester Sall Award of IEEE Trans. Consumer Electronics* (2005), and a *Distinguished Researcher Award (Chercheur de Haut Niveau)* from the French Ministry (2001). He served as Associate Editor for 5 International journals, including IEEE TRANS. IMAGE PROCESSING, IEEE TRANS. SYSTEMS, MAN, AND CYBERNETICS (1999–2006). He has published over 350 technical articles and graduated over 30 Ph.D. and Research Masters students. His research interests include radar imaging and signal processing, image processing, vision, machine learning, and pattern recognition.