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On implementation of bounding surface plasticity models with no overshooting effect in solving boundary value problems

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Abstract

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On implementation of Bounding Surface Plasticity Models with No Overshooting Effect in Solving Boundary Value Problems

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Abstract: This paper presents a new scheme that can be used to overcome the overshooting effect, one of the well known problems occurs during application of bounding surface plasticity models in numerical analysis of boundary value problems. The scheme is based on definition of clouds of loading surfaces with a specific margin of strains within which unloading does not accompanied kinematic hardening. The basic concept of the scheme is introduced and the methodology and the relevant step by step algorithm to implement this scheme are presented. This scheme has been incorporated in the UNSW bounding surface model and implemented in a finite difference code and used to simulate cyclic triaxial tests as well as complicated monotonic and dynamic boundary value problems. The satisfactory performance of the scheme is demonstrated and its efficiency is discussed thorough these simulations.

Keywords: bounding surface plasticity; overshooting; cyclic loading; granular soil

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1. INTRODUCTION

The concept of bounding surface plasticity models was first introduced by Dafalias and Popov [1]&[2] and independently by Krieg [3] to account for the kinematic hardening and softening of metals. Since development of this concept in the late 1970's, a large number of constitutive models have been proposed based on the concept of bounding surface, both in structural discipline for metals and concrete (e.g. Fardis *et al.* [4]) and in geotechnical engineering for geomaterials (e.g. Dafalias and Hermann [5] for cohesive soils and Bardet [6] for sands).

Despite their good performances, application of bounding surface plasticity models in solving boundary value problems requires excessive computational efforts, due partly to the complexity of mapping rules adopted in some models which requires the size and location of the maximum loading surfaces to be recorded. Another problem is perhaps overshooting issue which often encounters in application of bounding surface models in solving boundary value problems. Although this problem has been reported in implicit integration schemes, it is more pronounced in explicit integration schemes, such as forward Euler methods, utilized by finite element/difference codes [7].

A bounding surface model based on the concept of the critical state soil mechanics was developed at The University of New South Wales to model the stress-strain behaviour of sands [8] and clays [9]. This model was extended later to include the cyclic response of unsaturated granular materials [10]. In this model, hereafter referred to as UNSW constitutive model, a rigorous mapping rule, passing through stress reversal points, was introduced to predict the stress-strain behaviour under loading and unloading conditions. The mapping rule adopted in the original model could not be efficiently applied in a boundary value problem with highly variable loading paths due to its complex mapping procedure and memory requirements. To tackle this problem, Kan *et al.* [11] introduced a single stress point radial mapping rule which has a simpler procedure and is more amenable to complex loading paths.

In this paper a general method is presented for the treatment of overshooting. A margin for the successive loading surfaces is introduced through which no change in the centre of homology will occur for small unloading/reloading stress paths. This range is based on a critical threshold for the accumulation of plastic shear strain. The method is then implemented in conjunction with UNSW constitutive model to solve boundary value problems. A forward Euler explicit integration scheme is adopted throughout. The implemented model is used to simulate some boundary value problems to demonstrate the effectiveness of the proposed treatment.

2. OVERSHOOTING PROBLEM

2.1. Background

Overshooting is a phenomenon that occurs in most of bounding surface plasticity models when a small load cycle is encountered in a general loading path. Overshooting results in a stiffer behavior than that anticipated under the general loading path, as shown in Figure 1. This is due to the fact that any stress reversal changes the size and the location of the loading surface and, as a consequence, a large stiffness modulus is calculated upon any stress reversal.

A small unloading/reloading may occur at a point sufficiently far from the point of application of a large cyclic loading, or may occur during earthquake loading, or may be part of a general loading path. It also occurs in a finite element or finite difference simulation of a monotonic loading in a boundary value problem. The numerically obtained stress path for a point in a large domain oscillates around the correct stress path due to stress redistributions resulted from hardening/softening of adjacent elements, as shown in Figure 2. Such an approximation to the true solution may include many small unloading/reloading loops which are obviously

unreal and should be treated in such a way that the numerical approximation does not overshoot the true behavior.

In bounding surface plasticity models, for any given stress state on the loading surface, a proper mapping rule associates the stress point to a corresponding image point on the bounding surface. The stiffness modulus is calculated as a function of the distance of the actual stress point on the loading surface and its image point on the bounding surface from the center of homology to the loading and the bounding surfaces. Upon any stress reversal, the center of homology and the location of the loading surface are changed and the size of loading surface is reduced. The proportional distances of the center of homology to the actual stress point and its image will change, which in turn results in a larger stiffness modulus. If loading in the initial direction is resumed after a small unloading, the initial size of the loading surface may not be recovered if it is not kept in the memory and, as a consequence, the load deflection curve “overshoots” the one that would have been predicted for the initial loading path without any stress reversal. Dafalias [12], who first recognized the overshooting phenomenon, suggested the accumulated reverse plastic strain to be used as a measure of change in the plastic behavior. Only if the accumulated reverse plastic strain in a loading direction exceeds a pre-defined threshold, then the center of homology is updated and the stress reversal is considered as true unloading. Evaluation of the performance of this method was limited to uniaxial loading where the generated plastic strain is in the direction of loading. Fardis *et al.* [4] proposed another method to overcome the problem of overshooting. In this method if the distance between the actual stress point and its image on the bounding surface becomes less than any previously achieved value, then any further loading is considered as a continuation of the previous initial loading for which the minimum distance was recorded. In other words, any further loading is considered as a continuation of the previous initial loading if the size of the loading surface becomes larger than any previous loading surfaces. This

solution scheme however may result in a discontinuous plastic modulus, as was noted by Dafalias [12], since a particular plastic modulus could be related to many different centers of homology.

Tseng and Lee [13] used the length of the elastic unloading chord between the current stress point and the previous position at which the elastic unloading initialized as a criterion for updating the location of the loading surface and the center of homology. If the loading path comes back to its original path after several cycles of elastic loading and unloading without causing any plastic deformation, then no change in loading surface is considered. Despite its good performance in simulation of metal behaviour under one-dimensional loading, this method has an inherent limitation as it can only be applied for cases where elastic deformations take place upon unloading and reloading. This method is not applicable in simulation of geomaterial behavior where elasto-plastic deformation often occurs upon unloading and reloading.

Montans and Borja [7] introduced an implicit integration algorithm based on the radial return scheme in their proposed bounding surface plasticity model. To overcome the overshooting phenomena they used a virtual positioning of the bounding surface upon stress reversal. Their method is based on the concepts of Masing's rule which requires stable closed loops upon unloading/reloading. The Masing's rule states that the stress-strain curve developed during unloading/reloading should have a homological ratio of 2 to the monotonic stress-strain curve. Despite its simplicity, this method seems to be applicable only to Masing's type cyclic loading (e.g. in Montans[14]). Furthermore, this method has only been used with simple loading surfaces (e.g. circles for Prager's translation or Von Mises criterion) and without provisions for generation/dissipation of pore pressures.

Although most of the methods proposed to deal with overshooting problem are applicable to uniaxial loading, Lee *et al.* [15] proposed a method for two-dimensional plain strain loading

conditions. In this method stress reversal is assumed to occur when the angle between the two subsequent stress increment vectors on the loading surface exceeds a pre-defined value. This definition of stress reversal is not in accordance with the traditional definition used in the theory of plasticity, which based on loading direction and a plastic multiplier (Pastor *et al.* [16]) or the angle between the vector of elastic stress increment and the vector normal to the yield/bounding surface (Sloan *et al.* [17]).

The problem of overshooting can be minimized if the hardening rule is defined properly, as described by Dafalias [12]. The hardening rule and the mapping rule of the UNSW bounding surface model (Russell and Khalili [8]) have been defined in such a way that upon any small cyclic loading the initial loading surface is recovered to a great extent (Khalili *et al.* [18]). As a consequence, the initial loading path is continued after it encounters a small unloading-reloading cycle. To demonstrate the performance of UNSW model, the results of simulation of a series of conventional triaxial tests on a loose sample of Toyoura sand ($p'_0 = 100kPa$) are shown in Figure 3. Figure 3(a) shows three load paths selected in the simulation; one is a general monotonic loading without any load reversal, another includes a number of small unloading/reloading cycles in the general loading, and the third includes a large unloading/reloading loop. The results of the simulations (Figure 3b) show that the load deflection curve of the loading with small loops matches perfectly to that of the general monotonic loading, exhibiting no overshooting effects. For the load path with a large unloading/reloading loop, strain hardening behavior is captured during the reloading phase, which is expected to occur for loose sand (e.g. Gajo and Muir wood [19]).

Figure 4 presents a succession of loading surfaces predicted by UNSW model for a monotonic loading path with and without a small unloading/reloading loop. Two cases of unloading/reloading are considered, in one case unloading occurs in the opposite direction of the loading as shown in Figure 4(a) and in the other unloading is in an arbitrary direction as

shown in Figure 4(b). Loading surfaces are shown in bold lines for monotonic loading and in dotted lines for monotonic loading with small cycles. The monotonic loading passes through points 1-2-4 while the unloading/reloading paths pass through points 1-2-3-2-4. As it may be seen UNSW bounding surface model can recover the loading surface after a small unloading/reloading occurs regardless of the direction of unloading stress path. The loading surfaces developed at point 4 for cases with small loops are of the same size as that developed for monotonic loading but shifted slightly toward the bounding surface. In general the mapping scheme in UNSW model does not exhibit overshooting in simulation of laboratory tests using a single element. However, overshooting still occurs in simulation of boundary value problems.

In a finite element or finite difference solution to a boundary value problem, redistribution of stresses in elements adjacent to a point during stress integration causes oscillation of the predicted stress path for the point, which resembles many small unloading/reloading loops. This oscillation does not occur if only one element is used in an analysis. To illustrate the effects of such a condition on the performance of a bounding surface model, a simple element test which simulates the behavior of a dense sample of Hostun sand subjected to conventional triaxial stress path is considered here. The oscillation of the stress path around the true loading is simulated by a series of stepping stress paths with different increments of Δq (or $\Delta p'$), as shown in Figure 5. These pseudo stress paths show ideal uniform oscillations, as compared to those expected at a point in the domain of a boundary value problem during a finite element/difference analysis. However, they can demonstrate the problem of overshooting that may occur in a numerical analysis using multiple elements. The steps in pseudo paths are intentionally assumed to be vertical and horizontal so that at each step the change in load direction and consequently the potential for stress reversal will be more pronounced.

The results of the simulations for the true stress path and the three pseudo stress paths with $\Delta q / p'_0 = 0.1, 0.05, 0.025$ are shown in Figure 6. It can be seen that all the three pseudo stress paths overshoot the actual variations of the stress ratio (q / p'_0) and the volumetric strain versus axial strain. Overshooting is larger for the pseudo stress path with smaller steps which in fact oscillates closer to the true stress path. This indicates that a selection of smaller load steps does not improve the results of the integration scheme since smaller steps lead to a larger number of stress reversals and amplifies the overshooting effects. Whenever stress reversal occurs, the stiffness modulus is set to a value close to the elastic modulus. The cumulative effects of higher stiffness, in a stress path with a larger number of stress reversals, result in a greater overshooting effect.

It is clear that even if the hardening rule and the integration scheme of bounding surface plasticity models are defined in such a way that they do not exhibit overshooting in the scale of single element tests, the models are still susceptible to overshooting when used in numerical modeling of boundary value problems, when a large number of elements are used to model the domain and when stress redistribution results in small scale unloading/reloading loops at each load increment. If model parameters for a soil are obtained/calibrated by modeling triaxial tests in a single element analysis and used in simulation of a boundary value problem, the overshooting results in a stiffer behavior than expected or observed. In the present study attempts are made to obtain model parameters from independent element tests and used in the analysis of boundary value problems. This can only be achieved if the overshooting problem is treated properly in the analysis.

2.2. Proposed Approach

The method proposed to solve the problem of overshooting inherent to application of the bounding surface plasticity models in solution of boundary value problems is presented here. This method can be categorized as a generalization of the one suggested by Dafalias [12] in $p' - q$ loading space.

In this method a critical threshold for the accumulated plastic shear strain ($\Delta\varepsilon_q^p$) is defined as a measure for updating the position of the loading surface and the associated center of homology. If unloading occurs at any stage during an analysis, the last position of the center of homology is kept in memory. The accumulated plastic shear strain is calculated from the point of stress reversal as $\delta\varepsilon_q^p = \int \dot{\varepsilon}_q^p$, where $\dot{\varepsilon}_q^p$ is the incremental plastic shear strain. If reloading occurs before $\delta\varepsilon_q^p$ exceeds the critical threshold, the unloading is assumed to be superficial and too small to affect the behavior of the material. In this case the analysis is continued with the last record of the center of homology, as if no stress reversal has occurred. If $\delta\varepsilon_q^p$ exceeds the critical threshold, unloading is assumed to be real and sufficiently large to affect the behavior. Therefore, the center of homology and the loading surface are updated and the analysis is continued.

The process of implementation of the proposed method can be described by an example. Consider a cyclic test with small and large unloading/reloading cycles, as shown in Figure 7. In this figure cycles 2-3-4 and 8-9-10 are very small and superficial and should not affect the general response of the sample, while cycles 5-6-7 and 12-13-14 are large and should be treated like a real load cycle. Variations of the deviatoric stress (q) versus axial strain (ε_a) and the mean effective stress (p') are shown in Figure 7. It is assumed that the plastic shear strain accumulated from point 2 to point 3 is less than the critical threshold. Therefore point 2

will not be taken as a new center of homology for stress reversal 2-3. Therefore, the previously recorded center of homology, i.e. point 0, will be used for path 2-3-4-5. The next stress reversal occurs at point 5. $\delta\epsilon_q^p$ accumulated from point 5 is assumed to become larger than the critical threshold before reaching point 6. In this case the center of homology is updated from point 0 to point 5 as soon as $\delta\epsilon_q^p$ exceeds the critical threshold. $\delta\epsilon_q^p$ accumulated from point 6 is also assumed to be larger than the critical threshold. Therefore, point 6 is taken as the center of homology until stress path reaches the next stress reversal, at point 8. Since the load cycle 8-9-10 is also small, point 6 remains the center of homology until stress path reaches point 11 where a large stress reversal occurs. Point 11 is taken as the center of homology as soon as $\delta\epsilon_q^p$ becomes larger than the critical threshold along the stress path 11-12. The load cycle 12-13-14 is large and therefore point 12, and subsequently point 13 become the center of homology.

2.3. The cloud of loading surfaces

Figure 8 shows a depiction of the evolution of loading surfaces in the proposed method. For the sake of simplicity the general shape of the bounding and loading surfaces is presented as circles in $p'-q$ plane. Also shown in Figure 8 are the plastic strain axes where ϵ_p^p and ϵ_p^q are the volumetric and deviatoric plastic strains, respectively.

For a given stress path from A to B , a radial mapping rule originating from point H (the center of homology) results in image points $\bar{A}$ and $\bar{B}$ on the bounding surface. In this loading path the position of point H is kept unchanged which indicates that path $A-B$ is a small superficial stress reversal and will not be treated as a true stress reversal (otherwise center of

homology should be moved to point A). During this unloading, the two components of plastic strain, denoted as $\delta\epsilon_p^p$ and $\delta\epsilon_q^p$, accumulated from A to B can be calculated as:

$$\delta\epsilon_p^p = \int_A^B \dot{\epsilon}_p^p \quad (1)$$

$$\delta\epsilon_q^p = \int_A^B \dot{\epsilon}_q^p \quad (2)$$

where $\dot{\epsilon}_p^p$ and $\dot{\epsilon}_q^p$ are the components of incremental plastic strains.

Let's assume that when stress reversal occurs at point B the accumulated plastic strain calculated from point A , $\delta\epsilon_q^p$, is equal to the critical threshold. Line L_1L_2 is parallel to the ϵ_q^p axis and any point on L_1L_2 has the same accumulated plastic shear strain, $\delta\epsilon_q^p$, as for point B . Therefore, a cloud of loading surfaces exists; all have their center of homology located at point H and intersecting line L_1L_2 . Any stress reversal occurs within the cloud of loading surfaces has an accumulated plastic shear strain less than the critical threshold, $\Delta\epsilon_q^p$, and therefore is assumed to be superficial which does not change the center of homology. The extent of the cloud of loading surfaces depends on the specified critical threshold for the accumulated plastic shear strain, $\Delta\epsilon_q^p$. The effects of the plastic volumetric strain ($\delta\epsilon_p^p$) is neglected in this context, since it is widely believed that the behavior of geomaterials in any cyclic or dynamic loading is influenced mainly by the deviatoric plastic strain (e.g. Ishihara *et al.* [20]). The magnitude of the plastic shear strain threshold depends on the response of the soil fabric to cyclic loading and is a material parameter. However any value of $\Delta\epsilon_q^p$ within the range of 10^{-5} to 10^{-6} was found to be adequate in capturing small superficial stress reversals and preventing overshooting.

3. IMPLEMENTATION OF THE PROPOSED METHOD

The proposed method was implemented in UNSW bounding surface plasticity model in order to evaluate its performance in simulation of boundary value problems. The main concepts of UNSW constitutive model are briefly described here followed by application of the model in a few boundary value problems. A more detailed description of UNSW model can be found in Kan *et al.* [11].

3.1. UNSW Bounding Surface Plasticity Model

In UNSW bounding surface plasticity model, the shape of the bounding surface was obtained based on a host of experimental data and is described by the following expression:

$$F(\bar{p}', \bar{q}, \bar{\theta}, \bar{p}_c') = \left(\frac{\bar{q}}{M_{cs}(\bar{\theta}) \bar{p}'} \right)^N - \frac{\ln(\bar{p}_c' / \bar{p}')}{\ln R} = 0 \quad (3)$$

The stress conditions on the bounding surface are denoted using a superimposed bar throughout. The parameter $\bar{p}_c'$ controls the size of the bounding surface and is a function of the plastic volumetric strain. The material constant R represents the ratio of $\bar{p}_c'$ and the mean effective stress at the intercept of the bounding surface and the Critical State Line (CSL) in the $q-p'$ space, as shown in Figure 9. The constant N controls the curvature of the bounding surface. The critical state line in the $v-\ln p'$ plane is approximated by two linear segments and a dimensionless state parameter (ξ) is defined as a measure of consistency of the soil at its current state;

$$\xi = v - v_{cs} \quad (4)$$

where v is the specific volume at the current stress, p' , and v_{cs} is the specific volume at the critical state corresponding to p' . The CSL in the $q-p'$ plane is defined as a straight line passing through the origin, with slope of M_{cs} as a function of Lode angle, θ .

The size of the bounding surface ($\bar{p}'_c$) is taken as a function of the Limiting Isotropic Compression Line (LICL) which in turn is defined to be parallel to the CSL with a constant shift in $v - \ln p'$ plane along the recompression line, as shown in Figure 9.

The loading surface is assumed to be of the same shape as the bounding surface. For first time loading these two surfaces are homologous about the origin of the stress coordinate system. The state of stress, σ'_1 , is always located on the loading surface. An image for the state of stress can be found on the bounding surface, $\bar{\sigma}'_1$, as shown in Figure 10. For unloading and reloading conditions, the center of homology moves to the last point of stress reversal. The image of the stress point in the $p'-q$ plane is located using the Pegasus method (Dowell and Jarratt [21] and Sloan *et al.* [17]). Upon stress reversal, a new loading surface is formed with the new center of homology, as shown in Figure 10 and results in an image $\bar{\sigma}'_2$ for the new stress state, σ'_2 . To maintain similarity with the bounding surface, the loading surfaces undergo kinematic hardening during loading and unloading.

The unit normal vector at the image point, defining the direction of loading, is given by:

$$\mathbf{n} = \frac{\partial F / \partial \bar{\sigma}'}{\|\partial F / \partial \bar{\sigma}'\|} \quad (5)$$

The plastic potential defines the direction of plastic strain increments and is generally expressed using a plastic flow rule relating the plastic dilatancy ($\dot{\epsilon}_v^p / \dot{\epsilon}_q^p$) to the stress ratio q / p' . In this model, the plastic potential, g , is defined as

$$g(p', q, \bar{\theta}, p_o) = \tilde{t}q + \frac{AM_{cs}(\bar{\theta})p'}{A-1} \left(\left(\frac{p'}{p_o} \right)^{A-1} - 1 \right) \quad \text{for } A \neq 1 \quad (6)$$

$$g(p', q, \bar{\theta}, p_o) = \tilde{t}q + M_{cs}(\bar{\theta})p' \ln \left(\frac{p'}{p_o} \right) \quad \text{for } A = 1 \quad (7)$$

where p_o controls the size of the plastic potential, though its value is not important in the model since only derivatives of g are required, $\tilde{t}$ is a scalar, the sign of which controls the direction of plastic flow in the deviatoric plane (i.e. +1 for compressive loading and -1 for extensive loading), and A is a material parameter. The direction of plastic flow is defined by:

$$\mathbf{m} = \frac{\partial g / \partial \boldsymbol{\sigma}'}{\|\partial g / \partial \boldsymbol{\sigma}'\|} \quad (8)$$

The hardening modulus h consist of two components

$$h = h_b + h_f \quad (9)$$

where h_b is the plastic modulus at $\bar{\boldsymbol{\sigma}}'$ on the bounding surface, and h_f is the plastic modulus at $\boldsymbol{\sigma}'$ on the loading surface, and defined as a function of the distance between $\boldsymbol{\sigma}'$ and $\bar{\boldsymbol{\sigma}}'$:

$$h_b = \frac{\nu}{(\lambda - \kappa) \ln R} \frac{m_p}{\|\partial F / \partial \bar{\boldsymbol{\sigma}}'\|} \quad (10)$$

$$h_f = \frac{\nu p'}{(\lambda - \kappa)} \left(\frac{\bar{p}'_c}{p'_c} - 1 \right) k_m (\eta_p - \tilde{t}\eta) \quad (11)$$

The modulus h_f is zero on the bounding surface (i.e. where $\bar{p}'_c = p'_c$) and infinity at the point of stress reversal.

Within the above equations $m_p = \partial g / \partial \bar{p}' / \left\| \partial g / \partial \boldsymbol{\sigma}' \right\|$ and λ is the current slope of the LICL in $v - \ln p'$ plane, $\bar{p}'_c$ and p'_c define the sizes of the bounding and loading surfaces, respectively, η_p is the slope of the peak strength line in the $q - p'$ plane, and k_m is a scaling parameter controlling the steepness of the response in the $q - \varepsilon_q$ plane. The slope of the peak strength line is a function of the state parameter, ξ , and the slope of the critical state line, and is given by:

$$\eta_p = \tilde{t} (1 - k\xi) M_{cs} \quad (12)$$

where k is a material parameter. Manzari and Dafalias [22] have used a similar expression for peak stress ratio in their model. The scaling parameter k_m can be taken as a material constant for some soils, though it is strongly influenced by the initial state parameters and the loading direction. It is defined as

$$k_m = k_{mo} \langle 1.0 - \beta_1 \exp(\xi_0) \rangle (p'_0)^{\beta_2} \quad (13)$$

where k_{mo} , β_1 and β_2 are material parameters.

3.2. Stress Integration Scheme and Overshooting Issues

UNSW bounding surface plasticity model has been implemented in the explicit finite difference code FLAC [23]. The general explicit forward Euler integration scheme is used with an optional automatic sub-stepping technique (Sloan *et al.*, [17], Sloan [24]). In order to avoid overshooting during small superficial stress reversals, the method proposed in this study has been implemented in the integration algorithm. The accumulated plastic shear strain is calculated from:

$$\delta\epsilon_q^p = \int \dot{\epsilon}_q^p = \int \Lambda \frac{\partial g}{\partial q} \quad (14)$$

where Λ is a plastic strain multiplier defined as:

$$\Lambda = \frac{\mathbf{n}^T \mathbf{D}^e \dot{\boldsymbol{\epsilon}}}{h + \mathbf{n}^T \mathbf{D}^e \mathbf{m}} \quad (15)$$

and $\mathbf{D}^e$ is the elastic stiffness matrix. Defining $\partial g / \partial q = \tilde{t}$, the incremental plastic shear strain is calculated as:

$$\delta\epsilon_q^p = \int \Lambda \tilde{t} = \tilde{t} \int \frac{\mathbf{n}^T \mathbf{D}^e \dot{\boldsymbol{\epsilon}}}{h + \mathbf{n}^T \mathbf{D}^e \mathbf{m}} \quad (16)$$

The plastic shear strain accumulated over a number of loading increments is calculated simply by summing up the incremental plastic shear strains, assuming the stress increments in the forward integration scheme are sufficiently small:

$$\delta\epsilon_q^p = \tilde{t} \sum \frac{\mathbf{n}^T \mathbf{D}^e \dot{\boldsymbol{\epsilon}}}{h + \mathbf{n}^T \mathbf{D}^e \mathbf{m}} \quad (17)$$

The criterion for stress reversal is defined based on the sign of the plastic strain multiplier, a well established procedure in literature (e.g. Pastor *et al.*, [16]) which is well suited to the numerical integration. Whenever $\Lambda < 0$, the following actions are taken:

A. If $\delta\epsilon_q^p$ with reference to the last true stress reversal is greater than the prescribed threshold, $\Delta\epsilon_q^p$, then:

A.1. Store the current center of homology (H) as (H_p) and store its corresponding ϵ_q^p .

A.2. Update the center of homology to the current stress state.

A.3. Set the current stress state as reference for calculation of the accumulated plastic shear strain.

A.4. Record the maximum deviatoric stress if unloading is identified.

B. If $\delta\epsilon_q^p$ is less than $\Delta\epsilon_q^p$, the stress reversal is a small superficial one and the following steps should be followed to avoid overshooting:

B.1. As long as the current deviatoric stress is less than the maximum deviatoric stress recorded previously; use the current H as reference point for mapping of the image point and for calculation of the accumulated plastic shear strain. This is to insure that the stress path of a small loop has not yet been closed or reached the general loading path.

B.2. When the deviatoric stress becomes greater than maximum deviatoric stress recorded, use the previous center of homology H_p , and its corresponding ϵ_q^p as reference points for mapping of the image point and for calculation of the accumulated plastic strain; respectively. This case indicates that a small loop is closed and reached the general loading path and therefore its effects on the location of the center of homology or the size of loading surface should be ignored.

4. APPLICATION AND RESULTS

A series of cyclic and dynamic triaxial tests are simulated first, mainly for calibration of the material parameters used later in boundary value problems, and also to demonstrate the robustness of the bounding surface model. Then two boundary value problems, one under static loading and the other under dynamic loading, are simulated and the results are compared with experimental data.

4.1. Strain controlled cyclic triaxial tests on an artificial soil

Desai and Siriwardane [25] have reported the results of an extensive study on constitutive modelling of an artificial soil. This soil consists of a mixture of fire clay (50%) and zircon silt (50%), moisturized with a mineral oil as a binder. This soil can be classified as sandy silt which exhibits highly compressive characteristics.

A series of triaxial tests has been conducted on this soil, among which three conventional drained cyclic tests are selected for simulations in this study. The material parameters used in these simulations are summarized in Table 1. The parameter k_m is taken as 10 for first time loading and 50 for any subsequent unloading/reloading. The initial mean effective stresses of the samples were 6.89, 13.78 and 34.45 *kPa* with initial specific volumes of 1.85, 1.80 and 1.75 for tests CTC-01, -02 and -03; respectively.

The results of the simulations are compared with the experimental data in Figure 11, in both $q - \varepsilon_a$ and $\varepsilon_p - \varepsilon_a$ planes. In the $q - \varepsilon_a$ plane, while there is a slight discrepancy between the predicted performance and the experimental data in test CTC-03 where the sample is under higher confining pressure, the model predictions are in excellent agreement with the observed data. In the $\varepsilon_p - \varepsilon_a$ plane an excellent agreement is observed between the predicted and the experimental data up to an axial strain of about 9%. The predicted volumetric strains at the end of tests CTC-01 and -02 are lower than those measured in the experiments.

4.2. Stress controlled cyclic triaxial tests on Nevada sand

Nevada sand was used in VELACS project (Arulmoli *et al.* [26]) under both 1g laboratory conditions and in centrifuge experiments.

Two cyclic triaxial tests on loose ($D_r = 40\%$) and dense ($D_r = 60\%$) Nevada sand are considered for calibration purposes and to demonstrate the robustness of the model. Both tests were conducted under load controlled conditions, using a sinusoidal wave with frequency of around 1 Hz. An initial deviatoric stress offset was applied to the samples, which represents the real condition of the soil in the field.

The material parameters used in the simulation are shown in Table 1. The parameter k_{mo} is taken as 6.2 for first time loading and 115.8 for any subsequent unloading/reloading, and β_1 and β_2 are taken 0.95 and 0.34, respectively, for all loading and unloading/reloading conditions. The initial conditions for the loose sample were $p'_0 = 40kPa$ and $\nu_0 = 1.736$. The sample was loaded for 11 seconds by cyclic deviatoric stress varied from -14.5 to 22.3 kPa. The initial conditions for the dense sample were $p'_0 = 80kPa$ and $\nu_0 = 1.656$. The sample was loaded for 14 seconds under cyclic deviatoric stresses varied from -22.1 to 37.1 kPa.

The results of the simulation of loose and dense samples of Nevada sand are compared with experimental data in Figure 12 and Figure 13, respectively. These figures show that the predicted behaviour is in good agreement with the observed experimental data. The model captures the failure of the samples by liquefaction in which the effective normal stress decreases progressively until the stress path reaches the critical state and the material becomes unstable when excess pore pressure approaches the initial mean effective stress.

4.3. Stress controlled dynamic triaxial test on Niigata sand

Ishihara and Yasuda [27] performed a series of stress-controlled dynamic tests on Niigata sand. The dynamic stresses had irregular amplitude, applied on samples of loose sand by means of a closed loop servo-hydraulic loading system. The time histories of the N-S

component of horizontal accelerations recorded at the time of the Niigata earthquake (1964) were converted to shear stress time history and applied to the triaxial samples of Niigata sand.

In one of these tests the sample was subjected to deviatoric stresses with irregular amplitude and many small unloading/reloading components, as shown in Figure 14. This test is intentionally considered in this study to demonstrate the capability of UNSW model in treating small stress reversals. The sample was initially consolidated under $p'_0 = 147 \text{ kPa}$ to obtain an initial specific volume of $v_0 = 1.781$ and then was subjected to the deviatoric stress record under undrained conditions.

The material parameters used for the simulation are presented in Table 1. Due to the lack of information on the properties of the Niigata sand required for the constitutive model, most of the material parameters are assumed to be the same as those of Nevada sand. M_{cs} was estimated based on the tests on frozen dense samples of Niigata sand as reported by Yoshimi *et al.* [28]. k_m is also calibrated based on another test on Niigata sand performed by Ishihara and Yasuda [27] under sinusoidal loading on a sample with the same initial conditions, and is taken as 2 and 19 for first time loading and unloading/reloading; respectively.

The results of the numerical simulation are presented in Figure 15. Figure 15(a) shows the predicted variation of excess pore pressure, u , with time and compares it with those recorded in the experimental test. These results seem to offer good agreements in u –time space, despite the fact that typical material parameters have been used in the simulation. The variation of pore pressure ratio, r_u , with axial strain is shown in Figure 15(b). r_u is defined as the ratio of the excess pore pressure, u , to the initial mean effective stress, p'_0 . No experimental data has been published in r_u – ε_a space to be used for comparison. However the progressive axial deformation predicted by the simulation when pore pressure ratio exceeds

0.6 is similar to that observed in the behavior of loose Nevada sand in VELACS project (Arulmoli *et al.* [26]).

4.4. Settlement of a foundation on artificial soil

The proposed method and UNSW bounding surface plasticity model are used to predict the behaviour of a strip footing as reported by Desai *et al.* [29]. The soil used in this experiment was the artificial sandy silt described previously in this study. The experiment was conducted on a rectangular box 900 mm long, about 200 mm high and 115 mm wide, as shown in Figure 16(a). The footing, with a width of 75 mm, was loaded incrementally and the vertical displacements of the footing were recorded.

Figure 16(b) shows the finite difference mesh used in this study which models half of the footing-soil system. The vertical far-side boundary and the center line were assumed to be smooth, allowing only vertical movements, while the base boundary was assumed to be rough. The material parameters used for the simulation of this artificial soil in the triaxial tests performed previously, and summarized in Table 1, are also used in this simulation.

The predicted displacement of the footing versus applied vertical load are presented in Figure 17 and compared with the experimental data. Also shown in Figure 17 is the result of a numerical analysis using Mohr-Coulomb model, assuming a mobilized friction angle of 36° and dilation angle of 20° . It can be seen that the simulation with UNSW model captures well the experimental data throughout the whole loading phase. The Mohr-Coulomb model shows a very stiff behavior but predicts an ultimate bearing capacity close to that proposed by Desai and Siriwardane [25] for this soil. Obviously the behavior of this soil cannot be classified by elastic-perfectly plastic models and therefore a clear ultimate capacity could not be obtained in the experimental test.

As was discussed before, the overshooting effect due to the integration scheme is an inherent phenomenon in simulation of boundary value problems using bounding surface plasticity models. In order to show this effect in the foundation problem, an analysis was performed without any treatment for overshooting. Figure 17 shows the predicted load-deflection curve for the case where overshooting has not been treated in the analysis. The stiffer response of this case is a result of overshooting phenomenon. Figure 18 compares the variation of deviatoric stress versus the mean effective stress for two zones in the domain predicted by analyses with and without overshooting treatment. Zone#1 is located under the footing and zone#3 is adjacent to the loaded area, as shown in Figure 16(b). Simulation without treatment show significant overshooting effects in both zones. Although the effect of overshooting is not clear in zone#1 at the beginning of loading, the effect becomes significant in the later stages, leading to divergence of the stress path towards the critical state line. Also there is a noticeable difference between the results of analyses for zone#3 from the early stage of loading, where without treatment of overshooting the stress path for this zone coincides with the critical state line.

Figure 19 presents the variation of vertical displacement in the foundation soil, as predicted by the numerical simulation. It shows large movements occur in the vicinity and below the footing while very little movement is predicted in the regions away from the footing. The predicted displacement is similar to that observed in the experiment, as reported in Desai *et al.* [29]. This indicates that soil is very compressible and the footing essentially punches into the soil, the soil undergoes significant volumetric change under the footing without showing a clear ultimate capacity where large displacement occurs with insignificant change in capacity.

4.5. Seismic behaviour of liquefiable soil in centrifuge test

The centrifuge Model #1 of the VELACS project [26] has been used here to verify the capability of the model and the proposed method of preventing the overshooting problem. The centrifuge test simulates the seismic response of a liquefiable loose soil. The test setup and the instrumentation of the model in prototype scale are shown in Figure 20(a). The centrifuge model was formed in a laminar box and was subjected to 50g acceleration. In prototype scale, the experiment represents a 10m deep Nevada sand layer, with a relative density of 40%. The dynamic excitation applied at the base of the laminar box simulating a peak horizontal acceleration amplitude of 0.23g, a total duration of approximately 12s, and a predominant frequency of about 2Hz in prototype scale (Figure 21). The response of the model was monitored with eight accelerometers, eight pore pressure transducers, and six LVDTs for measuring displacements.

The centrifuge model was idealized with a plane strain model in the numerical analysis, as shown in Figure 20(b). Fluid flow (i.e. seepage) was allowed to occur only at the top, while the bottom and the lateral boundaries were set to be impermeable. The lateral boundaries of the numerical model were set free to move, while tied to each other in order to simulate the laminar box.

The initial conditions of the model before spin up were simulated by a uniform specific volume of 1.724, as was reported by Taboada and Dobry [30]. The initial static equilibrium was then achieved in the analysis assuming a coefficient of horizontal earth pressure at rest of $K_0 = 0.5$, which simulates the linear variation of the initial stresses with depth. The base of the model was subjected to the prescribed input motion in the horizontal direction as show in Figure 21. The Initial coefficient of permeability was set to 2.1×10^{-5} m/s, as suggested by Taboada and Dobry [31]. However, the permeability of the sand varies during liquefaction

tests as shown by many investigators. Arulanandan and Sybico [32] conducted a permeability test on the same soil and model using electrical resistivity probes and showed that the initial permeability increases rapidly as the soil approaches initial liquefaction and then decreases more smoothly thereafter, all due to fabric change after the onset of liquefaction. This concept was also utilized by Manzari and Arulanandan [33] and Andrianopoulos *et al.* [34] in their simulations. The variation of permeability of the sand during centrifuge test used in the numerical analysis is shown in Figure 22. This figure depicts the observation made by Arulanandan and Sybico [30] and matches the one used by Manzari and Arulanandan [33].

A coupled dynamic analysis was performed using the material parameters listed in Table 1. These parameters were calibrated previously in this study and used in the simulation of triaxial tests on Nevada sand.

The time histories of the excess pore pressures predicted at different elevations by the analysis are compared with those recorded in the centrifuge test in Figure 23. The initial effective vertical stresses at different elevations are also shown in this figure. The records of excess pore pressures show initial liquefaction in the upper 5 m of the sand, where very high excess pore pressures, in the order of the initial effective vertical stresses, are developed. Liquefaction occurs faster in the upper layers of the model. The excess pore pressures developed at points deeper than 5 m were not sufficient to initiate liquefaction. The variations of pore pressures predicted by the numerical analysis match well with those measured. At all points the predicted excess pore pressures reduces at the end of excitation, as observed in the experiment. The variations of the excess pore pressures with depth during the excitation are shown in Figure 24 and compared with those observed in the experiment. The predicted depth of the liquefied zone is exactly the same as that observed in the test at the end of excitation ($t = 12\text{sec}$). The predicted pore pressures in the liquefied zone are slightly higher than those observed in the test.

Figure 25 compares the time history of the vertical displacement (settlement) recorded at the top of the model with that predicted by the numerical analysis. While the numerical analysis predicts slightly higher displacements within the period of 4 to 12 seconds, the final settlement predicted by the analysis is in very good agreement with those measured. It should be noted that, apart from Manzari and Arulanandan [33] and Lacy [35], most of the numerical simulations of this centrifuge test have predicted settlements of less than 70 mm and therefore the results of the current simulation can be regarded as excellent compared to other predictions reported in the literature.

It should be noted that if overshooting is not treated, the pore pressure will be predicted to increase substantially within the first 2 seconds of excitation and the soil liquefies quickly.

5. CONCLUSIONS

The overshooting phenomenon is an important issue in application of bounding surface plasticity models in solving boundary value problems. Overshooting occurs when a small load cycle is encountered within a general load path. A small load cycle may be in the form of an unloading-reloading loop of small extents. It may also occur even in numerical analysis of problems under monotonic loading conditions when the numerically obtained stress path oscillates around the true stress path. A novel approach is presented in this paper which is based on the concept of cloud of loading surfaces within which any unloading/reloading does not have any effect on the size of the loading surface or the location of the center of homology. This scheme has been implemented in UNSW bounding surface plasticity model and its performance and robustness were demonstrated through simulations of two boundary value problems; a footing test on dense artificial sandy silt and a centrifuge dynamic test on loose Nevada sand. The model parameters used in these simulations were obtained through triaxial tests on the artificial sandy silt and Nevada sand. The results of these simulations were shown

to be in good agreement with those obtained from experiments if the proposed scheme to for treatment of overshooting problem is activated in the analysis. Without the treatment the numerical simulation of footing test shows stiffer behavior and the centrifuge simulation results in generation of large pore pressures at the beginning of the analysis and divergence of the solution.

It should be pointed out that in application of bounding surface models in analysis of boundary value problems, the model parameters can be selected in such a way that the predicted behavior matches well with the observed response. In such cases the stiffer behavior that would have been predicted for a problem due to overshooting is offset by calibrating material parameters that would result in a softer behavior for the soil. However, one of the novelties of the study presented here is that the model parameters have been calibrated by numerical analysis of triaxial tests and then used in the simulation of the footing and the centrifuge tests and good agreements between the predicted and the observed performances were obtained. This could not have been made possible if the overshooting phenomenon was not treated in the solution algorithm and its effects were not eliminated from the solutions.

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Table 1: Material constants used in all simulations

Soil	Reference	κ	ν	M_{cs}	λ_0	λ_{cr}	p'_{cr} (kPa)	Γ_0	N	R	k	A
Artificial Soil (Silty Sand)	Desai and Siriwardane [25]	0.001	0.35	1.60	0.11	N/A	N/A	2.04	1.44	2.0	2.0	1.0
Nevada Sand	Arulmoli <i>et al.</i> [26]	0.003	0.33	1.25	0.022	0.24	2000	1.91	1.5	3.0	2.0	1.5
Niigata Sand	Ishihara and Yasuda [27] Yoshimi <i>et al.</i> [28]	0.003	0.33	1.70	0.022	0.24	2000	1.91	1.5	3.0	2.0	1.5