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Strengthening hollow reinforced concrete columns with fibre reinforced polymers

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Strengthening Hollow Reinforced Concrete Columns With Fibre Reinforced Polymers

A thesis submitted in fulfillment of the requirements for the award of the degree of

DOCTOR OF PHILOSOPHY

From

UNIVERSITY OF WOLLONGONG

By

Veysel YAZICI, Civil Engineer (MSc)

School of Civil, Mining and Environmental Engineering

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THESIS DECLARATION

I, Veysel Yazici, hereby declare that all material in this thesis, submitted in fulfillment of the requirements of the award of Doctor of Philosophy, in the School of Civil, Mining and Environmental Engineering, University of Wollongong, is wholly my own work unless otherwise referenced or acknowledged. This document has not been submitted for qualifications at any other academic institution.

Veysel YAZICI

Date:
To my beloved wife and son
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Lastly, I want to thank my wife Arife YAZICI and my little son Erkal Baran YAZICI for their support during all stages of my PhD study.
PUBLICATIONS ARISING FROM THIS THESIS

Journal Publications:


Conference Papers:


ABSTRACT

Hollow Reinforced Concrete (RC) columns have been preferred to solid columns to reduce the cost of structures, and to decrease their self-weight where it is technically necessary. In spite of their widespread use, even modern design codes do not address any specific problems related to hollow section columns. This study aims to predict the behaviour of hollow RC columns confined with FRP wraps. A stress-strain model valid for both solid and hollow confined cylinders was developed. A modelling study was undertaken to generate the axial load-bending moment diagrams and the loading lines which give the axial load versus bending moment values under axial compression from early service loads up to the failure of FRP wrapped hollow RC columns using a moment magnification approach. To validate the modelling study, an experimental study was carried out to see the effect of column height, FRP wrapping configuration and loading eccentricity. A total of 18 hollow RC columns with the same cross section geometry and internal steel reinforcement were cast and tested under axial compressive loading in the Structural Engineering Laboratories of the University of Wollongong. Nine of these sample columns were 500 mm in height (short columns) and the other nine were 885 mm tall (tall columns). Each group of sample columns were divided into three sub-groups of three and a different FRP wrapping configuration was applied on each group. The sample columns were then tested under axial compressive load with 0, 25 and 50 mm eccentricities. The application of the eccentric load was achieved by special loading heads and knife-edges designed and manufactured for this study. The test results and the modelling study were seen to be in good agreement showing that the theoretical model leads to reliable results.
ABBREVIATIONS

\( A_c \) : Concrete area in a RC column cross section (mm\(^2\)).

\( A_s \) : Total area of longitudinal steel reinforcement in a RC column cross section (mm\(^2\)).

\( cc \) : Thickness of concrete cover from the surface to the internal steel reinforcement (mm).

\( D_h \) : Hollow diameter for hollow cross-sections (mm).

\( D_o \) : Outside diameter (mm).

\( D_{s,\text{helic}} \) : Diameter of helical steel bar in a RC column cross-section (mm).

\( D_{s,\text{long}} \) : Nominal diameter of longitudinal steel bars in a RC column cross-section (mm).

\( E_I \) : Flexural stiffness of a column cross-section (MPa.mm\(^4\)).

\( E_s \) : Elasticity modulus of longitudinal steel reinforcement (MPa).

\( f'_{cc} \) : Maximum axial stress for confined concrete (MPa).

\( f'_{co} \) : Compressive strength of unconfined concrete (MPa).

\( f_{frp} \) : Tensile rupture strength of FRP sheets determined by coupon test (MPa).

FRP : Fibre reinforced polymer.

\( f_{y,\text{helic}} \) : Yield strength of helical steel bar (MPa).

\( f_{y,\text{long}} \) : Yield strength of longitudinal steel bars (MPa).

\( H \) : Height of the RC column (mm).

\( K_N \) : Normalised stiffness of FRP confinement.

\( n_f \) : Number of vertical FRP strips.

\( n_{frp} \) : Number of layers of FRP sheets in concrete confinement.

\( n_{s,\text{long}} \) : Number of longitudinal steel reinforcement bars in a RC column cross-section.

\( P_c \) : Euler’s buckling load on a column (kN).

RC : Reinforced concrete.

\( t_f \) : Total thickness of FRP confinement (mm).
$t_{frp}$ : Thickness of a single layer of FRP sheet used for confinement of concrete (mm).

UOW : University of Wollongong.

$w_{frp}$ : Width of FRP strips (mm).

$\varepsilon_{cc}$ : Axial strain value corresponding to the maximum axial stress for confined concrete.

$\varepsilon_{co}$ : Axial strain value corresponding to the maximum axial compressive stress ($f'_{co}$) for unconfined concrete.

$\varepsilon_{frp}$ : Tensile strain of FRP sheets corresponding to the tensile strength determined by coupon test.
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Chapter 1: INTRODUCTION

1.1. General
Worldwide application and vast experience in reinforced concrete (RC) technology for the last 100 years have shown that RC structures exhibit a reliable and durable behaviour when they are properly designed, built and maintained. However, porosity and cracking of concrete, as well as corrosion of steel reinforcement can lead to deficiency of existing RC structures especially when they are exposed to extreme and hostile environmental conditions and excessive loads. Other causes of structural deficiency are design mistakes, increase in load carrying needs, change of use of the structure, and adoption of more stringent design codes especially in seismic zones.

It is a well known fact that the infrastructure of a country needs to be maintained either by demolishing the deficient structures and building new ones, or by rehabilitating the old ones with appropriate techniques to meet the current requirements for the continuity of modern civilization. Unavoidably, countries which have had well established structural infrastructure for a long time are also the ones to have problems with it due to aging of existing structures. Australia’s infrastructure condition was assessed to be in urgent need of rehabilitation especially for the highway bridges (Engineers Australia 1999). However, no cost figure has been estimated for the rehabilitation.

In USA, it was estimated that 40% of the 575000 highway bridges were either structurally deficient or obsolete, and 25% were over 50 years old and unsuitable for the current or projected traffic needs (Marshall et al. 1996). The estimated cost for repairing all deteriorating bridges was US$78 billion; however, only $5 billion a year was available for such repairs (Teng et al. 2000).

Increasing costs of construction materials and labour, together with the global financial hardship make it more difficult to handle this costly problem even for the most developed countries. Current financial conditions force the engineers to come up with new repair and upgrading techniques using alternative methods and materials to extend the service life of structures which may be a more economical and effective solution than building a new structure.
1.1.1. **RC Columns in Structures**

Many structures are erected on RC columns. The strength, ductility and durability of RC columns strongly affect the overall behaviour of structures, thus understanding the behaviour of individual columns under various loading conditions is of great importance. This study focuses on RC column retrofitting, thus beam and slab retrofitting are left out of discussion.

RC columns are generally referred to as vertical load carrying structural members. However, it is also known that this kind of classification is merely an idealization as columns may also carry shear and bending moments depending on their position in the structure, uneven settlements, workmanship errors and seismic loads. RC columns are constructed as highway bridge peers, vertical load carrying members in buildings or underground piles. They are constructed in many different cross-sectional geometries and heights using different types and amounts of concrete and steel reinforcement depending on the structural necessity and material availability.

To keep RC columns from further deterioration due to materials’ vulnerability to hostile environment conditions and to restore their strength, ductility and durability to a technically accepted level, there are a number of retrofitting techniques developed and tested. Steel jacketing and Fibre Reinforced Polymer (FRP) wrapping are the most commonly used methods (Priestley *et al.* 1996). These methods may also be applied to undamaged or new RC columns to increase their load carrying capacity and ductility.

In order to develop a general picture of RC column retrofitting, a brief description of steel jacketing and FRP wrapping techniques are described in Chapters 2 and 3, respectively.

1.2. **Scope**

Many studies have been carried out to establish a reassurance over this relatively new material and retrofitting technique, RC column strengthening by FRP wrapping, over the last 20 years. However, nearly in all of them, the experimental works were carried out only on solid cross-sectioned short columns leaving hollow cross-sectioned columns out of discussion, which were already left in the dark even in existing design codes for RC concrete columns. This study aims at explaining the behaviour of hollow RC columns wrapped with FRP materials under axial
compressive loads with varying eccentricities.

Due to various possibilities of combinations of RC column-FRP wrapping hybrid system, cross-sectional geometry, positioning of steel reinforcement in the hollow RC columns and loading types, this study is limited to the following:

1. The hollow RC columns under consideration have longitudinal and transverse steel reinforcement as one layer near the outside face.

2. Only short term behaviour of FRP wrapped hollow RC columns under axial compression with changing eccentricities is investigated. Behaviour of FRP wrapped hollow RC columns under dynamic and shear loads; creep and shrinkage due to sustained loads is left out of this study.

3. Due to more uniform distribution of FRP confinement, only circular hollow RC columns are considered.

4. In the experimental part, the effects of FRP wrapping configurations, loading eccentricities and the height on the behaviour of hollow RC columns are investigated.

1.3. Objectives

The objective of this study is to investigate the short term behaviour of circular hollow RC columns wrapped with Carbon-FRP sheets under concentric, and eccentric compression. The main objectives are as follows:

1. Develop a unified stress-strain model for FRP confined concrete usable for both solid and hollow cross-sectioned concrete columns.

2. Develop axial load-moment-curvature relations analytically using the proposed stress-strain model for FRP wrapped hollow RC columns, and verify the model with the tests on the experimental part of this study.

3. Develop moment-axial load interaction equations for calculating the maximum load carrying capacity of FRP wrapped hollow RC columns.

4. Apply moment magnification method on FRP confined hollow RC columns to predict the loading lines under compressive loads with a given eccentricity.

5. Investigate the effects of various parameters (material strengths, geometry and height) on the behaviour of FRP wrapped hollow RC columns.
1.4. **Organization of Study**

This study basically focuses on four main aspects of FRP retrofitted hollow RC columns under short term loads;

1. Modelling the stress-strain behaviour of FRP confined concrete for plain solid and hollow concrete columns.

2. Developing moment-curvature relationship and moment-axial load interaction diagrams of hollow RC columns under concentric, eccentric and pure bending loads.

3.Determining the effects of FRP wrapping on the behaviour of hollow RC columns under concentric and eccentric axial loads both analytically and experimentally. The study will also handle the effect of slenderness on FRP wrapped hollow RC columns.

In Chapter 2, the effects of lateral confinement on concrete behaviour are discussed. How the equations developed for active confinement could also be used for steel confined concrete is explained. The behavioural differences of hollow RC columns from the solid ones is emphasized. Steel jacketing as a retrofitting technique is described.

In Chapter 3, a brief history of Fibre Reinforced Polymer materials is given together with its typical material characteristics, constituents and production processes. Chapter 3 also contains a literature review of previously proposed FRP confined concrete models by other researchers.

In Chapter 4, a new unified stress-strain model for solid and hollow core plain concrete columns is developed using the previously reported experimental results of other researchers.

In Chapter 5, based on the new unified stress-strain model proposed, axial load-moment-curvature relations and axial load-moment interaction diagrams are developed and presented considering the nonlinear behaviour of constituting materials. Loading lines for short and slender RC columns are calculated using a moment magnification method similar to the ones described in AS3600 (2009) and ACI 318 (2011) to see the axial load carrying capacity change under various slenderness of RC columns and loading eccentricities.

To check the proposed relationships and methods in Chapters 5, an experimental study is carried out and presented in Chapter 6. The experimental results from
Chapter 6 and modelling results from Chapters 4 and 5 are compared and discussed in Chapter 7 to check the validity of the analytical modelling of FRP wrapped hollow RC columns.

In Chapter 8, a summary, concluding remarks and recommendations for future research studies are expressed.
Chapter 2: USE OF STEEL IN CONCRETE CONFINEMENT

2.1. Introduction
Today, concrete is undoubtedly one of the most commonly used construction material in the world. There have been many successful studies to explain the concrete behaviour and to develop engineering solutions regarding its use in civil engineering applications. However, its behaviour under changing loading, geometry and reinforcement conditions is still yet to be understood fully.

This chapter aims to give a brief introduction to the effects of lateral confinement provided by internal steel reinforcement and external steel jacketing on the compressive behaviour of concrete.

2.2. Confinement Provided by Internal Helix Steel Reinforcement
Though reinforcement of concrete was first applied to overcome tension stresses formed in the concrete beams and slabs, it was as early as 1906, when Considère (1906) acknowledged the beneficial effect of lateral confinement on the concrete strength and deformation capacity and the idea of using steel reinforcement for concrete column confinement has been generally accepted since then.

![Figure 2.1.](image)

Figure 2.1. (a) Confinement of concrete column core by steel helix, (b) an isolated part of pitch height, (c) equivalent jacket thickness of steel reinforcement.

In order to use steel reinforcement as a lateral confinement device, the steel bars are bent to form a helix and placed in the concrete column formwork before the casting.
takes place. In this way, it is aimed that the steel helix will be trapped within the concrete body and undergo a compatible radial expansion with the concrete as the column is subjected to axial loads providing a confinement to the concrete core (Figure 2.1 (a)).

Assuming that the helical reinforcement is smeared around the core concrete to form a continuous steel jacket (Figure 2.1 (b), (c)), the equivalent thickness \( t \) is calculated by assuming the volume of helix steel in the pitch \( s \) is equal to the volume of the equivalent steel jacket as shown in Equation 2.1.

\[
A_{\text{helix}}\pi D = t\pi D_s
\]

\[
\Rightarrow t = \frac{A_{\text{helix}}}{s}
\]

where \( A_{\text{helix}} \) is the cross sectional area of helical steel bar, \( s \) is the pitch of the helix reinforcement and \( D \) is the diameter of the concrete core confined by helix reinforcement steel.

When the tensile stress-strain relationship for the steel bars are simplified to an elastic-perfectly plastic behaviour as shown in Figure 2.2, the helix steel reinforcement can be assumed to be providing a constant confinement pressure after it yields as it expands.

![Figure 2.2. Simplified stress-strain behaviour of steel used for confinement.](image)

The constant confinement pressure \( (f_c) \) can be calculated using Equation 2.2 assuming plane stress conditions. Writing a force equilibrium equation in the \( y \) direction (Figure 2.3) gives;
\[
\int_0^\pi R_f \sin \theta d\theta = 2 f_{ys} t
\]
\[
\Rightarrow 2R_f = 2 f_{ys} t
\]
\[
f_i = \frac{f_{ys} t}{R}
\]

Where \( f_{ys} \) = yield strength of helix reinforcement, \( D \) = diameter of the concrete core inside the steel helix, \( t \) = the equivalent thickness of steel helix (calculated using Equation 2.1).

Figure 2.3. Stresses acting on half-loop steel jacket due to dilatation of concrete under axial compression.

### 2.3. RC Column Retrofitting by Steel Jacketing

This study is concerned with FRP confined RC columns. However, one of the popular and relatively cheap methods of strengthening RC columns. As such this section presents a brief overview of steel jacketing.

Steel jacketing has been a popular retrofitting method for the deficient RC columns due to already established ultimate strength and strain equations for steel confined concrete.

Steel jacketing is simply applied to the deficient RC column in two half shells with a slightly larger diameter than the RC column itself. The parts are welded together over the areas to be retrofitted and the gap between the RC column and steel jacket is filled with cement grout (Priestley et al. 1996). The jacket provides a passive lateral
confinement similar to internal steel helix reinforcement mentioned above as the column tends to expand laterally under axial loads.

2.4. Behaviour of Hollow RC Columns

It has been well known that, confined solid concrete columns can sustain larger compressive strains than unconfined ones without significant loss of compressive strength (Richart et al. 1928, Sheikh and Uzumeri 1982, Mander et al. 1988a, 1988b). These studies led to design codes determining the required amount of transverse steel reinforcement for solid RC columns to provide enough confinement for ductility under service and seismic loads. Interestingly, none of the existing design codes addresses specific problems attributed to the design of hollow columns. Confinement mechanism of transverse reinforcement in hollow RC columns has been assumed as the same as solid RC columns and designs were done accordingly.

For RC columns with small cross-sectional size, it is more convenient to place the longitudinal and transverse steel reinforcement as one layer near the outside face to achieve a simple arrangement of reinforcement (Figure 2.4 (a)). However, Zahn et al. (1990) showed that this arrangement of reinforcement for hollow columns results in a brittle behaviour in hollow RC columns since the concrete inside the transverse reinforcement is not confined as well as solid RC columns.

Mander et al. (1983) and Whittaker et al. (1987) have investigated the flexural strength and available ductility of large rectangular and circular hollow RC columns with two layers of longitudinal and transverse steel reinforcement placed near both the inside and outside faces of the section (Figure 2.4 (b)). These columns were shown to perform in a ductile manner during cyclic lateral loading in the inelastic range due to confinement provided by the transverse reinforcement. However, the amount of transverse reinforcement to achieve a ductile behaviour in hollow RC columns was higher than solid RC columns which will make the assumed advantages of using hollow RC columns questionable.
Figure 2.4. Typical cross sectional geometry of (a) a small hollow RC columns, and (b) a large hollow RC columns.

Since the existing studies on the hollow RC columns reveal that the design codes addressing the reinforcement of solid RC columns may not be suitable for hollow RC columns, previously designed and constructed structures having hollow RC columns may have to be retrofitted for the strength and ductility requirements.

2.5. Models for Active and Passive Confinement

2.5.1. Introduction

Though the steel confinement provides a passive confinement to the concrete under compressive stress, the theory behind the design equations were based on active confinement tests. Thus, it is convenient to mention also the active confinement studies in this study.

There are many studies done on both confinement types, however Richart et al.’s (1928, 1929) and Mander et al.’s (1988a,b) studies are emphasised in this study due to their common use in design equations.

2.5.2. Active Confinement Models

Concrete behaviour under axial compression and active confinement stress was studied by various researchers (Richart et al. 1928, Balmer 1949, Gardner 1969, Smith et al. 1989, Xie et al. 1995, Imran and Pantazopoulou 1996, Candappa et al. 1999, Sfer et al. 2002). In these studies, concrete cylinders were tested under axial compression ($\sigma_1$) in a pressurized chamber where the hydraulic confinement pressure
was kept constant throughout the test. The chamber pressure was first increased to the desired level \( f_i \) and the axial stress was applied to the concrete column while the confinement pressure was kept constant (Figure 2.5).

![Figure 2.5. Axial testing of a concrete cylinder under constant hydrostatic pressure.](image)

If the loading path of these tests are drawn on a graph which has the horizontal axis as confinement stress \( \sigma_3 \), and vertical axis as axial stress \( \sigma_1 \), it can be seen that, between the Points O and M, the axial stress is increasing under the given level of confinement (Figure 2.6 (a,b)). However, the level of confinement stress is only enough to keep the axial stress increasing until a level of \( f'_{cc} \) at Point M. Thus, if a point of loading lies on the Path O-M, it implies that the confined concrete strength is yet to be reached. After Point M, the axial stress applied to the concrete column decreases despite the unchanged level of confinement stress until Point F, where the concrete column’s axial load capacity is significantly decreased and accepted to have failed. The loading path of an unconfined concrete column under axial compression lies only on the vertical axis of Figure 2.6 (a) and passes through the Points O, unconfined compressive strength at Point A and an accepted level of failure stress at Point B, respectively.
To determine the axial stress value at Point M and the corresponding axial strain under a given constant confinement pressure, Richart et al. (1928) proposed Equations 2.3 and 2.4 respectively which were developed empirically. The unconfined concrete strength used in this study varied between 20 MPa and 50 MPa.

\[
\frac{f'_{cc}}{f'_{co}} = 1 + 4.1 \frac{f_l}{f'_{co}} \tag{2.3}
\]

\[
\frac{\varepsilon_{cc}}{\varepsilon_{co}} = 1 + 5 \left( \frac{f_{cc}}{f'_{co}} - 1 \right) \tag{2.4}
\]

where \(f'_{co}\) and \(\varepsilon_{co}\) are the unconfined compressive strength and the corresponding axial strain value of concrete, respectively; \(f'_{cc}\) and \(\varepsilon_{cc}\) are the confined strength and the corresponding axial strain value of the concrete, respectively; and \(f_l\) is the lateral confinement stress applied by the hydrostatic pressure in the chamber.

Mander et al. (1988a) proposed Equation 2.5 for calculating the confined concrete strength \((f'_{cc})\) at a given constant confining stress \(f_l\), thus Equation 2.5 can be assumed to give the position of Point M for any given constant confinement pressure. The non-linear relationship between the confined concrete strength and the constant confinement pressure given in Equation 2.5 was based on the ultimate surface
developed previously by Elwi and Murray (1979) and was calibrated using triaxial (constant confinement pressure) test data. However, the equation proposed by Mander et al. (1988a) for the corresponding axial strain was identical to Richart et al.’s (1928) Equation 2.4. Since this model was a parametric model, theoretically there is no concrete strength limitation for Equation 2.5.

\[
\frac{f_c}{f_{co}} = 2.254 \sqrt{1 + 7.94 \frac{f_t}{f_{co}} - 2 \frac{f_t}{f_{co}} - 1.254}
\]

Although these two ultimate strength equations (Equations 2.3 and 2.5) were developed by different researchers using different approaches, they yield very similar predictions for small confinement stresses. In fact, up to confinement stress levels \(f_t/f_{co}\) of 0.7, Mander et al.’s (1988a) model predicts the values of confined concrete strength values with absolute differences of below 20% compared to Richart et al.’s (1928) model (Figure 2.7).

![Figure 2.7. Confined concrete strength predictions using models proposed by Richart et al. (1928) and Mander et al. (1988a), and the absolute difference between the predictions.](image-url)
2.5.3. Passive Confinement Models

Both Equations 2.3 and 2.5 were also applied to concrete columns passively confined by steel bars and found to be suitable for predicting the confined strength of passively confined concrete (Richart et al. 1929, Mander et al. 1988b).

For steel confined concrete, the ultimate confinement stress is reached when the confinement steel reaches its yielding strain (assuming a simplified stress-strain behaviour for steel under tensile stress as shown in Figure 2.2), thus providing a constant confinement after this point which has been assumed as an analogy between active and passive confinement of concrete (Richart et al. 1929, Mander et al. 1988b).

The explanation for the suitability of a confinement model developed using a constant confinement pressure (active confinement) to steel confined concrete (passive confinement) was given by Imran and Pantazopoulou (1996) and Lan and Guo (1997). These researchers reported that the confined concrete strength was essentially independent of the shape of the loading path. The path independency of confined concrete strength implies that either Richart et al.’s (1928) or Mander et al.’s (1988a) equations (Equations 2.3 and 2.5, respectively) can still be taken as a
boundary for passively confined concrete as shown in Figure 2.8 for Richart et al.’s (1928) model.

The critical point for the applicability of active confinement models to passive confinement is to determine the confinement stress ratio ($f_i/f_{co}$) at the time of maximum axial stress.

For the case of passively confined concrete, the radial expansion of the concrete is very low until the unconfined concrete strength is reached, i.e. until this point the confinement stress is very low. However, after the axial stress value of $f_{co}$ is exceeded (1 on the vertical axis of the graph on Figure 2.8) the confinement pressure rapidly increases due to radial expansion of concrete cylinder with the increasing axial stress and the loading path begins to intersect the constant pressure loading paths. The intersection point values and the exact shape of the loading path depends on the rate of radial expansion of passively confined concrete and the stress-strain behaviour of confining material under tensile stress. Iterative methods to determine the relationship between any axial stress level and the corresponding confinement stress on the concrete by using the compatibility equations between the concrete and the confining materials to establish a full stress-strain graph of passively confined concrete was adopted successfully by a few researchers including Spoelstra and Monti (1999), Fam and Rizkalla (2001), Lignola et al. (2008). Determining the intersection points of loading paths other than the one corresponding to the confined strength of concrete ($f_{co}'$) is kept out of the scope of this study.

2.6. Summary

The previous studies on the effect of active and passive confinement (provided by steel bars) on the compressive behaviour of solid concrete columns were discussed briefly in order to demonstrate how the subject evolved until the introduction of Fibre Reinforced Polymers. The behavioural difference of hollow RC columns to the solid ones were also mentioned.

In the next chapter, first the material properties of FRP materials are described. FRP materials are compared to steel reinforcement, and advantages and disadvantages of using FRP as a concrete confinement material are discussed in detail. The reported outcomes of FRP confined concrete studies in the literature are presented in the last part of the next chapter.
Chapter 3: USE OF FRP IN CONCRETE CONFINEMENT

3.1. Introduction
For the last two decades, use of FRP materials to retrofit the existing structures has been an alternative to the use of steel for the same purposes. This chapter explains the FRP making materials and makes a comparison of FRP materials to structural steel in the first part. The second part of this chapter gives a literature review for the FRP confined concrete models.

3.1.1. Fibre Reinforced Polymer Making Materials
As their name implies, fibre reinforced polymer materials generally consist of two phases: the reinforcing fibre element, and the supporting polymer matrix (Figure 3.1). They are produced in many different forms and shapes such as bars, plates and thin sheets.

The reinforcing fibre element generally comes in the forms of continuous long fibres and short discontinuous fibres. Carbon, glass and aramid (also named as Kevlar® by DuPont) fibres are the most common types of fibres used in composites. Most commercially available fibres’ diameter range between 3 to 20 μm (Daniel and Ishai 1994).

![Figure 3.1. Two phases of fibre reinforced polymer materials.](image)

The supporting polymer matrix binds the embedded fibre together and provides the load transfer between them as well as providing them a protection from environmental factors.

There are mainly two different types of polymer matrix materials: thermoset and thermoplastic matrices. Thermoset matrix is a product of irreversible chemical
transformation of a resin system into an amorphous cross-linked polymer matrix. Epoxy resins are the most commonly used resin systems to produce thermoset matrices. The thermoplastic matrices do not undergo any chemical transformation during the production process and they exhibit a notably higher elongation capacity than thermosets before failure (Barbero 1999, Cheng 2000). Naming of fibre reinforced polymer materials is done according to the polymer matrix used, type, continuity and embedding orientation of reinforcing fibres as illustrated in Figure 3.2.

**Figure 3.2. Classification of fibre reinforced polymer materials (Daniel and Ishai 1994).**

In most composite products, a large number of fibres are used in continuous form lying in the same direction to form unidirectional composite lamina, layer or ply. By this way, the tensile strength of the fibre reinforced polymer composites are utilized in the most useful way in the desired direction of application. For concrete confinement, the concrete cylinder specimens are wrapped with FRP sheets with the
fibre orientation in the hoop direction to provide maximum confinement effect similar to steel jacketing application.

In this study, only unidirectional carbon fibre reinforced polymer sheets are focused on for strengthening purposes. Thus, the terms “Composite” and “Fibre Reinforced Polymer material” or FRP are used interchangeably to refer the composites in the sheet form consisting of unidirectional high strength carbon fibre and polymer resin matrix.

The lateral confinement pressure provided by FRP wrapping on the concrete, \( f'_l \), can be calculated using a similar approach used for deriving Equation 2.2, however this time tensile rupture strength \( (f_{fp}) \) and the thickness of FRP wrapping \( (t_{pp}) \) are used in the equation instead of the yield strength \( (f_{ys}) \) and the equivalent thickness of steel confinement \( (t) \), respectively. The linear stress-strain behaviour of FRP until failure implies that the confinement stress that FRP wrapping will provide is not reaching a constant value unlike steel, but it is continuously increasing until the failure of FRP wrapping (Figure 3.3), thus the maximum confinement stress that can be exerted to concrete can be calculated using Equation 3.1, where \( D \) is the outside diameter of the wrapped column.

\[
f'_l = \frac{2f_{fp}t_{pp}}{D}
\]  

Figure 3.3. Stress-strain relationship for Carbon-FRP, Glass-FRP and steel (Teng et al. 2002).
3.2. Comparison of Steel Jacketing and FRP wrapping on the Compressive Behaviour of Concrete

Steel jacketing and FRP wrapping are the most obvious technical solutions for RC column retrofitting. When compared to each other, FRP wrapping has advantages over steel jacketing since FRP materials are non-corrosive, have high strength-to-weight ratios, and they are easier to transport and handle than steel sheets. Moreover, FRP materials can be combined in various ways to create a jacket of desired mechanical properties to achieve optimum solutions to RC column retrofitting. Disruption of normal use of the structure is less for FRP wrapping method compared to steel jacketing.

There are also disadvantages of FRP wrapping method. Firstly, current cost of FRP materials is higher than that of steel which will increase the initial cost of FRP wrapping method. Vulnerability to mechanical damage, such as vandal attacks, after installation is another disadvantage of FRP wrapping but it can be overcome by applying a protective coat of plaster over the FRP retrofitted column. Lack of a universally accepted design code also makes it difficult for the structural engineers to choose FRP wrapping as an alternative over steel jacketing. Structural engineers’ hesitation to use FRP can be understandable considering how cautious the construction industry is over the new methods and materials. It is obvious that any new technique involves risk because there cannot be a cumulative record of previous successful applications (Hollaway and Leeming 2000).

3.3. FRP Confined Concrete Models

Introduction of FRP materials raised the question whether the already existing confined concrete models would be suitable to use for this new material since these models were developed considering the confinement provided by steel jacketing.

The earliest attempts to investigate the behaviour of FRP confined concrete were the experimental studies carried out by Kurt (1978) and Fardis and Khalili (1981, 1982). Ease of tailorability of FRP confinement gave way to research studies to investigate different confining methods experimentally such as confinement of concrete columns with FRP spirals (Ahmad et al. 1991, Nanni and Bradford 1995); continuous FRP wraps which were applied to concrete columns after the columns were cast (Harmon and Slattery 1992, Picher et al. 1996, Miyauchi et al. 1997, Watanabe et al. 1997, Kono et al. 1998, Matthys et al. 1999, Toutanji 1999, Rochette and Labossiere 2000,
Shawahy et al. 2000, Micelli et al. 2001, Rousakis 2001); and FRP tubes which were prepared before and the concrete columns were cast using these tubes as formwork (Mirmiran and Shahawy 1997, La Tegola and Manni 1999, Saafi et al. 1999, Fam and Rizkalla 2000).

The attempts to develop models to express the ultimate axial stress ($f_{ucc}$) and the corresponding axial strain ($\varepsilon_{ucc}$) can be classified into two groups: steel confinement based models and empirical or analytical models. A summary of the FRP confinement models mentioned in this study is given in Table 3.1.

### 3.3.1. Steel Confined Concrete Based Models

Researchers assumed that the analytical models developed for steel confined concrete (Richart et al. 1928, Newman and Newman 1969, Mander et al. 1988a) can be extended to FRP confined concrete. These models were based on the ultimate strength surfaces developed using triaxial test data, and thus predicting the strength enhancement of confined concrete as a function of confining pressure $f_c$, which is assumed to be constant during the compression tests.

Fardis and Khalili (1981) used the triaxial strength envelopes proposed by Richart et al. (1928) and Newman and Newman (1969) to explain the ultimate stress and strain of FRP confined concrete using the maximum confinement pressure, $f_c$, that the FRP can exert when it reaches its tensile rupture stress which is given by Equation 3.1 although these equations were originally developed assuming a constant confining pressure. Other than checking the suitability of Richart et al.’s (1928) model for FRP confined concrete, Fardis and Khalili (1981) also developed the ultimate stress and strain equations (see Equations 3.2 and 3.3, respectively) and checked against the results of the compression tests done on GFRP wrapped small size cylinders of sizes 75x150mm and 100x200mm (diameter x height). The confinement ratios, $f_c / f_{ucc}$, were between 0.1 and 0.6, and Fardis and Khalili (1981) found a reasonable agreement between the strength predictions using the developed equations and the test results, however they made no comparisons for the predicted and experimental ultimate strain values.
\[
\frac{f'_{cc}}{f_{co}} = 1 + 3.7 \left( \frac{f_t}{f_{co}} \right)^{0.86}
\]

\[\varepsilon_{cc} = \varepsilon_{co} + 0.0005 \left( \frac{E_l}{f_{co}} \right) \text{ where } E_l = \frac{2E_{frp}f_{frp}}{D}, \text{ (D: Diameter of specimens)}\]

Saadatmanesh et al. (1994) used the confinement model proposed by Mander et al. (1988a) to explain the stress-strain behaviour of FRP confined concrete. The authors used Equation 3.1 to express the confinement stress just as Fardis and Khalili (1981). Saadatmanesh et al. (1994) did not compare the model against any experimental data and the study was only a parametric study.

3.3.2. Empirical or Analytical FRP Confined Concrete Models

Empirical or analytical FRP confined concrete models are the models developed by using the best fitting of experimental data to express particularly the ultimate stress, \(f'_{cc}\), and corresponding axial strain, \(\varepsilon_{cc}\), except for the model proposed by Spoelstra and Monti (1999).

Miyauchi et al. (1997) tested 100x200 mm and 150x300 mm concrete cylinder specimens wrapped with Carbon FRP. The equation proposed by Miyauchi et al. (1997) to estimate the strength of FRP confined concrete (Equation 3.4) was similar to Equation 2.3 proposed by Richart et al. (1928) in being linearly related to the confinement stress provided by the FRP confinement. The only difference was that, the active confinement provided by the hydrostatic pressure was replaced by the maximum possible confinement that can be provided by FRP wrap as given in Equation 3.1, and the coefficient \(k_1\) was lowered by 15 percent from 4.1 to the value of 3.485. The confinement ratios in Miyauchi et al.’s (1997) tests, \(f_t / f_{co}\), were ranging from 0.1 to 0.5. For the axial strain corresponding to the strength of the concrete, two different equations were proposed for the two different unconfined concrete strengths used in their experiments (Equation 3.5). Miyauchi et al. ’s (1997) did no measurement for the lateral strains, thus, it is not possible to determine whether the hoop strain at the failure of FRP wrapping actually matched the value of the rupture strain of FRP in coupon tests. In fact, premature failure of FRP wraps is a
commonly reported phenomenon in most of the available literature on FRP wrapped concrete. This problem is explained in details in the next chapter.

\[
\frac{f'_{cc}}{f'_{co}} = 1 + 3.485 \left( \frac{f_t}{f'_{co}} \right) \quad 3.4
\]

\[
\frac{\varepsilon_{cc}}{\varepsilon_{co}} = 1 + 10.6 \left( \frac{f_t}{f'_{co}} \right)^{0.373} \quad \left( f'_{co} = 30MPa \right) \quad 3.5
\]

\[
\frac{\varepsilon_{cc}}{\varepsilon_{co}} = 1 + 10.5 \left( \frac{f_t}{f'_{co}} \right)^{0.525} \quad \left( f'_{co} = 50MPa \right)
\]

Kono et al. (1998) carried out tests on CFRP wrapped concrete cylinders of 32.3 to 34.8 MPa strength with the dimensions of 100x200mm and developed empirical equations for the estimation of peak stress (Equation 3.6) and corresponding axial strain (Equation 3.7) of FRP wrapped concrete cylinders under compression. Instead of using the ratio \( f_t / f'_{co} \) as in the most of other models, Kono et al. (1998) linearly correlated the ratios \( f'_{cc} / f'_{co} \) and \( \varepsilon_{cc} / \varepsilon_{co} \) to the maximum confinement pressure \( f_t \). The maximum confinement ratio, \( f_t / f'_{co} \), was ranging between 0.37 to 1.19 which was quite high compared to previous models. Kono et al.’s (1998) measurements of lateral strains revealed that when the failure took place, the FRP hoop strains only reached to 38 to 59% of rupture strains observed in the coupon samples tested under uniaxial tension.

\[
\frac{f'_{cc}}{f'_{co}} = 1 + 0.0572 f_t \quad 3.6
\]

\[
\frac{\varepsilon_{cc}}{\varepsilon_{co}} = 1 + 0.280 f_t \quad 3.7
\]
Figure 3.4. Parameters of Samaan et al. (1998) bilinear confinement model.

Samaan et al. (1998) tested a total of 30 CFRP wrapped concrete cylinders of size 152.5x305mm (6 x 12 in.) and unconfined strength of 29.64 to 31.97 MPa under uniaxial compression. In Samaan et al.’s (1998) tests, the $f_l / f_{co}'$ ratios ranged from 0.31 to 0.84. Apart from the ultimate stress and the corresponding axial stress equations (Equations 3.8 and 3.9 respectively), Samaan et al. (1998) also developed a bilinear continuous stress-strain model to express the response of FRP-confined concrete to axial compression as shown in Figure 3.4. The first linear part had an identical slope to that of unconfined concrete which is the modulus of elasticity of unconfined concrete $E_c$, and after a transition stress value, $f_o$ (Equation 3.10), the stress strain graph had a different slope, $E_2$ (Equation 3.11).

\[
\frac{f_{cc}'}{f_{co}'} = 1 + 6.0 \left( \frac{f_{o}^{0.7}}{f_{co}'} \right) \quad \text{3.8}
\]

\[
\varepsilon_{cc} = \left( f_{cc}' - f_{o} \right) \quad \frac{E_2}{E_2} \quad \text{3.9}
\]

\[f_o = 0.872 f_{co}' + 0.371 f_l + 6.258 \quad \text{3.10}\]

\[E_2 = 245.61 f_{co}'^{0.2} + 0.6728 E_l \quad \text{3.11}\]

\[E_l = \frac{2E_{fp} t_f}{D} \quad \text{3.12}\]
Samaan et al.’s (1998) measurements of lateral strains during axial loading of the specimens revealed that, for this set of experiments, FRP hoop strains at failure were very close to that of FRP in uniaxial tensile stress, unlike the Kono et al.’s (1998) results. Another notable aspect of this study was that, the ultimate strain, $\varepsilon_{\text{uc}}$, did not depend on the unconfined strain at peak, $\varepsilon_{\text{co}}$, but instead it was expressed as a function of the ultimate confinement stress, $f_\text{f}$, and the stiffness of the confining device, $E_\text{f}$, which can be calculated using the Equation 3.12; where $E_{\text{frp}}$ is the modulus of elasticity of FRP wrap determined by uniaxial tensile testing, $t_\text{f}$ is the total thickness of FRP confinement, and $D$ is the diameter of the confined cylinder.

Toutanji (1999) tested 76x305mm concrete cylinders with unconfined strength of 30.93 MPa and wrapped with Glass FRP (GFRP) and CFRP to yield confinement ratios ($f_\text{f} / f_{\text{co}}$) varying from 0.30 to 0.83. He proposed a model which divided the stress-strain response of FRP confined concrete into two distinct regions as shown in Figure 3.5. The stress-strain response in the first region was found to be similar to the unconfined concrete which was explained by the authors with the insignificant lateral expansion of the confined concrete before it reaches its unconfined strength ($f_{\text{co}}$) and the corresponding strain value ($\varepsilon_{\text{co}}$).

![Figure 3.5. Stress-strain model by Toutanji (1999).](image)
The equations for the maximum axial stress and the corresponding axial strain values proposed by Toutanji (1999) are given in Equations 3.13 and 3.14, respectively. These equations were obtained by regression analysis on results of tests conducted by Toutanji (1999), assuming that the FRP wrapped cylinders would fail when the lateral strain value equals to the FRP rupture strain obtained from coupon tests.

\[
\frac{f_{cc}'}{f_{co}} = 1 + 3.5 \left( \frac{f_t}{f_{co}} \right)^{0.85} \tag{3.13}
\]

\[
\frac{\varepsilon_{cc}}{\varepsilon_{co}} = 1 + (310.57 \varepsilon_{frp} + 1.90) \left( \frac{f_{cc}'}{f_{co}} - 1 \right) \tag{3.14}
\]

Saafi et al. (1999) proposed a model which was nearly identical to Toutanji’s (1999), however the confinement of the tested samples was obtained by FRP tubes rather than wraps. The equations for the maximum axial stress and the corresponding axial strain values proposed by Saafi et al. (1999) are given in Equations 3.15 and 3.16, respectively.

\[
\frac{f_{cc}'}{f_{co}} = 1 + 2.2 \left( \frac{f_t}{f_{co}} \right)^{0.84} \tag{3.15}
\]

\[
\frac{\varepsilon_{cc}}{\varepsilon_{co}} = 1 + (537 \varepsilon_{frp} + 2.6) \left( \frac{f_{cc}'}{f_{co}} - 1 \right) \tag{3.16}
\]

Different values of coefficients in Saafi et al.’s (1999) model from Toutanji’s (1999) model were assumed to be the result of conducting regression analysis on the FRP-tube encased concrete cylinders.

The model proposed by Spoelstra and Monti (1999) is developed using a different approach than the other models. Spoelstra and Monti (1999) used an iterative approach to determine the axial stress and strain values to the lateral strain of FRP confined concrete. At each step, an axial strain was induced and the corresponding lateral strain was calculated using the approach previously proposed by Pantazopoulou and Mills (1995) and the confinement stress, \( f_t \), was calculated using this lateral strain value. For each confinement stress value, the corresponding Mander et al. ’s (1988a) curve was used and the axial stress-strain point was calculated. The failure point is marked when a calculated axial strain value matches the ultimate
strain value as defined in Equation 2.4 for the corresponding level of confinement stress. Spoelstra and Monti (1999) developed the ultimate stress and the corresponding axial strain value Equations 3.17 and 3.18, respectively, after producing results for 600 cases and carrying out regression analysis on the analysis results. The distinct feature of this model is that, unlike other models, it can predict continuous stress-strain curves with a descending branch which can be observed for low levels of FRP confinement.

\[
\frac{\sigma_{ce}^\prime}{\sigma_{co}^\prime} = 0.2 + 3 \left( \frac{f_i}{f_{co}^\prime} \right)^{0.5}
\]

\[
\varepsilon_{ce}^\prime = 2 + 1.25 \frac{E_{co}}{f_{co}^\prime} \varepsilon_{frp} \sqrt{\frac{f_i}{f_{co}^\prime}}
\]

Xiao and Wu (2000) tested a total of 36 concrete cylinders of size 152x305mm with unconfined compressive strengths from 27.6 to 48.2 MPa under compression. The cylinders were wrapped with CFRP to obtain confinement ratios \((f_i / f_{co}^\prime)\) ranging from 0.14 to 0.70. The Equations 3.19 and 3.20 giving the confined strength and corresponding strains, respectively, were calibrated empirically against the test results obtained. The actual hoop strains at failure were measured to be ranging from 50 to 80% of the rupture strain values of FRP coupons tested under uniaxial tensile stress. However, the equations were calibrated against the coupon test values thus they did not incorporate with the premature failure of FRP wraps during the tests.

\[
\frac{\sigma_{ce}^\prime}{\sigma_{co}^\prime} = 1.1 + \left[ 4.1 - 0.75 \frac{f_i^2}{E_i} \right] \frac{f_i}{f_{co}^\prime}
\]

\[
\varepsilon_{ce}^\prime = \varepsilon_{frp}^\prime - 0.0005 \sqrt{\frac{f_{co}^\prime}{E_i}}\gamma, \quad \text{where} \quad E_i = \frac{2E_{frp}l_{frp}}{D}, \quad (D: \text{Diameter of specimens})
\]
Table 3.1. Summary of FRP confined concrete models.

<table>
<thead>
<tr>
<th>Model</th>
<th>( f'_{cc} ) (theoretical)</th>
<th>( \varepsilon_{cc} ) (theoretical)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Steel-based models</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(Fardis and Khalili 1981)</td>
<td>( f'<em>{cc} = 1 + 4.1 \left( \frac{f_i}{f</em>{co}} \right) )</td>
<td>( \varepsilon_{cc} = \varepsilon_{co} + 0.0005 \left( \frac{E_i}{f_{co}} \right) )</td>
</tr>
<tr>
<td>(Saadatmanesh et al. 1994)</td>
<td>( f'<em>{cc} = 2.254 \left( \frac{f_i}{f</em>{co}} \right) - 2 \left( \frac{f_{cc}}{f_{co}} \right) - 1.254 )</td>
<td>( \varepsilon_{cc} = 1 + 5 \left( \frac{f_{cc}}{f_{co}} - 1 \right) )</td>
</tr>
<tr>
<td><strong>Empirical or Analytical Models</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(Miyachi et al. 1997)</td>
<td>( f'<em>{cc} = 1 + 3.485 \left( \frac{f_i}{f</em>{co}} \right) )</td>
<td>( \varepsilon_{cc} = 1 + 10.6 \left( \frac{f_i}{f_{co}} \right) )</td>
</tr>
<tr>
<td>(Kono et al. 1998)</td>
<td>( f'_{cc} = 1 + 0.0572 f_i )</td>
<td>( \varepsilon_{cc} = 1 + 0.280 f_i )</td>
</tr>
<tr>
<td>(Samaan et al. 1998)</td>
<td>( f'<em>{cc} = 1 + 6.0 \left( \frac{f_i}{f</em>{co}} \right)^{0.7} )</td>
<td>( \varepsilon_{cc} = f'_{cc} )</td>
</tr>
<tr>
<td>(Toutanji 1999)</td>
<td>( f'<em>{cc} = 1 + 3.5 \left( \frac{f_i}{f</em>{co}} \right)^{0.85} )</td>
<td>( \varepsilon_{cc} = 1 + 310.57 \varepsilon_{frp} + 1.90 \left( \frac{f_{cc}}{f_{co}} - 1 \right) )</td>
</tr>
<tr>
<td>(Saafi et al. 1999)</td>
<td>( f'<em>{cc} = 1 + 2.2 \left( \frac{f_i}{f</em>{co}} \right)^{0.84} )</td>
<td>( \varepsilon_{cc} = 1 + 537 \varepsilon_{frp} + 2.6 \left( \frac{f_{cc}}{f_{co}} - 1 \right) )</td>
</tr>
<tr>
<td>(Spoelstra and Monti 1999)</td>
<td>( f'<em>{cc} = 0.2 + 3 \left( \frac{f_i}{f</em>{co}} \right)^{0.5} )</td>
<td>( \varepsilon_{cc} = 2 + 1.25 \frac{E_{co}}{f_{co}} \varepsilon_{frp} \sqrt{\frac{f_i}{f_{co}}} )</td>
</tr>
<tr>
<td>(Xiao and Wu 2000)</td>
<td>( f'<em>{cc} = 1.1 + \left[ 4.1 - 0.75 \frac{f</em>{co}}{E_i} \right] \frac{f_i}{f_{co}} )</td>
<td>( \varepsilon_{cc} = \frac{E_{frp}}{7 \left( \frac{f_{co}}{E_i} \right)^{0.8}} )</td>
</tr>
</tbody>
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where \( D \) is the diameter of specimens, \( f_{co} \) is the concrete compressive strength, \( f_i \) is the FRP tensile strength, \( E_{frp} \) is the FRP modulus of elasticity, and \( E_i \) is the internal strain.
3.3.3. Discussion on FRP Confinement Models

Although there are many more FRP confined concrete models reported, the abovementioned ones are the mostly cited models in the relevant research studies. Most of the FRP confinement models in the literature are empirical in nature and they are calibrated against their own sets of experimental results, thus a generally accepted analytical method has not been established. Besides, these models were developed using very small solid concrete specimens of normal compressive strength with no internal steel reinforcement, and all tests were conducted under concentric loads. Thus, their efficiency for real size structural reinforced concrete columns or hollow core columns are still to be investigated in detail. The subject of the interaction between the internal steel reinforcement and the FRP wrapping has never been studied to the author’s knowledge.

Yazici and Hadi (2009) investigated the effect of eccentric loading on the behaviour of CFRP wrapped hollow RC columns and found that none of the previously proposed models are fit to explain the behaviour of such columns under given loading conditions. Yazici and Hadi (2009) proposed a biaxially confined concrete model using Mander et al.’s (1988a) equations and this stress-strain model was found to be effective in predicting the axial load carrying capacities of FRP wrapped hollow RC columns. This study aims to develop Yazici and Hadi’s (2009) approach and to propose a more general confinement model which can be applied to both solid and hollow RC columns wrapped with FRP.

3.4. Summary

Despite being a relatively new material in structural engineering, FRP materials have been proven to be potentially effective in strengthening concrete structural members. Studies have shown that, columns externally wrapped with FRP sheets have demonstrated higher load carrying and deformation capacities than the unwrapped ones. Besides, loading eccentricity largely affects the strength and ductility gain of concrete. All stress-strain models which were developed for FRP confined concrete deal with solid concrete columns. Thus, a new approach to predict the stress-strain behaviour of hollow core columns is necessary.

In Chapter 4, a unified FRP confined concrete model which can be used to explain the behaviour of solid and hollow concrete columns is developed and proposed. The
developed model will be used to predict the moment curvature diagrams, axial load-moment interaction diagrams of the hollow RC column cross-sections and loading paths of short and tall hollow RC columns under eccentric loading in Chapter 5.
Chapter 4: DEVELOPING A UNIFIED FRP CONFINEMENT MODEL FOR SOLID AND HOLLOW COLUMNS

4.1. Introduction

Strengthening reinforced concrete columns by FRP wrapping has become a common practice of passive confinement application, however there is still no design guidelines or standards agreed upon internationally. Currently, “Guide for the design and construction of externally bonded FRP systems for strengthening concrete structures” reported by ACI Committee 440 (ACI440.2R-08 2008) is the most accepted guideline for design on the subject.

Combined with the absence of a confinement model describing the behaviour of hollow reinforced concrete columns, the subject of FRP confinement of RC hollow columns poses a great challenge in the column strengthening practice.

In this chapter, a unified stress-strain model for both solid and hollow concrete columns is developed for prediction purposes and its accuracy against previously reported data by various researchers is evaluated.

4.2. A Unified Model for FRP Confined Solid and Hollow Columns

In the previous version of the ACI440.2R guideline (2002), ACI 440.2R Committee had recommended to use Equation 2.5 which was proposed by Mander et al. (1988a) for prediction of the FRP confined strength of concrete. Although this equation was originally proposed for constant lateral confinement and tested to be suitable for steel confined concrete.

Considering the loading path independency reported by Imran and Pantazopoulou (1996) and Lan and Guo (1997), using Equation 2.5 would have given good estimate for the FRP confined concrete strength assuming that FRP confinement fails at the same hoop strain as it does in uniaxial coupon tests. However, FRP confinement’s failure strain values were observed to be varying between 58 to 61 percent of the coupon test strains (mainly based on Carbon-FRP confinement) by a number of researchers (De Lorenzis and Tepfers 2001, Matthys 2001, Pessiki et al. 2001, De Lorenzis and Tepfers 2003, Harries and Carey 2003, Lam and Teng 2003a) due to the multiaxial state of stress the FRP confinement is under and possible stress concentrations throughout the surface of concrete column. In order to be able to use Mander et al.’s (1988a) Equation 2.5 to predict the confined strength of the concrete,
the actual confining stress \((f_c)\) at the time of confined concrete’s peak axial stress should be known since this equation was derived assuming that the confinement stress is definitely known and equal to a constant confining pressure.

In the current version of ACI 440.2R (2008), the premature failure of FRP confinement is accounted for and rather than Mander et al.’s (1988a) Equation 2.5, it is recommended to predict the confined strength of concrete for circular cross-sections by using the Equations 4.1 and 4.2 as proposed by Lam and Teng (2003), for especially FRP confined concrete, with an additional reduction factor of \(\psi_f=0.95\).

\[
\frac{f_{cc}}{f_{co}} = 1 + \psi_f \frac{f_i}{f_{co}} \tag{4.1}
\]

\[
f_i = \frac{2E_{frp}t_f e_{frp}}{D} \tag{4.2}
\]

Where \(E_{frp}\) is the modulus of elasticity, \(t_f\) is the total thickness, \(e_{frp}\) is the effective strain value at the time of failure of FRP confinement; and \(D\) is the diameter of concrete cylinder. The effective strain value of FRP confinement is given by

\[
e_{frp} = \kappa_e e_{fu} \tag{4.3}
\]

where \(\kappa_e\) is the strain efficiency factor for which a value of 0.55 is recommended for circular cross sections and \(e_{fu}\) is the ultimate tensile strain value of FRP confinement material measured in coupon tests.

Based on the tests done by Lam and Teng (2003a, 2003b), ACI 440.2R (2008) recommend that the confinement ratio \((f_i/f_{co})\) should be larger than 0.08 in order to assure a non-descending stress-strain graph due to softening of concrete, which can be taken as a lower limit for confinement ratio.

For circular cross-sections, ACI 440.2R (2008) recommends the use of Equation 4.4 which was adopted from “Design Guidance for Strengthening Concrete Structures Using Fibre Composite Materials” by Concrete Society (2004) for the prediction of the axial strain of concrete corresponding to the maximum axial stress \(e_{co}\), however this value was restricted by ACI 440.2R (2008) to a maximum value of 0.01 in order
to prevent excessive cracking and to maintain the structural integrity, as a serviceability criterion.

\[
\varepsilon_{cc} = \varepsilon_{co} \left( 1.50 + 12 \left( \frac{f_l}{f_{lc}} \right) \left( \frac{\varepsilon_{fc}}{\varepsilon_{co}} \right)^{0.45} \right)
\]  

4.4

Assuming a \( \varepsilon_{co} \) value of 0.002 and an average \( \varepsilon_{fc} \) value of 0.0145 (calculated average of the relevant values given in Table 4.1) for Carbon FRP for Equation 4.4 and calculating \( \varepsilon_{fc} \) value from Equation 4.3, the upper boundary for the confinement ratio \( \left( f_l / f_{lc} \right) \) was calculated as 0.16 using the limitation of \( \varepsilon_{co} \) to 0.01 in Equation 4.4. Thus, the lower and upper boundaries of \( f_l / f_{co} \) are given in Equation 4.5.

\[
0.08 \leq \frac{f_l}{f_{co}} \leq 0.16
\]  

4.5

This study assumes that when the premature failure of FRP confinement is accounted for, the ultimate strength and the corresponding axial strain of FRP confined concrete can be predicted using the previously proposed model for active and passive confinement of concrete by Richart et al. (1928, 1929). The reason of preference of Richart et al. (1928, 1929) is that, in practice achieving passive confinement ratios \( \left( f_l / f_{co} \right) \) as large as 0.7 is quite difficult considering the larger size of real concrete columns compared to the test specimens and the cost of confinement materials. As explained in Chapter 2 above, up to the confinement ratio of 0.7 difference of predicted values using Richart et al.’s (1928) and Mander et al.’s (Mander et al. 1988a) models is below 20%. Moreover, when it is considered that ACI 440.2R (2008) recommends the upper and lower limits given in Equation 4.5 in designs, using Richart et al.’s (1928) model for predicting the behaviour of passively confined concrete gives more conservative results than Mander et al.’s (1988a) model.

Richart et al.’s (1928) Equations 2.3 and 2.4 are modified to account for the premature failure of FRP confinement and used for the prediction of the strength and the corresponding axial strain of FRP confined concrete.
In Equations 2.3 and 2.4, instead of using the confinement ratio, $f_i/f_{co}$, a normalized confinement stiffness ($K_N$) given by Equation 4.6 will be used to express the confined concrete strength and corresponding strain.

$$K_N = \frac{2E_{fu}t_f}{Df_{co}} \quad 4.6$$

The confinement ratio expression ($f_i/f_{co}$) appearing in Equations 2.3 and 2.4 can be re-written as shown in Equation 4.7.

$$\frac{f_i}{f_{co}} = \kappa \varepsilon_{fu} K_N \quad 4.7$$

Inserting Equation 4.7 into Equation 2.3 and 2.4 and using $\kappa = 0.55$ and an average value of 0.0145 for $\varepsilon_{fu}$ calculated by taking the average value of Carbon FRP coupon failure strains of 88 samples shown in Table 4.1, the equations take simple forms as shown in Equations 4.8 and 4.9. The upper and lower limits of confinement ratios ($f_i/f_{co}$) recommended by ACI 440.2R (2008) for design are also re-written in terms of $K_N$ and given in Equation 4.10.

$$f_{co}' = \left(1 + 0.033K_N \right) f_{co}' \quad 4.8$$

$$\varepsilon_{co}' = \left(1 + 0.17K_N \right) \varepsilon_{co} \quad 4.9$$

$$10 \leq K_N \leq 20 \quad 4.10$$

Equation 4.10 suggests that for $K_N$ values less than 10, the stress-strain graph will probably exhibit a descending branch after reaching $f_{co}'$ and for $K_N$ values greater than 10, the failure will occur on the ascending branch. Equation 4.10 also suggests that, $K_N$ values should also be limited to 20 in order to limit the axial strain of confined concrete to 0.01 to prevent excessive cracking of concrete elements and to maintain structural integrity.
4.3. Forming a Database for the Verification of Proposed Model

A database was formed for the FRP confined concrete tests for the verification of the ultimate stress-strain equations proposed in this study. As the majority of the experimental studies used CFRP as confinement material, only the tests reported to have been done on the CFRP wrapped concrete cylinders were taken into the database for comparison. Thus, the model, which is derived by modifying Richart et al.’s (1928) model, proposed in this study is only valid for CFRP confined concrete cylinders. It is noted that the strength and strain characteristics of CFRP sheets were determined by the researchers themselves. The studies using producer supplied values for FRP materials were excluded from this study.

In Table 4.1, using the reported dimensions of concrete cylinders (all concrete cylinders have 2:1 height to diameter ratio), concrete strengths and FRP characteristics determined by coupon tests, $K_n$ values were calculated for each confined concrete cylinder using Equation 4.6. Wherever reported, the shape of observed stress-strain graphs was also classified as Type 1 if it shows no descending branch after $f_{cc}'$ and Type 2 if a descending branch is observed after $f_{cc}'$ as shown in Figure 4.1.

![Simplified stress-strain graphs and the corresponding normalized loading paths of FRP-confined concrete reported in the literature.](image)

Figure 4.1. Simplified stress-strain graphs and the corresponding normalized loading paths of FRP-confined concrete reported in the literature.
Table 4.1. Database of Carbon-FRP confined concrete cylinders for model development.

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<th>Diameter D (mm)</th>
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<th>$e_{cc}$ (%)</th>
<th>$t_f$ (mm)</th>
<th>$E_f$ (GPa)</th>
<th>$f_f$ (MPa)</th>
<th>$K_N$</th>
<th>$f_{cc}'$ (MPa)</th>
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NR: not reported by the authors.

* Tasdemir et al.’s (1998) Equation 4.11 was used since $\varepsilon_{co}$ was not reported by the authors
In Table 4.1, for the cases where $\varepsilon_{co}$ value was not specified, Equation 4.11 proposed by Tasdemir et al. (1998) was used.

$$
\varepsilon_{co} = \left( -0.067 f_{cc}^{'2} + 29.9 f_{cc}^{'2} + 1053 \right) \times 10^{-6} \tag{4.11}
$$

### 4.4. Verification of the Proposed Model and Comparison

In this part, the accuracy of the predictions made by the proposed model is calculated for both strength and corresponding strain values, and compared to the predictions by Mander et al. (1988a) and ACI440.2R (2008) models accounting for the premature failure of FRP confinement by using Equation 4.7.

The confined strength ($f_{cc}^{'}$) and the corresponding strain ($\varepsilon_{cc}$) values were calculated using the relevant equations of each model and compared with the test results of FRP confined concrete test results shown in Table 4.1, whenever a corresponding result is reported by the researchers.

The accuracy of each model was then quantitatively evaluated by computing the average absolute percent error ($err_{ave}$) for confined strength ($f_{cc}^{'}$) and the corresponding strain ($\varepsilon_{cc}$) values as follows:

$$
err_{ave} = \frac{\sum_{i=1}^{N} |err_i|}{N} \tag{4.12}
$$

$$
err_i = \frac{f_{cc,P}^{'} - f_{cc}^{'}}{f_{cc}^{'}} \times 100 \quad \text{or} \quad err_i = \frac{\varepsilon_{cc,P} - \varepsilon_{cc}}{\varepsilon_{cc}} \times 100 \tag{4.13}
$$

where, $err_i$ = percent error in the predicted values of confined concrete strength ($f_{cc,P}^{'}$) or the corresponding predicted axial strain values ($\varepsilon_{cc,P}$), $N$ = number of tests reporting the $f_{cc}^{'}$ or $\varepsilon_{cc}$ values which are the experimentally observed values of confined strength and the corresponding axial strain values, respectively. It should be noted that all 85 tests shown in Table 4.1 reported $f_{cc}^{'}$ values, however only 64 of them reported corresponding $\varepsilon_{cc}$ values. For $f_{cc}^{'}$ or $\varepsilon_{cc}$, the accurate prediction line is achieved by assuming an ideal model which predicts exactly the same values as experimental results which can be expressed as a straight line making 45° angle to the experimental results axis.
The accuracy of the models were also compared by calculating the sum of squared errors over the the $f_{cc}'$ or $\varepsilon_{cc}$ values reported in Table 4.1. The error for each data point was calculated by the difference of the experimental value and the predicted value. The model providing a smaller sum of squared errors was accepted as a better model than the other two models.

### 4.4.1. Comparison of $f_{cc}'/f_{co}'$ Predictions

For each test on Table 4.1, $f_{cc}'/f_{co}'$ values were calculated using Equations 2.5, 4.1 and 4.8 for Mander et al. (1988a), ACI 440-2R (2008) and the proposed model, respectively. The observed results and predicted values are plotted on Figure 4.2.

![Figure 4.2. Observed vs predicted values for $f_{cc}'/f_{co}'$ using different strength models.](image)

To measure and compare the accuracy of used models, Equations 4.12 and 4.13 were used. Other than an overall comparison taking all tests into account, a separate comparison was made for tests with $K_y$ values between 10 and 20 because of the ACI 440-2R (2008) recommendations for stress and strain limitations which were explained above.
It has been seen that the proposed model gave better predictions for the $f'_{cc}/f'_{co}$ values than the other two models for both the overall and $K_N$ values between 10 and 20 with average absolute errors of 7.23% and 7.44%, respectively. Equation 4.1 which was recommended by ACI 440-2R (2008) gave the second best predictions for the $f'_{cc}/f'_{co}$ values on the overall and for $K_N$ values between 10 and 20 with average absolute errors of 8.79% and 8.26%, respectively. Although the premature failure of FRP confinement was accounted for, Mander et al.’s (1988a) Equation 2.5 could predict $f'_{cc}/f'_{co}$ values with higher average absolute errors on the overall and for $K_N$ values between 10 and 20 than the other two models (12.21% and 14.15%, respectively).

Figure 4.3. Accuracy comparisons for $f'_{cc}/f'_{co}$ predictions amongst the strength models by average absolute error.

The accuracy of the three models on the $f'_{cc}/f'_{co}$ values over the given database was also compared by calculating the sum of squared errors for each model (Figure 4.4). A smaller sum of squared errors also indicate that the proposed model is more accurate than the other two models in predicting $f'_{cc}/f'_{co}$ values over the given database. The order of the accuracy of the models remained the same for the sum of squared errors method.
Figure 4.4. Accuracy comparisons for $f'_{cc}/f'_{co}$ predictions amongst the strength models by the sum of squared errors.

It should also be noted that, for experimental $f'_{cc}/f'_{co}$ values larger than 3.0, the proposed model gave unconservative predictions whereas Mander et al.’s (1988a) Equation 2.5 and ACI 440-2R (2008) recommended Equation 4.5 yielded conservative predictions. The unconservative predictions may be attributed to the linear expression of the original model in terms of $f_i/f'_{co}$ value as given by Equation 2.3 which has a constant slope of 4.1. ACI 440-2R (2008) recommended Equation 4.5 is also a linear equation in terms of $f_i/f'_{co}$, however its slope is 3.3 (when $\psi=1.0$) and it always gives a more conservative prediction than the Richart et al.’s (1928) model. Mander et al.’s (1988a) Equation 2.5 is expressed as a non-linear function of $f_i/f'_{co}$ and it has a decreasing slope as $f_i/f'_{co}$ increases. In practice, achieving confinement ratios so as to get $f'_{cc}/f'_{co}$ ratios higher than 3.0 is quite difficult and requires large amount of confinement material. Thus, proposed model’s unconservative predictions for experimental values of $f'_{cc}/f'_{co}$ higher than 3.0 can be ignored.
4.4.2. Comparison of $\varepsilon_{cc}/\varepsilon_{co}$ Predictions

The experimentally reported $\varepsilon_{cc}/\varepsilon_{co}$ values were plotted against the three models’ predictions (Figure 4.5).

The calculated average absolute error values for $\varepsilon_{cc}/\varepsilon_{co}$ were found to be larger than that of $f'_{cc}/f'_{co}$ for all three models under discussion, however ACI 440-2R (2008) model was slightly better than the other two models on the overall database with an average absolute error of 29.30%. For $K_N$ values between 10 and 20, all three models gave similar results close to 30% (Figure 4.6).

![Figure 4.5. Observed vs predicted values for $\varepsilon_{cc}/\varepsilon_{co}$ using different strength models.](image-url)
Figure 4.6. Accuracy comparisons for $\varepsilon_{cc}/\varepsilon_{co}$ predictions amongst the models by average absolute error method.

Figure 4.7. Accuracy comparisons for $\varepsilon_{cc}/\varepsilon_{co}$ predictions amongst the models by the sum of squared errors.

When the accuracy of the models are compared by the sum of squared errors (Figure 4.7), ACI 440-2R (2008) model was again found to be better than the other two models in predicting the $\varepsilon_{cc}/\varepsilon_{co}$ values over the given database. Though Mander et
al.’s (1988a) model gave second best predictions for the overall database of $\varepsilon_{cc}/\varepsilon_{co}$ values, for $K_N$ values between 10 and 20, the proposed model was better than Mander et al.’s (1988a) model and gave second best predictions after ACI 440-2R (2008) model for this range.

4.4.3. Stress-Strain Graph Type versus $K_N$

The observed data in Table 4.1 indicate that the value of $K_N$ affects the shape of stress-strain graph in an expected way that, for $K_N$ values less than 10, there is a possibility to obtain a Type 2 (Figure 4.1(b)) stress-strain graph as observed in entries 17, 18 and 19 of Table 4.1 reported by Xiao and Wu (2000). Although the entries 28, 28 (Matthys et al. 1999), 58 and 59 (Jiang and Teng 2007) had $K_N$ values smaller than 10, Type 1 (Figure 4.1.(a)) stress-strain graphs were reported by the researchers, however the corresponding $K_N$ values were close to 10. For the entries 65, 66, 67, 77, 78 and 79 (Howie and Karbhari 1994) stress-strain graph shape was not reported.

All observed stress-strain graphs of concrete cylinders with $K_N$ values greater than 10 are reported to be of Type 1 (Figure 4.1(a)).

4.5. Extending the Proposed Model for Hollow Concrete Columns Confined with FRP

Having shown that the proposed model can predict the maximum axial stress and the corresponding axial strain values with no less accuracy than the other two models, it was attempted to extend this model for the hollow concrete columns with FRP confinement.

Since the model was proposed for solid specimens with circular cross-sections confined with Carbon-FRP, Equations 4.8 and 4.9 were modified by multiplying the right hand side of the equations by a coefficient, $\beta$ given by Equation 4.14, to account for the different confinement mechanism in the hollow columns which were explained by Fam and Rizkalla (2001), Lignola et al. (2008) and Yazici and Hadi (2009) previously.

$$\beta = \left(1 - \frac{D_i^2}{D_o^2}\right)$$  \hspace{1cm} 4.14
where $D_i = \text{hollow core diameter of concrete cylinder}$ and $D_o = \text{the outer diameter of the concrete cylinder}$. Hence the modified forms of the Equations 4.8 and 4.9 become

\[
f'_{cc} = (1 + 0.033K_N) \beta f'_{co} \tag{4.15}
\]
\[
\varepsilon_{cc} = (1 + 0.17K_N) \beta \varepsilon_{co} \tag{4.16}
\]

where $f'_{co}$ and $\varepsilon_{co}$ are the unconfined concrete strength and the corresponding axial strain value obtained from standard solid concrete specimens. The reason of multiplying Equations 4.8 and 4.9 by the factor $\beta$ is that, the tests done by researchers (Zahn et al. 1990, Fam and Rizkalla 2001, Modarelli et al. 2005) on hollow concrete columns have suggested that the efficiency of confinement decreases as the hollow diameter increases. Moreover, the tests done by Modarelli et al. (2005) also suggested that, unconfined hollow concrete specimens do not reach the concrete strength values of solid specimens even though they are made from the same batch of concrete. Thus it is seen appropriate to decrease the expected values of $f'_{cc}$ and $\varepsilon_{cc}$ by multiplying Equations 4.8 and 4.9 by $\beta$ to account for the hole effect on the ultimate strength and the corresponding axial strain values of concrete specimens. However, while calculating the value of $K_N$ using Equation 4.6, the outer diameter ($D_o$) was used and $f'_{co}$ and $\varepsilon_{co}$ values were taken as the compressive strength and the corresponding axial strain of standard solid concrete cylinders (100x200 mm or 150x300 mm), respectively.

Since the model was developed using the Carbon-FRP wrapped concrete cylinders, only the tests done on hollow cylinders confined by CFRP could be used for calculation of accuracy. Thus, the test results reported by Modarelli et al. (2005) in Table 4.2 were used in this part. Two different batches of concrete were used in this study and they were tested to have 28.35 MPa and 38.24 MPa compressive strengths ($f'_{co}$) with corresponding axial strains ($\varepsilon_{co}$) of 0.0049 and 0.0063, respectively.

The experimental results versus the predicted values of $f'_{cc}/f'_{co}$ and $\varepsilon_{cc}/\varepsilon_{co}$ using Equations 4.15 and 4.16 were plotted in Figure 4.8 and Figure 4.9, respectively. As in the case of solid samples, $f'_{cc}/f'_{co}$ calculations yielded a better accuracy than $\varepsilon_{cc}/\varepsilon_{co}$ calculations. The average absolute error in $f'_{cc}/f'_{co}$ predictions on the
overall and for $K_N$ values between 10 and 20 were calculated to be 5.97% and 6.95%, respectively. The average absolute error values in $\varepsilon_{cc}/\varepsilon_{co}$ predictions on the overall database and for $K_N$ values between 10 and 20 were 26.29% and 23.42%, respectively. However the number of the experiments for CFRP confined hollow concrete cylinders reported in the literature may be limited, it should be noted that the proposed model was developed using the solid cylinder test results and still observed to give good results in hollow cylinders after introducing the coefficient $\beta$ into the Equations 4.8 and 4.9.

Table 4.2. The experimental results for CFRP confined hollow concrete cylinders from Modarelli et al. (2005)

<table>
<thead>
<tr>
<th>Label</th>
<th>Dimensions (mm)</th>
<th>$f_{co}$ (MPa)</th>
<th>Hollow Diameter (mm)</th>
<th>$t_f$ (mm)</th>
<th>$E_f$ (GPa)</th>
<th>$K_N$</th>
<th>$f_{cc}'$ $/ f_{co}$</th>
<th>$\varepsilon_{cc}'$ $/ \varepsilon_{co}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>CC2</td>
<td>300x150</td>
<td>28.35</td>
<td>50</td>
<td>0.165</td>
<td>221</td>
<td>17.1</td>
<td>1.59</td>
<td>4.59</td>
</tr>
<tr>
<td>CC3</td>
<td>300x150</td>
<td>28.35</td>
<td>50</td>
<td>0.330</td>
<td>221</td>
<td>34.3</td>
<td>1.79</td>
<td>5.20</td>
</tr>
<tr>
<td>CC4</td>
<td>300x150</td>
<td>28.35</td>
<td>50</td>
<td>0.495</td>
<td>221</td>
<td>51.4</td>
<td>2.52</td>
<td>6.30</td>
</tr>
<tr>
<td>CC6</td>
<td>300x150</td>
<td>38.24</td>
<td>50</td>
<td>0.165</td>
<td>221</td>
<td>12.7</td>
<td>1.25</td>
<td>2.42</td>
</tr>
<tr>
<td>CC7</td>
<td>300x150</td>
<td>38.24</td>
<td>50</td>
<td>0.330</td>
<td>221</td>
<td>25.4</td>
<td>1.55</td>
<td>2.71</td>
</tr>
<tr>
<td>CC8</td>
<td>500x250</td>
<td>28.35</td>
<td>150</td>
<td>0.165</td>
<td>221</td>
<td>10.3</td>
<td>0.94</td>
<td>3.22</td>
</tr>
<tr>
<td>CC9</td>
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<td>28.35</td>
<td>150</td>
<td>0.330</td>
<td>221</td>
<td>20.6</td>
<td>1.20</td>
<td>3.37</td>
</tr>
<tr>
<td>CC10</td>
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<td>28.35</td>
<td>150</td>
<td>0.495</td>
<td>221</td>
<td>30.9</td>
<td>1.27</td>
<td>4.57</td>
</tr>
<tr>
<td>CC11</td>
<td>500x250</td>
<td>38.24</td>
<td>150</td>
<td>0.165</td>
<td>221</td>
<td>7.6</td>
<td>0.83</td>
<td>1.94</td>
</tr>
<tr>
<td>CC12</td>
<td>500x250</td>
<td>38.24</td>
<td>150</td>
<td>0.330</td>
<td>221</td>
<td>15.3</td>
<td>1.02</td>
<td>2.38</td>
</tr>
</tbody>
</table>

Figure 4.8. Experimental versus predicted values of $f_{cc}' / f_{co}$ of CFRP confined hollow concrete cylinders.
Figure 4.9. Experimental versus predicted values of $\varepsilon_{cc}/\varepsilon_{co}$ for CFRP confined hollow concrete cylinders.

Figure 4.10. Average absolute error of $f'_{cc}/f'_{co}$ and $\varepsilon_{cc}/\varepsilon_{co}$ predictions using the proposed model.

4.6. **Continuous Stress-Strain Relationship Expression**

Having proved the efficiency of ultimate strength and strain equations derived for this study, a continuous stress-strain relationship is needed to utilize these equations in the moment-curvature relationship calculations.
The stress-strain model proposed by Hognestad et al. (1955) was utilized for a continuous representation of both confined and unconfined concrete stress-strain relationship. This model was originally proposed to describe the compressive behaviour of unconfined concrete. However, with a slight modification it can be used to describe the behaviour of FRP confined concrete.

In Hognestad et al.’s (1955) model, the unconfined concrete stress-strain graph is expressed in two distinct parts. The first part (ascending part) is expressed as a second order function of axial strain until the unconfined concrete strength value is reached (Figure 4.11) as given in Equation 4.17.

\[
\sigma_c = \beta f_{co} \left[ \frac{2\varepsilon_c}{\beta \varepsilon_{co}} - \left( \frac{\varepsilon_c}{\beta \varepsilon_{co}} \right)^2 \right] \text{ where } 0 \leq \varepsilon_c \leq \beta \varepsilon_{co}
\]

where \( \sigma_c \) is the axial stress and \( \varepsilon_c \) is the level of corresponding axial strain and \( \varepsilon_{co} \) is the axial strain value corresponding to unconfined concrete strength \( (f_{co}) \) which is assumed to be equal to 0.002.

The model was kept in its original shape for the curved part, however the descending part was replaced by a linear part ascending or descending depending on the value of \( K_x \) and \( \beta \). The experimental data reported by Modarelli et al. (2005) also suggests that a FRP confined cylinder may not even reach its unconfined concrete strength if the values of \( K_x \) and \( \beta \) are not large enough (see Table 4.2, entries CC8 and CC11). Because the concrete strength \( (f_{co}) \) and the corresponding axial strain value \( (\varepsilon_{co}) \) equations (Equations 4.15 and 4.16, respectively) were validated in Section 4.5, a further validation for the continuous stress-strain equation was not done.
Figure 4.11. Continuous stress-strain graphs for both unconfined and FRP confined concrete using Hognestad et al.’s (1955) model.

Since the effect of FRP confinement is assumed to be negligible until the axial stress increases $f'_{co}$ and the corresponding axial strain value to $\varepsilon_{co}$, the first part of stress-strain graphs of both unconfined and FRP confined concrete can be expressed with Equation 4.17.

A general equation for the second part of stress-strain graphs for both unconfined and confined concrete can be expressed as;

$$\sigma_c = \beta f_{co} + s (\varepsilon_c - \beta \varepsilon_c) \quad (\beta \varepsilon_c \leq \varepsilon \leq \varepsilon_{cu})$$  

where $s$ is the slope of the second part of the stress strain graph, $f_{cu}$ is the ultimate axial stress at which the concrete is assumed to be failed, and $\varepsilon_{cu}$ is the corresponding axial strain value.

For unconfined concrete, $f_{cu}$ was assumed to be $0.85 f'_{co}$ and the corresponding axial strain, $\varepsilon_{cu}$, was assumed to be $0.0038 \beta$ by the original model proposed by
Hognestad et al.’s (1955), where $\beta$ is introduced for hollow cross sections in this study. In the original Hognestad et al.’s (1955) model $\beta$ was equal to 1.0 since only the solid concrete specimens were considered.

4.7. Summary
In this chapter, a unified stress-strain model was developed by using the confinement provided by FRP wraps accounting for the premature failure of FRP confinement. The developed model was shown to be effective for predicting the ultimate stress and the corresponding strain value of FRP confined solid and hollow columns. A comparison amongst the proposed model, ACI 440-2R (2008) recommended equations and Mander et al. (1988a) model was made. The proposed model was found to be better in predicting the ultimate strength values than the other two models, and as effective as the other two models in predicting the ultimate strain values using a large database of reported tests in the literature.

In Chapter 5, the proposed stress-strain model is used to calculate a set of thrust-moment-curvature diagrams for the cross section of RC columns from which Moment-Axial load interaction diagrams will be derived. Using a similar but simplified moment magnification method described in AS 3600 (2009) and ACI 318 (2011), the effect of column height, eccentricity, FRP confinement configuration, concrete strength, and initial reinforcing steel amount are taken into account to calculate the theoretical loading path of axially loaded hollow RC columns.

Using the data reported in the literature, the accuracy of the proposed stress-strain model and the moment-curvature diagrams are investigated.
Chapter 5: MOMENT-CURVATURE ANALYSIS AND MODELLING OF FRP-CONFINED RC COLUMNS

5.1. Introduction

RC columns are generally subjected to bending moments as well as axial loads as discussed above. Thus, the flexural stiffness plays an important role in RC column behaviour.

The flexural stiffness \( (EI) \) can be defined as the slope of Moment-Curvature \( (M-\phi) \) curve. For members made of elastic materials, the flexural rigidity can be assumed to have a constant value. However, for the members made of non-linear or inelastic materials, the flexural stiffness \( (EI) \) is not constant due to non-linear relationship of moment and curvature. In such cases, the instantaneous value of flexural stiffness is used in the analyses.

RC columns are made of concrete, a non-linear inelastic component, and steel reinforcement, an elastic-perfectly plastic (in simplified behaviour), thus the expected Moment-Curvature relationship is of non-linear characteristics.

In this chapter, the Moment-Curvature analysis is carried out for FRP confined circular solid and hollow RC cross-sections using the stress-strain relationship developed in Chapter 4 for the FRP confined concrete, and an idealized stress-strain relationship for the reinforcing steel.

In the first part of the analytical modelling, a computer program written in MS Excel-VBA, which will be called as Program P-M hereafter, is used to derive Moment-Curvature relations of circular RC cross-sections for 41 different constant load levels including pure bending moment with zero axial load and the concentric load carrying capacity without a bending moment. For the concentric loading capacity, the moment and the corresponding curvature on the cross-section is assumed to have zero values. Since the stress-strain model developed in Chapter 4 can be used for both unconfined and confined RC columns, the program is capable of generating Moment-Curvature curves for both cases. The calculated Moment-Curvature curves of the column cross-section are then used to determine the Moment-Axial load (P-M) interaction diagram of the same cross-section.

In the second part of the analysis, secondary moments on the column is taken into consideration for calculating the non-linear loading paths of RC columns by a
simplified Moment Magnification Method similar to the one described in AS3600-01 (2009) and ACI 318 (2011) using another computer program written in MS Excel-VBA, which will hereafter be called as Program LL.

Finally, a numerical example is presented to see how the behaviour of a hollow RC column changes with changing column heights for a given cross-section and FRP confinement configuration.

5.2. Assumptions

To simplify the derivation of the Moment-Curvature ($M - \phi$), and the Axial load-Moment interaction (P-M) curves for unconfined or FRP confined RC column sections, some commonly accepted assumptions are made which are given below;

1. Plane sections remain plane after deformation.
2. Strain compatibility amongst concrete, steel and FRP is assumed, implying a perfect bond amongst the constituting materials.
3. Concrete is assumed to have no tensile strength.
4. Concrete is assumed to fail at the failure axial stress ($f_{cu}'$) with the corresponding axial strain ($\varepsilon_{cu}'$). For FRP confined concrete, these values are the maximum axial stress ($f_{cc}'$) and the corresponding axial strain value ($\varepsilon_{cc}'$) calculated using Equations 4.15 and 4.16, respectively. For the unconfined concrete, the assumed values of the ultimate stress and axial strain are $0.85f_{cu}'\beta$ and $0.0038\beta$, respectively as shown in Figure 4.11.
5. The continuous compressive stress-strain Equations 4.17 and 4.18 derived in Chapter 4 are adopted.
6. Reinforcing steel is assumed to behave as an elastic-perfectly plastic material in both in tension and compression as shown in Figure 2.2.
7. FRP in hoop and axial directions is assumed to have no stiffness under compression, that is it shows no resistance to compression. It is assumed to exhibit a linear stress-strain distribution until failure under tensile stresses.
5.3. **RC Column Geometry and Material Properties**

A typical cross-section used to generate Moment-Curvature curves is shown in Figure 5.1, however the number of longitudinal steel bars and longitudinal FRP strips can be a different number than six.

![Figure 5.1. General cross-section of RC columns used to derive Moment Curvature curves.](image)

A typical cross section of an RC column is assumed to have a concrete part with a compressive strength (measured on solid samples) of $f_{c0}$ and a corresponding axial strain of $\varepsilon_{c0}$ and an inner diameter $D_i$ and an outer diameter $D_o$. The cross-section has a clear concrete cover, $cc$, from the surface to the internal helix steel reinforcement. The overall height of the column is $H$.

The number of longitudinal internal steel reinforcement bars is $n_{s,\text{long}}$ and each of these bars has a nominal diameter of $D_{s,\text{long}}$ and a tensile yield strength of $f_{y,\text{long}}$.

The helical steel reinforcement bars have a diameter of $D_{s,\text{helix}}$ and a tensile yield strength of $f_{y,\text{helix}}$.

The RC column is assumed to have been retrofitted with FRP sheets in axial direction (vertical direction) by $n_f$ discrete single layer FRP strips, and in hoop direction by $n_{frp}$ layers of FRP sheets. A single layer of FRP sheet has a thickness of $t_{frp}$ and a width of $w_{frp}$ with a tensile strength of $f_{frp}$ and a corresponding axial strain of $\varepsilon_{frp}$ which are measured by coupon tensile tests.
In case of a solid cross-section, inner diameter ($D_i$) is taken as zero. Likewise, if no longitudinal FRP strips are used, the number of longitudinal FRP strips is taken as zero. If the RC column is not confined by FRP, the number of FRP layers is taken as zero.

5.4. **Calculations by Program P-M**

The MS Excel-VBA code, Program P-M, reads the cross-sectional geometry and the material characteristics from MS Excel spreadsheet cells which were manually entered by the user before. It calculates the ultimate strength ($f'_{cu}$) and the corresponding axial strain value ($\varepsilon_{cu}$) of the concrete for the given geometry and material properties using the confined concrete model proposed in the previous chapter, Moment-Curvature relationship for a series of constant axial loads, and Axial load-Bending moment interaction envelope curve for the given cross-section using the calculated Moment-Curvature values. The calculation procedure for Program P-M is illustrated below (Figure 5.2). The MS Excel VBA code of Program P-M is given in Appendix A.

The output of the Program P-M is printed on the same MS Excel file. To demonstrate the effect of axial load on the moment curvature relationship of the cross-section, the calculated $P - M - \phi$ curves for 0 to 0.9$P_{ult}$ axial load levels with an increasing step of 0.1$P_{ult}$ are plotted using MS Excel graphing tool.

$P - M$ interaction diagram values are also calculated and printed on the same MS Excel file. The $P - M$ interaction diagram for the given cross-section is then plotted using the MS Excel graphing tool.
5.4.1. Calculation of \( f'_{cu} \) and \( \varepsilon_{cu} \)

In this part, first the normalized confinement stiffness \( K_N \) is calculated using Equation 4.6, however, the thickness of FRP wrapping \( t_f \) is calculated by multiplying the number of wrapping layers \( n_{fp} \) and the thickness of a single layer.
of FRP ($t_{frp}$) sheet. The confined concrete strength ($f_{cc}'$) and the corresponding axial strain value ($\varepsilon_{cc}$) are calculated using Equations 4.15 and 4.16, respectively.

### 5.4.2. Derivation of Moment-Curvature Relationships for RC Cross-sections

The response of the given cross-section of RC column to an external load $P(i)$ is assumed to be equivalent to the linear sum of the response of the components; concrete, longitudinal steel bars and the longitudinal FRP strips as shown in Figure 5.3. The helical steel reinforcement is assumed to have no effect on the behaviour of a RC column after FRP wrapping. For RC columns the effect of helical steel confinement becomes substantial when the cover concrete is lost due to axial loads. However for the FRP wrapped RC columns, the cover concrete is not lost until the rupture of FRP confinement, thus leaving the helical steel confinement negligible until the failure. This assumption was adopted from Cheng (2000). Moreover, the longitudinal FRP strips are assumed to be equivalent to FRP bars having the same material characteristics and cross-sectional areas placed at the surface of the cross-section.

![Figure 5.3. Separation of the FRP confined RC column cross-section into its components.](image)

Since the concrete behaviour is altered by the FRP confinement in the hoop direction, the concrete component of the cross-section is assumed to have a stress-strain relationship that can be expressed by the model proposed in Chapter 4. FRP in the hoop direction is not shown on the concrete component in Figure 5.3 since its effect on the concrete is already shown in the stress-strain relationship of the concrete. The response of the cross-section of RC column to the external loadings is assumed to be the summation of the response of each component.
In the calculation of the response of the RC cross-section, basic principles of superposition, force equilibrium of external and internal forces and strain compatibility are used.

![Figure 5.4. General loading on the RC column cross-section.](image)

In general case, the RC column is assumed to be under an axial loading and a bending moment at the same time, however the bending moment is assumed to be due to the eccentricity of the axial load as shown in Figure 5.4.

In this study, 41 equally spaced constant axial load levels were used in Moment-Curvature curves varying from zero axial load (\( P = 0 \), pure bending moment) to concentric load carrying capacity (\( P = P_{ult} \)) which can be calculated by Equation 5.1.

\[
P_{ult} = f'_{c,\text{max}} A_c + f_{y,\text{long}} A_s
\]

Where \( f'_{c,\text{max}} \) is the maximum axial compressive stress of the concrete, \( A_c \) is the cross-sectional area of concrete component, \( f_{y,\text{long}} \) is the yield strength of longitudinal steel reinforcement bars, and \( A_s \) is the total cross-sectional area of longitudinal steel bars in the RC column cross-section.

For unconfined hollow RC columns, the value of \( f'_{c,\text{max}} \) can still be found using Equation 4.15 by inserting a zero value for \( K_N \). Thus, a general equation for the maximum expected axial compressive stress of the concrete in the unconfined RC columns (not wrapped by FRP in hoop direction) can be expressed as Equation 5.2.
\[ f'_{c,\text{max}} = \beta f'_{co} \quad 5.2 \]

For the FRP confined RC columns, \( f'_{c,\text{max}} \) is assumed to be equal to \( f'_{cc} \) which can be calculated using Equation 4.15.

For the concentric load capacity \( P_{ult} \), no moment curvature calculation was done since the cross-section was assumed to deform uniformly and such a load could only be applied to centroid of the cross-section without an eccentricity. Thus Moment-Curvature values are assumed to be equal to zero for \( P_{ult} \).

Thus a general expression for the axial load level can be expressed as in Equation 5.3.

\[ P(i) = \frac{P_{ult}(i)}{40} \quad i = 0,1,2,\ldots,40. \quad 5.3 \]

Since the axial load is assumed to be constant for Moment-Curvature relationship, the curves can also be named as Axial thrust-Moment-Curvature curves \( (P - M - \phi) \). Each constant axial load \( P(i) \) is assumed to act on an eccentric loading line on the column cross-section to produce a compressive strain \( (\varepsilon_c(j)) \) in the outermost concrete fibre as shown in Figure 5.5. It is assumed that, for each eccentricity the \( P(i) \) is applied on the cross-section, there is a unique strain distribution, that is a unique combination of maximum compressive strain value \( (\varepsilon_c(j)) \), and a depth of neutral axis value \( (d_N(i, j)) \).

Figure 5.5. Axial load acting on the RC column cross-section and the corresponding axial-strain distribution.
The maximum axial strain value of concrete \( (\varepsilon_c(j)) \) is assumed to be varying between 0.0002 and the ultimate axial strain value \( (\varepsilon_{cu}) \) with an equal increments of 0.0002. The value of \( \varepsilon_{cu} \) is assumed to be equal to 0.0038 for unconfined concrete (unwrapped columns) and \( \varepsilon_{cc} \) for FRP confined concrete (FRP wrapped columns) which is calculated using Equation 4.16. The number of \( \varepsilon_c(j) \) values \( (n_{\text{strain}}) \) are determined by the expression stated in Equation 5.4, where the expression “int” stands for the integer part of the division.

\[
n_{\text{strain}} = \text{int}(\frac{\varepsilon_{cu}}{0.0002}) \tag{5.4}
\]

Thus, for each axial load level \( (P(i)) \), the maximum axial strain values of concrete \( (\varepsilon_c(j)) \) can be expressed as

\[
\varepsilon_c(j) = \frac{\varepsilon_{cu}}{n_{\text{strain}}} \quad j = 1,2,3,..,n_{\text{strain}} \tag{5.5}
\]

For each \( \varepsilon_c(j) \) value considered, a neutral axis depth, \( d_N(i,j) \), is assumed and the corresponding force response of confined concrete \( (F_c(i,j)) \), longitudinal steel \( (F_s(i,j)) \) and the longitudinal FRP strips \( (F_{frp}(i,j)) \) are calculated using the axial strain distribution corresponding to the axial strain level \( (\varepsilon_c(j)) \) under consideration and the assumed \( d_N(i,j) \) as shown in Figure 5.5. The value of \( d_N(i,j) \) is iterated to achieve a sum of \( \sum F_R = F_c(i,j) + F_s(i,j) + F_{frp}(i,j) \) (force response of the cross section) within 10 kN of the external load level \( P(i) \), that is also expressed in Equation 5.6.

\[
|\sum F_R - P(i)| \leq 10 \text{ kN} \tag{5.6}
\]

A maximum of 10 kN difference was adopted as a convergence criterion instead of a percentage of \( P(i) \) to avoid the loss of precision in calculation of cross-section response to the applied load especially for high axial load levels.

While iterating \( d_N(i,j) \), its value is assumed to be starting from 1 mm and increasing with an increment of 0.5 mm after each unsuccessful iteration. When the response of
the cross section is close enough to the external load applied \( P(i) \) as stated in Equation 5.6, the strain distribution and the depth of the neutral axis is assumed to be found correctly, thus the curvature of the cross-section under a given load \( P(i) \) and maximum axial strain level \( \varepsilon_c(j) \) is calculated using Equation 5.7.

\[
\phi(i, j) = \frac{\varepsilon_c(j)}{d_N(i, j)} \tag{5.7}
\]

The corresponding bending moment on the cross-section \( M(i, j) \) is then calculated by summing up the moments of the concrete component \( M_c(i, j) \), longitudinal steel component \( M_s(i, j) \) and the longitudinal FRP strips \( M_f(i, j) \) about the horizontal centreline.

\[
M(i, j) = \sum M_R(i, j) = M_c(i, j) + M_s(i, j) + M_f(i, j) \tag{5.8}
\]

The force and moment response of concrete, longitudinal steel bars and the longitudinal FRP strips are explained in detail in the following parts.

### 5.4.2.1. Calculation of Concrete Response

The concrete response for a given strain distribution is generally calculated by using an equivalent rectangular stress block approach in most RC design calculations. However, in this study, a rectangular stress block approach is not adopted since the necessary coefficients to transform a non-uniform stress distribution into a uniform equivalent stress distribution together with the application point of the resultant force on a circular hollow cross-section RC confined with FRP is not known. Besides, for the Moment-Curvature relationship calculations, the use of maximum axial strains other than the ultimate axial strain \( \varepsilon_{cu} \) values combined with the existence of the hollow part in the cross-section make it more complicated to use an equivalent rectangular stress block approach.

Thus, in the calculation of the response of concrete component corresponding to \( \varepsilon_c(j) \) and \( d_N(i, j) \), the cross-section is divided into a finite number of horizontal strips with a thickness of \( \Delta h \) as shown in Figure 5.6. The number of the strips \( m_{strip} \) is found by using Equation 5.9.
\[ m_{\text{strips}} = \text{int} \left( \frac{D_o}{\Delta h} \right) \]

Where \( D_o \) is the outside diameter of the concrete cross-section, \( \Delta h \) is the chosen strip height.

The position of each strip from the top of the cross-section \( (X_{\text{top}}(k)) \) and from the horizontal centreline \( (X_{\text{centre}}(k)) \) are calculated using the Equations 5.10 and 5.11 respectively.

\[
X_{\text{top}}(k) = (k - 0.5)\Delta h \quad k = 1,2,3,\ldots,m_{\text{strips}}
\]

\[
X_{\text{centre}}(k) = \frac{D_o}{2} - X_{\text{top}}(k) \quad k = 1,2,3,\ldots,m_{\text{strips}}
\]

Axial strain of each concrete strip is assumed to be constant throughout the strip and equal to the value at the centre of each strip. Thus, the axial strain value of any strip \( (\varepsilon_{c,\text{strip}}(k)) \) is calculated by Equation 5.12.

\[
\varepsilon_{c,\text{strip}}(i,j,k) = \varepsilon_c(j) - \frac{\varepsilon_c(j)}{d_N(i,j)} X_{\text{top}}(k)
\]

where

\( i = 0,1,2,\ldots,39 \)
\( j = 1,2,3,\ldots,n_{\text{strain}} \)
\( k = 1,2,3,\ldots,m_{\text{strips}} \).

After calculating the axial strain of each strip corresponding to the values of \( P(i), \varepsilon_c(j) \) and \( d_N(i,j) \), the stress value on the centre of each strip can be
calculated using the continuous stress-strain equations proposed in Chapter 4 which can be re-written as

\[ \sigma_{e,\text{strip}}(i, j, k) = 0 \]

for \( \varepsilon_{e,\text{strip}}(i, j, k) \leq 0 \)

\[ \begin{align*}
\sigma_{e,\text{strip}}(i, j, k) &= \beta f_{c0} \left( \frac{2 \varepsilon_{e,\text{strip}}(i, j, k)}{\beta \varepsilon_{c0}} - \left( \frac{\varepsilon_{e,\text{strip}}(i, j, k)}{\beta \varepsilon_{c0}} \right) \right) \\
&\quad \text{for } 0 \leq \varepsilon_{e,\text{strip}}(i, j, k) \leq \beta \varepsilon_{c0}
\end{align*} \] \hspace{1cm} (5.13)

\[ \begin{align*}
\sigma_{e,\text{strip}}(i, j, k) &= \beta f_{c0} + s \left( \varepsilon_{e,\text{strip}} - \beta \varepsilon_{c0} \right) \\
&\quad \text{for } \beta \varepsilon_{c0} \leq \varepsilon_{e,\text{strip}}(i, j, k) \leq \varepsilon_{cu}
\end{align*} \]

where

\( i = 0, 1, 2, ..., 39 \)

\( j = 1, 2, ..., n_{\text{strain}} \)

\( k = 1, 2, ..., m_{\text{strips}} \)

\( s = \) slope of the second part of the stress-strain equation which can be calculated using Equation 4.19 in Chapter 4.

The force response from a concrete strip \( f_{c,\text{strip}}(i, j, k) \) can be calculated by multiplying the axial stress value calculated \( \sigma_{e,\text{strip}}(i, j, k) \) and the area of the strip \( a_{\text{strip}}(k) \) itself.

\[ f_{c,\text{strip}}(i, j, k) = \sigma_{e,\text{strip}}(i, j, k) a_{\text{strip}}(k) \] \hspace{1cm} (5.14)

The strip area can be found by multiplying the strip height \( \Delta h \) by the strip width \( w_{\text{strip}}(k) \) (Equation 5.15).

\[ a_{\text{strip}}(k) = \Delta h w_{\text{strip}}(k) \] \hspace{1cm} (5.15)

However, the width of each strip is different since the cross-section is circular. Besides, in the case of a hollow cross-section, some strips intersect the hollow part (as shown in Figure 5.7) for which the hollow part of these strips should be subtracted from the overall area of the strip in order to find the effective area.
Thus, for a concrete strip on the cross section the width of a concrete strip can be calculated as

\[
\begin{align*}
\text{if } & D_i = 0 \\
\text{if } & D_i \neq 0 \\
\text{for } & D_o \leq Xc_{center}(k) \leq D_i \\
\text{for } & 0 \leq Xc_{center}(k) \leq D_i
\end{align*}
\]

\[
w_{strip}(k) = 2 \sqrt{\left(\frac{D_o}{2}\right)^2 - \left(Xc_{center}(k)\right)^2}
\]

and

\[
w_{strip}(k) = 2 \left[\sqrt{\left(\frac{D_o}{2}\right)^2 - \left(Xc_{center}(k)\right)^2} - \sqrt{\left(\frac{D_i}{2}\right)^2 - \left(Xc_{center}(k)\right)^2}\right]
\]

where \( k = 1, 2, 3, \ldots, m_{strips} \).

The overall concrete section response corresponding to \( \varepsilon_c(j) \) and assumed \( d_N(i, j) \) is calculated by summing up the response of all concrete strips as shown in Equation 5.17.

\[
F_c(i, j) = \sum_{k=1}^{m_{strip}} f_{c,strip}(i, j, k)
\]

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The moment response of each strip about horizontal centreline \( m_{c,\text{strip}(i,j,k)} \) can be calculated using Equation 5.18.

\[
m_{c,\text{strip}(i,j,k)} = f_{c,\text{strip}(i,j,k)} X_{c\text{centre}(k)}
\]

It should be noted that, above the horizontal centreline, a concrete strip under a compressive force would yield a positive moment whereas the one below the centreline would yield a negative moment according to Equation 5.18. The formulation of modelling was designed in such a way that the signs of strains, stresses, forces and positions would yield correct signs for both forces and moments at the same time (e.g., a tensile force on a strip below the center line will also yield a positive moment).

Thus, the overall moment response of the concrete component is calculated by summing up the moments produced by the concrete strips (Equation 5.19).

\[
M_c(i,j) = \sum_{k=1}^{m_{\text{strip}}} m_{c,\text{strip}(i,j,k)}
\]

### 5.4.2.2. Calculation of Longitudinal Steel Reinforcement Response

Given that the RC column cross-section has a number of longitudinal steel bars \( n_{s,\text{long}} \), with a nominal diameter \( D_{s,\text{long}} \), the position of each bar can be determined using polar coordinates of each bar, which is \( (r_s, \theta_s(p)) \) as shown in Figure 5.8. The angle is in Radians, and the polar angle for each steel bar is calculated using Equation 5.20.

\[
\theta_s(p) = \frac{2\pi(p-1)}{n_{s,\text{long}}} \quad p = 1,2,3,\ldots,n_{s,\text{long}}
\]

![Figure 5.8. Positioning of longitudinal steel bars.](image-url)
The position of a steel bar from the horizontal centreline \( X_{s_{\text{centre}}}(p) \) is then calculated as

\[
X_{s_{\text{centre}}}(p) = \sin(\theta_s(p)) r_s \quad p = 1,2,3,\ldots,n_{s,\text{long}} \tag{5.21}
\]

where \( r_s \) is the radius measured from the centre of the cross-section to the centre of the steel bar, which is calculated using Equation 5.22;

\[
r_s = \frac{D_u}{2} - cc - D_{s,\text{helix}} - \frac{D_{s,\text{long}}}{2} \tag{5.22}
\]

The value of \( r_s \) is the same value for all longitudinal steel bars. The position of each bar from the top can be calculated using Equation 5.23.

\[
X_{s_{\text{top}}}(p) = \frac{D_u}{2} - X_{s_{\text{centre}}}(p) \quad p = 1,2,3,\ldots,n_{s,\text{long}} \tag{5.23}
\]

The axial strain on each longitudinal bar is then calculated using a similar approach to the calculation of axial strains on concrete strips. For a given external load \( P(i) \), the maximum axial strain in the outermost concrete fibre \( \varepsilon_c(j) \) and a neutral axis depth \( d_{\text{N}}(i,j) \), the axial strain on a steel bar is calculated assuming a linear strain distribution throughout the cross-section (Equation 5.24).

\[
\varepsilon_s(i,j,p) = \varepsilon_c(j) - \frac{\varepsilon_c(j)}{d_{\text{N}}(i,j)} X_{s_{\text{top}}}(p)
\]

where

\[
i = 0,1,2,\ldots,39 \\
j = 1,2,3,\ldots,n_{\text{strain}} \\
p = 1,2,3,\ldots,n_{s,\text{long}}
\]

The stress in each bar \( \sigma_s(i,j,p) \) is then calculated by assuming a simplified elastic-perfectly plastic stress-strain relationship for the longitudinal steel bars as mentioned in Chapter 2 and illustrated with Figure 2.2 which can be expressed as

\[
\begin{align*}
\text{for } |\varepsilon_s(i,j,p)| &\leq \varepsilon_{y_{\text{long}}} & \sigma_s(i,j,p) &= \varepsilon_s(i,j,p) E_s \\
\text{for } |\varepsilon_s(i,j,p)| &> \varepsilon_{y_{\text{long}}} & \sigma_s(i,j,p) &= f_{y_{\text{long}}}
\end{align*}
\]

where \( \varepsilon_{y_{\text{long}}} \) is the yield strain of longitudinal steel bars, \( E_s \) is the elasticity modulus which is assumed to be 200000 MPa, and \( f_{y_{\text{long}}} \) is the yield strength of longitudinal
The steel bars. The steel bars are assumed to behave similarly in tension and compression.

The force response of each bar \( f_s(i, j, p) \) is then calculated by multiplying the axial stress and the cross sectional area of the steel bar \( a_s \) as in Equation 5.26.

\[
f_s(i, j, p) = \sigma_s(i, j, p) a_s \quad p = 1, 2, 3, ..., n_{s,\text{long}}
\]

where the cross-sectional area of a steel bar can be calculated by Equation 5.27.

\[
a_s = \frac{\pi}{4} D_{s,\text{long}}^2
\]

The overall response of the longitudinal steel bars \( F_s(i, j) \) is calculated by summing the force response of all the longitudinal bars according to the given strain distribution as in Equation 5.28.

\[
F_s(i, j) = \sum_{p=1}^{n_s} f_s(i, j, p)
\]

The moment response of each longitudinal steel bar about the horizontal centreline \( m_s(i, j, p) \) can be calculated using Equation 5.29.

\[
m_s(i, j, p) = f_s(i, j, p) X_{s,\text{centr}}(p) \quad p = 1, 2, 3, ..., n_{s,\text{long}}
\]

Thus, the overall moment response of the longitudinal steel component \( M_s(i, j) \) is calculated by summing up the moments produced by the longitudinal steel bars (Equation 5.30).

\[
M_s(i, j) = \sum_{p=1}^{n_{s,\text{long}}} m_s(i, j, p)
\]

5.4.2.3. Calculation of Longitudinal FRP Reinforcement Response

The number of the vertical FRP strips on the surface of the RC column is \( n_f \) which are evenly distributed over the surface (Figure 5.9). For the simplicity of the calculations, each vertical FRP strip is transformed to an equivalent FRP bar with the
same cross-sectional area as the FRP strip \((t_{frp} \cdot w_{frp})\) (Figure 5.10), and placed on the surface with their centre coinciding with the surface line of the RC column.

Figure 5.9. Positions of vertical FRP strips.

Figure 5.10. Cross-sectional shape transformation for the vertical FRP strips.

The positions of FRP bars (see Figure 5.9) are calculated in a similar way with the longitudinal steel bars using Equations 5.31, 5.32, and 5.33.

Where \(s = 1, 2, 3, \ldots, n_f\)

\[
\theta_f(s) = \frac{2\pi}{n_f}(s - 1) 
\]

\[
X_{f,\text{centre}}(s) = \sin(\theta_f(s))r_f 
\]

\[
X_{f,\text{top}}(s) = \frac{D_o}{2} - X_{f,\text{centre}}(s) 
\]

where \(r_f\) is equal to the outer radius of the concrete cross-section \(\left(\frac{D_o}{2}\right)\).

For a given external load \(P(i)\), the maximum axial strain in the outermost concrete fibre \(\varepsilon_f(j)\) and a neutral axis depth \(d_N(i, j)\), the axial strain on a vertical FRP bar is calculated assuming a linear strain distribution throughout the cross-section (Equation 5.34).
\[ \varepsilon_j(i,j,s) = \varepsilon_e(j) - \frac{\varepsilon_e(j)}{d_h(i,j)} X_{j_{lep}}(s) \]

where

\[ i = 0,1,2,\ldots,39 \]
\[ j = 1,2,3,\ldots, n_{\text{strain}} \]
\[ s = 1,2,3,\ldots, n_f \]

The stress in each FRP bar (\( \sigma_j(i,j,s) \)) is then calculated by assuming that FRP only has a stiffness under tensile stress and shows no resistance to compressive stresses, which can be expressed as

for \( \varepsilon_j(i,j,s) \geq 0 \)
\[ \sigma_j(i,j,s) = 0 \text{ MPa} \]

for \( \varepsilon_j(i,j,s) < 0 \)
\[ \sigma_j(i,j,s) = E_f \varepsilon_j(i,j,s) \text{ MPa} \]

where \( E_f \) is the elasticity modulus (in MPa) of FRP strips under tensile loading.

The force response of each vertical FRP bar (\( f_j(i,j,s) \)) is then calculated by multiplying the axial stress and the cross sectional area of the FRP bar (\( a_f \)) as in Equation 5.36.

\[ f_j(i,j,s) = \sigma_j(i,j,s)a_f \quad s = 1,2,3,\ldots, n_f \]

where the cross-sectional area of a vertical FRP bar can be calculated by Equation 5.37.

\[ a_f = t_f w_f \]

The overall response of the longitudinal FRP bars (\( F_j(i,j,s) \)) is calculated by summing the force response of all the longitudinal FRP strips according to the given strain distribution as in Equation 5.38.

\[ F_j(i,j) = \sum_{s=1}^{n_f} f_j(i,j,s) \]
The moment response of each longitudinal FRP strip about the horizontal centreline \( m_f(i,j,s) \) can be calculated using Equation 5.39.

\[
m_f(i,j,s) = f_j(i,j,s)X_{center}(s) \quad s = 1,2,3,\ldots,n_f
\]

Thus, the overall moment response of the longitudinal FRP component \( M_f(i,j) \) is calculated by summing up the moments produced by the longitudinal FRP strips (Equation 5.40).

\[
M_f(i,j) = \sum_{s=1}^{n_f} m_f(i,j,s)
\]

5.4.3. **Axial Load-Bending Moment (P-M) Interaction curves**

Having calculated \( M(i,j) - \phi(i,j) \) values for a given \( P(i) \) load level in the previous part, Program P-M determines the maximum value of \( M(i,j) \) values for the \( P(i) \) from the corresponding \( M - \phi \) graph, and names it as \( M_{\text{max}}(i) \) (Figure 5.11(a)). The \( M_{\text{max}}(i) \) and \( P(i) \) values are used to form an envelope curve which is also called as Axial Load-Moment (\( P - M \)) interaction diagram for the given cross-section (Figure 5.11(b)).

![Figure 5.11](image)

Figure 5.11. (a) \( M - \phi \) curve for an axial load level \( P(i) \), (b) Corresponding point on \( P - M \) diagram.

The \( P - M \) interaction curves are used to determine the maximum axial load and the corresponding bending moment values on a RC cross-sections. When an axial load and applied bending moment combination on a cross-section is within the \( P - M \) interaction diagram, the RC column cross-section is assumed to be yet to reach its
capacity. The $P-M$ curve itself gives the maximum applicable axial load and bending moment values. No loading combination is allowed outside the $P-M$ diagrams.

### 5.5. Calculations by Program LL

The calculations by Program P-M gives the $P-M$ interaction diagram for a given cross-section as explained above. However, under eccentric loading, RC columns undergo lateral deformations ($\delta$) which causes the bending moment to be greater than the multiplication of the axial load and the eccentricity for the mid-height level (Figure 5.12).

![Diagram](image)

**Figure 5.12.** (a) Eccentric loading of RC column, (b) Deformed shape of eccentrically loaded column.

For short columns, the value of lateral deformation is small and generally neglected, thus the loading line is assumed to be linear as shown with a line segment OA in Figure 5.13. For the columns with a considerable height, the value of $\delta$ can be expected a larger value than that of short ones, causing the loading lines to intersect the $P-M$ interaction diagram at a smaller axial load as shown with Curve OC in Figure 5.13.
The purpose of Program L-L is to determine the loading line of RC columns (solid or hollow, confined or unconfined), and calculate the maximum axial load level that can be applied to a RC column with an eccentricity $e$.

Though the secondary moments cause the tall columns to fail earlier than the short ones, calculation of the lateral deformation for a given axial load level and eccentricity poses a difficulty due to nonlinearity of RC making materials. However, the equations governing the linear elastic column behaviour are utilized in this study to explain the behaviour of RC columns with simple modifications. A similar approach was adopted by AS 3600 (2009) and ACI 318 (2011) to account for the secondary moments on RC columns. The method adopted is explained below.

For a column made of an ideally linear elastic material, the bending moment and the curvature values are linearly related as shown in Equation 5.41, where the value of $EI$ is constant due to material linearity.

$$ M = EI \phi $$  \hspace{1cm} 5.41

For the mid-height of this elastic column, the increased bending moment due to lateral deformation $\delta$ is calculated by Equation 5.42;

$$ ML = M_p \delta_{mag}. $$  \hspace{1cm} 5.42
where $ML$ is the bending moment on the cross-section at the mid-height level which also contains secondary moment, $M_p$ is the primary bending moment which is equal to $P.e$, and $\delta_{mag}$ is the moment magnification factor corresponding to the axial load level $P$. For the pin-ended elastic column with equal eccentricities, Timoshenko and Gere (1961) gave the magnification factor as

$$\delta_{mag} = \frac{a^*}{1 - \frac{P}{P_C}}$$  \hspace{1cm} 5.43

where $a^*$ is a constant slightly larger than unity and depends on the ratio of $P/P_C$, $P$ is the axial load and $P_C$ is the critical axial load which is taken to be the Euler’s buckling load, expressed as:

$$P_C = \pi^2 \frac{EI}{H^2}$$  \hspace{1cm} 5.44

where $H$ is equal to the height of pin connected column.

It must be noted that Equations 5.43 and 5.44 are given for columns made of elastic materials. However, RC columns are composite members consisting of materials (steel and concrete) whose behaviours are different than elastic materials which makes it quite difficult to determine a $EI$ (flexural stiffness) value at the load level equal to the critical buckling load. Warner et al. (1993) used the ratio of balanced moment ($M_b$) divided by the corresponding curvature value of the cross section ($\phi_b$) (Equation 5.45) which equals to the instantaneous flexural stiffness ($EI_b$) of the RC column cross section, and used it as the critical flexural stiffness in Equation 5.44 without a mathematical proof, just because of its convenience. The same approach was adopted in this thesis.

To calculate the $P_c$ for the RC columns using the Equation 5.44, the column stiffness ($EI$) was calculated using the strain distribution of the balanced failure case illustrated in Figure 5.14.
The cross-section moment response for the balance failure ($M_B$) is calculated using the strain distribution in Figure 5.14, and the corresponding $EI$, which is named as $(EI)_B$, is calculated as

$$(EI)_B = \frac{M_B}{\phi_B}$$

5.45

where $\phi_B$ is the curvature value for the same strain distribution. The $P_C$ is calculated using Equations 5.44 and 5.45, and $\delta_{mag}$ for a given axial load is calculated using Equation 5.43 with replacing $a^*$ value with unity.

Program L-L calculates the loading line coordinates ($PL(i)\ ML(i)$), accounting for the slenderness of the RC columns. The magnified moments ($ML(i)$) for a given eccentricity ($e$) and service loads ($PL(i)$) are calculated until the loading line intersects the $P-M$ interaction diagram. The calculation procedure of Program L-L is illustrated in Figure 5.15.
The MS Excel VBA code for Program LL is given in Appendix B.

The last axial load value before intersecting the $P-M$ interaction diagram is accepted as the axial load capacity of the given RC column. In Program P-M, the axial load values had increased from zero to $P_{ult}$ with equal increments of $P_{ult}/40$. Having used the same axial load levels for loading line calculations, the maximum
error by stopping the loading line calculations at the last axial load before intersecting the $P - M$ interaction diagram will be $+P_{ult}/40$ (2.5% of $P_{ult}$) which can be assumed as a relatively small error for axial load carrying capacity calculation.

5.6. Numerical Example for the Model
In this section, a numerical example for the calculations done by Program P-M and B is illustrated. The expected behaviour (according to the model explained above) of a series of RC columns with the same cross-sections but different heights and FRP wrapping configurations are demonstrated.

5.6.1. Unconfined Hollow RC Column
The cross-section of the example RC column before the FRP confinement is illustrated in Figure 5.16.

![Cross-section of the example RC column before FRP confinement.](image)

The unconfined cross section has an outer diameter ($D_o$) of 500 mm, inner diameter ($D_i$) of 200 mm. The unconfined concrete strength obtained from standard solid specimens is 70 MPa ($f_{co}$), and the corresponding axial strain is 0.002. The cross-section has 10 longitudinal steel bars ($n_{s,long} = 10$) with 16 mm diameter ($D_{s,long} = 16 \text{ mm}$) of 500 MPa tensile strength ($f_{yl} = 500 \text{ MPa}$), and a helix reinforcement made of 10 mm diameter steel bar ($D_{s,helix} = 10 \text{ mm}$) of 250 MPa...
tensile strength ($f_{yh} = 250 \text{MPa}$) with 100 mm pitch. The modulus of elasticity for steel reinforcement is taken as 200000 MPa ($E_{s,\text{long}} = E_{s,\text{helix}} = 200000 \text{MPa}$).

The height of horizontal concrete strips is taken as 5% of the outer diameter is ($\Delta h = 0.05D_o$).

In the first part of the analysis, the moment-curvature curves for a series of axial load is calculated and plotted on a graph (Figure 5.17) and using the calculated moment curvature values, the Axial Load-Bending Moment (P-M) interaction diagram of the cross-section is produced using Program P-M (Figure 5.18 and Figure 5.19).

Using two different axial loading eccentricity values (25 mm and 50 mm) and four different column heights of 0 mm (for the cross-section), 1000 mm, 5000 mm and 10000 mm, the loading lines of RC column are calculated using Program LL (Figure 5.18 and Figure 5.19).

![Figure 5.17. Moment-Curvature curves for the unconfined cross-section.](image-url)
Figure 5.18. P-M diagram and the loading lines of unconfined RC column for 25 mm loading eccentricity.

Figure 5.19. P-M diagram and the loading lines of unconfined RC column for 50 mm loading eccentricity.
5.6.2. FRP Wrapped Hollow RC Column in Hoop Direction

The cross-section given in Figure 5.16 is assumed to be confined by three layers of Carbon FRP strips in the hoop direction only. The CFRP has a thickness \((t_f)\) of 0.5 mm per layer and a strip width \((w_f)\) of 75 mm. The FRP sheets used in wrapping is assumed to have a unidirectional tensile strength \((f_{frp})\) of 1000 MPa with a corresponding failure strain \((\varepsilon_{fu})\) of 0.015 which gives a modulus of elasticity \((E_f)\) value of 66667 MPa.

Program P-M gives the Moment-Curvature curves (Figure 5.20) and the P-M interaction Diagram of the FRP confined (hoop direction) RC column cross-section as shown in Figure 5.21 and Figure 5.22.

Program L-L gives the loading lines of the FRP confined column for the same column heights as the unconfined RC column (Figure 5.21 and Figure 5.22).

![Figure 5.20. Moment-Curvature curves for the FRP confined (hoop direction) cross-section.](image-url)
5.6.3. FRP Wrapped Hollow RC Column in Hoop and Vertical Directions

To show the effect of vertical strips in FRP strengthening of RC columns, 10 vertical single layer FRP strips are assumed to be attached on the surface before confining the example RC column with three layers of FRP in the hoop direction.

Program P-M gives the Moment-Curvature curves (Figure 5.23) and the P-M interaction Diagram of the FRP confined (hoop and vertical directions) RC column.
cross-section as shown in Figure 5.24 and Figure 5.25. Program LL gives the loading lines of the FRP confined column (hoop and vertical directions) for the same column heights as the unconfined RC column (Figure 5.24 and Figure 5.25).

Figure 5.23. Moment Curvature curves for the FRP confined (hoop and vertical directions) cross-section.

Figure 5.24. P-M diagram and the loading lines of FRP confined (hoop and vertical directions) RC column for 25 mm loading eccentricity.
5.7. Comparison of Analysis Results for Example RC Column

Assuming the efficiency of the modelling at this stage of the study, the results of the analysis done by Program P-M and Program LL can be used to explain the effect of FRP confinement on the Moment-Curvature relationships, P-M diagrams, and the loading lines.

5.7.1. Effect of FRP Confinement on the Moment-Curvature Relationship

The FRP confinement in only hoop direction is predicted to yield higher bending moment values than unconfined RC section for the same level of curvature values whereas the bending moment values seem to be unaffected for the lower axial load levels for the same curvature values. The introduction of vertical strips together with the FRP wrapping in the hoop direction yielded higher bending moment values for the same level of curvature values. Moreover, the ultimate curvature values are calculated to be lower than FRP confined (in hoop direction) section. Thus, to increase bending moment capacity for lower axial load levels and to limit the excessive curvature values due to FRP wrapping in hoop direction, vertical FRP strips can be used. A similar result was observed and reported for the effect of vertical FRP strips on the behaviour of RC columns by Hadi and Yazici (2011).
5.7.2. Effect of FRP Confinement on the Axial Load-Bending Moment Interaction Diagrams

The calculated P-M diagrams for the given cross section for unconfined and FRP confinement configurations in Figure 5.26 show that, compared to unconfined P-M diagram FRP confined (hoop direction) cross-section provides higher axial load and bending moment capacities in the compression failure zone while providing similar results for the tension failure zone. The calculated balanced failure points are also plotted on the corresponding P-M diagram in dark coloured points in Figure 5.26. It can be concluded that the confinement in the hoop direction is effective mainly in compression failure zone. However, the vertical FRP strips affected the predicted behaviour of FRP confined RC column’s P-M diagram in the tension zone as well. The axial load values corresponding to balanced failure case seem unaffected regardless of the application of FRP confinement.

![Figure 5.26. P-M diagrams for the example cross-section.](image)

5.7.3. Effect of FRP Confinement and Column on the Loading Line of RC Columns

The calculated loading lines for the example RC column imply that the effect of FRP wrapping diminishes as the loading eccentricity increases for a given height. It can also be seen from the loading lines that, the height of the column also affects the efficiency of strengthening by FRP confinement in a negative way by increasing the
secondary moments on the RC columns, thus it should also be taken as a main variable of FRP strengthening of RC columns.

For this specific example RC cross section, the two FRP strengthening configurations, loading eccentricity and column heights resulted in close failure loads and bending moment values. However the calculated failure points were within the P-M envelope with a better margin for FRP confinement with vertical strips than FRP confinement in the hoop direction only. In general, both FRP confinement configurations increased the axial load carrying capacities in the compression failure zones, specifically in concentric loading case.

5.8. Summary

In this chapter, an analytical approach to FRP confined RC columns is described. Using the stress-strain model developed in the previous chapter, the moment-curvature relationships for a series of axial load levels and P-M diagrams of a RC cross-section are calculated using Program P-M. The loading lines corresponding to the loading eccentricity and the column height is calculated by Program L-L, which uses a simplified moment magnification method for RC columns.

The following chapter gives the details of the experimental study carried out to validate the analytical model described above. The results of experimental part is compared to analytical predictions and the efficiency of the model is discussed at the end of the next chapter.
Chapter 6: EXPERIMENTAL STUDY

6.1. Introduction
Although the modelling explained in Chapter 5 can do calculations for all combinations of solid and hollow, and FRP confined and unconfined RC columns, in the experimental part of this study, only the hollow RC columns were tested under monotonic compression with 0, 25 and 50 mm eccentricities.

In this chapter, the experimental part of this study is explained and the results are discussed in order to explain the effects of FRP confinement configuration, column height and loading eccentricity on the behaviour of hollow RC columns. Predictions made by the analytical modelling are compared with the experimental results.

6.2. Experimental Design
The experimental part of this study is designed to show the effects of FRP wrapping configurations on the hollow RC columns of different heights and axial loading eccentricities. For this purpose, 18 hollow RC column samples were cast and tested.

Figure 6.1. Typical geometry and cross-section dimensions of sample columns before FRP strengthening.

To see the effect of the column height, the column samples were cast in two different heights. The shorter RC columns, which had a height of 500 mm, were called “Short Columns” and the taller ones which had a height of 885 mm were called “Tall
Columns” (Figure 6.1 (a)). All sample columns had the same cross-sectional geometry before FRP strengthening process (Figure 6.1 (b)). The maximum height of a sample that can be tested in the 500 tonnes Universal testing machine, which was used for axial loading of the column specimens, limited the maximum height of the sample columns to 885 mm. All tests were carried out at the Civil Engineering Laboratory of the University of Wollongong.

The labeling, reinforcement details, geometry of the cross-section and the height of the RC columns tested are summarized in Table 6.1.

<table>
<thead>
<tr>
<th>Column label</th>
<th>Reinforcement type</th>
<th>Height (mm)</th>
<th>Outside Diameter, $D_{out}$ (mm)</th>
<th>Hollow core diameter, $D_{in}$ (mm)</th>
<th>Testing eccentricity (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SU0</td>
<td>Internal steel</td>
<td>500</td>
<td>150.0</td>
<td>56.0</td>
<td>0</td>
</tr>
<tr>
<td>SU25</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>25</td>
</tr>
<tr>
<td>SU50</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>50</td>
</tr>
<tr>
<td>SF0</td>
<td>Internal steel+ FRP wrapping in hoop direction</td>
<td>500</td>
<td>150.0</td>
<td>56.0</td>
<td>0</td>
</tr>
<tr>
<td>SF25</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>25</td>
</tr>
<tr>
<td>SF50</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>50</td>
</tr>
<tr>
<td>SFV0</td>
<td>Internal steel+ FRP wrapping in hoop and longitudinal direction</td>
<td>500</td>
<td>150.0</td>
<td>56.0</td>
<td>0</td>
</tr>
<tr>
<td>SFV25</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>25</td>
</tr>
<tr>
<td>SFV50</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>50</td>
</tr>
<tr>
<td>TU0</td>
<td>Internal steel</td>
<td>885</td>
<td>150.0</td>
<td>56.0</td>
<td>0</td>
</tr>
<tr>
<td>TU25</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>25</td>
</tr>
<tr>
<td>TU50</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>50</td>
</tr>
<tr>
<td>TF0</td>
<td>Internal steel+ FRP wrapping in hoop direction</td>
<td>885</td>
<td>150.0</td>
<td>56.0</td>
<td>0</td>
</tr>
<tr>
<td>TF25</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>25</td>
</tr>
<tr>
<td>TF50</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>50</td>
</tr>
<tr>
<td>TFV0</td>
<td>Internal steel+ FRP wrapping in hoop and longitudinal direction</td>
<td>885</td>
<td>150.0</td>
<td>56.0</td>
<td>0</td>
</tr>
<tr>
<td>TFV25</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>25</td>
</tr>
<tr>
<td>TFV50</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>50</td>
</tr>
</tbody>
</table>

The first letters “S” or “T” in the naming code stands for short column and tall column, respectively. The following letter “U” shows that the column is not wrapped with FRP. The letter and “F” shows that the hollow RC column is wrapped with three layers of unidirectional FRP in the hoop direction only, whereas the symbol “FV” means there are also single layer vertical FRP strips attached to the surface of Hollow RC column as well as three layers of FRP wrapping in the hoop direction.
6.2.1. Materials Used in the Experiments

The materials used to make the tested columns are concrete, reinforcing longitudinal and helical steel bars and unidirectional Carbon FRP sheets. Testing of each group of material is explained below.

6.2.1.1. Concrete Testing

It was aimed to cast the columns with a concrete having a 28 day compressive strength of 80 MPa at the design stage. Three cylindrical specimens with 100 mm diameter and 200 mm height were cast from the same batch of concrete used to cast the column specimens. After the concrete was set, the specimens were taken out of the moulds and immersed into the curing tank to provide the specimens with enough amount of water to continue hydration and gain maximum strength. At the 28th day after the casting, concrete specimens were tested to failure under compressive loading. To prevent premature cracking of concrete specimens under compressive loading, a high strength plaster, Hydrostone, was prepared with a plaster:water ratio of 3.5:1 (specified by producer to give an approximate compressive strength of 80 MPa) and used as capping to prevent premature cracking and to ensure an even transfer of load over the concrete sample loading surface during the test. Before testing average diameter, height and weight of each specimen was measured.

The concrete specimens were tested in the Avery compression testing machine under a constant rate of strain of 17.5%. Avery compression testing machine does not record load-deformation during the test. It only shows the load applied to the specimen at any time during the test and the maximum load applied to the specimen up to that time. Thus, at the time of failure, the load suddenly drops with the brittle failure of the specimen but the gauge stays at the maximum load applied to that specimen during the test. Since the geometry of the specimen is already known, the compressive strength of the concrete was found simply by dividing the maximum load to the cross sectional area of the cylindrical specimen.

Results of seven day and 28 day compressive strength tests are given in Table 6.2 below. As can be seen from the results, the 28 day concrete strength came out to be slightly lower than was intended in the design stage. Thus, the $f'_{co}$ value is measured to be equal to 76.5 MPa and the corresponding axial strain value is taken as 0.002.
Table 6.2. 28th day concrete testing results.

<table>
<thead>
<tr>
<th>Sample No.</th>
<th>Age (days)</th>
<th>Measured Average Diameter (mm)</th>
<th>Measured Height (mm)</th>
<th>Mass (g)</th>
<th>Cap</th>
<th>Load (kN)</th>
<th>Compressive Strength (MPa)</th>
<th>Average Compressive Strength (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>28</td>
<td>100</td>
<td>200</td>
<td>3850.6</td>
<td>HSP*</td>
<td>585</td>
<td>74.5</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>28</td>
<td>101</td>
<td>200</td>
<td>3715.9</td>
<td>HSP*</td>
<td>608</td>
<td>75.9</td>
<td>76.5</td>
</tr>
<tr>
<td>3</td>
<td>28</td>
<td>100</td>
<td>200</td>
<td>3716.9</td>
<td>HSP*</td>
<td>621</td>
<td>79.1</td>
<td></td>
</tr>
</tbody>
</table>

* HSP: High Strength Plaster.

6.2.1.2. Tensile Testing for Reinforcing Steel Bars

300 mm length samples were taken from N12 longitudinal steel reinforcement bars, and R6 helical reinforcement bars, and tested under tensile load to failure. The testing was done with the Instron 8033 tensile testing machine located in the Civil Engineering Laboratory at the University of Wollongong. Tensile load applied to the steel bars was monotonically increased up to the failure. The load applied (in N) and corresponding deformation (mm) in the specimens were recorded via an electronic data acquisition system connected to the Instron 8033 tensile testing machine.

Three samples of 6 mm diameter plain bar and 12 mm diameter deformed bar were tested under tensile loading. The summary of the tensile test results are displayed in Table 6.3. Tensile stress-strain graphs of 6 mm diameter plain steel bar and 12 mm diameter deformed steel bar are shown in Figure 6.2 and Figure 6.3, respectively.

Table 6.3. Summary of steel reinforcement tensile testing results.

<table>
<thead>
<tr>
<th>Sample Code</th>
<th>Diameter (mm)</th>
<th>Yield Load (N)</th>
<th>Yield Strength (MPa)</th>
<th>Average (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>R6-1</td>
<td>6</td>
<td>13157</td>
<td>465.3</td>
<td></td>
</tr>
<tr>
<td>R6-2</td>
<td>6</td>
<td>13061</td>
<td>461.9</td>
<td>466.1</td>
</tr>
<tr>
<td>R6-3</td>
<td>6</td>
<td>13324</td>
<td>471.2</td>
<td></td>
</tr>
<tr>
<td>N12-1</td>
<td>12</td>
<td>55564</td>
<td>491.3</td>
<td></td>
</tr>
<tr>
<td>N12-2</td>
<td>12</td>
<td>56376</td>
<td>498.5</td>
<td>494</td>
</tr>
<tr>
<td>N12-3</td>
<td>12</td>
<td>55656</td>
<td>492.1</td>
<td></td>
</tr>
</tbody>
</table>
Based on the average results of tensile tests, the yield strength of longitudinal steel bars \( f_{y,\text{long}} \) is 494 MPa and yield strength of the helical steel reinforcement \( f_{y,\text{helix}} \) is 466 MPa.
6.2.1.3. Tensile testing of Carbon FRP sheets

The Carbon FRP sheet, SIKA-Wrap© FRP fabric, was bought as a single layer unidirectional fabric from the supplier company, and its fibre orientation is arranged along its longitudinal axis (Figure 6.4). Multiple layer FRP wrapping was obtained by adhering the layers with appropriate glue material, ie epoxy resin. The Carbon FRP sheets came in 75 mm width continuous rolls.

The FRP sheets were used to wrap the RC columns in three layers in the hoop direction, and the total thickness of the wrapping is assumed to be the number of the wrapping layers times the thickness of a single layer in this study. Thus, it was seen adequate to test single layer of FRP coupons, however these coupons were treated with Epoxy which was used to adhere the FRP wrapping to the surface of the RC columns.

ASTM-D3039 (2006), Standard Test Method for Tensile Properties of Polymer Matrix Composite Materials was used to prepare the samples and to determine the tensile strength of FRP coupons. For unidirectional FRP sheets, ASTM-D3039 (2006) specified the dimensions of the samples as shown in Figure 6.5.
After the preparation of CFRP coupon samples, their actual dimensions were measured, such as width, testing length and thickness. Tensile load was applied by the Instron 8033 tensile testing machine in the Engineering Laboratory at the UOW (Figure 6.6). The average thickness of the coupon samples ($t_{frp}$) was 0.88 mm.

The stress-strain graphs of tested three FRP coupons are shown in Figure 6.7. Though the FRP materials are known to exhibit a linear stress-strain diagrams until failure, the tests results of FRP coupons revealed that, due to testing conditions the
coupons may exhibit non-linear stress-strain paths especially at the higher levels of stress and strain values. The possible sources of different stress-strain path may include stress concentrations due to the gripping effect of the testing machine, local failure of Carbon fibres, and uneven epoxy treatment of coupon samples. However, at the lower level of stress-strain values, difference in stress-strain graphs is negligible. In Figure 6.7, it can be seen that up to 0.01 strain and 300 MPa stress value, all the coupons exhibited a similar elasticity moduli, thus an average value for FRP elasticity modulus \( (E_{frp}) \) of 30000 MPa was adopted in this study. Though the average failure strain \( (\varepsilon_{frp}) \) and stress \( (f_{frp}) \) values for the three coupon tests are calculated as 0.01978 and 588.45 MPa, respectively, the modelling was based on the average elastic modulus \( (E_{frp}) \) of FRP coupons which is taken as the initial common slope of the stress-strain graphs. Thus, in the modelling part, \( E_{frp} \) value is assumed to be equal to 30000 MPa.

![Figure 6.7](image)

**Figure 6.7. Stress-strain graphs of single layer FRP coupons.**

### 6.3. Implementation of Experiments

Before the RC columns were cast, the formworks were prepared and using the same materials tested, the internal steel reinforcement was prepared for each RC column and placed in the formworks before pouring the concrete in the formworks. After the setting of the concrete, the formworks were removed and the RC columns were wrapped with the FRP sheets as described in Table 6.1.
Each part of RC column preparation is described below.

6.3.1. Preparation of Formwork

The main formwork for the hollow RC columns tested in the experimental program was cut out of PVC pipes. For the outer formwork, a PVC pipe with an inner diameter of 150 mm was chosen. For forming the hollow part, a PVC pipe with 56 mm outer diameter was used. The length of the pipe used to form the hollow part (56 mm diameter pipe) was kept 150 mm longer than the RC column height to make it possible to pull the pipe after the setting of the concrete. The length of larger diameter pipe was kept as the same height as the RC column height.

![Figure 6.8. Fixing PVC formwork on the wooden base.](image)

After preparing the PVC formwork, a wooden base was prepared out of plywood and the PVC formwork was fixed on to the wooden base using steel straps and screws as seen in Figure 6.8. All the formwork surfaces to be in contact with concrete were lubricated with white petroleum gel before concrete pouring.

6.3.2. Preparation of Internal Steel Reinforcement

The internal steel reinforcement consisted of a continuous helical steel reinforcement of R6 bar and six N12 longitudinal bars. The length of longitudinal steel bars and the outer diameter of helical steel were arranged in order to achieve 15 mm clear concrete cover from the surface of RC columns.
Helical steel was prepared by forming a coil with 108 mm inner diameter out of 6 m length of R6 bars with 50 mm pitch (Figure 6.9). The coiling of 6 mm bars was done by a local company, Illawarra Springs at Port Kembla, Wollongong.

![Figure 6.9. Coils made of 6 m length bars.](image)

The coils were cut to form the helical reinforcement matching the specified height of the RC columns (Table 6.1) with a 15 mm clear cover from the bottom and the top.

The longitudinal steel bars were cut from N12 steel. For each RC column six bars were cut with the lengths matching the height of the column specified in Table 6.1 with a 15 mm concrete cover from the top and the bottom.

The steel reinforcement was prepared as a cage by putting the helical steel and the longitudinal steel together with steel wires. To measure the axial strains at the mid-height level of the RC columns during the experiments, strain gauges were attached to the middle of four longitudinal steel bars, two at each side of the loading line (Figure 6.10). For the indirect measurement of hoop strains, two strain gauges were attached at mid-height of helical reinforcement one at each side of the loading line as shown in Figure 6.10.
Before the strain gauges were attached, the surfaces of the steel reinforcement were prepared by grinder and sandpaper to achieve a clean and smooth surface. The strain gauges were attached to the surface of steel bars using an industrial grade super-glue. The gauges were sealed by non-corrosive silicon to prevent mechanical damage during concrete pouring (Figure 6.11).

To ensure the clear cover distance 15 mm, small pieces of steel bars (spikes) of 15 mm lengths were cut out from 6 mm diameter steel bar and welded to the bottom of
longitudinal bars and to the sides of helical steel before placing the steel cage into the formwork (Figure 6.12).

Figure 6.12. Steel spikes welded to the steel cage for clear cover.

6.3.3. Casting of Hollow RC columns

After fixing the PVC formworks on the wooden base and placing the steel reinforcement cages, nine short and nine tall columns were cast using the concrete supplied by a local company which was ordered to have 80 MPa 28th day compressive strength. To achieve such a high compressive strength, a superplasticiser was added to the fresh concrete by the supplier which was already prepared with a low water/cement ratio. The concrete was brought to the laboratory by a truck-mixer and carried inside the laboratory using wheelbarrows to where the formworks for the columns were prepared beforehand. The slump value of the concrete was measured to be 180 mm by following the instructions explained in AS1012.3.1 (1998). The achieved average 28th day compressive strength was 76.5 MPa, as mentioned in material tests part above,
The concrete was poured manually into the moulds using scoops and compacted in layers using an immersion vibrator with 22 mm head diameter. When the moulds were completely filled with compacted concrete, the surface was levelled smoothly using a trowel. All concrete residue on the screws which were used to fix the formwork on to the timber base was cleared with a wire brush and water at this stage since it would be very difficult to remove it after the setting of concrete.

After placing of concrete, a wet Hessian was placed over the columns to prevent moisture loss which was essential for sufficient hydration of cement in the concrete. After three days, the outer moulds were removed using the previously cut joints on the PVC pipe. The inner PVC pipes were pulled out of RC columns by means of a hydraulic jack. The sample columns were kept under wet Hessian for 28 days to ensure enough amount of moisture was provided for hydration.

6.3.4. FRP Wrapping of Columns

No FRP wrapping was applied to three of hollow RC columns from both the short and the tall groups, however, the top and bottom parts of unconfined columns were strengthened by a single layer of FRP to prevent premature cracking of concrete during axial compression tests (Figure 6.13). Likewise, three of each short and tall RC column group were wrapped with three layers of FRP sheets in the hoop direction by a wet lay-up process. The last three of each short and tall group of columns were first reinforced with six discrete FRP layers in the vertical direction and three layers of FRP wrapping was applied in the hoop direction. In FRP wrapping in the hoop direction, 100 mm overlapping was applied. No overlapping was applied in the vertical direction. All RC columns after FRP wrapping application is shown in Figure 6.14.
Figure 6.13. Strengthening of the top and the bottom parts of unconfined RC columns to prevent premature failure.

Figure 6.14. RC columns after FRP wrapping process.
FRP sheets were adhered to the columns applying a wet lay-up process with epoxy resin. After forming each layer, the columns were left to develop enough bond strength with the CFRP layer for a couple of hours and the following layers were applied after that. The epoxy resin was prepared by mixing a small amount of resin and hardener in a ratio of 3:1 just before applying to the surface of the column since the hardening of epoxy happens very quickly. More resin was prepared as needed during CFRP wrapping stage.

6.3.5. Preparing Columns for Testing

The sample column labels, dimensions, FRP wrapping configurations and the testing eccentricities are summarized in Table 6.4.

Table 6.4. FRP wrapping configurations and testing conditions for RC columns.

<table>
<thead>
<tr>
<th>Column label</th>
<th>FRP Wrapping Configuration</th>
<th>Height (mm)</th>
<th>Testing eccentricity (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SU0</td>
<td>No FRP wrapping</td>
<td>500</td>
<td>0</td>
</tr>
<tr>
<td>SU25</td>
<td></td>
<td></td>
<td>25</td>
</tr>
<tr>
<td>SU50</td>
<td></td>
<td></td>
<td>50</td>
</tr>
<tr>
<td>SF0</td>
<td>Three layers of FRP wrapping in hoop direction</td>
<td>500</td>
<td>0</td>
</tr>
<tr>
<td>SF25</td>
<td></td>
<td></td>
<td>25</td>
</tr>
<tr>
<td>SF50</td>
<td></td>
<td></td>
<td>50</td>
</tr>
<tr>
<td>SFV0</td>
<td>Six vertical FRP strips and three layers of FRP wrapping in hoop direction</td>
<td>500</td>
<td>0</td>
</tr>
<tr>
<td>SFV25</td>
<td></td>
<td></td>
<td>25</td>
</tr>
<tr>
<td>SFV50</td>
<td></td>
<td></td>
<td>50</td>
</tr>
<tr>
<td>TU0</td>
<td>No FRP wrapping</td>
<td>885</td>
<td>0</td>
</tr>
<tr>
<td>TU25</td>
<td></td>
<td></td>
<td>25</td>
</tr>
<tr>
<td>TU50</td>
<td></td>
<td></td>
<td>50</td>
</tr>
<tr>
<td>TF0</td>
<td>Three layers of FRP wrapping in hoop direction</td>
<td>885</td>
<td>0</td>
</tr>
<tr>
<td>TF25</td>
<td></td>
<td></td>
<td>25</td>
</tr>
<tr>
<td>TF50</td>
<td></td>
<td></td>
<td>50</td>
</tr>
<tr>
<td>TFV0</td>
<td>Six vertical FRP strips and three layers of FRP wrapping in hoop direction</td>
<td>885</td>
<td>0</td>
</tr>
<tr>
<td>TFV25</td>
<td></td>
<td></td>
<td>25</td>
</tr>
<tr>
<td>TFV50</td>
<td></td>
<td></td>
<td>50</td>
</tr>
</tbody>
</table>

All RC columns were tested at the laboratories of the School of Civil, Mining and Environmental Engineering at the University of Wollongong. A 500 Tonne Denison compression machine was used to test the specimens.

The axial compression was applied to the sample columns using special loading heads and knife edges designed by the Author. The engineering drawings of the knife
edges and the loading heads are given in Figure 6.15 and Figure 6.16, respectively (all dimensions are in mm).

The loading heads and knife edges were manufactured in the Workshops of Engineering Faculty of UoW (Figure 6.17, Figure 6.18, Figure 6.19). The loading heads and the knife edges were manufactured from 4140 grade high strength steel with minimum 740 MPa tensile strength and EN26 grade alloy steel with a minimum 820 MPa tensile strength, respectively.

Figure 6.15. Engineering drawings of the knife edge (all units in mm).
Figure 6.16. Engineering drawings of loading heads (all units in mm).

Figure 6.17. Manufacturing of knife edge.
The axial compression load was applied to the RC columns after attaching loading heads on both ends of the column. For 25 mm and 50 mm eccentric loading tests, the axial compression was applied on the loading lines defined by the relevant grooves on the loading heads using the loading knives.

The loading heads were attached to both ends of the column to be tested using high strength plaster.
6.3.6. Data Measurement During Tests

During the axial compression of the hollow RC columns, the external responses of the RC column such as the axial load \( (P) \), axial deflection \( (\Delta) \) and lateral deflection \( (\delta) \) values at the mid-height were automatically measured and recorded every two seconds (Figure 6.20). Beside the external response of the hollow RC column, the readings from the axial and hoop strain values were also measured by the strain gauges attached to the steel reinforcement and recorded at were also recorded every two seconds simultaneously with the external response of the hollow RC column.

The axial load \( (P) \) was measured by the load cell of the 500 tonne Denison Compression Machine whose position is shown with ① in Figure 6.21. Since the upper plate is positioned according to the height of the tested sample using the switch and does not move during the loading, the axial deflection \( (\Delta) \) of the hollow RC column is measured by a LVDT device attached to the lower plate of the Loading machine assuming that the axial deformation of the tested sample would be equal to the displacement of the lower plate which is shown with ② in Figure 6.21.

![Figure 6.20. External measurements during sample column testing.](image-url)
The lateral deformations at the mid-height are measured by a laser displacement gauge as shown in Figure 6.22.

The strain values on the longitudinal and helical steel bars at the mid-height level of the tested specimens (shown with ③ in Figure 6.21) were also measured and recorded by the data acquisition system during the tests.

As illustrated in Figure 6.21, the control of the testing and the data acquisition were conducted by using two different computers. System Control PC was used to control the mode of testing (deformation control was applied for tests), rate of deformation,
to start and stop the tests and record the axial deformation and axial load data during the tests. The Data Acquisition PC was used to gather the data from the deformation and strain gauges, as well as the load cell of the testing machine. It should be noted that, axial deformation and load data are measured and recorded by both computers. The measured data are stored in the data acquisition system and converted to MS Excel spreadsheets after the tests for further analyses.

6.4. Testing of Hollow RC columns
The prepared hollow RC columns were tested under axial compression with 0, 25 and 50 mm loading eccentricities. All tests were conducted under displacement control with 1mm/min displacement rate.

Load-deformation graphs are drawn for each group of RC columns using the measured data during the tests. The Moment-Axial Load interaction diagrams are calculated by the Program P-M and the expected maximum axial load values are calculated by Program L-L using the moment magnification method for each group as described in the previous chapter. The observed maximum axial load values are plotted on the same graph to see the analytical model’s suitability to the experimental results.

The maximum axial compressive strain values measured by the strain gauges attached to the internal longitudinal steel bars are also compared to see the effect of loading eccentricity and the FRP confinement configuration. It was assumed that the readings taken by the strain gauges were close enough to the strain values of the surface of the RC columns. It is to be noted that, all the strain gauges attached to the helical steel reinforcement failed before the maximum axial load level was achieved for all sample columns. Thus, a comparison of hoop strain values cannot be made for the tests.

The test results are presented according to the height and the FRP confinement configurations of the tested columns.

6.4.1. Short Columns
In this part, the experimental results of short hollow RC columns (Groups SU, SF and SFV) are presented together with the predicted load carrying capacities using the model proposed in the previous chapter.
6.4.1.1. Unconfined Short Columns (Group SU)

The unconfined short hollow RC columns were tested under 0, 25 and 50 mm eccentricities. SU0 underwent a concrete failure which took place at the surface and the top of the column. SU25 had a concrete failure due to the bending effect however, the failure took place slightly away from the mid-height level. SU50 failed near the top and the compression side of the column. All the sample columns in Group SU failed due to concrete failure as seen in Figure 6.23.

The measured axial and horizontal deformation values are plotted against the axial load in Figure 6.24. The axial shortening measured by the LVDT device attached to the lower loading plate are assumed to be positive whereas the deformation in the horizontal direction measured by the laser displacement gauge are assumed to be negative. It must be noted that the horizontal deformation values for eccentrically loaded column samples denote the flexural deformation whereas they denote the increase of radius for concentrically (e=0 mm) loaded column. All the measurements were taken at the mid-height level of the tested specimens.
Figure 6.24. Deformations vs. Axial load graph for Group SU.

To be able to make comparisons, the observed values of maximum axial load ($P_{\text{max}}$), corresponding axial ($\Delta_{\text{max}}$) and lateral deformation ($\delta_{\text{max}}$) values of each tested columns in this experimental study is normalised using the corresponding values of the sample column SU0. The normalised values of maximum axial load, corresponding axial and lateral deformations and the axial compressive strain values are given in Table 6.5.

Table 6.5. Normalised results of axial loading tests for Group SU.

<table>
<thead>
<tr>
<th>Sample label</th>
<th>Norm. axial load</th>
<th>Norm. axial deformation</th>
<th>Norm. lateral deformation</th>
<th>Norm. max. compressive strain</th>
</tr>
</thead>
<tbody>
<tr>
<td>SU0</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>SU25</td>
<td>0.55</td>
<td>1.07</td>
<td>2.39</td>
<td>0.52</td>
</tr>
<tr>
<td>SU50</td>
<td>0.34</td>
<td>0.97</td>
<td>5.17</td>
<td>0.60</td>
</tr>
</tbody>
</table>

where $P_{N,\text{max}}$, $\Delta_{N,\text{max}}$, $\delta_{N,\text{max}}$ and $\varepsilon_{N,\text{max}}$ are the normalised maximum axial load, corresponding axial deformation and the lateral deformation values, and the axial compressive strain values respectively, according to the corresponding values of SU0.

As seen in Figure 6.24, the axial load carrying capacity of Group SU columns have dropped considerably as the loading eccentricity increased. The normalised results in Table 6.5 also show that the axial load capacity has dropped as the loading
eccentricity increased. The axial deformation capacity $\Delta_N$ values were not affected significantly as the eccentricity increased, whereas the $\delta_N$ values were observed to increase with the increasing loading eccentricity.

The axial compressive strain values ($\varepsilon_c$) measured by the strain gauges attached to the longitudinal steel bars are also compared by normalising the readings according to the axial strain value of SU0 corresponding to the maximum axial load up to the maximum axial load level. The normalised axial compressive strains are plotted against the normalised axial load ($P_N$) as shown in Figure 6.25.

![Figure 6.25. Normalised compressive strain vs axial load graph for Group SU columns.](image)

For the eccentrically loaded sample columns, the maximum compressive strain value was assumed equal to $\varepsilon_{cu}$ in the modelling study, which is equal to $\varepsilon_{co}$ for unconfined concrete, and $\varepsilon_{cc}$ for FRP confined concrete. However, the test results reveal that, the maximum value of compressive strain corresponding to the maximum load cannot reach $\varepsilon_{co}$ for the eccentrically loaded short columns, and the failure occurs before $\varepsilon_{co}$ value as seen in Figure 6.25.
The Axial load-Moment interaction diagrams calculated by Program P-M using the given geometry and the measured material properties, and the predicted maximum axial load and the corresponding moment values using the Program P-M are plotted on the same graph together with the experimentally observed maximum axial load and the bending moment values (including the secondary moments) on Figure 6.26.

Figure 6.26. Predicted P-M diagram, predicted and observed values of maximum axial load and bending moment values of group SU.

The experimental results for Group SU were observed to be lower than the predicted values for all eccentricities. The difference between the prediction and the observed axial load capacity was less than that of eccentrically loaded column samples. The premature failure of concrete at lower axial strains for the eccentrically loaded columns (as mentioned above) may have been a possible source of prediction error. Another source of error may be the occurrence of failures at different levels other than the expected mid-height failures. At such failures, the measured lateral deformations, and the axial strain values are different from the actual values corresponding to the maximum axial load.

6.4.1.2. Short Columns Wrapped with FRP in the Hoop Direction (Group SF)

The short hollow RC columns wrapped with FRP in the hoop direction (Group SF) were tested under 0, 25 and 50 mm eccentricities. SF0 failed due to FRP confinement failure under compression away from mid-height level. Since the $K_N$ value for the
FRP confinement is calculated as 13.8, the failure of the column and the FRP confinement was simultaneous for SF0 as predicted by the stress-strain model proposed.

All the sample columns in Group SF failed due to FRP and concrete failure away from the mid-height level as seen in Figure 6.27.

![Group SF sample columns after testing.](image1)

Specimen SF25 also failed due to FRP failure close to the top of the column. FRP confinement of SF50 also failed near the top of the column without undergoing a significant flexural deformation.

![Deformations vs. Axial load graph for Group SF.](image2)
The measured axial and horizontal deformation values are plotted against the axial load in Figure 6.28. The normalised values of maximum axial load, corresponding axial and lateral deformations and the axial compressive strain values are given in Table 6.6.

<table>
<thead>
<tr>
<th>Sample label</th>
<th>Norm. axial load $P_{N,\text{max}}$</th>
<th>Norm. axial deformation $\Delta_{N,\text{max}}$</th>
<th>Norm. lateral deformation $\delta_{N,\text{max}}$</th>
<th>Norm. max. compressive strain $\varepsilon_{N,\text{max}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>SU0</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>SF0</td>
<td>1.66</td>
<td>3.74</td>
<td>-1.61</td>
<td>4.39</td>
</tr>
<tr>
<td>SF25</td>
<td>0.80</td>
<td>2.94</td>
<td>15.56</td>
<td>1.44</td>
</tr>
<tr>
<td>SF50</td>
<td>0.34</td>
<td>1.17</td>
<td>2.39</td>
<td>0.36</td>
</tr>
</tbody>
</table>

As seen in Figure 6.28, FRP wrapping in the hoop direction resulted a considerable axial load carrying capacity for concentrically loaded sample column SF0. The increase in the load carrying capacity was 66% compared to SU0. The normalised results in Table 6.6 also show that the axial load capacity has dropped as the loading eccentricity increased. For 50 mm eccentricity, no axial load carrying capacity increase was observed, both SU50 and SF50 could carry only 34% of the axial load carrying capacity of SU0.

The axial deformation corresponding to maximum axial load values ($\Delta_{N,\text{max}}$) were significantly increased compared to SU0, however the amount of increase dropped with the increasing eccentricity.

The lateral deformation corresponding to maximum axial load ($\delta_{N,\text{max}}$) value for SF0 was measured to be negative implying that the column’s surface was getting further away from the laser displacement gauge due to uneven deformations and settlements. The failure was observed to take place away from the mid-height for SF0. SF25 exhibited a significant increase in $\delta_{N,\text{max}}$ compared to SU0 and SU25. The $\delta_{N,\text{max}}$ value of SF50 was measured to be less than both SF0 and SF50, implying a possible unforeseen premature failure of column.

The axial compressive strain readings corresponding until the maximum axial load ($\varepsilon_{c,\text{max}}$) measured by the strain gauges attached to the longitudinal steel bars are also...
compared by normalising the readings according to the axial strain value of SU0 corresponding to the maximum axial load. The normalised axial compressive strains are plotted against the normalised axial load ($P_N$) as shown in Figure 6.29.

![Figure 6.29. Normalised axial strain vs axial load graph for Group SF columns.](image)

The normalised load-axial strain graph of SF50 did not exhibit a relatively flat part compared to the initial steep slope. This situation was interpreted as a premature failure of SF50.

The P-M interaction diagram calculated by Program P-M, predicted maximum axial load and the corresponding moment values using the Program L-L are plotted on the same graph together with the experimentally observed maximum axial load and the bending moment values (including the secondary moments) on Figure 6.30.
The axial load carrying capacity of SF0 was predicted 16% lower than the observed value. For SF25, the axial load carrying capacity was predicted to be only 3.5% lower than the observed maximum axial load whereas the corresponding bending moment value was 27% lower than the observed value. The difference in the predicted and experimental bending moments imply that, for the maximum axial load the lateral deformation is larger than the assumption made by the moment magnification method. Both the predicted axial load capacity and the corresponding bending moment values were larger than the experimentally observed values for SF50.

6.4.1.3. Short Columns Wrapped with FRP in the Hoop and Vertical Direction (Group SFV)

The short hollow RC columns wrapped with FRP in the hoop direction and vertical FRP strips (Group SFV) were tested under 0, 25 and 50 mm eccentricities. The failed sample columns are shown in Figure 6.31.
The measured axial and horizontal deformation values are plotted against the axial load in Figure 6.32. The normalised values of maximum axial load, corresponding axial and lateral deformations and the axial compressive strain values are given in Table 6.6.

SFV0 failed due to FRP confinement failure under compression away from mid-height level close to the top of the column. The failure of the column and the FRP confinement was simultaneous. The axial load capacity of SFV0 was close to that of SF0 which can be interpreted as the vertical strips did not contribute to load carrying capacity under compression significantly (Table 6.7).

SFV25 failed at the mid-height level by FRP rupture after undergoing a large lateral deformation due to the applied eccentricity. The rupture of FRP confinement occurred after the peak load was observed (Figure 6.32). The axial load carrying capacity was the same as that of SF25, thus it can be said that the vertical strips did not contribute to the load carrying capacity of SFV25. However, \( \Delta_{N,\text{max}} \) and \( \delta_{N,\text{max}} \) values were less than those of SF25 for SFV25 due to possible limitation of deformations due to vertical FRP strips. The axial compressive strain value corresponding to the maximum axial load (\( \varepsilon_{N,\text{max}} \)) was measured to be larger than that of SF25, however it must be remembered that the failure had not occurred at mid-height level for SF25.
Figure 6.32. Deformations vs. Axial load graph for Group SFV.

Table 6.7. Normalised results of axial loading tests for Group SFV.

<table>
<thead>
<tr>
<th>Sample label</th>
<th>Norm. axial load</th>
<th>Norm. axial deformation</th>
<th>Norm. lateral deformation</th>
<th>Norm. max. compressive strain</th>
</tr>
</thead>
<tbody>
<tr>
<td>SU0</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>SFV0</td>
<td>1.56</td>
<td>2.54</td>
<td>1.90</td>
<td>1.12</td>
</tr>
<tr>
<td>SFV25</td>
<td>0.80</td>
<td>2.17</td>
<td>13.31</td>
<td>2.35</td>
</tr>
<tr>
<td>SFV50</td>
<td>0.54</td>
<td>2.11</td>
<td>13.77</td>
<td>1.56</td>
</tr>
</tbody>
</table>

Similar to SF50, the failure of SFV50 also took place away from the mid-height level, close to the top of the column. Axial load carrying capacity, corresponding axial and lateral deformations and compressive strain values were measured to be larger than those of SF50 (Table 6.7).

The normalised results in Table 6.7 also show that the axial load capacity has dropped as the loading eccentricity increased just as the previous groups of short columns. Unlike SF50, SFV50 showed a 59% axial load capacity increase compared to SU50.

The normalised axial compressive strain readings until the maximum axial load ($\varepsilon_{c,max}$) measured by the strain gauges attached to the longitudinal steel bars are compared. All the normalised load-axial strain graphs of Group SFV exhibited a
relatively flat part compared to the initial steep slope. SFV0 exhibited a strain reversal at an axial load level of $1.2P_N$, which was interpreted as a possible debonding between the concrete and the longitudinal steel bars where the strain gauges are attached.

The P-M interaction diagram calculated by Program P-M, predicted maximum axial load and the corresponding moment values using the Program LL are plotted on the same graph together with the experimentally observed maximum axial load and the bending moment values (including the secondary moments) on Figure 6.34.

The axial load carrying capacity of SFV0 was predicted 10.7% lower than the observed value of SFV25, the axial load carrying capacity was predicted to be only 3.6% lower than the observed maximum axial load whereas the corresponding bending moment value was 24% lower than the observed value, similar to SF25 results. For SFV50, the prediction of axial load capacity was 9.4% lower than the observed value. The corresponding bending moment for SF50 was predicted to be 20.3% lower than the experimentally observed value.

Figure 6.33. Normalised axial strain vs axial load graph for Group SFV.
Figure 6.34. Predicted P-M diagram, predicted and observed values of maximum axial load and bending moment values of Group SFV.

The results for the eccentric tests of Group SFV implies that, similar to group SF, the model’s prediction of the lateral deformations were lower than the experimental outcomes. However, the axial load carrying capacities were better predicted for eccentric tests of short columns.

6.4.2. Tall Columns

To see the effect of column height on the axial load carrying capacity of hollow columns, group TU, TF and TFV were tested under the same eccentricities and FRP wrapping configurations as the short columns. The height of the tall columns had to be limited to 885 mm because of the maximum height limitation of the testing machine.

The sample columns were tested under 0, 25 and 50 mm eccentricities with axial deformation rate of 1mm/min. applied axial load, axial deformation, lateral deformation and the strain values from the strain gauges were recorded during the experiments.

The experimental results and the predictions for each of the tested column are presented according to the FRP confinement configurations the same way as the short columns.
6.4.2.1. Unconfined Tall Columns (Group TU)

The sample columns in Group TU were tested under 0, 25 and 50 mm axial loading eccentricities. The failure modes and positions were similar to those of Group SU (Figure 6.35). The failed sample columns after testing and axial load-deformation graphs of the tests are shown in Figure 6.35 and Figure 6.36, respectively.

TU0 failed due to concrete cover spalling and longitudinal steel bar buckling. The normalised summary of results are presented in Table 6.8. Axial load carrying capacity of TU0 was slightly smaller than SU0. The axial compressive strain value corresponding to the axial load carrying capacity is 52% higher than that of SU0 whereas the corresponding axial deformation is 16% higher than that of SU0, which can be expected due to the greater height of TU0. Lateral deformation corresponding to the axial load capacity is measured to be only 6% of the lateral deformation value of SU0 for its axial load capacity, which can be explained with the excessive deformations of the surface of unconfined columns at this level of loading leading to unrealistic readings of lateral deformations by the laser displacement gauge.

TU25 failed due to concrete failure under flexural deformations slightly away from the mid-height (Figure 6.35). The difference in the axial load capacity between TU25 and SU25 was negligible, thus the height of the column did not affect the load.
capacity for the unconfined columns for 25 mm eccentricity. The axial and lateral deformations corresponding to the axial load capacity were measured to have increased compared to those of both SU0 and SU25, but the corresponding compressive strain value of TU25 was lower than those of SU0 and SU25.

Figure 6.36. Deformations vs. Axial load graph for Group TU.

Table 6.8. Normalised results of axial loading tests for Group TU.

<table>
<thead>
<tr>
<th>Sample label</th>
<th>Norm. axial load</th>
<th>Norm. axial deformation</th>
<th>Norm. lateral deformation</th>
<th>Norm. max. compressive strain</th>
</tr>
</thead>
<tbody>
<tr>
<td>SU0</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>TU0</td>
<td>0.96</td>
<td>1.16</td>
<td>0.06</td>
<td>1.52</td>
</tr>
<tr>
<td>TU25</td>
<td>0.56</td>
<td>1.28</td>
<td>6.65</td>
<td>0.28</td>
</tr>
<tr>
<td>TU50</td>
<td>0.26</td>
<td>0.51</td>
<td>6.98</td>
<td>0.22</td>
</tr>
</tbody>
</table>

The axial load capacity, corresponding axial deformation and compressive strain values for TU50 have dropped dramatically compared to those values of SU0 and SU50, which can be due to the combined effect of the increased column height and loading eccentricity.
The normalised compressive strain plotted against the normalised axial strain graph (Figure 6.37) shows that, TU25 and TU50 exhibited no second relatively flat part as SU25 and SU50 did. In fact, the failure of TU25 and TU50 was observed to be abrupt and brittle.

Figure 6.37. Normalised axial strain vs axial load graph for Group TU.

Figure 6.38. Predicted P-M diagram, predicted and observed values of maximum axial load and bending moment values of Group TU.
Similar to group SU, the Program L-L predicted the concentric axial load capacity (for TU0) higher than the experimental value (7% above). The axial load carrying capacities of TU25 and TU50 were also overpredicted by 9.8% and 39.6%, respectively compared to the experimental results.

6.4.2.2. Tall Columns Wrapped with FRP in the Hoop Direction (Group TF)

The sample columns in Group TF were tested under axial compression with 0, 25, and 50 mm loading eccentricities. The failed columns and their axial load-deformation graphs are shown in Figure 6.39 and Figure 6.40, respectively. A summary of test results for group TF is given in Table 6.9.

TF0 failed at the mid-height level and close to the top of the column as a result of FRP rupture (Figure 6.39). The failure took place at the maximum axial load after observing a relatively milder slope than the initial slope of the load-deformation line as predicted by the stress-strain model since $K_N$ was 13.8. The axial load capacity, corresponding axial and lateral deformation and axial compressive strain values were all larger than those of SU0 (Table 6.9).

TF25 failed due to tension in the concrete at the mid-height level. The FRP confinement remained intact after the failure (Figure 6.39). The axial load carrying
capacity of TF25 was quite similar to that of SF25 and SFV25 (0.82 of \( P_{N,\text{max}} \) of SU0). However, \( \Delta_{N,\text{max}}, \delta_{N,\text{max}} \) and \( \varepsilon_{N,\text{max}} \) of TF25 were all larger than those of SU0.

TF50 failed due to concrete failure close to the top of the sample column. The failure was sudden and brittle, and no considerable increase in the load carrying capacity was observed compared to SU0 similar to SF50.

![Figure 6.40. Deformations vs. Axial load graph for Group TF.](image)

Table 6.9. Normalised results of axial loading tests for Group TF.

<table>
<thead>
<tr>
<th>Sample label</th>
<th>Norm. axial load ( P_{N,\text{max}} )</th>
<th>Norm. axial deformation ( \Delta_{N,\text{max}} )</th>
<th>Norm. lateral deformation ( \delta_{N,\text{max}} )</th>
<th>Norm. max. compressive strain ( \varepsilon_{N,\text{max}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>SU0</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>TF0</td>
<td>1.77</td>
<td>6.98</td>
<td>12.61</td>
<td>3.82</td>
</tr>
<tr>
<td>TF25</td>
<td>0.82</td>
<td>2.23</td>
<td>25.58</td>
<td>1.12</td>
</tr>
<tr>
<td>TF50</td>
<td>0.37</td>
<td>0.80</td>
<td>9.33</td>
<td>0.60</td>
</tr>
</tbody>
</table>

When the normalised axial compressive strain values \( \varepsilon_{N} \) of Group TF are compared, it is seen that as the eccentricity of loading increased, the maximum strain values \( \varepsilon_{N,\text{max}} \) are decreased (Figure 6.41). It can also be concluded that with the
increasing eccentricity, the sample columns in Group TF behaved more brittle, that is the column failed at smaller strain levels of $\varepsilon_{N,max}$ with a small or no milder second slope part in the load-strain graph.

Figure 6.41. Normalised axial strain vs axial load graph for Group TF.

Figure 6.42. Predicted P-M diagram, predicted and observed values of maximum axial load and bending moment values of Group TF.
The prediction of axial load capacities were 21.1% and 14.5% lower for TF0 and TF25, respectively. The predicted axial load capacity was 11.8% higher than the experimental result for TF50. Though TF25 showed a larger lateral deformation than the Program L-L’s prediction (resulting in larger bending moment capacity than predicted), TF50 exhibited a smaller lateral deformation capacity than the prediction (Figure 6.42).

6.4.2.3. Tall Columns Wrapped with FRP in the Hoop and Vertical Directions (Group TFV)

The sample columns in group TFV were tested under axial compression loading with 0, 25 and 50 mm eccentricities. The failed columns and their axial load-deformation graphs are shown in Figure 6.43 and Figure 6.44, respectively. A summary of the test results for Group TFV is given in Table 6.10.

TFV0 was failed due to FRP rupture close to mid-height of the column (Figure 6.43). The failure took place after observing a milder slope in the load-axial deformation graph (Figure 6.44), at the maximum load level. The $P_{N,\text{max}}$, $\Delta_{N,\text{max}}$,
and values were all increased considerably for the TFV0 compared to SU0 (Table 6.10). The $P_{N,\text{max}}$ of TFV0 was slightly lower than that of TF0, however $\Delta_{N,\text{max}}$, $\delta_{N,\text{max}}$ and $\epsilon_{N,\text{max}}$ values were substantially lower than those values of TF0.

Figure 6.44. Deformations vs. Axial load graph for Group TFV.

Table 6.10. Normalised results of axial loading tests for Group TFV.

<table>
<thead>
<tr>
<th>Sample label</th>
<th>Norm. axial load $P_{N,\text{max}}$</th>
<th>Norm. axial deformation $\Delta_{N,\text{max}}$</th>
<th>Norm. lateral deformation $\delta_{N,\text{max}}$</th>
<th>Norm. max. compressive strain $\epsilon_{N,\text{max}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>SU0</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>TFV0</td>
<td>1.68</td>
<td>5.23</td>
<td>6.63</td>
<td>1.32</td>
</tr>
<tr>
<td>TFV25</td>
<td>0.83</td>
<td>2.32</td>
<td>16.28</td>
<td>1.64</td>
</tr>
<tr>
<td>TF50</td>
<td>0.48</td>
<td>1.42</td>
<td>19.84</td>
<td>0.64</td>
</tr>
</tbody>
</table>

TFV25 had an axial load capacity similar to the other FRP wrapped sample columns with the same loading eccentricity (83% of the axial load capacity of SU0). The $\Delta_{N,\text{max}}$, $\delta_{N,\text{max}}$ and $\epsilon_{N,\text{max}}$ values of TFV25 were all increased compared to SU0 (Table 6.10).

The axial load capacity of TFV50 was only 48% of that of SU0, however $\Delta_{N,\text{max}}$, $\delta_{N,\text{max}}$ values were substantially increased for TFV50 compared to SU0 (Table 6.10).
All sample columns in Group TFV have exhibited a relatively milder second slope in their $P_N-\varepsilon_N$ graphs (Figure 6.45). The axial load capacity of sample columns were observed to have decreased with the increasing eccentricity. The maximum axial compressive strain value ($\varepsilon_{N,\text{max}}$) for TFV25 was larger than that of TFV0, which was considered to be the effect of longitudinal strips attached.

Figure 6.45. Normalised axial strain vs axial load graph for Group TFV.

Figure 6.46. Predicted P-M diagram, predicted and observed values of maximum axial load and bending moment values of Group TFV.
Program LL have predicted the axial load capacity of sample columns 20.6%, 11.7% and 5.2% lower than the experimental results for TFV0, TFV25 and TFV50, respectively (Figure 6.46). The bending moment values corresponding to $P_{N,\text{max}}$ values of TFV25 and TFV50 were larger than the prediction, similar to sample columns which have demonstrated a milder second slope in their axial load-axial deformation graphs above.

### 6.5. Discussion of Experimental Outcomes

The experimental part of this study was designed to demonstrate the effect of loading eccentricity, FRP wrapping configuration and the column height on the behaviour of hollow RC columns.

The sample columns to investigate the effects of abovementioned factors were prepared and tested according to the details given in Table 6.4.

Table 6.11 gives an overall summary of the experimental results. It should be noted that these results correspond to the maximum axial load observed during the testing of each sample column, and they are normalised according to the results measured for SU0.

<table>
<thead>
<tr>
<th>Sample label</th>
<th>Norm. axial load, $P_{N,\text{max}}$</th>
<th>Norm. axial deformation, $\Delta_{N,\text{max}}$</th>
<th>Norm. lateral deformation, $\delta_{N,\text{max}}$</th>
<th>Norm. max. compressive strain, $\varepsilon_{N,\text{max}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>SU0</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>SU25</td>
<td>0.55</td>
<td>1.07</td>
<td>2.39</td>
<td>0.52</td>
</tr>
<tr>
<td>SU50</td>
<td>0.34</td>
<td>0.97</td>
<td>5.17</td>
<td>0.60</td>
</tr>
<tr>
<td>SF0</td>
<td>1.66</td>
<td>3.74</td>
<td>-1.61</td>
<td>4.39</td>
</tr>
<tr>
<td>SF25</td>
<td>0.80</td>
<td>2.94</td>
<td>15.56</td>
<td>1.44</td>
</tr>
<tr>
<td>SF50</td>
<td>0.34</td>
<td>1.17</td>
<td>2.39</td>
<td>0.36</td>
</tr>
<tr>
<td>SFV0</td>
<td>1.56</td>
<td>2.54</td>
<td>1.90</td>
<td>1.12</td>
</tr>
<tr>
<td>SFV25</td>
<td>0.80</td>
<td>2.17</td>
<td>13.31</td>
<td>2.35</td>
</tr>
<tr>
<td>SFV50</td>
<td>0.54</td>
<td>2.11</td>
<td>13.77</td>
<td>1.56</td>
</tr>
<tr>
<td>TU0</td>
<td>0.96</td>
<td>1.16</td>
<td>0.06</td>
<td>1.52</td>
</tr>
<tr>
<td>TU25</td>
<td>0.56</td>
<td>1.28</td>
<td>6.65</td>
<td>0.28</td>
</tr>
<tr>
<td>TU50</td>
<td>0.26</td>
<td>0.51</td>
<td>6.98</td>
<td>0.22</td>
</tr>
<tr>
<td>TF0</td>
<td>1.77</td>
<td>6.98</td>
<td>12.61</td>
<td>3.82</td>
</tr>
<tr>
<td>TF25</td>
<td>0.82</td>
<td>2.23</td>
<td>25.58</td>
<td>1.12</td>
</tr>
<tr>
<td>TF50</td>
<td>0.37</td>
<td>0.80</td>
<td>9.33</td>
<td>0.60</td>
</tr>
<tr>
<td>TFV0</td>
<td>1.68</td>
<td>5.23</td>
<td>6.63</td>
<td>1.32</td>
</tr>
<tr>
<td>TFV25</td>
<td>0.83</td>
<td>2.32</td>
<td>16.28</td>
<td>1.64</td>
</tr>
<tr>
<td>TFV50</td>
<td>0.48</td>
<td>1.42</td>
<td>19.84</td>
<td>0.64</td>
</tr>
</tbody>
</table>
The testing of the columns lasted about six months from start to finish. The short columns were tested before the tall ones. The loading heads and knives designed by the Author and manufactured in the UoW Engineering Faculty Workshop are used for applying the axial loads to the columns by the 500 Tonne Denison Machine.

6.5.1. Effect of Loading Eccentricity on the Strength and Deformation Capacity of Hollow RC columns

The experimental results of each group show that, the axial load carrying capacity \( P_{N,\text{max}} \) decreased as the loading eccentricity increased. However, for all 50 mm eccentric loading tests, the failure took place away from the mid-height of the sample columns resulting in smaller readings in the deformations and strain readings \( \delta_{N,\text{max}} \) and \( \varepsilon_{N,\text{max}} \), respectively which were taken at the mid-height level.

The sample columns SF50, TU25, TU50 and TF50 were observed to fail close to the top of the column where the concrete cover is. From the \( P_N - \delta_N \) graphs of these sample columns, it can be seen that, there is no second part with a milder slope for these columns which was interpreted as that the concrete cover at the top of the column acted as a weak plane since there is no steel reinforcement in this part.

The axial deformation corresponding to the axial load capacity \( \Delta_{N,\text{max}} \) values also show a tendency to decrease as the eccentricity of loading increased with the exception of SU25 and TU25, whereas the lateral deformations measured at the mid-height level \( \delta_{N,\text{max}} \) values were observed to increase as the eccentricity increased with the exceptions of SF0, SF50 and TF50 the latter two of which were assumed to have failed prematurely as mentioned above.

The axial compressive strain values \( \varepsilon_{N,\text{max}} \) corresponding to the axial load capacity of sample columns were observed to be getting smaller as the loading eccentricity increased. This condition is worth attention since in the calculation of eccentrically loaded RC columns, it is generally assumed that at the axial load capacity the maximum compressive strain is equal to the strain value corresponding to the (unconfined or confined) compressive strength of the concrete.
6.5.2. Effect of FRP Wrapping on the Strength and Deformation Capacity of Hollow RC Columns

The experimental results summarized in Table 6.11 shows that, FRP wrapping has increased the axial load and deformation capacity of concentrically loaded sample columns. Besides, the concentrically loaded sample columns with vertical FRP strips did not show a considerable increase in the load capacity.

As the eccentricity is increased to 25 mm, the load capacity of columns drops to about 80% of the axial load capacity of SU0 regardless of the FRP wrapping configuration. However, compared to SU25, the axial load capacity is considerably increased (from 55% to 80% of that of SU0, a 45% increase). The deformation characteristics ($\Delta_{N,\text{max}}$, $\delta_{N,\text{max}}$, and $\varepsilon_{N,\text{max}}$) of FRP wrapped columns under 25 mm loading eccentricity were observed to have increased compared to unwrapped sample columns. The vertical FRP strips in SFV25 and TFV25 were observed to have limited the lateral deformations, however the axial load capacities were observed to remain at the same level as SF25 and TF25.

Under 50 mm eccentricity, the effect of FRP wrapping on the axial load capacity was observed to have diminished for SF50 and TF50 as they exhibit axial load capacities close to SU50. The sample columns SFV50 and TFV50 (which have vertical FRP strips) were observed to have a larger axial load capacities than the SF50 and TF50. The FRP wrapping configurations were seen to increase the deformation capacities of sample columns with 50 mm eccentricity considerably.

6.5.3. Effect of Column Height on the Strength and Deformation Capacity of Hollow RC Columns

The effect of column height were considered to be demonstrated by testing sample columns with two different heights. However, the due to the limitation of column height to 885 mm because of the testing machine dimensions, the P-δ effect was limited and the desired effect could not be demonstrated properly. The tall sample columns’ load carrying and deformation capacities were seen to be quite similar to those of short columns. It is concluded that, much taller sample columns with the same cross-section geometry should have been tested. As seen from Table 6.11, the
axial load capacities for the same FRP configurations and eccentricity values look quite similar for short and tall columns.

6.6. Summary
A total of 18 hollow RC columns were cast and tested to demonstrate the effects of variables column height, FRP configuration and loading eccentricity on the behaviour of hollow RC columns.

The axial load-bending moment diagrams and maximum axial load and the corresponding bending moment values were predicted by using the Program P-M and Program L-L, respectively developed in Chapter 5, which utilizes the stress-strain model for FRP wrapped columns developed in Chapter 4.

The model was found to be not suitable to predict the axial load and bending moment values of unconfined sample columns due to premature failures mainly occurring at the tips of the columns due to the lack of enough confinement at those areas.

All sample columns tested under 50 mm eccentricity failed in an unexpected way, either by premature failure, or by failure taking place away from the mid-height resulting in unreliable comparisons with the model predictions.

The axial load capacities were predicted better than the corresponding bending moment values. In the cases where the bending moment values were overestimated, the sample columns were seen to have undergone a premature failure away from the mid-height, close to the ends, whereas in the cases where the bending moment values were underestimated the sample columns were observed to have undergone larger horizontal deformations resulting in larger secondary moments.

In general, the FRP wrapping were shown to have increased the axial load and deformation capacity of hollow RC columns, however the increase in the axial load capacity was observed to diminish with the increasing loading eccentricity.
Chapter 7: DISCUSSION OF RESULTS

7.1. Introduction
The experimental results of this study have shown that the axial load carrying and deformation capacities of hollow RC columns are increased considerably after FRP confinement, however the effect on the axial load capacity diminishes as the loading eccentricity increases.

Prediction of the behaviour of FRP wrapped hollow RC columns were proven quite difficult as the tests on the sample columns exhibited many unexpected factors such as the failure of the columns at locations other than mid-height and existence of a weak plane in the concrete cover at the top or bottom of the sample columns as seen in 50 mm eccentricity tests.

Discussion on the results of experimental and modelling study are presented below.

7.2. Analytical Results vs Experimental Results
The experimental and the modelling study were aimed to demonstrate and predict the behaviour of the isolated hollow RC columns with pin ended connections with a single loading eccentricity and confined by FRP materials. Ideally, these columns were expected to bend symmetrically under a single eccentricity loading at both ends and fail at the mid-height level. However, in reality many unforseen factors such as the non-homogeneous nature of the concrete, existence of weak planes throughout the volume of the sample columns (eg. in the cover concrete as explained above), uneven confinement of the FRP due to workmanship errors, possible errors in applying the eccentricity caused unexpected failures of sample columns. It should be noted that, in the modelling part described in Chapter 5, no material factors were used in order to calculate the maximum expected capacities.

In spite of all the factors mentioned above, the axial load capacities of FRP confined sample columns (SF0, SFV0, TF0 and TFV0) for concentric loading were predicted with a safety margin (Figure 6.30, Figure 6.34, Figure 6.42 and Figure 6.46, respectively).

The analytical model predictions for 25 mm eccentricity for FRP wrapped hollow RC columns (SF25, SFV25, TF25 and TFV25) were found to be closer to the experimental results for axial load capacity, whereas the corresponding bending
moment values were underpredicted. This situation implies that the FRP wrapped sample columns underwent larger lateral deformations than the moment magnification method predicted and the actual loading lines for 25 mm eccentricity went beyond the predicted P-M diagrams until the load carrying capacity is reached as it can be seen in Figure 7.1 for TF25.

![Graph showing comparison of predicted vs. experimental loading lines for TF25.](image)

Figure 7.1. Comparison of predicted vs. experimental loading lines for TF25.

For 50 mm eccentric loading tests, the FRP wrapped columns SF50 and TF50 failed before the analytical model’s predictions which due to failures described as “primitive” at the concrete cover region near the top of the columns. The experimental loading line for TF50 (in Figure 7.2) shows that the column had failed before reaching the axial load and the bending moment values predicted by the model. However, the predicted and experimental loading lines were quite similar before the failure of the sample columns.
For the FRP wrapped columns with vertical FRP strips under 50 mm eccentricity (SFV50 and TFV50), the analytical model predictions for axial load capacities were quite close to the experimental values. However, the corresponding bending moment
were larger than the predicted one. The experimental loading line were seen to be quite similar to the predicted one.

7.3. Overall Suitability of Analytical Model for Prediction Purposes
Due to premature failures taking place in the sample columns, the experimental results and the predictions using the analytical model did not provide a conclusion for the unconfined sample columns and FRP confined sample columns loaded under 50 mm eccentricity.

The predictions for axial load capacity were found suitable for the FRP wrapped hollow RC column samples with 25 mm loading eccentricity.

For 50 mm eccentricity, the proposed model yielded non-conservative results for the sample columns without vertical FRP strips. However, these columns were assumed to have failed prematurely.

The predicted loading lines were found to be quite similar to the experimentally observed ones which means that the lateral deformations of eccentrically loaded FRP confined hollow RC columns can be predicted for the service loads using the moment magnification approach.

7.4. Summary
The effects of FRP wrapping configuration, loading eccentricity and column height on the behaviour of hollow RC columns were investigated in the experimental and modelling parts of this study.

The FRP wrapping were seen to improve the axial load capacity of sample columns considerably for concentric loading. For eccentric loading, the increase of axial load capacity decreases as the eccentricity increases. However, FRP wrapping was observed to increase the deformation capacity of hollow RC columns for all eccentricities.

The moment magnification method which was originally used for the columns made of non-linear materials was seen to be effective in predicting the behaviour of FRP wrapped hollow columns. Thus, the stress-strain model, the Program P-M and Program LL were found to be effective in predicting the behaviour of hollow RC columns wrapped with FRP.
The next chapter concludes this study and gives a summary of the implications of this study together with the possible future studies to develop the experimental and modelling approaches on the subject.
8.1. Introduction
The main objective of this study was to make a contribution to fill the gap of knowledge on the behaviour of FRP wrapped hollow RC columns as no prediction or design guideline addresses to hollow RC column behaviour though their common use.

A unified stress-strain model for both solid and hollow columns were developed by modifying the Richart et al.’s (1928) Equations by using the Normalised Stiffness Approach in Chapter 4. The developed model was further modified to fit both solid and hollow concrete sections confined by FRP. The developed model was validated using a large database formed by the previously reported experimental results in the literature.

Two MS Excel VBA codes were developed to predict the Axial Load-Bending Moment (P-M) interaction diagrams and loading lines of FRP wrapped RC columns, Program P-M and Program LL, respectively in Chapter 5. Since these programs were valid for both solid and hollow columns, they were used for prediction of behaviour of hollow RC columns in this study.

To validate the analytical model which consists of Program P-M and Program LL, an experimental study was designed and implemented in Chapter 6. A total of 18 hollow RC columns with the same steel reinforcement and cross-section geometry were cast and tested to determine the effects of FRP wrapping configurations, loading eccentricity and the column height.

8.2. Conclusions of the Study
Conclusions drawn from the theoretical and experimental parts of this study are summarised below.

1. The unified stress-strain model proposed in Chapter 4 compared well with the test results reported by other researchers for both solid and hollow concrete confined by FRP given in Table 4.1 and Table 4.2. The model gave the maximum compressive stress and the corresponding axial strain value. The model did not give the ultimate stress and the strain values on the descending part, if it exists. However, it was stated
that for $K_n$ values less than 10 the stress strain graph is expected to exhibit an ascending part.

2. The experimental results showed a good comparison to the axial load capacity of sample columns wrapped with FRP and tested under concentric and 25 mm eccentric loading, whereas the corresponding bending moment values were predicted with less accuracy than the axial load capacities.

3. The predicted loading lines for hollow RC columns showed good comparisons for the service loads (before the failure point) which makes the model applicable for low levels of load.

4. For the eccentric loading tests, special loading heads and knives were designed and manufactured in the workshops of the Engineering Faculty of UoW. The eccentric loads were applied to the sample columns using these apparatus. The smaller eccentricity (25 mm) was applied to the columns better than the 50 mm eccentricity since unexpected failures took place as the eccentricity was increased to 50 mm. The clear concrete cover at the top and bottom of the sample columns where there is no steel reinforcement was thought to be the reason of weak planes when these failures occurred for 50 mm eccentricity. It is recommended that the ends of the RC columns be reinforced to prevent premature failures for the future studies involving eccentric testing of FRP wrapped RC columns.

5. For the FRP wrapped columns with vertical FRP strips (SFV50 and TFV50) the axial load capacity increase was larger compared to the ones without vertical FRP strips (SF50 and TF50) for 50 mm eccentric loading. Thus, it can be concluded that, the vertical FRP strips help to increase the axial load capacity for larger eccentricities by delaying the premature failure.

6. In order to demonstrate the effect of column height, taller hollow RC columns were needed, as the tall columns tested behaved similarly as the short ones. However, the dimensions of the testing machine were a limitation for testing larger columns.

7. From the Interaction Diagrams constructed using the analytical model (Figure 5.26) and experimental results presented, it is seen that the effect of FRP confinement on the moment capacity can be ignored and the diagram can be
constructed by calculating the increased axial load capacity based on the confined concrete properties.

8.3. Possible Areas for Future Research
The possible future research areas are suggested as follows:

1. The effect of column height can be investigated by testing taller columns, as the model proposed already gives the predicted loading lines for any given height as demonstrated at the end of Chapter 5 in the numerical example. Testing taller sample columns can be done by using a larger loading machine.

2. The effect of hollow core diameter can be investigated in larger cross-sections. The stress-strain model proposed implies that even the concentrically loaded FRP wrapped columns may exhibit a confined strength of concrete smaller than the unconfined strength of concrete used. The tests results of Modarelli et al. (2005) on the hollow columns also show this problem. Thus, the critical hollow core diameter ratio for FRP wrapping can be described by a series of tests on the hollow RC columns.

3. A similar study to this study can be done on the rectangular cross-sectioned hollow RC columns to see the effects of the same variables used in this study.
REFERENCES

ACI440.2R-02 (2002). "Guide for the design and construction of externally bonded FRP systems for strengthening concrete structures", American Concrete Institute, Farmington Hills.


Engineers Australia (1999). "A report card on the nation's infrastructure: investigating the health of Australia's water systems, roads, railways and bridges", Barton, ACT.


APPENDICES

APPENDIX A: VBA CODE FOR PROGRAM P-M

Sub pmcurvature()
'
' pmcurvature Macro
'Macro recorded 30/03/2011 by Faculty of Engineering
'
'GEOMETRY OF CROSS-SECTION
Dim Dout As Single
Dim Din As Single
Dim cc As Single
Dim Dialong As Single
Dim nlong As Integer
Dim Diahelix As Single
Dim Pitch As Single
Dim stripheight As Integer
Dim numFRPhoop As Integer ' number of FRP layers in hoop direction
Dim numvertFRP As Integer 'number of vertical FRP strips
Dim w As Single 'width of vertical FRP strips

'MATERIAL PROPERTIES

'concrete
Dim fco As Single
Dim eco As Single
' Dim fcoo As Single

Dim fcc As Single
Dim ecc As Single

'FRP single layer properties
Dim ffrp As Single
Dim strainfrp As Single
Dim Efrp As Single
Dim tfrp As Single

'Steel properties
Dim fylong As Single
Dim Eslong As Single
Dim fyhelix As Single
Dim Eshelix As Single

'giving values to variables
Dout = Cells(2, 2).Value
Din = Cells(3, 2).Value
cc = Cells(4, 2).Value
Dialong = Cells(5, 2).Value
nlong = Cells(6, 2).Value
Diahelix = Cells(7, 2).Value
Pitch = Cells(8, 2).Value
stripheight = Cells(9, 2).Value
numFRPhoop = Cells(10, 2).Value
numvertFRP = Cells(11, 2).Value
w = Cells(12, 2).Value
fco = Cells(2, 6).Value  
eco = Cells(3, 6).Value  

'FOLLOWING VALUES ARE FOR A SINGLE LAYER ONLY

ffrp = Cells(4, 6).Value  
strainfrp = Cells(5, 6).Value  
Efrp = ffrp / strainfrp  
tfrp = Cells(7, 6).Value  

'steel material properties

fylong = Cells(8, 6).Value  
Eslong = Cells(9, 6).Value  
fyhelix = Cells(10, 6).Value  
Eshelix = Cells(11, 6).Value  

'CALCULATING VARIABLES

Dim Ac As Single 'total concrete area  
Dim Ast As Single 'total steel area  
Dim Pult As Single 'axial load carrying capacity  
Dim KN As Single  

KN = 2 * numFRPhoop * tfrp * Efrp / (Dout * fco)  

If KN = 0 Then  
fcc = 0.85 * fco  
ecc = 0.0038  
Else  
fcc = (0.033 * KN + 1) * fco * (1 - Din ^ 2 / Dout ^ 2)  
ecc = (0.17 * KN + 1) * eco * (1 - Din ^ 2 / Dout ^ 2)  
End If  

'POSITION OF VERTICAL FRP STRIPS FROM THE CENTERLINE

Dim frpteta() As Single  
Dim vertFRPpos() As Single  
Dim vertFRPpostop() As Single  
Dim vertFRParea() As Single  

ReDim frpteta(numvertFRP) As Single  
ReDim vertFRPpos(numvertFRP) As Single  
ReDim vertFRPpostop(numvertFRP) As Single  
ReDim vertFRParea(numvertFRP) As Single  

For f = 1 To numvertFRP  
vertFRParea(f) = w * tfrp 'her bir vertical FRP stripin alani  
frpteta(f) = 8 * Atn(1) / numvertFRP * (f - 1)  
vertFRPpos(f) = Sin(frpteta(f)) * Dout / 2  
vertFRPpostop(f) = Dout / 2 - vertFRPpos(f)  
Next f  

'POSITION OF STEEL BARS from the centerline of CROSS-SECTION up positive, down negative.

Dim teta() As Single 'positive angle from the centerline, first bar on the center right  
Dim steelpos() As Single 'from centre  
Dim steelpostop() As Single 'from top
Dim steelarea() As Single
ReDim teta(nlong) As Single
ReDim steelpos(nlong) As Single
ReDim steelpostop(nlong) As Single
ReDim steelarea(nlong) As Single

For k = 1 To nlong
  steelarea(k) = 0.25 * 4 * Atn(1) * Dialong ^ 2 'area of each steel bar
  teta(k) = 8 * Atn(1) / nlong * (k - 1) 'angles OK!
  steelpos(k) = Sin(teta(k)) * (Dout / 2 - cc - Diahelix - Dialong / 2)
  steelpostop(k) = Dout / 2 - steelpos(k)
Next k

'POSITIONS OF CONCRETE STRIPS FROM CENTERLINE

Dim Xc() As Single 'strip position from centerline
Dim strippostop() As Single 'strain calculation icin concrete strip positions from the top.
Dim stripwidth() As Single
Dim striparea() As Single
Dim numstrips As Integer
numstrips = Dout / stripheight

ReDim Xc(numstrips) As Single
ReDim strippostop(numstrips) As Single
ReDim stripwidth(numstrips) As Single
ReDim striparea(numstrips) As Single

For j = 1 To numstrips
  Xc(j) = Dout / 2 - (j - 0.5) * stripheight
  strippostop(j) = Dout / 2 - Xc(j)
  If Sqr((Xc(j)) ^ 2) > Din / 2 Then
    stripwidth(j) = 2 * Sqr((Dout / 2) ^ 2 - (Xc(j)) ^ 2)
  Else
    stripwidth(j) = 2 * (Sqr((Dout / 2) ^ 2 - (Xc(j)) ^ 2) - Sqr((Din / 2) ^ 2 - (Xc(j)) ^ 2))
  End If
  striparea(j) = stripheight * stripwidth(j)
Next j

'SECTION CAPACITY CALCULATIONS FOR CONCENTRIC LOAD

Ac = 0.25 * 4 * Atn(1) * (Dout ^ 2 - Din ^ 2)
Ast = nlong * 0.25 * 4 * Atn(1) * Dialong ^ 2
Pult = (fcc * Ac + fylong * Ast) / 1000

'CALCULATING P-M-CURVATURE TABLES FOR 41 LOAD LEVELS
Dim n As Integer
n = ecc / 0.0002 'number of outer concrete strain levels curvature values will be calculated

Dim P(41) As Single
Dim MMAX(41) As Single
Dim dna() As Single
Dim M() As Single
Dim strainP() As Single
Dim curv() As Single
Dim c As Variant 'depth of neutral axis
Dim stripstrain() As Single ' strain level at each concrete strip
Dim stripstress() As Single
Dim stripforce() As Single
Dim stripmoment() As Single
Dim steelstrain() As Single
Dim steelstress() As Single
Dim steelforce() As Single
Dim steelmoment() As Single
Dim vertFRPstrain() As Single
Dim vertFRPstress() As Single
Dim vertFRPforce() As Single
Dim vertFRPmoment() As Single

'declaration of component forces and moments
Dim Fconcrete As Single
Dim Mconcrete As Single
Dim Fsteel As Single
Dim Msteel As Single
Dim Fvertfrp As Single
Dim Mvertfrp As Single

Dim Totalforce As Single 'total force response of the cross-section for the assumed dna
Dim Totalmoment As Single 'total moment response of the cross-section for the assumed dna

ReDim strainP(n) As Single
ReDim dna(41, n) As Single 'for given p(i);for each outer strain value, there is one dna

    For v = 1 To n 'from the first strip to the last one
        strainP(v) = ecc / n * v 'strain value for each strip
    Next v

ReDim M(41, n) As Single
ReDim curv(41, n) As Single

ReDim stripstrain(numstrips) As Single
ReDim stripstress(numstrips) As Single
ReDim stripforce(numstrips) As Single
ReDim stripmoment(numstrips) As Single

ReDim steelstrain(nlong) As Single
ReDim steelstress(nlong) As Single
ReDim steelforce(nlong) As Single
ReDim steelmoment(nlong) As Single

ReDim vertFRPstrain(numvertFRP) As Single
ReDim vertFRPstress(numvertFRP) As Single
ReDim vertFRPforce(numvertFRP) As Single
ReDim vertFRPmoment(numvertFRP) As Single

'MOMENT-CURVATURE CALCULATIONS

For i = 0 To 39
    P(i) = Pult / 40 * (i)
    MMAX(i) = 0 'reset before starting calculations for each P(i)
For A = 1 To n 'number of outer strip strain levels
c = 1 'mm initial value of neutral axis depth at the beginning of each iteration

Do
  c = c + 0.5
  Fconcrete = 0
  Mconcrete = 0
  Fsteel = 0
  Msteel = 0
  Fvertfrp = 0
  Mvertfrp = 0

'FOR CONCRETE
  For t = 1 To numstrips
    stripstrain(t) = strainP(A) - strainP(A) / c * strippostop(t)
    If stripstrain(t) <= 0 Then
      stripstress(t) = 0
    Else
      If stripstrain(t) <= eco Then
        stripstress(t) = fco * (2 * stripstrain(t) / eco - (stripstrain(t) / eco) ^ 2)
      Else
        stripstress(t) = fco + (fcc - fco) / (ecc - eco) * (stripstrain(t) - eco)
      End If
    End If
    stripforce(t) = striparea(t) * stripstress(t) / 1000
    Fconcrete = Fconcrete + stripforce(t)
    stripmoment(t) = stripforce(t) * Xc(t) / 1000
    Mconcrete = Mconcrete + stripmoment(t)
  Next t

'FOR LONGITUDINAL STEEL
  For z = 1 To nlong
    steelstrain(z) = strainP(A) - strainP(A) / c * steelpostop(z)
    If Abs(steelstrain(z)) <= fylong / Elong Then
      steelstress(z) = steelstrain(z) * Elong
    Else
      steelstress(z) = Abs(steelstrain(z)) / steelstrain(z) * fylong
    End If
    steelforce(z) = steelarea(z) * steelstress(z) / 1000
    Fsteel = Fsteel + steelforce(z)
    steelmoment(z) = steelforce(z) * steelpos(z) / 1000
    Msteel = Msteel + steelmoment(z)
  Next z

'FOR VERTICAL FRP STRIPS
  For q = 1 To numvertFRP
    vertFRPstrain(q) = strainP(A) - strainP(A) / c * vertFRPpostop(q)
    If vertFRPstrain(q) > 0 Then
      vertFRPstress(q) = 0 'no compressive strength assumed for vertical frp
    Else
      vertFRPstress(q) = Efrp * vertFRPstrain(q)
    End If
    vertFRPforce(q) = vertFRParea(q) * vertFRPstress(q) / 1000
    Fvertfrp = Fvertfrp + vertFRPforce(q)
    vertFRPmoment(q) = vertFRPforce(q) * vertFRPpos(q) / 1000
    Mvertfrp = Mvertfrp + vertFRPmoment(q)
Next q
Totalforce = Fconcrete + Fsteel + Fvertfrp

Loop Until Abs((P(i) - Totalforce)) <= 10 Or c = 20 * Dout

If c < 20 * Dout Then
  dna(i, A) = c
  curv(i, A) = strainP(A) / dna(i, A)
  M(i, A) = Mconcrete + Msteel + Mvertfrp
Else
  dna(i, A) = 20 * Dout
  curv(i, A) = 0
  M(i, A) = 0
End If

If M(i, A) > MMAX(i) Then
  MMAX(i) = M(i, A)
End If

Cells(1, 15 + 2 * (i + 1)).Value = 0
Cells(1, 16 + 2 * i).Value = 0
Cells(A + 1, 15 + 2 * (i + 1)).Value = Format(M(i, A), "0.00")
Cells(A + 1, 16 + 2 * (i + 1)).Value = curv(i, A)
  Next A

Cells(i + 29, 2).Value = Format(MMAX(i), "0.00")
Cells(i + 29, 1).Value = Format(P(i), "0.00")

Next i

P(40) = Pult
MMAX(40) = 0 ' set by the writer
Cells(69, 2).Value = Format(MMAX(40), "0.00")
Cells(69, 1).Value = Format(P(40), "0.00")

End Sub
APPENDIX B: VBA CODE FOR PROGRAM LL

Sub loadingline()

Range("d29:h69").Select
    Selection.ClearContents

'P-M interaction diagram coordinates

Dim P(41) As Single
Dim MMAX(41) As Single

' cross section Loading line coordinates

Dim e As Single 'eccentricity
Dim PL(41) As Single
Dim ML(41) As Single

'coordinates of loading line
Dim Pload(41) As Single
Dim Mload(41) As Single

Dim mmag(41) As Single
Dim Pc As Single 'Eulers critical buckling load
Dim L As Single 'effective height of the column

'BALANCED MOMENT AND CURVATURE VALUES

Dim Mb As Single
Dim curvb As Single
Dim EIb As Single

Mb = Cells(19, 2).Value
curvb = Cells(19, 4).Value

e = Cells(2, 10).Value
L = Cells(13, 2).Value

EIb = Mb / curvb / 1000 'EI value at the balanced failure point

Pc = (4 * Atn(1)) ^ 2 * EIb / (L / 1000) ^ 2

Dim numCROSS As Integer
Dim numRC As Integer

For i = 1 To 41 '41th values equal to concentric loading (last value of P-M diagram)
    MMAX(i) = Cells(i + 28, 2).Value
    P(i) = Cells(i + 28, 1).Value

'LOADING LINE P-M VALUES

PL(i) = P(i)
ML(i) = P(i) * e / 1000

'RC COLUMN LOADING LINE

Pload(i) = P(i)
mmag(i) = 1 / (1 - Pload(i) / Pc)
Mload(i) = Pload(i) * (e / 1000) * mmag(i) 'moment magnified due to secondary moment effects

Next i
numCROSS = 0 'number of load points within P-M for cross-section or very short column

    Do
    numCROSS = numCROSS + 1
    Loop While MMAX(numCROSS) >= ML(numCROSS)

    For c = 1 To numCROSS - 1
        Cells(c + 28, 5).Value = Format(ML(c), "0.00")
        Cells(c + 28, 4).Value = Format(PL(c), "0.00")
    Next c

    'the last value of Pload before intersecting the P-M curve

numRC = 0 'number of load points within P-M for RC column, slenderness included

    Do
    numRC = numRC + 1
    Loop While MMAX(numRC) >= Mload(numRC)

    For A = 1 To numRC - 1
        Cells(A + 28, 8).Value = Format(Mload(A), "0.00")
        Cells(A + 28, 7).Value = Format(Pload(A), "0.00")
    Next A

End Sub
APPENDIX C: TEST RESULTS

In the attachment.