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Abstract

Einstein first proposed that a river flow can be divided into three parts, corresponding to the banks and its bed, respectively, but he did not explain why the flow is dividable and how to divide the flow, in other words the flow division is only a mathematical treatment to simplify his analysis. Since Einstein's proposition there have been many researches and debates on this topic, many division lines have been proposed, but there is no specially designed experimental research to verify the physical existence of division lines, and these division lines have not been tested against the experimental data. For this purpose, an experiment in a rectangular open channel was conducted to measure whether zero-shear stress exists in an open channel except its existence on the free surface. The measured results reveal that zero-shear stress indeed exists below the free surface, and some proposed equations of division line agree well with the profile of the measured zero-shear line, thus it is clarified that Einstein's hypothesis is not only useful to simplify the mathematical treatment, but also it has the physical basis, i.e., zero-shear division line. As far as the authors know, in the literature, this is the first experimental proof that the division lines indeed exist in channel flows.

Keywords

division line, boundary shear stress, 3-D flow division, reynolds shear stress, secondary currents

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Experimental Study on the Validity of Flow Region Division

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Abstract:

Einstein first proposed that a river flow can be divided into three parts, corresponding to the banks and its bed, respectively, but he did not explain why the flow is dividable and how to divide the flow, in other words the flow division is only a mathematical treatment to simplify his analysis. Since Einstein's proposition there have been many researches and debates on this topic, many division lines have been proposed, but there is no specially designed experimental research to verify the physical existence of division lines, and these division lines have not been tested against the experimental data. For this purpose, an experiment in a rectangular open channel was conducted to measure whether zero-shear stress exists in an open channel except its existence on the free surface. The measured results reveal that zero-shear stress indeed exists below the free surface, and some proposed equations of division line agree well with the profile of the measured zero-shear line, thus it is clarified that Einstein's hypothesis is not only useful to simplify the mathematical treatment, but also it has the physical basis, i.e., zero-shear division line. As far as the authors know, in the literature, this is the first experimental proof that the division lines indeed exist in channel flows.

Keywords: Division line, boundary shear stress, 3-D flow division, Reynolds shear stress, Secondary Currents

1. Introduction

In river and environmental engineering, the method of flow division has been often used to estimate sediment transport, boundary shear stress and to eliminate the sidewall effects. If the flow region can be accurately partitioned, the bed and side-wall shear stress may be separated from the total shear stress (Yang and Tan, 2008). For this reason, partitioning flow is very important in studies of bed-load transport, channel migration of bank erosion. The physical existence of division lines in open channel and closed duct flow is of great interest to a wide range of people.

Leighly (1932) first proposed that a channel could be split into sub-regions by division lines crossing the isovels orthogonally. He pointed out that in the absence of secondary currents, the boundary shear stress acting on the bed must be balanced by the downstream component of the weight of water contained within the bounding orthogonals. Keulegan (1938) also contributed to the early development of this subject. He suggested that the flow region should be partitioned with straight division lines which bisected the base angles. Einstein (1942) probably is the first one who believed that river flows are also dividable, after him the hydraulic radius separation method has been widely used in practical studies and engineering practice. He suggested that the flow region could be divided into three sub-sections, two for sidewalls and the other for the bed. Unfortunately, Einstein and others did not provide any theoretical explanation why the flow region is dividable or how it is divided. Chien and Wan (1999) attempted to explain Einstein's method in terms of the energy transport mechanism, stating that the energy present in each of the flow regions would be transferred as heat at the boundary. This mechanism has received little attention from researchers and academics since its publication.

Since the 1990s, several division line theories have been proposed by Daido (1992), Yang and Lim (1997, 1998, 2002, 2005) and Guo and Julien (2005), which have considerably advanced our understanding of flow partitioning. Although the concept of flow division is widely accepted, there are several differing methods of calculating division lines. Daido (1992) attempted to use the Prandtl-Karman equation to obtain an expression for the division line in a rectangular open channel, developing two different configurations depending on the aspect ratio. Interestingly, the division line obtained by Daido's method produces a concave shaped boundary shear stress distribution on the sidewall which is contradictory to the experimental findings of Knight (1985). Yang and Lim (1997, 1998, 2002, 2005) hypothesised that turbulent energy must be transported through the minimum relative distance. This model divides the flow in a rectangular or trapezoidal channel with linear lines. This theory does not account for the effects of secondary currents and was validated with experimental data. Guo and Julien (2005) developed the first approximation of the division line expression, following this study, Yang and Lim (2006) advanced the theory of Guo and Julien by deriving the second approximation of their division line expression. All proposed methods of partitioning flow proposed are mathematical treatments only and are not based on experimental data, some of them believe that the flow can be divided by straight lines, e.g., Keulegan (1938); Yang and Lim (1997); others use curves to divide the flow region, i.e., Leighly (1932), Chien and Wan (1999), Daido (1992), Guo and Julien (2005), Yang et al. (2005).

Obviously, there is a debate in the research community whether the division line is only a mathematical method or the flow is really dividable from the physical point of view; the debate can be also extended to whether a rectangular and trapezoidal channel is divided by straight line or curves. Researchers and engineers are keen to know which model in the literature can yield the best agreement with the experimental data. The purpose of this study is to experimentally test if rectangular open channel flow can be partitioned, and if so, to determine the physical meaning of the division line. Additionally, it is important to determine if the division lines are linear or curved. In this study, experimentally obtained data recorded by 2D LDA system specifically for this study as well as data from previous research by Melling and Whitelaw (1976) and Tracy (1965) will be analysed as they did not report whether the flow region is dividable from their experiments.

2. Review of existing division lines

The following section provides a brief review of the existing division line theories which have been considered in this study. The division lines proposed by the previous researchers are:

Keulegan Method (KM): Keulegan (1938) proposed that the flow in a polygonal channel should be separated into three areas by division lines bisecting the base angles as shown in Fig. 1. Unfortunately, no theoretical explanation was given.

Daido Method (DM): Daido (1992), performed extensive research in order to derive an equation for the division line in a rectangular open channel flow. Utilising the Prandtl-Karman equation, he proposed that the velocity at the division line could be calculated using the log-law from the bed and side wall as follows

$$\frac{u_1}{\overline{u_{*_b}}} = \frac{1}{\kappa}\ln\left(\frac{9\overline{u_{*_b}}y}{\nu}\right) \quad (1)$$

$$\frac{u_2}{\overline{u_{*_w}}} = \frac{1}{\kappa}\ln\left(\frac{9\overline{u_{*_w}}z}{\nu}\right) \quad (2)$$

where u_1 and u_2 are the velocity valid in the bed and side wall regions, he assumed that the condition of division line should be $u_1 = u_2$. $\overline{u_{*_b}}$ and $\overline{u_{*_w}}$ are the shear velocities on the bed and side walls, respectively, κ is the Karman constant and ν is the kinematic viscosity.

On the division line $u_1 = u_2$ must be satisfied, thus one can obtained the following relationship

$$\alpha = \frac{\overline{u_{*_b}}}{\overline{u_{*_w}}} \quad (3)$$

$$z = \frac{\left(9\overline{u_{*_b}}/\nu\right)^{\alpha}}{9\overline{u_{*_w}}/\nu}\, y^{\alpha} \quad (4)$$

Guo and Julien Method (GJM): Guo and Julien (2005) determined the bed and sidewall shear stresses in a rectangular open channel flow through the solving of the continuity and momentum equations. Conformal mapping was used to partition the flow and the following relationship was derived:

$$\frac{\pi z}{2b} = \tan^{-1}\exp\left(-\frac{\pi y}{b}\right) \quad (5)$$

where b is the channel width.

The GJM considers the control volume partitioned by the curved division line by their first approximation as illustrated in Fig. 1 in which the effect of secondary currents was not taken into account. Yang and Lim (2005), advanced the theory of Guo and Julien by deriving the second approximation of their division line expression, in which they claimed that the effect of secondary currents is included. Thus, the second approximation of their division line is given by:

$$\frac{\pi y}{2b} = \tan^{-1} \exp\left(-\frac{\pi z}{b}\right) - \frac{\pi z}{b} \frac{\exp(-\pi z/b)}{1+\exp(-2\pi z/b)} + \lambda_1 \frac{\pi z}{2b} \exp\left(-\frac{z}{b}\right)\left(1-\frac{z}{2b}\right) \quad (6)$$

<u>*where*</u> $\lambda_1 = \pi/4$.

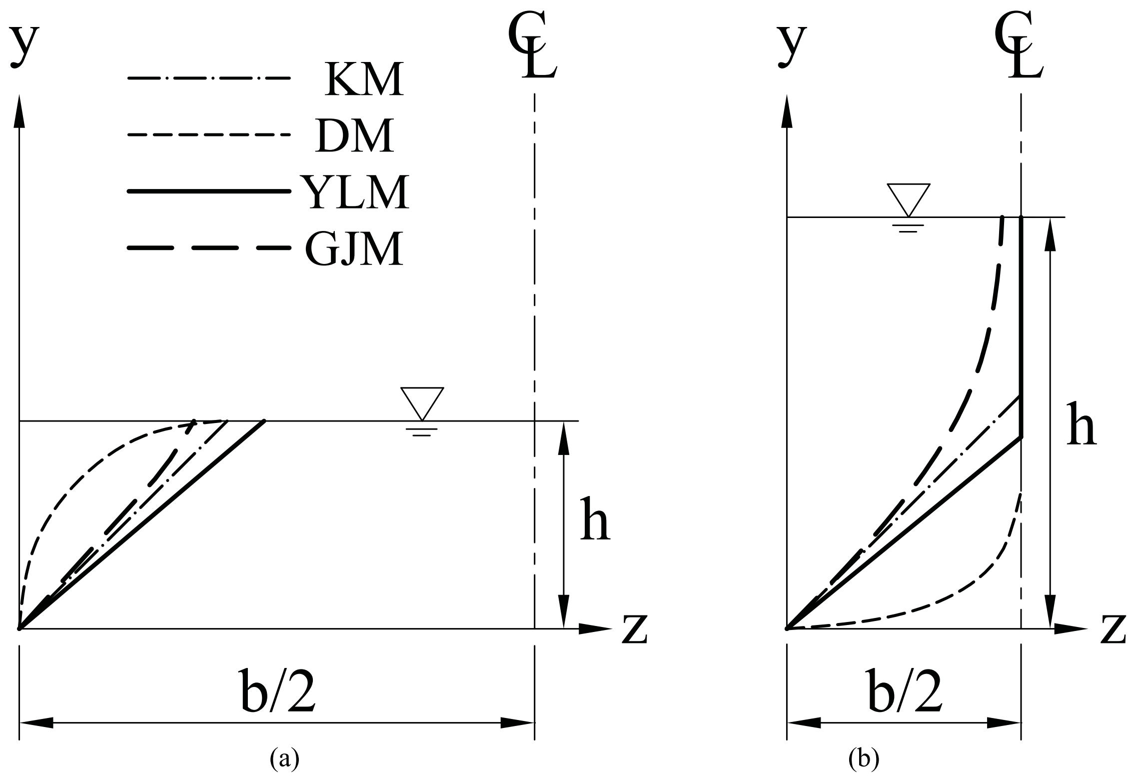

Fig. 1 Division lines as proposed by previous researchers, (a) b/h ≥ α; (b) b/h ≤ α, in which KM stands for Keulegan's method; DM for Daido's method, YLM for Yang and Lim's method, GJM for Guo and Julien.

<u>*Yang and Lim Method (YLM):*</u> Yang and Lim (1997) hypothesised that turbulent energy must be transported through the minimum relative distance. This model divides the flow in a rectangular or trapezoidal channel by linear lines. For a smooth channel as shown in Fig. 1, there are two different configurations depending on the aspect ratio of a rectangular channel. The critical aspect ratio, α, is used to determine if the channel is wide or narrow. As calculated in Yang and Lim (1997), the critical aspect ratio for rectangular open channel flow is 2.

From the minimum relative distance concept it can be assumed that division line will exist at locations where the relative distance to the bed is equal to the relative distance to the wall. In wide open channel flows $(b/h \geq 2)$, the slope of the division line is dependent upon the k term which can be calculated by:

$$k^3 + \left(\frac{2h}{b}\right) k - 2 = 0; \; \frac{b}{h} \geq \alpha \tag{7}$$

where $k = z/y$.

In narrow open channel flows $(b/h \leq 2)$, the slope of the division line is dependent upon the k_1 term which is given by

$$k_1{}^3 + \left(\frac{b}{2h}\right) k_1 - 2 = 0; \; \frac{b}{h} \leq \alpha \tag{8}$$

where $k_1 = y/z$.

3. Physical Meaning of Division Line

The previous section clearly defines several differing methods which can be used to determine the division line locations mathematically. Theoretically, the division line interface should satisfy the condition of zero-shear stress condition, if the flow is dividable, then the shear stress on the interface or division line must be zero. Therefore, the accurate determination of the locations of zero-shear stress in the flow cross-section is very important. Yang (2009), shows that the shear stress on an interface should be expressed by:

$$\tau_{x\boldsymbol{n}} = \mu \frac{\partial u}{\partial \boldsymbol{n}} - \rho u v_{\boldsymbol{n}} - \rho \overline{u'v'_{\boldsymbol{n}}} \tag{9}$$

where $\boldsymbol{n}$ represents the normal direction of the division line, $\rho \overline{u'v'_{\boldsymbol{n}}}$ is the Reynolds shear stress and $uv_{\boldsymbol{n}}$ is the momentum flux caused by secondary currents, and $v_{\boldsymbol{n}}$ is the velocity component of secondary flow normal to the interface. Therefore, the prerequisite condition that flow is dividable as assumed by Einstein and others should be $\tau_{xn} = 0$ along the division line. As the magnitude of streamwise velocity, u, is significantly higher than the vertical velocity, v, small errors in measurement could incur a significant error the analysis. As the 2-D ADV can measure horizontal and vertical velocities, for convenience, along the division line, (9) can be approximately rewritten in the following form:

$$\tau_{xy} = \mu \frac{\partial u}{\partial y} - \rho uv - \rho\overline{u'v'} \qquad (10)$$

If sufficient locations are found within the flow where $\tau_{xy} = 0$, the physical existence of division lines in experimental fluid flow could be verified. It can be seen that there are many assumptions and definitions of the division lines available in the literature, it is needed to conduct some specially designed experiments to validate them.

4. Experimental Setup

Shear stress in a channel flow can be accurately measured using the laser Doppler anemometer (LDA) that measures the fluid velocity based on the random sampling of individual velocity events which occur when particles pass through the measuring volume. In 1964 the LDA was first introduced by Yeh and Cummins, and has since been used comprehensively in investigations of fluid flow. In just under 50 years, the LDA has been developed in to an extremely useful research tool in the field of fluid dynamics. Through the use of LDA, accurate measurements throughout the entire flow region may be obtained (Nezu and Nakagawa 1993). In this study, particle velocities were measured using a Dantec two component Laser Doppler Anemometry (LDA) system. The probe was aligned normal to the x-axis. A scale diagram of the experimental arrangement can be seen in Fig. 2. The experiment was conducted in the hydraulic engineering laboratory, Univ. of Wollongong, the flume is 30cm wide, 40 cm high and 21m long, the vertical profiles were measured from the centreline to a side-wall, and the measuring points in each profile is shown in Fig. 3. The cross section for measurement was located 6m downstream of the channel entrance where a honeycomb was placed to regulate the inflow. A detailed list of experimental conditions is shown in tabulated form in Table 1 below. All experimental measurements were collected in a steady, uniform and fully developed turbulent flow.

5. Experimental Results

The measured dimensionless streamwise velocity (u/U_m) is shown as a contour plot in Fig. 4. The maximum velocity occurs in the centre of the channel and is depressed slightly below the free surface. This phenomenon is referred to as 'velocity dip', and has been linked to the action of secondary currents.

Interestingly, a significant bulge in isovels occurs towards to the bottom corner of the channel. This is known as the 'corner effect', and has been observed in open channel flows by many experimenters, like Nezu et al. (1993). Importantly, the central flow region appears to be free from the sidewall effect and the corner effect.

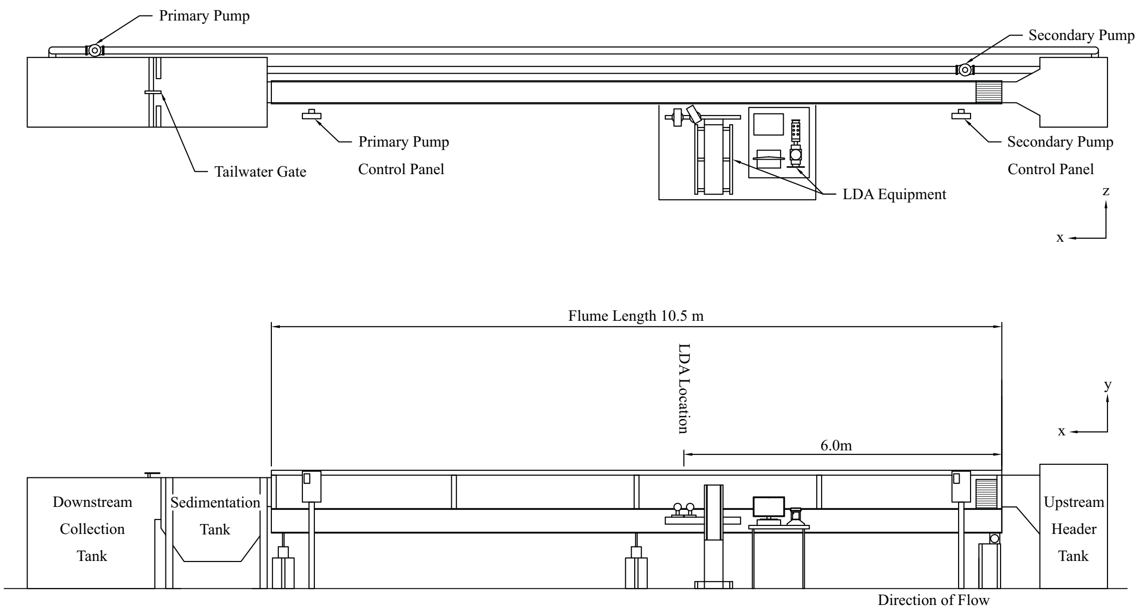

Fig. 2. Experimental arrangement

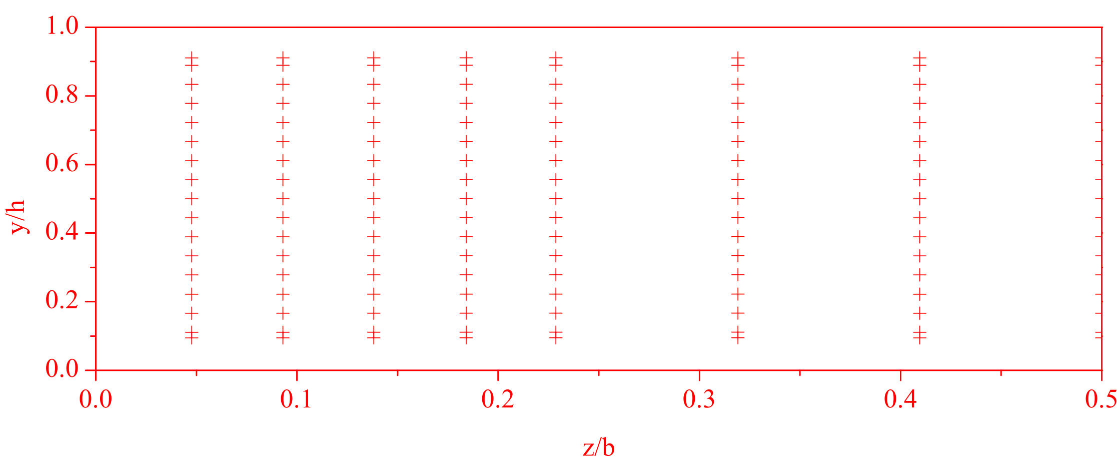

Fig. 3. LDA sampling mesh

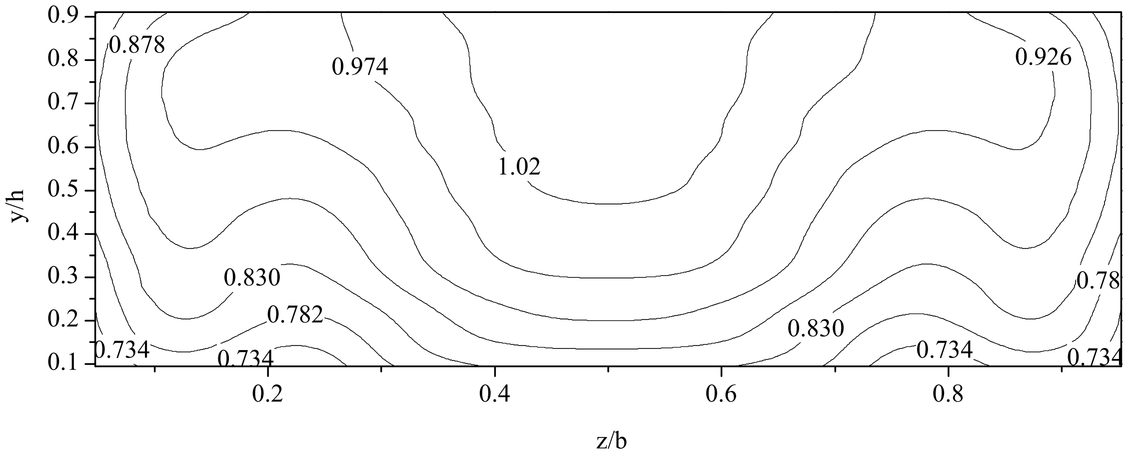

Fig. 4. Dimensionless streamwise velocity contour u/U_m

In this instance, the shear stress at each sample location within the cross section has been calculated with Eq. (10). From these calculations it is evident that there are numerous locations within the cross section where the shear stress is equal to zero, and locations of measured zero-shear are included in Fig. 5 and are represented by solid black squares. These division lines suggested by KM, DM, YLM and GJM are also included in Fig. 5 for comparison. From this figure one may conclude that there exist indeed zero-shear inside the flow domain, thus it proves the idea that flow is dividable proposed by Einstein and others, indicating the physical existence of the division line in the experimental flow. One may also recognize that all existing division lines can approximate the zero-shear line, but it seems that the YLM result yields reasonably good agreement among the existing division lines proposed by the KM, DM and YLM. To validate the conclusion that there generally exist a zero-shear line in a channel flow, it is necessary to analyse the measured shear stress by other researchers.

Melling and Whitelaw (1976). To further validate this physical existence of division lines in rectangular open channel flow, data from research conduction by Melling and Whitelaw (1976) has also been analysed in this study. The experimentation was performed in a straight rectangular duct of uniform cross section and water was chosen as the working fluid. Extensive measurements of instantaneous particle velocities in the streamwise and transverse directions were taken using a laser Doppler anemometry system. Relevant experimental information is shown in Table 1.

The extraction of u, v and $\overline{u'v'}$ data was required so that the locations of zero shear stress could be calculated with Eq. 10. These locations of zero shear stress as well as the proposed division lines from the KM, DM, YLM and GJM are presented in Fig. 5, in which only the results from one quarter of the duct is included. From this figure, the physical existence of division lines is validated further and it can be seen that the physical division line is approximately linear. The locations of zero shear stress do not lie directly on any of the proposed division lines, but are best approximated by the profile obtained from the YLM.

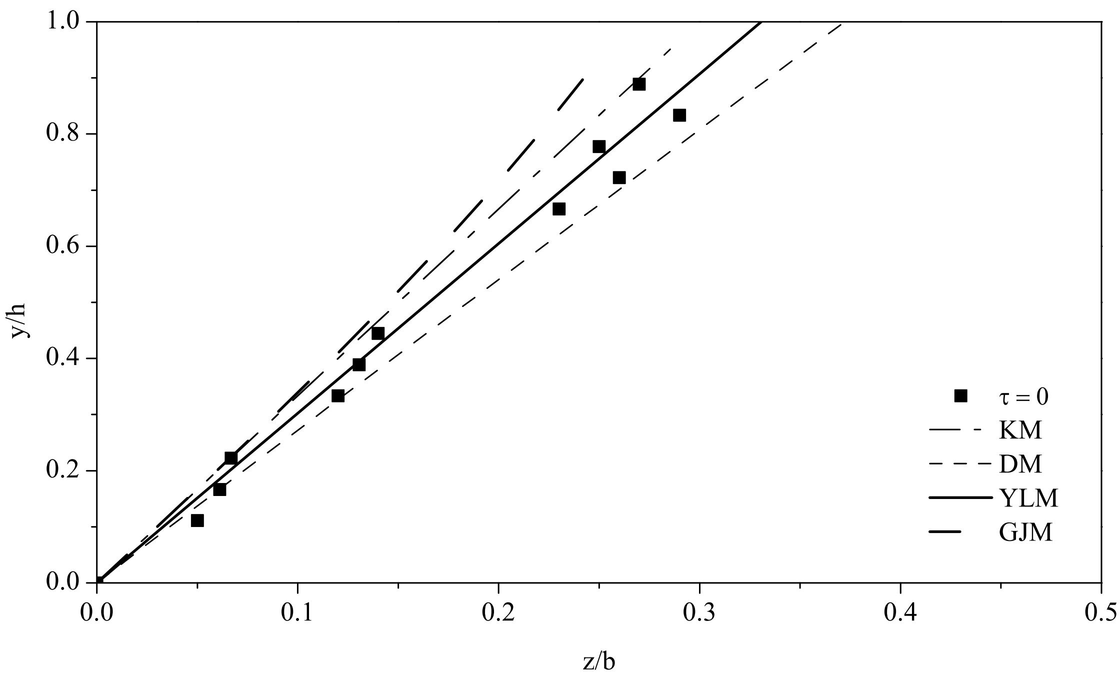

Fig. 5. Division line comparison – Experimental data, in which KM stands for Keulegan's method; DM for Daido's method, YLM for Yang and Lim's method, GJM for Guo and Julien.

Tracy (1965), Data from experimentation by Tracy (1965) has also been analysed in this study. In Tracy's duct flow experimentation, air was supplied to the apparatus location within a wind tunnel, and the velocity was controlled by adjusting the current supplied to the fan. The experimental duct was 5 inches (0.127m) wide and 32 inches (0.812m) high. Turbulence measurements were recorded using a constant-current hot-wire anemometer. Tracy measured the turbulent flow structures including $u, v, w, \overline{u'v'}$and $\overline{u'w'}$ from both the near boundary region and the main flow region. Table 3 outlines the relevant experimental parameters.

Extensive data has been presented in this paper, the most relevant of which being the distribution of $u'v'/u_*^2$ which has been reproduced in Fig. 7. From this plot, the locations of zero Reynolds shear may be easily identified and accurately extracted for comparison in this study.

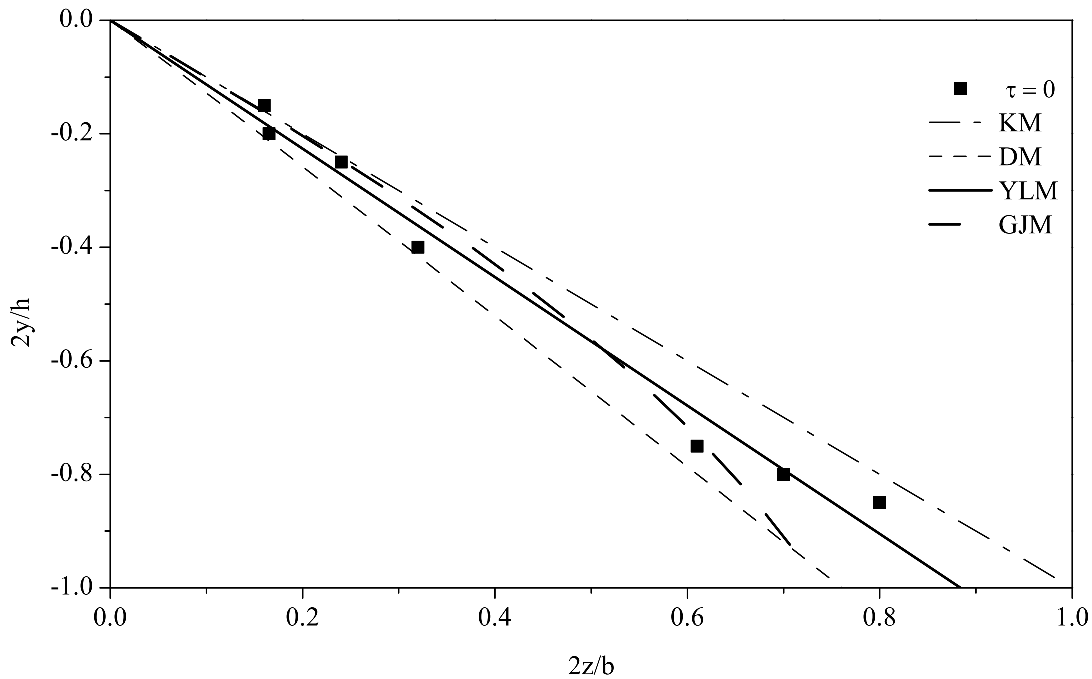

Fig. 6. Division line comparison – Melling and Whitelaw (1976), in which KM stands for Keulegan's method; DM for Daido's method, YLM for Yang and Lim's method, GJM for Guo and Julien.

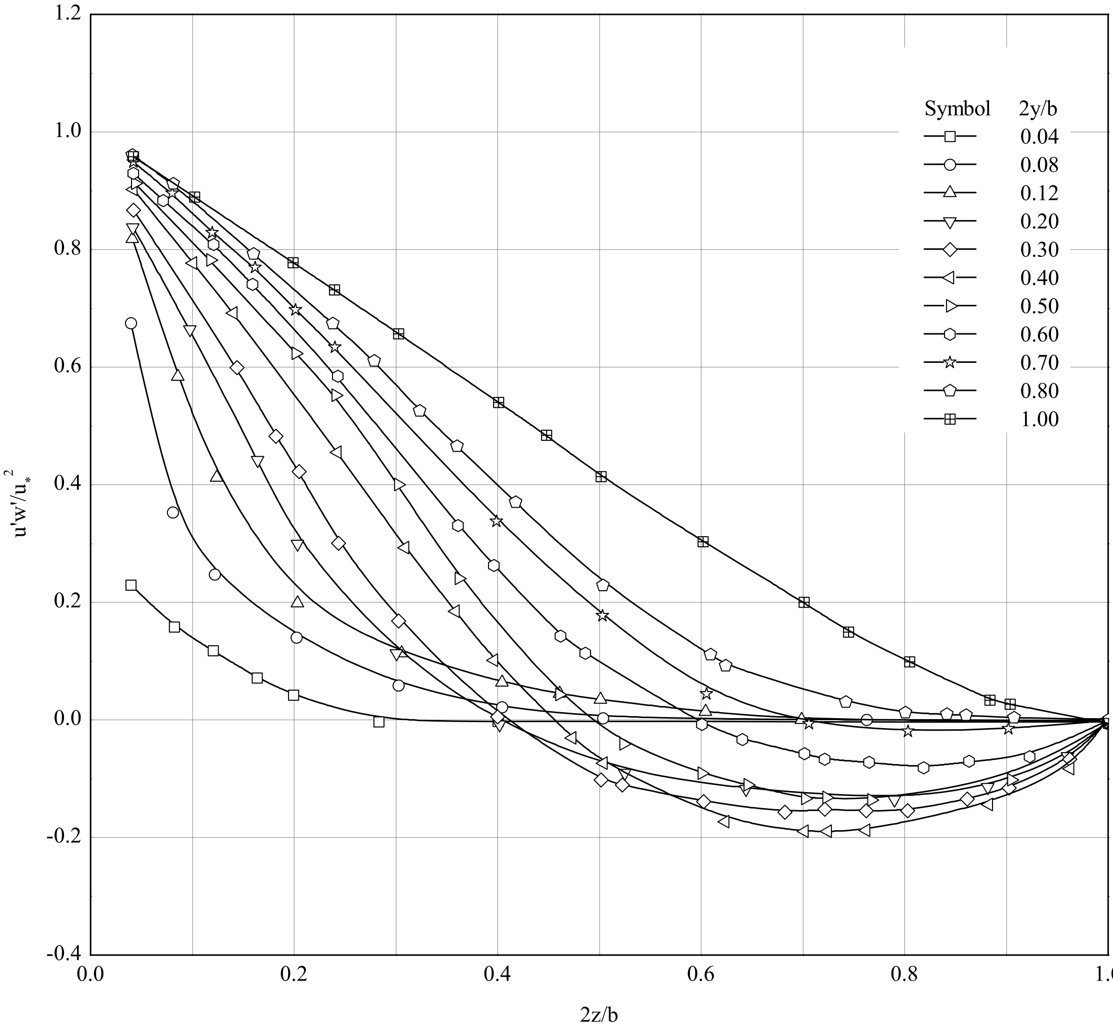

Fig. 7. Distribution of $-u'v'/u_*^2$ – Tracy (1965)

Although these researchers listed above measured the distribution of Reynolds shear stress, but none of them analysed the existence of division lines based on their data, thus it is needed to conduct a comparison of division lines and locations of zero Reynolds shear stress measured by these researchers. Tracy's data is illustrated in Fig. 8. It can be seen that a large number of zero shear stress locations are presented in this figure. Again, one may acknowledge the physical existence of the division line within the flow cross-section. The division line location obtained from the KM gives the most accurate representation of the physical division line present in this experimental data. The division lines obtained from the YLM also provides a good approximate of the physical division line. The GJM division line is very close to the locations of zero shear stress over the outer 0.6 $2z/b$, but deviates significantly in the central flow region.

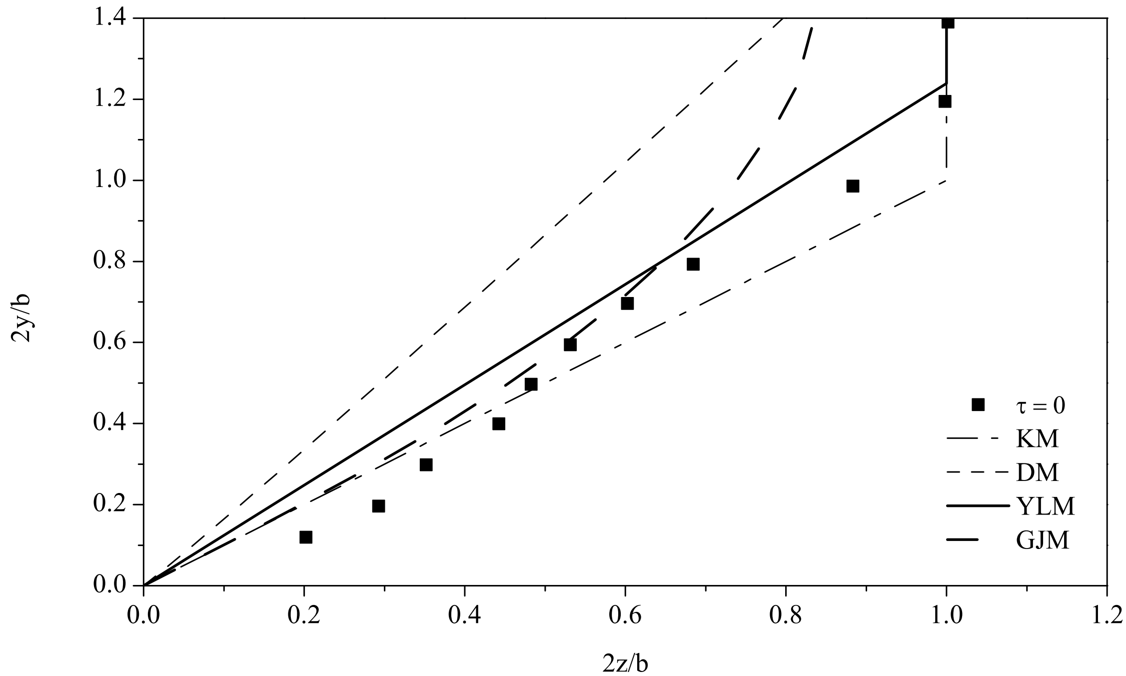

Fig. 8. Division line comparison – Tracy (1965), in which KM stands for Keulegan's method; DM for Daido's method, YLM for Yang and Lim's method, GJM for Guo and Julien.

6. Conclusions

The experimental data recorded at the Hydraulics Laboratory, University of Wollongong for this study as well as the experimental data recorded previously by Melling and Whitelaw (1976) and that of Tracy (1965) have been used to prove the physical existence of division lines and to make a critical comparison of several existing methods of partitioning rectangular flow cross-sections. As far as the authors know, in the literature, this is the first experimental proof that the division lines indeed exist in channel flows.

From this study the following conclusions may be made:

1. Division lines are present and can be physically located in rectangular open channel flow as is clearly shown by all three data sets.
2. Division lines in rectangular open channel flow are approximately linear.

3. This study utilised smooth rectangular open channel flows to make a comparison of the KM, DM, YLM and GJM of determining division line locations. The results of these comparisons suggest the existence of linear division lines in close proximity to those produced by the KM and YLM.

At present the authors are currently extending their research to include an increased number of aspect ratios and both convex and concave channel boundaries. With this increased number of data sets, further comparisons of the division line theories will performed and the conclusions made here will be validated further.

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References

Chien N. and Wan Z. (1999). "Mechanics of sediment transport". , ASCE, Reston, VA.

Daido A. (1992). "Effect of Aspect Ratio of Channel and Sediment Density on Transport Phenomena". Proc. International Symposium on Hydraulic Research in Nature and laboratory, Wuhan, China.

Einstein H.A. (1942). "Formulas for the Transportation of Bed-Load". Transactions ASCE.ASCE, 107, 561-597.

Guo J. and Julien P.Y. (2005). "Shear stress in smooth rectangular open-channel flows". J. Hydr. Eng., ASCE, 131 (1), 30-37.

Keulegan C.H. (1938). "Laws of turbulent flow in open channels". National Bureau of Standards, Research Paper RP1151, 1938, 707-741.

Knight D.W. and Harish S.P. (1985). "Boundary Shear in Smooth Rectangular Ducts". J. Hydraul. Div. ASCE 111(1). 29-47.

Leighly J.B. (1932). "Toward a theory of the morphologic significance of turbulence in the flow of water in streams". University of California Publications in Geography, 6, 1-22.

Melling A. and Whitelaw J.H. (1976). "Turbulent flow in a rectangular duct". J. Fluid Mech., 78(2), 289-315.

Nezu I. and Nakagawa H. (1993). "Turbulence in Open-Channel Flows". IAHR, A.A. Balkema Publishers, Brookfield, USA.

Nezu, I., Tominaga, A., and Nakagawa, H. (1993). ‘‘Field measurements of secondary currents in straight rivers.’’ J. Hydr. Engrg., ASCE, 119 (5), 598–614.

Tracy H.J. (1965). "Turbulent flow in a three-dimensional channel". J. Hydraulics Div., 91(6), 9-35.

Yang S.Q and Lim S.Y. (1998). "Boundary shear stress distributions in smooth rectangular open channel flows". Proc. Instn Civ. Engrs, Wat., Marit & Energy, 130, 163-173.

Yang, S.-Q., Lim, S.-Y., McCorquodale, J.A. (2005). “Investigation of near wall velocity in 3-D smooth channel flows”. Journal of Hydraulic Research 43 (2) , 149-157.

Yang S.Q. and Lim S.Y. (1997). "Mechanism of Energy Transportation and Turbulent Flow in a 3D Channel". J. Hydraulics Eng., 123(8), 684-692.

Yang S.Q. and Lim S.Y. (2002). "Discussion on a geometrical method for computing the distribution of boundary shear stress across irregular straight open channels". J. Hydraulic Research, 40(3), 535-542.

Yang S.Q. and Lim S.Y. (2005). "Boundary shear stress distributions in trapezoidal channels". J. Hydraulic Research, 43(1), 98-102.

Yang, S.Q. and Lim S.Y. (2006). “Discussion on Shear Stress in Smooth Rectangular Open-Channel Flows”. Journal of Hydraulic Engineering, ASCE , 132(6), 629-631.

Yang, S.Q., and Tan, S.K., (2008), “Flow resistance over mobile bed in an open-channel flow”, Journal of Hydraulic Engineering 134 (7) , 937-947.

Yeh, Y. and Cummins, H. Z. (1964). “Localized Fluid Flow Measurements with a He-Ne Laser Spectrometer”, Appl Phys Lett. 4, 176.