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### Abstract

This paper examines a practical tactical liner ship route schedule design problem, which is the determination of the arrival and departure time at each port of call on the ship route. When designing the schedule, the availability of each port in a week, i.e., port time window, is incorporated. As a result, the designed schedule can be applied in practice without or with only minimum revisions. This problem is formulated as a mixed-integer nonlinear nonconvex optimization model. In view of the problem structure, an efficient holistic solution approach is proposed to obtain global optimal solution. The proposed solution method is applied to a trans-Atlantic ship route. The results demonstrate that the port time windows, port handling efficiency, bunker price and unit inventory cost all affect the total cost of a ship route, the optimal number of ships to deploy, and the optimal schedule.

### Keywords

port, time, windows, design, schedule, liner, route, ship

### Disciplines

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# Liner Ship Route Schedule Design with Port Time Windows

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## Abstract

This paper examines a practical tactical liner ship route schedule design problem, which is the determination of the arrival and departure time at each port of call on the ship route. When designing the schedule, the availability of each port in a week, i.e., port time window, is incorporated. As a result, the designed schedule can be applied in practice without or with only minimum revisions. This problem is formulated as a mixed-integer nonlinear non-convex optimization model. In view of the problem structure, an efficient holistic solution approach is proposed to obtain global optimal solution. The proposed

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solution method is applied to a trans-Atlantic ship route. The results demonstrate that the port time windows, port handling efficiency, bunker price and unit inventory cost all affect the total cost of a ship route, the optimal number of ships to deploy, and the optimal schedule.

**Keywords:** container liner shipping; schedule design; containership scheduling; port time windows; mixed-integer nonlinear programming

# 1 Introduction

Liner shipping mainly involves the transportation of containerized cargos (containers) such as manufactured products, food, and garment (Øvstebø et al., 2011). Unlike tramp shipping, liner shipping services have fixed sequences of ports of call and fixed schedules, i.e., arrival and departure times at each port of call (Karlaftis et al., 2009; Norstad et al., 2011; Rakke et al., 2011). Liner services are announced in advance to attract potential customers. For example, Fig. 1 shows a liner service named Atlantic Gulf Mexico Service (AGM) provided by Orient Overseas Container Line (OOCL, 2013). The ports of call and schedule are published in the website of OOCL. Customers can arrange the delivery of their cargo based on the available date of the cargo at the origin port and the expected arrival date at the destination port.

Liner shipping operations are similar to public transport services (Yan et al., 2013, Liu. et al., 2013). Schedule design for a liner service (ship route) is a tactical-level planning decision that is made every three to six months. To design a schedule of a ship route, the first factor to be considered is the service availability of the ports. Since a port needs to provide services for a number of liner shipping companies and a number of ships, it cannot guarantee the availability of services whenever a ship arrives. For instance, a port may be able to provide services on Monday, Tuesday, and Friday, and is fully occupied on Wednesday, Thursday, Saturday, and Sunday. We use the term “port time window” to refer to the time in a week that a port can provide services to ships. Hence, schedule design is subject to the constraint



**Port Rotation**

Westbound: Le Havre - Antwerp - Rotterdam - Bremerhaven - Charleston - Miami - Veracruz - Altamira  
 Eastbound: Veracruz - Altamira - Houston - Miami - Le Havre - Antwerp - Rotterdam - Bremerhaven

Transit Time (Days)					Transit Time (Days)						
Europe to USA/Mexico (Westbound)					Mexico/USA to Europe (Eastbound)						
From	To	Charleston (Mon)	Miami (Wed)	Veracruz (Sun)	Altamira (Tue)	From	To	Le Havre (Sun)	Antwerp (Tue)	Rotterdam (Wed)	Bremerhaven (Fri)
Le Havre (Mon)		15	17	21	23	Veracruz (Mon)		20	22	23	25
Antwerp (Tue)		13	15	19	21	Altamira (Wed)		18	20	21	23
Rotterdam (Thu)		11	13	17	19	Houston (Fri)		16	18	19	21
Bremerhaven (Fri)		10	12	16	18	Miami (Mon)		13	15	16	18

Figure 1: AGM service provided by OOCL (2013)

of port time windows. Moreover, because of the fast growth of container trade and the long time required for the construction/expansion of port capacity, ports tend to be more congested. Another factor for port unavailability is that some ports do not provide services at all times due to social and cultural reasons. As a result, it is important to consider the availability of ports in schedule design. Otherwise the designed schedule may be infeasible in reality.

The design of schedule is also influenced by other factors because different schedules result in different ship costs, bunker costs, and inventory costs. Liner services are usually weekly, which means that the round-trip journey time (weeks) of a ship route is equal to the number of ships deployed on it (Alvarez, 2009). As a result, sailing at a higher speed will reduce the round-trip journey time, thereby the number of ships required and the ship cost. However, a higher speed implies a higher bunker

cost: the daily fuel consumption of ships increases approximately proportional to the sailing speed cubed (Ronen, 2011). At the same time, a higher speed leads to a shorter transit time of containers from origin to destination, and thereby a lower inventory cost (Notteboom, 2006). Consequently, in schedule design a liner shipping company must balance the trade-off between ship cost, bunker cost, and inventory cost, subject to the port time window constraints.

## 1.1 Literature review

According to reviews of Christiansen et al. (2004), Christiansen et al. (2013) and Meng et al. (2014), most studies on liner shipping operations focus on network design, ship deployment, and container routing with fixed schedules or without considering the schedules, e.g., Fagerholt (1999), Shintani et al. (2007), Gelareh and Meng (2010), Meng and Wang (2010), Bell et al. (2011), Meng and Wang (2011), Reinhardt and Pisinger (2012), Dong and Song (2012) and Brouer et al. (2013a). In the scarce literature related to the design of ship schedules, at the tactical level, Mourão et al. (2001) analyzed a small hub-and-spoke network consisting of two routes, i.e., a feed route and a main route, and one origin-to-destination pair of ports, by assuming that all containers must be transshipped at the hub port in the feeder route. They examined the schedules of the two ship routes, and compared each alternative on the basis of the inventory costs of the containers to be shipped. Qi and Song (2012) designed an optimal containership schedule for a liner ship route to minimize the total expected fuel consumption. The time spent at port was treated

as a random variable, and a certain level of service, in terms of the probability that the containership would arrive at a port no later than the published arrival time, had to be maintained. Wang and Meng (2012a) designed a robust schedule for a liner ship route in which uncertainties in port operations and schedule recovery by fast steaming were captured endogenously. Wang et al. (2014) developed a dynamic programming approach to design a schedule for a single ship route with port time windows. However, they assumed that each port on the ship route can only be visited once, whereas in reality many ship routes have ports that are visited twice. Wang and Meng (2011) investigated the schedule design and container routing problem in a general liner shipping network with many ports and many ship routes. However, the sailing speed is not a decision variable. Wang and Meng (2012b) extended the work of Wang and Meng (2011) by incorporating the optimization of speed. Neither Wang and Meng (2011) nor Wang and Meng (2012b) have considered the port time windows.

At the operational level, Yan et al. (2009) developed a container routing model from the perspective of a liner shipping company with the objective of maximizing operating profit while considering the arrival time of ships at ports. They performed a case study utilizing operating data from a major Taiwanese marine shipping company. Brouer et al. (2013b) proposed a vessel schedule recovery problem to evaluate a given disruption scenario and to select a recovery action balancing the tradeoff between increased bunker consumption and the impact on cargo in the remaining network and the customer service level. The model was applied to four real-life cases from Maersk Line and cost savings of up to 58% were achieved by the suggested so-



lutions compared to realized recoveries of the real life cases. It should be mentioned that none of the above studies have taken into consideration the port time windows in schedule design or schedule recovery.

Another category of relevant studies is focused on port operations, e.g., Golias et al. (2010), Du et al. (2011) and Zhen et al. (2011). Both quay-side operations including berth allocation and quay crane assignment and yard-side operations such as yard template planning and yard truck scheduling have been extensively investigated. These models usually assume that each ship has a desired arrival time, and a penalty cost is imposed if the allocated arrival time deviates from the desired one. In other words, the models have actually assumed that the liner shipping schedules are known so that port operations can be modeled and optimized from the viewpoint of port operators.

The above literature review shows that a general liner ship route schedule design with port time windows, where the ship route may visit a port twice, is a new research topic. It incorporates both shipping operations and port operations in the planning decision and hence has practical significance for liner shipping companies.

## **1.2 Objectives and contributions**

The objective of this paper is to address the general liner ship route schedule design problem with port time windows (SDPTW). We assume that a port can be visited at most twice in a week on the ship route, and a ship can only be served by one berth. We design the arrival time at each port of call on a ship route that satisfies the port

time window constraints while minimizing the sum of ship cost, bunker cost, and inventory cost. The designed schedule is feasible in that it takes into account port time windows. The designed schedule is also optimal because the total cost of ships, bunker, and inventory is minimized. Therefore, this problem is of significant value for liner shipping companies.

The contributions of the paper to the state-of-the-art literature and practice are three-folds: first, it takes the initiative to address the practical liner ship route schedule design problem with port time windows where a port can be visited twice. Second, it develops a holistic solution approach that obtains the global optimal solution by taking advantage of the problem structure. Third, a number of interesting managerial insights from case studies are obtained and these managerial insights provide guidelines for liner shipping companies to make planning decisions.

## 2 Problem description

Consider a ship route such as the AGM service in Fig. 1. The port rotation of the ship route has a total of  $N$  ports of call. Define a set  $I := \{1, 2, \dots, N\}$ . We can arbitrarily choose one port of call as the first, and let  $p_i$  represent the physical port of the  $i$ th port of call,  $i \in I$  (Lam and Gu, 2013, Yap and Lam, 2013). For instance, if we let Le Havre be the first port of call, the AGM service can be coded as follows: 1 (Le Havre)  $\rightarrow$  2 (Antwerp)  $\rightarrow$  3 (Rotterdam)  $\rightarrow$  4 (Bremerhaven)  $\rightarrow$  5 (Charleston)  $\rightarrow$  6 (Miami)  $\rightarrow$  7 (Veracruz)  $\rightarrow$  8 (Altamira)  $\rightarrow$  9 (Houston)  $\rightarrow$  10 (Miami)  $\rightarrow$  1 (Le Havre).  $p_6 = p_{10} = \text{Miami}$ . We define the voyage from the  $i$ th

port of call to the  $(i + 1)$ th as leg  $i$ ; leg  $N$  is the voyage from the  $N$ th port of call to the first one. Ships may also transit canals on a voyage leg (Li et al., 2012, Qu and Meng, 2012).

## 2.1 Ship cost, bunker cost and inventory cost

We assume that a string of  $m$  homogeneous containerships are deployed on the ship route to maintain a weekly service frequency, where  $m$  is a decision variable. The highest possible sailing speed of the ships is denoted by  $V^{\max}$  (knot). Represent by  $t_i^{\text{port}}$  the fixed time (day) a ship spends at port of call  $i$ , and  $L_i$  (n mile) the length of leg  $i$ . Let  $v_i$  be the sailing speed (knot) of ships on leg  $i$ .  $v_i$  is a decision variable. To maintain a weekly service frequency, we have the relation:

$$\sum_{i \in I} \frac{L_i}{24v_i} + \sum_{i \in I} t_i^{\text{port}} = 7m \quad (1)$$

In Eq. (1), the left-hand side is the round-trip journey time (day), and the right-hand side is the number of ships times 7 days/week. Denote by  $C^{\text{ship}}$  (USD/week) the fixed operating cost of a ship, including capital cost, manning cost and consumable but not bunker cost. Hence, the weekly operating cost of ships is  $C^{\text{ship}}m$ .

Eq. (1) implies that when the speed is higher, fewer ships need to be deployed to maintain the same weekly service frequency. However, a higher speed implies a larger amount of bunker consumed. To take into consideration the bunker cost, we let  $g_i(v_i)$  (tons/n mile) be the bunker consumption per nautical mile at the speed  $v_i$  on leg  $i$ . Based on the results in existing studies (Bell and Bichou, 2008; Kontovas

and Psaraftis, 2011; Psaraftis and Kontovas, 2010, 2013; Ronen, 2011), we assume that  $g_i(v_i)$  is a power function of the form:

$$g_i(v_i) = a_i(v_i)^{b_i}, i \in I \quad (2)$$

where  $a_i$  and  $b_i$  are two coefficients calibrated from operating data and satisfy  $a_i > 0$  and  $b_i > 1$ . Denote by  $\alpha$  (USD/ton) the bunker fuel price. The weekly bunker cost is  $\alpha \sum_{i \in I} L_i g_i(v_i) = \alpha \sum_{i \in I} L_i a_i (v_i)^{b_i}$ .

Besides the ship cost and bunker cost, the inventory cost of containers should also be incorporated. In fact, a lower speed (slow-steaming) would increase the transit time of containers, and thereby the inventory cost. We let  $\bar{V}_i$  be the number of containers (twenty-foot equivalent units, or TEUs) transported on leg  $i$ , and  $\beta$  be the unit inventory cost (USD per TEU per h). Since the time spent at each port of call is constant, we only consider the inventory cost associated with sailing time at sea (sea time). Therefore, the total inventory cost is  $\sum_{i \in I} \beta \bar{V}_i L_i / v_i$ .

## 2.2 Liner ship route schedule

We use “day” as the unit for liner ship route schedule design as liner shipping companies publish their schedules in terms of days, see Fig. 1. We define the time 00:00 of a certain Sunday as time 0 (day), and hence 00:00 on Monday is time 1, and 00:00 next Tuesday is time 7+2=9. The time of departure  $t_i^{\text{dep}}$  at port  $i$  is

determined by the time of arrival  $t_i^{\text{arr}}$  and the fixed port time  $t_i^{\text{port}}$ , that is:

$$t_i^{\text{dep}} = t_i^{\text{arr}} + t_i^{\text{port}}, i \in I \quad (3)$$

Because of the weekly service frequency, without loss of generality, we let

$$0 \leq t_1^{\text{arr}} \leq 6 \quad (4)$$

Moreover, we define the time when the ship returns to the 1st port of call as  $t_{N+1}^{\text{arr}}$ , that is:

$$t_{N+1}^{\text{arr}} := t_1^{\text{arr}} + 7m \quad (5)$$

The *schedule* of a liner ship route is the vector defined below:

$$(t_i^{\text{arr}}, i \in I; m) \quad (6)$$

In the above schedule, there is the number of ships  $m$  because  $(t_i^{\text{arr}}, i \in I)$  cannot define the inter-arrival time from the last port of call to the first. Of course, the schedule can also be uniquely determined by vector  $(t_i^{\text{arr}}, i \in I; t_{N+1}^{\text{arr}})$ .

Because liner ship routes provide weekly services, to simplify the notation, we define  $W$  to be a set that contains all days in a week, that is,

$$W := \{0, 1, 2, 3, 4, 5, 6\}$$

where 0 represents Sunday, 1 represents Monday, etc.

## 2.3 Port time windows

To account for the availability of ports, we must consider the availability of each berth at each port. This is because some ports are visited twice a week on a ship route, such as the port of Miami on AGM, and a port usually has more than one berth.

To formulate the availability of ports, first, we let  $I_1$  be the set of ports of call, that correspond to ports that are visited only once. If a port is visited twice, supposing that the first visit is the  $j$ th port of call, we use  $j'$  to represent the second visit. We further let  $I_2$  represent all the ports of call that correspond to the first call at a port that is visited twice and  $I'_2$  represent all the ports of call that correspond to the second call at a port that is visited twice. Take the AGM service as an example. We have  $I = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ ,  $I_1 = \{1, 2, 3, 4, 5, 7, 8, 9\}$ ,  $I_2 = \{6\}$ , and  $I'_2 = \{10\}$ . Mathematically, the following relations hold:

$$I'_2 = \{j' \in I | j \in I_2\}$$

$$I = I_1 \cup I_2 \cup I'_2$$

### 2.3.1 Berth time windows

A port may have several berths, and each berth has its own time window. Hence, we let  $B_i$  be the set of berths at the physical port  $p_i$  (the  $i$ th port of call) and the available days in a week at berth  $b \in B_i$  (berth time window) is represented by  $\Omega_i^b$ ,

$\Omega_i^b \subseteq W$ . For instance,  $\Omega_i^b = \{1, 2, 4\}$  means that berth  $b$  at the  $i$ th port of call is free on Monday, Tuesday and Thursday. We further define a parameter  $\delta_{it_i^{\text{arr}}}^{t'}$  that equals 1 if the ship that arrives at port of call  $i$  on day  $t_i^{\text{arr}}$  needs to be served on day  $t'$ ,  $t' \in W$ . For example, if  $t_i^{\text{port}} = 2$ , then we have  $\delta_{i0}^0 = \delta_{i0}^1 = 1$ ,  $\delta_{i0}^2 = 0$  (a ship that arrives on day 0, i.e., Sunday, needs to be berthed on day 0 and day 1);  $\delta_{i8}^0 = 0$ ,  $\delta_{i8}^1 = 1$ ,  $\delta_{i8}^2 = 1$  (a ship that arrives on day 8, i.e., the next Monday, needs to be berthed on Monday and Tuesday). Evidently, if the ship arrives on Sunday and is served by berth  $b \in B_i$ , the berth must be available on both Sunday and Monday. Mathematically, if a ship arrives at port of call  $i$  on day  $t$ , the following is the set of days in a week that the ship needs to be served:

$$\Pi_{it} := \{t' \in W \mid \delta_{it}^{t'} = 1\}$$

In the above example, we have  $\Pi_{i0} = \{0, 1\}$  and  $\Pi_{i8} = \{1, 2\}$ . Not every berth can serve ships at any time because of limited berth time windows. A berth  $b \in B_i$  whose time window is  $\Omega_i^b = \{1, 2, 4\}$  cannot serve the ship if it arrives on day 0 because the berth is not available on Sunday, or  $\Pi_{i0} \not\subseteq \Omega_i^b$ . In sum, a berth  $b \in B_i$  can serve a ship that arrives on day  $t$  only if the following relation holds:

$$\Pi_{it} \subseteq \Omega_i^b$$

Therefore, the set of possible arrival days in a week at berth  $b$  of port of call  $i$  can be written as:

$$\hat{\Omega}_i^b = \{t \in W \mid \Pi_{it} \subseteq \Omega_i^b\}, i \in I, b \in B_i$$

Apparently, if the port time  $t_i^{\text{arr}} = 1$ , there will be more possible arrival days than  $t_i^{\text{arr}} = 2$ .

### 2.3.2 Feasible arrival days at ports and berths

For ports of call in  $I_1$ , we simply let  $\hat{\Omega}_i$  be the set of possible arrival days in a week at the port of call considering all the berths. We have:

$$\hat{\Omega}_i = \bigcup_{b \in B_i} \hat{\Omega}_i^b, i \in I_1$$

For example, suppose that the  $i$ th port of call has five berths and berths 1, 3 and 5 are busy all the time (there is no time windows) and berth 2 has the time window  $\Omega_i^2 = \{1, 2\} \cup \{5\}$  and berth 4 has  $\Omega_i^4 = \{4\}$ . Suppose further that the port time  $t_i^{\text{port}} = 2$ . Hence, the set of possible arrival days in a week at each berth is:

$$\hat{\Omega}_i^1 = \emptyset, \hat{\Omega}_i^2 = \{1\}, \hat{\Omega}_i^3 = \emptyset, \hat{\Omega}_i^4 = \emptyset, \hat{\Omega}_i^5 = \emptyset$$

Therefore, the set of possible arrival days in a week at the port is:

$$\hat{\Omega}_i = \bigcup_{b \in B_i} \hat{\Omega}_i^b = \{1\}$$

We can let  $\Omega_i := \bigcup_{b \in B_i} \Omega_i^b$  be the time window at port of call  $i$ . However, we cannot use  $\Omega_i$  to calculate the set of possible arrival days  $\hat{\Omega}_i$ . For instance, in the above example if we use the combined port time window  $\Omega_i = \{1, 2\} \cup \{4, 5\}$ , we will reach



the wrong conclusion that the ship can arrive either on Monday or on Thursday. In fact, if the ship arrives on Thursday, it has to be moved from berth 4 to berth 2 on Friday. This involves considerable cost and time that prohibit such an operation in practice.

For ports of call  $j \in I_2$ ,  $B_j$  is the set of berths at port of call  $j$  and  $B_{j'}$  is the set of berths that correspond to the second call at the port.  $B_j \equiv B_{j'}$  as  $p_j = p_{j'}$ . Because the port is visited twice, it may be of little value to come up with a set of feasible arrival days similar to  $\hat{\Omega}_i, i \in I_1$ . We have to directly consider the sets  $\hat{\Omega}_j^b$  and  $\hat{\Omega}_{j'}^b, b \in B_j$ .

A berth cannot serve more than one ship at the same time. Suppose that port of call  $j \in I_2$  has only one berth  $b \in B_j$  with  $\Omega_j^b = \{1, 2, 3\}$ . Assume that  $t_j^{\text{port}} = t_{j'}^{\text{port}} = 2$ . Suppose further that the ship uses the berth  $b$  when it arrives at port of call  $j \in I_2$  at time  $t_j^{\text{arr}} = 1$  (Monday) and still uses the berth when it arrives at port of call  $j' \in I_2$  at time  $t_{j'}^{\text{arr}} = 2+7 = 9$  (next Tuesday). Evidently, both arrivals are feasible, because  $t_j^{\text{arr}} \in \hat{\Omega}_j^b$  and  $t_{j'}^{\text{arr}} \in \hat{\Omega}_{j'}^b$ . However, their combination is infeasible because the berth cannot serve two ships on day 2. Mathematically, the combination is infeasible because  $\delta_{j1}^2 = 1$  and  $\delta_{j'9}^2 = 1$ . In sum, if two arrivals  $j$  and  $j'$  use the same berth  $b \in B_j$ , the following relation must hold:  $\delta_{jt_j^{\text{arr}}}^{t'} + \delta_{j't_{j'}^{\text{arr}}}^{t'} \leq 1, \forall t' \in W$ .

### 3 Mathematical model

The ship route schedule design problem with port time windows aims to determine the optimal arrival time and the berth to use at each port of call on a ship route

that satisfies the berth time window constraints to minimize the total cost including ship cost, bunker cost, and inventory cost. Before presenting the model, we list the notation below.

### Variables

$m$	Number of ships deployed on the ship route
$t_i^{\text{arr}}$	Arrival time (day) at the $i$ th port of call
$t_{N+1}^{\text{arr}}$	The time (day) when the ship returns to the 1st port of call
$t_i^{\text{dep}}$	Departure time (day) from the $i$ th port of call
$v_i$	Sailing speed (knot) on leg $i$
$z_j^b$	A binary variable that equals 1 if and only if the ship uses berth $b$ when it arrives at port of call $j \in I_2$ , $b \in B_j$
$z_{j'}^b$	A binary variable that equals 1 if and only if the ship uses berth $b$ when it arrives at port of call $j' \in I'_2$ , $b \in B_{j'}$

### Parameters

$\alpha$	The bunker fuel price (USD/ton)
$\beta$	The unit inventory cost of containers (USD per TEU per h)
$\hat{\Omega}_i$	The set of feasible arrival days in a week at the $i$ th port of call, $i \in I$
$\hat{\Omega}_j^b$	The set of feasible arrival days in a week at berth $b$ at the $j$ th port of call, $j \in I_2$

$\hat{\Omega}_{j'}^b$	The set of feasible arrival days in a week at berth $b$ at the second call at the port $p_j$
$B_j$	The set of berths at port of call $j$
$B_{j'}$	The set of berths that correspond to the second call at the port $p_j$
$C^{\text{ship}}$	The weekly operating cost of a ship (USD/week)
$g_i(v_i)$	The bunker consumption per nautical mile at the speed $v_i$ on leg $i$ (tons/n mile)
$I$	Set of ports of call, $I := \{1, 2, \dots, N\}$
$I_1$	The set of ports of call that correspond to ports that are visited only once
$I_2$	The set of ports of call that correspond to the first call at a port that is visited twice
$I_2'$	The set of ports of call that correspond to the second call at a port that is visited twice
$L_i$	Length (n mile) of the leg $i$
$N$	Number of ports of call on the ship route, $N =  I $
$p_i$	The physical port that corresponds to the $i$ th port of call on the ship route
$t_i^{\text{port}}$	Time (day) a ship spends at port of call $i$
$\bar{V}_i$	Number of containers (TEUs/week) transported on leg $i$
$V^{\text{max}}$	Maximum speed of the ships (knots)

$m^{\max}$  Maximum number of ships deployed on the ship route

$\mathbb{Z}^+$  Set of nonnegative integers

The SDPTW can be formulated as:

$$[\text{SDPTW}] \quad \min C^{\text{ship}} m + \alpha \sum_{i \in I} L_i g_i(v_i) + \sum_{i \in I} \beta \bar{V}_i \frac{L_i}{v_i} \quad (7)$$

subject to:

$$t_i^{\text{dep}} = t_i^{\text{arr}} + t_i^{\text{port}}, i \in I \quad (8)$$

$$0 \leq t_1^{\text{arr}} \leq 6 \quad (9)$$

$$t_{i+1}^{\text{arr}} \geq t_i^{\text{arr}} + t_i^{\text{port}} + \left\lceil \frac{L_i}{24V^{\max}} \right\rceil, i \in I \quad (10)$$

$$t_{N+1}^{\text{arr}} = t_1^{\text{arr}} + 7m \quad (11)$$

$$v_i = \frac{L_i}{24(t_{i+1}^{\text{arr}} - t_i^{\text{dep}})}, i \in I \quad (12)$$

$$0 \leq v_i \leq V^{\max}, i \in I \quad (13)$$

$$m \in \{1, 2, 3, \dots, m^{\max}\} \quad (14)$$

$$t_i^{\text{arr}} \in \mathbb{Z}^+, i \in I \quad (15)$$

$$(t_i^{\text{arr}} \bmod 7) \in \hat{\Omega}_i, i \in I_1 \quad (16)$$

$$z_j^b = 1 \Rightarrow (t_j^{\text{arr}} \bmod 7) \in \hat{\Omega}_j^b, \forall j \in I_2, \forall b \in B_j \quad (17)$$

$$z_{j'}^b = 1 \Rightarrow (t_{j'}^{\text{arr}} \bmod 7) \in \hat{\Omega}_{j'}^b, \forall j' \in I'_2, \forall b \in B_{j'} \quad (18)$$

$$z_j^b z_{j'}^b = 1 \Rightarrow \delta_{jt_j^{\text{arr}}}^{t'} + \delta_{j't_{j'}^{\text{arr}}}^{t'} \leq 1, \forall j \in I_2, \forall b \in B_j, \forall t' \in W \quad (19)$$

$$\sum_{b \in B_j} z_j^b = 1, \forall j \in I_2 \quad (20)$$

$$\sum_{b \in B_{j'}} z_{j'}^b = 1, \forall j' \in I_2' \quad (21)$$

$$z_j^b \in \{0, 1\}, \forall j \in I_2, \forall b \in B_j \quad (22)$$

$$z_{j'}^b \in \{0, 1\}, \forall j' \in I_2', \forall b \in B_{j'} \quad (23)$$

The objective function (7) minimizes the sum of ship cost, bunker cost, and inventory cost. Constraint (8) defines the departure time from each port of call. Constraint (9) eliminates symmetric solutions. Constraint (10) confirms that the sailing speed cannot exceed  $V^{\max}$ . Constraint (11) defines the time when the ship returns to the 1st port of call after one round-trip. Constraint (12) calculates the sailing speed on each leg. Constraint (13) enforces the lower and upper limits on the sailing speed. Constraint (14) indicates that the number of ships is a positive integer. Constraint (15) indicates that the arrival time at each port of call is a nonnegative integer. Constraint (16) imposes the port time window constraints at ports that are visited once. Constraints (17) and (18) are berth time window constraints at the ports that are visited twice. Constraint (19) imposes that a berth cannot serve two ships at the same time. Constraints (20) and (21) require that a ship uses exactly one berth each time it visits a port. Constraints (22) and (23) define  $z_j^b$  and  $z_{j'}^b$  as binary variables, respectively.

## 4 Solution method

The model [SDPTW] is a mixed-integer nonlinear non-convex optimization problem. It is difficult to solve because (i) it has both continuous and discrete variables; (ii) it has nonlinear objective function (7) and constraint (12); (iii) the “mod” operator leads to a disjoint domain. After carefully examining the properties of the problem, we develop a holistic solution approach. We first relax the port time window constraints in Subsection 4.1.1. The relaxed mixed-integer nonlinear programming model is transformed to a mixed-integer linear programming model in Subsection 4.1.2. We solve the mixed-integer linear programming model to obtain the optimal solution. If the port time window constraints are violated, we add constraints to exclude such a solution. The above process is repeated until a feasible solution, which is optimal, is found, as elaborated in Subsection 4.2.

### 4.1 Relaxed models

#### 4.1.1 Relaxing port time window constraints

First, we relax the port time window constraints and obtain a relaxed problem (RP):

$$[\text{RP}] \quad \min C^{\text{ship}}m + \alpha \sum_{i \in I} L_i g_i(v_i) + \sum_{i \in I} \beta \bar{V}_i \frac{L_i}{v_i} \quad (24)$$

subject to:

$$t_i^{\text{dep}} = t_i^{\text{arr}} + t_i^{\text{port}}, i \in I \quad (25)$$

$$0 \leq t_1^{\text{arr}} \leq 6 \quad (26)$$

$$t_{i+1}^{\text{arr}} \geq t_i^{\text{arr}} + t_i^{\text{port}} + \left\lceil \frac{L_i}{24V^{\text{max}}} \right\rceil, i \in I \quad (27)$$

$$t_{N+1}^{\text{arr}} = t_1^{\text{arr}} + 7m \quad (28)$$

$$v_i = \frac{L_i}{24(t_{i+1}^{\text{arr}} - t_i^{\text{dep}})}, i \in I \quad (29)$$

$$0 \leq v_i \leq V^{\text{max}}, i \in I \quad (30)$$

$$m \in \{1, 2, 3, \dots, m^{\text{max}}\} \quad (31)$$

$$t_i^{\text{arr}} \in \mathbb{Z}^+, i \in I \quad (32)$$

Note that the difficult “mod” operator is relaxed. In other words, we assume that a berth is always available whenever a ship visits a port.

#### 4.1.2 An equivalent mixed-integer linear programming model

[RP] is a mixed-integer nonlinear programming (MINLP) model. In view of its special structure, we transform it to an equivalent mixed-integer linear programming (MILP) model and the MILP model can be solved by off-the-shelf MILP solvers. To this end, we first define the reciprocal of the speed as a new variable:

$$u_i := \frac{1}{v_i}, i \in I \quad (33)$$

Hence, [RP] is transformed to another MINLP model:

$$[\text{MINLP}] \quad \min C^{\text{ship}}m + \alpha \sum_{i \in I} L_i g_i(1/u_i) + \sum_{i \in I} \beta \bar{V}_i L_i u_i \quad (34)$$

subject to:

$$t_i^{\text{dep}} = t_i^{\text{arr}} + t_i^{\text{port}}, i \in I \quad (35)$$

$$0 \leq t_1^{\text{arr}} \leq 6 \quad (36)$$

$$t_{i+1}^{\text{arr}} \geq t_i^{\text{arr}} + t_i^{\text{port}} + \left\lceil \frac{L_i}{24V^{\text{max}}} \right\rceil, i \in I \quad (37)$$

$$t_{N+1}^{\text{arr}} = t_1^{\text{arr}} + 7m \quad (38)$$

$$u_i = 24(t_{i+1}^{\text{arr}} - t_i^{\text{dep}})/L_i, i \in I \quad (39)$$

$$u_i \geq 1/V^{\text{max}}, i \in I \quad (40)$$

$$m \in \{1, 2, 3, \dots, m^{\text{max}}\} \quad (41)$$

$$t_i^{\text{arr}} \in \mathbb{Z}^+, i \in I \quad (42)$$

Now the only nonlinear term is  $g_i(1/u_i)$  in Eq. (34), which has the following form:

$$g_i(1/u_i) = a_i(u_i)^{-b_i} \quad (43)$$

$g_i(1/u_i)$  is a convex function shown in Fig. 2a. Eq. (39) indicates that  $u_i$  can only



take a limited number of values because  $t_{i+1}^{\text{arr}} - t_i^{\text{dep}}$  is a positive integer and is not greater than  $7m^{\text{max}}$ . Hence, we obtain a tangent line at each of the possible values of  $u_i$ . In particular, as shown in Fig. 2b, we let  $u_i^\kappa$  denote the possible values of  $u_i$ :

$$u_i^\kappa = 24\kappa/L_i, \kappa = 1, 2, \dots, 7m^{\text{max}}$$

The tangent lines at the points are:

$$a_i(u_i^\kappa)^{-b_i} - a_i b_i (u_i^\kappa)^{-b_i-1} (u_i - u_i^\kappa), \kappa = 1, 2, \dots, 7m^{\text{max}}$$

We use variable  $\bar{g}_i$  to represent the bunker consume on leg  $i$  for formulating the tangent lines, and we have :

$$\bar{g}_i \geq a_i(u_i^\kappa)^{-b_i} - a_i b_i (u_i^\kappa)^{-b_i-1} (u_i - u_i^\kappa), \kappa = 1, 2, \dots, 7m^{\text{max}}$$

The model [MINLP] can be transformed to a MILP model after introducing the intermediate variable  $\bar{g}_i$ , which is an auxiliary variable that is not smaller than the bunker consumption per nautical mile on leg  $i$ :

$$[\text{MILP}] \quad \min C^{\text{ship}} m + \alpha \sum_{i \in I} L_i \bar{g}_i + \sum_{i \in I} \beta \bar{V}_i L_i u_i \quad (44)$$

subject to:

$$\bar{g}_i \geq a_i(u_i^\kappa)^{-b_i} - a_i b_i (u_i^\kappa)^{-b_i-1} (u_i - u_i^\kappa), \kappa = 1, 2, \dots, 7m^{\text{max}}, i \in I \quad (45)$$

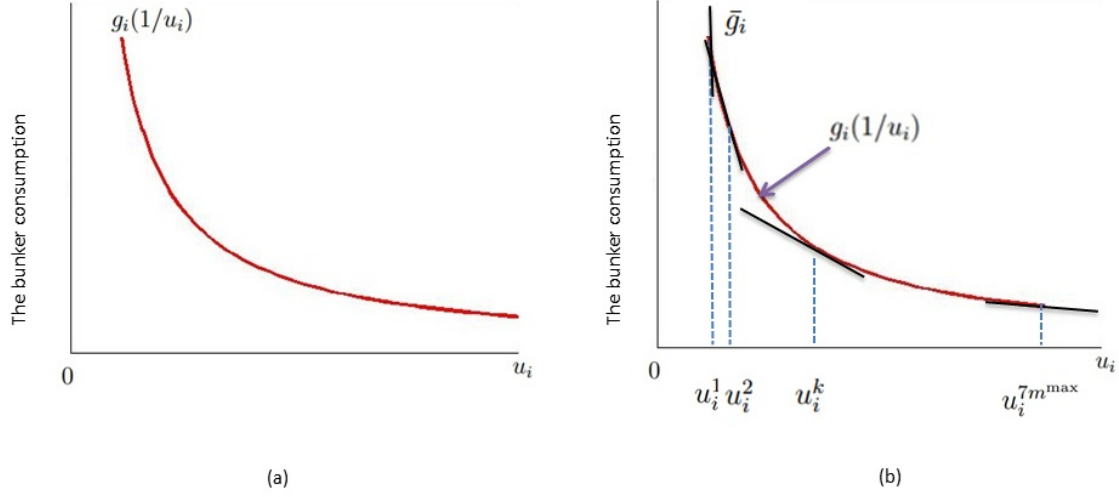


Figure 2: Linerization

$$t_i^{\text{dep}} = t_i^{\text{arr}} + t_i^{\text{port}}, i \in I \quad (46)$$

$$0 \leq t_1^{\text{arr}} \leq 6 \quad (47)$$

$$t_{i+1}^{\text{arr}} \geq t_i^{\text{arr}} + t_i^{\text{port}} + \left\lceil \frac{L_i}{24V^{\text{max}}} \right\rceil, i \in I \quad (48)$$

$$t_{N+1}^{\text{arr}} = t_1^{\text{arr}} + 7m \quad (49)$$

$$u_i = 24(t_{i+1}^{\text{arr}} - t_i^{\text{dep}})/L_i, i \in I \quad (50)$$

$$u_i \geq 1/V^{\text{max}}, i \in I \quad (51)$$

$$m \in \{1, 2, 3, \dots, m^{\text{max}}\} \quad (52)$$

$$t_i^{\text{arr}} \in \mathbb{Z}^+, i \in I \quad (53)$$

**Theorem 1.** Model [RP] and model [MILP] are equivalent. In other words, if

$(m^*, v_i^*, t_i^{arr*})$  is an optimal solution to [RP] and the optimal objective value is  $C_{RP}$ , then  $(\hat{m} = m^*, \hat{u}_i = 1/v_i^*, \hat{t}_i^{arr} = t_i^{arr*}, \hat{g}_i = g_i(v_i^*))$  is a feasible solution to [MILP] and the resulting objective value is equal to  $C_{RP}$ . If  $(\hat{m}, \hat{u}_i, \hat{t}_i^{arr}, \hat{g}_i)$  is an optimal solution to [MILP] and the optimal objective value is  $C_{MILP}$ , then  $(m^* = \hat{m}, v_i^* = 1/\hat{u}_i, t_i^{arr*} = \hat{t}_i^{arr})$  is a feasible solution to [RP] and the resulting objective value is equal to  $C_{MILP}$ .

*Proof.* The difference between model [RP] and model [MILP] is the linearization of  $g_i(v_i)$  to  $\bar{g}_i$ . As we use tangent lines to approximate the nonlinear function  $g_i(1/u_i)$ , and the tangent lines are not above the nonlinear function, Eq. (45) may underestimate the bunker consumption but will not overestimate it. Therefore [MILP] may underestimate the total cost, but will not overestimate it. That is,  $C_{MILP} \leq C_{RP}$ .

Now we prove that  $C_{MILP} \geq C_{RP}$ . If  $(\hat{m}, \hat{u}_i, \hat{t}_i^{arr}, \hat{g}_i)$  is an optimal solution to [MILP], the integrality of  $\hat{t}_i^{arr}$ , the integrality of the departure times, and Eq. (50) imply that  $\hat{u}_i$  is the same as one  $u_i^\kappa$ ,  $\kappa = 1, 2, \dots, 7m^{\max}$ . Note that there is no approximation error caused by the tangent lines at the points  $u_i^\kappa$ ,  $\kappa = 1, 2, \dots, 7m^{\max}$ . In other words, at these points  $\bar{g}_i$  does not underestimate  $g_i(v_i)$ . Hence, the resulting objective value to [RP] of the solution  $(m^* = \hat{m}, v_i^* = 1/\hat{u}_i, t_i^{arr*} = \hat{t}_i^{arr})$  is equal to  $C_{MILP}$ . This means that  $C_{MILP} \geq C_{RP}$ . Consequently, model [RP] and model [MILP] are equivalent.  $\square$

## 4.2 Global optimization method

### 4.2.1 Reformulation

Still, [RP] or [MILP] is not the original model. Suppose that the optimal solution to [RP] is denoted by  $(t_i^{\text{arr}*}, i \in I)$ . If  $(t_i^{\text{arr}*}, i \in I)$  satisfies berth time windows at all ports, then this optimal solution is also optimal to the original [SDPTW]. Otherwise, it is infeasible. Enlightened by this observation, we develop a solution method that excludes infeasible solutions from [RP] by adding linear constraints.

First, we add to [RP] the following constraints:

$$\bar{t}_i^{\text{arr}} = t_i^{\text{arr}} - 7k_i, \forall i \in I \quad (54)$$

$$0 \leq \bar{t}_i^{\text{arr}} \leq 6, \forall i \in I \quad (55)$$

$$k_i \in \{0, 1, 2, \dots, m-1\}, \forall i \in I \quad (56)$$

It is easy to see that the above constraints are equivalent to:

$$\bar{t}_i^{\text{arr}} = t_i^{\text{arr}} \pmod{7}, \forall i \in I \quad (57)$$

Note that because  $t_i^{\text{arr}}$  and  $k_i$  are defined to be integers,  $\bar{t}_i^{\text{arr}}$  is automatically an integer.

Next, we rewrite  $\bar{t}_i^{\text{arr}}$  using binary variables:

$$\bar{t}_i^{\text{arr}} = k_i^0 + 2k_i^1 + 4k_i^2, \forall i \in I \quad (58)$$

$$k_i^0, k_i^1, k_i^2 \in \{0, 1\}, \forall i \in I \quad (59)$$

It is clear that Eqs. (58)–(59) imply that given  $\bar{t}_i^{\text{arr}}$ , there is a unique binary vector  $(k_i^0, k_i^1, k_i^2)$  and vice versa. For example, if  $\bar{t}_i^{\text{arr}} = 5$ , we have  $k_i^0 = 1$ ,  $k_i^1 = 0$ , and  $k_i^2 = 1$ . If  $k_i^0 = 0, k_i^1 = 0, k_i^2 = 1$ , we have  $\bar{t}_i^{\text{arr}} = 4$ . We now define a new model: reformulated MILP problem (RMILP):

[RMILP]:

[MILP] with constraints (54)–(56) and (58)–(59).

**Theorem 2.** *The two models [RMILP] and [MILP] are equivalent.*

*Proof.* First, the only difference of the two models is that [RMILP] has more constraints than [MILP]. Therefore, [RMILP] is at least as tight as [MILP]. Hence, we only need to prove that the additional constraints in [RMILP] does not confine the domain of [MILP]. In other words, we need to prove that for any feasible solution  $(t_i^{\text{arr}}, i \in I; m)$  to [MILP], we can find a vector  $(k_i, k_i^0, k_i^1, k_i^2, i \in I)$  such that all the constraints (54)–(56) and (58)–(59) are satisfied.

As  $(t_i^{\text{arr}}, i \in I; m)$  is feasible to [MILP], we have  $0 \leq t_i^{\text{arr}} \leq 7m - 1, i \in I$ . Now it is easy to see that there always exists a vector  $(k_i, k_i^0, k_i^1, k_i^2, i \in I)$  such that all the constraints (54)–(56) and (58)–(59) are satisfied.  $\square$

#### 4.2.2 Linear constraints excluding infeasible solutions

Since [RMILP] is a mixed-integer linear optimization model, we can apply off-the-shelf MILP solvers to solve it. If the resulting solution is infeasible (that is, in-

compatible with the available port time windows or berth time windows), we add a linear constraint that excludes this solution while keeping all other solutions, and solve [RMILP] with the added linear constraints. This process is repeated until a feasible solution is found, and this solution is also optimal.

Suppose that  $(t_i^{\text{arr}*}, i \in I)$  is the optimal solution to [RMILP],  $\bar{t}_i^{\text{arr}*} = t_i^{\text{arr}*} \bmod 7$ , and it corresponds to  $(k_i^{0*}, k_i^{1*}, k_i^{2*}, i \in I)$ . We now elaborate on how to check the feasibility of  $(t_i^{\text{arr}*}, i \in I)$ .

At a port of call  $i \in I_1$ , if  $\bar{t}_i^{\text{arr}*} \in \hat{\Omega}_i$ , then  $t_i^{\text{arr}*}$  is feasible at the port of call; otherwise it is infeasible. If  $t_i^{\text{arr}*}$  is infeasible, to exclude it as well as other infeasible arrival times  $t_i^{\text{arr}}$  satisfying  $t_i^{\text{arr}} \bmod 7 = \bar{t}_i^{\text{arr}*}$  at port of call  $i \in I_1$  from model [RMILP], we add the following constraint:

$$\begin{aligned} k_i^0(1 - k_i^{0*}) + (1 - k_i^0)k_i^{0*} + k_i^1(1 - k_i^{1*}) + (1 - k_i^1)k_i^{1*} + \\ k_i^2(1 - k_i^{2*}) + (1 - k_i^2)k_i^{2*} \geq 1 \end{aligned} \quad (60)$$

This constraint will exclude not only solution  $t_i^{\text{arr}*}$  but also all  $t_i^{\text{arr}}$  satisfying  $t_i^{\text{arr}} \bmod 7 = \bar{t}_i^{\text{arr}*}$ . For example, if the ship cannot arrive on Tuesday, then  $t_i^{\text{arr}} = 2$  is infeasible, and  $t_i^{\text{arr}} = 2 + 7 = 9$ ,  $t_i^{\text{arr}} = 2 + 2 \times 7 = 16$  and  $t_i^{\text{arr}} = 2 + 3 \times 7 = 23$  are all infeasible. All these infeasible  $t_i^{\text{arr}}$  correspond to the same  $\bar{t}_i^{\text{arr}*} = 2$ , and correspond to the same  $(k_i^{0*}, k_i^{1*}, k_i^{2*}) = (0, 1, 0)$ . Hence, Eq. (60) becomes:

$$k_i^0 + (1 - k_i^1) + k_i^2 \geq 1 \quad (61)$$

Evidently,  $(k_i^0, k_i^1, k_i^2) = (0, 1, 0)$  is the only solution that violates this constraint.

At  $i \in I_2 \cup I'_2$ , if  $\bar{t}_i^{\text{arr}*} \in \bigcup_{b \in B_i} \hat{\Omega}_i^b$ , then this arrival time  $t_i^{\text{arr}*}$  is feasible; otherwise it is infeasible and we can use a constraint similar to Eq. (60) to exclude it from [RMILP]. However, for  $j \in I_2$ , even if both  $t_j^{\text{arr}*}$  and  $t_{j'}^{\text{arr}*}$  are feasible, their combination may not be feasible. We need to check whether there exists a berth allocation plan such that the combination  $(t_j^{\text{arr}*}, t_{j'}^{\text{arr}*})$  is feasible. Similar to the above analysis, checking the feasibility of  $(t_j^{\text{arr}*}, t_{j'}^{\text{arr}*})$  is actually checking the feasibility of  $(\bar{t}_j^{\text{arr}*}, \bar{t}_{j'}^{\text{arr}*})$ .

**Algorithm 1: Check feasibility of  $(\bar{t}_j^{\text{arr}*}, \bar{t}_{j'}^{\text{arr}*})$**

Step 1. Set  $b_1 = 1$ .

Step 2. If  $b_1 > |B_j|$ ,  $(\bar{t}_j^{\text{arr}*}, \bar{t}_{j'}^{\text{arr}*})$  is infeasible, return. If  $\bar{t}_j^{\text{arr}*} \notin \hat{\Omega}_j^{b_1}$ , set  $b_1 = b_1 + 1$  and go to Step 2.

Step 3. Set  $b_2 = 1$ .

Step 4. If  $b_2 > |B_j|$ , set  $b_1 = b_1 + 1$  and go to Step 2. Else if  $\bar{t}_{j'}^{\text{arr}*} \notin \hat{\Omega}_j^{b_2}$ , set  $b_2 = b_2 + 1$  and go to Step 4. Else go to Step 5.

Step 5. If  $b_1 \neq b_2$ ,  $(\bar{t}_j^{\text{arr}*}, \bar{t}_{j'}^{\text{arr}*})$  is feasible, return. Else

Step 5.0. Set  $t' = 1$ ;

Step 5.1. If  $t' \geq 7$ ,  $(\bar{t}_j^{\text{arr}*}, \bar{t}_{j'}^{\text{arr}*})$  is feasible, return. Else if  $\delta_{j t_j^{\text{arr}*}}^{t'} + \delta_{j' t_{j'}^{\text{arr}*}}^{t'} = 2$ , the berth cannot serve two ships on day  $t'$ , and hence set  $b_2 = b_2 + 1$  and go to Step 4. Else set  $t' = t' + 1$  and go to Step 5.1.  $\square$

Note that in Algorithm 1, we have at most  $|B_j|^2$  possible berth allocation plans. Hence it is not difficult to check the feasibility of  $(\bar{t}_j^{\text{arr}*}, \bar{t}_{j'}^{\text{arr}*})$ .

If we cannot find a berth allocation plan such that  $(\bar{t}_j^{\text{arr}*}, \bar{t}_{j'}^{\text{arr}*})$  is feasible, then we exclude the solution  $(\bar{t}_j^{\text{arr}*}, \bar{t}_{j'}^{\text{arr}*})$  from [RMILP] by adding the following constraint:

$$\begin{aligned}
& k_j^0(1 - k_j^{0*}) + (1 - k_j^0)k_j^{0*} + k_j^1(1 - k_j^{1*}) + (1 - k_j^1)k_j^{1*} + \\
& k_j^2(1 - k_j^{2*}) + (1 - k_j^2)k_j^{2*} + k_{j'}^0(1 - k_{j'}^{0*}) + (1 - k_{j'}^0)k_{j'}^{0*} + \\
& k_{j'}^1(1 - k_{j'}^{1*}) + (1 - k_{j'}^1)k_{j'}^{1*} + k_{j'}^2(1 - k_{j'}^{2*}) + (1 - k_{j'}^2)k_{j'}^{2*} \geq 1
\end{aligned} \tag{62}$$

Similar to constraint (60), this constraint excludes all the arrival times  $(t_j^{\text{arr}}, t_{j'}^{\text{arr}})$  satisfying  $t_j^{\text{arr}} \bmod 7 = \bar{t}_j^{\text{arr}*}$  and  $t_{j'}^{\text{arr}} \bmod 7 = \bar{t}_{j'}^{\text{arr}*}$ .

### 4.2.3 Overall algorithm

We now elaborate on the overall solution algorithm that obtains the global optimal solution to model [SDPTW].

#### Algorithm 2: Overall global optimization algorithm

Step 0. Define set  $\Psi_1 := \emptyset$  that will contain constraints (60) and set  $\Psi_2 := \emptyset$  that will contain constraints (62).

Step 1. Solve [RMILP] with constraints (60) defined by set  $\Psi_1$  and constraints (62) defined by set  $\Psi_2$ . The optimal solution is denoted by  $(m^*, v_i^*, t_i^{\text{arr}*}, \bar{t}_i^{\text{arr}*}, k_i^*, k_i^{0*}, k_i^{1*}, k_i^{2*}, i \in I)$ .



Step 2. Check each port  $i \in I = I_1 \cup I_2 \cup I'_2$ . If  $t_i^{\text{arr}*}$  is infeasible, add to set  $\Psi_1$  the constraint (60), go to Step 1;

Step 3. Check each port  $j \in I_2$ . If the combination of arrival times  $(t_j^{\text{arr}*}, t_{j'}^{\text{arr}*})$  is infeasible, add to set  $\Psi_2$  the constraint (62), go to Step 1;

Step 4. The solution  $(m^*, v_i^*, t_i^{\text{arr}*}, i \in I)$  is a feasible and optimal solution to [SDPTW].  
Stop.  $\square$

**Theorem 3.** *Algorithm 2 terminates in a finite number of iterations.*

*Proof.* In each iteration of Algorithm 2, we either exclude one  $\bar{t}_i^{\text{arr}*}$ , or a combination of  $(\bar{t}_j^{\text{arr}*}, \bar{t}_{j'}^{\text{arr}*})$ . The total number of  $\bar{t}_i^{\text{arr}*}$  and combinations of  $(\bar{t}_j^{\text{arr}*}, \bar{t}_{j'}^{\text{arr}*})$  does not exceed  $7|I| + 7^2|I_2|$ , which is finite. Hence, Algorithm 2 terminates in a finite number of iterations.  $\square$

**Theorem 4.** *The feasible solution  $(m^*, v_i^*, t_i^{\text{arr}*}, i \in I)$  obtained in Step 4 of Algorithm 2 is optimal to [SDPTW].*

*Proof.* Model [RMILP] is a relaxed problem of the original model [SDPTW] in that some constraints (i.e. port time window constraints) are removed but the objective function does not change. We add more and more constraints (60) and (62) in each iteration of Algorithm 2. However, these constraints do not exclude any feasible solution. Hence, the feasible solution  $(m^*, v_i^*, t_i^{\text{arr}*}, i \in I)$  obtained in Step 4 of Algorithm 2, which by definition is optimal to [RMILP] with the generated constraints (60) and (62), and by definition is feasible to port time window constraints, is also optimal to [SDPTW].  $\square$

## 5 Case study

We carry out case studies based on the AGM ship route in Fig. 1 to evaluate the applicability of the proposed models and algorithms. The AGM ship route consists of 10 ports of call in a round trip. The port of Miami is visited twice and hence there are a total of 9 physical ports. We assume that these 9 ports have a total of 30 berths, whose available times are shown in Table 1.

We assume that 5000-TEU ships are deployed on the ship route. The operating cost  $C^{\text{ship}} = 500,000$  USD/week. The max speed  $V^{\text{max}} = 30$  knots, the bunker price  $\alpha = 400$  USD/ton, the unit inventory cost  $\beta = 1$  USD per TEU per hour and the maximum number of ships  $m^{\text{max}} = 20$  ships. The port time, length, bunker consumption function  $g_i(v_i)$ , and container number on each leg are shown in Table 2.

### 5.1 Performance of the solution algorithm

We apply the proposed global optimization algorithm (Algorithm 2) to design the schedule of the AGM ship route of OOCL. The models are all solved by matlab calling CPLEX 12.2 on a 3.2 GHz Dual Core laptop with 4 GB of RAM. The algorithm finds the optimal solution after 26 iterations in about 1 minute. Hence, the algorithm is efficient for addressing problems of realistic scales.

The number of ships and the total cost in each iteration are shown in Fig. 3. As more and more constraints are added, the optimal objective value (the total cost) is non-decreasing. It is interesting to notice that the total cost does not change in the first seven iterations. This is because of “symmetrical solutions” as follows.

Table 1: Available time at each port

Port ID	Port	Berth	Sun	Mon	Tue	Wed	Thu	Fri	Sat
1	Le Havre	1	free	free			free	free	
		2			free				
		3				free			free
2	Antwerp	1	free	free	free				
		2							free
3	Rotterdam	1		free		free	free		
		2	free	free		free	free		
4	Bremerhaven	1	free	free			free	free	
		2			free				
		3				free			free
5	Charleston	1	free		free				
		2			free	free	free		free
		3		free			free	free	
		4	free	free		free	free		
6	Miami	1	free	free			free	free	
		2			free				
		3				free			free
		4	free		free				
		5				free			
		6		free			free	free	
7	Veracruz	1	free	free		free	free		
		2	free		free				
		3	free		free				
8	Altamira	1		free			free		
		2	free	free			free	free	
		3			free				
		4	free			free	free		free
9	Houston	1		free			free		
		2	free	free			free	free	
		3			free				

In the first iteration, the optimal solution of  $(\bar{t}_i^{\text{arr}}, i \in I)$  is  $(3, 6, 1, 3, 6, 2, 6, 1, 3, 0)$  (of course, it is infeasible to the original problem). Since  $\bar{t}_1^{\text{arr}} = 3$  is infeasible,

Table 2: Parameters in the case study

Port of call	Port	Port time	Length	Bunker function	# containers
1	Le Havre	2 days	252	$0.001(v_1)^2$	3500
2	Antwerp	1 day	149	$0.001(v_2)^{2.1}$	3600
3	Rotterdam	1 day	225	$0.001(v_3)^{2.3}$	3900
4	Bremerhaven	1 day	4014	$0.001(v_4)^2$	5000
5	Charleston	2 days	435	$0.001(v_5)^2$	4900
6	Miami	2 days	1012	$0.001(v_6)^2$	4000
7	Veracruz	1 day	233	$0.001(v_7)^2$	3500
8	Altamira	1 day	512	$0.001(v_8)^{2.1}$	3800
9	Houston	1 day	970	$0.001(v_9)^{2.3}$	4100
10	Miami	2 days	3922	$0.001(v_{10})^2$	4950

constraint (60) excludes it and solution  $(4, 0, 2, 4, 0, 3, 0, 2, 4, 1)$  is obtained in the second iteration (of course, it is still infeasible to the original problem). Comparing these two solutions, we find that the second solution differs from the first one in that the arrival times at all ports of call are postponed by one day. Hence, the ship cost, bunker cost, and inventory cost do not change. Similarly, the solution obtained in the third iteration simply postpones the arrival times at all ports of call by two days. Repeating in a similar manner, the optimal number of ships and the total cost in the first seven iterations are the same.

In the eighth iteration, the solution of  $(\bar{t}_i^{\text{arr}}, i \in I)$  is  $(0, 4, 6, 1, 3, 6, 3, 5, 0, 4)$ . We observe that in this solution, the inter-arrival times between two adjacent ports of call are different from the previous seven solutions. For example, in the eighth solution,  $\bar{t}_2^{\text{arr}} - \bar{t}_1^{\text{arr}} = 4$ , which means that  $t_2^{\text{arr}} - t_1^{\text{arr}}$  is equal to 4 plus an integer number of weeks. However, in the first seven solutions,  $\bar{t}_2^{\text{arr}} - \bar{t}_1^{\text{arr}} = 3$ . Consequently, in the eighth solution, the sailing speed and the inventory cost on leg 1 are changed.

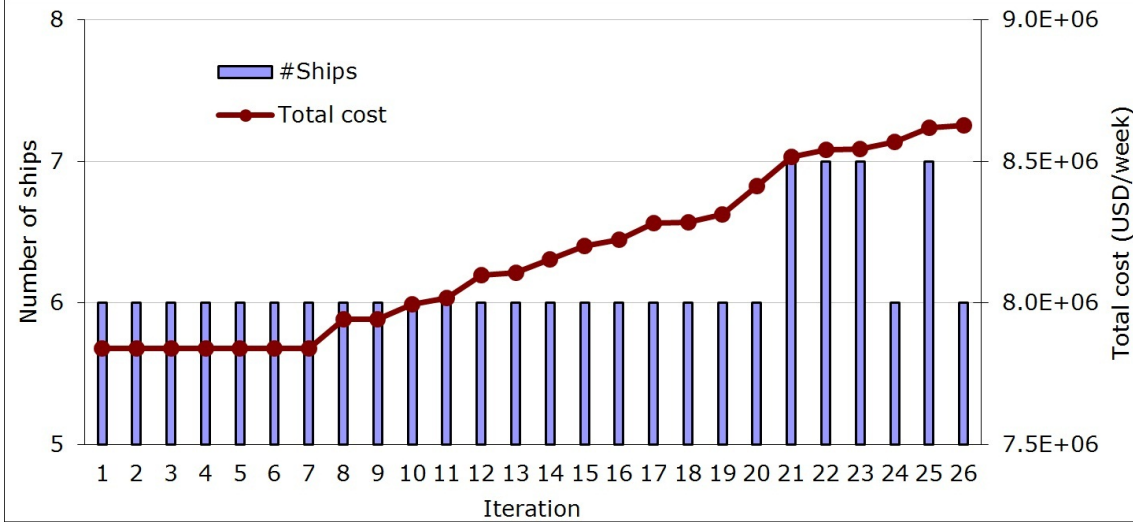


Figure 3: Number of ships and total cost in each iteration

In the first 22 iterations, exactly one constraint (60) is added to the model in Step 2 of Algorithm 2. The solution of  $(\bar{t}_i^{\text{arr}}, i \in I)$  in the 23rd iteration is  $(4, 1, 3, 5, 3, 0, 4, 6, 2, 0)$ . The arrival time at each single port of call all satisfies the port time windows. However, the arrival times at the 6th and 10th ports of call, which correspond to the same physical port, i.e., Miami, are both Sunday. As  $t_6^{\text{port}} = t_{10}^{\text{port}} = 2$ , both calls must use berth 1 of the port according to Table 1. As a result, their combination is infeasible, and hence in Step 3 of Algorithm 2, one constraint (62) is added to exclude such a combination. The solution in the 24th iteration is  $(0, 6, 1, 3, 3, 0, 4, 6, 1, 5)$ . Now  $\bar{t}_{10}^{\text{arr}} = 5$  is infeasible. Hence, one constraint (60) in Step 2 of Algorithm 2 is added to exclude the solution. In the 25th iteration, the solution is  $(4, 1, 3, 5, 0, 4, 2, 4, 0, 4)$ , and the combination  $(\bar{t}_6^{\text{arr}} = 4, \bar{t}_{10}^{\text{arr}} = 4)$  is again infeasible and therefore one constraint (62) is added. In the 26th iteration, the solution of  $(\bar{t}_i^{\text{arr}}, i \in I)$  is  $(0, 6, 1, 3, 3, 0, 4, 6, 1, 4)$ , which does not violate any port

time window constraint. Hence, this solution is optimal to the original problem. The optimal solution of  $(t_i^{\text{arr}}, i \in I \cup \{N + 1\})$  is  $(0, 6, 8, 10, 17, 21, 25, 27, 29, 32, 42)$ .

The number of ships in each iteration does not show any trend in Fig. 3. The optimal number of ships to deploy is 6.

## 5.2 Impact of port time windows

In this section, we examine the impact of the availability of a port on the optimal schedule and the total cost. We take the example of the port of Miami, which is visited twice a week with  $t_6^{\text{port}} = t_{10}^{\text{port}} = 2$ . Both Miami and the liner shipping company are interested in looking at the result if more available time is provided at Miami. We hence examine 7 berth availability cases of Miami, as shown in Table 3. Note that a berth is not included in a case of Table 3 if it is busy the whole week. From case 1 to case 7, more and more available times are provided. For example, in case 1 we must have  $\bar{t}_6^{\text{arr}} = 0, \bar{t}_{10}^{\text{arr}} = 0$ ; in case 2 either  $\bar{t}_6^{\text{arr}}$  or  $\bar{t}_{10}^{\text{arr}}$  can be 5; in case 3 it is further possible that either  $\bar{t}_6^{\text{arr}}$  or  $\bar{t}_{10}^{\text{arr}}$  is 6, etc.

Table 3: Different cases of available time at Miami

Case	Berth	Sun	Mon	Tue	Wed	Thur	Fri	Sat
1	1	free	free					
	4	free	free					
2	1	free	free					
	3						free	free
3	4	free	free					
	1	free	free					
4	3						free	free
	4	free	free					free
	1	free	free					
5	2					free	free	
	3						free	free
	4	free	free	free				free
6	1	free	free		free	free		
	2					free	free	
	3						free	free
7	4	free	free	free				free
	1	free	free		free	free		
	2					free	free	
	3						free	free
	4	free	free	free				free
	6			free	free			

The results of the total cost and optimal number of ships deployed for the 7 berth availability cases are shown in Fig. 4. More available days at Miami leads to a lower total cost: the total cost is reduced by 603,738 USD/week from case 2 to case 6. Fig. 4 also demonstrates that the number of available days at a port may affect the optimal number of ships deployed. The optimal ship route schedule is shown in Table 4. We observe that the optimal arrival time at Miami and its neighboring ports may change if the time windows at the port of Miami change.

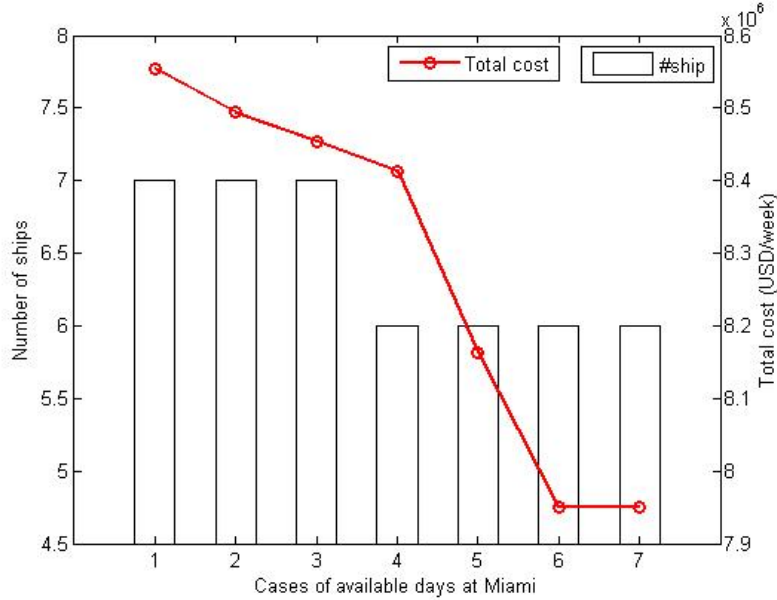


Figure 4: Impact of port time windows

### 5.3 Consequence of port efficiency

The port time  $t_i^{\text{port}}$  to a large extent depends on the container handling efficiency. Therefore, port operators seek to improve efficiency by optimizing quay-side and



Table 4: Impact of port time window on the optimal schedule

Port of call	Port	Cases 1	Case 2	Cases 3	Case 4	Case 5	Cases 6,7
1	Le Havre	4	4	4	0	4	4
2	Antwerp	7	7	7	6	7	7
3	Rotterdam	10	10	10	8	10	10
4	Bremerhaven	12	12	12	10	12	12
5	Charleston	24	23	23	17	21	21
6	Miami	28	26	26	20	25	24
7	Veracruz	32	32	31	24	29	28
8	Altamira	34	34	33	26	31	30
9	Houston	37	37	36	28	33	32
10	Miami	42	42	41	32	36	36
1	Le Havre	53	53	53	42	46	46

yard-side operations. To investigate the effect of port handling efficiency, we change the port time at Miami to generate four scenarios. In the first scenario, we assume both two visits use the port for only one day. The second scenario assumes the port time for the first visit (coming from Charleston) is one day and the second visit (coming from Houston) is two days. In the third scenario, the first visit needs two days to serve and one day is needed for the second visit. The last scenario is generated by assuming both visits need two days to serve which is the same as subsections 5.1 and 5.2. Table 5 shows the four port time scenarios.

Table 5: The scenarios of port times of the two calls at Miami

Scenario	Port time (day)	
	1st visit	2nd visit
1-1	1	1
1-2	1	2
2-1	2	1
2-2	2	2

We carry out numerical experiments for each combination of the 7 berth availability cases of Miami in Table 3 and the 4 scenarios of port time in Table 5. Hence, we have a total of 28 experiments, and the total costs and the number of ships to deploy are shown in Fig. 5 and Fig. 6, respectively. Consistent with the previous subsection, Fig. 5 indicates that the total cost always decreases when the port has more available days. Fig. 6 shows that the number of ships is reduced with more available days at Miami. Generally, by comparing the 4 port time scenarios, we find that improving port efficiency may reduce the total cost for liner shipping companies. However, because we do not include the inventory cost of containers associated with port time in objective functions of the models, a higher port efficiency may lead to a higher total cost if the reduced port time does not help to reduce the round-trip time of the ship route (at least by one week). For example, in berth availability case 2, the total cost of scenario 1-2 (the first visit is one day and the second visit is two days) is larger than the total cost of scenario 2-2 (both visits are two days). The reduced one day port time moves from the port time to sailing time on leg 4, which leads to more inventory cost at sea without reducing the round-trip time of the ship route. This result can be seen in Table 6, which reports the optimal schedules of the 4 scenarios under berth availability case 2 and the sailing time on each leg  $i$ , i.e.,  $t_{i+1}^{\text{arr}} - t_i^{\text{dep}}$ . It should be noted that if we include the inventory cost of containers associated with port time, then the total cost of scenario 1-2 is always lower than that of scenario 2-2.

Finally, we note that the reduction of the round-trip time of the ship route is always an integer number of weeks, which corresponds to the reduction in the

number of ships to deploy. In some cases, reducing port time leads to a smaller number of ships deployed, for example, under berth availability case 2, the optimal number of ships is 6 in scenario 1-1 and the number is 7 in scenario 2-2, as shown in Fig. 6.

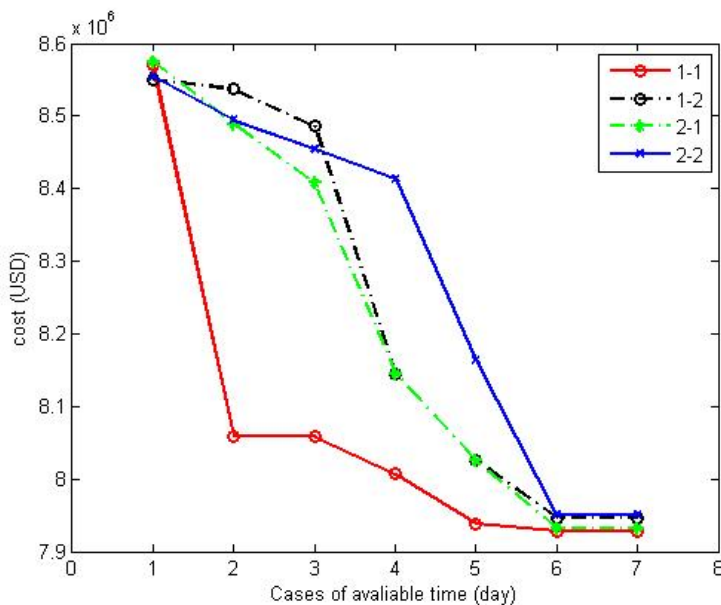


Figure 5: Impact of port time at Miami on the total cost

## 5.4 Result of bunker prices

The bunker price is volatile and hence we examine the sensitivity of the solution with different bunker prices (USD/ton) from 300, 400, 500, 600, 700, to 800. The parameters are the same as Table 1 and Table 2. The results are shown in Fig. 7. We observe that a higher bunker price always leads to a higher total cost for liner shipping companies. In addition, Fig. 7 shows that there is a rise in the number of

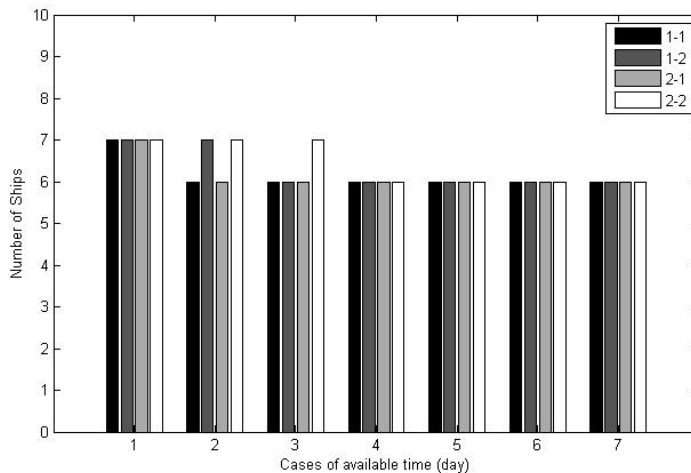


Figure 6: Impact of port time at Miami on the number of ships

Table 6: Optimal schedules of the 4 scenarios under berth availability case 2

Port of call	Arrival time $t_i^{\text{arr}}$				Sailing days on leg $i$			
	1-1	1-2	2-1	2-2	1-1	1-2	2-1	2-2
1	4	4	0	4	1	1	4	1
2	7	7	6	7	2	2	1	2
3	10	10	8	10	1	1	1	1
4	12	12	10	12	10	11	6	10
5	23	24	17	23	1	1	2	1
6	26	27	21	26	2	4	2	4
7	29	32	25	32	1	1	1	1
8	31	34	27	34	1	2	1	2
9	33	37	29	37	2	4	3	4
10	36	42	33	42	9	9	8	9
1	46	53	42	53				
Total					30	36	29	35

ships used when the bunker price becomes higher. This is because when more ships are deployed, the sailing speed can be lower, resulting in lower bunker consumption,

which is more significant when the bunker price is higher.

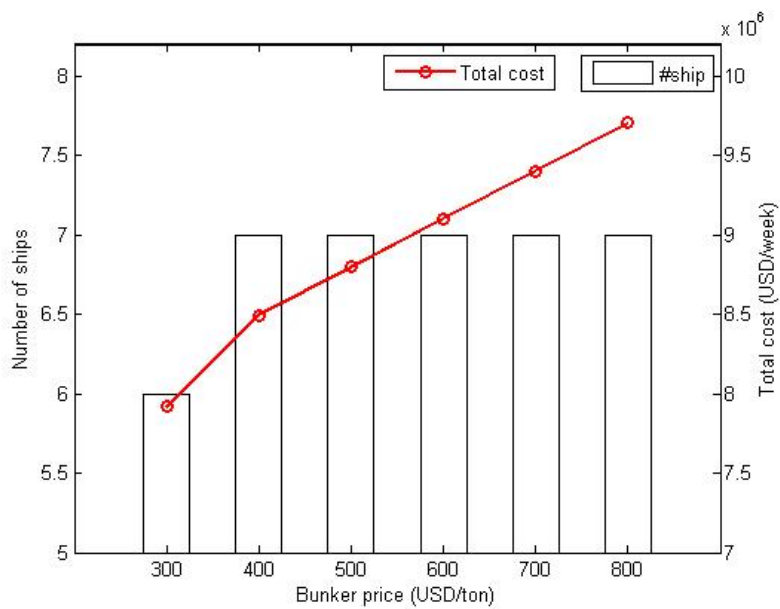


Figure 7: Result of bunker prices on the total cost and the number of ships

## 5.5 Effect of inventory cost

Finally, the unit inventory cost  $\beta$  may affect the ship route schedule and then the total cost. We change  $\beta$  from 1, 1.2 through to 2 and the results of 6 experiments are shown in Fig. 8. The number of ships decreases when the unit inventory cost rises. This shows that when the cargoes in the containers are more valuable, ships should sail at a higher speed. The total cost increases almost linearly (not strictly linearly) when the unit inventory cost rises. This indicates that as a result of increase in the unit inventory cost, the total cost of a liner shipping company is higher.

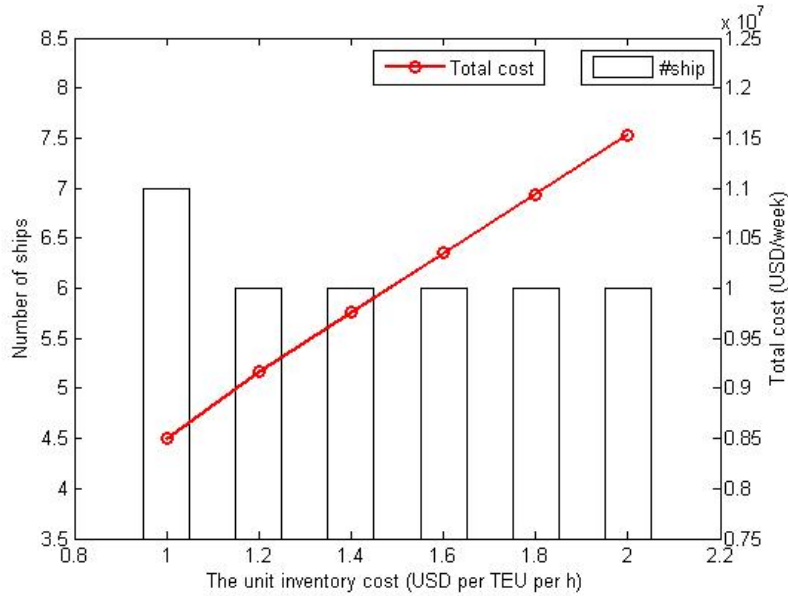


Figure 8: Effect of unit inventory cost on the total cost and the number of ships

## 6 Conclusions and Future work

This paper has studied the practical liner ship route schedule design problem with port time windows where a port can be visited twice in a week. This is a significant tactical planning decision problem for liner shipping companies because it considers the availability of ports when planning liner shipping services. As a result, the designed schedule can be applied in practice without or with only minimum revisions. This problem is formulated as a mixed-integer nonlinear non-convex optimization model. In view of the problem structure, we developed a holistic solution approach. In this approach, at first the port time window constraints are relaxed to obtain a mixed-integer nonlinear programming model, which is subsequently transformed to a mixed-integer linear programming model. This mixed-integer linear model is repeatedly solved by adding the violated port time window constraints until a feasible solution is obtained. This feasible solution is proved to be the global optimal solution to the problem.

We have conducted extensive numerical experiments based on the AGM ship route of OOCL. According to the results, the solution approach could efficiently find the optimal solution, which demonstrates its applicability to realistic problems. The availability of ports affects the the total cost of liner shipping companies, the optimal number of ships deployed, and the ship route schedule. Therefore, it is important for liner shipping companies to consider port time windows in liner ship route schedule design. In addition, due to the importance of some of the parameters on the sailing schedule, we conducted sensitivity analysis of port efficiency, bunker price and unit

inventory cost. Useful managerial insights are obtained. Firstly, improving port efficiency generally will reduce the total cost for liner shipping companies. However, because of the weekly service of a ship route, improving port efficiency may but does not necessarily lead to a reduction of the round-trip time of a ship route and thereby a smaller number of ships deployed. Secondly, increasing bunker price leads to a higher total cost and a rise in the number of ships deployed. Third, the number of ships to deploy drops and the total cost increases with a rise in the unit inventory cost.

In future we will look at the schedule design problem with port time windows for a liner shipping network. In a liner shipping network, container transshipment operations at a port can occur when this port is visited by ships at least twice a week. As a consequence, the dwell time of containers at transshipment ports and the resulting inventory cost should be taken into account. This problem is more challenging and interesting, and new solution approaches should be developed.

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