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Wenting Chen

Jiangnan University, wtchen@uow.edu.au

Xinjiang He

University of Wollongong, xinjiang@uow.edu.au

Sha Lin

Zhejiang Gongshang University, sl945@uowmail.edu.au

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Pricing credit default swaps with Parisian and *Parasian* default mechanics

Wenting Chen ^{*} Xin-Jiang He [†] Sha Lin [‡]

Abstract

This paper proposes Parisian and *Parasian* default mechanics for modeling the credit risks of the CDS (credit default swap) contracts. Unlike most of the structural models used in the literature, our new model assumes that the default will occur only if the price of the reference asset stays below a certain level for a pre-described period of time. To work out the corresponding CDS price, a general pricing formula containing the unknown no-default probability is derived first. It is then shown that the determination of such a probability is equivalent to the valuation of a Parisian or *Parasian* down-and-out binary options, depending on how the time is recorded. After the option price is solved with a θ finite difference scheme, the CDS price is obtained through the derived general pricing formula. Finally, some numerical experiments are carried out to study the effects of the new default mechanics on the CDS prices.

AMS(MOS) subject classification.

Keywords. Credit default swaps, Parisian-type options, binary options, finite difference method

^{*}Department of Business, Jiangnan University, Jiangsu, China

[†]School of Mathematics and Applied Statistics, University of Wollongong NSW 2522, Australia

[‡]Corresponding author. School of Finance, Zhejiang Gongshang University, Hangzhou 310018, China (ls92zj@163.com)

1 Introduction

Nowadays, it is known that credit risk is one of the most important types of risk in financial markets. To effectively manage this kind of risk, the credit derivatives are introduced and developed. Among them, the most basic one is the so-called credit default swap (CDS). This kind of financial derivative is usually traded over the counter, and is an agreement between two parties aiming to offer protection against credit risk through periodic payments from the buyer to the seller, and compensation in case of default from the seller to the buyer. The CDS has attracted lots of attention since the last two decades due to the huge demand for the effective management of credit risks.

In the literature, two kinds of models, namely, the reduced-form models and the structural models, are widely adopted for modelling the credit risk contained in a CDS contract. Proposed by Jarrow & Turnbull [6], the former kind of models developed quickly and were followed by a great number of authors [4, 8]. These models are mathematically appealing in the sense that the probability of default can be extracted from market prices. However, one of the main drawbacks is that they can not capture the wide range of default correlations. On the other hand, the structural default models, as another alternatives, although mathematically complicated, are able to provide correlation between different firms as they use the evolution of the asset price to determine the time when the default occurs. Typical models in this category include the Merton model [9], which assumes that the reference asset follows a geometric Brownian motion and the default can occur only at the expiry of the CDS contract, the Malherbe model [3], which replaces the geometric Brownian motion by a Poisson process, and the Zhou model [11], which uses a jump-diffusion process to model the evolution of the reference asset.

It needs to be pointed out that Merton's assumption that the default can only occur at the expiry is unrealistic, as has already been pointed out by a number of authors [1, 2]. The first-passage time model which assumes that the default occurs when the price of the

reference asset first hits the default barrier is favored by a number of authors (see [5, 7, 10] and the references therein). However, based on the observation that fluctuations of asset values are quite common in financial markets, it is still not appropriate to assume that the default will occur as soon as the value of the reference asset touches the default barrier.

To incorporate this issue, in this paper, we introduce new default mechanics with both Parisian and *Parasian* specifications. Similar to the Parisian and *Parasian* options described in [12], these two types of default share the same feature that there is a separate “clock” to record the total time that the price of the reference asset is below the default barrier. The main difference between these two types of specifications is how the time is recorded. Specifically, for the Parisian type, the clock accumulates the time in a row and resets itself to zero each time the price crosses the default barrier, whereas for the *Parasian* type, the clock never needs to be reset to zero. For simplicity of reference, we may sometimes in this paper refer to these two specifications as “Parisian-type” when there is no need to distinguish them.

To work out the price of the CDS under the proposed default model, an analytical expression for the CDS price is derived first, which contains the no-default probability to be further determined. It is then shown that this unknown probability is equivalent to the price of a Parisian-type down-and-out binary option. With the θ finite difference scheme, the partial differential equation (PDE) system governing the price of this option is numerically solved, and the price of the corresponding CDS is finally obtained. From various numerical experiments, it is interesting to notice that the Parisian-type default mechanics has a significant impact on the CDS price.

The rest of the paper is organized as follows. In Section 2, the newly proposed default mechanics is introduced, and a numerical scheme is proposed to solve for the CDS price. In Section 3, numerical experiments are conducted to examine some properties of the new CDS prices. Concluding remarks are given in the last section.

2 Mathematical formulation

How and when to default is a key question in designing credit derivatives. In case of the CDS, Merton assumes that the default would occur only at the expiration of the contract [9]. His assumption is obviously not realistic. Later on, most authors modified Merton's assumption by assuming that the default could occur at any time during the life span of the CDS [1, 2]. According to the fact that fluctuations in asset prices are often observed in financial markets, this assumption is however, still not appropriate.

In this section, Parisian-type default mechanics will be introduced to model the credit risks of the CDS contracts. According to the issues to be addressed, this section is further divided into three subsections. In the first subsection, a general pricing formula for the price of the CDS will be derived. In the second subsection, new default models will be introduced, whereas in the last subsection, the θ finite difference scheme will be proposed to solve for the CDS price.

2.1 General pricing formula

In this subsection, we shall consider the pricing of the CDS under a general default model. By “general”, it means the result obtained in this subsection can be used for the pricing of the CDS under any reasonable default models. This will pave the way for solving the CDS price with the new default mechanics that will be introduced later. It needs to be remarked that the price of the CDS refers to the spread, i.e., the regular fee that the buyer pays to the seller, instead of being its value as usual, and is often quoted as the ratio of the price of the reference asset.

To determine the price of the CDS, we need to analyze its cash flow first. It is known that the CDS buyer pays regularly the protection fee to the seller before the default occurs. In this case, if we denote c as the spread of the CDS and $p(S, t)$ as the probability of no default occurring before the time t , it is not difficult to show that the expectation of the

amount of payment from the buyer to the seller between $t - dt$ and t is equal to $cMp(S, t)dt$ with M being the face value of the reference asset. As a result, the present value of the cash flow of the buyer (denoted by V_1) is

$$V_1 = \sum_t [e^{-rt} cMp(S, t)dt] = cM \int_0^T e^{-rt} p(S, t)dt,$$

where r is the risk-free interest rate. On the other hand, once the default occurs, the seller should pay $(1 - R)M$ to the buyer with R being the recovery rate. With the probability of the default taken place between $[t, t + dt]$ being

$$[1 - p(S, t + dt)] - [1 - p(S, t)] = p(S, t) - p(S, t + dt) = -dp(S, t),$$

it is obvious that the present value of the cash flow of the seller (denoted by V_2) is

$$V_2 = \sum_t [-e^{-rt}(1 - R)Mdp(S, t)] = -(1 - R)M \int_0^T e^{-rt} dp(S, t).$$

To be fair to both parties, the value of a CDS is zero when it is initiated. Therefore, we have $V_1 = V_2$, implying that

$$cM \int_0^T e^{-rt} p(S, t)dt = -(1 - R)M \int_0^T e^{-rt} dp(S, t).$$

From the above equation, it is clear that the spread c is meaningless if the no-default probability remains zero during the life span of the contract. Excluding this case, the price

of the CDS can be solved as

$$\begin{aligned}
c &= \frac{(1-R) \int_0^T -e^{-rt} dp(S, t)}{\int_0^T e^{-rt} p(S, t) dt}, \\
&= \frac{(1-R) \left[-e^{-rt} p(S, t) \right]_0^T - r \int_0^T e^{-rt} p(S, t) dt}{\int_0^T e^{-rt} p(S, t) dt}, \\
&= \frac{(1-R)[1 - e^{-rT} p(S, T)]}{\int_0^T e^{-rt} p(S, t) dt} - r(1-R). \tag{2.1}
\end{aligned}$$

The above derivation has used the fact that $p(S, 0) = 1$, for $S \neq L$. It should also be noticed that the spread of the CDS will be zero when the no-default probability remains 1.

2.2 New default models

Clearly, once $e^{-rt} p(S, t)$ for any $t \in [0, T]$ is determined, the price of the CDS contract can be obtained straightforwardly through (2.1). Hence, the left work is to calculate $e^{-rt} p(S, t)$.

As pointed out previously, most of the work in the literature adopts the so-called first-passage time model for credit evaluation [7, 10]. Under this model, a firm defaults when its asset value first hits the default barrier L . For the completeness of the paper, the derivation of $p(S, t)$ is provided in Appendix A. From the appendix, it is clear that the spread of the CDS is equal to zero when $S \rightarrow \infty$ or $T \rightarrow 0$, and is meaningless when the reference asset hits the default barrier. Indeed, $S = L$ can be viewed as a singularity of the CDS price under the first-passage time model.

However, it should be pointed out that the default assumption under the first-passage time model is too rigid, because fluctuation is a common feature of the price of the reference asset. The price may happen to fall below the default barrier and will return back quickly. Therefore, to be more close to the real situation, we assume that the default would occur only if the price of the reference asset stays below the default barrier for a pre-described

period of time.

Under this new default mechanics, a new state variable J called the barrier time, is needed to record the time the asset price is spent below the default barrier L . Like the Parisian and *Parasian* options, there are two ways for J to record the time. For the Parisian default mechanics, the state variable J starts to accumulate values at the same rate as the passing time t when the reference is below L , and is reset to zero and remains zero each time the asset price crosses L . Mathematically, it can be expressed as:

$$\begin{cases} J = 0, & dJ = 0, & S > L, \\ dJ = dt, & & S \leq L. \end{cases}$$

If the barrier time is not reset to zero each time the reference price crosses L , this type is referred to as the *Parasian* type. In both cases, when J reaches the trigger value $\bar{J}$, the company will default. We remark that under the new default assumption, the no-default probability will not be zero at the default barrier, and thus calculating the spread of the CDS in this case is still meaningful. Indeed, as will be shown in later sections, the Parisian-type default mechanics has made the no-default probability curve smoother, resulting in $S = L$ no long the singularity. It should also be remarked that, under the current model, the spread of the CDS is still zero when $S \rightarrow \infty$ or $T \rightarrow 0$. These two zero-spread cases are indeed “model-free”, and are in line with the financial clause set for a CDS contract.

With the introduction of the new default mechanics, the probability of no default before the current time t could be expressed as

$$p(t) = P(J_s < \bar{J}, \forall 0 \leq s \leq t) = E(I_{J_s < \bar{J}, \forall 0 \leq s \leq t}).$$

Consequently, we obtain

$$e^{-rt}p(t) = e^{-rt}P(J_s < \bar{J}, \forall 0 \leq s \leq t) = e^{-rt}E(I_{J_s < \bar{J}, \forall 0 \leq s \leq t}). \quad (2.2)$$

Now, denote $V(S, T-t) = e^{-rt}p(t)$. From (2.2), it is clear that $V(S, t) = e^{-r(T-t)}E(I_{J_s < \bar{J}, \forall 0 \leq s \leq T-t})$. Therefore, for any $t \in [0, T]$, $V(S, t)$ is the price of a Parisian-type down-and-out binary option with t being the current time and S being the underlying price.

From the relationship between the no-default probability and the price of the CDS, it is clear that the pricing of CDS is now equivalent to that of a Parisian-type down-and-out binary option with barrier L and the trigger device J . Once we have obtained the price of such a particular option, the spread of the CDS could be determined straightforwardly through (2.1).

In the following subsection, the PDE system governing the price of this special option will be established, and the numerical scheme to solve for the established system will also be presented.

2.3 Parisian-type binary options

As demonstrated in the last subsection, the valuation of the CDS under the new default model has now transferred to the pricing of a Parisian-type down-and-out binary option. To solve for the option price, the governing PDE system needs to be established, which will be the main issue of this subsection. After the PDE system is established, appropriate numerical schemes will also be designed and presented.

We now assume that the price of the reference asset S follows a geometric Brownian motion as

$$\frac{dS}{S} = rdt + \sigma dW_t, \quad (2.3)$$

where σ is the volatility and W_t is a standard Brownian motion. Let V_1 and V_2 be the price of the option for $S > L$ and $S \leq L$, respectively. By using a similar argument as shown in

[12], we obtain, for the Parisian case, V_1 and V_2 satisfy

$$\mathcal{A}_1 \begin{cases} \frac{\partial V_1}{\partial t} + \mathbb{L}V_1 = 0, \\ V_1(S, T - \bar{J}) = V_{BS}^{binary}(S, \bar{J}), \\ \lim_{S \rightarrow \infty} V_1(S, t) = e^{-r(T-t)}, \\ \lim_{S \rightarrow L} V_1(S, t) = \lim_{S \rightarrow L} V_2(S, t, 0), \end{cases} \quad \mathcal{A}_2 \begin{cases} \frac{\partial V_2}{\partial t} + \frac{\partial V_2}{\partial J} + \mathbb{L}V_2 = 0, \\ V_2(S, t, \bar{J}) = 0, \\ \lim_{S \rightarrow 0} V_2(S, t, J) = 0, \\ \lim_{S \rightarrow L} V_2(S, t, J) = \lim_{S \rightarrow L} V_2(S, t, 0) = \lim_{S \rightarrow L} V_1(S, t), \end{cases}$$

$$\text{Connectivity condition : } \lim_{S \rightarrow L} \frac{\partial V_1}{\partial S}(S, t) = \lim_{S \rightarrow L} \frac{\partial V_2}{\partial S}(S, t, 0). \quad (2.4)$$

Here, $\mathcal{A}_1$ is defined on $t \in [0, T - \bar{J}]$, $J \in [0, \bar{J}]$, $S \in [L, \infty)$, and $\mathcal{A}_2$ is defined on $t \in [J, T - \bar{J} - J]$, $J \in [0, \bar{J}]$, $S \in [0, L]$. The operator $\mathbb{L}$ is defined as

$$\mathbb{L} = \frac{1}{2}\sigma^2 \frac{\partial^2}{\partial S^2} + rS \frac{\partial}{\partial S} - rI,$$

with I being the identity operator.

Contrary to the “resetting” feature of the Parisian case, the *Parasian* specification has no reset of the barrier time J . Again, following the argument in [12], a properly closed PDE system governing the price of the *Parasian* down-and-out binary option can be found as

$$\mathcal{A}_1 \begin{cases} \frac{\partial V_1}{\partial t} + \mathbb{L}V_1 = 0, \\ V_1(S, T - \bar{J} + J; J) = V_{BS}^{binary}(S, \bar{J} - J), \\ \lim_{S \rightarrow \infty} V_1(S, t; J) = e^{-r(T-t)}, \\ \lim_{S \rightarrow L} V_1(S, t; J) = \lim_{S \rightarrow L} V_2(S, t, J), \end{cases} \quad \mathcal{A}_2 \begin{cases} \frac{\partial V_2}{\partial t} + \frac{\partial V_2}{\partial J} + \mathbb{L}V_2 = 0, \\ V_2(S, t, \bar{J}) = 0, \\ \lim_{S \rightarrow 0} V_2(S, t, J) = 0, \\ \lim_{S \rightarrow L} V_2(S, t, J) = \lim_{S \rightarrow L} V_1(S, t; J), \end{cases}$$

$$\text{Connectivity condition : } \lim_{S \rightarrow L} \frac{\partial V_1}{\partial S}(S, t; J) = \lim_{S \rightarrow L} \frac{\partial V_2}{\partial S}(S, t, J), \quad (2.5)$$

Note that $\mathcal{A}_1$ is defined on $t \in [J, T - \bar{J} + J]$, $J \in [0, \bar{J}]$, $S \in [L, \infty)$, and $\mathcal{A}_2$ is defined on $t \in [J, T - \bar{J} + J]$, $J \in [0, \bar{J}]$, $S \in [0, L]$.

It should be pointed out that both (2.4) and (2.5) can be solved analytically by using a similar approach as adopted in [12]. We still prefer to use numerical approaches here because the main concern of the current work is to study how the Parisian-type default mechanics will affect the CDS price. Also, numerical approaches might be more efficient than the analytical scheme for the current case, as pointed out in [12].

To solve for (2.4) and (2.5) effectively, the following transforms are adopted: $x = \ln S$, $\tau = T - t$, and $JJ = \bar{J} - J$. Our computation is then discretized with $N + 1$ uniformly-sized grids in the x direction, and $M + 1$ uniformly-sized grids in both the τ and JJ directions. Therefore, by adopting the θ scheme, we obtain,

$$V_{1,i}^{n+1} = V_{1,i}^n - \theta[\alpha_p \Delta \tau V_{1,i+1}^{n+1} + \beta \Delta \tau V_{1,i}^{n+1} + \alpha_m \Delta \tau V_{1,i-1}^{n+1}] - (1 - \theta)[\alpha_p \Delta \tau V_{1,i+1}^n + \beta \Delta \tau V_{1,i}^n + \alpha_m \Delta \tau V_{1,i-1}^n],$$

for $i = \bar{i}, \dots, N$, where $\bar{i}$ is the smallest integer that exceeds $\frac{\ln(L)}{\Delta x}$, $\alpha_p = -\frac{\sigma^2}{2(\Delta x)^2} + \frac{\sigma^2 - 2r}{2\Delta x}$, $\beta = \frac{\sigma^2}{(\Delta x)^2} + r$, and $\alpha_m = -\frac{\sigma^2}{2(\Delta x)^2} - \frac{\sigma^2 - 2r}{2\Delta x}$. On the other hand, for V_2 , we have

$$\begin{aligned} V_{2,i,j+1}^{n+1} &= V_{2,i,j}^n - \theta[\alpha_p \Delta \tau V_{2,i+1,j+1}^{n+1} + \beta \Delta \tau V_{2,i,j+1}^{n+1} + \alpha_m \Delta \tau V_{2,i-1,j+1}^{n+1}] \\ &\quad - (1 - \theta)[\alpha_p \Delta \tau V_{2,i+1,j+1}^n + \beta \Delta \tau V_{2,i,j+1}^n + \alpha_m \Delta \tau V_{2,i-1,j+1}^n]. \end{aligned}$$

For the Parisian case, the discretized values of V_1 and V_2 are connected by $V_{1,\bar{i}}^n = V_{2,\bar{i},1}^n$, whereas for the Parasian case, this discretized reset condition is not required.

Once we have worked out all the values of V_1 and V_2 , the spread of the CDS at the point $(i\Delta x, n\Delta \tau, j\Delta JJ)$ can be approximated by

$$c_{i,j}^n = \frac{2(1 - R)[1 - V_{i,j}^1]}{\Delta \tau (V_{i,j}^1 + 2 \sum_{m=2}^M V_{i,j}^m + V_{i,j}^{M+1})} - r(1 - R).$$

With the numerically approximated CDS prices available, we shall conduct some analysis on how the Parisian-type default mechanics will affect the CDS price. This will be the main concern of the next section.

3 Numerical examples and discussions

In this section, we shall present the numerical results as well as some useful discussions. Our emphasis will be put on studying how the Parisian-type default mechanics affects the CDS price. Unless otherwise stated, the values of the parameters adopted in this section are listed as follows. The risk-free interest rate r and the volatility σ are equal to 0.05 and 0.1 respectively. The default barrier L is set to 12 and the trigger time $\bar{J}$ is 0.5(year). The CDS contract will be expired in $T = 1$ (year).

As demonstrated earlier, the key step in solving the CDS price is the determination of the no-default probability, which is equivalent to the pricing of down-and-out binary options. Therefore, we shall first examine how the Parisian-type default mechanics will affect that option price. Depicted in Fig 1 is the comparison among the down-and-out binary option prices with and without Parisian-type features. Clearly, the option price without Parisian-type features, denoted as the BS price, is the lowest, while the Parisian price is always higher than the *Parasian* one for any given J . This is quite reasonable, because the “out” feature is weakened by the Parisian-type features, and thus Parisian-type options are less vulnerable to be knocked out than the corresponding option under the BS model. Consequently, their values should be higher than the BS price, if all the other terms are the same. Similarly, since the “out” feature is amplified by the *Parasian* feature, the Parisian price should be higher than the corresponding *Parasian* price.

Now, we turn to investigate the impacts of the new default mechanics on the CDS price. In Fig 2, we display the CDS prices as a function of the reference asset S with both the Parisian and *Parasian* default mechanics. From these two figures, it is clear that the CDS

Figure 1: Comparison of the down-and-out binary options at $J = 0$.

price is a decreasing function of the price of the reference asset S . This can be explained by the fact that when S is increasing, the no-default probability becomes smaller, as shown in Fig 1, resulting in the CDS being less useful in protecting the company from default. On the other hand, these two figures also suggest that the CDS price is negatively correlated with the barrier time J . This is also reasonable, because the company is more likely to default as J increases. In this case, the CDS will be more demanded and its price will increase. One should note that due to the reset mechanics, the Parisian specification provides a smooth transition only at $J = 0$, and moreover, its Delta value increases dramatically to infinity as the barrier time becomes closer to the trigger value $\bar{J}$, as shown in Fig 2(a). In contrast, for the *Parasian* case, the CDS price is smooth across L for all J values, as shown in Fig 2(b), simply because J does not need to be reset each time S crosses L .

The introduction of Parisian-type default mechanics brings in a new parameter called the trigger value $\bar{J}$. Therefore, it is necessary to study its impact on the CDS prices, which is presented in Fig 3 (a) and Fig 3 (b) for the Parisian and *Parasian* cases, respectively. In these two figures, the CDS price is displayed as a function of the current time t with

Figure 2: CDS prices with Parisian-type default mechanics at different J values

different values of $\bar{J}$. Clearly, the CDS price increases when $\bar{J}$ decreases. In addition, it will approach to the corresponding BS price when $\bar{J}$ becomes closer to zero. This could be explained by the fact that smaller $\bar{J}$ value will enlarge the probability of the occurrence of the default of the company, resulting in more expensive CDS price. On the other hand, one can also observe from these two figures that the CDS price is not a monotonic function of the current time t . This is also reasonable, as the current time is not correlated with the no-default probability, and thus it cannot affect the CDS price.

Depicted in Fig 4 is the comparison of the CDS prices calculated from the first-passage time model and the Parisian-type default models. It can be clearly observed from this figure that the *Parasian* specification has made the CDS more expensive than the corresponding Parisian case, and moreover, the CDS price calculated directly from the first-passage time model is the highest. Financially, the Parisian-type feature makes the firm less likely to default than the first-passage time model, and thus the CDS price under the latter is the highest to against default. Furthermore, the cumulative feature of the *Parasian* case will result in smaller no-default probability, and thus the *Parasian* CDS price is expected to become higher than the Parisian one.

Before closing this section, we point out that a model calibration process is required before the new default model can be safely applied to real financial markets. Such a process is often carried out based on closed-form analytical solutions rather than numerical

Figure 3: Comparison of the CDS prices under the first-passage time model and Parisian-type default models. The time barrier $J = 0.5$, and the current reference asset price is $S = 13$.

Figure 4: CDS prices at $J = 0$ and $S = 13$ with different trigger values $\bar{J}$.

approaches. This is because the latter often produce unavoidable systematic errors and require a bit of time to be implemented, probably resulting the model calibration process extremely time-consuming and the results unreliable. However, in the current work, the prices are determined purely numerically. Moreover, the main aim of the current work is to analyze the new default mechanics from a theoretic point of view. In view of these, we shall leave the calibration of the model and empirical studies to a forthcoming paper, in which the closed-form analytical solution for the CDS price is to be derived by using the “moving window” technique proposed in [12].

4 Conclusion

In this paper, we propose Parisian-type default mechanics for CDS contracts by assuming that the default would only occur if the price of the reference asset stays under a certain level for a pre-described period of time. By establishing an equivalence between the prices of the CDS and Parisian-type down-and-out binary options, the required CDS price is finally obtained through a θ scheme. Numerical experiments suggest that the CDS price is significantly affected by the newly proposed default mechanics. Since this is the first time that a Parisian-type default is applied to model the credit risk, several future research directions are expected. Firstly, it is very promising to derive closed-form analytical solution for the CDS price by using the “moving window” technique proposed in [12]. With this kind of solution, the calibration of the new default model will be greatly facilitated. Secondly, we will continue to analyze default correlations under the new default assumption. Last but by no means the least, in addition to the Parisian-type default mechanics, other market factors, such as regime-switching and stochastic volatility, will also be taken into consideration.

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6 Appendix A

In this appendix, we shall derive the CDS price under the first-passage time model.

Under this model, a firm defaults when its asset value first hits the default barrier [7]. Let $X_t = \ln S_t$ and τ be the first time that the reference asset hits the default barrier L , i.e., $\tau = \inf\{t \geq 0, X(t) \leq \ln L\}$. In addition, let $f(y, t; x)$ be the first-passage probability density function of X_t starting at x , i.e.,

$$f(y, t; x)dy = \text{Prob}\{y \leq X(t) \leq y + dy, t \leq \tau \mid X(0) = x\}.$$

Therefore, the no-default probability before time t can be expressed as

$$p(x, t) = \int_R I(y \geq \ln L) f(y, t; x) dy,$$

where $I(\cdot)$ is the indicator function. According to the Fokker-Planck equation of $f(y, t; x)$, it is not difficult to show that the no-default probability p satisfies

$$\begin{cases} \frac{\partial p}{\partial \tau} = \frac{1}{2}\sigma^2 \frac{\partial^2 p}{\partial x^2} + (r - \frac{1}{2}\sigma^2) \frac{\partial p}{\partial y}, \\ p(x, 0) = I(x \geq \ln L), \\ p(\ln L, t) = 0. \end{cases}$$

By solving the above PDE system, the no-default probability under the first-passage time can be derived as

$$p(S, t) = N(d_1) - \left(\frac{L}{S}\right)^{\frac{2r}{\sigma^2}-1} N(d_2), \quad (6.1)$$

where $d_1 = \frac{\ln S - \ln L + (r - \frac{1}{2}\sigma^2)t}{\sigma\sqrt{t}}$, and $d_2 = \frac{\ln L - \ln S + (r - \frac{1}{2}\sigma^2)t}{\sigma\sqrt{t}}$. With p available, the CDS price can finally be determined through (2.1).

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