Cross-domain password-based authenticated key exchange revisited

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Abstract
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Cross-Domain Password-Based Authenticated Key Exchange Revisited

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We revisit the problem of secure cross-domain communication between two users belonging to different security domains within an open and distributed environment. Existing approaches presuppose that either the users are in possession of public key certificates issued by a trusted certificate authority (CA), or the associated domain authentication servers share a long-term secret key. In this article, we propose a generic framework for designing four-party password-based authenticated key exchange (4PAKE) protocols. Our framework takes a different approach from previous work. The users are not required to have public key certificates, but they simply reuse their login passwords, which they share with their respective domain authentication servers. On the other hand, the authentication servers, assumed to be part of a standard PKI, act as ephemeral CAs that certify some key materials that the users can subsequently use to exchange and agree on as a session key. Moreover, we adopt a compositional approach. That is, by treating any secure two-party password-based key exchange (2PAKE) protocol and two-party asymmetric-key/symmetric-key-based key exchange (2A/SAKE) protocol as black boxes, we combine them to obtain generic and provably secure 4PAKE protocols.

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1. INTRODUCTION

There are many cross-domain communication scenarios, such as email communication, mobile phone communication, and instant messaging, where the information being communicated may need to be protected against both passive and active attackers. In these scenarios, a user is typically registered to some kind of domain server, such as email exchange server or home location register (in the cases of email and mobile phone communications, respectively). Moreover, two communicating parties from different domains very often neither share a password nor possess a public key certificate. Hence, although two-party and three-party authenticated key exchange protocols have been extensively studied and widely deployed in the real world, see for example Neuman and Tso [1994], Jablon [1996], Bellare et al. [2000a], Boyko et al. [2000], and

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Abdalla et al. [2005], it is not clear how they can be directly applied to establish a secure cross-domain communication channel.

In this work, we consider the use of an authenticated key exchange protocol to establish a session key such that two users can securely transmit information from one domain to another. We assume that each security or administrative domain\(^1\) has (at least) a trusted domain server acting as an authentication server governing a group of users. Each user within the domain shares only a password with the server and the user does not necessarily own a public key certificate. Further, we assume that a domain server makes available its public key to other domain servers in the form of a public key certificate, that is, the server is connected to a public key infrastructure (PKI). In such a setting, we focus on enabling a user from one domain to establish a secure communication channel with another user from a different domain through their respective domain servers. Our approach makes use of both password-based and public-key cryptographic techniques for authentication and key exchange. We call this work four-party password-based authenticated key exchange (4PAKE) instead of client-to-client password-based authenticated key exchange (C2C-PAKE), as suggested in the literature [Byun et al. 2002; Yin and Bao 2006], because we thought that the latter might be confused with 3-party C2C-PAKE (in which both communicating parties are registered to the same domain server). We believe that our aforementioned communication model is more realistic, user-friendly, and scalable than that of related previous work, for example, public key Kerberos [Hur et al. 2001; Zhu and Tung 2006] and C2C-PAKE [Byun et al. 2007; Yin and Bao 2006]. The later protocols either require a user to obtain a public key certificate, or assume that the domain servers corresponding to the communicating users share a long-term secret key. We elaborate more on previous work in Section 2 and their limitations in Section 4.

The primary contribution of this article is a new generic framework for designing 4PAKE protocols that address the aforementioned requirements and limitations of Kerberos and C2C-PAKE. Our framework is based on a compositional approach and can be regarded as a form of compiler combining and transforming two building blocks—(1) a secure two-party password-based authenticated key exchange (2PAKE) protocol, and (2) a secure two-party asymmetric-key or symmetric-key authenticated key exchange (2A/SAKE) protocol—into a secure four-party password-based authenticated key exchange protocol. We describe the cryptographic primitives and building blocks used in this work in Section 3 and give an overview of our approach in Section 4. The details of our new framework and some concrete instantiations of generic 4PAKE protocols are presented in Section 5.

Moreover, we define a security model for our generic 4PAKE protocols based on the Real-Or-Random (ROR) security model [Abdalla et al. 2005] and show that our protocols are secure in the model. Our security definitions and analyses are shown in Sections 6 and 7, respectively. Our security proofs rely solely on the security properties of the cryptographic primitives on which our protocols are based, and do not make use of the Random Oracle model [Bellare and Rogaway 1993a]. We also implement our protocols and provide some empirical performance evaluation in Section 8. Our experimental results show that all of the 4PAKE protocols have only slightly higher (between 1.05\(\times\) and 1.1\(\times\)) client-side computational overhead than that of public key Kerberos. However, in return, our protocols are approximately 1.1\(\times\) to 1.2\(\times\) faster than public key Kerberos from the server’s perspective, and thus, can scale better. Further, our approach is more usable in the sense that the user only needs to remember a password, and flexible in terms of reuse of existing two-party key exchange protocols.

\(^1\)Here a domain is a logical network that defines a group of system users, computers, and/or servers, such that it is easy for a system administrator to enforce security policies and to protect these entities.
2. PREVIOUS WORK

The design and analysis of authentication and key exchange protocols are fundamental research problems in information security and have been extensively studied over the last three decades (see for example Diffie and Hellman [1976], Lampson et al. [1992], Bellare and Rogaway [1993b], Wobber et al. [1994], Law et al. [2003], and many others). In this article, we focus on authenticated key exchange protocols for protecting cross-domain communication between two users belonging to different security domains.

2.1. Kerberos

Initially developed by an MIT research team led by Miller and Neuman, Kerberos\(^2\) [Neuman and Ts'o 1994; Neuman et al. 2005] is now a widely deployed network authentication protocol. The most current version, Kerberos 5, is supported by all major operating systems, including Solaris, Linux, MacOS, and Microsoft Windows.

In Kerberos, each domain (also known as realm) is governed by a Key Distribution Center (KDC), which in turn, provides user authentication and ticket-granting services. Each user shares a password with its KDC, while local application servers that are accessible to the user share (long-term) symmetric keys with the KDC. Kerberos then allows single sign-on that authenticates clients to multiple networked services, such as remote hosts, file servers, and print spoolers. This can be summarised in three rounds of communication between the client (typically acting on behalf of a user), and different principals as follows:

1. The client first performs a password-based login to its local KDC, i.e., authentication server, and obtains a ticket-granting ticket (TGT);
2. The TGT is then forwarded to a ticket-granting server in order to obtain a service ticket;
3. The client finally presents the service ticket to the application servers to get access to networked services.

This standard Kerberos protocol makes use of highly efficient symmetric key techniques. However, one security weakness is that a password-derived symmetric key is used in the first round of the protocol between the client and the KDC. This opens up the possibility of allowing a passive attacker to eavesdrop the protocol messages (transmitted in the first round) and perform an off-line password guessing attack. In other words, the strength of the user authentication may be only as strong as the user’s ability to choose and remember a strong password.

Public key cryptography for initial authentication in Kerberos (PKINIT) has thus been proposed by Zhu and Tung [2006] to add flexibility, security, and administrative convenience by replacing the password-based authentication with signature-based authentication between the client and the KDC. (The symmetric key operations in the second and third rounds of the protocol are retained.) The client and the KDC do not share a secret now. Each of them is assigned a public-private key pair instead, and they must then generate their respective signatures over the messages communicated in the first round. While the PKINIT extension offers stronger user authentication, it adds complexity to the protocol since we now require a public key infrastructure (PKI) and each user needs to manage her public-private key pair.

Moreover, Kerberos can be used to achieve cross-realm authentication (PKCROSS) by using public key techniques. This is useful when a client from a domain wishes to access networked services offered by another domain (that is governed by a remote domain).
KDC). Here, the two corresponding KDCs exchange messages following closely the
PKINIT specification [Zhu and Tung 2006]. This avoids the unnecessary administrative
burden of maintaining cross-realm, shared symmetric keys. The (simplified) basic
PKCROSS protocol is as follows [Hur et al. 2001].

(1) The client submits a request to its local KDC for credentials associated with the
remote realm.3
(2) The local KDC submits a request (using the standard PKINIT) to the remote KDC
to obtain a cross-realm TGT.
(3) The remote KDC responds as with PKINIT, and the local KDC passes the cross-
realm TGT to the client.
(4) The client then submits a request directly to the remote KDC and proceeds with
the second and third rounds of the standard Kerberos protocol using symmetric
key techniques.

Our work is closely related to PKCROSS, in the sense that we also deal with cross-
domain authentication and secure communication. However, our proposal of 4PAKE is
based on rather different design principles. We will elaborate on this in Section 4.

2.2. Client-to-Client Key Exchange

The idea of extending password-based key exchange between two users from the same
domain to the cross-domain setting was also studied by Byun et al. [2002], and Yin and
Bao [2006]. It was often known as client-to-client password-based authenticated key
exchange (or key agreement), thus the acronym C2C-PAKE (or C2C-PAKA). The key
concept of C2C-PAKE is based on the cross-realm Kerberos protocol [Neuman and Ts’o
1994], in which each realm (or domain) has a KDC, and two users from two distinct
realms establish a common secret key through their respective KDCs. We note that,
however, existing proposals for C2C-PAKE make use of a symmetric key approach.
Both KDCs that are involved in a C2C-PAKE protocol run are assumed to be sharing a
long-term symmetric key. This leads to a similar limitation in symmetric key manage-
ment that PKCROSS aims to address. More recent work on C2C-PAKE with improved
efficiency and/or security can be found in Byun et al. [2007], Feng and Xu [2009], and
Wu and Zhu [2009].

3. PRELIMINARIES

3.1. Cryptographic Primitives

We first describe some cryptographic primitives required for our protocols. In our
description, we let \( \kappa \) denote a security parameter.

**Message Authentication Codes.** A message authentication code (MAC) scheme is a
tuple of polynomial-time algorithms \((\text{GEN}, \text{MAC}, \text{VER})\) such that:

- \( \text{GEN}(1^\kappa) \), the key generation algorithm, takes as input the security parameter \( 1^\kappa \)
  and outputs a key \( K \);
- \( \text{MAC}(K; m) \), the MAC tag generation algorithm, takes as input a key \( K \) and a mes-
  sage \( m \in \{0, 1\}^* \), and outputs a tag \( \mu \);
- \( \text{VER}(K; m; \mu) \), the verification algorithm, takes as input a key \( K \), a message \( m \) and
  a tag \( \mu \); it outputs 1 if \( \mu \) is a valid tag for message \( m \) under key \( K \), or 0 otherwise.

3Note that the local KDC can authenticate the client using a password-based approach or PKINIT. However,
as explained, the latter is usually a preferred choice since it is more secure than the former.

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We consider security against strong existential unforgeability under a chosen-message attack (SUF-CMA). The adversary attacking a MAC scheme should not be able to create a new valid message-tag pair with non-negligible probability, even after seeing many such valid pairs [Bellare et al. 2000a]. Let $\text{SUCC}_{\text{MAC}}$ denote the event in which the adversary, $\mathcal{A}$, is able to output a message $m$ along with a tag $\mu$ such that: (1) $\text{VER}(K;m;\mu) = 1$, and (2) $\mathcal{A}$ had not previously requested a tag $\mu$ on the message $m$. (Clearly, we assume that $\mathcal{A}$ has no knowledge of the key, $K$.) The advantage of $\mathcal{A}$ in violating the strong existential unforgeability of the MAC scheme under chosen-message attacks [Bellare et al. 2000a] is defined as $\text{Adv}_{\text{MAC}}^{\text{suf-cma}}(\mathcal{A}) = \Pr[\text{SUCC}_{\text{MAC}}]$. The associated advantage function, $\text{Adv}_{\text{MAC}}^{\text{suf-cma}}(t, q_{\text{mac}}, q_{\text{ver}})$, is then defined as the maximum value of $\text{Adv}_{\text{MAC}}^{\text{suf-cma}}(\mathcal{A})$ over all $\mathcal{A}$ with time-complexity at most $t$, and asking at most $q_{\text{mac}}$ and $q_{\text{ver}}$ queries to the tag generation and verification oracles, respectively.

**Digital Signatures.** A signature scheme is a tuple of polynomial-time algorithms $(\text{GEN}, \text{SIG}, \text{VER})$ satisfying the following.

- $\text{GEN}(1^n)$, the key generation algorithm, takes as input the security parameter $1^n$ and outputs a pair of public/private keys $(pk, sk)$;
- $\text{SIG}(sk;m)$, the signing algorithm, takes as input a private key $sk$ and a message $m \in \{0, 1\}^*$, and outputs a signature $\sigma$;
- $\text{VER}(pk;m;\sigma)$, the verification algorithm, takes as input a public key $pk$, a message $m$, and a signature $\sigma$; it outputs 1 if $\sigma$ is a valid signature for message $m$ under key $sk$, or 0 otherwise.

A signature scheme is considered secure against existential unforgeability under an adaptive chosen-message attack (EUF-CMA), if the adversary attacking the scheme could not create a new valid message-signature pair with non-negligible probability without knowing the corresponding signing key. This is so even if the adversary is allowed to ask for signing of multiple messages chosen adaptively [Goldwasser et al. 1988]. Let $\text{SUCC}_{\text{Sig}}$ denote the event in which adversary $\mathcal{A}$ is able to output a forged signature $\sigma$ for a message $m$ such that: (1) $\text{VER}(pk;m;\sigma) = 1$, and (2) $\mathcal{A}$ had not previously requested a signature on the message $m$ from the signing oracle. The advantage of $\mathcal{A}$ in violating the existential unforgeability of the signature scheme under adaptive chosen-message attacks [Goldwasser et al. 1988] is defined as $\text{Adv}_{\text{Sig}}^{\text{euf-cma}}(\mathcal{A}) = \Pr[\text{SUCC}_{\text{Sig}}]$. The associated advantage function, $\text{Adv}_{\text{Sig}}^{\text{euf-cma}}(t, q_{\text{sig}}, q_{\text{ver}})$, is then defined as the maximum value of $\text{Adv}_{\text{Sig}}^{\text{euf-cma}}(\mathcal{A})$ over all $\mathcal{A}$ with time-complexity at most $t$, and asking at most $q_{\text{sig}}$ queries to the signing oracle and at most $q_{\text{ver}}$ queries to the verification oracle.

**Authenticated Encryption.** An authenticated encryption (AE) scheme (a symmetric-key encryption scheme that provides both privacy and authenticity/integrity) is a tuple of polynomial-time algorithms $(\text{GEN}, \text{ENC}, \text{DEC})$ satisfying the following.

- $\text{GEN}(1^n)$, the key generation algorithm, takes as input the security parameter $1^n$ and outputs a symmetric key $sk$;
- $\text{ENC}(sk;m)$, the encryption algorithm, takes as input a secret key $sk$ and a message $m \in \{0, 1\}^*$, and outputs a ciphertext $c$;
- $\text{DEC}(sk;c)$, the decryption algorithm, takes as input a secret key $sk$ and a ciphertext $c$; it outputs $m$ if $c$ is authenticated, or 0 otherwise indicating an invalid ciphertext.

In authenticated encryption, we consider both privacy and authenticity/integrity [Bellare and Namprempre 2000]. In terms of privacy, we consider the conventional
We define the advantage of an adversary against a key derivation function as 
\[ \text{Adv}_{\text{KD}}(D) = |\Pr[D^{\text{KD}(K, \ell, n)}(\kappa, \ell, n) = 1] - \Pr[D^{\text{RF}(\cdot)}(\kappa, \ell, n) = 1]|, \]
where \( t \) denotes the maximum running time of \( D \) and \( q_{\text{kd}} \) is the maximum number of oracle queries \( D \) is allowed to make. The associated advantage function, \( \text{Adv}_{\text{KD}}(D, q_{\text{kd}}) \), is then defined as the maximum value of \( \text{Adv}_{\text{KD}}(D) \) over all \( D \) with time-complexity at most \( t \), and asking at most \( q_{\text{kd}} \) oracle queries.

Noninteractive Key Derivation. A noninteractive key derivation scheme consists of two polynomial-time algorithms (\( \text{GEN}, \text{NKD} \)) satisfying the following:

- \( \text{GEN}(1^n) \), the key generation algorithm, takes as input the security parameter \( 1^n \) and outputs a pair of public/private keys \( (pk, sk) \);
- \( \text{NKD}(pk_A, sk_B) \), the noninteractive key derivation function, takes as input a public key \( pk_A \) and a private key \( sk_B \), and outputs a symmetric key \( K_{AB} \in \{0, 1\}^n \).

There are two security requirements involved in a noninteractive key derivation scheme:

- completeness, \( \text{NKD}(sk_A, pk_B) = \text{NKD}(sk_B, pk_A) \);  
- key privacy, \( \{pk_A, pk_B, \text{NKD}(sk_A, pk_B)\} \) is computationally indistinguishable from \( \{pk_A, pk_B, U_n\} \), where \( U_n \) denotes a string selected uniformly at random from \( \{0, 1\}^n \).

We define the advantage of an adversary \( D \) against a noninteractive key derivation scheme as
\[ \text{Adv}_{\text{NKD}}(D) = |\Pr[D(pk_A, pk_B, \text{NKD}(sk_A, pk_B)) = 1] - \Pr[D(pk_A, pk_B, U_n) = 1]|. \]

The associated advantage function, \( \text{Adv}_{\text{NKD}}(D, t) \), is then defined as the maximum value of \( \text{Adv}_{\text{NKD}}(D) \) over all \( D \) with time-complexity at most \( t \).
3.2. Two-Party Authenticated Key Exchange

A two-party authenticated key exchange (2AKE) protocol consists of two probabilistic polynomial time algorithms.

- $\text{GEN}(1^\kappa)$, the key generation algorithm, takes as input the security parameter $1^\kappa$ and outputs a long-term key $K_U$ for entity $U$;
- $\text{P} (K_A, K_B)$, a protocol execution algorithm that can be executed by two distinct parties holding $K_A$ and $K_B$, respectively.

Depending on the type of long-term key, one can divide 2AKE protocols into three major categories: (1) two-party password-based authenticated key exchange (2PAKE), (2) two-party asymmetric-key authenticated key exchange (2AAKE), and (3) two-party symmetric-key authenticated key exchange (2SAKE).

Many 2AKE protocols are extensions of the classic Diffie-Hellman key exchange [Diffie and Hellman 1976] by incorporating an authentication component into the protocol. Here authentication can be achieved by using passwords (2PAKE), asymmetric-keys (2AAKE), or symmetric-keys (2SAKE). Take for example, the 2PAKE protocol presented in Bellare et al. [2000a], which uses a password-derived key to encrypt the Diffie-Hellman key exchange message flows so that only the two users sharing the same password can decrypt the messages and compute the Diffie-Hellman key.

In what follows, we present the security model for each type of 2AKE protocol. We first give an overview of the Real-Or-Random (ROR) model for 2PAKE [Abdalla et al. 2005]. We then define a security model in the Find-Then-Guess (FTG) sense for 2AAKE and 2SAKE.

3.2.1. Password-Based. Let us first recall two existing security models related to password-based authenticated key exchange protocols: the Find-Then-Guess (FTG) and the Real-Or-Random (ROR) models.

The FTG model (sometimes also known as the BPR2000 model) was proposed by Bellare et al. [2000a] to measure the indistinguishability of a session key from a random key. In the FTG model, an adversary is allowed to pose multiple queries to a reveal oracle (in addition to other oracles, for example execute and send oracles). The reveal oracle is used to model the misuse of session keys by a user. However, the adversary is restricted to ask only a single query to the test oracle.

Abdalla et al. [2005] then proposed the ROR model, which is very similar to the FTG model, except that the former does not make use of a reveal oracle. This means that the adversary no longer has access to the reveal oracle to learn session keys of user instances. However, the adversary is allowed to pose as many test queries as it wishes to different instances. Note that in the ROR model, the test oracle (instead of the reveal oracle) is used to model the misuse of keys by a user.

We remark that the recently proposed ROR model is strictly stronger than the FTG model in the password-based setting.\(^4\) Hence, we adopt the ROR model for password-based protocols in this article.

In the 2PAKE setting, we assume that each protocol participant is either a client $C \in C$ or a server $S \in S$. The set of all users or participants $U$ is the union $C \cup S$. We also assume that each client $C \in C$ holds a password $pwd_C$, while each server $S \in S$ holds a vector $pwd_S = (pwd_C)_{C \in C}$ with an entry for each client [Bellare et al. 2000a]. Here, $pwd_C$ and $pwd_S$ are regarded as the long-lived keys of client $C$ and server $S$.

\(^4\) A protocol proved secure in the ROR model is also secure in the FTG model. The reverse, however, is not necessarily true. See Abdalla et al. [2005] for further details about the relation between the ROR and FTG models.
As with a typical security model, an adversary $\mathcal{A}$ interacts with protocol participants only via oracle queries. Such queries model the adversary’s capabilities in a real attack. During a protocol execution, there may be many concurrent running instances of a participant. We denote an instance $i$ of a protocol participant $U \in \mathcal{U}$ by $U^i$. A protocol session between a client instance and a server instance is labeled by a unique session identifier. As pointed out by Bellare et al. [2000b], a session identifier can be constructed based on the partial protocol messages exchanged between the client and the server instances before the acceptance. Two instances, $U^i_1$ and $U^i_2$, are said to be partners if the following conditions are met [Bellare et al. 2000a]:

1. both $U^i_1$ and $U^i_2$ accept,\(^5\)
2. both $U^i_1$ and $U^i_2$ share the same session identifiers;
3. the partner identifier for $U^i_1$ is $U^i_2$, and vice versa;
4. no instance other than $U^i_1$ and $U^i_2$ accepts with a session identifier equal to that of $U^i_1$ or $U^i_2$.

The oracle queries in the ROR security model for 2PAKE are then classified as follows [Abdalla et al. 2005].

— EXECUTE($C^i, S^j$). This query models a passive attack, in which the adversary eavesdrops on an honest execution of the protocol between a client instance $C^i$ and a server instance $S^j$. The output of the query comprises messages that were exchanged during the honest execution of the protocol.

— SEND($U^i, m$). This query models an active attack, in which the adversary may intercept a message and then either modify it, create a new one, or simply forward it to the intended participant. The output of the query is the message that the participant instance $U^i$ would generate upon receipt of message $m$.

— TEST($U^i$). This query models the misuse of a session key by a user. Let $b$ be a bit chosen uniformly at random at the beginning of an experiment defining indistinguishability in the ROR model. The output of the query is then the session key for participant instance $U^i$ if $b = 1$ or a random key from the same domain if $b = 0$. However, if no session key is defined for instance $U^i$, then return the undefined symbol $\bot$.

We note that the adversary is allowed to ask multiple queries to the TEST oracle in the ROR model (this is in contrast with the FTG model, which allows only a single query to the TEST oracle). All TEST queries must be made on fresh instances (which have not revealed their session keys) and they should be answered using the same value for the hidden bit $b$ (chosen at the beginning of the experiment). This implies that the keys returned by the TEST oracle are either all real or all random. Moreover, in the case where the returned key is random, the same random value should be returned for TEST queries that are asked to two instances, which are partnered [Abdalla et al. 2005]. The goal of the adversary is to guess the value of the hidden bit $b$ used to answer TEST queries. The adversary is considered successful if it guesses $b$ correctly. Let $\text{SUCC}$ denote the event in which an adversary is successful. The advantage of an adversary $\mathcal{A}$ in violating the indistinguishability of the 2PAKE protocol in the ROR sense is

$$\text{Adv}_{2\text{PAKE},D}^{\text{ROR}}(\mathcal{A}) = 2 \cdot \Pr[\text{SUCC}] - 1,$$

\(^5\)An instance $U^i$ goes into an accept mode after it has received the last expected protocol message.
when passwords are drawn from a dictionary $\mathcal{D}$. The associated advantage function is then
\[
\text{Adv}_{\text{2PAKE, } \mathcal{D}}^{\text{ror}}(t, q_e, q_s, q_t) = \max_{A} \{\text{Adv}_{\text{2PAKE, } \mathcal{D}}^{\text{ror}}(A)\},
\]
where the maximum is over all $A$, with time-complexity at most $t$, and making at most $q_e$ execute queries, $q_s$ send queries, and $q_t$ test queries.

We say that a 2PAKE protocol is secure in the ROR model if the advantage $\text{Adv}_{\text{2PAKE, } \mathcal{D}}^{\text{ror}}(t, q_e, q_s, q_t)$ is only negligibly larger than $cn/|\mathcal{D}|$, where $c$ is a constant, $n$ is the number of active sessions, and $|\mathcal{D}|$ is the size of the dictionary $\mathcal{D}$.

### 3.2.2. Asymmetric-Key

A two-party asymmetric-key based authenticated key exchange (2AAKE) protocol has a similar objective as with a 2PAKE protocol, that is, to agree on a session key between a pair of communication parties. The long-lived keys of each protocol participant $U \in \mathcal{U}$ are now, however, a public key $pk_U$ and the corresponding private key $sk_U$, instead of a password.

Generally speaking, an adversary in the Find-Then-Guess model is allowed to submit EXECUTE, SEND, REVEAL, CORRUPT, and TEST queries. The first two types of queries (EXECUTE and SEND) are similar to those for 2PAKE in the ROR model. The others are defined as follows [Bellare and Rogaway 1995].

- **REVEAL($U^i$)**. This query models leakage of information on specific session keys. If a session key is not defined for instance $U^i$ or if a TEST query was asked to either $U^i$ or its partner, then return $\bot$. Otherwise, return the session key held by the instance, a passive attack, in which the adversary eavesdrops on an honest execution of the protocol between a client instance $U^i$.

- **CORRUPT($U$)**. This query models the capability of an adversary being able to learn the long-term secrets of clients. The output of the query is the long-lived private key $sk_U$ of participant $U$.

- **TEST($U^i$)**. Let $b$ be a bit chosen uniformly at random at the beginning of an experiment defining indistinguishability in the FTG model. Denote $V^i$ the partner (if it exists) of $U^i$. If no session key for instance $U^i$ is defined, or if a REVEAL query was asked to either $U^i$ or $V^i$, or if the adversary has made a CORRUPT($U$) or CORRUPT($V$) query, then return $\bot$. Otherwise, the output of the query is the session key, for instance $U^i$ if $b = 1$ or a random key from the same domain if $b = 0$.

As explained before, the adversary can query only once to the TEST oracle. However, the goal of the adversary is till the same, that is, to guess the value of the hidden bit $b$ used to answer the TEST query. Let Succ denote the event in which an adversary guesses $b$ correctly. The advantage of an adversary $A$ in violating the indistinguishability of the 2AAKE protocol in the FTG sense, $\text{Adv}_{\text{2AAKE}}^{\text{ftg}}(A)$, and the associated advantage function $\text{Adv}_{\text{2AAKE}}^{\text{ftg}}(t, q_e, q_s, q_r, q_c)$ are then defined as in the password-based setting where $q_r$ and $q_c$ denote the maximum number of reveal and corrupt queries, respectively.

We say that a 2AAKE protocol is secure in the FTG model if the advantage $\text{Adv}_{\text{2AAKE}}^{\text{ftg}}(t, q_e, q_s, q_r, q_c)$ is negligible (in the associated security parameter).

### 3.2.3. Symmetric-Key

The security definition of two-party symmetric-key based authenticated key exchange (2SAKE) is almost identical to that of 2AAKE. The only

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6A session is said to be active if it involves SEND queries by the adversary.
difference is that in a 2SAKE protocol, each protocol participant $A \in U$ shares with another participant $B \in U \setminus \{A\}$, a symmetric long-lived secret key $K_{AB} (= K_{BA})$.

4. OUR MOTIVATIONS AND APPROACH

Our concern here is with secure communication between two users from different administrative or security domains. While this is not a new security problem per se, we observe that current approaches fall short of being able to offer a satisfactory solution.

A main design goal of Kerberos was to allow single sign-on such that a user is able to access multiple networked services without the need to repeatedly enter her login password. Hence, the emphasis is on authenticating a user once and allowing secure access to multiple services within a given time period. On the other hand, our work focuses on authenticated key agreement between two users of separate domains. This fundamental difference leads to other differences in terms of the protocol message flows and the security architecture. For example, in Kerberos, we assume that a user shares a password, while each application server shares a long-term symmetric key with their KDC; while in our case, we have a more symmetric situation, where each user shares a password with her respective domain authentication server.

Furthermore, Kerberos seems to be more suited to a rather closed distributed environment, where it is feasible to establish, distribute, and maintain long-term secret keys between a KDC and a group of application servers, and between the KDC and other remote KDCs. Although this is compensated with the PKINIT and PKCROSS extensions, PKIs are known to be difficult to deploy for various practical reasons, such as cost, registration process, trust establishment, key revocation, and management of user private keys [Ellison and Schneier 2000; Price 2005]. We believe that what is needed here is a PKI that is more user-friendly, that is, which hides the complexity of public key management from a user's viewpoint.

Existing work on C2C-PAKE does provide some useful insights on how to construct an efficient authenticated key agreement protocol between local and a remote user. Unfortunately, however, these protocols rely on long-term symmetric keys shared between KDCs. This may not be a practical and scalable approach, particularly in an open distributed environment, because establishment and management of shared keys between KDCs can be a complicated and costly process.7

From a security perspective, the complexity of a cross-domain authenticated key exchange often complicates its security analysis. As shown in Butler et al. [2006] and Backes et al. [2011], the security analysis of Kerberos, with or without PKINIT, is rather complex and involved. This sometimes may hinder a security flaw in the protocol from being detected early. For example, the IETF Internet Draft for PKINIT was first circulated in 1996, but it was only after almost a decade later when Cervesato et al. [2008] reported a man-in-the-middle attack in PKINIT. Similarly, designing a secure C2C-PAKE protocol seems to be a non-trivial task. Most of the C2C-PAKE protocols found in the literature have security flaws. These include offline password guessing attacks [Phan and Goi 2005; Wang et al. 2004], undetectable online password guessing and unknown key-share attacks [Phan and Goi 2006], insider attacks (by malicious clients and servers) [Chen 2003; Phan and Goi 2005, 2006], and password-compromise impersonation attacks [Cao et al. 2009]. Recent proposals [Feng and Xu 2009; Wu and Zhu 2009] attempt to address these security problems.

7Administrators must maintain separate keys for every domain which wishes to exchange authentication information with another domain, implying $n(n-1)$ keys for $n$ domains, or they must utilise a hierarchical arrangement of domains, which may increase network traffic and complicate the trust model by requiring evaluation of transited domains [Hur et al. 2001].
Taking all of these observations into consideration, we propose a generic 4PAKE framework and give a set of example protocols that we believe are suitable for secure cross-domain communication between two users. In our framework, intuitively, each domain server possesses a public key certificate that is publicly available to other domain servers (as with the case of Web servers used for many e-commerce or online applications). In a protocol run, the servers corresponding to two communicating users certify some key materials that are associated with the users so that the latter can subsequently exchange the key materials and agree on a session key. Clearly, we assume that the user trusts the remote server to only certify key materials submitted by an authenticated user. We achieve this through a 2PAKE protocol between the user and her domain server. Hence, the user needs to remember only a password and does not have to deal with public key management. Once both the users have received the certified key materials (in the form of a signature) from their respective servers, they exchange the key materials following a two-party AKE protocol.

Inspired by the work of Abdalla et al. [2005] on a compositional approach to three-party password-based authenticated key exchange, we adopt a similar approach to our 4PAKE protocol, which comprises 2PAKE and 2A/SAKE as the building blocks. This is to simplify the security analysis of our protocol. By making use of secure 2PAKE and 2A/SAKE, we treat them as black-boxes and our analysis then focuses only on the input and output parameters of these two-party protocols. Moreover, using a compositional approach, we have the flexibility to choose any secure 2PAKE and 2A/SAKE protocols, implying that one can simply build a 4PAKE protocol based on existing deployed 2PAKE and 2A/SAKE protocols. In fact, each domain may deploy a different 2PAKE protocol, yet a user from one domain can securely establish a secret key with another user of a different domain. Further, if a serious security flaw is found on one of the building blocks, we simply replace it with another secure two-party protocol without changing the entire four-party protocol.

5. GENERIC FOUR-PARTY KEY EXCHANGE PROTOCOLS

In this section, we first present our generic framework for 4PAKE. Using our framework, we then give three concrete instantiations denoted by 4PAKEv1, 4PAKEv2, and 4PAKEv3. Succinctly, 4PAKEv1 is a generic protocol constructed from 2PAKE and 2AAKE, while 4PAKEv2 is a variant of 4PAKEv1 that considers key reuse. In 4PAKEv3, we consider the case where 2SAKE (instead of 2AAKE) is used in order to derive a more efficient protocol than 4PAKEv1.

5.1. Notation

The notation used in our 4PAKE protocols is described in Table I. We assume that \((pk_X, sk_X)\) and \((epk_X, esk_X)\) are both asymmetric key pairs. We write \(ssk_{X_1, X_2} \leftarrow 2PAKE(pwd_{X_1, X_2})\) to denote the execution of a 2PAKE protocol, where the protocol is
run between $X_1$ and $X_2$, and which takes $pwd_{X_1,X_2}$ as input and establishes $ssk_{X_1,X_2}$. We then use $ssk_{X_1,X_2} \leftarrow 2AAKE\left( esk_{X_1}, epk_{X_1}, \sigma_{X_1} \right), \left( esk_{X_2}, epk_{X_2}, \sigma_{X_2} \right)$ to denote the execution of a signature-based 2AAKE protocol, where the protocol is run between $X_1$ and $X_2$. It takes as input an asymmetric key pair $(esk_X, epk_X)$ and a signature $\sigma_X$ (ensuring the authenticity of the public key $epk_X$) of each entity, and establishes $ssk_{X_1,X_2}$. Moreover, we use $ssk_{X_1,X_2} \leftarrow 2SAKE\left( sk_{X_1}, X_2 \right)$ to denote the execution of a MAC-based 2SAKE protocol between $X_1$ and $X_2$. It takes as input, a shared symmetric key $sk_{X_1,X_2}$ and outputs a session key $ssk_{X_1,X_2}$. Also, we define a session identifier $sid$ as $(id_A, epk_A, id_B, epk_B)$ in all the 4PAKE protocols shown in this article. (A similar definition of $sid$ is used in [Bellare and Rogaway 1993b].)

We label a protocol message flow by $M_x$, where $x$ indicates the $x$-th message flow. We use subscripts to differentiate protocol messages that are created and transmitted in parallel, for example $M_{xA}$ and $M_{xB}$. We assume that the execution of any of our 4PAKE protocols involves a pair of clients, denoted by $A, B \in C$, and their respective servers, denoted by $S_A, S_B \in S$. Here $C$ and $S$ are sets of possible clients and servers, respectively.

### 5.2. Generic Framework

We begin by describing a generic framework for constructing 4PAKE protocols. Our framework consists of the following two phases.

1. **SETUP.** In the initial phase, each server $S$ generates its long-term public-private key pair $(pk_S, sk_S)$, and each client $C$ chooses a password $pwd_C$ that is shared with its domain server.

2. **4PAKE EXECUTION.** The second phase is divided into the following steps, as illustrated in Figure 1.
   - **Initialisation** ($M_1, M_2$). Should two clients $A$ and $B$ wish to securely communicate with each other, they first exchange their identity information, $id_A, id_B$, and their ephemeral public keys, $epk_A, epk_B$. If $A$ and $B$ already have knowledge of each other’s identity information prior to the start of the protocol (e.g., they may obtain such information from a higher application layer), then this step can be skipped.
   - **Local authentication and key exchange** ($M_3$). $A$ and $B$ each runs a 2PAKE protocol with their own domain (authentication) server to establish a temporary session key for their communication in the next step.
   - **Token acquirement** ($M_4, M_5$). Subsequently, $A$ and $B$ each obtain an authentication token issued by their respective domain servers via the temporary session key established in the previous step.
   - **Two-party authentication and key exchange** ($M_6$). Last, $A$ and $B$ use their respective authentication tokens and ephemeral public/private keys to execute the 2A/S/PAKE protocol in order to establish a session key.

**Remark 1.** In scenarios where clients $A$ and $B$ may already have knowledge of each other’s identity information prior to the start of a 4PAKE protocol, we can remove the first two message flows ($M_1, M_2$). Hence the signature $\sigma$ created by each domain server does not include the remote domain client’s ephemeral public key (but its local domain client’s). We note that, however, the ephemeral public key $epk$ chosen by a client can be seen as a challenge to the other communicating client. Thus, the removal of $M_1 & M_2$ opens up the possibility of a replay attack. For example, an adversary who somehow managed to learn $A$’s ephemeral secret key $esk_A$ can reuse $\sigma_{S_A}$ to establish a session key with $B$; or even if $A$ has been revoked by $S_A$ after obtaining $\sigma_{S_A}$, $A$ can still reuse...
the signature to continue to establish secure channels with \( B \) without \( B \) knowing it. Hence, it is essential to include a timestamp \( ts \) in the signature, such that \( \sigma_{SA} \) is now being tied to a specific time, allowing \( B \) to check the freshness of the signature.

**Remark 2.** We assume that each domain server has access to the authenticated credentials (e.g., public key certificates) of other domain servers. Moreover, each client possesses a copy of its server’s credential in order to verify the authentication token issued by the server in \( M_5 \). This is not necessarily required to be done in advance. In practice, the servers can distribute their credentials to their clients during the execution of 2PAKE in \( M_3 \), or alternatively in \( M_5 \), by using a MAC algorithm in the same way as \( M_4 \). In that case, secure and authenticated key exchange can be achieved if neither the client nor the server is corrupted. However, to prevent a stronger password-compromise impersonation attack, the clients must obtain their respective servers’ credentials through out-of-band mechanisms.

**Remark 3.** Furthermore, we stress that there is no interaction between servers \( S_A \) and \( S_B \) during a protocol run. This seems to be a very attractive property since we can avoid overloading the servers with high communication cost in an open, distributed environment should they need to exchange messages in the protocol. The savings in terms of communication bandwidth is significant compared to PKCROSS, for example. (See Section 8 for further details of our performance evaluation.)

### 5.3. 4PAKEv1 (from 2PAKE and 2AAKE)

Our first instantiation of 4PAKE protocol, denoted by 4PAKEv1, entails piggybacking 2PAKE (password-based) and 2AAKE (asymmetric-key based) protocols. This is illustrated in Figure 2.

In \( M_1 \) & \( M_2 \), as described before, clients \( A \) and \( B \) exchange information about their identities (\( id_A, id_B \)), and ephemeral public keys (\( epk_A, epk_B \)), such as

---

### Figure 1. A generic framework for 4PAKE.

<table>
<thead>
<tr>
<th>( M_1 )</th>
<th>( M_2 )</th>
<th>( M_3 )</th>
<th>( M_4 )</th>
<th>( M_5 )</th>
<th>( M_6 )</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A</strong></td>
<td><strong>( S_A )</strong></td>
<td><strong>( S_B )</strong></td>
<td><strong>B</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>([id_A, epk_A])</td>
<td><strong>([id_B, epk_B])</strong></td>
<td>( ssk_{A,S_A} \leftarrow 2PAKE(pwd_{A,S_A}))</td>
<td>( id_A, id_B, epk_A, [epk_B], \mu_A )</td>
<td>( token_{SA} \leftarrow 2A/S/PAKE((esk_A, epk_A, token_{SA}), (esk_B, epk_B, token_{SB})))</td>
<td></td>
</tr>
</tbody>
</table>

where

- \([\ldots\ldots]\): optional component
- \( \mu_A = MAC(ssk_{A,S_A}; id_A, id_B, epk_A, [epk_B], [ts_{SA}]) \)
- \( \mu_B = MAC(ssk_{B,S_B}; id_B, id_A, epk_B, [epk_A], [ts_{SB}]) \)

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8In such an attack, a client’s password is assumed to be known to the adversary, which in turn, attempts to impersonate the target server by distributing a fake credential.

9We can assume, in practice, that the identity information contains information about its associated domain. For example, if \( id_A \) is an IP address, then it also gives information about the domain to which \( id_A \) belongs.
4PAKEv2 (Key Reuse)

In our second example protocol, denoted by 4PAKEv2, we consider a scenario where key reuse is tolerated. Our goal is to simplify 4PAKEv1 such that some of the steps can be skipped should a client wish to reconnect to the same remote client within a predefined period of time. To achieve this, we now require the client to generate two
ephemeral key pairs: one can be reused over multiple sessions, while the other can be used just for a session and regenerated for each new session.

Let \((epk^0, esk^0)\) and \((epk^1, esk^1)\) be the two key pairs generated by the client, and let \(sk_{CA}\) be the secret key of a CA. We also let \([sk > pk]\) denote a certificate signed using secret key \(sk\) over public key \(pk\); and \([sk > pk] \rightarrow [sk' > pk']\) denote a certificate chain rooted at \(sk\). Our protocol is then illustrated in Figure 3.

Here, we treat \(\sigma_{SA}\) as a public key certificate with respect to \(epk^0_{SA}\), which is issued by the domain server \(S_A\) and has a validity period of more than just a session, for example, a day, week, or month. The client then creates another new pair of ephemeral keys \((esk^1_A, epk^1_A)\) that are taken as input for the signature-based 2AAKE protocol. Particularly, a certificate-chain of the form

\[
[sk_{CA} > pk_{SA}] \rightarrow [sk_{SA} > epk^0_A] \rightarrow [esk^0_A > epk^1_A]
\]

is created in this setting. This way, the authenticity of ephemeral public key \(epk^1_A\) is assured by a signature under \(esk^0_A\), which can be verified by \(B\) using \(pk_{SA}\) and \(epk^0_A\), during the execution of 2AAKE.

Should \(A\) want to establish a session key with \(B\) again within the validity period \((ts_{SA})\) of \(epk^0_{SA}\), \(A\) simply generates a new \((epk^1_A, esk^1_A)\) key pair and runs the 2AAKE protocol with \(B\) directly without going through \(M1\) to \(M5\). Henceforth, we use 4PAKEv2* to denote a 4PAKEv2 protocol that executes only the last message flow \(M6\). We show, in our performance evaluation in Section 8, that 4PAKEv2* is roughly 2× faster than PKCROSS and our other 4PAKE protocols.

Notice that in fact, more generally, we can use the domain information of \(B\) instead of \(id_B\) in \(\mu_A\) and \(\sigma_{SA}\), such that \(A\) can reuse the \((epk^0_{SA}, esk^0_{SA})\) key pair and \(\sigma_{SA}\) to establish a secure communicate session with any client from the same domain as \(B\).

### 5.5. 4PAKEv3 (from 2PAKE and 2SAKE)

We now give an example that relies on 2PAKE and 2SAKE to improve the computational efficiency of 4PAKEv1.

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**Fig. 3.** The 4PAKEv2 protocol.
can simply compute a common secret key, which is a Diffie-Hellman key, based on the Hellman component be achieved if, for example, the server's long-term public key is of the form of a Diffie-Hellman key with a remote server via a noninteractive key derivation scheme. This can generate a long-term shared key between the Diffie-Hellman key will be of the form $g^{sk_k}$, where $sk_k$ is the corresponding secret key. This is so since we can simply compute a common secret key, which is a Diffie-Hellman key, based on the local server's secret key and the remote server's public key. In the case of $S_A$ and $S_B$, the Diffie-Hellman key will be of the form $g^{sk_kA}sk_kB$. The secret key shared between the two servers will then be used to derive a presession key for the clients, such that the latter can use the presession key to further exchange and establish a session key using a 2SAKE. Our protocol is illustrated in Figure 4.

The domain servers $S_A$ and $S_B$ first use a noninteractive key derivation scheme to generate a long-term shared key $lsk_{S_A,S_B}$. This key is then used to generate a short-term presession key $ssk_{S_A,S_B}$ based on the session-specific data $(id_A, epk_A, id_B, epkB)$. After the establishment of a local session key $ssk_{S_A,S_A}$ in $M3$, the client $A$ and the domain server $S_A$ use a key derivation function to generate two sub-keys $ssk_{0,SA}^{0}$ and $ssk_{0,SA}^{1}$. The former sub-key is used as a MAC key in $M4$ to let $A$ deliver the session-specific data to $S_A$, while the latter is used as an encryption key in $M5$ to let $S_A$ securely deliver the pre-session key $ssk_{S_A,S_B}$ to $A$. The same procedures are also performed between $B$ and $S_B$. This way, $A$ and $B$ will both obtain the pre-session key $ssk_{S_A,S_B}$ at the end of step 5. Finally, in $M6$, $A$ and $B$ use the shared pre-session key to execute a MAC-based 2SAKE protocol. A good example of a MAC-based 2SAKE is the REKEY protocol of Canetti and Krawczyk [2001].
Alternative. Here we briefly sketch another example of 4PAKE that aims to optimise the implementation cost of the 4PAKEv3 protocol. The core idea is that we now construct a 4PAKE protocol from 2PAKE as the only building block. That is, 2PAKE is used not only between the client and the domain server, but also between the two clients. Our motivation for doing this is that implementing a single protocol is much cheaper than implementing two separate protocols (2PAKE and 2SAKE/2AAKE) in terms of memory space or circuit size, particularly if the protocol runs within a constrained hardware device, for example, smart card and mobile phone. Such a protocol is very similar to 4PAKEv3. The only difference between the two protocols is that we replace $M_6$ in Figure 4 with $ssk_{A,B} \leftarrow 2PAKE(ssk_{S_A,S_B})$, where $ssk_{S_A,S_B}$ is regarded as the password shared between clients $A$ and $B$.

6. SECURITY MODEL

We now define an ROR security model for 4PAKE by extending the work of Abdalla et al. [2005] for the three-party case. In the 4PAKE setting, we assume that each protocol participant is a client $U \in \mathcal{U}$ or a trusted server $S \in \mathcal{S}$. A protocol execution involves two client-server pairs from two distinct security domains. Each client shares a password with its domain server. (As with the two-party case, each client $U \in \mathcal{U}$ holds a password $pwd_U$, while each server $S \in \mathcal{S}$ holds a vector $pwd_S = \langle pwd_U \rangle_{U \in \mathcal{U}}$ with an entry for each client.) We also assume that a server has access to public information about other servers, such as their identities, public keys, and so forth.

6.1. Indistinguishability of Session Keys

In order to model insider attacks, the set of clients $\mathcal{U}$ comprises two disjoint sets: $\mathcal{C}$, the set of honest clients, and $\mathcal{E}$, the set of malicious clients. We assume that all passwords of clients from the set $\mathcal{E}$ are known by the adversary [Abdalla et al. 2005].

The notion of partnering (between two clients) in the four-party setting is similar to that for the two-party setting (between client and server), and thus will not be further discussed here.

The oracle queries in the ROR security model for 4PAKE are defined as follows.

— **EXECUTE**($U_1^{i_1}, S_1^{i_1}, U_2^{i_2}, S_2^{i_2})$. This query models a passive attack in which the adversary eavesdrops on an honest execution of the protocol between client instances, $U_1^{i_1}$ and $U_2^{i_2}$, and trusted server instances, $S_1^{i_1}$ and $S_2^{i_2}$. The output of the query comprises messages that were exchanged during the honest execution of the protocol.

— **SENDCLIENT**($U^i, m$). This query models an active attack in which the adversary may intercept a message and then either modify it, create a new one, or simply forward it to the intended participant. The output of the query is the message that the client instance $U^i$ would generate upon receipt of message $m$.

— **SENDSERVER**($S^i, m$). This query models an active attack against a server. The output of the query is the message that the server instance $S^i$ would generate upon receipt of message $m$.

— **TEST**($U^i$). This query models the misuse of a session key by a user. Let $b$ be a bit chosen uniformly at random at the beginning of an experiment defining indistinguishability in the ROR model. The output of the query is then the session key for participant instance $U^i$ if $b = 1$ or a random key from the same domain if $b = 0$. 

10Note that in Section 3.2, the set $\mathcal{U}$ includes both clients and servers. In the four-party case, however, the set $\mathcal{U}$ is restricted to only clients, since the goal of a 4PAKE protocol is to establish secure channels between two clients (rather than between a client and a server).
However, if no session key is defined for the client instance $U^j$, then return the undefined symbol $\bot$.

The advantage of an adversary $A$ in violating the indistinguishability of the 4PAKE protocol in the ROR sense, $\text{Adv}_{4PAKE,\text{ROR}}(A)$, and the associated advantage function $\text{Adv}_{4PAKE,\text{ROR}}(t,q_s,q_t)$ are then defined as in the two-party setting.

Note that for simplicity of presentation, we will not consider the notion of perfect forward secrecy [Bellare et al. 2000a] in this article. Defining such a notion is a straightforward exercise.

### 6.2. Key Privacy With Respect to Servers

We stress that the servers involved in a 4PAKE protocol run in our ROR security model are trusted and assumed to be honest-but-curious. Since the servers have access to all the passwords within their respective security domains, it seems impossible to prevent any of them from impersonating a client from the same domain to another client of a different domain. However, in the security model, we allow the servers to launch passive attacks against any clients by intercepting their protocol messages.

We adopt the definition of key privacy from Abdalla et al. [2005], which says that the session key shared between two instances should be known to only these two instances and no one else (including the trusted servers). Moreover, the adversary is allowed access to all passwords of the clients in the set $U$, and the EXECUTE and SENDCLIENT oracles, but not the SENDSERVER oracle (which can be easily simulated by the adversary using the passwords). In order to capture the adversary’s ability to distinguish a real session key from a random key, the adversary is allowed access to a TEST PAIR oracle defined as follows [Abdalla et al. 2005].

— TestPair($U^i_1, U^j_2$). Let $b$ be a bit chosen uniformly at random at the beginning of the experiment defining the notion of key privacy. If $b = 1$, the output of the query is the actual key shared between client instances $U^i_1$ and $U^j_2$ for a session in which the adversary performed only passive attacks. Else if $b = 0$, a random key from the same domain is output. However, if client instances $U^i_1$ and $U^j_2$ do not share the same key, then return the undefined symbol $\bot$.

Let $A$ be an adversary that is given the passwords of all users and is allowed to ask multiple queries to the EXECUTE, SENDCLIENT, and TESTPAIR oracles in an experiment defining the key privacy of the 4PAKE protocol. The advantage of the adversary $A$ in violating the key privacy of the protocol, $\text{Adv}_{4PAKE}^{kp}(A)$, and the associated advantage function $\text{Adv}_{4PAKE}^{kp}(t,R)$, are defined as before.

### 7. SECURITY ANALYSIS

Before presenting our security proofs, we first give a high level description of the proof idea with regards to the generic framework illustrated in Figure 1.

We adopt a game-hopping approach, which starts with the original game, or Game $G_0$, defined in our 4PAKE model (as described in the last section), and ends with Game $G_5$. These games proceed in the following manner.

— In Game $G_1$, we replace the key $ssk_{A,S_A}$ with a random key. The difference between $G_1$ and $G_0$ will be negligible to the adversary if the 2PAKE protocol is secure.
— Game $G_2$ proceeds in a similar way as Game $G_1$, except that we make the same modification to $ssk_{B,S_B}$, that is, replacing it with a random key.
— In Game $G_3$, we further change Game $G_2$ by rejecting all the MACs forged by the adversary in $M_4$. Due to the unforgeability of the MAC, the difference between $G_3$ and $G_2$ is also negligible.

— In Game $G_4$, we do a similar modification by rejecting all the authentication tokens forged by the adversary in $M_5$. The difference between $G_4$ and $G_3$ is again negligible if the token is unforgeable.

— In Game $G_5$, we replace the key $ssk_{A,B}$ with a random key. The adversary will not notice such a change if the $2A/2S/2PAKE$ protocol is secure.

— Finally, in Game $G_5$, it is obvious that the adversary has no advantage since a random session key will be returned in each TEST query regardless of the value of $b$.

7.1. 4PAKEv1

We first give a security proof for 4PAKEv1 from which we can also relate and derive the security proofs for 4PAKEv2 and 4PAKEv3.

7.1.1. Indistinguishability of Session Keys. As the following theorem states, our 4PAKEv1 protocol shown in Figure 2 is secure in the ROR model (as defined in Section 6), provided that the underlying primitives it uses are secure.

**Theorem 1.** Let $4PAKEv1$ be the four-party password-based authenticated key exchange protocol. Let $q_{exe}$ be the number of queries to the EXECUTE oracle of the $4PAKEv1$ protocol and $q_{test}$ be the number of TEST queries. Let also $q_{send}^{M_x}$ denote the number of SEND CLIENT or SEND SERVER queries related to message $M_x$ of the $4PAKEv1$ protocol for $x \in \{1, 2, 3A, 3B, 4, 5, 6\}$. Then

$$\text{Adv}_{4PAKEv1}^{\text{ror}}(t, q_{exe}, q_{send}^{M_x}, q_{test}) \leq 2 \cdot \text{Adv}_{2PAKE, D}^{\text{ror}}(t, q_{exe}, q_{send}^{M_3A}, q_{exe} + q_{send}^{M_3A}) + 2 \cdot \text{Adv}_{2PAKE, D}^{\text{ror}}(t, q_{exe}, q_{send}^{M_3B}, q_{exe} + q_{send}^{M_3B}) + 2 \cdot q_{M_4}^{send} \cdot \text{Adv}_{\text{MAC}}^{\text{suCMA}}(t, 2, 0) + 4 \cdot \text{Adv}_{\text{Sig}}^{\text{suCMA}}(t, q_{M_5}^{send}, 0) + 2 \cdot q_{test} \cdot \text{Adv}_{2AAKE}^{\text{ror}}(t, q_{exe}, q_{send}^{M_1, M_2, M_6}, q_{send}, 0)$$

assuming the 2PAKE and 2AAKE protocols, and the MAC and signature schemes used in the protocol are secure.

**Proof of Theorem 1.** Let $A$ be an adversary against the indistinguishability of $4PAKEv1$ in the ROR sense. Our security proof is a sequence of security games simulated using techniques from Abdalla et al. [2005].

As described earlier, we start with the real security game against the $4PAKEv1$ protocol, and end with a game in which the adversary’s advantage is zero, and for which we can bound the difference in the adversary’s advantage between any two consecutive games. For each game $G_n$, we define $\text{SUCC}_{n}$ to be the event in which the adversary correctly guesses the hidden bit $b$ used in the TEST queries (as defined in Section 6).

Game $G_0$. This is the original attack game with respect to a given polynomial-time adversary $A$. By definition, we have

$$\text{Adv}_{4PAKEv1}^{\text{ror}}(A) = 2 \cdot \text{Pr}[\text{SUCC}_0] - 1.$$

Game $G_1$. In this game, we model the adversary almost exactly the same as in game $G_0$. The only difference between these two games is that here, we replace the session key $ssk_{A,S_A}$ output by 2PAKE by a random key $ssk'_{A,S_A}$ in all of the sessions involving honest users. We show that the difference in success probability of the adversary $A$
between games $G_0$ and $G_1$ is at most the probability of breaking the security of the underlying 2PAKE protocol between $A$ and $S_A$.

**Lemma 2.** $|\Pr[\text{Succ}_1] - \Pr[\text{Succ}_0]| \leq \text{Adv}^{\text{2PAKE},D}_{\text{2PAKE}}(t, q_{\text{exe}}, q_{\text{send}}, q_{\text{exe}} + q_{\text{send}})$.

**Proof of Lemma 2.** In order to prove this lemma, we simulate an adversary $A_{\text{2PAKE}}$ against the indistinguishability of the 2PAKE protocol using a distinguisher, $A_1$, between games $G_0$ and $G_1$. Adversary $A_{\text{2PAKE}}$ first selects a bit $b$ uniformly at random. It also chooses a password for each client in the system except $A$, according to the distribution $D$ and generates an asymmetric-key pair for each server participating in the protocol. It then starts answering oracles queries from $A_1$ as follows.

- **SENDCLIENT** queries. If $A_1$ makes a query on an instance of the 2PAKE protocol run between $A$ and $S_A$, then $A_{\text{2PAKE}}$ responds by sending the corresponding query to its SEND oracle (as defined in the 2PAKE security model). If the query forces the given instance $A$ or $S_A$ to accept, then we also ask a TEST query to that instance, unless such a query had already been made to its partner. The output of the TEST query is subsequently used as the session key shared between $A$ and $S_A$.

- On the other hand, if $A_1$ issues a SENDCLIENT query targeting an instance of the 2PAKE protocol run between $B$ and $S_B$, $A_{\text{2PAKE}}$ responds using the password of client $B$ that it has chosen at the beginning of the simulation.

- All remaining SENDCLIENT queries by $A_1$ can be answered either using the session key shared between $A$ and $S_A$ or the session keys generated during the execution of the 2PAKE protocol between $B$ and $S_B$.

- **SENDSERVER** queries. $A_{\text{2PAKE}}$ can respond to these queries using the generated asymmetric-key pairs for servers by acting as the required signing oracles.

- **EXECUTE** queries. $A_{\text{2PAKE}}$ can easily answer these queries using its own EXECUTE oracle and the output of the relevant TEST queries, just as how SENDCLIENT and SENDSERVER queries are responded.

- **TEST** queries. $A_{\text{2PAKE}}$ uses the bit $b$ it has previously selected and the session keys that it has computed to answer these queries.

Let $b'$ be the output of $A_1$. If $b' = b$, then $A_{\text{2PAKE}}$ outputs 1. Otherwise, it outputs 0. Let $b^*$ denote the random bit in the 2PAKE experiment for $A_{\text{2PAKE}}$. Then we have

\[
\begin{align*}
\text{Adv}^{\text{2PAKE},D}_{\text{2PAKE}}(A_{\text{2PAKE}}) &= 2(\Pr[b = b'] \cdot \Pr[b^* = 1] + \Pr[b' \neq b] \cdot \Pr[b^* = 0] \cdot \Pr[b^* = 0]) - 1 \\
&= \Pr[b = b'] \cdot \Pr[b^* = 1] + \Pr[b' \neq b] \cdot \Pr[b^* = 0] - 1 \\
&= \Pr[b = b'] \cdot \Pr[b^* = 1] - \Pr[b' = b] \cdot \Pr[b^* = 0] \\
&= \Pr[\text{Succ}_0] - \Pr[\text{Succ}_1].
\end{align*}
\]

The lemma follows by noticing that $A_{\text{2PAKE}}$ has at most time-complexity $t$ and makes at most $q_{\text{exe}}$ queries to its EXECUTE oracle, at most $q_{\text{send}}$ queries to its SEND oracle, and at most $q_{\text{exe}} + q_{\text{send}}$ queries to its TEST oracle.

**Game $G_2$.** We modify the previous game by replacing the session key $ssk_{B,S_B}$ output by 2PAKE by a random key $ssk'_{B,S_B}$ in all of the sessions involving honest users. Using similar arguments for proving the lemma in the previous game, we can prove the following lemma.

**Lemma 3.** $|\Pr[\text{Succ}_2] - \Pr[\text{Succ}_1]| \leq \text{Adv}^{\text{2PAKE},D}_{\text{2PAKE}}(t, q_{\text{exe}}, q_{\text{send}}, q_{\text{exe}} + q_{\text{send}})$. 

ACM Transactions on Information and System Security, Vol. 16, No. 4, Article 15, Publication date: April 2014.
**Game G₃.** We now further modify the previous game as follows. Game G₃ is exactly the same as game G₂, except that in G₃, we modify the way the oracle instances respond to SEND CLIENT queries on M₄ of our 4PAKEv1 protocol. If the adversary makes a SEND CLIENT query containing a new MAC message-tag pair (forgery) not previously generated by an oracle, then we consider the MAC tag invalid and force the instance in question to terminate without accepting. As the following lemma shows, the difference between the current and previous games should be negligible if we use a secure MAC scheme.

**LEMMA 4.** \(|\Pr[\text{SUCC}³] - \Pr[\text{SUCC}²]| \leq q₄^M_{send} \cdot \text{Adv}^{\text{cma}}_{\text{MAC}}(t, 2, 0).\)

**PROOF OF LEMMA 4.** We use a hybrid argument to prove this lemma. We define a sequence of hybrid experiments \(V_i\), where \(0 ≤ i ≤ q₄^M_{send}\). (Note that we do not need to take into account EXECUTE queries here, because they are used to simulate only passive attacks.) In experiment \(V_i\), queries (to the SEND CLIENT oracle) in the first \(i\) sessions involving honest clients \(A\) and \(B\) are answered as in game G₃, and all other queries in the remaining sessions are answered as in game G₂. We remark that the hybrid experiments at the extremes (when \(i = 0\) and \(i = q₄^M_{send}\)) are equivalent to games G₂ and G₃, respectively. Let \(P_i\) be the probability of the event SUCC in experiment \(V_i\). Since \(P₀ = \Pr[\text{SUCC}²]\) and \(Pₘ₄ = \Pr[\text{SUCC}₃]\), it follows that

\[|\Pr[\text{SUCC}₃] - \Pr[\text{SUCC}²]| = \sum_{i=1}^{q₄^M_{send}} |P_i - P_{i-1}|.\]

Hence, it suffices to show that \(|P_i - P_{i-1}|\) is at most \(\text{Adv}^{\text{cma}}_{\text{MAC}}(t, 2, 0)\), in order to prove the lemma. This can be achieved by assuming the existence of a distinguisher \(A_i₃\) for experiments \(V_{i-1}\) and \(V_i\), and using it to build an adversary \(A_i_{mac}\) for breaking the security of the MAC scheme.

The description of the adversary \(A_i_{mac}\) is as follows. For the first \(i - 1\) sessions, the adversary \(A_i_{mac}\) chooses random values for the MAC key and is therefore can perfectly simulate the oracles given to \(A_i₃\), while imposing the restriction as defined for game G₃. In the \(i\)-th session, \(A_i_{mac}\) makes use of its MAC tag generation and verification oracles to answer queries from \(A_i₃\). In this session, if adversary \(A_i₃\) asks a SEND CLIENT query containing a message-tag pair not previously generated by adversary \(A_i_{mac}\), then \(A_i_{mac}\) halts and outputs the pair as its forgery. However, if no such pair is generated by \(A_i₃\), we output a failure indication. For all remaining sessions, \(A_i_{mac}\) simulates all oracles exactly as in game G₂, using actual MAC keys, to answer queries from \(A_i₃\).

Let \(F₁\) be the event in which a message-tag pair is considered valid in experiment \(V_{i-1}\) but invalid in experiment \(V_i\). It is then not difficult to see that \(\Pr[F₁]\) is at most the probability that adversary \(A_i_{mac}\) can forge a new message-tag pair under a chosen-message attack. Since \(A_i_{mac}\) has time-complexity \(t\) and makes at most two queries to its MAC tag generation oracle (to answer the SENDCLIENT queries from \(A_i₃\) in one session) and no queries to its verification oracle, we have \(\Pr[F₁] ≤ \text{Adv}_{\text{MAC}}^{\text{cma}}(t, 2, 0)\). One also sees that

\[\Pr[\text{SUCC}_{V_{i-1}} \land \neg F₁] = \Pr[\text{SUCC}_{V_i} \land \neg F₁]\]

since experiments \(V_{i-1}\) and \(V_i\) proceed identically until \(F₁\) occurs. Therefore, by Lemma 1 of [Shoup 2002] (also known as the Difference Lemma), we have

\[|\Pr[\text{SUCC}_{V_{i-1}}] - \Pr[\text{SUCC}_{V_i}]| ≤ \Pr[F₁].\]
Our lemma then follows by noticing that there are at most \( q_{send}^{M4} \) experiments, where \( M4 = M4A + M4B \).

**Game G4.** In this game, we modify the way the oracle instances respond to \textsc{SendClient} queries on \( M5 \) of our 4PAKEV1 protocol. This implies that if the adversary makes a \textsc{SendClient} query containing a new signature not previously generated by an oracle, then we consider the signature invalid and force the instance in question to terminate without accepting. The following Lemma shows that the difference between \( G3 \) and \( G4 \) is negligible.

**Lemma 5.** \[ |\Pr[\text{SUCC4}] - \Pr[\text{SUCC3}]| \leq \text{Adv}^{\text{euf-cma}}_{\text{Sig}}(t, q_{send}^{M5}, 0). \]

**Proof of Lemma 5.** Let \( A_{\text{sig}} \) denote an adversary against the digital signature scheme. \( A_{\text{sig}} \) receives a public key \( pk \) of the digital signature scheme and simulates the game as follows.

\( A_{\text{sig}} \) chooses a random client \( C \in \{A, B\} \) and guesses that a forge event would happen on \( C \). \( A_{\text{sig}} \) assigns \( pk \) as the public key of the server \( SC \), and generates the public/private key pairs for all the other servers and the passwords for all the clients honestly. \( A_{\text{sig}} \) then simulates the game \( G3 \) for the adversary \( A_{\text{2PAKE}} \). When a signature of \( SC \) is required to respond a \textsc{SendServer} query, \( A_{\text{sig}} \) makes a query to its signing oracle to obtain a valid signature and uses it to answer the \textsc{SendServer} query. Let \( F_2 \) denote the event that the adversary makes a \textsc{SendClient} query containing a valid signature with respect to \( SC \) and which is not previously returned by \( A_{\text{sig}} \). We then have

\[ |\Pr[\text{SUCC4}] - \Pr[\text{SUCC3}]| \leq \Pr[F_2] \leq 2 \cdot \text{Adv}^{\text{euf-cma}}_{\text{Sig}}(t, q_{send}^{M5}, 0). \]

**Game G5.** This game is identical to the previous game, except that we replace the session key \( ssk_{AB} \) (output by the 4PAKEV1 protocol) by a random key \( ssk'_{AB} \) in all of the sessions. As the following lemma shows, the difference in success probability between the current and previous games is at most the probability of breaking the security of the underlying signature-based 2AAKE protocol between \( A \) and \( B \).

**Lemma 6.** \[ |\Pr[\text{SUCC5}] - \Pr[\text{SUCC4}]| \leq q_{test} \cdot \text{Adv}^{\text{rg}}_{\text{2AAKE}}(t, q_{exe}, q_{send}^{M1,M2,M6}, q_{test}, 0). \]

**Proof of Lemma 6.** Again, we prove the lemma by a hybrid argument. Let \( V_i (0 \leq i \leq q_{test}) \) denote a variant of the game \( G4 \) such that for the first \( i \) \textsc{Test} queries, the real session keys \( ssk_{AB} \) are returned to the adversary, while for the remaining \textsc{Test} queries, random session keys are returned. Then we have \( V_0 = G5 \) and \( V_{q_{test}} = G4 \). If there exists an adversary \( A_5 \) that can distinguish \( G4 \) and \( G5 \) with advantage \( \epsilon \), then there must exist an index \( i \) such that \( A_5 \) can distinguish \( V_i \) and \( V_{i+1} \) with advantage at least \( \epsilon/q_{test} \).

Given such a distinguisher \( A_5 \), we can construct an adversary \( A_{\text{2AAKE}} \) against the indistinguishability of the signature-based 2AAKE protocol. \( A_{\text{2AAKE}} \) first selects a password for each client and uses the public keys \( pk_{SA} \) and \( pk_{SB} \) in the 2AAKE game as the public keys for \( SA \) and \( SB \) in the simulated game for \( A_5 \). \( A_{\text{2AAKE}} \) then generates asymmetric-key pairs for the other servers in the system. Next, \( A_{\text{2AAKE}} \) responds to queries from \( A_5 \) as follows.

\[ \text{SendClient} \text{ and SendServer queries.} \quad A_{\text{2AAKE}} \text{ answers the SendClient and SendServer queries from } A_5 \text{ by using pwd}_A \text{ and pwd}_B, \text{ together with queries to its own Send oracle. Note that since no forgery event with respect to} \]
\( \mu_A, \mu_B, \sigma_{SA}, \sigma_{SB} \) would occur in game G4, \( A_{2AAKE} \) can successfully embed the answers from its own SEND oracle into the simulated game for \( A_5 \).

- EXECUTE queries. \( A_{2AAKE} \) can easily answer these queries by using its own EXECUTE oracle and \( pwd_A, pwd_B \).
- TEST queries. For the first \( i \) TEST queries made by \( A_5 \), \( A_{2AAKE} \) uses its REVEAL oracle to obtain the real session keys and use them to answer the TEST queries. For the \( i+1 \)-th TEST query made by \( A_5 \), \( A_{2AAKE} \) uses the key obtained from its own TEST oracle to answer the query. For the rest of the TEST queries made by \( A_5 \), \( A_{2AAKE} \) responds with random keys.

It is obvious that if \( A_5 \) can distinguish \( V_i \) from \( V_{i+1} \), then \( A_{2PAKE} \) can successfully guess the value of \( b \) in the 2AAKE game.

The lemma follows by noticing that \( A_{2AAKE} \) has at most time-complexity \( t \) and asks at most \( q_{exe} \) queries to its EXECUTE oracle, at most \( q_{send}^{M5M6} \) queries to its SEND oracle, at most \( q_{test} \) queries to its REVEAL oracle, and at most 1 query to its TEST oracle.

Clearly, \( \Pr[\text{SUCC}_5] = \frac{1}{2} \). All the above lemmas yield the result in Theorem 1.

7.1.2. Key Privacy against Servers. As explained in Section 6, we assume that the servers are honest-but-curious. Hence, we should also show that if the 4PAKEv1 protocol is executed as expected and does not abort, the servers should not gain any knowledge about the resulting session key.

**Theorem 7.** Let 4PAKEv1 be the four-party password-based authenticated key exchange protocol. Then

\[
\text{Adv}^{kp}_{4PAKEv1}(t, q_{exe}, q_{send}^{M5M6}, q_{test}) \leq 2 \cdot q_{test} \cdot \text{Adv}^{rg}_{2AAKE}(t, q_{exe}, q_{send}^{M1M2M6}, q_{test}, 0),
\]

where parameters are defined as in Theorem 1, and assuming the 2PAKE and 2AAKE protocols, and the MAC and signature schemes are secure.

**Proof of Theorem 7.** We can use arguments similar to those in game G5 in the proof of Theorem 1. Thus, we do not repeat the details here.

Succinctly, assuming that the 2AAKE protocol used between the clients is secure, that is, inherits the indistinguishability property, the servers should not be able to distinguish an accepted session key between users A and B from a random key. This holds if users A and B were honest users and the servers performed only passive attacks.

7.2. 4PAKEv2

The security proof for the 4PAKEv1 protocol can be easily extended to prove the security of 4PAKEv2 by following the same sequence of games outlined at the beginning of this section. The only difference is that we now adopt a signature-based authenticator, which relies on time-stamps [Tin et al. 2004]. We obtain the same security bounds for 4PAKEv2 as with those for 4PAKEv1.

7.3. 4PAKEv3

We now provide a sketch of the security proof for our 4PAKEv3 protocol, which is based on 2PAKE and 2SAKE, by using the same techniques (particularly game-hopping and hybrid argument) we have used in proving the security for 4PAKEv1.

**Theorem 8.** Let 4PAKEv3 be the four-party password-based authenticated key exchange protocol based on 2SAKE. Let \( q_{exe} \) be the number of queries to the EXECUTE oracle of the 4PAKEv3 protocol and \( q_{test} \) be the number of TEST queries. Let also \( q_{send}^{Mx} \)
denote the number of SENDCLIENT or SENDSERVER queries related to message \( M_x \) of the 4PAKEv3 protocol for \( x \in \{1_A, 1_B, 2, 3, 4\} \). Then
\[
\text{Adv}^\text{ror}_{\text{4PAKEv3}}(t, q_{\text{exe}}, q^M_{\text{send}}, q_{\text{test}}) \leq \\
2 \cdot \text{Adv}^\text{ror}_{\text{2PAKE}}(t, q_{\text{exe}}, q^M_{\text{send}}) + 2 \cdot \text{Adv}^\text{ror}_{\text{2PAKE}}(t, q_{\text{exe}}, q^M_{\text{send}}) + \\
2 \cdot \text{Adv}^\text{ror}_{\text{2PAKE}}(t, q_{\text{exe}}, q^M_{\text{send}}) + 2 \cdot \text{Adv}^\text{ror}_{\text{NKD}}(t) + \\
2 \cdot q^M_{\text{send}} \cdot \text{Adv}^\text{euf-cma}_{\text{AE}}(t, 1, 0) + \text{Adv}^\text{ind-cpa}_{\text{AE}}(t, 1) + \\
2 \cdot q_{\text{test}} \cdot \text{Adv}^\text{ind-cma}_{\text{2SAKE}}(t, 1, q^M_{\text{send}}, q^M_{\text{send}}, 0, 0)
\]
assuming the 2PAKE and 2SAKE protocols, and the KD and NKD functions, and the authenticated encryption scheme used in the protocol are secure.

**Proof of Theorem 8 (Sketch).** The adversarial games \( G_0, G_1, G_2 \) are the same as in the proof of Theorem 1. However, we define two additional games \( G_{2.1} \) and \( G_{2.2} \) between \( G_2 \) and \( G_3 \).

**Game \( G_{2.1} \).** In this game, we replace the keys \( ssk^0_{A,SA} \) and \( ssk^1_{A,SA} \) with two independent and random keys. From the assumption that the key derivation function KD is secure and by a hybrid argument, we obtain
\[
|\Pr[\text{Succ}_{2.1}] - \Pr[\text{Succ}_{2}]| \leq q^M_{\text{send}} \cdot \text{Adv}_{\text{KD}}(t, 2).
\]

**Game \( G_{2.2} \).** Similarly, in this game, we replace \( ssk^0_{B,SB} \) and \( ssk^1_{B,SB} \) with two independent and random keys and we have
\[
|\Pr[\text{Succ}_{2.2}] - \Pr[\text{Succ}_{2.1}]| \leq q^M_{\text{send}} \cdot \text{Adv}_{\text{KD}}(t, 2).
\]

Game \( G_3 \) is the same as in the proof of Theorem 1. We then define two additional games \( G_{3.1} \) and \( G_{3.2} \) between Game \( G_3 \) and \( G_4 \).

**Game \( G_{3.1} \).** In this game, we replace the values of NKD\( (sk_{SA}, pk_{SB}) \) and NKD\( (sk_{SB}, pk_{SA}) \) with a random key. Then from the security of the noninteractive key derivation scheme, we have
\[
|\Pr[\text{Succ}_{3.1}] - \Pr[\text{Succ}_{3}]| \leq \text{Adv}_{\text{NKD}}(t).
\]

**Game \( G_{3.2} \).** Here, we further replace the values of \( ssk_{SA,SB} \) and \( ssk_{SB,SA} \) in each session with a random key. Once again, since KD is a secure key derivation function, the difference between game \( G_{3.1} \) and game \( G_{3.2} \) can be bounded by
\[
|\Pr[\text{Succ}_{3.2}] - \Pr[\text{Succ}_{3.1}]| \leq q^M_{\text{send}} \cdot \text{Adv}_{\text{KD}}(t, 1).
\]

In game \( G_4 \), we modify the way the oracle instances respond to SENDCLIENT queries on \( M_5 \): if the adversary makes a SENDCLIENT query containing a new ciphertext not previously generated by an oracle, then we consider the ciphertext invalid and force the instance in question to terminate without accepting. By following a proof similar to that of Lemma 5, we have
\[
|\Pr[\text{Succ}_4] - \Pr[\text{Succ}_3]| \leq q^M_{\text{send}} \cdot \text{Adv}_{\text{euf-cma}}^\text{ind-cma}(t, 1, 0).
\]
We then define an additional game \( G_{4.1} \) between game \( G_4 \) and game \( G_5 \).
Game \( G_{4.1} \). In this game, we replace the key \( ssk_{SA,SB} (= ssk_{SB,SA}) \) used in the 2SAKE protocol with a random key \( rsk \) that is independent of \( c_{SA} \) and \( c_{SB} \). Since \( \mathcal{AE} \) is IND-CPA secure, by a hybrid argument, we have
\[
| \Pr[\text{SUCC}_{4.1}] - \Pr[\text{SUCC}_4] | \leq q_{\text{send}}^M \cdot \text{Adv}_{\mathcal{AE}}^{\text{ind-cpa}}(t, 1).
\]

Finally, in game \( G_5 \), we replace the session keys \( ssk_{A,B} \) in all the sessions with random keys. By following a hybrid argument similar to that of Lemma 6, we have
\[
| \Pr[\text{SUCC}_5] - \Pr[\text{SUCC}_{4.1}] | \leq q_{\text{test}} \cdot \text{Adv}_{\mathcal{2SAKE}}^{\text{ror}}(t, 1, q_{\text{send}}^M, 0, 0).
\]

Notice that the number of queries in the bound is different from that in Lemma 6, this is because in 4PAKEv3 the 2SAKE protocol will use a different presession key \( ssk_{SA,SB} \) in each session, while in 4PAKEv1 the 2AAKE protocol will use the same public keys \( pk_A \) and \( pk_B \) in all the sessions.

It is obvious that \( \Pr[\text{SUCC}_5] = \frac{1}{2} \). Combining all together yields the result in Theorem 8.

We can also use an almost identical approach to prove the security of a variant of our 4PAKEv3 protocol that makes use of only 2PAKE (as described at the end of Section 5.5) to obtain similar bounds as above except the following:
\[
| \Pr[\text{SUCC}_5] - \Pr[\text{SUCC}_{4.1}] | \leq q_{\text{test}} \cdot \text{Adv}_{\mathcal{2PAKE},2_{skA,SB}}^{\text{ror}}(t, 1, q_{\text{send}}^M, 1),
\]
where \( |ssk_{A,B}| \) denotes the length of \( ssk_{A,B} \).

### 8. EFFICIENCY ANALYSIS

In this section, we evaluate and compare the communication and computational costs of our 4PAKE protocols against PKCROSS [Hur et al. 2001]. For this purpose, we instantiate concrete 2PAKE, 2AAKE, and 2SAKE based on SPEKE [Jablon 1996], the signature-based authenticator of Canetti and Krawczyk [2001], and the MAC-based authenticator of Abdalla et al. [2005], respectively.

**Notation.** We use PENC/PDEC, SIG/VER, and GEXP to denote public key encryption/decryption, public key signing/verifying, and group exponentiation, respectively. As before, we let ENC denote symmetric-key (authenticated) encryption, and MAC denote MAC tag generation. We then use \( |\lambda| \) to denote the length of the output of an algorithm or a function \( \lambda \), for example, \( |\text{ENC}| \) denotes the output size of the ENC algorithm. Moreover, we let \( C \leftrightarrow C \) denote the interaction between two clients in 4PAKE (or a client and a service/application server in PKCROSS); let \( S \leftrightarrow S \) denote the interaction between two servers in 4PAKE (or two KDCs in PKCROSS); and let \( C \leftrightarrow S \) represent the interaction between a client and a server (or KDC).

#### 8.1. Communication Overhead

Our generic protocols may understandably be less efficient than one that is based on a standard, noncompositional approach; although in return, we achieve protocol interoperability by reusing existing two-party protocols and our protocol is easier to analyse. However, Table II shows that our generic protocols are still better than PKCROSS in terms of the total numbers of message flows exchanged between \( C \leftrightarrow S \), \( C \leftrightarrow C \), and

\[\text{M4}.\]

We assume that in real world implementation, the last message of the SPEKE protocol (between a client and a server) can be combined with \( \text{M4} \).
Table II. Message Flows in PKCROSS and our Protocols

<table>
<thead>
<tr>
<th>Protocol</th>
<th>C ↔ S</th>
<th>C ↔ C</th>
<th>S ↔ S</th>
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</thead>
<tbody>
<tr>
<td>PKCROSS</td>
<td>6</td>
<td>2</td>
<td>2</td>
<td>10</td>
</tr>
<tr>
<td>4PAKEv1/v2/v3</td>
<td>4</td>
<td>4</td>
<td>0</td>
<td>8</td>
</tr>
<tr>
<td>4PAKEv2*</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>2</td>
</tr>
</tbody>
</table>

Table III. Communication Complexity of PKCROSS and our Protocols

<table>
<thead>
<tr>
<th>Protocol</th>
<th>C ↔ S</th>
<th>C ↔ C</th>
<th>S ↔ S</th>
</tr>
</thead>
<tbody>
<tr>
<td>4PAKEv2*</td>
<td>0</td>
<td>4[Stg] + 2[GEXP]</td>
<td>0</td>
</tr>
<tr>
<td>4PAKEv3</td>
<td>4[GEXP] + 2[ENC] + 2[MAC]</td>
<td>2[GEXP] + 2[MAC]</td>
<td>0</td>
</tr>
</tbody>
</table>

Table IV. Cryptographic Operation Counts in PKCROSS and our Protocols

<table>
<thead>
<tr>
<th>Protocol</th>
<th>Operation count</th>
</tr>
</thead>
<tbody>
<tr>
<td>PKCROSS</td>
<td>1 PENC + 1 Sig + 2 VER</td>
</tr>
<tr>
<td></td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>1 PENC + 1 Sig + 2 VER</td>
</tr>
<tr>
<td></td>
<td>1 VER + 4 GExp</td>
</tr>
<tr>
<td></td>
<td>1 Sig + 2 GExp</td>
</tr>
<tr>
<td></td>
<td>1 Sig + 2 VER + 4 GExp</td>
</tr>
<tr>
<td></td>
<td>1 Sig + 2 GExp</td>
</tr>
<tr>
<td></td>
<td>1 Sig + 1 VER + 2 GExp</td>
</tr>
<tr>
<td></td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>4 GExp</td>
</tr>
<tr>
<td></td>
<td>3 GExp</td>
</tr>
</tbody>
</table>

S ↔ S. This is not surprising given that all of our protocols do not require interaction between the local and the remote servers; while Kerberos is designed such that a client must obtain a ticket-granting ticket and a service ticket through a remote server (KDC) before it can access a target service. Also, clearly, our 4PAKEv2* requires the smallest number of message flows since authentication tokens are reused multiple times within a predefined validity period.

Delving into the bandwidth requirement, Table III compares the communication complexity of PKCROSS and our protocols.

In summary, the communication complexity for C ↔ S in our protocols are comparable to that of PKCROSS (except that our 4PAKEv2* has zero communication cost). However, all of our protocols have higher bandwidth requirement for C ↔ C. On the contrary, no communication overhead is incurred by our protocols with respect to S ↔ S.

8.2. Computational Overhead

First, we evaluate the computational complexity of our protocols by counting the computationally expensive operations (PENC/PDEC, Sig/VER, and GEXP) in a protocol run, and comparing them with those for PKCROSS. This is summarised in Table IV. From the table, we see that the client-side computational overhead in our protocols is expected to be slightly higher, while the server-side overhead is expected to be slightly lower, than that of PKCROSS.
Table V. Raw Cryptographic Computation Times inMilliseconds (ms)

<table>
<thead>
<tr>
<th>Operation</th>
<th>Time</th>
<th>Operation</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>RSA-1024 Enc</td>
<td>0.0389</td>
<td>DH-1024 Exp</td>
<td>0.2093</td>
</tr>
<tr>
<td>RSA-1024 Dec</td>
<td>0.2274</td>
<td>AES/CBC-128 Enc</td>
<td>0.0030</td>
</tr>
<tr>
<td>RSA-1024 Sig</td>
<td>0.2568</td>
<td>AES/CBC-128 Dec</td>
<td>0.0041</td>
</tr>
<tr>
<td>RSA-1024 Ver</td>
<td>0.0170</td>
<td>SHA-1</td>
<td>0.0018</td>
</tr>
<tr>
<td>DH-1024 Gen</td>
<td>0.0798</td>
<td>HMAC (SHA-1)</td>
<td>0.0029</td>
</tr>
</tbody>
</table>

We then verify the efficiency differences between PKCROSS and our protocols through concrete experiments. Our experimental results are based on implementation of PKCROSS and our protocols using C++ and the OpenSSL 1.0.1 crypto library.\footnote{http://www.openssl.org/} Particularly, we access and make use of the \texttt{libcrypto} library through the EVP (envelope) functions provided by OpenSSL. The experiments run on a machine equipped with an Inter 3.30GHz quad-core i3-2120 CPU, 8GB of RAM, and Ubuntu Linux 12.04.2 LTS. Table V presents the base time for our chosen cryptographic algorithms for the protocols.

Protocol Execution Time. We measure the computation time for each protocol. The protocol is run and an average execution time is calculated over 1000 iterations. (Here both the client and the server are simulated on the same machine, thus we do not consider network latency. However, we will come back to this issue in Section 9.1 when we discuss an application of 4PAKE to secure communications over the Internet.) Figure 5 shows the results of our experiments. We also give a breakdown of the protocol execution time, that is, the client-side and the server-side\footnote{In PKCROSS, unlike our protocols, the local server has different computational overhead than the remote server (by only roughly 0.1ms). For simplicity of exposition, our results are based on the computational cost of only the local server.} overhead. The computational overhead includes cryptographic operations and a number of management functions for initialising and clearing data structures.

From the graph on the left of Figure 5, it is clear that the protocol execution times for 4PAKEv1 and 4PAKEv3 are very close to that of PKCROSS. Although 4PAKEv2 is less efficient than PKCROSS, it is run only once within a predefined period of time;
each subsequent execution of $4PAKEv2^*$ is approximately $2 \times$ faster than PKCROSS. As expected, the client-side computational cost in all of our protocols is slightly higher (approximately between $1.05 \times$ and $1.1 \times$) than that of PKCROSS. This is because the clients in our protocols may have to perform not only public key signing/verifying, but also group exponentiation (during Diffie-Hellman key agreement). This is a trade-off between performance and usability (since the clients in our protocol avoid public key management and depend only on passwords). We also stress that our protocols, when relying on the Diffie-Hellman key exchange technique, additionally provide forward secrecy—a property not achievable by PKCROSS. In fact, if we do not make use of the Diffie-Hellman key exchange, our protocols are expected to be more efficient than PKCROSS from the client's perspective. For example, in $4PAKEv3$, we can use the REKEY protocol of Canetti and Krawczyk [2001] as the 2SAKE protocol to avoid performing group exponentiation, and hence having a more lightweight client.

On the other hand, we also confirm that the server-side computational cost in our protocols (except $4PAKEv2^*$ with no overhead) is slightly lower (approximately between $0.8 \times$ and $0.9 \times$) in comparison with that of PKCROSS. The higher cost in PKCROSS is due to the extra interaction between the local and the remote servers.

**Scalability of Server.** We simulate the server for each protocol and see how well it scales with the addition of more computational resources in the form of CPU cores. In these experiments, we artificially restrict computation (through a tool called cpulimit\(^{14}\)) to a subset of the possible cores from one to four of the cores supported by our machine. We then measure the number of protocol sessions that can be executed by the server in a second.

The graph on the right of Figure 5 shows the throughput scaling of our four-core machine. For all protocols, the addition of a CPU core increases approximately between $1.4 \times$ and $2.3 \times$ the number of executed protocol sessions per second; while the use of all four cores increases the throughput by roughly $5 \times$ in comparison with just one core. More importantly, our experiments show that all our protocols are more scalable than PKCROSS. For example, the server running our $4PAKEv3$ protocol with a quad-core CPU can handle up to almost 2000 sessions per second. This is about $25\%$ more than the number supported by PKCROSS, which is upper bounded at approximately 1600 with a similar amount of computational resources. Using a similar empirical analysis, we deduce that $4PAKEv1$ and $4PAKEv2$ are approximately $14\%$ more scalable than PKCROSS.

9. APPLICATIONS

Our approach of $4PAKE$ is applicable to many cross-domain authenticated key exchange scenarios.

9.1. Client-to-Client TLS

One of the most common password-based user authentication mechanisms is the use of username/passwords through the TLS (also known as SSL) protocol [Dierks and Rescorla 2008]. This method has been widely used in online applications, such as Web-based emails, online booking, and Internet banking, for mutual authentication and key establishment between a user and a server. However, this approach is restricted to only the two-party client-server setting. Using our compositional approach, we

\(^{14}\)http://cpulimit.sourceforge.net/
envisage that a client-to-client TLS (C2C-TLS) protocol in the four-party setting can be constructed in a natural way by using the following building blocks:

— hybTLS, the hybrid server-authenticated TLS handshake and username/password approach (between a client and a server);15

— dhTLS, the Diffie-Hellman authenticated key exchange TLS handshake protocol (between two clients) [Dierks and Rescorla 2008].

In our C2C-TLS protocol, a client first authenticates to its domain server using the hybTLS protocol and obtains its credential (or authenticated data) in the form of a signature. Using the credential, the client then performs the dhTLS protocol with the intended remote client. To instantiate the C2C-TLS protocol from our generic 4PAKE framework of Figure 1, we simply replace the 2PAKE protocol by the certTLS protocol, and the 2AAKE protocol by the dhTLS protocol, such that \((epk_A, esk_A)\) and \((epk_B, esk_B)\) are now ephemeral Diffie-Hellman keys. Hence, \(token_{SA}\) and \(token_{SB}\) are of the form of signed Diffie-Hellman keys created by \(SA\) and \(SB\), respectively.

We build a prototype of our C2C-TLS protocol in order to evaluate its feasibility in real world applications. The prototype is based on the OpenSSL API and the BIO (abstraction) library, and written in C++. We choose the DES-CBC3-SHA (168 bits) and the DHE-RSA-AES256-SHA (256 bits) TLSv1/SSLv3 cipher suites for hybTLS and dhTLS, respectively. We then conduct experiments to quantify how much time is required for a client to set up a secure communication channel with a remote client using our C2C-TLS protocol.

In our experiments, we simulate a local client using our machine (with the same specification described in Section 8.2), which is connected to the Internet at 100 Mbits/s. We also simulate a local domain server using the Webmail server of Nanyang Technological University (NTU), Singapore. The local client performs hybTLS with the NTU Webmail server, which in turn, artificially generates and returns an authentication token. (The token is in fact locally generated using the same machine on which the client runs.) The local client then runs dhTLS with a target remote client. For our purposes, we use four geographically distributed SSL-supported Web servers of educational institutions: Singapore Management University – SG, the University of Melbourne – AU, the University of Bristol – UK, and New York University – US, as the artificial remote clients. Each of these Web servers already possesses signed Diffie-Hellman keys, required to perform dhTLS. Hence, we treat the Web server as a remote client without considering the corresponding domain server in the context of 4PAKE. This will not have any noticeable impact on our results, since our goal is to determine the network latency experienced by the local client. It suffices to run hybTLS between the local client and the NTU Webmail server, and dhTLS between the local client and one of the distributed Web servers.

Our experimental results were obtained hourly for a period of 15 hours from 09:00 to 23:00 (GMT+8), on August 20, 2013. These are shown in Figure 6. We have network latency ranging from approximately 0.14s, when communicating with another client within SG, to approximately 1.7s, when connecting to a remote client in the UK. This is only slightly higher than a standard server-authenticated HTTPS connection between a client and a Web server for online applications, that is, conventional two-party key exchange. For example, establishing an HTTPS connection (using the AES256-SHA

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15Typically, a user first establishes a secure TLS channel with a server by performing the server-authenticated TLS handshake (using the server's public key certificate). Note that at this point, the user is still not authenticated by the server. The user then transmits her authentication information, such as a username and a password, to the server (in plaintext) through the TLS channel, so that the server can verify the authenticity of the user.
cipher suite) with the Web server of the University of Bristol requires approximately 1.5s.

9.2. Mobile Phone Communication

4PAKE can also be used to secure communications between mobile clients that subscribe to different mobile service providers. In mobile networks, such as the 2G and 3G telecommunication networks, each mobile client subscribes to a Home Location Register (HLR). The mobile client also shares a secret key (stored in the SIM card) with the HLR, and uses this key to authenticate itself to the HLR when the mobile device is turned on. Meanwhile, different HLRs are connected via wired networks.

Our 4PAKE protocols fit nicely into the mobile network structure. That is, two mobile clients that subscribe to different HLRs can establish a secure communication channel by executing the protocol. Further, our protocols provide key privacy with respect to servers, that is, HLRs. The latter is a desirable feature that is not supported by current mobile communication systems.

9.3. Instant Messaging

Yet another example application of 4PAKE is to secure instant messaging (IM) between two users. Exchanging messages and monitoring availability of a list of users in real time through IM services have been very popular for a relatively long time. There are many free public domain IM services, such as AOL Instant Messenger (AIM), ICQ, MSN Messenger (Windows Messenger in XP), and Yahoo! Instant Messenger (YIM). Mannan and van Oorschot [2006] proposed the use of a three-party password-based authenticated key exchange for securing public IM, assuming that two communication parties are using the same IM service. However, it is not uncommon that two users, wishing to communicate with each other, subscribe to two different IM services. Our four-party key exchange approach is just what is needed to secure communication in such a scenario.

REFERENCES


16http://www.3gpp.org/


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