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Abstract

Track transitionssuch as bridge approaches, road crossings and shifts from slab track to ballasted track are common locations wheretrack degradation accelerates due to dynamic and high impact forces; as a consequence there is higher differential settlement. Thesetypes of discontinuities cause an abrupt change in the structural responseof the track due mainly tovariations in stiffness and track damping. Track transitionzones are prone to an accelerated deterioration of track material and geometrythat leads to increased maintenance costs.Track deteriorationalso leads to vehicle degradation due to enhanced acceleration, low frequency oscillation, and high frequency vibrations. While ballastdeterioration is amajor factor affecting thestability and longevity of rail tracks, the cost of tackling transitionrelatedproblems that detract from passenger comfort is also high.A good transition zone lessens the impact of dynamic load of moving trains by minimising theabrupt variations in track stiffnessand ensuring asmooth and gradual change from a less stiff (ballasted track)to a stiff (slabtrack) structure. This paper presents a critical review of various problems associated with transition zones and the measures adopted to mitigate them; it also includes critical review of research work carried out using large-scale laboratory testing, mathematical and computational modelling and field measurements on track transition zones.

Keywords

ballasted, tracks, transition, review, experimental, zones:, modelling, approaches, improved, performance

Disciplines

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Improved Performance of Ballasted Tracks at Transition Zones: A Review of Experimental and Modelling Approaches

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Abstract: Track transitions such as bridge approaches, road crossings and shifts from slab track to ballasted track are common locations where track degradation accelerates due to dynamic and high impact forces; as a consequence there is higher differential settlement. These types of discontinuities cause an abrupt change in the structural response of the track due mainly to variations in stiffness and track damping. Track transition zones are prone to an accelerated deterioration of track material and geometry that leads to increased maintenance costs. Track deterioration also leads to vehicle degradation due to enhanced acceleration, low frequency oscillation, and high frequency vibrations. While ballast deterioration is a major factor affecting the stability and longevity of rail tracks, the cost of tackling transition related problems that detract from passenger comfort is also high. A good transition zone lessens the impact of dynamic load of moving trains by minimising the abrupt variations in track stiffness and ensuring a smooth and gradual change from a less stiff (ballasted track) to a stiff (slab track) structure. This paper presents a critical review of various problems associated with transition zones and the measures adopted to mitigate them; it also includes critical review of research work carried out using large-scale laboratory testing, mathematical and computational modelling and field measurements on track transition zones.

Keywords: Transition zone, Ballasted track, Bridge deck, Transportation geotechnics, Modelling approach, Differential settlement, Stiffness variation, Impact load, Field measurement

1. Introduction

Track or railway transitions are locations along the track characterized by the presence of an abrupt variation of their stiffness, such as rail tracks change from a stiff structure (slab track) to soft structure (ballasted track) or vice versa. They occur when a conventional track changes to slab track to cross a roadway, a waterway (canal, river, etc.), or valleys through bridges, culverts or level crossings. Such transitions can be due to a sudden change in track substructural components (as in the case of slab track to ballast track transitions, bridge approaches, etc.), track superstructural components (as at special trackwork, level crossings, tie types, etc.) or both [96]. Figure 1 provides some examples of track transitions as a result of sudden change in substructural components. Figure 1a shows two rail track transitions indicating a ballast track to slab track and a slab track to ballast track on both sides of Berry Bridge crossing Tannery road in NSW, Australia. In contrast, Figure 1b illustrates a ballast track to slab track transition with an obvious alignment error that is often a problem associated with such transitions. Figure 2 provides some examples of track transitions as a result of sudden change in the superstructural components. Figure 2a shows a single level crossing on a conventional track at Unanderra, NSW, Australia, whereas, Figure 2b illustrates several other types of such transitions that can generate extreme dynamic loadings attributed to associated gaps and discontinuities causing variations on the rail running surface [96].

Transitions create a sudden change in the structural properties of tracks due to variations in track stiffness, track damping and subgrade reactions. This abrupt change leads to differential settlements and increased dynamic loading that accelerates track degradation through the successive deterioration of track geometry and materials [9, 10, 24, 31, 60, 95, 128]. The sudden change in structural properties at track transition can have an adverse effect on the rail deflections, dynamic loads and track acceleration due to the moving wheel loads. This effect can be seen in Figure 3 that has been reproduced from the modelling data given in references [34] and [186]. Figure 3 illustrates how these values vary suddenly in a short length at the junction point of a ballasted and slab track, while loads move from the ballast track to the slab track. It has been suggested that such abrupt variation in track acceleration causes oscillations or vibrations that further cause destructive effects [186]. If there is no proper intervention the wear and tear of track and vehicle components will increase as ride quality for all types of rail traffic will decrease, and this includes accelerated ballast degradation (breakage). The consequences will affect railway operations through restrictions in train speed, delays in train schedules, further passenger discomfort and higher maintenance costs [10, 31, 56, 95, 97, 127, 137, 159, 188, 194].

The cost of tackling problems associated with track transitions to maintain the smooth operation of

70 railways is often very high [64, 137, 139], for instance the annual maintenance costs for track
71 transitions is approximately 200 million dollars in the USA and 110 million dollars in Europe [64,
72 139, 162]. According to Nicks [120], US\$26 million per year is spent just repairing bridge related
73 transitions, while in Spain; a major portion of their overall investment goes to track maintenance and
74 infrastructure materials [137]. Previous studies also show that the cost of maintaining track transitions
75 (at track discontinuities) is much higher (up to eight times) than normal conventional track [58, 79,
76 84, 95, 133, 137, 166, 167].

77 Transition zones are provided at track junctions to alleviate the problems associated with structural
78 discontinuities [56, 79, 133, 137, 194] and to mitigate the dynamic effect of moving loads through
79 smooth and gradual stiffness transitions [4]. With the increasing demand for long and heavy haul
80 trains to travel at fast speeds, crossing bridge decks, concrete culverts or tunnels with stiff foundations
81 towards softer soils or very soft estuarine plains, the precise and economic design of transition zones
82 is a challenge for designers and practising engineers.

83 This article reviews various aspects of railway transition zones by first defining a track transition and
84 its importance with respect to the structural integrity of track catering for passenger and heavy haul
85 trains (in introduction). Second, reviewing the various problems associated with track transition to
86 identify their causes and consequences on railway operations, and third, investigating the multiple
87 measures taken to minimise and mitigate these problems with reference to their limitations and
88 effectiveness. The design and modelling of tracks at transition zones, including large-scale laboratory
89 testing and prototype experiments, mathematical and computational modelling and field
90 measurements is also discussed, and detailed comparisons of computational modelling and field
91 measurements are also provided in tabular forms. The paper concludes with future research
92 recommendations for improved track design.

93 **2. Problems of track transitions**

94 Major problems associated with railway transitions include (i) differential settlement, (ii) enhanced
95 dynamic load, and (iii) accelerated track deterioration; these problems are summarised as follows.

96 Differential settlement (also referred to as geometric irregularity) is the result of uneven deformation
97 on both sides of track transitions where sections of ballasted tracks undergo more settlement than the
98 stiff side, such as a slab track on a bridge, which is normally designed for minimal settlement [48, 79,
99 137]. Field investigations mostly show the maximum deformations at any specific point under
100 repeated train loading. This maximum deformation includes the elastic component (i.e. fully
101 recoverable upon unloading)) as well as plastic deformations which remain irrecoverable and
102 continue to accumulate over successive load applications. Usually this occurs within a very short
103 time (e.g. in 120ms), where the deformation attains its maximum value and returns to the residual

104 value [119]. Furthermore, the smaller values of differential settlement suggest the structural
105 behaviour to be elastic, whereas the large differential settlements indicate plastic track response
106 especially that of the substructural components [137]. It is also noted that some research based on
107 computational modelling addresses the occurrence of transient deformations where the materials
108 considered in the analysis are assumed fully elastic (i.e. small strain behaviour).

109 A detailed comparison of the differential settlement of various rail transitions is given in [Figure 4](#).
110 [Figure 4a](#) shows a sudden increase in vertical displacement [56, 57, 138, 189] when a slab track
111 changes to a ballast track, giving rise to differential settlement at this location. [Figure 4b](#) compares
112 the field measurements of two studies [43, 97, 174] for vertical displacements on each side of a bridge
113 where the sudden variation in values is obvious. [Figure 4c](#) shows the increasing trend of rail
114 displacement along the approach zones towards bridges at three different sites that could be due to
115 hanging sleepers [172]. [Figure 4d](#) compares the vertical displacement at concrete culverts [25, 133]
116 where approach slabs have been provided on each side. In this specific [Figure 4](#), the part (a) indicates
117 the elastic settlements that have been obtained by load application for a shorter duration, whereas (b),
118 (c) and (d) include the plastic deformations as well.

119 Differential settlement also leads to the development of dips, bumps and undulations near the junction
120 of tracks at transition zones, which is another source of passenger discomfort and increasing
121 maintenance costs [43, 45, 62, 84, 120]. Fara [43] calls the development of dips and bumps, “Jump
122 and Bump” and reports they can occur at both sides of a bridge at track transitions, as shown in [Figure](#)
123 [5](#). While dips and bumps in railway and highway bridge approaches have been seen by various
124 researchers [16, 100, 120, 187], in the USA, more than 50% of all bridge transitions face dip/bump
125 problems, with average bumps being 33 mm high and 5.2m long [120].

126 The amplification of dynamic loads at track transitions due to abrupt changes in the structural
127 properties of tracks is another major problem associated with rail transitions. Sudden variations in
128 stiffness and differential movement at track junctions often increase the dynamic force at track
129 transition under vehicle loading [26, 50, 106, 123, 194]. Mishra et al. [106] measured wheel loads
130 using strain gauges at two bridge approaches and found an increase of up to 100% or more in the
131 dynamic force on top of sleepers at bridge approaches compared to ballasted tracks. They recorded
132 both the plastic and elastic deformations of every individual substructural layer through multi-depth
133 deflectometer systems, but they considered only the effect of elastic (transient) response for obtaining
134 the wheel load. This could be due to poor sleeper support at the bridge approach section, which affects
135 the dynamic response of train suspension [106]. Lei and Mao [91] showed that differential settlement
136 at track transitions leads to higher dynamic forces at the wheel-rail interface than sudden changes in
137 track stiffness; the results found in [8, 50, 102] are similar. A detailed comparison of modelling result
138 of enhanced dynamic loads in terms of wheel-rail interaction forces for various track transitions [92,

139 120, 175, 189] is given in [Figure 6](#); here the very high contact forces at the wheel rail interfaces within
140 the transition zone area are a clear indication of enhanced load impact at the transition.

141 The differential settlement and dynamic load at a transition zone are directly connected, and any
142 increase in dynamic load would results in a corresponding increase in differential settlement, a
143 process that seems to be exacerbated when moving trains with increased axle loads and faster speeds
144 are involved [8, 45, 46, 50, 62, 64, 92, 103, 139, 194]. The cycle of track transition problems and its
145 main components is drawn in [Figure 7](#) (inspired by Paixão [122]). This figure illustrates the inter-
146 dependency of enhanced dynamic load and differential settlement and their relationship to track
147 degradation. If these problems are not addressed properly, they can lead to enhanced track
148 deterioration and increased maintenance costs [95, 107, 137]. [Figure 7](#) also shows the various causes
149 of these problems and the probable consequences of not intervening properly; further details of these
150 causes and consequences are discussed in subsequent sections.

151 It is known that rail tracks deteriorate faster at transition zones than normal ballasted tracks [30, 95],
152 and this deterioration is triggered by the uneven settlement at rail transition zones which also increases
153 the track degradation process. Track degradation includes rail corrugation and wear, track level
154 irregularities, cracking sleepers, loosening ballast and rail fastenings, and hanging sleepers [8, 9, 30,
155 89, 91, 110, 128, 173, 189]. Moreover, the ballast breakage and particle migration adjacent to the
156 sleeper may also lead to hanging or swinging sleepers due to increasing dynamic loads and differential
157 settlement at transitions [5, 25, 31, 48, 58, 64, 92, 95, 106, 120, 123, 133, 154, 162, 194]. According
158 to Pita et al. [127], track deterioration is mainly influenced by the performance of its components at
159 track transitions subjected to higher dynamic loads and the frequent movement of high-speed trains.
160 Track degradation also leads to vehicle degradation due to enhanced acceleration, low frequency
161 oscillation, and high frequency vibrations [9, 30, 41, 93, 102, 190].

162 **3. Major causes of track transition problems**

163 Uneven stiffness and damping between two different subgrade materials, the variation of moisture
164 and geotechnical causes are the primary sources of track degradation at any transition zone [79, 84,
165 95, 120]. Gallage et al. [48] divided the causes of transition related problems into two categories: (i)
166 primary causes such as variations in stiffness and damping, geotechnical issues, subgrade failure,
167 excessive plastic deformation, progressive shear failure, soil water response, and wetting and
168 shrinking cycles, and (ii) secondary causes such as train loads and speed, traffic conditions,
169 embankment heights and types of bridge abutments.

170

171 The abrupt variations in stiffness at track transition are the major reasons for track problems [22, 23,
172 45, 62, 79, 84, 92, 98, 103, 107, 113]. [Figure 8](#) shows a typical example of variations in track stiffness

173 where the total track stiffness k_o (ballast track) suddenly changes to k_n (slab track on a bridge deck);
174 these sudden variations cause differential settlement and expedite track degradation [10, 31]. High
175 values of track stiffness can also cause hanging sleepers as sleeper-ballast contact decreases and the
176 gap between ballast and sleepers increases [25]. A detailed comparison of variations in track
177 stiffness/modulus at various sites is given in Figure 9. Figure 9a shows the sudden variations in track
178 modulus on both sides of the bridge [133], whereas Figure 9b shows how the track modulus/stiffness
179 increases when a track changes from being less stiff to stiffer [54, 120, 166]. Note that stiff tracks
180 such as bridges have higher modulus values than tracks that are not as stiff.

181 Stiffer tracks can reduce track settlement and increase longevity but they are vulnerable to track
182 deterioration because the stiffness increases the contact forces between wheel and rail and at sleeper-
183 ballast interfaces that could increase dynamic pressures acting on track substructure [10, 24, 98].
184 Track stiffness is defined as the load needed to produce a unit deflection in track and is denoted by
185 k , (kN/mm); it can be static (remains constant) or dynamic (depending on the load and excitation
186 frequency) [131, 137]. Track modulus is sometimes used instead of track stiffness because it can be
187 defined as the load to produce a unit deflection per unit length of rail [133]. The value of track stiffness
188 depends on the type of material and height of track embankments [51]. Figure 10 shows the track
189 stiffness at various locations on a west coast line in Sweden; note that the track on a pile-deck bridge
190 is almost twice as stiff as the normal track. The influence that sub-ground (formation soils) has on
191 track stiffness is also evident, hence the rapid change in stiffness for various types of track [30].

192 Ballast degradation at track transitions is one of the main causes of progressive track deterioration
193 [72, 79, 95]. This degradation occurs as ballast deforms due to volumetric compaction (ballast
194 compaction and particle breakage) and frictional sliding (lateral movement of ballast particles under
195 sleepers) mechanisms [25, 29, 70, 78, 140, 144, 158, 164, 174]. There is always more ballast
196 degradation on the ballasted track side due to ballast fouling (contamination by fines), plastic
197 deformation, chemical actions, and variations in moisture and temperature [22, 66, 154, 175, 194].
198 However, no such degradation occurs on a slab track [4]. Because of this, the subsequent differential
199 settlement does increase the dynamic loads and ballast stresses at transition zones [107, 137]. These
200 sudden variations of induced stresses on ballast aggregates continually increases the rate of ballast
201 degradation. A comparison of the various ballast pressure/stresses for different track transitions [120,
202 171, 174] is given in Figure 11. This figure shows that the amplitude of ballast stresses suddenly
203 changes at the bridge abutments on both sides, thus indicating the abrupt variation in measured
204 stresses at these locations where the stiffness is greater.

205 Since the settlement of the capping and subgrade layers is mostly permanent (i.e. plastic deformation)
206 it does contribute to problems such as hanging sleepers at transition zones because the vertical
207 movement of ballast particles that leads to differential settlement and enhanced dynamic loads [85,

107, 109, 137, 162]. Impeded track drainage and poor compaction of low quality backfill materials, as well as limited accessibility beside these structures, can also accelerate subgrade settlement [95, 120, 130, 137]. A detailed study on the settlement of subgrade under heavy axle loads can be found in [94].

Sudden variations in track damping characteristics plays an important role in the development of differential settlements at track transitions [107] and also defines dynamic interaction at the wheel-track interface [24, 40], which is why damping of track components at transition zones influences the vehicle-track-subgrade dynamic response and helps to reduce track vibrations [90, 149]. Track damping helps to dissipate the energy produced by large dynamic loads from fast moving vehicles, whereas a sudden change from a highly damped track (track on an embankment) to a low damped track (slab track on a bridge deck) can cause damage from wheel impact (due to surface deterioration) or wheel bounce (due to variations in stiffness) phenomena. The energy imparted onto embankment tracks can be dissipated through its structural components and the subgrade and surrounding ground, and while the ballast layer in a ballasted-deck bridge track will dissipate some of the energy and most of it will still reach the bridge structures [139].

4. Mitigation measures to track transition problems

A good transition zone must be able to minimise the impact of dynamic loads applied by moving trains. Different approaches for providing a smooth and gradual transition have been proposed and implemented through laboratory experiments, model testing, field investigations and mathematical and numerical modelling; they are reviewed and discussed in the following sections.

Transition wedges are widely used to smooth the tracks at transition areas; these wedge shaped backfills are combinations of cement bond granular materials (CBM), unbound granular material (UGM), graded gravels with some percentage of cement, simple graded gravels, and well graded coarse grained soils [25, 44, 59, 79, 95, 125, 126]. When this system is used at bridge approaches, there is an immediate improvement in the dynamic response of the overall track system under moving train loads [147]. This technique focuses mainly on selecting materials with a variety of characteristics (type, modulus, stiffness, cementation, among others) for the transition wedge and its geometry (thickness, slope, layer distribution, among others) so there is a gradual transition from soft to stiff material in the transition zones. Recommendations for such selections based on variations from soft to stiff, and even from sleeper to sleeper, can be found in [50]. At present there is no universal standard for the design of a transition wedge, so different countries choose their own set of parameters. A comparison of various transition wedges, including their material configurations and geometric shapes that are used in different countries, is described in [44].

Varying the size and spacing of sleepers is another common approach for reducing abrupt change in

242 track stiffness at transition zones; in this approach the length, width and height of sleepers gradually
243 increase, while the spacing between them gradually decreases when track structures proceed from
244 being less stiff to more stiff [112, 113, 133]. Larger sleepers have proven to be good at mitigating
245 ballast settlement and contact pressure between ballast and sleepers [113, 147], but not as good at
246 reducing the dynamic load factor [120]. Unlike maintaining the uniformity and compaction of ballast,
247 this approach to mitigation does not help to increase track stiffness, but it may reduce vertical
248 displacement and induced stresses by distributing train loads over wider areas [137].

249 Sleepers made from composite, plastic, or rubber materials can be used at the transition zone [113,
250 133, 139]. Rubber sleepers that can adjust the sleeper/ballast stiffness are better at reducing the ballast
251 vibration of high speed railway lines [113]. Frame sleepers, where every two sleepers are connected
252 by additional supports to distribute the load over a wider area can also be used in the transition zone
253 [3]. Nicks [120] studied the effect that three different sleepers (wooden, concrete and plastic) had on
254 the dynamic response of bridge approaches in dip and bump cases and found that wooden sleepers
255 will help to mitigate the bumps and dips better than the other materials.

256 Rail pads have recently been used to reduce the vibration and noise from impact loads under train
257 movements and improve the damping properties of track substructure [30, 133, 152]. According to
258 Namura and Suzuki [113], installing softer rail pads on the stiffer side of track transitions makes the
259 rider smoother; studies into the use of soft pads on the stiff side of transitions can be found in [56,
260 166] and studies on the use of rubber pads that are as stiff as the bridge approaches is given in [83].
261 Note that thermoplastic elastomer rail pads/seat plates that are used in railway maintenance works are
262 temperature dependent, so this property must be considered in design practice [20]. The temperature-
263 dependency of static stiffness of various types of rail pads can be found in [176], where a nonlinear
264 variation of static stiffness of rail pads was observed with temperature ranging from -40°C to 70°C.

265 Under sleeper pads (USPs) are increasingly being used to mitigate the problems associated with
266 transition zones because they actually reduce ballast degradation, enabling the stiffness on the stiffer
267 side (concrete deck) to match the softer (ballasted track) side, and minimise the dynamic load impact
268 [2, 30, 76, 112, 115, 120, 152, 166]. However, placing USPs on ballasted track at bridge approaches
269 may not be that effective in reducing the stiffness variation, as it makes the softer side of the bridge
270 transition even softer [120]; however, it can reduce the ballast stresses significantly due to the increase
271 in sleeper-ballast contacts leading to reduced ballast degradation [115]. The use of resilient material
272 mats (rubber mats) under slab track has proven to reduce track vibration [52]. A summary of the
273 various effects of under sleeper pads can be found in [102, 178], and a detailed investigation for their
274 effectiveness through laboratory experiments and numerical simulations can be found in [80, 115].

275 Adding an extra rail (auxiliary rail) in between or on the sides of the main rails at the ballasted track

276 section of the transition zone will help to distribute the dynamic loads evenly [34, 56, 84, 113, 133];
277 one example of this is where guard rails are extended from the bridge abutments to the bridge
278 approaches [133]. This technique helps to improve the bending stiffness of the track and reduce ballast
279 stresses by distributing the load to the sleepers [147]. According to Shahraki et al. [147], auxiliary
280 rails improve the dynamic response of track by providing a smooth transition over the sudden changes
281 in stiffness. In some cases, auxiliary rails may not be as good at reducing the dynamic response
282 compared to some other mitigation approaches, so proper consideration should be given to its benefit-
283 cost ratio before making a final selection [133, 148]. However, two extra rails along the transition
284 zone are the optimum number of rails needed to decrease rail deflections [56].

285 In some cases, concrete confinement walls (wing walls) are installed along the approaches to reduce
286 ballast loosening [133, 177]; these walls increase the lateral confinement on ballast and thus reduce
287 the problems associated with track deformation; these walls also confine the subgrade layers and
288 further decrease track settlement [177]. However, there may be a large increase in track modulus due
289 to increased confinement and associated ballast breakage and this must be considered during design
290 [137]. Apart from wing walls, Nicks [120] installed steel bars of varying lengths between sleepers
291 into the subgrade to increase the confinement and strength of ballast; this approach is much better at
292 reducing subgrade stress and the track deflection, and ultimately mitigating the development of dips.

293 While increasing the thickness of ballast and sub-ballast (capping) at transition zones will enhance
294 track performance, it might also cause excessive track settlement [90, 94, 95]. However, if there are
295 bumps in the track, increasing the thickness of ballast at the bridge approaches is the best approach
296 because the extra depth helps to attenuate stress and reduce the deviatoric stresses applied on the
297 substructural layers [120, 141]. Moreover, increasing the ballast thickness also increases the track
298 modulus; Selig and Li [143] report an increase in the track modulus from 24 MPa to 34 MPa after
299 increasing the ballast from 0.3 m to 1.07 m thick.

300 The use of resin and polyurethane compound to glue ballast aggregates to reduce track settlement
301 [55, 82, 113, 180-183] has been tested. In fact according to Kennedy et al. [82], an almost 99%
302 reduction in permanent settlement can be achieved with polymer treated tracks because a ballast track
303 performs almost the same as a slab track. Similarly, reinforcing ballast with 3D polyurethane and 3D
304 polymer not only improves the efficiency and safety of a railway track, it also helps to reduce the cost
305 of track maintenance [180, 181]. However, Stanislav et al. [153] while investigating the effectiveness
306 of expanding polyurethane resin at bridge transition zone, found no improvement in track dynamic
307 performance. While there is no convincing evidence to indicate the life-span of this polymeric
308 material and its resistance to harsh track environments including UV damage, this method also raises
309 the question about the benefits of highly angular ballast particles (i.e. intrinsic friction in the
310 microscale) that may be subdued by bonding of particles.

311 Approach slabs (submerged approach structure) are often used on both sides of buried structures such
312 as viaducts, culverts or bridges to reduce the high impact loads associated with sudden changes in
313 track stiffness [19, 26, 114, 139, 165, 167]. It is a common practice in European railways to have
314 concrete slab transitions (both horizontal and inclined) between a ballasted track and a concrete
315 culvert to provide a smooth transition [25]. However, the recent research by Coelho et al. [26] shows
316 that track on an approach slab has about four and eight times higher vertical displacements than on
317 an embankment or culvert, respectively. It is reported that these higher displacements could be due
318 to hanging sleepers on the approach slab and the tracks rocking under train movement due to a
319 pivoting action around the edges of the stiff culverts, all of which leads to high impact loading.

320 Placing a layer of hot mix asphalt (HMA) under ballast to improve its performance is another proven
321 mitigation technique for transition zones [86, 95, 133, 155, 177]. When an HMA layer is placed under
322 ballast and protected from the effects of climate it can increase the life of the track substructure and
323 enhance track performance by reducing stress at the ballast/capping interface and reduce the
324 maintenance cycles [135]. Moreover, since HMA is impervious, it prevents water from seeping into
325 the underlying subgrade layer [95], so the drainage capacity of tracks increases. An HMA layer may
326 also help to strengthen the substructure layer by improving its load bearing capacity and further
327 reducing the stresses acting on the subgrade [142].

328 Improving the load bearing capacity of track embankments with soil treatment such as grouting,
329 dynamic compaction, soil cement, geosynthetics, geocells, cement gravels, etc., has been widely
330 adopted [95, 130, 137, 153]. Using geocells (honeycomb structure) within the sub-ballast (capping)
331 layer will help to improve track performance by increasing the stiffness of infilled aggregates [94]. A
332 variety of mitigation techniques commonly used at transition zones, especially bridge approaches, to
333 improve the embankment soil can be found in [79, 129, 130, 153]; they can be divided into three
334 categories: (i) Mechanical (excavation and replacement, preloading and surcharge, dynamic
335 compaction), (ii) Hydraulic (sand drains, prefabricated drains, surcharge loading), and (iii)
336 Reinforcement (columns, stone and lime columns, geo-piers, concrete injected columns, deep soil
337 mixing columns, deep foundations, in-situ: compacted piles, continuous flight auger cast piles, driven
338 piles: timber and concrete piles, geosynthetics, geotextiles / geogrids, geocells).

339 Improving the foundation of track embankments using piles made from reinforced concrete, steel,
340 gravel, timber, sand column, and stone column, etc., can be very helpful in mitigating transition zone
341 problems by increasing track stiffness and reducing settlement on the softer side of the transition [88,
342 137, 146, 168, 179]. However, this solution may not be cost effective because it depends mainly on
343 the length of the piles and the material used [133]. The effectiveness of piles at a transition zone can
344 be enhanced by arranging them in a proper pattern, and by varying their lengths depending on whether
345 the structures are soft or stiff, as shown in [86]; the length of any transition zone can be optimised

346 by these arrangements.

347 Other mitigation measures may include lightweight fills [101, 130, 146], precast prestressed crossings
348 (PPC) [113], increasing the length of the stiffness transition zone [137] and improving the treatment
349 of subbase materials [139]. The use of lightweight fills (expanded polystyrene, geofoam lightweight
350 concrete or aggregate, among others) at transition zones (i.e. bridge approaches) reduces the dead
351 weight (self-weight load) of embankments, which further increases their stability and reduces track
352 settlement [101, 130, 146]. Although this approach has been widely used for the approaches to
353 highway bridges, it can also be used for railways provided that the selected material is suitable (i.e.
354 high stiffness, strength, compressibility, etc.) [130]. Similarly, precast prestressed crossings (PPC),
355 which are approximately 1m long concrete blocks with larger sleepers or rubber sleepers at each ends
356 towards the ballast track, have also been used for level crossings [113].

357 For a smooth and gradual transition, more than one mitigation approach can be used to improve track
358 performance [137]. For example, the cost of maintenance has been reduced considerably using
359 sleepers of varying lengths, and transition slabs [133]. A combination of auxiliary rail, pads with
360 varying stiffness, and geo-grids have been used by Kang et al. [81], and no abnormal response was
361 observed. Similarly, longer rubber sleepers result in a larger base plane which, through the fastening
362 system, also helps to avoid loose sleepers; this has proven to be the best countermeasure against
363 differential settlement used by Namura and Suzuki [113].

364 Some countermeasures are better at fulfilling the desired function while others are either partly
365 effective or completely unsuccessful; for example, according to Read and Li [133], pads under rails
366 and slab is the best way to reduce structural stiffness, whereas Seara and Correia [142] and Read and
367 Li [133] indicate that longer sleepers with a reduced spacing in the transition zones do not increase
368 structural stiffness. However, a Hot Mix Asphalt (HMA) layer definitely improves the load-bearing
369 capacity and reduces the stress in subgrade [142], but it does not improve the behaviour of ballast on
370 rigid pavements [83, 133].

371 **5. Research into track transition zones**

372 Along with the ever-increasing demand for high-speed passenger and heavy haul freight trains goes
373 the increasing need to design transition zones that deliver smooth and gradual changes of track
374 stiffness at track junctions. In this regard, rail tracks have undergone dynamic analysis to better
375 understand the response of track at transition zones under moving loads, as well as the associated
376 track problems and possible countermeasures. This dynamic analysis of railway track transition zones
377 sets out to understand how traffic loads affect track components in terms of stresses, strains and
378 deformation using established theories on the interaction between vehicle components and the track.
379 These models are powerful enough to predict the performance of track structure as well as making

380 designs safe and economical [156].

381 According to Esveld [39], dynamic analysis is the interaction between an applied load and the
382 structure where the structural components react according to their inherent frequencies (governed by
383 mass elastic properties) to the applied load and large amplifications occur when the frequencies of
384 these structural components become equal to their natural frequencies. With transition zones, the
385 properties of components such as track damping and stiffness, rail modulus and inertia, train loads,
386 etc., will vary in time and space, whereas the effect of train load on track components depends on the
387 type (static, dynamic, cyclic, etc.), and velocity in which it is being applied. Other factors that must
388 also be considered in track dynamic analysis are the mass (providing resistance to geometric changes
389 under applied loading), inertia (proving resistance against velocity), damping (energy absorption)
390 characteristics, stiffness (providing resistance to deflection), and the mechanical and geometric
391 properties of track components [156].

392 A concise review of ongoing research into the dynamic analysis of tracks at transition zones via
393 laboratory experiments, mathematical and computational modelling, and field investigations is
394 presented and discussed in the following sections.

395 **5.1 Laboratory testing and prototype modelling**

396 A number of laboratory experiments on rail track and track components to investigate the properties
397 and performance of different materials/components under various situations have been carried out
398 worldwide, and an extensive number of outcomes in the field of railway engineering have been
399 published by various researchers [35-37, 63, 69, 73, 75, 80, 99, 116, 118].

400 However, very little work has been carried out in laboratories to model transition zones due to
401 limitations of size and composition. Momoya et al. [111] performed some laboratory experiments on
402 railway track transitions between ballasted embankments and the concrete box culverts. This model
403 was a 1/5th scale model of a transition onto which a moving load was applied onto rail sleepers by
404 electric-hydraulic actuators to simulate an actual load from a 10-car train with four axles each. The
405 four models tested were (a) without any buffering, (b) with an approach block, (c) with an approach
406 slab, and (d) with a resilient mat. Results were based on track settlement, the hanging sleeper
407 phenomenon, deformation of the ballast layer, and the relationship between the mobilised friction
408 angle and ballast settlement. The conclusion of this extensive laboratory study was that the
409 countermeasures are expected to reduce any large local settlement and an approach block will reduce
410 settlement by almost one half.

411 Likewise, Namura and Suzuki [113] performed the cyclic loading tests on a 1/5th scale model to
412 evaluate the effectiveness of precast prestressed crossing (PPC). The model represents the transition

413 between ballast track (consisting of sleeper, ballast and subgrade) and the slab track (consisting of
414 concrete block and subgrade). Train loading was simulated by movement loading device with 15
415 actuators (nine on ballasted track and six on PPC) considering the loading pattern of single wheel
416 load running cyclically at a speed of 1.2 m/s. Track dynamic analysis was carried out to investigate
417 the effect of rail fastening system on the reaction forces and vertical displacements. With no rail
418 fastening system, loose sleepers were observed on the ballasted track side soon after track
419 maintenance by tamping (i.e. after 1000 passes of movement loading), however, no such loose
420 sleepers were occurred even after 20000 passes of movement loading in case of rail fastening
421 provision. It was concluded that the rail fastening system provides a better alignment to longitudinal
422 irregularities, which minimises the disturbance of ballast components caused by tamping work.

423 **5.2 Mathematical modelling approach**

424 There have been a number of researchers used the theory of beams on elastic foundations (BOEF) to
425 model railway tracks and transitions [28, 33, 47, 81, 145, 185]. This theory is based mainly on Euler-
426 Bernoulli beam (rail of infinite length) or Timoshenko beam resting on a Winkler foundation. The
427 mathematical framework for the motion of a track built on a viscoelastic foundation using this theory
428 can be found in [28]. Previous studies [34, 56, 95, 107, 126, 165] used the Euler-Bernoulli to model
429 a transition zone, while some researchers used a Timoshenko beam to consider transverse shear
430 deformation and beam vibration theory [4, 59, 113]. However, after comparing these two
431 conventional approaches for various cases, Czyczula et al. [28] concluded that if either monotonic or
432 moving loads are considered, the results through the Timoshenko beam are almost the same as the
433 Euler- Bernoulli beam. A detailed comparison of deflection of rail beams predicted by different
434 theories subjected to varying train speeds can be found in [28].

435 The use of BOEF theory to analyses the dynamic response of railway substructure has several
436 limitations. First, a foundation with distributed Winkler springs for soil reactions only gives
437 approximate results if the speed of a moving load (train speed) is less than the critical velocity [166];
438 second, a Winkler springs foundation assumes there is no deformation of the adjacent soil elements,
439 which does not always represent an actual rail track embankment [169]; third, granular materials
440 (ballast, sub-ballast) under track substructure do not exhibit tension, whereas springs have some
441 tension [169]; fourth, this approach does not consider the interaction between train and track while
442 representing the train loading by a constant moving load [166, 169]. Moreover, the load-deformation
443 response of track is frequently been assumed to be linear [120], whereas a highly non-linear response
444 of ballasted tracks under dynamic loading often occurs, especially in stage-1 (rapid) settlement [29,
445 65, 66, 70, 140].

446 In spite of lacking of a comprehensive model to predict the actual response of rail track while

447 considering the complex nature of track substructure [156], the BOEF model has been used
 448 extensively in practice, albeit using an analytical approach to solve the dynamic response of tracks at
 449 transition zones is limited because the problem of sudden changes in track stiffness is complex.
 450 Walker and Indraratna [169] recently used a semi-analytical approach to solve the moving loads at
 451 transition zones; this model considers a Euler-Bernoulli beam (pinned) of finite length on viscoelastic
 452 foundations and the approach considers the spatial variation of rail characteristics (i.e. damping, mass,
 453 bending stiffness and cross-sectional area) as well as track stiffness and damping. The governing
 454 equation for a moving load used in this study is given as:

$$455 \quad \frac{EI\partial^4 w}{\partial x^4} + \frac{\rho A\partial^2 w}{\partial t^2} + c \frac{\partial w}{\partial t} + kw = -F\delta(x - vt) \quad (1)$$

456
 457 where, E = modulus of elasticity (N/m²), I = second moment of area (m⁴), ρ = rail density (kg/m³), A
 458 = cross sectional area (m²), k = track stiffness (kN/m), c = damping (Ns/m²), w = track deflection
 459 (mm), F = dynamic load (kN), v = train speed (m/s) and δ = Dirac-delta function

460 Equation 1 is solved for normalised track displacement (w), using the semi-analytical spectral
 461 Galerkin method that assumes ‘ n ’ terms truncated series. A general transition from soft (low stiffness
 462 value, k_1) to stiff track (higher stiffness value, k_2) over a given transition length (L_t) is investigated
 463 under single and multiple moving loads. The differential settlements are simulated by comparing the
 464 deflections on each side of the transition. The deflection amplification factor (DAF_{wrt1}), is calculated
 465 using Eq.2, which considers various speed ratios ($\alpha=v/v_{cr}$), damping ratios ($\beta=c/c_{cr}$), and stiffness
 466 ratios (k_1/k_2); these ratios can be determined as:

$$467 \quad DAF_{wrt1} = \left(\frac{k_1}{k_2}\right)^{3/4} \times DAF_2 \quad (2)$$

$$468 \quad \frac{\alpha_2}{\alpha_1} = \left(\frac{k_1}{k_2}\right)^{1/4} \quad (3)$$

$$469 \quad \frac{\beta_2}{\beta_1} = \left(\frac{k_1}{k_2}\right)^{1/2} \quad (4)$$

470 To find an optimum length for a transition zone, Walker and Indraratna [169] examined the beam
 471 deflection of various transition length ratios with a characteristic length (L_c), as described in Eq.5;
 472 they concluded that the minimum transition length should be 8-10 times of system’s characteristic
 473 length (L_c) to avoid stiffness transition deflection spikes. One of the main outcomes of this study is
 474 how valid the conventional theory of BOEF is for long transitions with gradual changes in stiffness;
 475 the conclusion is that the dynamic response of transition zones can be described adequately with this
 476 theory. Furthermore, the presented model is validated by comparing the results of maximum

displacement at the transition zone with field data, as shown in Figure 12; this took place using a case study by simulating the actual variations, further details can be found in [169]. This figure shows that as the distance from the abutment increases, track stiffness decreases and peak displacement increases. This figure also indicates the abrupt variation in track stiffness and displacement at the junction between the bridge approach and the abutment.

$$L_c = \sqrt[1/4]{\frac{4EI}{k}} \quad (5)$$

Mass spring-dashpot models have been used in previous studies to model a multilayer track system [11, 32, 56, 149]. Sometimes these models are simplified by using over-all track stiffness and damping values by combining the values of all structural components and layers, as suggested in [10]. To understand the nature of the transition between a ballast track and a slab track, a simplified mass spring-dashpot model can be developed, as shown in Figure 13. In this model, the total stiffness of the track is represented by the “spring” with spring constants k_b and k_s for ballast track and slab track respectively, while damping of the track structure is represented as dashpots. However, to study the effect of every individual track supporting layer, full layered models can be used because they simulate all the supporting layers and also incorporate the additional elements for USPs, elastic and soft pads, geogrid, geotextile, and polystyrene, among others [39, 42, 90, 126, 139].

Varandas [166] presented a linear mathematical model for the response analysis of inhomogeneous foundation using the two-layer mass spring-dashpot system shown in Figure 14. In this model two Euler-Bernoulli beams, one for the rail and another for the concrete slab are linked together by visco-elastic elements to represent rail pads of fill material. The whole system is supported by soil represented by a visco-elastic foundation. The stiffness of the upper and lower visco-elastic elements is assumed to change abruptly at the $x=0$ section from K_{11} and K_{21} to K_{12} and K_{22} respectively. The vertical displacements are defined as $U_{ij}(x,t)$, as mentioned in Figure 14, and are calculated using the dynamic equilibrium equations for forced vibration of beams by considering the load is acting on the left side. The governing equations for the model are given in Appendix A. This mathematical model considers an inhomogeneous foundation so it can be applied to rail transitions for a dynamic response analyse under train moving loads; it can also be utilised for the design of transition zones but it would require extensive calculations that may not be solved analytically.

5.3 Computational modelling approach

The numerical modelling approach is increasingly being used to simulate rail using fully calibrated numerical models of track transitions under various loading and boundary conditions [12, 56, 184]. Various countermeasures have been modelled and analysed using FEM (finite element method) or

509 DEM (discrete element method). In addition to the extensive use of FEM in rail track modelling
510 (Table 1), the DEM has also been increasingly used to study the micromechanical behaviour of
511 railway ballast because it can capture the discrete nature of particulate materials [14, 132, 161].
512 Furthermore, it is capable of examining the mechanical behaviour of granular assembly of arbitrarily
513 shaped discrete particles under quasi-static and dynamic conditions [61, 105, 117, 192]. A
514 comparison of several numerical and analytical models used to evaluate the dynamic response of
515 railway tracks under moving train loads can be seen in [64, 156]. Numerical modelling through proper
516 calibration and field validation is an appropriate tool to predict the dynamic response of any transition
517 zone with various design options, remedial measures, train speeds and loads. A detailed comparison
518 of several computational models of transition zones is given in Table 1.

519 One of the benefits of computational modelling is that a single model can be utilised to work out
520 multiple design options for a specific transition. For example, Sañudo et al. [136] placed sleepers at
521 six different locations using 2D FEM modelling and investigated the dynamic response of track in
522 each case to optimise the overall design. Likewise, in [174], a 3D FE model is used to analyse the
523 dynamic response of track transitions by considering the differential settlement, stiffness variation,
524 vehicle dynamics and hanging sleepers. Similarly using a 3D FE model, various subgrade fillings
525 have been investigated to explore the economic filling materials for a high-speed railway transition
526 zone [59].

527 Another use of numerical modelling is to investigate the effect that complex site situations can have
528 on the dynamic response of track. These situations may include large-scale excavation close to a track
529 transition, variations in the moisture of track substructure, and ballast fouling, among others [116,
530 150, 175]. Likewise, numerical modelling can be used to investigate the dynamic response of track
531 at various levels and locations of track components at any time. Mishra et al. [107] observed
532 deformation at various levels using a 3D FE model to fully calibrate it with field values measured
533 with multi-depth deflectometers.

534 The type of model and the modelling software/program influences how reliable and accurate is the
535 dynamic response analysis of transition zones. Various selection parameters include, (i) the type of
536 analysis required (static or dynamic), (ii) the inclusion of non-linearity and plasticity of material, (iii)
537 the calculation time, and (iv) the expected outcomes. Previous studies show the use of two types of
538 finite element programs: (i) vehicle modelling packages, and (ii) track modelling packages. The
539 vehicle modelling software packages concentrate more on vehicle dynamics while over simplifying
540 the modelling of ballast and subgrade materials, whereas track modelling software packages mostly
541 deal with a substructure model that over simplifies the vehicle model [120]. At transition zones, even
542 though the main variation is in the structural properties of track, utilising the model while considering
543 the vehicle and track responses would enable a better understanding of the dynamic response of track

544 under moving loads.

545 Likewise, selecting a vehicle model which considers various suspended, semi-suspended, and non-
546 suspended loads can also help to obtain a true dynamic response of track structure. Hunt and Winkler
547 [62] used four vehicle models with, (i) axle load only, (ii) axle and bogie, (iii) axle, bogie, and vehicle
548 body, and (iv) two axles and bogies with the same static axle loads and found similar settlement
549 results from every model; they then conclude there is no effect on the settlement growth rate, even
550 for closely spaced axles. Paixão et al. [125] found a similar track response for a train with different
551 cars in terms of the wheel/rail interaction and vertical displacements; they conclude that a 2-car model
552 can be just as practical as a full train model, and therefore very useful at reducing the calculation time.
553 However, a simplified (one-bogie) vehicle model is not always appropriate for considering responses
554 such as the pitching motion of a vehicle [4].

555 The choice of models depends on the complexity of the analysis and the required precision. 2D
556 models are incapable of modelling the train load distribution in a longitudinal direction, so 2D plane
557 strain model with continuous support has been considered for a transversal track profile instead of
558 real field conditions with the discrete support of rails by sleepers. However, a 3D model can overcome
559 these limitations [147], which is why Galvín et al. [53] suggested using 3D models that include track
560 non-linearity to obtain an accurate response of track transitions under moving loads.

561 Paixão et al. [125] used a 2D numerical model to examine how backfill settlement affects train and
562 track interaction by measuring the wheel-rail contact force in the transition zone. They used four
563 scenarios of maximum backfill settlement, 1mm, 5mm, 10mm and 15mm, and found large interacting
564 forces in each case due to the negativity of existing settlement (hanging sleeper phenomenon). Similar
565 results can also be found in various other studies [102, 191, 193].

566 Numerical modelling can be utilised to investigate the response of track due to various train speeds
567 and loads. Coelho et al. [25] concluded that train speeds up to a certain limit (160 km/h for that
568 specific case) had limited impact on the track but as the speed became critical (180km/h) the response
569 of track became higher due to resonance. Likewise, in more recent research, Labrado Palomo et al.
570 [86] investigated the effect of train speeds on four different kinds of approaches at embankment-
571 bridge transition using a 3D finite element model. The characteristic parameters of ballast, sub-ballast
572 and soil were optimised through model calibration with field results, and then the model was
573 successfully validated. It is found that the peak and average particle velocities for vehicle speeds of
574 100 km/h are higher than at 160 km/h and 220 km/h, possibly due to a match between train speed and
575 the critical speed of the entire system. However, Heydari-Noghabi et al. [56] found an increasing
576 trend of track displacements for various sections of track as the train speed and loads increase, as
577 shown in [Figure 15](#). Note here that as the train speed ([Figure 15a](#)) and loads ([Figure 15b](#)) increase,

578 track displacement also increases. Moreover, ballasted track has a higher displacement than a slab
579 track. [Figure 15](#) also shows that the auxiliary rails help to smooth the differential settlement at the
580 transition zone.

581 Numerical modelling can also analyse how the direction of train movement will affect the dynamic
582 response of track in terms of enhanced train-track interactions at transition zones. Many authors
583 believe that trains passing through a transition zone from soft to stiff medium such as embankment
584 to bridge are the worst-case scenarios [113, 125, 174]. This could be due to trains moving from a
585 deformable structure to a non-deformable structure (i.e. concrete bridge) which enhances the impact
586 load. However, Chen [22] found more settlement when moving from a stiff to soft transition zone
587 because the boundary conditions for his model could be case specific. Despite this, some authors
588 suggest that the effect of train direction on the dynamic behaviour of track is minor, which is the case
589 when the quality of tracks is high and there are no sharp variations in the track support conditions [4,
590 5, 124]

591 A 3D finite element model has been used by Hu et al. [59] to evaluate how effective different filling
592 materials are for a wedge-shaped backfill at a tunnel-culvert transition zone. Three different materials
593 are used; (i) graded gravel with 5% cement, (ii) graded gravel with no cement ($c=160\text{kPa}$, $\phi=39.5$),
594 and (iii) well graded coarse grained soil with less than 30% of fine grained soil ($c=200\text{kPa}$, $\phi=41.8$);
595 their properties were calculated through laboratory (for new materials) and field testings (for in-situ
596 material). In every case, maximum deflection occurs under maximum allowed values, however lower
597 wheel loads are used in this study.

598 In [7-9] the train and track interaction has been investigated by applying a 3D finite element model
599 to the track transition mechanism. This model incorporates variations in stiffness and considers the
600 non-linear behaviour of ballast and subgrade. The conclusion is that simple variations in stiffness at
601 track transitions is not the primary cause of transition problems, it is the soft subgrade, voids, and
602 other faults at transition zones that increase the interaction forces as train speeds increase that cause
603 passenger discomfort. It is therefore suggested that difference in deflection at the junction of two
604 different tracks over a 4-10 m long transition will lead to a smooth transition.

605 To study the dynamic response of bump at bridge approaches, a detailed investigation using a 3D
606 finite element model that incorporates train and track structure/substructure is given in [120]. The
607 response of this track is observed by varying the sizes of the bumps and dips, the thickness of the
608 ballast, the sleeper material, train speed direction, and the type of abutment and length of the sleepers.
609 It is found that the enhanced load impact and ballast/subgrade pressure due to variations in the track
610 modulus cause dips and bumps to develop; this further increases the dynamic response of track at
611 bridge transitions.

612 **5.4 Field measurements**

613 A number of field investigations have been carried out to evaluate the effectiveness of the approaches
614 used to control differential settlement at the transition zones; these studies took place mainly in the
615 USA, Europe, Japan, and China. Various instruments are used to measure/investigate the response of
616 rail track at transition zones in real time scenarios, they include multi-depth deflectometers (MDD),
617 uniaxial and triaxial accelerometers, strain gauges, pressure cells, settlement pegs, video gauge
618 systems, position sensitive devices, geophones, inclinometers, linear variable displacement
619 transducers (LVDTs), among others [15, 26, 71, 126, 153, 155, 174]. Furthermore, the structural
620 health monitoring of rail tracks and transition zones is being carried out with the help of conventional
621 data measuring coaches and advanced techniques including digital image correlation (DIC) device
622 and satellite synthetic aperture radar (InSAR) system which is developed by Wang et al. [170].

623 Stone blowing is a process of adjusting the track geometry by adding the crushed rock to ballast
624 surface under the lifted sleeper. It is a relatively new method involving less damage to sleepers as
625 compared to the tamping process where adjustment is achieved by ballast rearrangement to fill the
626 voids under the lifted sleeper [153]. The effectiveness of stone blowing instead of tamping was
627 investigated by Boler et al. [15] where comparison was made by analysing the performance of track
628 before and after stone blowing; the data shows that stone blowing led to an almost 60% reduction in
629 transient peak displacement, and moreover, the vertical acceleration and gaps at the sleeper-ballast
630 interface also decreased due to stone blowing. The results obtained by vertical space curves through
631 track geometry car measurements also indicate the increased effectiveness and longevity of this
632 remedial measure.

633 Paixão et al. [126] concluded that Under Sleeper Pads, (USPs) at the transition zones are effective
634 based on the passage of 40 different types of trains passing the fully instrumented zone. It is noted
635 that USPs will reduce the track stiffness values by a considerable amount. Fortunato et al. [44] used
636 a wedge shaped approach at the transition zone and concluded that a gradual transition of vertical
637 stiffness can be achieved with this approach, however, passenger trains at higher speeds cause more
638 acceleration at sleepers than heavy freight at lower speeds. They also presented the various
639 characteristics of wedge shaped countermeasures that are commonly used at transition zones in
640 several countries around the world.

641 Stark and Wynn [155] presented a report on the use of geosynthetic reinforcement systems in the
642 railway transition zones to mitigate the differential settlement at these locations. They concluded from
643 this ongoing research that ballast reinforcement in transition approaches with a geoweb underlay
644 helps to mitigate transition problems by providing enhanced ballast confinement and improved load
645 distribution. They also showed there is a large reduction in cost and installation time when geoweb

underlay is used; in fact this research shows that geosynthetic reinforcement will help to reduce differential settlement because it can increase (when used at approaches) and decrease (when used under the bridge abutment) the stiffness values.

Coelho et al. [26] presented the results of field measurements for a track crossing with a concrete culvert by showing that the design for the transition was not optimal. The box culvert is almost 1.5 m deep from the track and is made from a sand embankment over soft soils. There are 4m long by 300mm thick approach slabs on both sides of the culvert above which sand is placed up to the ballast under the actual track. The vertical displacement, axle load, and average track stiffness are measured by geophones (mounted on top of wooden sleepers), uniaxial accelerometers (within ballast), triaxial accelerometers (within soil below track), strain gauges and a high-speed camera. The hanging sleepers in the transition zone that are the result of long term track differential settlements are the main sources of the track displacement that caused increased impact loading and accelerated track degradation.

A summary of the most recent field investigations of the transition zones in terms of the project description, transition types, countermeasure used and the outcome of the overall research, is given in Table 2.

6. Recommendations for improved track design

Despite the effort that has gone into studying the performance of tracks at transition zones using advanced modelling techniques, an optimal solution to transition related problems is still not fully understood [79, 137], hence the need to find an effective and low cost solution (to eradicate/minimise the problems), with minimum disruption to traffic and a longer life [64]. The main aim of designing these transition zones is to maintain track quality while reducing the maintenance cycles and costs [137]. In order to use computational models properly for predicting the true dynamic track performance, the model parameters require realistic calibration either using large-scale laboratory simulations or instrumented field trials.

Vertical track stiffness at transition zones is mainly influenced by the type of materials used in the embankment, and its slope and height in the transition zone. Natural ground beneath an embankment also affects the stiffness of track depending on the material used in the embankment, so it should be replaced if it is highly compressible [49-51]. Therefore, to design a transition zone efficiently, a proper selection of materials along with the shape and height of the embankment should be considered, as should a proper consideration of natural ground characteristics.

While the inconsistent dynamic response of ballast particles with respect to the point of load application indicates its particulate nature [107], and while finite element modelling (FEM) cannot

679 model discrete particles due to continuum based solutions, discrete element modelling (DEM) can
680 model irregular-shaped particles, the angularity of granular material, and particle breakage; therefore
681 particle to particle contact for wheel load interactions can be considered properly in dynamic track
682 modelling using DEM. In particular, the DEM application may be most appropriate where particle
683 degradation is exacerbated due to impact, and while these track sections can be modelled separately
684 using DEM, most part of the track can generally be modelled as a continuum using FEM. A FEM-
685 DEM coupled model could be the best approach to investigate the ballasted rail track dynamic as
686 suggested by Nishiura et al. [121] who developed a sleeper model by DEM for viscoelastic multi-
687 body dynamics and coupled it with rail model (FEM) providing greater insight into the dynamic
688 response of ballasted railway tracks under impact loading.

689 Ideally, ballast assemblies should be tested in a prototype scale to determine how enhanced dynamic
690 loadings at transition zones will affect the deformation and degradation of ballast; this is because
691 testing smaller particles with small equipment will affect the internal angle of friction (shearing
692 resistance) of the granular assembly, and hence the rate of volumetric strain during the shearing
693 process. A large-scale triaxial testing rig (300 mm diameter by 600 mm high) has been designed and
694 built at the University of Wollongong ([Figure 16a](#)); it is custom made to minimise boundary effects,
695 so it has been widely used to evaluate the deformation and degradation of ballast with reference to
696 the origin, size, and shape of aggregates used in Australian tracks. Key factors affecting ballast
697 degradation were found to be as loading characteristics (i.e. monotonic, cyclic), frequency and
698 confining pressure. Details of the components of this apparatus and its measuring techniques can be
699 found elsewhere [67, 74, 77, 87, 151]. A similar apparatus can also be found in [27] that can be used
700 to investigate the resilient behavior of track ballast with particle size up to 63mm.

701 A large-scale process simulation testing apparatus (PSTA) has been used to study the response of
702 ballast track components under realistic cyclic loading ([Figure 16b](#)). This PSTA can accommodate
703 specimens 800 mm long by 600 mm wide by 600 mm high, these dimensions will mimic a typical
704 unit cell section of Australian standard gauge tracks [13, 68, 80, 115]. The PSTA can also apply a
705 dynamic load up to 100 kN with frequency up to 40 Hz, simulating typical Australian passenger and
706 heavy haul freight trains traveling up to 200 km/h. Large-scale constant normal stiffness (CNS) direct
707 shear tests ([Figure 16c](#)) have been designed to study the interface between ballast-rubber
708 mats/geosynthetics and to determine the internal friction angle of ballast. The recently funded
709 National Facility for Cyclic Testing High-Speed Rail (FCTHSR; [Figure 16d](#)) is now being
710 commissioned with double axle loading to mimic heavy haul operations. With axle loads up to 40-
711 tonnes and a frequency range from 5-40 Hz (i.e. speeds of 55-220 km/h), the FCTHSR will
712 accommodate a range of cyclic loading patterns to evaluate the actual performance of ballast under
713 different structural and geotechnical conditions especially for transition zones.

714 A large-scale permeameter has been designed to measure the hydraulic conductivity of ballast
715 contaminated with fouling materials such as coal and subgrade mud [69, 160]. This chamber will
716 accommodate ballast specimens of 500mm in diameter by 300-500mm high (Figure 16e). A full-scale
717 model track (dimensions: 4.76 m by 3.48 m by 0.79 m) has been built at UOW to study the behaviour
718 of fouled ballast (Figure 16f). This model track is used to determine how ballast fouling can influence
719 ground penetrating radar (GPR) data, while capturing the moisture content and loading frequency
720 [157]. These unique and novel testing devices help us to examine and quantify the influence of
721 induced train loading characteristics on the strength, deformation, and degradation of ballast, hence
722 could be utilised for design optimization of transition zones.

723 Likewise, the CEDEX track box; a full-scale (1:1) testing facility for railway tracks in Spain [104],
724 can be utilized to optimize various maintenance works at transition zones especially for slab track to
725 ballast track transitions. This track box is 21m long, 4m deep and 5m wide and has a capacity to
726 model the full rail track section for various train loads and speeds up to 450km/h considering static
727 and dynamic loading conditions. This facility has been used in the past to study various aspects of
728 rail track performance such as track lateral stability, vertical stiffness, short and long-term
729 settlements, and track dynamic response under high-speeds trains among others, the details of which
730 can be found in [36-38].

731 The full-scale laboratory testing facility developed by the University of Nottingham [17] for railway
732 track can also be utilized to investigate the performance of railway ballast under impact loading
733 caused by differential settlement at transition zones. This facility involves three sleepers and the
734 cyclic load of 94kN can be applied directly to the sleepers through hydraulic actuators. The permanent
735 settlement as well as transient deflection can be measured in addition to the transient stresses and
736 degree of particle degradation, the results of various tests performed on this apparatus can be found
737 elsewhere [6, 18, 21]. Similarly, the Southampton railway testing facility [2] can also be utilized to
738 study the track dynamic response especially the permanent (plastic) settlement of ballast particles that
739 can cause the differential settlement as suggested by Abadi et al. [1].

740 To produce a decent design, the difference between the response of track before and after a transition
741 zone is set as low as possible. Various factors found in the literature that affect the structural response
742 of track transitions are summarised in Figure 17; these factors should receive enough attention to
743 achieve the appropriate design of transition zones in terms of selecting different design techniques
744 and approaches. To make a precise model, every individual component must be modelled separately
745 by considering characteristics such as elastic/plastic, linear/non-linear, continuum/particulate
746 (discrete), and their interaction with neighbouring components. These factors can be addressed
747 selecting proper modelling techniques such as 1D, 2D or 3D and commercially available advanced
748 modelling software packages. Note that each model and modelling software has some limitations that

749 should be considered before designing track transitions.

750 **7. Conclusions**

751 This paper presents a detailed review of rail track transitions, various associated issues and their
752 solutions. After defining the importance of rail transitions, their related problems are discussed,
753 including their causes and effects on railway operations. Numerous mitigations measures to improve
754 the performance of ballasted tracks at transition zones are also discussed, with special reference to
755 their effectiveness. The ongoing research into designing transition zones to minimise the effect of
756 abrupt changes in the structural properties of track is critically reviewed by considering large-scale
757 laboratory testing, mathematical and computational modelling, and field investigations.
758 Recommendations for the performance of the ballasted track at transitions are also presented after
759 reviewing the various design approaches. Following are the conclusions that can be drawn from this
760 review of track transition zones:

761

- 762 • Differential settlement and enhanced dynamic loads are the main problems associated with
763 track transition and are thus responsible for track degradation. The major cause of these
764 problems is the abrupt change in stiffness, which can only be controlled by designing smooth
765 and gradual transition zones.
- 766 • Most current design practices are based mainly on empiricism established through trial and
767 error. These methods can be unreliable if the empirical parameters are calibrated only to local
768 subgrade and ballast properties, and should not be extended to any track section without
769 exercising caution for different soil characteristics and dynamic loading conditions. Transition
770 zones should be designed to cope with the required variations in stiffness and possible initial
771 settlement, which may vary depending on the case. If the variations in stiffness are known,
772 the length of the transition zone and the type of materials can be established to provide a
773 smooth and gradual variation in track stiffness at the junction.
- 774 • One of many reasons for not having a precise and economical design of transition zones is
775 because the problem is complex due to the interaction of several structures and structural
776 components. Since conventional rail track structure consists of various structural components,
777 it is a composite structure, but in transition zones, this complexity is enhanced due to the
778 sudden variations in the structural properties of track. This makes prediction of the dynamic
779 performance of the overall structure a challenging task because every component behaves
780 differently under various loading conditions. Furthermore, the interaction between these
781 components (vehicle-track-structure) makes the model more complex.
- 782 • There is a lack of effective computational model to study the dynamic response of transition
783 zones due to the complexity of the problem. Similarly, implementing semi-analytical

approaches based on the transformation of time and frequency is difficult owing to the sub-structural inhomogeneity of track. Various mitigation measures have been utilised but without any theoretical reasoning, and therefore they are not overcoming the need for frequent maintenance.

- It is observed that deformation in most models is very small and is in the elastic range, whereas in actual field investigations, high deformation occurs more frequently with the passage of time. The main reason for these discrepancies could be the assumed linear and elastic nature of the structural components, but the ballast, sub-ballast and subgrade layers do not behave elastically or in a linear fashion under repeated train loads; this leads to plastic deformation and permanent settlement i.e. bumps/dips. This plastic deformation has been overlooked in most transition models, which is another source of error for predicting the dynamic response of track.

8. Appendix A

This appendix provides the governing equations used by [166] to solve the linear mathematical model for the response analysis of inhomogeneous foundation using two-layer mass spring-dashpot model. The governing equation for $x < 0$ is as:

$$\begin{cases} \frac{E_1 I_1 \partial^4 u_{11}}{\partial x^4} + m_1 \frac{\partial^2 u_{11}}{\partial t^2} + c_1 \left(\frac{\partial u_{11}}{\partial t} - \frac{\partial u_{21}}{\partial t} \right) + k_{11}(u_{11} - u_{21}) = -F\delta(x - vt) \\ \frac{E_2 I_2 \partial^4 u_{21}}{\partial x^4} + m_2 \frac{\partial^2 u_{21}}{\partial t^2} + c_1 \left(\frac{\partial u_{21}}{\partial t} - \frac{\partial u_{11}}{\partial t} \right) + k_{11}(u_{21} - u_{11}) + k_{21}u_{21} + c_2 \frac{\partial u_{21}}{\partial t} = 0 \end{cases}$$

and for $x > 0$ is as:

$$\begin{cases} \frac{E_1 I_1 \partial^4 u_{12}}{\partial x^4} + m_1 \frac{\partial^2 u_{12}}{\partial t^2} + c_1 \left(\frac{\partial u_{12}}{\partial t} - \frac{\partial u_{22}}{\partial t} \right) + k_{12}(u_{12} - u_{22}) = 0 \\ \frac{E_2 I_2 \partial^4 u_{22}}{\partial x^4} + m_2 \frac{\partial^2 u_{22}}{\partial t^2} + c_1 \left(\frac{\partial u_{22}}{\partial t} - \frac{\partial u_{12}}{\partial t} \right) + k_{12}(u_{22} - u_{12}) + k_{22}u_{22} + c_2 \frac{\partial u_{22}}{\partial t} = 0 \end{cases}$$

These equations for vertical displacement in each section are solved by considering the interface conditions at $x=0$. The solution for these equations of differential settlements is a complex process that involves many assumptions and the substitution of many variables, it can be found in [163, 166].

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Table 1: Comparison of computational modelling approaches and summarised key research findings on track transition zones

Reference	Model type	Numerical method	Software	Analysis type	Beam type	Foundation type	Model length (m)	Transition length (m)	Model calibration	Model validation	Transition type	Train speed (km/h)	Axle load (kn)	Parameters studied	Train direction	Mitigation measures	Innovations / Findings
Aggestam and Nielsen, 2019 (Sweden) [4]	2D	FEM	-	Dyn.	TS	LE	60	23.4	-	Yes	BTS ~ BTCd	250-350	170	CF, BS	Both	SS, RP	Considering certain responses of vehicle, especially car body pitching motion, simplified (one bogie) vehicle model is not appropriate. Train direction has almost no effect on the dynamic response of track for an optimised transition design
Hu et al., 2019 (China)[59]	3D	FEM	ANSYS	Dyn.	TS	EP	52.2	46.2	No	Yes	BsTR ~ BTCc	350	140	VtD, TV, TA, BS	Both	WSB	Graded gravel with 5% cement is the best filling material, followed by simple graded gravel, and then well graded coarse grained soil
Esmaeili et al., 2018 (Iran)[34]	2D	FEM	ANSYS	Dyn.	EB	LE	42.2	18	No	Yes	BTS ~ BTCc	120-340	160, 200	RD, TA, RpF	SoTSt	AR	Rail deflections and track acceleration decrease with stiffer rail pads & damping but increase with increased train speed, auxiliary rails transition is smoother and the dynamic performance improved
Koch et al., 2018 (Hungary) [85]	3D	FEM	PLAXIS	Dyn.	RB	LE	96	18	-	-	BTS ~ BTCd	80, 250	125	VtD, TV	SoTSt	BCW	Vertical displacement and Velocity amplitudes decrease with depth. Transition zone design is more important for higher train speed than lower speeds
Labrado Palomo et al., 2018 (Spain) [86]	3D	FEM	ANSYS	Dyn.	SB	LE	54	36	Yes	Yes	BTS ~ BTCd	50-300	24	TV, TA	SoTSt	CW, HMA, SRG, SRP	Geogrids are not the right approach to modify the vertical stiffness of track. Peak vibration increases with an increase in train speeds. concrete wedge is more effective technique, then HMA wedge and the Piles
Paixão et al., 2018 (Portugal) [126]	3D	FEM	Pegasus/MATLAB	Dyn.	EB	NLVE	75	17.4	Yes	Yes	BTS ~ BTCv	220	250	VtD, CF, BS	SoTSt	WSB, USPs	USPs reduce the vibration transmitted to the ballast & sleeper-ballast contact forces so as the ballast degradation & diff. settlement in the long run
Wang and Markine, 2018a (Netherlands) [171]	3D	FEM	LS-DYNA	Dyn.	HL	NLE	120	Varies	-	Yes	BTS ~ BTCd	200	142	VtD, WL, BS	SoTSt	WS, LS, WS+LS	The dips in transition zones increase, expand & propagate farther away from bridges with loading cycles. Number of iteration steps have strong effect on settlement prediction
Wang and Markine, 2018b (Netherlands) [174]	3D	FEM	LS-DYNA	Dyn.	HL	LE	120	Varies	-	Yes	BTS ~ BTCd	72-288	178	VtD, WL, BS	Both	WS, LS, WS+LS	Diff. settlement & train speed, enhance the dynamic effect (wheel loads, ballast stress), longer sleepers are recommended for economy and longer & wider for performance, fasteners are effective but complex
Heydari-Noghabi et al., 2017 (Iran) [56]	3D	FEM	MATLAB	Dyn.	EB		52.2	18	Yes	Yes	BsTR ~ BTCc	120-300	180-250	RD, CF	StTSO	AR	Track deflection increases with an increase in train speed and vehicle load. Auxiliary rails reduce rail deflection, the optimum no. of additional rails is 2
Sañudo et al., 2017 (Spain) [136]	2D	FEM	ANSYS	Dyn.	-	LE	200	Varies	-	-	BTS ~ BTCd	300		VtD, BS, TA	Both	SS	The position & separation of sleepers influences the vertical acceleration and vibration, while slab to ballast track train movement is more critical
Wang et al., 2017 (Netherlands) [175]	3D	FEM	LS-DYNA	Dyn.	HL	LE	120	Varies	Yes	Yes	BTS ~ BTCd	144	142	VtD, WL	Both	MCE	Strong connection between track degradation & the high moisture condition causes increased dynamic wheel loads and reduced stiffness at both sides of the bridge
Paixão et al., 2016 (Portugal) [125]	2D	FEM	ANSYS	Dyn.	RB	NLE	172.5		Yes	Yes	BTS ~ BTCd	220	132	VtD, CF, TA	Both	WSB,	Soft to stiff train movements are more critical when considering track degradation and wheel/rail contact loss, wedge-shaped backfill is a good approach for train-track system

Table 1 (continued)

Reference	Model type	Numerical method	Software	Analysis type	Beam type	Foundation type	Model length (m)	Transition length (m)	Model calibration	Model validation	Transition type	Train speed (km/h)	Axle load (kn)	Parameters studied	Train direction	Mitigation measures	Innovations / findings
Varandas et al., 2016 (Portugal) [165]	3D	FEM	Pegasus MATLAB	Dyn.	EB	NLE	47.75	4.1	Yes	Yes	BTS ~ BTCc	130	174	VtD, BS	Both	AS	Ballast stresses are more than their strength and create a tendency towards sliding/rolling. The flow ability of ballast is due to high shear stresses. Ballast settlement depends mainly on the movement and inclination of the approach slab
Real et al., 2016 (Spain) [134]	3D	FEM	ANSYS	St. & Dyn.	SB	LE	60	20	Yes	Yes	BTS ~ BsTAS ~ BsTCd	35	90	TM, TA	Both	AR, RP	Stiffness variation between concrete and asphalt slab tracks can be smoothen by using rubber mats. No contribution from the addition of auxiliary rail in smoothening the transition. No significant effect on track dynamic response
Shahraki et al., 2015 (Germany) [147]	3D	FEM	ANSYS	Dyn.	RB	LE	80	7.5	-	-	BTS ~ BsTCd	300	180	VtD, TA, BS	SoTSt	LS,AR, WSB	More improvement in the dynamic performance of track with auxiliary rails, improved subgrade helps the entire system to perform better. Larger sleepers are the most effective way of reducing ballast stress
Mishra et al., 2014 (Illinois) [108]	2D, 3D	FEM, DEM	GEOTRACK BLOKS3D	Dyn.	EB	LE	2.22	-	Yes	Yes	-	177-241	270	BS, BPA	-	RP	Ballast is not a continuum medium therefore ballast particles accelerate at different speeds depending on its location with respect to load application
Chen, 2013 (UK) [22]	3D	DEM	PFC ^{3D}	Dyn.	-	LE	2.1	1.9	-	Yes	-	25-380	160-320	VtD, TA	Both	SRG	Stiff to soft train movement causes larger settlement than soft to stiff, multi-step stiffness is better at reducing the differential settlement, while geogrid in ballast over a soft subgrade is ineffective
Banimahd et al., 2012 (UK) [7, 9]	3D	FEM	GEOTRACK	Dyn.	-	LE, NLE	60	4~10	-	-	BTS ~ BTCd	180-250	170	VtD, CF, TA	SoTSt	-	Soft subgrade, Voids and other faults at transition zones produce increased interaction forces, while increased train speed causes passenger discomfort. Simple variations in stiffness is not the primary issue, a 4-10 m long transition based on deflection is enough
Gallego et al., 2011 (Spain) [49-51]	3D	FEM	ANSYS	St	SB	EP	7.2	4.8	-	Yes	BTS ~ BTCd	300	180	VtD, TM	Both	WSB	Effect of the type of material in natural ground and the embankment, and the height of an embankment on the vertical track stiffness. The vertical stiffness of track should be important when designing a transition zone
Coelho et al., 2011 (Netherlands) [25]	3D	FEM	-	Dyn.	RB		20	5.0	-	Yes	BTS ~ BTCc	96 - 200	124 - 193	VtD, BS	Both	AS, MCE	Dynamic response found to be 20% higher than static, Higher response for soft to stiff train movement, development of hanging sleepers and their effect on the long-term performance of track
Varandas et al., 2011 (Portugal) [167]	1D	-	-	Dyn.	EB	NLE	60	5.0	-	Yes	BTS ~ BTCc	120 - 130	108 - 174	VtD, CF	Both	AS	Hanging sleepers were observed on both sides of the culvert, ballast settlement above the approach slab is caused by sleeper loading and ballast flow in a horizontal direction
Galvín et al., 2010 (Spain) [52, 53]	3D	FEM	-	QS & Dyn	EB	NLE	90	15.4	-	Yes	BTS ~ BsTCd	298	152	TV,	SoTSt	RP, Hsb	Rail pads used in ballast-less tracks play very important role in the vibration induced under moving train loads
Nicks, 2009 (Texas) [120]	3D	FEM	LS-DYNA	Dyn.	EB	LE	16	1.6-8.4	No	Yes	BTS ~ BTCd, BTS ~ BsTCd	32 - 161	292	CF, TA, VtD, BS	Both	CMS, USPs, SB, RP	Going from soft to stiff structure causes higher impact loads, increased velocity has more impact for bumps than dips, track modulus is linearly proportional to soil modulus, ballast deck bridge is better than the ballast-less deck, steel bars of varying lengths will reduce track deflection and subgrade pressure

Table 1 (continued)

Reference	Model type	Numerical method	Software	Analysis type	Beam type	Foundation type	Model length (m)	Transition length (m)	Model calibration	Model validation	Transition type	Train speed (km/h)	Axle load (kn)	Parameters studied	Train direction	Mitigation measures	Innovations / Findings
Witt, 2008 (Sweden) [178]	3D	FEM	LS-DYNA	Dyn.	RB	LE	18	-	-	-	BTS ~ BTCd	324	215	CF	SoTSt	RP, USPs	Medium strength USPs (with vertical stiffness of 400kn/mm) are better at reducing the wheel/rail contact forces, Softer USPs help to reduce the ballast contact forces
Namura and Suzuki, 2007 (Japan) [113]	3D	FEM	-	Dyn.	TB	LE	~5	-	-	Yes,	BTS ~ BsTCd	100-300	~160	WL,	SoTSt	LS, RS	Ballast settlement increased with the use of longer sleepers (more) and the resilient sleepers (less), length of approach track should be more than 22 m. Resilient sleepers are best at reducing ballast vibration
Read and Li, 2006 (Colorado) [133]	2D	FEM	GEOTRACK	QS	SB	LE	115	0	-	-	-	80	54, 67, 100	VtD, BS, TM	-	CW, AR, HMA, WS+LS	Concrete slab is the best approach, followed by HMA, and then additional rails. Longer and wider sleepers at reduced spacing have an insignificant effect
Li and Davis, 2005 (Colorado) [95]	3D	FEM	NUCARS	Dyn.	EB	LE	-	30	-	-	BTS ~ BTCd	160	178	WL, CF	-	-	Variations in stiffness lead increase the variations in dynamic load and wheel-rail interaction forces
Lei and Mao, 2004 (China) [91]	2D	FEM	-	Dyn.	SB	LE	231	20	Yes	-	BTS ~ BTCd	60-300	170	CF, TA	SoTSt	-	Variations in vertical stiffness have no direct effect on wheel/rail dynamic interaction forces. Permanent settlement is the main cause of transition related problems. Suggestions for irregularity angle and length of transition zone
Hunt and Winkler, 1997 (UK) [62]	2D	DEM	-	Dyn.	SB	LE	10	2.0	-	-	BTS ~ BTCd	150	110	VtD, CF	SoTSt	USPs	Rate of track settlement mainly depends on the Initial settlement (voids under sleepers) owing to accelerated settlement under impulsive loads

Numerical Method (**FEM**: Finite Element Method, **DEM**: Discrete Element Method)

Analysis Type (**St.**: Static, **Dyn.**: Dynamic, **QS**: Quasi-static),

Beam Type (**TS**: Timoshenko, **EB**: Euler-Bernoulli, **HL**: Hughes-Liu, **SB**: Simple Beam, **RB**: Rectangular Beam),

Foundation Type (**LE**: Linear Elastic, **NLE**: Non-linear Elastic, **LVE**: Linear Visco-elastic, **NLVE**: Non-linear Visco-elastic, **EP**: Elastoplastic)

Transition Type (**BTR**: Ballast Track on Rock, **BTS**: Ballast Track on Soil, **BTCd**: Ballast Track on Concrete deck, **BsTR**: Ballast-less Track on Rock, **BsTS**: Ballast-less Track on Soil, **BsTCd**: Ballast-less Track on Concrete deck, **BTCc**: Ballast Track on Concrete Culvert, **BTCv**: Ballast Track on Concrete Viaduct, **BsTAS**: Ballast-less Track on Asphalt Slab),

Parameter Studied (**VtD**: Vertical displacement/deflection, **TV**: Track Velocity, **TA**: Track Acceleration, **BS**: Ballast/subgrade Stresses, **WL**: Wheel Load, **TM**: Track Modulus/Stiffness, **RD**: Rail Deflection, **RpF**: Railpad Force, **CF**: Contact Forces, **BPA**: Ballast Particles Acceleration),

Train Direction (**SoTSt**: Soft to Stiff, **StTSO**: Stiff to Soft),

Mitigation Measures (**WSB**: Wedge-Shaped Backfill, **AR**: Auxiliary Rails, **SRP**: Soil Reinforcement with Piles, **HMA**: Hot Mix Asphalt Wedge/layer, **USPs**: Under Sleeper Pads, **RP**: Rail Pads, **BCW**: Backfill Confinement with Walls, **CW**: Concrete Wedge/slab, **AS**: Approach Slab, **SRG**: Soil Reinforcement with Geogrid, **WS**: Wider Sleeper, **LS**: Longer Sleeper, **RS**: Resilient Sleepers, **CMS**: Changing Material of Sleepers, **SS**: Sleeper Spacing/Location, **MCE**: Moisture Content Effect, **SB**: Steel Bars, **HSb**: Hydraulic sub-base)

Table 2: Summarised outcomes of field measurements on track transition zones

Author	Description	Transition type	Transition length (m)	Train speed (km/h)	Loading (kN)	Train direction	Track type	Parameters studied	Instrumentation	Outcomes
Boler et al., 2019 (USA) [15]	Stone blowing	BTS ~ BTCd		177	150		PT	VtD, WL, TA, BS	MDD, SG	60% reduction in transient displacements, Reduction in vertical acceleration and in gaps at sleeper-ballast interface, increased effectiveness and longevity
Paixão et al., 2018 (Portugal) [126]	WSB+USPs	BTS ~ BTCv	20	220	250		MT	RD, TA, SD	SG, LU, PSD, Acm, LVDT	Variations in the vertical stiffness of various sections along the transition, USPs with t=7mm reduced the stiffness by 30% of embankment but increases by 22% for UGM, USPs with t=10mm reduced the stiffness by 27% for CBM but remains same for CBM and concrete
Wang and Markine, 2018 (Netherlands) [174]	methodology for comprehensive analysis of railway transition zones	BTS ~ BTCd	24	104	186	Both	PT	VtD	VGS	Maximum displacement close to bridge is higher than the far end, maximum displacement is larger for embankment ~ bridge case than the bridge ~ embankment case of train movement, Differential settlement at transition zones causes hanging sleepers
Zuada Coelho et al., 2018 (Netherlands) [26]	Approach slab	BTS ~ BTCc	4	65-106	76-142	Both	PT	VtD, WL, TS	Gp, Acm, VC, SG	Large differences in displacement and increased wheel loads show the presence of hanging sleepers. Non symmetric response and non-uniform distribution of track displacements indicates inefficiency at the transition zone. Pivoting about the culvert generates enhanced impact loading
Stark and Wynn, 2018 (USA) [155]	Reinforcement (Geoweb, HMA, Grout)	BTS ~ BTCd	15		30 MGT		FT	VtD, TA	VC, Acm	Geosynthetic reinforced transitions are less expensive, perform well for freight loads and reduce differential vertical displacements. Geoweb can be used as an alternative to HMA. With Geosynthetic reinforcement stiffness increases (in approaches) and decreases (while used under bridge abutments)
Wilk et al., 2016 (USA) [177]	Sleeper support effect, BCW, HMA	BTS ~ BsTCd	8.2	16, 40, 177	280	Both	PT, FT, MT	VtD, WL, TA, VF	Acm	For tracks with better sleeper support, peak acceleration remains below 5g for various types of trains, speeds, and loads, indicating smooth load transfer between track components, whereas poor sleeper support means that acceleration varies from 5g to 10g. Accelerometer data indicates a qualitative analysis of the track support and not quantitative
Mishra et al., 2014 (Illinois) [107]	Railpads,	BTS ~ BTCd		177-241	135	Both	MT	VtD, WL, TA	MDD, SG	Data obtained for track deformation and corresponding loading through field instrumentation is utilised to calibrate a 3D track dynamic model
Fortunato et al., 2013 (Portugal) [44]	Wedge-shaped approach	BTS ~ BTCd	20	220	66-125	SoTSt	MT	VtD	SG, LU, PSD, Acm, LVDT, IT	Passenger trains at higher speed cause more acceleration amplitude at sleepers than heavy freight at lower speeds. Using wedge-shaped approach leads to a gradual transition of vertical stiffness, and settlement seems to stabilise after one year of construction
Namura and Suzuki, 2007 (Japan) [113]	Railpads, subgrade stabilization, Auxiliary rails, Ggling resin	BTS ~ BTCd	20	100-300		Both	PT	WL, TA	SG, TLV	Variations in loads occur on ballast track at transition zone. Application of rail pads has no effect on track irregularities. Axle load variation at track transition found which increased with increasing speed. More variations in acceleration while travelling from a ballast track to a slab track
Li and Davis, 2005 (Colorado) [95]	HMA, geocell, Thick ballast, cement stabilised	BTS ~ BTCd	30	160	178	Both	FT	RD, VtD, TS	SE, TLV	More track geometry degradation at bridge approaches, No improvement in track performance, a very high initial settlement rate of 100 to 180 mm in just six months (80 MGT), Rubber mats reduce track stiffness and increase track damping

Description (WSB: Wedge-Shaped Backfill, AR: Auxiliary Rails, SRP: Soil Reinforcement with Piles, HMA: Hot Mix Asphalt Wedge/layer, USPs: Under Sleeper Pads, RP: Rail Pads, BCW: Backfill Confinement with Walls, CW: Concrete Wedge/slab, AS: Approach Slab, SRG: Soil Reinforcement with Geogrid, WS: Wider Sleeper, LS: Longer Sleeper, RS: Resilient Sleepers, CMS: Changing Material of Sleepers, SS: Sleeper Spacing/Location, MCE: Moisture Content Effect, SB: Steel Bars, HSb: Hydraulic sub-base)

Transition Type (BTR: Ballast Track on Rock, BTS: Ballast Track on Soil, BTCd: Ballast Track on Concrete deck, BsTR: Ballast-less Track on Rock, BsTS: Ballast-less Track on Soil, BsTCd: Ballast-less Track on Concrete deck, BTCc: Ballast Track on Concrete Culvert, BTCv: Ballast Track on Concrete Viaduct), Train Direction (SoTSt: Soft to Stiff, STSo: Stiff to Soft), Track Type (PT: Passenger Track, FT: Freight Track, MT: Mixed Traffic/Track), Parameter Studied (VtD: Vertical displacement/deflection, TV: Track Velocity, TA: Track Acceleration, BS: Ballast/subgrade Stresses, WL: Wheel Load, TM: Track Modulus, RD: Rail Deflection, RpF: Railpad Force, CF: Contact Forces, BPA: Ballast Particles Acceleration),

Instrumentation (MDD: Multidepth Deflectometers, SG: Strain gauges, LU: LASER Units, PSD: Position Sensitive Devices, Acm: Accelerometers, LVDT: Linear Variable Differential Transducer, VGS: Video Gauge System, VC: Video Cameras, Gp: Geophones, IT: Inclinator Tubes, SE: Survey Equipments, TLV: Track Loading Vehicle)

(a) Track transitions at Berry bridge crossing Tannery road, NSW, Australia

(b) Slab track to ballast track transitions indicating alignment error (adopted from [96])

Figure 1: Rail track transitions due to sudden change in substructural components

(a) Level crossing on a conventional track at Unanderra, NSW, Australia

(b) Two crossing diamond, a level crossing, and a turnout (adopted from [96])

Figure 2: Rail track transitions due to the change of superstructural components

Figure 3: Variation in rail deflection, railpad force and track acceleration at track transition (data source: [34])

(a)

(b)

(c)

(d)

Figure 4: Comparison of differential settlements measured at various track transitions; (a) slab track to ballast track, (b) bridge crossing, (c) bridge approaches and (d) culvert crossing

Figure 5: Schematic diagram of the development of bump/dip at bridge approaches (modified after [43])

Figure 6: Variation in wheel rail interaction forces at track transition

Figure 7: Summarised track transition problems: causes and effects (inspired by [122])

Figure 8: Track stiffness variation from a ballasted track to a slab track at transition zones

(a)

(b)

Figure 9: Abrupt variation in track modulus/stiffness at various track transitions; (a) at bridge crossings, (b) soft to stiff track transition

Continuous stiffness measurements, West coast line in Sweden, east track w37 2001, 20 km/h 5,7 Hz

Figure 10: Variation in track stiffness for various track types along the railway track (adopted from [30])

Figure 11: Variation in measured ballast stresses at various track transitions

Figure 12: Peak displacement and stiffness distribution at transition zone (data source: [169])

Figure 13: Mass and spring-dashpot models for ballast track to slab track transition

Figure 14: Two layers mass spring-dashpot model for track transition

Figure 15: Rail deflection along the transition zone (a) for 180 kN vehicle load and various speeds, (b) for 200 km/h speed and various vehicle loads (adopted from [56]).

Figure 16: Selected large-scale ballast testing equipment at the University of Wollongong Australia

Figure 17: Summarised important factors for transition zone design considerations

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