Steady laminar natural convection of nanofluid under the impact of magnetic field on two-dimensional cavity with radiation

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Abstract
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ABSTRACT
In current investigation, steady free convection of nanofluid has been presented in occurrence of magnetic field. Non-Darcy model was utilized to employ porous terms in momentum equations. Working fluid is H₂O based nanofluid. Radiation effect has been reported for various shapes of nanoparticles. Impacts of shape factor, radiation parameter, magnetic force, buoyancy and shape impact on nanofluid treatment were demonstrated. Result demonstrated that maximum convective flow is observed for platelet shape. Darcy number produces more random patterns of isotherms.

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I. INTRODUCTION
Thermal behavior in presence of buoyancy force is an important need in manufacturing and production progressions such as cooling processes. Mostly in the production mediums, it is usually imagined in the development of micro systems. Wu and Wang deliberated time-periodic natural convective in an inclined permeable closure. Cheikh et al. scrutinized nanoliquids migration in a cavity shaped medium using rheological properties. An analysis of combined behavior of CuO-water nanoliquid within a tank inside a regime of corner heater was explained by Ismael et al. Some of the important and novel work related to natural convection is referred in Refs. 4–14.

Heat transfer can be improved as per need of the industry by changing the physical conditions, turbulent boundary layer and augmentation in thermal treatment involved fluid. Improving θ of
the ordinary liquid by adding nano sized particles is the finest process. Primarily, Maxwell\cite{15} familiarized the notion of probability of augmentation of thermal conductivity by consuming tiny particles which has specific restrictions as blocking. Secondly, Choi\cite{16} established that the $k$ of working fluid meaningfully can be enhanced by nanoparticles. Effects of Lorentz forces and migration of CuO-H$_2$O nanoliquid in a permeable semi annulus was conducted by Sheikholeslami et al.\cite{17} Enrichment of CO$_2$ nanofluids by absorption was discussed Zhang et al.\cite{18} Mixed convective nanofluid in an expelled cavity with fluid-solid interface of elastic-step type corrugation was demonstrated by Selimfendigil and Oztop.\cite{19} Kanafer et al.\cite{20} inspected nanomaterial migration with heat transfer. Various uses of nanomaterials were reported recently.\cite{21–32}

Sadig et al.\cite{33} initiated the simulation of micropolar nanofluid with oscillator. Stretched flow of Casson nanomaterial about an inclined permeable cylinder with slip effects was investigated by Usman et al.\cite{34} Ebad and Sharif\cite{35} considered the influence of magnetic force on nanomaterial phenomenon. To analysis entropy production of nanomaterial, numerical approach was applied by Sheremet et al.\cite{36} A number of advantages of nanofluids were described in numerous literatures.

Radiation was serious influence in physical science and planning uses. Inspiration of radiation impact on nanomaterial migration has been discovered by Sheikholeslami et al.\cite{17} Analysis of radiation with porous zone has been demonstrated by Raju et al.\cite{37} Salem et al.\cite{38} scrutinized migration of magnetized Jeffrey fluid round a non-fixed cone involving chemical reaction. Few important and relevant literatures for numerous physical characteristics like MHD and radiation are registered in Refs. 47–64. This inquiry intention to examine the behavior of thermal radiation on natural convective nanofluid through a permeable two dimensional enclosure. The characters of Da, Rd, nanoparticle concentration, and Hartmann number are revealed by Control Volume Finite Element Method. Effects of various important variables were deliberated in graphs.

II. EXPLANATION OF GEOMETRY

Fig. 1 exhibits the active geometric variables of present article. Triangle sample element has been depicted in this figure, too. Inner surface was under the uniform heat flux and outer one was cold. Cylinder is occupied with nanofluid and stimulated by uniform magnetic field.

III. FORMULAS AND APPROACH

In current investigation, laminar two dimensional free convection of nanofluid which is affected by magnetic field is deliberated. For porous media, non-Darcy model is involved. Therefore, the formulas are:

$$\frac{\partial v}{\partial y} + \frac{\partial u}{\partial x} = 0,$$

$$\frac{\mu_{nf}}{\rho_{nf}} \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) + B_0^2 v (\sin \lambda) \sigma_{nf} (\cos \lambda) - \frac{1}{\rho_{nf}} \frac{\partial P}{\partial x} - \frac{1}{\rho_{nf}} \frac{\mu_{nf}}{K} u = - (T_c - T) C_{nf} \cos \gamma + \frac{\partial T}{\partial y},$$

$$\frac{\mu_{nf}}{\rho_{nf}} \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) - (T_c - T) \beta_{nf} \cos \gamma - \frac{\partial P}{\partial y} - \frac{1}{\rho_{nf}} \frac{\mu_{nf}}{K} v + \sigma_{nf} B_0^2 \left[ -v (\cos \lambda)^2 + u (\sin \lambda) (\cos \lambda) \right] = \frac{\partial v}{\partial y} + \frac{\partial v}{\partial y},$$

$$\frac{1}{(\rho C_p)_{nf}} \frac{\partial T}{\partial y} + \frac{1}{(u T + v T)} (\frac{\partial T}{\partial y}) = k_{nf} (\rho C_p)^{-1}_{nf} \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right),$$

$$(\rho C_p)_{nf}, \rho_{nf}, (\rho \beta), \sigma_{nf} \text{ and } \sigma_{nf} \text{ are defined as:}$$

$$(1 - \phi) (\rho C_p)_{nf} + (\rho C_p)_{f}, \phi = (\rho C_p)_{nf},$$

$$\rho r = \rho (1 - \phi) / \rho_f + \phi, \rho r = \rho_{nf} / \rho_f,$$

$$\beta r = (\rho \beta)_{nf} \frac{(1 - \phi) / (\rho \beta)_{nf} + \phi, \beta r = (\rho \beta)_{nf} / (\rho \beta)_{f}}{\delta},$$

$$\sigma_{nf} = 1 + \frac{3(\delta - 1) \phi}{(\delta - 1) \phi + (\delta + 2)}, \delta = \frac{\sigma_{nf}}{\sigma_f}$$

FIG. 1. (a) Current porous domain, (b) CVFEM element.
To estimate \( \mu_{\text{eff}} \) and \( k_{\text{eff}} \):\(^{65}\)

\[
\mu_{\text{eff}} = \mu_{\text{static}} + \frac{k_{\text{Brownian}}}{k_f} \times \frac{\mu_f}{\Pr_f},
\]

\[
k_{\text{Brownian}} = 5 \times 10^4 \epsilon_p \rho_f g \left( \frac{d_p}{\phi(T)} \right) \sqrt{\frac{k_f T}{\rho_d d_p}},
\]

\[
k_f = \frac{mk_f + (k_f - k_p) \phi + k_f}{k_f - k_p} m \phi + (k_f - k_p) \phi + mk_f + k_p + k_f.
\]

To find the characteristics of testing fluid, we employed similar material which was utilized in Ref. \( 65 \).

We considered the Eq. (11) to remove pressure terms.

\[
\omega + \frac{\partial u}{\partial y} - \frac{\partial v}{\partial x} = 0, \frac{\partial \psi}{\partial y} = u.
\]

Presenting dimensionless measures:

\[
U = \frac{uL}{\alpha_{\text{sf}}}, \quad V = \frac{vL}{\alpha_{\text{sf}}}, \quad \theta = \frac{T - T_c}{\Delta T}, \quad \Delta \tau = \frac{\rho L^2}{\kappa_f}, \quad (XL, YL) = (x,y),
\]

\[
\Psi = \frac{\psi}{\alpha_{\text{sf}}}, \quad \Omega = \frac{\alpha L^2}{\alpha_{\text{sf}}},
\]

Corresponding to E. (12), we have:

\[
\Psi_{YY} + \Psi_{XX} = -\Omega,
\]

\[
U \frac{\partial \Omega}{\partial X} + V \frac{\partial \Omega}{\partial Y} = \Pr A_1 A_4 \left( \frac{\partial^2 \Omega}{\partial Y^2} + \frac{\partial^2 \Omega}{\partial X^2} \right) + \Pr Ha A_1 A_4 \left( \frac{\partial U}{\partial X} \cos \lambda \sin \lambda 
\]

\[
- \frac{\partial V}{\partial X} (\cos \lambda)^2 + \frac{\partial U}{\partial Y} (\sin \lambda)^2 - \frac{\partial V}{\partial Y} \cos \lambda \sin \lambda 
\]

\[
+ Pr Ra A_1 A_4 \left( \frac{\partial \theta}{\partial X} \cos y - \frac{\partial \theta}{\partial Y} \sin y \right) - Pr A_5 A_2 \Omega,
\]

\[
\left( 1 + \frac{4}{3} \frac{k_{\text{eff}}}{k_f} \right) \frac{\partial^2 \theta}{\partial Y^2} + \left( \frac{\partial^2 \theta}{\partial X^2} \right) = \frac{\partial \theta}{\partial Y} \frac{\partial \psi}{\partial X} + \frac{\partial \psi}{\partial Y} \frac{\partial \theta}{\partial X}.
\]

In Eq. (14) and (15), following parameters has been used:

\[
Ra = g(\gamma^2) \Delta TL / (\mu_f \alpha_f), \quad Ha = LB_0 \sqrt{\sigma_f / \mu_f},
\]

\[
A_1 = \frac{\rho_f}{\rho_f}, \quad A_2 = \frac{(\rho C_p)_{\text{sf}}}{(\rho C_p)_{\text{f}}}, \quad A_3 = \frac{(\rho C_p)_{\text{sf}}}{(\rho C_p)_{\text{f}}}, \quad A_4 = \frac{k_{\text{eff}}}{k_f}, \quad A_5 = \frac{\alpha_{\text{sf}}}{\alpha_f}, \quad A_6 = \frac{\sigma_{\text{sf}}}{\sigma_f}, \quad Pr = \nu f / \alpha_f.
\]

To solve Eq. (13) to (15), we considered the boundary conditions as below:

\[
\frac{\partial \theta}{\partial n} = 1.0, \quad @ r = r_a \quad (17)
\]

\[
\theta = 0.0, \quad @ r = r_{\text{out}}
\]

\[
\Psi = 0.0, \quad @ \text{every walls}
\]

To estimate rate of heat transfer the following equations can be used:

\[
Nu_{\text{loc}} = \frac{1}{\theta} \left( 1 + \frac{4}{3} \frac{k_{\text{eff}}}{k_f} \right) \frac{1}{Rd} \left( \frac{k_{\text{eff}}}{k_f} \right),
\]

\[
Nu_{\text{ave}} = \frac{1}{S} \int S N u_{\text{loc}} \, ds.
\]

Initially, Sheikholeslami\(^{66}\) utilized the emerging authoritative scheme (CVFEM) for problems related to heat transfer. Both Finite volume method (FVM) and Finite element method (FEM) played important role in the development of CVFEM.\(^{71-82}\) In the last iteration, Gauss-Seidel technique was engaged to compute the scalars. Various new powerful numerical methods were suggested in the world.\(^{43-64}\)

**IV. CODE VERIFICATION AND GRID DESCRIPTION**

In order to confirm the precision of existing FORTRAN code, the program has been engaged to compute former available

\[
\text{FIG. 2. Validation for nanofluid (Khanafer et al.\(^{35}\)).}
\]

| TABLE I. Changing of \( Nu_{\text{ave}} \) for different grids at \( Ra = 10^5, \phi = 0.04, Da = 100, Ha = 20 \) and \( Rd = 0.8 \). |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| \( 51 \times 151 \) | \( 61 \times 181 \) | \( 71 \times 211 \) | \( 81 \times 241 \) | \( 91 \times 271 \) |
| 7.12552 | 7.12941 | 7.13476 | 7.13501 | 7.13737 |
articles. As exposed in Fig. 2, the precision of this algorithm is guaranteed. Also, the consistent outputs must not reliant on grid. Table 1 revealed the outcomes of various meshes for different cases and proposed that the mesh size must be 71 × 211.

V. RESULTS AND DISCUSSION

In current report, nanomaterial management under the impact of magnetic forces was simulated. Fig. 2. Display the authentication for nanofluid \( Gr = 10^4, \phi = 0.1 \). We showed that the existing data were in exceptional arrangement with prior one. The inspiration of Darcy parameter on streamlines and isotherms for nanoliquid with and without magnetic field is analyzed numerically and portrayed in Figs. 3, 4, and 5. It is illustrious that aggregate the amount of Ha the array of streamlines is changed which can be obviously perceived from the central portion of the cavity. While, a minor escalation in the isotherm is detected for \( Ha = 0 \) or no magnetic fluid while the large values of Hartmann numbers partially \( Ha = 20 \) isotherms of the nanofluid and results the escalation in Nu. A declaration is being made about that the Hartmann number is essential to augment and significantly influence the streamlines and isotherms. One can perceive that there exists a formerly clock-wise eddy. An enhancement in \( Da \) reasons to create the secondary eddy which revolves in an anti-clockwise manner and the primary circle directed to the higher side. When magnetic employs, then it bases the central circle to become tougher and directed upwards. Allowing to \( \theta \) contour, it is establish that isotherms become more complex with zero magnetic force.

Changes of \( Nu_{ave} \) corresponding parameters have been demonstrated in Fig. 6 and Eq. (20):

\[
Nu_{ave} = 4.19 + 0.031 m + 1.57 R_d + 2.67 \log(R_a) + 0.64 Da - 0.09 Ha \\
+ 0.2 m Ha - 0.31 R_d Ha - 0.77 R_d Ha + 0.27 \log(R_a) R_d \\
- 0.59 \log(R_a) Ha + 0.10 Ha R_d - 2.74 \times 10^{-3} \ m^2
\]

(20)

Fig. 6 displays the impact of \( Ra, Ha, Rd, m, \) and \( Da \) on average Nusselt number, respectively. Thermal radiation is supportive to advance the flow due to convective. The manners of permeability are parallel with the \( Rd \). Hence, \( Nu_{ave} \) acts as an augmenting function for the permeability and \( Rd \). Also, as the Rayleigh number produce a reduction in the temperature which results in increasing \( Nu_{ave} \).

![Streamlines and Isotherms](image-url)

**FIG. 3.** Effect of \( Da \) on nanofluid behavior \( (Da = 100 (- - -) \) and \( Da = 0.01 (---) ) \) when \( Ra = 10^3, m = 5.7, Rd = 0.8 \).
FIG. 4. Impacts of $H_a$ on nanofluid flow when $Ra = 10^5$, $Da = 0.01$, $Rd = 0.8$, $m = 5.7$, $\phi = 0.04$.

FIG. 5. Impacts of $H_a$ on nanofluid flow when $Ra = 10^5$, $Da = 100$, $Rd = 0.8$, $m = 5.7$, $\phi = 0.04$. 
FIG. 6. Influences of $Ra$, $Ha$, $Rd$, $m$, $Da$ on $Nu_{ave}$.

$Ra = 10^4$, $Rd = 0.4$, $\phi = 0.04$, $m = 4.35$

$Da = 50$, $Rd = 0.4$, $m = 4.35$, $\phi = 0.04$

$Ra = 10^4$, $Ha = 10$, $Da = 50$, $\phi = 0.04$
VI. CONCLUSIONS

In this scientific analysis steady laminar natural convection of nanofluid with magnetized cavity is performed. Application takes more importance in appearance of radiation and porosity. Innovative numerical technique was adopted to stimulate the behavior of pertinent parameters. The graphical analysis is carried out for with and without magnetic effects. It is perceived that Hartmann number is important to augment and expressively influence the streamlines and isotherms. Convection enhances with an increase of thermal radiation in the system.

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NOMENCLATURE

- $\kappa$: thermal conductivity
- $C_p$: heat Capacity
- $\mu$: dynamic viscosity
- $R_d$: radiation parameter
- $D_a$: Darcy number
- $\beta$: thermal expansion coefficient
- $Ra$: Rayleigh number
- $Ha$: Hartmann number
- $Re$: Reynolds number
- $q_r$: radiation heat flux
- $m$: shape factor
- $p$: pressure
- $g$: gravitational acceleration vector

Greek symbols

- $\epsilon$: electric conductivity
- $\phi$: volume fraction
- $\sigma$: electrical conductivity

Subscripts

- $f$: base fluid
- $p$: particle
- $nf$: nanofluid

REFERENCES


M. Sheikholeslami, A. Arabkoohsar, I. Khan, A. Shafee, and Z. Li, “Impact of Lorentz forces on Fe3O4–water ferrofluid entropy and exergy treatment within a permeable semi annulus,” Journal of Cleaner Production 221, 885–898 (2019).


M. Sheikholeslami, M. A. Sheremet, A. Shafee, and Z. Li, “CVFEM approach for EHD flow of nanofluid through porous medium within a wavy chamber under the impacts of radiation and moving walls,” Journal of Thermal Analysis and Calorimetry 166, 1–10 (2019).

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