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Experimental testing and modelling of a rotary variable stiffness and damping shock absorber using magnetorheological technology

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Experimental testing and modelling of a rotary variable stiffness and damping shock absorber using magnetorheological technology

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Abstract

This paper presents a novel rotary shock absorber which combines the abilities of variable stiffness and variable damping by assembling a set of two magnetorheological (MR) damping units, one of which being placed in series with a rubber spring. This allows the damping and stiffness to be controlled independently by the internal damping and the external damping units, respectively. A test bench was established to verify the variable stiffness and damping functionality. The experimental results for variable damping test, variable stiffness test and co-working test are presented. At the amplitude 10 degree and the frequency 0.5 Hz, increases of 141.6% and 168.1% are obtained for damping and stiffness separately if increasing the corresponding current from 0 A to 1 A and from 0 A to 2 A, respectively. A mathematical model is then developed and verified to predict the changing of the damping and stiffness. The test results and the simulated model confirm the feasibility of the shock absorber with the ability of varying damping and stiffness simultaneously.

Key words: magnetorheological fluid damper, shock absorber, variable stiffness and damping,
1. Introduction

Magnetorheological fluid (MRF) is a kind of smart material that possesses controllable rheological properties under the application of a magnetic flux. This change presents itself as an apparent increase in viscosity, occurring rapidly within a few milliseconds (Muhammad et al., 2006; Carlson and Jolly, 2000). Without a magnetic field applied, MRF is in free-flowing state, sometimes referred to as the ‘off-state’. In this state, the plastic viscosity of the material is controlled only by the viscosity of the carrying liquid and particle volume fractions. In contrast, MRFs will change to a semisolid state (‘on-state’) when exposed to external magnetic field. In this state, MRFs exhibit field-dependent behaviour characterized by a variable yield stress, dependent on the applied magnetic field strength. After the yield stress is surpassed by an applied shear stress, MRFs revert back to the free flow state with nearly unchanged plastic viscosity (Zhu et al., 2012; Goncalves and Carlson, 2007). Apart from the advantage of a fast response, MRF also has the characteristics of low operational power consumption since a magnetic field induced yield stress of MRF can be achieved using an electromagnet with a low voltage and modest current (Olabi and Grunwald, 2007). Thus, since first introduced in 1984 (Rabinow, 1948), MRF attracted a lot of research attention in applications such as civil engineering (Dyke et al., 1996; Tse and Chang, 2004; Housner et al., 1997; Jung et al., 2004; Christie et al., 2019), medical rehabilitation and surgery (Liu et al., 2006; Nguyen et al., 2011b; Carlson et al., 2001; Gudmundsson et al., 2010), and automotive design (Sun et al., 2017; Sun et al., 2016; Tang et al., 2017; Sun et al., 2015; Neelakantan and Washington, 2005; Sassi et al., 2005).

Amongst all of the applications for MRFs, the most common mechanical design utilising the material is the Magnetorheological damper, referred to as MR damper. Depending on whether the input to these dampers is linear or angular, they may be classed as either linear, or rotary MR dampers. Linear MR dampers are commonly studied in the areas of vibration control. However, these tend to require larger installation space than rotary dampers. Additionally, the surface of the linear damper rod is usually
exposed to the environment, making the damper more-easily damaged by other objects (Els and Holman, 1999). Furthermore, the linear MR damper requires a relatively large amount of costly MR fluid which may increase cost to some extent (Giorgetti et al., 2010). Thus, the rotary MR dampers are considered as an advantageous alternative to linear versions in some cases, and are developed for many purposes. In addition, the rotary damper is inherently suitable in all rotary-movement-based applications, namely as braking devices and angular joints. As some examples of uses, rotary MR dampers can be used in the areas of position control and vibration attenuation. Sapiński et al. (Sapiński et al., 2017) demonstrated the application of a rotary MR damper in position control. The dynamic behaviour of the designed system was improved by using various control methods. Yang et al. (Yang et al., 2012) used a compact and light weight rotary MR damper in pathological tremor suppression.

In terms of the structure of rotary MR dampers, they are typically divided into: drum-type, disk-type, and hybrid-type rotary dampers, according to the location of the effective shear area (Imaduddin et al., 2013). In the case of drum-type dampers, Huang et al. (Huang et al., 2002) theoretically investigated the design method of these. The torque of which could be increased by expanding the effective area (Huang et al., 2002; Nam and Ahn, 2009), and optimizing magnetic field strength (Senkal and Gurocak, 2009; Kikuchi and Kobayashi, 2011). As for disk-type MR dampers, Li and Du (Li and Du, 2003) were among the first to present the development of such a device, using a single disk. Later, multi-disk MR dampers were researched (Park et al., 2006; Ismail et al., 2012) in order to improve performance and torque output by providing more shear surfaces. Combining these disk- and drum-type designs, a hybrid type MR damper was later introduced (Nguyen and Choi, 2011; Nguyen et al., 2011a). While other, more complex structures of rotary MR dampers do exist, those reported in literature typically serve only to provide variable damping, and this is what they have in common.

The use of rotary MR dampers to provide variable stiffness, rather than variable damping is quite a new concept. Furthermore, simultaneously providing variable stiffness and damping using rotary MR dampers has not been reported on. Variable stiffness and damping abilities are regarded as important features in some applications, especially in vibration control. Taking vibration control as an example,
the quantity of vibration energy dissipated in a system could be controlled by adjusting the damping, such that the vibration magnitude would be reduced. In addition, vibration could also be suppressed by varying the system’s stiffness, as it can shift the resonant frequency of the system away from a given excitation frequency. Considering the importance of both the stiffness variability and the damping variability, a new rotary damper capable of controlling its damping and stiffness should be developed and investigated systematically.

Following the previous work of the authors of this paper, on the development of a compact linear variable stiffness and damping damper [19], this paper presents a rotary variable stiffness and damping shock absorber based on MR technology. The proposed rotary shock absorber consists of a rubber spring and two rotary MR damping cylinders, one of which acting in series with a spring. As the two dampers are connected in parallel, the damping and stiffness of the device can be controlled independently. The remainder of this paper is organised as follows. The structure design, working principle and magnetic field simulation of the proposed shock absorber are introduced in the Section 2. In the following section, Section 3, the experimental test rig for the characterisation of the manufactured shock absorber is presented. The experimental results are presented and analysed in the Section 4. Following this, Section 5 focuses on the development of the mathematical model for the designed shock absorber. In addition, the numerical model parameters for the internal damper and the external damper were also identified in this section. The last section, Section 6, then presents the conclusions.

2. Structure and analysis of the MR Fluid rotary damper

2.1 The structure of the MR Fluid rotary damper

As shown in Figure 1, the proposed shock absorber consists of a set of two damper structures: a drum-type internal damper, and a disk-type external damper. The shaft is connected to the internal damper rotor and the plate. A motor can be mounted to the base to drive the proposed damper from the bottom, with the plate on the top serving as the output. A cylindrical rubber spring made of silicone rubber
(M4601A/B, Barnes crop.) is mounted between the plate and the external damper casing. As the internal damper casing is connected to the unmovable (fixed) plate and the external damper disk, both the internal damper casing and the external damper disk keep still while the internal damper rotor and the external damper casing may spin, following the shaft. Basically, all the parts in the yellow block are fixed and cannot rotate, while other parts are able to rotate following the shaft. Magnetorheological fluid (MRF-122EG, Lord corp.) is filled in the two 0.8 mm gap chambers of both internal and external dampers. The internal damper coil is seated in the internal damper rotor while the external damper coil is mounted to the external damper casing. Both of these two coils can generate magnetic flux to control the yield stress of the MR fluids contained inside. Here, internal damper current and external damper current are assigned as $I_1$ and $I_2$, respectively. In order to optimise the magnetic flux path, the part connecting the internal and the external dampers was made of aluminium with a low magnetic permeability while other parts are made of low carbon steel with a high magnetic permeability. In addition, aluminium was also used for both the internal damper lid and the casing to guide the magnetic flux to pass through MRF gap for the purpose of realising a high magnetic flux density across MRF working area. As shown in Figure 1, the captions of the rubber spring, the internal and the external damper coil, and the aluminium parts are in green, indicating non-ferromagnetic, while other parts with black caption are ferromagnetic.
2.2 Working principle

If no current is applied to both the internal damper coil and the external damper coil, all the parts except the ones in yellow block in Figure 1: the internal damper rotor, the plate, the rubber spring, and the external damper casing can rotate freely with the shaft. The rubber spring will only experience a small strain due to the inertial torque of the external damper. By providing current $I_1$ to the internal damper coil, variable damping is realised, with the relative movement between the internal damper rotor and the casing restrained by the semi-solid MR fluid between them. If even a small current $I_2$ is given to the external damper coil, the external damper casing would not be able to freely rotate from the external damper disk, due to the braking torque induced by the MR fluid. However, the plate is able to always move simultaneously with the shaft. As a result, the rubber spring would be stretched, which largely increases the stiffness of the entire device. As such, damping variability is achieved through the regulation of the internal damper current $I_1$, while the external damper current $I_2$ can control the
stiffness of the device.

2.3 Magnetic field simulation

In order to provide guidance to the damper design, the induced magnetic field of the proposed damper was simulated using COMSOL Multiphysics with a 2D axisymmetric study, with the results being shown in Figure 2(a). The internal damper coil has 500 turns with 0.5 mm diameter and 7.8 Ω resistance copper wire, while the external damper coil has 200 turns with 0.5 mm diameter, 10.9 Ω resistance copper wire. From Figure 2(a), it can be noticed that the aluminium part connecting the external damper disk and the internal damper casing effectively builds a barrier between the internal magnetic circulation and the external magnetic circulation, so the magnetic field interference between the two damping units is minimised. For the same reason, additional aluminium parts were added to the internal damper to guide the magnetic flux where desired. Thus, the magnetic flux of each can be separately controlled by the internal current $I_1$ and external current $I_2$, without mutual interference.

![Figure 2](image.png)

**Figure 2.** Magnetic field simulation: (a) modelled damper, and (b) average flux through the MRF for varied currents

For the internal damper, the working area of the MR fluid is on the upper and lower gaps (i.e. the surfaces in the radial direction) and the small gaps between the drum and the internal damper casing.
(in the axial direction). As for the external damper, the upper and lower gaps between the disk and the casing (in the radial direction) serve as the working area. Thus, the relationship between the mean magnetic flux in these working areas and the current is plotted in Figure 2(b). The magnetic flux of the MR fluid in both the internal damper and external damper increase continuously with the increasing current. The maximum mean flux of the external damper was found to be 0.918 T at 2.0 A. Given this appears sufficient for the MR fluid selected, the current range of 0–2.0 A was set for the external damper coil. In contrast, the magnetic flux of the internal damper shows saturation when current above 0.6 A, with the maximum mean fluxes at 1.0 A being 0.552 T and 0.594 T for the radial and axial directions, respectively. As a result, the current range of 0 ~ 1.0 A was chosen to be used in the tests of the internal damper coil.

3. Experimental setup for damper testing

![Figure 3. Schematic diagram for the experimental setup](image)

In order to verify the variable damping and stiffness features of the designed rotary MR damper, a test system was established to characterise the device. As it is shown in Figure 3, the device is mounted on an unmovable base connecting to the test bench frame. The damper is driven by an AC servomotor and drive system (Panasonic 1.3 N·m, MBDKT2510CA1 200 V) with a 40:1 planetary gearbox (After ATF
This servomotor is controlled using an NI myRIO to provide the position control signal, through a LabVIEW program prepared for this testing. The myRIO is also connected to a PC to provide the user interface for testing. When running the tests, the measured signals including displacement and torque are recorded by the myRIO, serving as both the controller and the data acquisition (DAQ) device. In addition, a DC power supply (GW INSTEK GPC – 3030D) is used to provide currents at different levels to both internal damper coil and the external damper coil.

The testing cases consist of damping variability and stiffness variability. For each case, sufficient tests were measured in order to ensure performance stability and uniformity. As the damping variability is controlled by the internal damper current $I_1$, the external damper current $I_2$ was set as 0 A in the damping variability test case. Similarly, for the stiffness variability test cases, the internal damper current $I_1$ was set as 0 A. The test input is selected as the sinusoidal signal $\varphi = A \sin(2\pi ft)$ to characterize both the damping and the stiffness of the device. $\varphi$, $A$, $f$ and $t$ represent the angular displacement, amplitude, frequency and time respectively. At the same time, the signals of displacement and the torque generated on the shaft were recorded through the motor driver. In addition, the co-working of the internal and external damping units were also conducted to further characterise the behaviour of the device.

4 Testing results and analysis

4.1 Damping variability

4.1.1 Current dependency

Figure 4 shows the damping variability in response to various internal damper currents: $I_1 = 0, 0.2, 0.4, 0.6, 0.8$ and $1.0$ A, along with an amplitude $A = 10$ degree, frequency $f = 0.5$ Hz external damper current $I_2 = 0$ A. The calculated equivalent damping coefficient as plotted in Figure 4(b) was found using the following equation (Li et al., 2013):

$$C_{t,eq} = \frac{EDC}{2\pi^2fA^2}$$  \hspace{1cm} (1)
where \( C_{t,eq} \) is the equivalent damping coefficient, EDC is the energy dissipated per cycle, being the enclosed area of each torque-displacement loop, \( f \) is the sinusoidal frequency in Hz, and \( A \) is the sinusoidal amplitude.

(a) Torque-displacement relation \((A = 10 \text{ degree}, f = 0.5 \text{ Hz}, I_2 = 0 \text{ A})\)

(b) Equivalent damping coefficient to internal currents \( I_1 \)

**Figure 4.** Variable damping behaviour under varied internal damper current \( I_1 \)

As shown in Figure 4(a), the enclosed area of torque-displacement loops, EDC, increases with the increase of the internal damper current \( I_1 \) from 0 A to 1.0 A. The calculated equivalent damping coefficient in Figure 4(b) increases proportionally with all other variables held constant. This equivalent damping showed an increase of 141.6% from 13.98 N·m·s·rad\(^{-1}\) to 33.78 N·m·s·rad\(^{-1}\) with the increase of current from 0 A to 1.0 A. The peak torque, shown in Figure 4(a), begins to saturate when the applied current is larger than 0.6 A, which is consistent with the magnetic field saturation phenomenon in the magnetic field studies conducted, as shown in Figure 2.
4.1.2 Amplitude dependent response

The amplitude dependent response of the damping variability at three amplitudes: \( A = 5, 10 \) and 15 degree under the combined conditions of internal damper current \( I_1 = 1.0 \) A, frequency \( f = 0.5 \) Hz and external damper current \( I_2 = 0 \) A is illustrated in Figure 5(a). The peak torque is almost the same for all the amplitude cases. Even though the energy dissipated in each loop, EDC, increases with the increasing current, the equivalent damping coefficient shows a decreasing trend in Figure 5(b) because the equivalent damping coefficient is inversely proportional to the square of amplitude, as shown in Equation (1).

![Figure 5](image)

(a) Torque-displacement relation  (b) Equivalent damping coefficient

**Figure 5.** Response to different amplitudes (\( I_1 = 1.0 \) A, \( f = 0.5 \) Hz, \( I_2 = 0 \) A)

4.1.3 Frequency dependent response

Figure 6 shows the response of the MR damper under the various loading frequencies: \( f = 0.5, 0.75 \) and 1.0 Hz, with the combined conditions of amplitude \( A = 10 \) degree, internal damper current \( I_1 = 0.5 \) A and external damper current \( I_2 = 0 \) A. It is noticed that the peak torque of the enclosed area, EDC, is almost the same at three frequencies for all test cases. However, as it is indicated in Figure 6(b), the equivalent damping coefficient is decreasing with the increasing of frequency \( f \) because the equivalent
damping coefficient is inversely proportional to this.

(a) Torque-displacement relation                   (b) Equivalent damping coefficient

Figure 6. Response to different frequencies ($A = 10$ degree, $I_1 = 0.5$ A, $I_2 = 0$ A)

4.2 Stiffness variability

In this section, the stiffness variability of the designed damper is tested. Effective stiffness was calculated for all the test cases using the equation (Li et al., 2013):

$$K_{\text{eff}} = \frac{T_{d,\text{max}} - T_{d,\text{min}}}{A_{\text{max}} - A_{\text{min}}}$$  \hspace{1cm} (2)

where $T_{d,\text{max}}$ and $T_{d,\text{min}}$ are the maximum and minimum torques at the maximum and the minimum amplitude displacements, $A_{\text{max}}$ and $A_{\text{min}}$, respectively.

4.2.1 Current dependency

Figure 7 presents the stiffness variability of the damper under different levels of external damper current: $I_2 = 0$, 0.5, 1.0, 1.5 and 2.0 A. Two sets of combined conditions: ‘$A = 5$ degree, $f = 0.5$ Hz, $I_1 = 0$ A’ and ‘$A = 10$ degree, $f = 0.5$ Hz, $I_1 = 0$ A’ were selected to illustrate the current dependent behaviour for varied amplitudes.
(a) Torque-displacement relation at small amplitude $A = 5$ degree ($f = 0.5$ Hz, $I_1 = 0$ A)

(b) Torque-displacement relation at large amplitude $A = 10$ degree ($f = 0.5$ Hz, $I_1 = 0$ A)

(c) Effective stiffness response for the two amplitude conditions: $A = 5$ degree and 10 degree

**Figure 7.** Variable stiffness under the varied external damping current $I_2$

As shown in Figure 7(a) and (b), the torque follows piecewise behaviour in response to a displacement input, with these loops being generated in a clockwise direction. Taking the test case of ‘$A = 5$ degree, $f = 0.5$ Hz, $I_1 = 0$ A, $I_2 = 0$ A’ in Figure 7(a) as an example, letters are placed about the figure at its vertices in a clockwise direction, starting with A at the maximum positive amplitude. Considering
loading for $A \rightarrow D$, starting with the section of AB, a torque is generated as the torque from the actuator overcomes the initial damping of the internal damper in order to allow the shaft to rotate. Then, focusing on section BC, if a current $I_2$ is applied to the external damper coil, this tends to stretch the rubber, through the engagement of the external damper. As the motor continues to actuate the device, the spring becomes loaded, with its stiffness defining the slope of section BC. After the point at which the spring torque exceeds the braking torque of the external damper, occurring at some displacement level, the rubber spring will stop stretching further and the external casing will move together with the plate without relative motion. This process describes the loop segment of CD, which is almost parallel to the x-axis, at which stage the stiffness in this process could be regarded as zero. The reverse process represented by $D \rightarrow A$ is symmetrical to the regular process $A \rightarrow D$, as the segments of DE, EF and FA are parallel to the segments of AB, BC and CD, respectively. It should also be noted that in Figure 7(a), the segments CD and FA tend to reduce in size until they are non-existent when current is high enough. This is due to the external damper tendency to not yield as damping torque becomes relatively large when compared to the spring torque. In contrast, under the $A = 10$ degree amplitude, no level of current tested is sufficient to prevent such yielding.

The effective stiffness of the damper is shown in Figure 7(c), as calculated using Eq. (2), which is simply the slope of the line AD. As illustrated, the stiffness of the designed damper increases with the increase of the external damper current $I_2$ for both test cases: ‘$A = 5$ degree, $f = 0.5$ Hz, $I_1 = 0$ A’ and ‘$A = 10$ degree, $f = 0.5$ Hz, $I_1 = 0$ A’. For the smaller amplitude $A = 5$ degree condition, the stiffness is increased 524% from $69.88 \text{ N} \cdot \text{m} \cdot \text{rad}^{-1}$ to $436.02 \text{ N} \cdot \text{m} \cdot \text{rad}^{-1}$ as the current is increased from 0 A to 2 A. In the case of $A = 10$ degree, a 618.1% increase of effective stiffness from $31.51 \text{ N} \cdot \text{m} \cdot \text{rad}^{-1}$ to $226.26 \text{ N} \cdot \text{m} \cdot \text{rad}^{-1}$ was achieved. The effective stiffness for the smaller amplitude $A = 5$ degree is larger than that of the $A = 10$ degree condition in general. This is simply due to the stiffness being an average across the entire displacement of the device, which includes more ‘zero-stiffness’ yielding of the external damper as displacement amplitude increases.
4.2.2 Amplitude dependent response

Figure 8 illustrates the amplitude dependency of the damper for the three amplitude conditions $A = 5$, 10 and 15 degree. The results of two sets of the combined conditions: ‘$I_2 = 1 \text{ A}, f = 0.5 \text{ Hz}, I_1 = 0 \text{ A}$’ and ‘$I_2 = 2 \text{ A}, f = 0.5 \text{ Hz}, I_1 = 0 \text{ A}$’ are presented.

(a) Torque-displacement relation at small current $I_2 = 1 \text{ A} (f = 0.5 \text{ Hz}, I_1 = 0 \text{ A})$

(b) Torque-displacement relation at big current $I_2 = 2 \text{ A} (f = 0.5 \text{ Hz}, I_1 = 0 \text{ A})$

(c) Amplitude response of effective stiffness at the two current conditions: $I_2 = 1 \text{ A}$ and $2 \text{ A}$
In Figure 8(a), the external damper is yielded for all the amplitude cases at the lower current $I_2 = 1$ A. The damping torque generated by the low current $I_2 = 1$ A is not sufficient to prevent yielding even for the smallest amplitude $A = 5$ degree. In contrast, as shown in Figure 8(b), the external damper is not yielded for the amplitude $A = 5$ degree case when a higher current $I_2 = 2$ A is applied until the amplitude increased to $A = 10$ degree. Consequently, the maximum torque of the amplitude $A = 5$ degree is slightly smaller than that of $A = 7.5$ and 10 degree, as in this case the constant yielding torque of the damper was not reached by the spring for this given external damper current and test frequency.

The effective stiffness for these tests are illustrated in Figure 8(c), which reveals a declining trend with increasing amplitude. Under a constant amplitude, because the maximum torque of the larger external damper current condition $I_2 = 2$ A is larger than that of the smaller current condition $I_2 = 1$ A, the effective stiffness of the large current conditions are larger than that of the smaller current conditions.

### 4.2.3 Frequency dependent response

Figure 9 provides the frequency dependent response of the proposed damper. Three frequencies $f = 0.5$, 0.75 and 1.0 Hz are prescribed for the test cases of ‘$I_2 = 1$ A, $A = 5$ degree, $I_1 = 0$ A’ and ‘$I_2 = 1$ A, $A = 10$ degree, $I_1 = 0$ A’. Relative movement between the external damper casing and external damper disk exists at both amplitude conditions. The torque-displacement loops are almost the same at the three frequency conditions for both the smaller and larger amplitude test cases. Thus, the effective stiffness is also almost the same for all the frequency conditions as illustrated in Figure 9(c).
(a) Torque-displacement relation at small amplitude $A = 5$ degree ($I_2 = 1$ A, $I_1 = 0$ A)

(b) Torque-displacement relation at big amplitude $A = 10$ degree ($I_2 = 1$ A, $I_1 = 0$ A)

(c) Frequency response of effective stiffness at two amplitude conditions: $A = 5$ degree and 10 degree

**Figure 9.** Stiffness response to various frequency
4.3 Simultaneous working of internal and external damping units

Figure 10. Simultaneously working for internal and external damping units

Figure 10 presents the test results when the internal damper unit and external damper unit work together. The external damper current \( I_2 \) is set as 0A and 2 A for two groups of test separately. In each group, internal damper current \( I_1 \) is applied with three levels of currents: 0 A, 0.5 A and 1 A. When the external damper current \( I_2 = 2 \) A, the maximum torque shows an increase from 42.3 Nm to 52.08 Nm as the internal damper current \( I_1 \) increases from 0 A to 1 A. This increasing trend is quite similar to that of the reference group in which the external damper current \( I_2 = 0 \) A.

5 Modelling and the parameter identification

In order to predict the variable damping and stiffness behaviour of the designed shock absorber, a mathematical model is established in this section. The parameters for both the internal damping unit and the external damping unit were also identified as followed.

5.1 Model establishment

As it is shown in Figure 10, a mathematical model which incorporates two Bingham components...
(Spencer Jr et al., 1997) is built to predict the damping and the stiffness variation of the designed MR damper. A coulomb friction element $T_1$ in parallel with a dashpot with viscous damping coefficient $c_1$ is used to describe the internal MR damper unit. Similarly, the external damper unit is characterized by another coulomb friction element $T_2$ in parallel with a dashpot with viscous damping coefficient $c_2$. In addition, a stiffness $k_r$ represents that of the rubber spring. For these parameters, $T_1$ and $c_1$ are controlled by the applied current to the internal damper $I_1$, while $T_2$ and $c_2$ vary in value due to the change of external damper current $I_2$. The angular displacements of the shaft or plate and the external damper casing are prescribed as $\varphi_1$ and $\varphi_2$, respectively.

![Mathematic model of the proposed rotary MR damper](image)

**Figure 10.** Mathematic model of the proposed rotary MR damper

The rotation torque $T_e$ can be written as

$$T_e = T_{in} + T_r$$

(3)

$$T_{in} = c_1 \dot{\varphi}_1 + T_1 \text{sign}(\dot{\varphi}_1)$$

(4)

$$T_r = k_r (\varphi_1 - \varphi_2)$$

(5)

where $T_{in}$ is the torque generated by the internal damper, and $T_r$ represents the rubber spring torque. The external damper torque can be calculated by
\[ T_{\text{ex}} = c_2 \dot{\phi}_2 + T_2 \text{sign}(\phi_2) \] (6)

The damper torques \( T_{\text{in}} \) and \( T_{\text{ex}} \) increase with the increase of currents \( I_1 \) and \( I_2 \), respectively. As illustrated of the segment BC and EF in Figure 7(a), during the loading of the rubber, the external damper casing keeps still. After the external damper torque yields due to that of the rubber, the external damper casing would move simultaneously with the plate without relative movement until the shaft reaches the minimum or maximum displacement. This process is represented by the segments of FA and CD in Figure 7(a). The torque of the rubber spring then stops increasing as the rubber elongation becomes constant. Thus,

If \(|T_r| < |T_{\text{ex}}|\)

\[ \dot{\phi}_2 = 0 \] (7)

Else if \(|T_r| = |T_{\text{ex}}|\)

\[ \dot{\phi}_2 = \dot{\phi}_1 \] (8)

### 5.2 Parameter identification

After building the model of the system, four parameters in the model: \( c_1, T_1, c_2, T_2 \) should be identified using the experimental data to predict the behaviour of the device. The procedure follows the least-square method in combination with the trust-region-reflective algorithm which is available in the MATLAB & Simulink. The goal of this method is to minimize the difference between the modelled results to the experimental results by adjusting the four parameters as shown in the following equation:

\[ S = \sum_{i=1}^{N} \sqrt{\frac{(T_{ei} - T_{mi})}{N}} \] (9)

where the \( S \) is the root mean square between the predicted and the experimental results, \( T_{ei} \) and \( T_{mi} \).
are the \(i\)th experimental torque and the modelled torque, and \(N\) is a neighbourhood which is defined as the trust region that could represent the behaviour of the equation (9). Obviously, by using this method, the model results will be approximate the experimental results to the greatest degree only if \(S\) reaches its minimum value.

As the damping variability and the stiffness variability are separately controlled by the internal damper current \(I_1\) and the external damper current \(I_2\), the internal damper parameters \(c_1, T_1\) at different currents are identified with the fixed external damper parameters \(c_2\) and \(T_2\) for the damping variable part, and vice versa. Adopting the experimental data of variable damping tests: \(A = 10\) degree, \(f = 0.5\) Hz, \(I_1 = 0\) A, 0.5 A and 1 A, and the data of variable stiffness tests: \(A = 10\) degree, \(f = 0.5\) Hz, \(I_2 = 0\) A, 1 A and 2 A, the parameters are identified using the builded model in the Simulink as shown in Table 1. The modelled results compared with the experiment results are also shown in Figures 11 and 12.

**Table 1.** Parameter identification results for the variable damping (the left half) and variable stiffness (the right half) at the conditions: \(A = 10\) degree and \(f = 0.5\) Hz

<table>
<thead>
<tr>
<th>Parameter (variable damping)</th>
<th>(T_1)</th>
<th>(c_1)</th>
<th>(T_2)</th>
<th>(c_2)</th>
<th>Parameter (variable stiffness)</th>
<th>(T_1)</th>
<th>(c_1)</th>
<th>(T_2)</th>
<th>(c_2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(I_1 = 0) A, (I_2 = 0) A</td>
<td>2.8</td>
<td>0.8</td>
<td>3.8</td>
<td>1.8</td>
<td>(I_1 = 0) A, (I_2 = 0) A</td>
<td>2.8</td>
<td>0.8</td>
<td>3.8</td>
<td>1.8</td>
</tr>
<tr>
<td>(I_1 = 0.5) A, (I_2 = 0) A</td>
<td>8.5</td>
<td>1.5</td>
<td>3.8</td>
<td>1.8</td>
<td>(I_1 = 0) A, (I_2 = 1) A</td>
<td>2.8</td>
<td>0.8</td>
<td>21.7</td>
<td>4.2</td>
</tr>
<tr>
<td>(I_1 = 1) A, (I_2 = 0) A</td>
<td>11.2</td>
<td>2.2</td>
<td>3.8</td>
<td>1.8</td>
<td>(I_1 = 0) A, (I_2 = 2) A</td>
<td>2.8</td>
<td>0.8</td>
<td>35.6</td>
<td>5.8</td>
</tr>
</tbody>
</table>
As illustrated in the left half part of Table 1, the external damper parameters $T_1$ and $c_1$ increase with an increasing internal damper current $I_1$, while $T_2$ and $c_2$ are constant with the constant zero external damper current $I_2$. Plotting the simulated result against the experimental result, the modelled variable damping behaviour in Figure 11 shows a good fit with the experimental data at various levels of $I_1$. For the modelled results, the area enclosed by the torque-displacement loop increases with the
increasing current, which predicts the variable damping ability very well. The parameters of the external damper, $T_2$ and $c_2$, also show a positive relationship with the increase of $I_2$. At the same time, the internal damper parameters $T_1$ and $c_1$ do not change for the fixing $I_1$. The variable stiffness capability is also well modelled for the effective stiffness increases with the increasing of $I_2$, as shown in Figure 12.

To verify that the proposed model with the identified parameters could predict the behaviours of the device. Two modelled results for the conditions: ‘$I_1 = 0.5$ A, $I_2 = 1$ A’ and ‘$I_1 = 1$ A, $I_2 = 2$ A’ were plotted in Figure 13 by using the parameters as shown in Table 2. The values of $T_1$ and $c_1$ are obtained from the cases with the same internal damper current $I_1$ while the values of $T_2$ and $c_2$ are obtained from the cases with the same external damper current $I_2$. For instance, within the parameters for the modelled case ‘$I_1 = 0.5$ A, $I_2 = 1$ A’, ‘$T_1 = 8.5$, $c_1 =1.5$’ and ‘$T_2 = 21.7$, $c_2 =4.2$’ are separately obtained from the corresponding parameters of the test cases of ‘$I_1 = 0.5$ A, $I_2 = 0$ A’ and ‘$I_1 = 0$ A, $I_2 = 1$ A’ as listed in Table 1. As it is shown in Figure 13, the modelled results have a very good accordance to the experimental data, which means the proposed model with the parameters in Table 2 could predict the behaviour of the device very well, facilitating future control efforts for such a device.

<table>
<thead>
<tr>
<th>Modelled cases</th>
<th>$T_1$</th>
<th>$c_1$</th>
<th>$T_2$</th>
<th>$c_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_1 = 0.5$ A, $I_2 = 1$ A</td>
<td>8.5</td>
<td>1.5</td>
<td>21.7</td>
<td>4.2</td>
</tr>
<tr>
<td>$I_1 = 1$ A, $I_2 = 2$ A</td>
<td>11.1</td>
<td>2.2</td>
<td>35.6</td>
<td>5.8</td>
</tr>
</tbody>
</table>

Table 2. Parameters used to verify the proposed model
Figure 13. Verification of the proposed model under co-working conditions by comparing the modelled results to the experimental results ($A = 10$ degree, $f = 0.5$ Hz)

6 Discussion

In this paper, a rotary variable stiffness and damping is designed, tested and modelled. Two points worth discussing in this work are listed as follows:

- **Existing problems:** Even though the proposed shock absorber could realize independent control of variable damping and stiffness, it still has drawbacks such as magnetic saturation as mentioned in section 2.3. This issue could be solved by further optimization of the internal damper, which may contribute a larger damping variation. In addition, the shock absorber would be more versatile in application if it had a more compact structure.

- **Potential applications:** For the proposed rotary shock absorber, it can be applied in the areas of vibration control and position control of the joints of manipulators. By altering the variable stiffness and damping of the device, the accuracy of position control can be improved, with the vibration reduced at the same time, providing an overall improvement to system dynamic properties.
7 Conclusion

This study includes the design of a novel shock absorber with variable damping and stiffness capabilities based on rotary MR damper. Experimental tests were conducted to verify such performance, and the test results show that the damping increased 141.6% from 13.98 N·m·s·rad^{-1} to 33. N·m·s·rad^{-1} as the current is increased from 0 A to 1.0 A with a 10 degree amplitude. In addition, a maximum of 618% increase of effective stiffness from 31.51 N·m·rad^{-1} to 226.26 N·m·rad^{-1} is achieved with the current shift from 0 A to 2 A at the amplitude of 10 degree. The accuracy of the established mathematical model in predicting the variable damping and stiffness characteristics of the damper was also verified with the identified parameters of internal and external MR dampers.

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References

Nam TH and Ahn KK. (2009) A new structure of a magnetorheological brake with the waveform


