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Performance analysis of the activated sludge model (number 1)

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Performance analysis of the activated sludge model (number 1)

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Abstract

The activated sludge process is widely used to treat both municipal sewage and a variety of industrial wastewaters. We investigate the steady-state behaviour of an activated sludge process. We use the activated sludge model number one, an internationally accepted model, to describe the biochemical, biological, and physical-chemical phenomena that occur inside the bioreactor. The treatment configuration consists of a single aerated reactor attached to a settling unit.

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Continuation methods are used to determine the steady-states of the model as a function of the hydraulic residence time. From these solutions we construct important operational parameters including the chemical oxygen demand, total suspended solids and total nitrogen. These are determined inside the bioreactor, in the effluent stream and in the wastage stream.

We show that there are two critical values of the hydraulic retention time. As the hydraulic retention time is increased through the first critical value heterotrophic biomass become viable. This bifurcation is associated with a substantial decrease in the chemical oxygen demand in the effluent stream and a corresponding increase in both the total suspended solids and total nitrogen in the reactor. Autotrophic biomass become viable as the hydraulic retention time is increased through the second bifurcation point. Associated with this bifurcation there are dramatic changes in the concentration of soluble ammonium nitrogen and soluble nitrate/nitrite inside the reactor; the former being converted to the latter.

Of particular practical interest is the value of the hydraulic retention time at which the chemical oxygen demand in the effluent stream is equal to a preset target value. We investigate how this value varies as either the composition of the influent stream or the recycle ratio is varied.

Keywords: activated sludge; mathematical modelling; nitrification; wastewater treatment.

1 Introduction

The activated sludge process is the most commonly used aerobic process for the biological treatment of both domestic and industrial wastewaters [22]. As such, most wastewater treatment plants contain a unit employing the activated sludge process. The key to this process is the presence of highly concentrated micro-organisms, typically 2-4 g/L, present in the form of flocs, which grow though consuming organic pollutants. The mixture of flocs and particulate matter is known, for historical reasons, as “activated sludge”. Without sufficient quantities of sludge, i.e. micro-organisms, the process cannot work.

The simplest reactor configuration for the activated sludge process entails the use of two units: an aerated biological reactor and a settling unit (or clarifier). In the former the pollutants are

degraded by microorganisms (the active agent that puts the ‘activated’ into ‘activated sludge’). Inside the bioreactor the microorganisms flocculate to form settleable solids. These solids are removed from the effluent stream by sedimentation in a settling unit and then returned to the aerated reactor in a more concentrated culture. The use of a clarifier ensures the presence of a culture of highly concentrated micro-organisms and is one of the keys to the success of the activated sludge process. Figure 1 shows a schematic for this process.

Modelling has become an important tool to develop an understanding of the processes that govern the behaviour of the activated sludge process. A widely used model that describes the biological processes occurring in the activated sludge process is the activated sludge model #1 (ASM#1) [12]. This consists of a biochemical model for the particulates and soluble materials and a sub-model describing nitrifying processes. Process variables included in the ASM#1 are defined in table 1.

The ASM1 model includes eight processes that are fundamental to the activated sludge process. These are: aerobic and anoxic growth of heterotrophic biomass, death of heterotrophic biomass, aerobic growth of autotrophic biomass, decay of autotrophic biomass, ammonification of soluble organic nitrogen, and hydrolysis of both entrapped particulate organic matter and entrapped organic nitrogen. Together these processes describe nitrogen and chemical oxygen demand within suspended-growth treatment processes, including mechanisms for nitrification and denitrification.

One significant feature of the biochemical model is the inclusion of two kinds of biomass: autotrophs ($X_{B,A}$) and heterotrophs ($X_{B,A}$). The autotrophs are assumed to be nitrifying bacteria. Their growth is associated with the conversion of soluble ammonium nitrogen (S_{NH}) into soluble nitrate and nitrite nitrogen (S_{NO}). The rate of growth of the heterotrophs is associated with the removal of soluble organic carbon (S_S). The heterotrophs are also responsible for the, non-growth related, biodegradation of both slowly biodegradable particulate matter (X_S) and biodegradable organic nitrogen (X_{ND}).

Through its inclusion of the eight fundamental processes the model has been found to give a good description of the activated sludge process *provided* that the wastewater has been characterised and the model calibrated. Wastewater characterisation requires the determination of 12 parameters. Model calibration involves determining two physical parameters (associated

with oxygen) and 19 parameters associated with microbial processes. Finally, these models are applicable for domestic or municipal wastewater, but not industrial wastewater which contain a significant portion of hardly biodegradable organic material.

We investigate the steady-state behaviour of the ASM#1 as a function of the hydraulic retention time. (The hydraulic retention time is defined as the volume of the reactor divided by the feed flow rate. It represents the average amount of time that a soluble compound remains in the reactor). From the steady-state values of the state variables we calculate the corresponding steady-state values of various quantities associated with the operation of the activated sludge process. These quantities are the chemical oxygen demand, the total suspended solids, total nitrogen and total inorganic nitrogen. Values for the first three of these quantities are determined inside the aerated reactor, in the influent stream and in the wastage stream. Besides these quantities we are interested in the following state variables: the concentration of the two biomass species ($X_{B,A}$ and $X_{B,H}$), the concentration of ammonium nitrogen (S_{NH}), the concentration of soluble nitrate and nitrite nitrogen (S_{NO}) and the dissolved oxygen concentration (S_O).

There have been many simulation studies of wastewater treatment plants using the ASM #1. Almost exclusively these have used direct numerical integration of the governing equations. However, as the resulting model consists of a system of coupled non-linear differential equations continuation methods are a more efficient tool to investigate steady-state behaviour. According to Ozturk and Teymour [18], as recently as 2014 there had been only three studies of ASM #1 which had used continuation methods to analyze water reclamation plants [9, 17, 18]. None of these studies investigated the reactor configuration shown in figure 1.

Recent developments in the mathematical modeling of the activated sludge process are reviewed in [11, 15].

2 Model

Section 2.1 presents the model equations. The performance of the activated sludge process can be evaluated in a variety of ways. Section 2.2 shows how these performance measures can be calculated from the state variables.

2.1 Model equations

In this section we present the model equations. The model consists of a system of differential equations for the concentrations of soluble and particulate materials inside the reactor, equations (1)–(12), expressions for the the reaction rates, equations (13)–(19), and an algebraic equation for the settling of particulates in the settling unit, equation (20).

Many authors using the ASM #1 assume that the process of aeration can be tightly controlled, meaning that the dissolved oxygen concentration is kept constant as operating conditions change. Alternatively, the differential equation for dissolved oxygen may be ignored if the oxygen concentration in the reactor is kept above 3mg l^{-1} ; under these conditions the oxygen concentration is not rate limiting [19]. A constant dissolved oxygen concentration is commonly assumed in simplified models for the activated sludge process [5]. We do not make this assumption. Rather, we assume that the operation of the aeration process is fixed. Consequently, in our model the dissolved oxygen remains a state variable.

The nomenclature for the model is defined in appendix A.

The rate of change of inert soluble organic material

$$\frac{dS_I}{dt} = \frac{q}{V} (S_{I,\text{in}} - S_I). \quad (1)$$

The rate of change of readily biodegradable soluble substrate

$$\begin{aligned} \frac{dS_S}{dt} = & \frac{q}{V} (S_{S,\text{in}} - S_S) - \frac{1}{Y_H} \cdot \mu_{\text{max,H}} \cdot M_2 \cdot M_{8h} \cdot X_{B,H} \\ & - \frac{1}{Y_H} \cdot \mu_{\text{max,H}} \cdot M_2 \cdot I_8 \cdot M_9 \cdot \eta_g \cdot X_{B,H} \\ & + k_h \cdot k_{\text{sat}} (M_{8h} + \eta_h \cdot I_8 \cdot M_9) X_{B,H}. \end{aligned} \quad (2)$$

The rate of change of particulate inert organic matter

$$\frac{dX_I}{dt} = \frac{q}{V} (X_{I,\text{in}} - X_I) + \frac{R_1 q}{V} (b_1 - 1) X_I. \quad (3)$$

The rate of change of slowly biodegradable particulate substrate

$$\begin{aligned} \frac{dX_S}{dt} = & \frac{q}{V} (X_{S,\text{in}} - X_S) + \frac{R_1 q}{V} (b_1 - 1) X_S + (1 - f_p) b_H X_{B,H} \\ & + (1 - f_p) b_A X_{B,A} - k_h \cdot k_{\text{sat}} (M_{8h} + \eta_h \cdot I_8 \cdot M_9) X_{B,H}. \end{aligned} \quad (4)$$

The rate of [change](#) of active heterotrophic particulate biomass

$$\begin{aligned} \frac{dX_{B,H}}{dt} = & \frac{q}{V} (X_{B,H,\text{in}} - X_{B,H}) + \frac{R_1 q}{V} (b_1 - 1) X_{B,H} + \mu_{\text{max},H} \cdot M_2 \cdot M_{8h} \cdot X_{B,H} \\ & + \mu_{\text{max},H} \cdot M_2 \cdot I_8 \cdot M_9 \cdot \eta_g \cdot X_{B,H} - b_H \cdot X_{B,H}. \end{aligned} \quad (5)$$

The rate of change of active autotrophic particulate biomass

$$\begin{aligned} \frac{dX_{B,A}}{dt} = & \frac{q}{V} (X_{B,A,\text{in}} - X_{B,A}) + \frac{R_1 q}{V} (b_1 - 1) X_{B,A} \\ & + \mu_{\text{max},A} \cdot M_{10} \cdot M_{8,a} \cdot X_{B,A} - b_A \cdot X_{B,A}. \end{aligned} \quad (6)$$

The rate of change of non-biodegradable particulate products arising from biomass decay

$$\begin{aligned} \frac{dX_P}{dt} = & \frac{q}{V} (X_{P,\text{in}} - X_P) + \frac{R_1 q}{V} (b_1 - 1) X_P + f_p \cdot b_H \cdot X_{B,H} \\ & + f_p \cdot b_A \cdot X_{B,A}. \end{aligned} \quad (7)$$

The rate of change of soluble oxygen

$$\begin{aligned} \frac{dS_O}{dt} = & \frac{q}{V} (S_{O,\text{in}} - S_O) + K_{L,a} (S_{O,\text{max}} - S_O) \\ & - \frac{(1 - Y_H)}{Y_H} \cdot \mu_{\text{max},H} \cdot M_2 \cdot M_{8h} \cdot X_{B,H} \\ & - \frac{(4.57 - Y_A)}{Y_A} \cdot \mu_{\text{max},A} \cdot M_{10} \cdot M_{8a} \cdot X_{B,A}. \end{aligned} \quad (8)$$

The rate of [change](#) of soluble nitrate and nitrite nitrogen

$$\begin{aligned} \frac{dS_{NO}}{dt} = & \frac{q}{V} (S_{NO,\text{in}} - S_{NO}) - \frac{(1 - Y_H)}{2.86 Y_H} \cdot \mu_{\text{max},H} \cdot M_2 \cdot I_8 \cdot M_9 \cdot \eta_g \cdot X_{B,H} \\ & + \frac{1}{Y_A} \cdot \mu_{\text{max},A} \cdot M_{10} \cdot M_{8a} \cdot X_{B,A}. \end{aligned} \quad (9)$$

The rate of [change](#) of soluble ammonium (NH_4^+ and NH) nitrogen

$$\begin{aligned} \frac{dS_{NH}}{dt} = & \frac{q}{V} (S_{NH,\text{in}} - S_{NH}) - i_{XB} \mu_{\text{max},H} \cdot M_2 \cdot M_{8h} \cdot X_{B,H} \\ & - i_{XB} \mu_{\text{max},H} \cdot M_2 \cdot I_8 \cdot M_9 \cdot \eta_g \cdot X_{B,H} \\ & - \left(i_{XB} + \frac{1}{Y_A} \right) \mu_{\text{max},A} \cdot M_{10} \cdot M_{8a} \cdot X_{B,A} + k_A \cdot S_{ND} \cdot X_{B,H}. \end{aligned} \quad (10)$$

The rate of change of soluble biodegradable organic nitrogen

$$\begin{aligned} \frac{dS_{\text{ND}}}{dt} &= \frac{q}{V} (S_{\text{ND,in}} - S_{\text{ND}}) - k_{\text{A}} \cdot S_{\text{ND}} \cdot X_{\text{B,H}} \\ &\quad + k_{\text{h}} \cdot k_{\text{sat}} (M_{8\text{h}} + \eta_{\text{h}} \cdot I_8 \cdot M_9) X_{\text{B,H}} \frac{X_{\text{ND}}}{X_{\text{S}}}. \end{aligned} \quad (11)$$

The rate of change of particulate biodegradable organic nitrogen

$$\begin{aligned} \frac{dX_{\text{ND}}}{dt} &= \frac{q}{V} (X_{\text{ND,in}} - X_{\text{ND}}) + \frac{R_1 q}{V} (b_1 - 1) X_{\text{ND}} \\ &\quad + (i_{\text{XB}} - f_{\text{p}} \cdot i_{\text{XP}}) b_{\text{H}} \cdot X_{\text{B,H}} + (i_{\text{XB}} - f_{\text{p}} \cdot i_{\text{XP}}) b_{\text{A}} \cdot X_{\text{B,A}} \\ &\quad - k_{\text{h}} \cdot k_{\text{sat}} \cdot (M_{8\text{h}} + \eta_{\text{h}} \cdot I_8 \cdot M_9) X_{\text{B,H}} \cdot \frac{X_{\text{ND}}}{X_{\text{S}}}. \end{aligned} \quad (12)$$

Reaction Rates

$$M_2 = \frac{S_{\text{S}}}{K_{\text{S}} + S_{\text{S}}}, \quad (13)$$

$$M_{8\text{a}} = \frac{S_{\text{O}}}{K_{\text{O,A}} + S_{\text{O}}}, \quad (14)$$

$$M_{8\text{h}} = \frac{S_{\text{O}}}{K_{\text{O,H}} + S_{\text{O}}}, \quad (15)$$

$$M_9 = \frac{S_{\text{NO}}}{K_{\text{NO}} + S_{\text{NO}}}, \quad (16)$$

$$M_{10} = \frac{S_{\text{NH}}}{K_{\text{NH}} + S_{\text{NH}}}, \quad (17)$$

$$I_8 = \frac{K_{\text{O,H}}}{K_{\text{O,H}} + S_{\text{O}}}, \quad (18)$$

$$k_{\text{sat}} = \frac{X_{\text{S}}}{K_{\text{X}} X_{\text{B,H}} + X_{\text{S}}}. \quad (19)$$

We assume that the settling unit captures all particulates. This gives the value of the concentrating factor as

$$b_1 = \frac{1 + R_1}{R_1 + w_1}, \quad (20)$$

where R_1 is the recycle ratio of the settling unit and w_1 is the fraction wasted.

2.2 Calculating some important quantities

In this section we show how the chemical oxygen demand (COD), the total suspended solids (TSS), and the total nitrogen (TN) can be evaluated in the aerated reactor, the effluent stream and the wastage stream. At the end of this section we characterise the COD, TSS and TN in the influent stream to the reactor.

Chemical oxygen demand in reactor (mg COD l⁻¹).

$$\text{COD} = S_S + S_I + X_S + X_{B,A} + X_{B,H} + X_I + X_P. \quad (21)$$

Chemical oxygen demand in effluent stream (mg COD l⁻¹).

$$\text{COD}_e = S_S + S_I. \quad (22)$$

Chemical oxygen demand in wastage stream (mg COD l⁻¹).

$$\text{COD}_w = S_S + S_I + b_1 (X_S + X_{B,A} + X_{B,H} + X_P + X_I). \quad (23)$$

Total suspended solids in reactor (mg SS l⁻¹).

$$\text{TSS} = c_1 (X_S + X_P + X_I) + c_2 (X_{B,A} + X_{B,H}). \quad (24)$$

Total suspended solids in effluent stream (mg SS l⁻¹).

$$\text{TSS}_e = 0. \quad (25)$$

Total suspended solids in wastage stream (mg SS l⁻¹).

$$\text{TSS}_w = b_{1,\max} \text{TSS}. \quad (26)$$

Total nitrogen (TN) in the reactor (g N l⁻¹)

$$\text{TN} = S_{\text{NO}} + S_{\text{NH}} + S_{\text{ND}} + X_{\text{ND}} + i_{\text{XB}} (X_{\text{BH}} + X_{\text{BA}}) + i_{\text{XP}} (X_{\text{P}} + X_{\text{I}}). \quad (27)$$

Total nitrogen (TN) in the effluent (g N l^{-1})

$$\text{TN}_e = S_{\text{NO}} + S_{\text{NH}} + S_{\text{ND}}. \quad (28)$$

Total nitrogen (TN) in the wastage stream (g N l^{-1})

$$\text{TN}_w = S_{\text{NO}} + S_{\text{NH}} + S_{\text{ND}} + b_1 [X_{\text{ND}} + i_{\text{XB}} (X_{\text{BH}} + X_{\text{BA}}) + i_{\text{XP}} (X_{\text{P}} + X_{\text{I}})]. \quad (29)$$

Using the default values in the appendix the influent is characterised as follows:

$$\text{COD}_{\text{in}} = 305 \text{ mg COD l}^{-1}.$$

$$\text{TSS}_{\text{in}} = 77.25 \text{ mg SS l}^{-1}.$$

$$\text{TN}_{\text{in}} = 40.180 \text{ mg N l}^{-1}.$$

3 Results

In this section we discuss the steady-state behaviour of our model. [Except for section 3.7](#) We fix the recycle ratio (R) at which the settling unit operates and treat the hydraulic retention time (HRT) as the primary bifurcation parameter. When the activated sludge process only contains a single reactor, as is the case here, a linear relationship exists between the sludge retention time (SRT) and the hydraulic retention time (τ),

$$\text{SRT} = \frac{\tau}{b_1 w} = \frac{\tau}{w} \cdot \frac{w + R}{1 + R}.$$

Consequently, in our steady-state diagrams we also indicate the corresponding sludge retention time.

The influent used in our simulations is characterised in table 2. Parameters associated with the activated sludge model, chiefly kinetic and stoichiometric values, are defined in table 3. Four miscellaneous parameters, associated with conversion of COD into TSS and the operation of the settling unit, are defined in table 4.

Steady-state diagrams have been found using the software package XPPAUT [7]. [Steady-state analysis in XPPAUT](#) requires two steps. In the first step the differential equations are integrated, using XPPAUT, until the solution converges to a stable steady-state solution. XPPAUT

contains the code for the standard bifurcation program AUTO. The steady-state solution is then used as an input into AUTO, which uses continuation methods, i.e. path-following, to trace the steady-state solution as an unfolding parameter is varied. The XPPAUT contains a user-friendly AUTO interface, so that the user is able to switch between the two without knowing very much about the latter. Steady-state diagrams have been drawn using the standard representation, i.e. stable solution branches and unstable solutions are denoted by solid and dashed lines respectively. In our figure we have, for clarity, redrawn the steady-state diagrams by removing unstable solutions.

3.1 Chemical oxygen demand

Chemical oxygen demand (COD) is an indicator of the amount of organic compounds in water, serving as an indirect measurement of the amount of pollution in a sample of water. Total chemical oxygen demand can be split into two components: soluble organic material ($S_S + S_I$) and particulate organic matter. Particulate organic matter, in turn, can be split into two components: biomass ($X_{B,A} + X_{B,H}$) and non-biomass ($X_S + X_I + X_P$). The operators of wastewater treatment plants are primarily interested with aqueous phase COD.

Figure 2 (a) shows the variation of the total chemical oxygen demand and its division into these three components as a function of the hydraulic retention time. In figure 2 (b) the chemical oxygen demand in each component has been normalised against the total chemical oxygen. Thus, this figure shows the fractional contribution of each of the three components.

Figure 2 indicates that if the hydraulic retention time is too small then the only stable solution corresponds to washout, i.e. the steady-state values of the particulate biomass species' are zero. Under these conditions the soluble chemical oxygen demand inside the reactor is equal to its value in the feed stream. The washout solution loses stability at a transcritical bifurcation when the hydraulic retention time is approximately 0.054 days. Figure 4 reveals that as the hydraulic retention time is increased through its value at the transcritical bifurcation that heterotrophic biomass, but not the autotrophic biomass, become viable. In the post-washout region it is convenient to partition the variation of the chemical oxygen demand with hydraulic retention time into two regions.

Returning to figure 2 observe that in the vicinity of the washout point there is a removal of soluble COD a narrow operating region. This rapid removal is associated with an equally rapid increase in the concentration of (heterotrophic) biomass. The significant removal of soluble COD at a hydraulic retention time of approximately 0.1 days (corresponding to a SRT of 0.36 days) illustrates how effectively organic compounds are degraded in the activated sludge process. The (heterotrophic) biomass concentration increases rapidly in this region, reaching a maximum value of $654 \text{ mg COD l}^{-1}$ at a hydraulic retention time of 0.183 days.

For higher values of the hydraulic retention time, beyond a value of approximately 0.2 days, there is a much more gradual decrease in the value of the soluble COD. In this second region the minimal impact of the hydraulic retention time upon the soluble COD in the reactor is attributed to endogenous decay of (heterotrophic) biomass [20]. Although the total biomass concentration initially decreases in this region there is a ‘kink’ in its curve at a hydraulic retention time of approximately 0.413 days, leading to a temporary increase. We see latter, figure 4, that as the hydraulic retention time is increased through its value at the ‘kink’ that autotrophic biomass becomes viable. However, the long term trend in this region is for a gradual decrease in the *total* biomass concentration.

Figure 2 (b) shows the fractional COD as a function of the hydraulic retention time. In the washout region 64.5% of the COD is due to non-biomass particulate matter and 35.5% is due to soluble organic compounds. Note that even though there is no biomass, the steady-state COD in the reactor is higher than that in the feed. The reason for this is that settling unit increase the concentration of particulates inside the reactor relative to the feed. For example, consider the steady-state value for the concentration of particulate inert organic matter. From equation (3) we have

$$\begin{aligned} X_i &= \frac{X_{I,\text{in}}}{1 - R_1 (b_1 - 1)}, \\ &= \frac{R + w}{w(1 + R)} \cdot X_{I,\text{in}}, \quad \text{using equation (20)}. \end{aligned}$$

Hence the steady-state value is an increasing function of the recycle ratio.

In region one the fractional biomass contribution rapidly increases to slightly over 90% of the total COD. In the second region there is a very slight decay in the fractional contribution of the biomass and a corresponding increase in the fractional non-biomass particulate matter. At a

hydraulic retention time of one day the fractional contributions are 91.7% (biomass particulate matter), 7.7% (non-biomass particulate matter) and 0.6% (soluble organics).

Figure 3 (a) shows the total COD in the effluent stream, the reactor and the waste stream as a function of the hydraulic retention time. As it is assumed that the settling unit captures all particulate matter, the total COD in the effluent stream is equal to the soluble COD inside the reactor, shown in figure 2 (a). The horizontal dotted black line corresponds to a ‘target’ COD value of $125 \text{ mg COD l}^{-1}$. This value is achieved at a hydraulic retention time of 0.055 days (1.32 hours). Note that in practice, the value taken for the hydraulic retention time is influenced by several factors. In many applications a value of around six hours is used.

As the effluent stream is devoid of particulate matter, i.e. it consists only of soluble COD, it is unsurprising that it has a significantly reduced COD compared to the values in both the reactor and the effluent stream. The significantly higher COD of the waste stream, as compared to that of the bioreactor, is due to the assumption that the settling tank captures all particulate matter. (This is the desired state of operation). Note that the use of a settling unit allows operation at a lower hydraulic retention time than would otherwise be the case.¹

3.2 Total suspended solids

Solids that are contained in water that can be trapped by a filter are called suspended solids. Figure 3 (b) shows the variation in the total suspended solids (TSS) in the effluent stream, the reactor and the waste stream. High concentrations of TSS are known to cause problems for the health of water sources, consequently having adverse consequences for aquatic life, downstream of wastewater treatment plants [3].

As it is assumed that the settling unit captures all particulate matter, the TSS in the effluent stream is equal to zero. Equation (26) shows that the TSS in the waste stream is a multiple of the TSS in the reactor. Equation (24) shows that the TSS in the reactor comprises the same five components as the particulate COD. To a first approximation the coefficients c_i are equal ($c_1/c_2 \approx 0.83$) so that the TSS is approximately the same as the particulate COD. This

¹If there is no recycle ($R = 0$) the HRT required to achieve the target COD in the effluent is approximately 343.8 hours.

accounts for the great similarity between the curves for the TSS in the reactor and effluent streams shown in figure 3 (b) and the corresponding curves for the COD in the reactor and effluent streams shown in figure 3 (a).

3.3 Concentration of biomass

Figure 4 shows the variation of the concentration of heterotrophic ($X_{B,H}$) and autotrophic ($X_{B,A}$) biomass with the hydraulic retention time. Comparing figure 4 with figure 2 (a) we see that the transcritical bifurcation associated with the rapid removal of soluble COD, at a hydraulic retention time of approximately 0.054 days, coincides with the appearance of heterotrophic bacteria whilst the ‘kink’ in the biomass concentration, at approximately 0.413 days, corresponds to a second transcritical bifurcation associated with the appearance of the autotrophic bacteria.

This figure shows that at hydraulic retention times greater than approximately 0.183 days, corresponding to longer SRT, the concentration of the heterotrophs gradually decrease. Autotrophs comprise only a tiny fraction of the total biomass, at a hydraulic retention time of 1 day they comprise 3% of the total biomass.

3.4 Total nitrogen

The total nitrogen can be split into two components: soluble nitrogen ($S_{NO} + S_{NH} + S_{ND}$) and nitrogen contained in particulate matter. For the purposes of calculating total nitrogen, the particulate matter is split into three components: biodegradable particulates (X_{ND}), biomass ($X_{B,A} + X_{B,H}$) and non-biomass ($X_s + X_I + X_P$). In calculating the nitrogen in the last two components, the appropriate conversion factors must be used — see equations (27)–(29).

Figure 5 shows the variation in the total nitrogen in the effluent stream, the reactor and the waste stream as a function of the hydraulic retention time. Following the transcritical bifurcation at which heterotrophic biomass becomes viable ($HRT \approx 0.054$ days) there is an increase in the total nitrogen inside the reactor and a corresponding decrease in the concentration of total nitrogen in the effluent stream. However, for values of the hydraulic residence time higher than approximately 0.1 days there is very little variation in either of these values.

The concentration of soluble nitrogen in the feed is 31 mg l^{-1} . At a hydraulic retention time of 1 day the concentration of soluble nitrogen in the effluent stream is reduced to 24.8 mg l^{-1} . This represents a 20% reduction in the total nitrogen. Thus even though the settling unit is assumed to capture all particulate matter, figure 5 shows that, unlike COD, total nitrogen is incompletely removed from the effluent stream. Pollution discharge regulations typically impose a maximum nitrogen concentration in the effluent stream of 10 mg N/l [1]. The activated sludge process simulated in figure 5 fails to meet such a requirement.

Nitrogen removal is achieved via two steps; nitrification, the conversion of soluble ammonium (NH_4^+) to nitrite (NO_2^-) and nitrate (NO_3^-) under aerobic conditions, and denitrification, the conversion of nitrate to gaseous nitrogen under anoxic conditions. In the ASM #1 model these two **pools** of inorganic nitrogen are denoted by S_{NH} and S_{NO} respectively. Figure 6 shows the variation of these two terms and their sum as a function of the hydraulic retention time. This figure shows that only one of the two steps required to remove nitrogen, nitrification, occurs. This is the reason for the failure to significantly lower the total nitrogen in the effluent stream.

The post-washout behaviour exhibited in figure 6 can be delimited into two regions. In the first of these, when the hydraulic retention time is between 0.054 days and 0.413 days, the concentration of soluble ammonium nitrogen increases: nitrification is not occurring. Within this region the concentration of soluble nitrite/nitrate initially decreases as the hydraulic retention time is increased through the lower transcritical bifurcation. At slightly higher values of the hydraulic retention time there is a very small increase in the concentration of soluble nitrite/nitrate.

As the hydraulic retention time is increased through 0.413 days (SRT 1.47 days), corresponding to the second transcritical bifurcation, figure 4 shows that autotrophic biomass become viable. The appearance of autotrophic bacteria is responsible for the onset of nitrification. As we enter into the second post-washout region there is a dramatic decrease in the concentration of soluble ammonium nitrogen and a corresponding increase in the concentration of soluble nitrite/nitrate nitrogen. Ozturk & Teymour [18] have similarly concluded that the ammonium content of the wastewater can not be significantly reduced in the absence of autotrophic biomass.

As the hydraulic retention time is increased from 0.413 to 1 day there is a small increase in the total soluble nitrogen: from 22.7 mg N l^{-1} to 24.1 mg N l^{-1} . The total nitrogen does not

decreases because the final step of the two-step process to remove nitrogen, denitrification, is not turned “on”. In a single tank operated under continuous aeration it is impossible to simultaneously obtain aerobic and anaerobic conditions. Consequently, nitrogen can not be efficiently removed in this reactor configuration [21].

It has frequently been observed experimentally that nitrification is an “all-or-nothing” affair [10, Figure 6.4 and accompanying discussion]: below a critical value of the SRT there is very little nitrification whilst for values of the SRT above the critical value nitrification rapidly approaches 100%. This behaviour is related to the very low half-saturation coefficient for the removal of soluble ammonium nitrogen by autotrophic biomass ($K_{\text{NH}} = 1.0 \text{ mg N l}^{-1}$) [10, chapter 6.3.1]. A critical SRT of three days is commonly mentioned in the literature for substantial nitrification.

For the settling unit parameters used in this paper, a SRT of three days corresponds to a hydraulic residence time of 0.84 days. Figure 6 shows that a hydraulic retention time of 1 day, corresponding to a SRT of 3.57 days, is sufficient to facilitate nitrification. Indeed at a hydraulic retention time of 1 day the concentration of ammonium in the reactor, 0.768 mg l^{-1} , is only 5% of the its value in the influent, 15.0 mg l^{-1} . The removal of 95% of the ammonium is comparable to available data [20].

3.5 Dissolved oxygen

Figure 7 shows the variation in the concentration of dissolved oxygen as a function of the hydraulic retention time. When the hydraulic retention time is between 0 and 0.054 days the oxygen concentration is an increasing function. This region corresponds to the washout steady-state solution being stable, c.f. figures 2 (a) and figure 4. As the hydraulic retention time increases the steady-state value increases from the value in the feed, $S_{\text{O},\text{in}} = 2 \text{ mg l}^{-1}$, towards the maximum concentration of soluble oxygen, $S_{\text{O},\text{max}} = 10 \text{ mg l}^{-1}$.

Following the first transcritical bifurcation, at which the washout solution losses stability, there is a dramatic increase in the concentration of heterotrophs, figure 4, and a corresponding dramatic decrease in the dissolved oxygen concentration. The oxygen concentration reaches a minimum value of 0.77 mg l^{-1} at a hydraulic retention time of 0.086 days. The oxygen concentration then increases to a maximum value of 7.00 mg l^{-1} at a hydraulic retention time

of 4.13 days. This hydraulic retention time at this value corresponds to that at the second transcritical bifurcation.

Following the second transcritical bifurcation, which gives rise to a population of autotrophs, the dissolved oxygen concentration again decreases. This decrease is not as dramatic as that associated with the formation of the heterotrophs — figure 4 shows that the increase in heterotrophs following the first transcritical bifurcation is much greater than the increase in autotrophs following the second transcritical bifurcation. The dissolved oxygen concentration reaches a minimum value of 5.86 mg l^{-1} at a hydraulic retention time of 0.474 days. Thereafter the oxygen concentration is an increasing function of the hydraulic retention time.

Thus the variation of dissolved oxygen in the reactor, figure 7, rhymes with the biomass growth pattern shown in Figure 4.

In practice, a minimum acceptable dissolved oxygen concentration is often set and a control system is implemented to maintain the dissolved oxygen concentration at this value [2]. If the dissolved oxygen concentration reduces below this target level, then measures are taken to increase its value. These measures effectively act to increase the value of the oxygen transfer coefficient ($K_{L,A}$).

In an aeration tank the set point for dissolved oxygen is often set at 2 mg l^{-1} to ensure that the biokinetic processes do not become oxygen limited, see for example [4]. In figure 7 the set point value of 2 mg l^{-1} is reached when the hydraulic retention time takes a value of either 6.176249×10^{-2} days or 12.36243×10^{-2} days, approximately 1.48 and 2.97 hours respectively. These values correspond to an oxygen transfer coefficient $K_{L,A} = 96 \text{ day}^{-1}$. It is to be expected that the values of the hydraulic retention time where the steady-state dissolved oxygen concentration is equal to its minimum value change as a function of the transfer coefficient.

In figure 8 the value of the hydraulic retention time at which the dissolved oxygen concentration is equal to the minimum value (2 mg/l) is unfolded as a function of the oxygen mass transfer number. As the latter increases, the two values of the hydraulic retention time approach each other, i.e. the range of hydraulic retention times over which the steady-state dissolved oxygen concentration is below the minimum acceptable value decreases. At a critical value of the oxygen mass transfer number, corresponding to a limit point bifurcation, the two values coalesce. This critical value is given by $K_{L,A} = K_{L,A,cr} \approx 129.825 \text{ (day}^{-1}\text{)}$. For values of the

oxygen mass transfer coefficient larger than this critical value the steady-state dissolved oxygen concentration will always be larger than the minimum value.

3.6 Influence of the influent composition upon the treatment process

In figure 9 we investigate how the composition of the influent changes two characteristics of the treatment process. These are the hydraulic retention time at which the heterotrophic biomass become viable and the value of the hydraulic retention time at which the chemical oxygen demand in the effluent stream is equal to the target value.

We change the influent composition by relating the concentration of readily biodegradable substrate in the feed stream ($S_{S,in}$) to the concentration of slowly biodegradable particulate substrate ($X_{S,in}$) via

$$S_{S,in} + X_{S,in} = 300.$$

We have chosen to vary the influent concentration in this way so as to maintain constant values of both the chemical oxygen demand in the influent and the ratio of the chemical oxygen demand to total nitrogen in the influent.

The default values for the two influent concentrations used in other figures are $S_{S,in} = 200 \text{ mg COD l}^{-1}$ and $X_{S,in} = 100 \text{ mg COD l}^{-1}$. The values of the HRT at the transcritical bifurcation and when the COD equals its target value are then given 0.054 days and 0.055 days respectively.

Figure 9 shows that the HRT at the transcritical bifurcation is an increasing function of the concentration of slowly biodegradable substrate. For values of the concentration below $177 \text{ mg COD l}^{-1}$ the value of the HRT decreases relatively slowly. As the value of the concentration increases through this value, the rate of change of the HRT at the transcritical bifurcation increases rapidly.

The value of the HRT when the chemical oxygen demand in the effluent stream is equal to the target value is a *decreasing* function of the concentration of slowly biodegradable substrate in the effluent stream. There is a critical value of this concentration at which the two curves intersect. For values of the concentration higher than this value, the chemical oxygen demand in the effluent stream is *always* lower than the target value. The critical value is when $X_{S,in} =$

177 mg COD l⁻¹, i.e. when $S_{S,in} = 123$ mg COD l⁻¹.

What is special about the value $S_{S,in} = 123$ mg COD l⁻¹? The significance of this number is revealed by considering the corresponding value of the chemical oxygen demand in the effluent stream along the no-washout branch. We have

$$\text{COD}_{e,\text{washout}} = S_S + S_I = 123 + 2 = 125 \text{ mg COD l}^{-1}.$$

This finding is a consequence of the modelling assumption that the settling unit operates perfectly, i.e. it captures all particulate matter. This is a standard assumption since settling units operate ideally when the flow rate and the total suspended solids in the influent stream are relatively constant [6]. This is the case for the steady-state operating conditions investigated in this paper.

3.7 The influence of the settling unit upon the treatment process

Here we repeat our analysis of the previous section, but vary how the settling unit is operated rather than the influent composition. In the ideal settling unit model the operation of the settling unit is represented by the value of the recycle ratio (R).

Figure 10 shows that both values of the hydraulic retention time are decreasing functions of the recycle ratio. However, the rate of decrease decreases as the value of the recycle ratio increases. For the standard value of the feed concentration the value of the HRT at the transcritical bifurcation gives a good approximation to the value at which the chemical oxygen demand in the influent is equal to that of the target value. For the standard value of the feed concentration increasing the recycle ratio from $R = 0.4$ to $R = 1.0$ has an insignificant effect upon the performance of the treatment process. As the value of the recycle ratio is increased from $R = 0.4$ to $R = 1.0$ the value of the HRT at the transcritical bifurcation decreases from 0.054 days to 0.0352 days (a decrease of twenty-seven minutes) whilst the value of the HRT when the chemical oxygen demand equals its target value decreases from 0.055 days to 0.0404 days (a decrease of twenty-one minutes).

4 Conclusion

In this paper we have employed continuation methods to investigate the steady-state behaviour of the activated sludge process. The process configuration employed was the simplest, consisting of a single aerated biological reactor attached to a settling unit. The biochemical model used was the well-known activated sludge model #1 (ASM#1) [12]. An important feature of our implementation of the ASM#1 is that, unlike many authors, we have not fixed the dissolved oxygen concentration but have allowed it to vary in response to changes in the other state variables.

Although this reactor configuration has often been simulated using the ASM#1, previous investigations have taken the form of direct numerical integration. Ours is the first investigation to deploy continuation methods. In this paper we have fixed the properties of the settling unit and investigated how important variables associated with the operation of the activated sludge process vary as the hydraulic retention time is varied. The variables considered were: the chemical oxygen demand (COD), total suspended solids (TSS), biomass concentration, total nitrogen (TN), soluble inorganic nitrogen, and the dissolved oxygen concentration.

We have demonstrated the importance of two transcritical bifurcations is governing the behaviour of the system. Process failure occurs for values of the hydraulic retention time lower than that at the first of the bifurcation points. As the value of the hydraulic retention time is increased through the first critical value heterotrophic biomass become viable and there is a dramatic decrease in the chemical oxygen demand in the effluent stream.

As the hydraulic retention time is increased through the value at the second of the bifurcation points autotrophic biomass become viable. The presence of autotrophic biomass ensures that soluble ammonium nitrogen is efficiently converted into soluble nitrate and nitrite nitrogen.

Having established the generic behaviour of the model we investigated how the process responds to variation in some of the model parameters. We first investigated the effect of the oxygen transfer coefficient upon the hydraulic retention times at which the set-point for dissolved oxygen is reached. We showed that for sufficiently high values of the transfer coefficient that the system is guaranteed to operate with a dissolved oxygen concentration higher than the set point.

We then investigated how the hydraulic retention times at the lower transcritical bifurcation and when the chemical oxygen demand in the effluent stream is equal to the target value change as the composition of the influent stream varies. We varied the composition of the influent stream in such a way that both its chemical oxygen demand and the ratio of the chemical oxygen demand to the total nitrogen remained constant. We found that the value of the hydraulic retention when $\text{COD}_e = 125 \text{ mg COD l}^{-1}$ decreased as the concentration of slowly biodegradable substrate (X_S) increased. Furthermore, there is a critical value of the slowly biodegradable substrate concentration above which the chemical oxygen demand in the effluent stream is always below the target value.

Finally, we investigated the effect of changing the recycle ratio. Although increasing the recycle ratio improves the performance of the process, the improvement for values larger than the default value ($R = 0.4$) are insignificant from a practical perspective.

In future work we intend to extend our reactor configuration to include multiple bioreactors, as well as including recycle units in addition to the use of a settling unit. The results presented here form a natural benchmark to assess the performance of these extended reactor configurations.

A Nomenclature

The notation X_{in} is used to denote the concentration of component X in the influent.

BOD	Biological oxygen demand	(mg l^{-1})
COD	Chemical oxygen demand	(mg l^{-1})
I_8	Inhibition kinetics for soluble oxygen	(—)
$K_{L,A}$	Oxygen transfer coefficient	(day^{-1})
K_{NH}	Ammonia half-saturation coefficient for autotrophic biomass	(mg N l^{-1})
K_{NO}	Nitrate half-saturation coefficient for denitrifying heterotrophic biomass	(mg N l^{-1})
$K_{O,A}$	Oxygen half-saturation coefficient for autotrophic biomass	($\text{mg O}_2 \text{ l}^{-1}$)
$K_{O,H}$	Oxygen half-saturation coefficient for heterotrophic biomass	($\text{mg O}_2 \text{ l}^{-1}$)

	biomass	
K_S	Substrate half-saturation coefficient for heterotrophic biomass	(mg COD l ⁻¹)
K_X	Half-saturation coefficient for hydrolysis of particulate biodegradable substrate	(—)
M_2	Monod kinetics for readily biodegradable soluble substrate	(—)
M_{8a}	Monod kinetics for the component S_O with respect to autotrophic biomass	(—)
M_{8h}	Monod kinetics for the component S_O with respect to heterotrophic biomass	(—)
M_9	Monod kinetics for soluble nitrate and nitrite nitrogen	(—)
M_{10}	Monod kinetics for soluble ammonium nitrogen	(—)
R	Recycle ratio of the settling unit	(—)
S_I	Concentration of inert soluble organic material	(mg COD l ⁻¹)
S_{ND}	Concentration of soluble biodegradable organic material	(mg N l ⁻¹)
S_{NH}	Concentration of soluble ammonium nitrogen	(mg N l ⁻¹)
S_{NO}	Concentration of soluble nitrate and nitrite nitrogen	(mg N l ⁻¹)
S_O	Concentration of soluble oxygen	(mg l ⁻¹)
$S_{O,max}$	Maximum concentration of soluble oxygen	(mg l ⁻¹)
S_S	Concentration of readily biodegradable soluble substrate	(mg COD l ⁻¹)
SS	Suspended solids	(mg l ⁻¹)
TN	Total nitrogen	(mg l ⁻¹)
TSS	Total suspended solids	(mg SS l ⁻¹)
V	Reactor volume	(l)
$X_{B,A}$	Concentration of active autotrophic particulate mass	(mg COD l ⁻¹)
$X_{B,H}$	Concentration of active heterotrophic particulate mass	(mg COD l ⁻¹)

X_{ND}	Concentration of particulate biodegradable organic nitrogen	(mg N l^{-1})
X_I	Concentration of particulate inert organic matter	(mg COD l^{-1})
X_P	Concentration of non-biodegradable particulate product arising from biomass decay	(mg COD l^{-1})
X_S	Concentration of slowly biodegradable particulate substrate	(mg COD l^{-1})
Y_A	Autotrophic yield coefficient	$(\text{g COD (g N)}^{-1})$
Y_H	Heterotrophic yield coefficient	$(\text{g COD (g COD)}^{-1})$
b_A	Autotrophic decay coefficient	(day^{-1})
b_H	Heterotrophic decay coefficient	(day^{-1})
b_1	Concentration factor for particulates in the settling unit	$(—)$
c_i	Conversion factors	g SS (g COD)^{-1}
f_P	Fraction of biomass yielding particulate products	$(—)$
i_{XB}	Nitrogen content in biomass	$(\text{mg N (mg SS)}^{-1})$
i_{XP}	Nitrogen content in inert particulate	$(\text{mg N (mg SS)}^{-1})$
k_A	Ammonification coefficient	$(\text{l mg (COD day)}^{-1})$
k_h	Hydrolysis coefficient	(day^{-1})
q	Feed flow-rate	(l day^{-1})
k_{sat}	Saturation kinetics	$(—)$
t	Time	(day)
w_1	Fraction of influent wasted.	$(—)$
η_g	Correction factor for anoxic growth of heterotrophs	$(—)$
η_h	Correction factor for anoxic hydrolysis	$(—)$
$\mu_{\text{max, A}}$	Maximum specific growth rate for autotrophs	(day^{-1})
$\mu_{\text{max, H}}$	Maximum specific growth rate for heterotrophs	(day^{-1})
τ	Hydraulic retention time ($\tau = \frac{V}{q}$)	(day)

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Component	Symbol	Units
Particulate matter		
1. Active autotrophic biomass	$X_{B,A}$	$[\text{mg COD l}^{-3}]$
2. Active heterotrophic biomass	$X_{B,H}$	$[\text{mg COD l}^{-3}]$
3. Particulate inert organic matter	X_I	$[\text{mg COD l}^{-3}]$
4. Particulate biodegradable organic nitrogen	X_{ND}	$[\text{mg N l}^{-3}]$
5. Particulate biomass decay products	X_P	$[\text{mg COD l}^{-3}]$
6. Slowly biodegradable substrate	X_S	$[\text{mg COD l}^{-3}]$
Soluble matter		
7. Inert soluble organic material	S_I	$[\text{mg COD l}^{-3}]$
8. Readily biodegradable substrate	S_S	$[\text{mg COD l}^{-3}]$
9. Nitrate and nitrite nitrogen	X_{NO}	$[\text{mg N l}^{-3}]$
10. NH_4^+ and NH_3 nitrogen	S_{NH}	$[\text{mg N l}^{-3}]$
11. Slowly biodegradable organic nitrogen	S_{ND}	$[\text{mg N l}^{-3}]$
Miscellaneous components		
12. Dissolved oxygen	S_O	$[\text{mg l}^{-3}]$

Table 1: Model components of the activated sludge model #1.

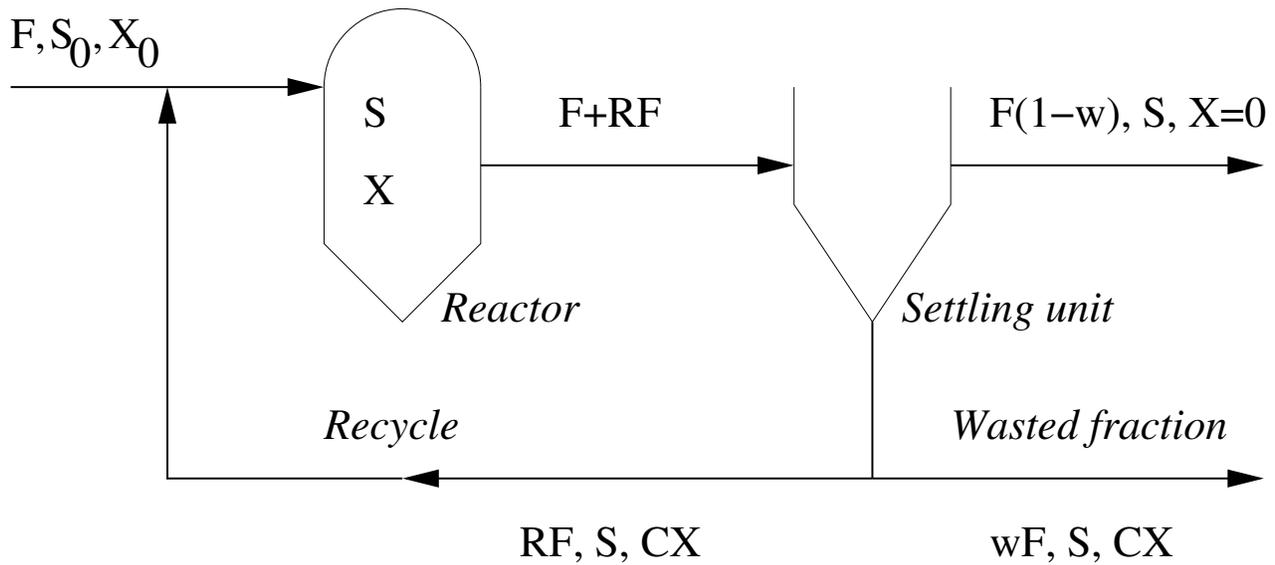


Figure 1: A bioreactor with recycle and separate wasting of biomass. Figure adapted from [16].

Parameter	Value	Reference
$S_{I,in}$	2 mg COD l ⁻¹	[23]
$S_{ND,in}$	15 mg N l ⁻¹	[23]
$S_{NH,in}$	15 mg N l ⁻¹	[23]
$S_{NO,in}$	1 mg N l ⁻¹	[23]
$S_{O,in}$	2 mg l ⁻¹	
$S_{S,in}$	200 mg COD l ⁻¹	[23]
$X_{I,in}$	3.0 mg COD l ⁻¹	[23]
$X_{ND,in}$	9 mg N l ⁻¹	[23]
$X_{S,in}$	100 mg COD l ⁻¹	[23]
$X_{B,A,in}$	0 mg COD l ⁻¹	[23]
$X_{B,H,in}$	0 mg COD l ⁻¹	[23]
$X_{P,in}$	0 mg COD l ⁻¹	

Table 2: Influent composition.

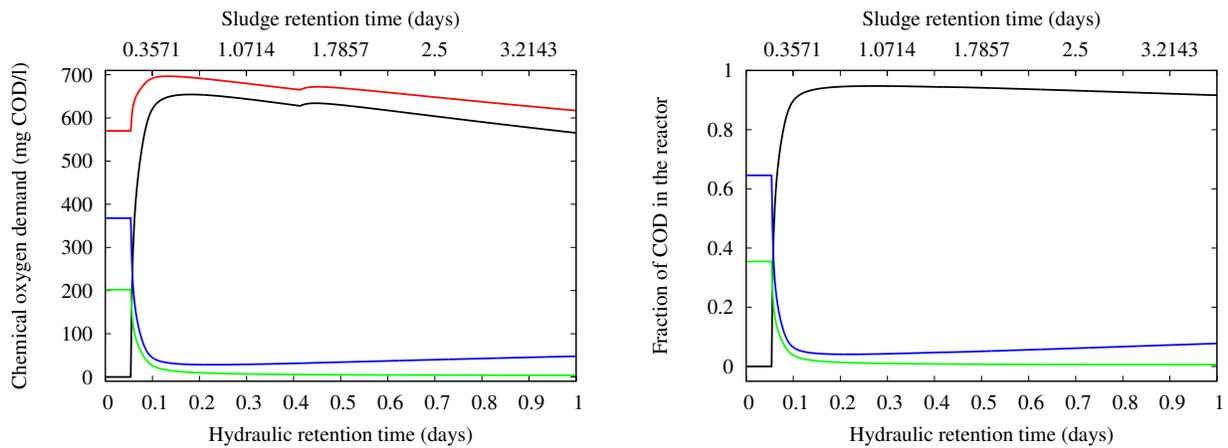


Figure 2: Chemical oxygen demand and fractional chemical oxygen demand inside the reactor. The components shown in figures (a)& (b) are: $X_{B,A} + X_{B,H}$ (brown), $X_S + X_I + X_P$ (blue), $S_S + S_I$ (green). In figure (a) the red line is total COD.

Parameter	Value	Reference
$K_{L,A}$	108 day ⁻¹	[8]
K_{NH}	1.0 mg N l ⁻¹	[12, 23]
K_{NO}	0.5 mg N l ⁻¹	[12, 23]
$K_{O,A}$	0.4 mg O ₂ l ⁻¹	[12, 23]
$K_{O,H}$	0.2 mg O ₂ l ⁻¹	[12, 23]
K_S	20.0 mg COD l ⁻¹	[12, 23]
K_X	0.03 (—)	[12, 23]
$S_{O,max}$	10 mg l ⁻¹	[8]
Y_A	0.24 g COD (g N) ⁻¹	[12, 23]
Y_H	0.67 g COD (g COD) ⁻¹	[12, 23]
b_A	0.05 day ⁻¹	[14, 23]
b_H	0.22 day ⁻¹	[14, 23]
f_P	0.08 (—)	[12, 23]
i_{XB}	0.086 mg N (mg SS) ⁻¹	[12, 23]
i_{XP}	0.06 mg N (mg SS ⁻¹)	[12, 23]
k_A	0.081 mg (COD day) ⁻¹	[12, 23]
k_h	3.0 day ⁻¹	[12, 23]
η_g	0.8 (—)	[12, 23]
η_h	0.4 (—)	[12, 23]
$\mu_{max,A}$	0.8 day ⁻¹	[12, 23]
$\mu_{max,H}$	6.0 day ⁻¹	

Table 3: Parameter values.

Parameter	Value	Reference
R_1	0.40	
c_1	$0.75 \text{ g SS (g COD)}^{-1}$	[13]
c_2	$0.90 \text{ g SS (g COD)}^{-1}$	[13]
w_1	0.1	

Table 4: Miscellaneous parameter values. The values for R_1 and w_1 give a concentration factor $b_1 = 14/5$.

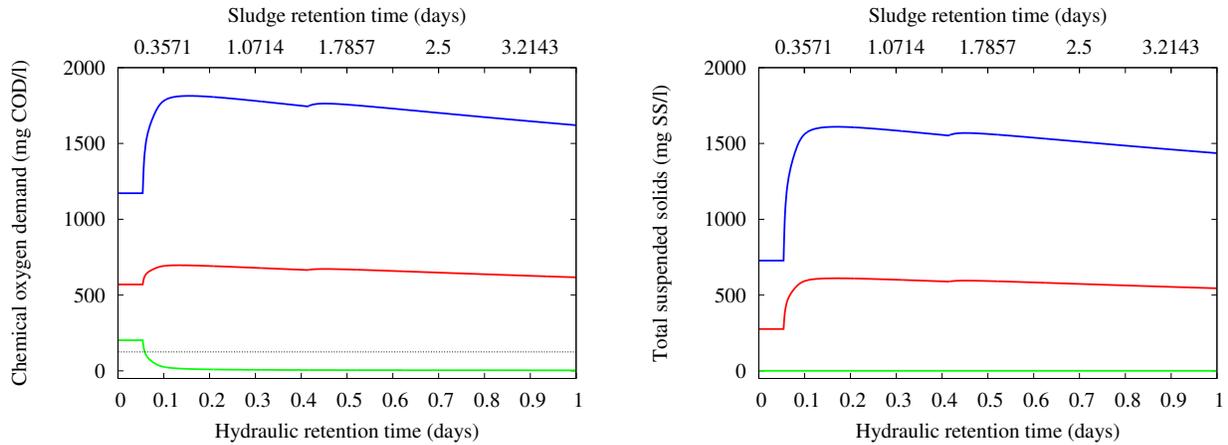


Figure 3: Chemical oxygen demand and total suspended solids inside the reactor (red), the effluent stream (green) and the waste stream (blue) of an aerated reactor connected to a settling unit. The target values are: $\text{COD}_t = 125 \text{ mg COD l}^{-1}$ and $\text{SS}_t = 35 \text{ mg SS l}^{-1}$.

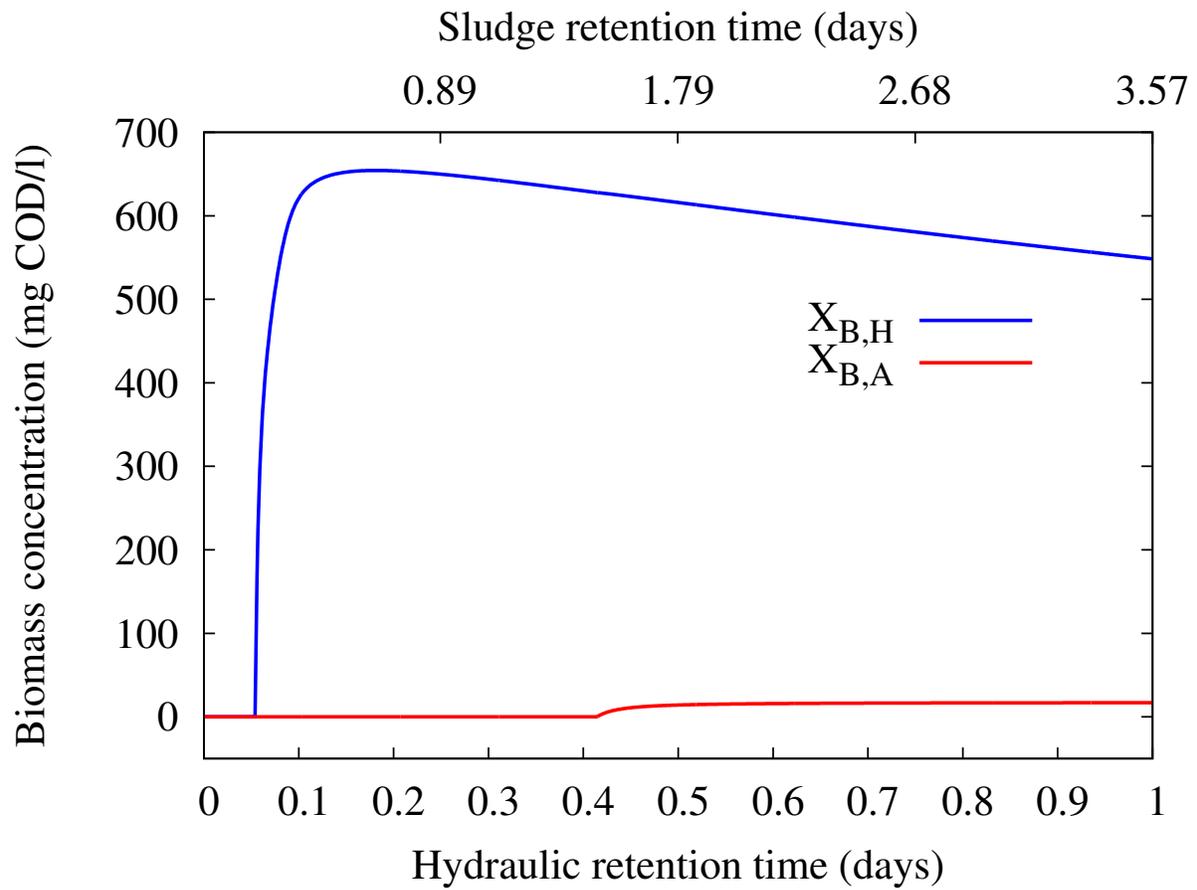


Figure 4: Variation of autotrophic ($X_{B,A}$) and heterotrophic ($X_{B,H}$), biomass in a reactor connected to a settling unit.

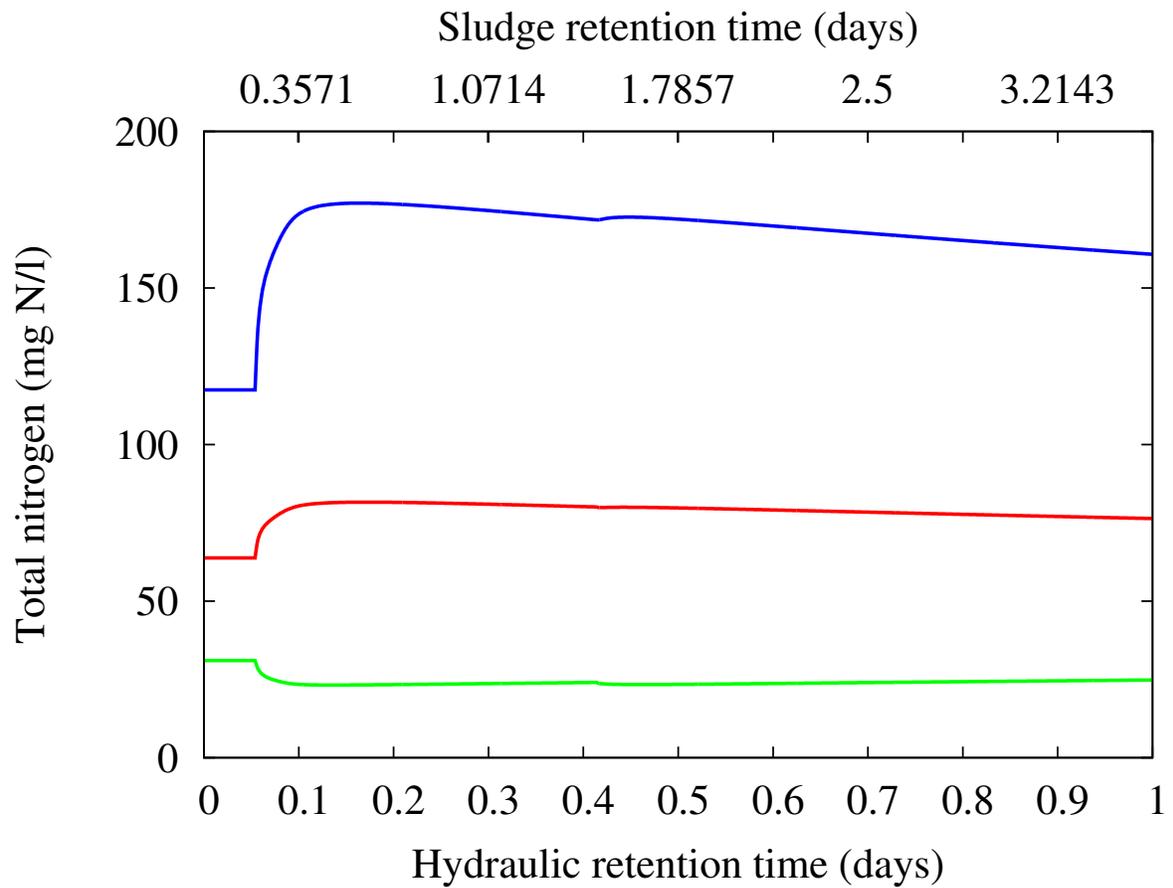


Figure 5: Total nitrogen inside the reactor (red), the effluent stream (green) and the waste stream (blue) of a reactor connected to a settling unit.

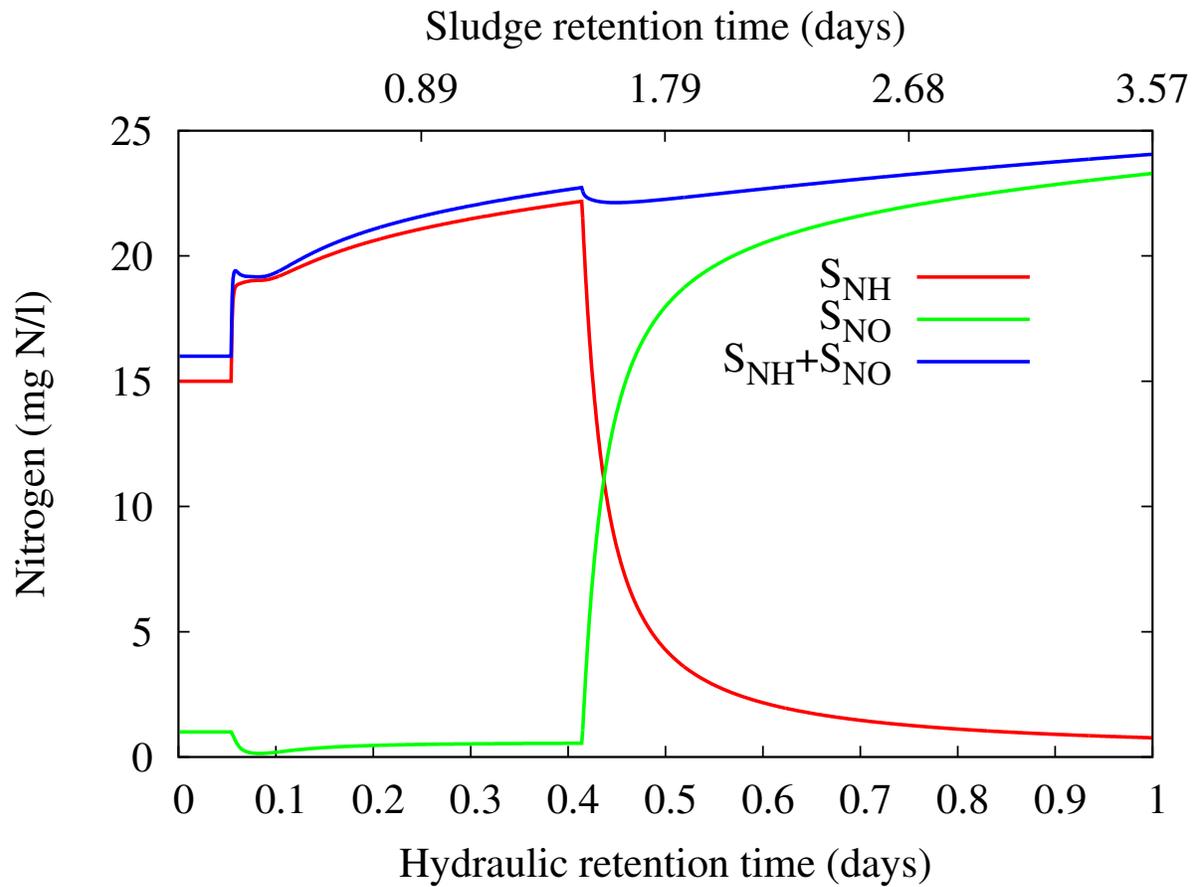


Figure 6: Variation of soluble inorganic nitrogen in a reactor connected to a settling unit. The variables plotted are the ammonium nitrogen (S_{NH}), the soluble nitrate and nitrite nitrogen (S_{NO}), and their sum ($S_{NH} + S_{NO}$).

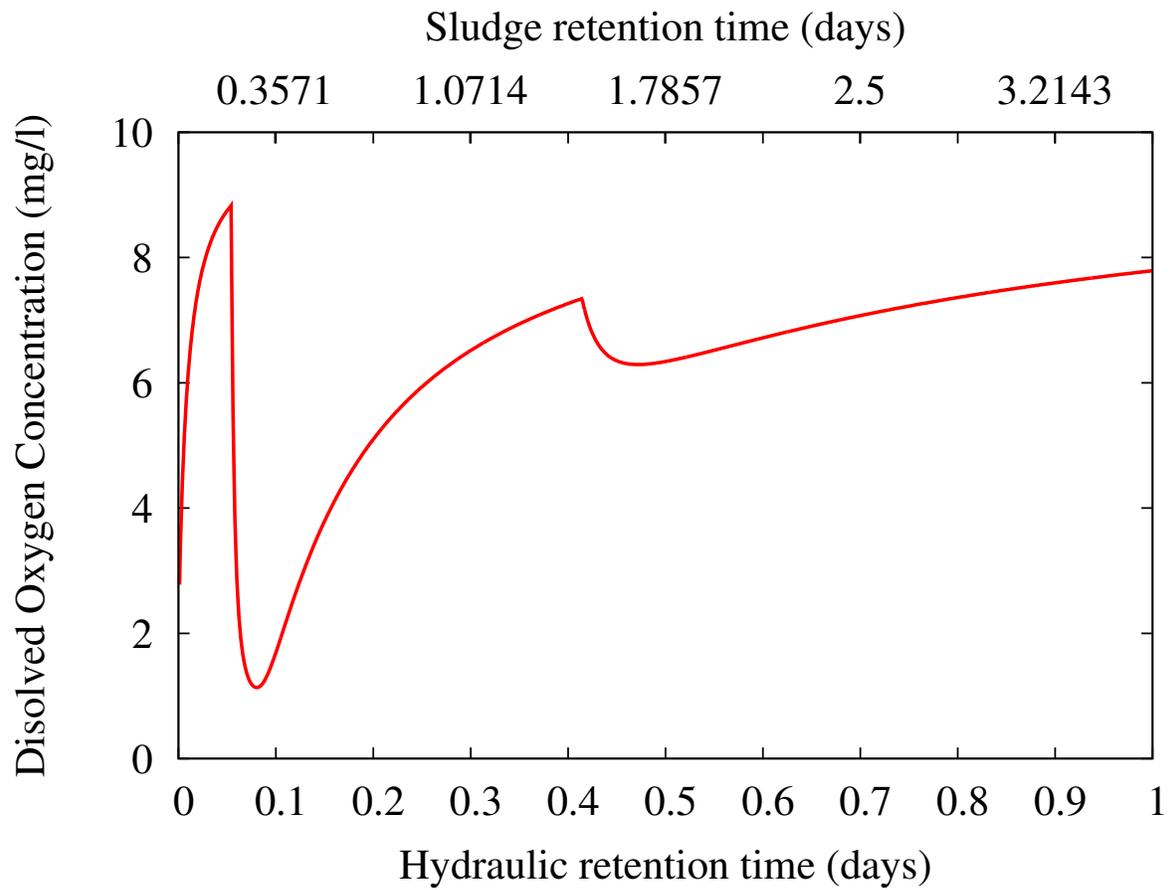


Figure 7: Variation of dissolved oxygen concentration (S_O) with the residence time in a reactor connected to a settling unit.

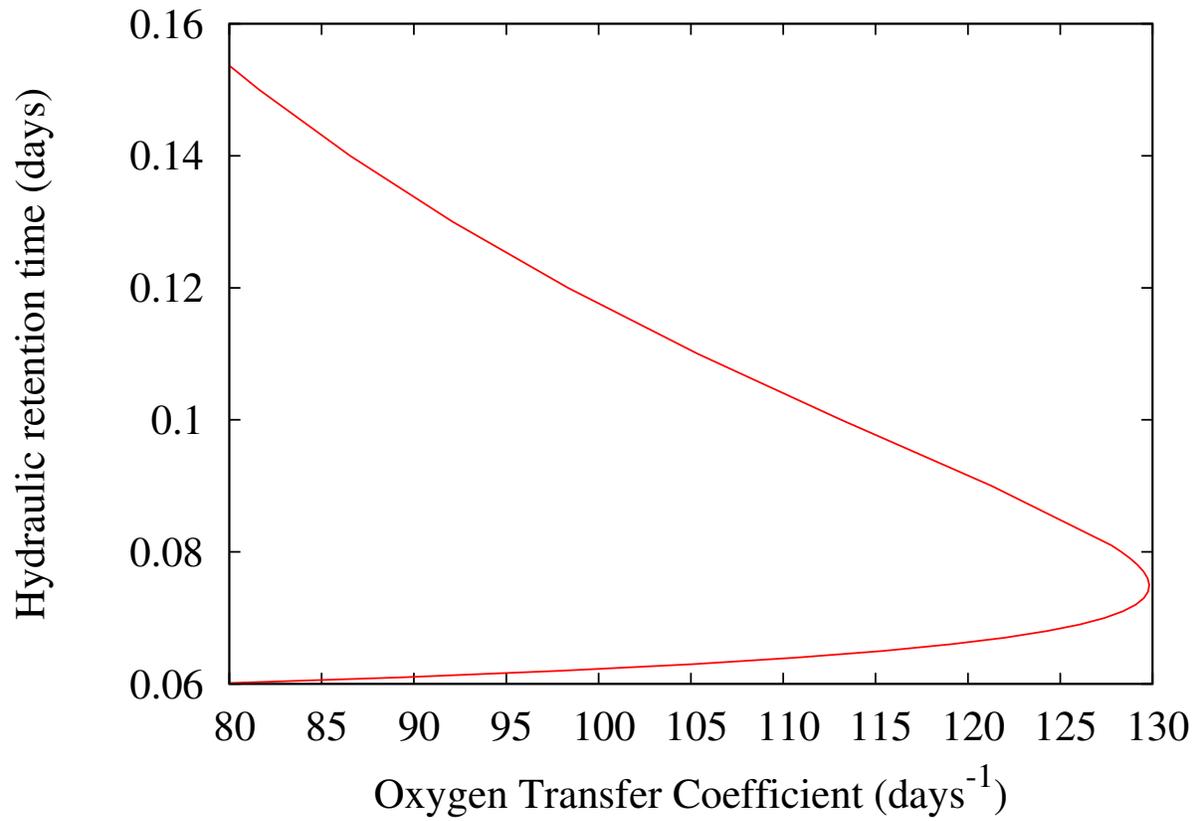


Figure 8: Variation of the hydraulic retention time (τ) at which the dissolved oxygen concentration is $2 \text{ mg O}_2 \text{ l}^{-1}$ as a function of the oxygen transfer coefficient ($K_{L,A}$).

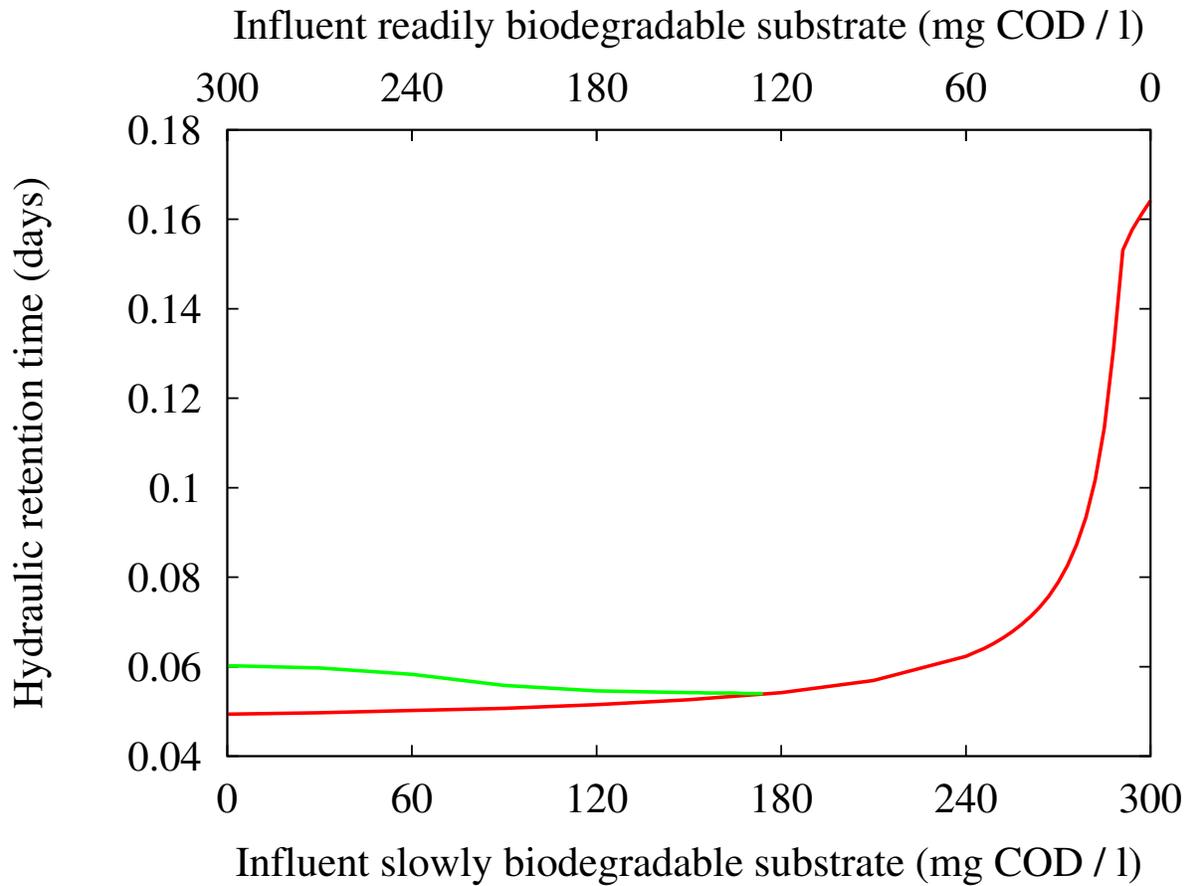


Figure 9: Variation of the HRT at which the heterotrophic biomass become viable (corresponding to a transcritical bifurcation, red line) and the HRT at which the chemical oxygen demand in the effluent stream is equal to the target value ($\text{COD}_t = 125 \text{ mg COD l}^{-1}$ green line) as a function of the composition of the influent.

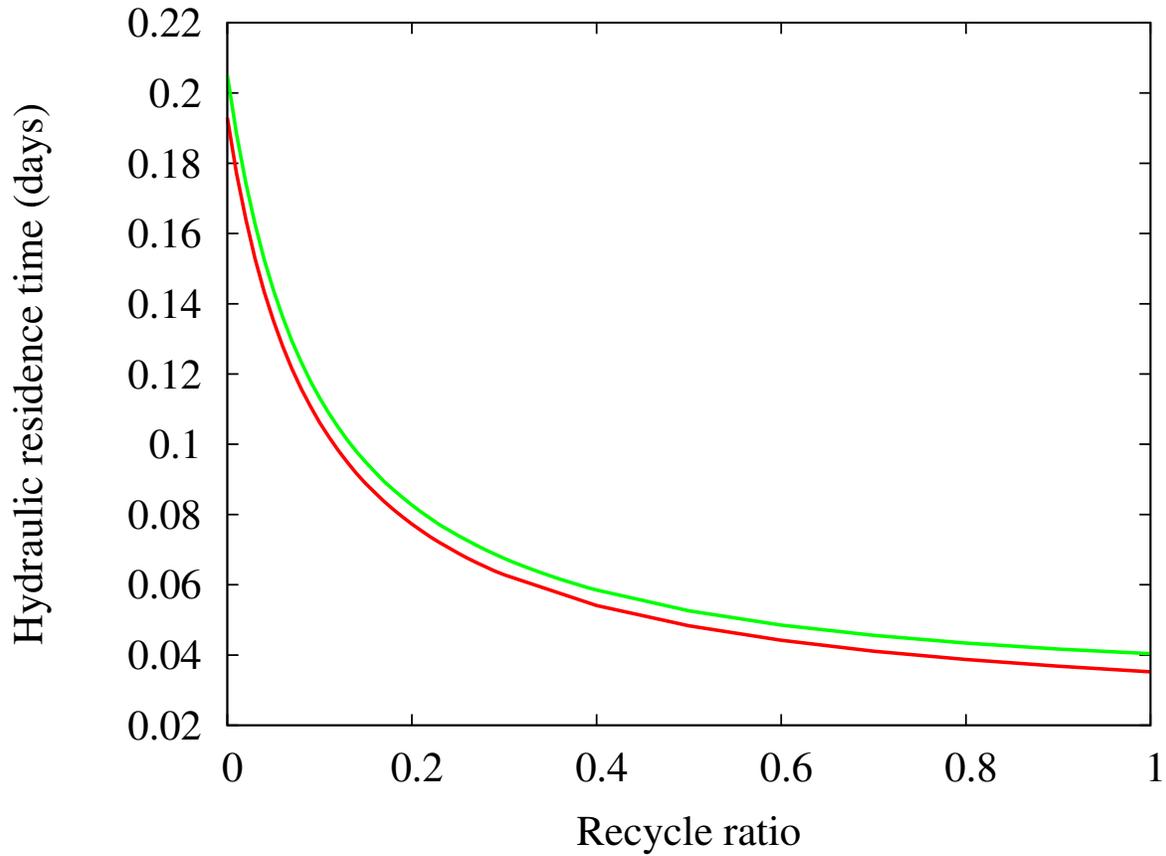


Figure 10: Variation of the HRT at which the heterotrophic biomass become viable (corresponding to a transcritical bifurcation, red line) and the HRT at which the chemical oxygen demand in the effluent stream is equal to the target value ($\text{COD}_t = 125 \text{ mg COD l}^{-1}$ green line) as a function of the recycle ratio ($0 < R \leq 1$).