

University of Wollongong

Research Online

---

Faculty of Engineering and Information  
Sciences - Papers: Part A

Faculty of Engineering and Information  
Sciences

---

1-1-2009

## Numerical study on self-similar pulses in mode-locking fiber laser by coupled Ginzburg-Landau equation model

Ting Lei  
*Nankai University*

Chenghou Tu  
*Nankai University*

Fuyun Lu  
*Nankai University*

Yixin Deng  
*Nankai University*

Enbang Li  
*Tianjin University, enbang@uow.edu.au*

Follow this and additional works at: <https://ro.uow.edu.au/eispapers>



Part of the [Engineering Commons](#), and the [Science and Technology Studies Commons](#)

---

Research Online is the open access institutional repository for the University of Wollongong. For further information contact the UOW Library: [research-pubs@uow.edu.au](mailto:research-pubs@uow.edu.au)

---

# Numerical study on self-similar pulses in mode-locking fiber laser by coupled Ginzburg-Landau equation model

## Abstract

A theoretical model is established to study the self-similar pulses in nonlinear polarization evolution (NPE) mode-locked fiber lasers. The propagation of pulse in single mode fibers and gain fibers are described by coupled Ginzburg-Landau equation (GLE). Two wave plates and a polarizer are considered to realize the NPE mechanism in simulation. This model describes the laser completely and provides some useful pulses' information. In our simulation the laser generates high quality self-similar pulses output. The region of steady self-similar pulses operation is found. The polarization states of different parts across the pulse are simulated along the laser cavity. It is found that polarization states across the pulse are modulated from elliptical to almost circular before the pulse passing through the polarizer.

## Keywords

laser, coupled, ginzburg, landau, equation, model, study, numerical, self, similar, pulses, mode, locking, fiber

## Disciplines

Engineering | Science and Technology Studies

## Publication Details

Lei, T., Tu, C., Lu, F., Deng, Y. & Li, E. (2009). Numerical study on self-similar pulses in mode-locking fiber laser by coupled Ginzburg-Landau equation model. *Optics Express*, 17 (2), 585-591.

# Numerical study on self-similar pulses in mode-locking fiber laser by coupled Ginzburg-Landau equation model

Ting Lei,<sup>1</sup> Chenghou Tu,<sup>1</sup>Fuyun Lu,<sup>1,\*</sup> Yixin Deng,<sup>1</sup> and Enbang Li<sup>2</sup>

<sup>1</sup>College of Physical Science and The Key Laboratory of Weak Light Nonlinear Photonics, Nankai University ,  
Tianjin 300071, Ministry of Education, People's Republic of China

<sup>2</sup>College of Precision Instrument and Optoelectronics Engineering, Tianjin University, Tianjin 300072, People's  
Republic of China

\*Corresponding author: [lufy@nankai.edu.cn](mailto:lufy@nankai.edu.cn)

**Abstract:** A theoretical model is established to study the self-similar pulses in nonlinear polarization evolution (NPE) mode-locked fiber lasers. The propagation of pulse in single mode fibers and gain fibers are described by coupled Ginzburg-Landau equation (GLE). Two wave plates and a polarizer are considered to realize the NPE mechanism in simulation. This model describes the laser completely and provides some useful pulses' information. In our simulation the laser generates high quality self-similar pulses output. The region of steady self-similar pulses operation is found. The polarization states of different parts across the pulse are simulated along the laser cavity. It is found that polarization states across the pulse are modulated from elliptical to almost circular before the pulse passing through the polarizer.

©2009 Optical Society of America

**OCIS codes:** (140.0140) Lasers and laser optics; (140.3538) Lasers, pulsed; (140.3560) Lasers, ring; (140.4050) Mode-locked lasers; (190.5530) Pulse propagation and temporal solitons.

---

## References and links

1. K. Tamura, H. A. Haus, and E. P. Ippen, "Self-starting additive pulse mode-locked erbium fiber ring laser," *Electron. Lett.* **28**, 2226-2228 (1992).
2. D. Anderson, M. Desaix, M. Lisak, and M. L. Quiroga-Teixeiro, "Wave breaking in nonlinear-optical fibers," *J. Opt. Soc. Am. B* **9**, 1358-1361 (1992).
3. L. E. Nelson, S. B. Fleischer, G. Lenz, and E. P. Ippen, "Efficient frequency doubling of a femtosecond fiber laser," *Opt. Lett.* **21**, 1759-1761 (1996).
4. L. M. Zhao, D. Y. Tang, and J. Wu, "Gain-guided soliton in a positive group-dispersion fiber laser," *Opt. Lett.* **31**, 1788-1790 (2006).
5. A. Chong, W. H. Renninger, and F. W. Wise, "Environmentally stable all-normal-dispersion femtosecond fiber laser," *Opt. Lett.* **33**, 1071-1073 (2008).
6. A. Chong, W. H. Renninger, and F. W. Wise, "All-normal-dispersion femtosecond fiber laser with pulse energy above 20nJ," *Opt. Lett.* **32**, 2408-2410 (2007).
7. D. Anderson, M. Desaix, M. Karlsson, M. Lisak, and M. L. Quiroga-Teixeiro, "Wave-breaking-free pulses in nonlinear-optical fibers," *J. Opt. Soc. Am. B* **10**, 1185- 1190 (1993).
8. F. Ö. Ilday, J. R. Buckley, W. G. Clark, and F. W. Wise, " Self-Similar Evolution of Parabolic Pulses in a Laser, " *Phys. Rev. Lett.* **92**, 213902 (2004).
9. A. Komarov, H. Leblond, and F. Sanchez, "Theoretical analysis of the operating regime of a passively-mode-locked fiber laser through nonlinear polarization rotation," *Phys. Rev. A (Atomic, Molecular, and Optical Physics)* **72**, 063811-063817 (2005).
10. B. G. Bale, J. N. Kutz, and F. Wise, "Analytic theory of self-similar mode-locking," *Opt. Lett.* **33**, 911-913 (2008).
11. K. M. Spaulding, D. H. Yong, A. D. Kim, and J. N. Kutz, "Nonlinear dynamics of mode-locking optical fiber ring lasers," *J. Opt. Soc. Am. B* **19**, 1045-1054 (2002).

12. C. H. Tu, Z. Li, T. Lei, Y. N. LI, W.G. Guo, D. Wei, H. Zhu, S. G. Zhang, and F.Y. Lu, "Pulse evolution characteristics in self-similar mode-locked fiber laser," *Chin. Phys. Lett.* **24**, 3175-3178 (2007).
13. A. Latkin, and S. Turitsyn, "Semi-Analytical Description of Parabolic Pulse Generation in the Normal-Dispersion Fibre Amplifiers," in *Proceedings of IEEE International Conference on Transparent Optical Networks Conference* (IEEE, 2006), pp. 259-262.
14. G. P. Agrawal, *Nonlinear Fiber Optics* (Academic Press, 2001), 203-209.
15. C. R. Menyuk, Nonlinear pulse propagation in birefringent optical fibers," *IEEE J. Quantum Electron* **23**,174-176 (1987).
16. D. S. Kliger, and J. W. Lewis, *Polarized light in optics and spectroscopy* (Academic, 1990).
17. M. E. Ferman, V. I. Kruglov, B. C. Thomsen, J. M. Dudley, and J. D. Harvey, "Self-Similar Propagation and Amplification of Parabolic Pulses in Optical Fibers," *Phys. Rev. Lett.* **84**, 6010 (2000).
18. J. Wu, D. Y. Tang, L. M. Zhao, and C. C. Chan, Phys. "Soliton polarization dynamics in fiber lasers passively mode-locked by the nonlinear polarization rotation technique," *Phys. Rev. E* **74**, 046605 (2006).
19. A. Komarov, H. Leblond, and F. Sanchez, "Quintic complex Ginzburg-Landau model for ring fiber lasers," *Phys. Rev. E (Statistical, Nonlinear, and Soft Matter Physics)* **72**, 025604-025604 (2005).

## 1. Introduction

Passive mode-locking fiber lasers have many advantages such as high stability, ultrashort pulse width, simple structure and compact size. Nonlinear polarization evolution is an important technique used in passive mode-locked fiber lasers [1]. The NPE mode-locked fiber lasers can work in several regimes: stable soliton, stretched-pulse, all-normal-dispersion pulses and self-similar pulse by adjusting parameters in the cavity. For stable soliton fiber lasers the energy of a single pulse is limited by the nonlinear phase shift induced by the high peak power. The pulse will break into multiple pulses when the energy rises to 0.1 nJ [2]. Stretched-pulses can reach an energy level which is one order of magnitude larger than that of stable soliton [3]. All-normal dispersion-pulses [4-6] and Self-similar pulses can reach a much higher pulse energy comparing with stretched pulse. Self similar pulse, which have a parabolic shape and a linear frequency chirp, have been proved to be wave-breaking free pulses in the propagation [7] and convenient for efficiency compression. Self-similar pulses evolution in a laser has been studied in theory and experiment [8].

These years, there is the work of analytic theory of self-similar mode-locking [9, 10], while we only focus on the numerical simulation in this paper. A number of theoretical models for simulating the NPE mode-locked fiber lasers have been established. These models are widely used in studying the solitons in fiber lasers. However, all these models have certain weakness. A fourth-order system of ordinary differential equations model simply presents the pulse dynamics after each round trip [11]. Some models solve the pulse propagation in the fiber with nonlinear Schrödinger equation (NLSE) ignoring the birefringence of the fiber [12]. The mode-locking mechanism of NPE modeled by a fast saturable absorber (SA) with a monotonic transmittance function provides few parameters to modulate the pulses [8].

In this paper, we propose a self similar fiber laser model utilizing NPE mode locking to simulate the pulse evolution and polarization states. We get self-similar pulses output when the polarization controllers are adjusted in certain regions. A parameter  $K$  is induced to measure the self-similar pulse evolution [13]. Self-similar pulses have various polarization states across the pulse from peak to wings. These simulation results agree with the former models and provide more pulses' information such as polarization states.

## 2. Model of NPR mode-locked fiber laser

A typical self-similar mode-locked fiber laser based on the NPE technique is shown in Fig. 1.

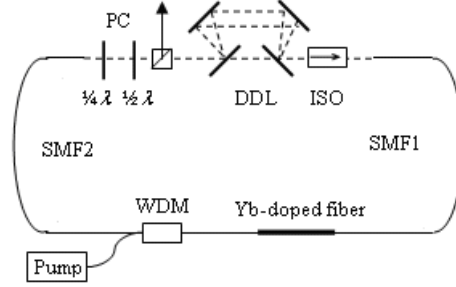


Fig. 1. Schematic diagram of a NPE mode-locked fiber laser

The ring cavity is made of a short section of Yb-doped fiber; two sections of single-mode fiber (SMF1, SMF2), a wavelength division multiplexer (WDM), a polarization controller (PC, consisting of two wave plates), a polarization beam splitter (PBS), an isolator (ISO) and a pair of gratings as the dispersion delay line (DDL). Pulses evolve in the single-mode fiber and are amplified in the Yb-doped fiber; output at the position of PBS. A pairs of gratings as DDL is used to compress the pulse after the PBS in the cavity. The pulse passing through the ISO is coupled into SMF1.

Pulses propagating in the SMF and gain fiber are well described by coupled NLSE [14].

$$\frac{\partial A_x}{\partial z} = -\beta_{1x} \frac{\partial A_x}{\partial t} - \frac{i\beta_2}{2} \frac{\partial^2 A_x}{\partial t^2} + \frac{g}{2} A_x + i\gamma(|A_x|^2 + A|A_y|^2) + iB\gamma A_x^* A_y^2 \exp(-2i\Delta\beta z) \quad (1.a)$$

$$\frac{\partial A_y}{\partial z} = -\beta_{1y} \frac{\partial A_y}{\partial t} - \frac{i\beta_2}{2} \frac{\partial^2 A_y}{\partial t^2} + \frac{g}{2} A_y + i\gamma(|A_y|^2 + A|A_x|^2) + iB\gamma A_y^* A_x^2 \exp(2i\Delta\beta z) \quad (1.b)$$

Where,  $A_x$  and  $A_y$  are the slow-varying envelopes of the electric fields polarized along the two principal axes (denoted as the x- and y-axes, and the x-axis is the slow axis) of the birefringent fiber,  $\gamma$  is the nonlinear coefficient, and  $\beta_2$  is the group velocity dispersion (GVD) parameter, In a silica-based fiber,  $A = 2/3$  and  $B = 1/3$ .  $\Delta\beta = \beta_{0x} - \beta_{0y} = 2\pi / L_B$  is the wave number difference between the two polarization modes,  $\beta_{1x}$  and  $\beta_{1y}$  are the group-velocities of the two polarization modes. For fibers with low birefringence, the two polarization modes with the same central wavelength have equal GVD parameter  $\beta_2$  and nonlinear coefficient  $\gamma$ .

Define  $A_x = u \exp(-i\Delta\beta z / 2)$ ,  $A_y = v \exp(i\Delta\beta z / 2)$ ,  $T = t - \beta_1 z$ . On the assumption that  $\beta_{1x} = \beta_{1y} = \beta_1$ , substitute  $A_x$ ,  $A_y$  and  $t$  with  $u$ ,  $v$  and  $T$ , we obtain coupled GLE to describe the pulses propagation [15].

$$\frac{\partial u}{\partial z} = \frac{i\Delta\beta}{2} u - \frac{i\beta_2}{2} \frac{\partial^2 u}{\partial T^2} + \frac{g}{2} u + i\gamma(|u|^2 + \frac{2}{3}|v|^2)u + \frac{i\gamma}{3} u^* v^2 \quad (2.a)$$

$$\frac{\partial v}{\partial z} = -\frac{i\Delta\beta}{2} v - \frac{i\beta_2}{2} \frac{\partial^2 v}{\partial T^2} + \frac{g}{2} v + i\gamma(|v|^2 + \frac{2}{3}|u|^2)v + \frac{i\gamma}{3} v^* u^2 \quad (2.b)$$

For gain fiber,  $g = g(t) \left( 1 + T_2^2 \frac{\partial^2 A}{\partial t^2} \right)$ ,  $T_2 = 2\pi / (ck^2 \Delta\lambda_g)$ ,  $\Delta\lambda_g$  is gain bandwidth,  $c$  is the

velocity of light and  $k$  is the wave vector. Saturable gain  $g(t) = \frac{g_0}{1 + \int (|A_x|^2 + |A_y|^2) dt / E_s}$ ,  $E_s$

is saturable energy and  $g_0$  is the small signal gain.

After the propagation in optical fibers, pulses pass through the polarization controllers. We consider a simple polarization controller which consists of a  $1/4\lambda$  wave plate and a  $1/2\lambda$  wave plate. The polarization state of pulses can be modulated by choosing the angles of the plates' fast-axes relative to the fiber fast-axis. Finally, pulses passing through the polarization beam splitter are changed into a linear polarization state and finish a round trip in the laser cavity. We define the relative angles of the polarization elements in the cavity (as shown in Fig. 2).

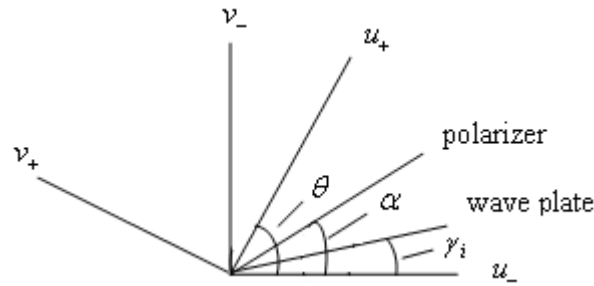


Fig. 2. Angles of polarization elements relative to the fiber fast-axis  $u_-$ .

Here  $u_{\pm}$  and  $v_{\pm}$  represent the alignment of the fiber fast-and slow-axes before (-) and after (+) passage through the polarization elements (polarization controller and PBS).  $\theta$  is the angle of the fiber fast-axis  $u_+$  relative to the fiber fast-axis  $u_-$ .  $\alpha$  is the angle of the polarizer relative to the fiber fast-axis  $u_-$ .  $\gamma_i$  are the angles of the wave plates fast axis relative to the fiber fast-axis  $u_-$ .  $\gamma_1$  is the angle of the  $1/4$  wave plate relative to the fiber fast-axis  $u_-$ ,  $\gamma_2$  is the angle of the  $1/2$  wave plate relative to the fiber fast-axis  $u_-$ . All these angles determine the modulation of pulses by wave plates in mode-locked fiber lasers.

### 3. Simulation and discussion

The propagations of the self-similar pulses in the SMF and Yb-doped fiber are described by coupled GLE as given in Section 2. The DDL compensates the dispersion and causes the most loss in the cavity. The functions of two wave plates and a polarizer are modeled by  $2 \times 2$  transfer matrixes [16]. According to lots of simulation, the laser can generate steady self similar pulses when we carefully choose the parameters in the cavity. The typical parameters used in the model shown in Fig. 1 are given in Table 1.

Table 1. Parameters used in the model of NPE mode-locked fiber laser.

|      | GVD ( $\text{ps}^2/\text{m}$ ) | $\gamma$ ( $\text{W}^{-1}\text{m}^{-1}$ ) | $g_0$ ( $\text{m}^{-1}$ ) | Length (m) | $\Delta\beta$ | $E_s$ (nJ) | $\Delta\lambda_g$ (nm) |
|------|--------------------------------|---|---------------------------|------------|---------------|------------|------------------------|
| SMF1 | 0.025                          | 0.0047                                    | 0                         | 2.3        | $10^{-6}$     |            |                        |
| Gain | 0.025                          | 0.0047                                    | 10                        | 0.3        | $10^{-6}$     | 3          | 45                     |
| SMF2 | 0.025                          | 0.0047                                    | 0                         | 0.2        | $10^{-6}$     |            |                        |
| DDL  | -0.062                         | 0   | -0.5                      | 1          | 0             |            |                        |

To identify the pulse evolution, a parameter  $K$  is induced to measure the evolution of pulses [13].  $K$  is defined as

$$K[\psi(z,t)] = \int_{-\infty}^{+\infty} t^2 |\psi(z,t)|^2 dt \cdot \left( \int_{-\infty}^{+\infty} |\psi(z,t)|^4 dt \right)^2 / \left( \int_{-\infty}^{+\infty} |\psi(z,t)|^2 dt \right)^5. \quad (3)$$

$\psi(z,t)$  is a function of slowly varying pulse envelope. For a parabolic pulse,  $K_0 = 0.0720$ . The closer  $K / K_0$  approaches 1 the better pulses evolve to self-similar pulses.

### 3.1 Modulations of wave plates

With the wave plates posed at various angles, we simulate the NPE mode-locked fiber laser and get self-similar pulses. With the angles fixed at  $\alpha = \pi/6$  and  $\theta = \pi/3$ , we modulate angles  $\gamma_1$  from  $0.35\pi$  to  $0.44\pi$ ,  $\gamma_2$  from  $0.05\pi$  to  $0.14\pi$  in the simulations. The energy of a single pulse and  $K / K_0$  parameter with various angles are shown in Fig. 3.

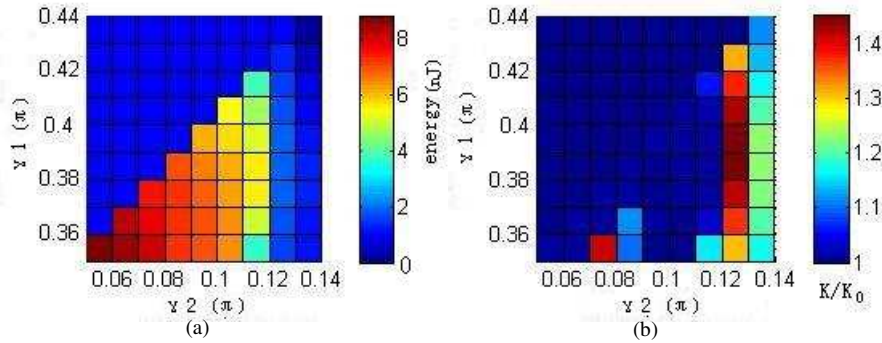


Fig. 3. (a) The energy of a single output pulse at various angles of wave plates. (b) The value  $K / K_0$  of output pulses at the polarizer at various angles of wave plates

In Fig. 3(a), an output pulse in a warm colored area has higher energy than that in a cold colored area. In Fig. 3(b) the pulses in areas marked by cold colors evolves closer to a parabolic pulse than that in areas marked by warm colors. It can be seen that there is a wide range to get a pulse with nearly parabolic shape as well as a high energy, which means the proposed fiber laser can stably output self similar pulse with a wide operation region. At the same time we can vary parameters  $\gamma_1$ ,  $\gamma_2$  to optimize output pulses (pulse shape and energy). The output pulse characters at PBS are determined by wave plates' positions and pulses' coupled evolution in optical fiber. Therefore, in this model the effect of the NPE is too complex to be described by a fast SA with a monotonic transmittance function. Although parameters (wave plates' angles) used in simulation are not exactly the same as those in experiments for the unknown orientations of axes in the birefringent optical fiber, they ensure that the self-similar laser can operate in this condition by adjusting the wave plates.

### 3.2 Intra-cavity pulses evolutions

Taking account into the simulation in 3.1, herein we choose parameters  $\gamma_1 = 0.4\pi$ ,  $\gamma_2 = 0.1\pi$ , the output pulse's characteristic is shown in Fig. 4.

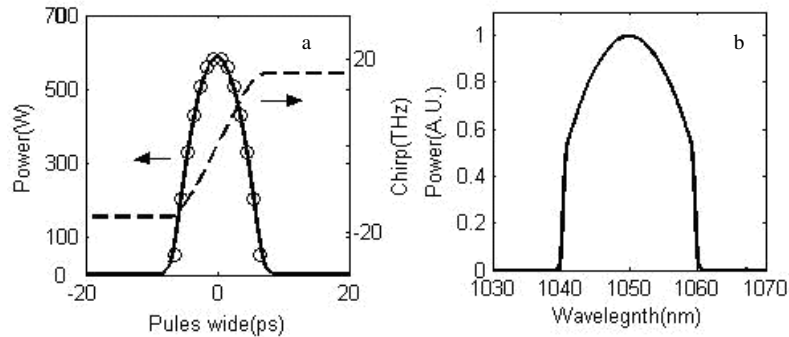


Fig. 4. (a) Temporal intensity profile (solid curve), chirp (dashed curve), and a parabolic fit (dotted curve) to the intensity profile. (b) Output power spectrum

From Figs. 4(a) and 4(b), the output pulse shape is almost parabolic ( $K = 0.0727$ ) with a linear chirp, which are known as characteristics of self-similar pulses [17]. The single pulse energy is 5.4nJ and the pulse is 9.6ps wide. This implies the pulse has a relative high energy and good parabolic pulse shape. Comparing with papers of wise group [8], the results in our simulation is close to the experiments, which indicates that our model is available and reliable. In addition this model can supply some useful information of pulses, especially the polarization states inside the pulses along the cavity, which will guide the experiment.

### 3.3 Polarization states characteristics of self-similar pulses

At the position after polarizer the pulse is linearly polarized. When passing through the SMF and the Yb-doped fiber, the polarization states at different parts of the pulse are no longer uniform. The polarization ellipse is described by two parameters: orientation and ellipticity. We calculate and depict the polarization states of different points across the pulse from the peak to the wings, which is shown in Fig. 5. There is a time interval of one pico-second between the two adjacent points. The outside ellipses represent the polarization states near central parts of the pulse; while the inner ellipses represent the states of the pulse wings. The pulse polarization state is modulated by every element in the cavity. In Figs. 5(a)-5(d) represent the polarization states after the polarizer, before the wave plates, after the 1/4 wave plate and after the 1/2 wave plate, respectively.

This polarization ellipses evolution of self-similar pulses is much different from that of the stable soliton in fiber lasers with negative dispersion. The stable soliton has uniform polarization states from peak to wings [18]. But for self-similar pulse, the pulse width and phase evolve along the cavity [8] and the polarization states at different positions are determined by the pulses evolution. In addition, we find that different parts of the pulses have various polarization ellipses. According to our simulation the peak and wings of a self-similar pulse have different polarization rotations and thus will experience different attenuation by the polarizer. The polarization state across the pulse is an accumulated effect, i.e., the integral property.

From Figs. 5(a)-5(d) it is clear that after the polarizer pulses polarization state evolves from linearity to ellipse. The 1/4 wave plate strongly changes the ellipticity of the ellipse, while the 1/2 wave plate only changes the orientation of the ellipse across the pulse. The energy of pulse passing through the polarizer is determined by the orientation of the polarizer and the polarization states of the pulse. In the self-similar regime the pulse width is at a level of tens of pico-seconds.

In our simulation the nearly circular polarization state helps the pulse to establish a steady mode-locking operation. The simulation agrees with the theoretical analysis by the group of



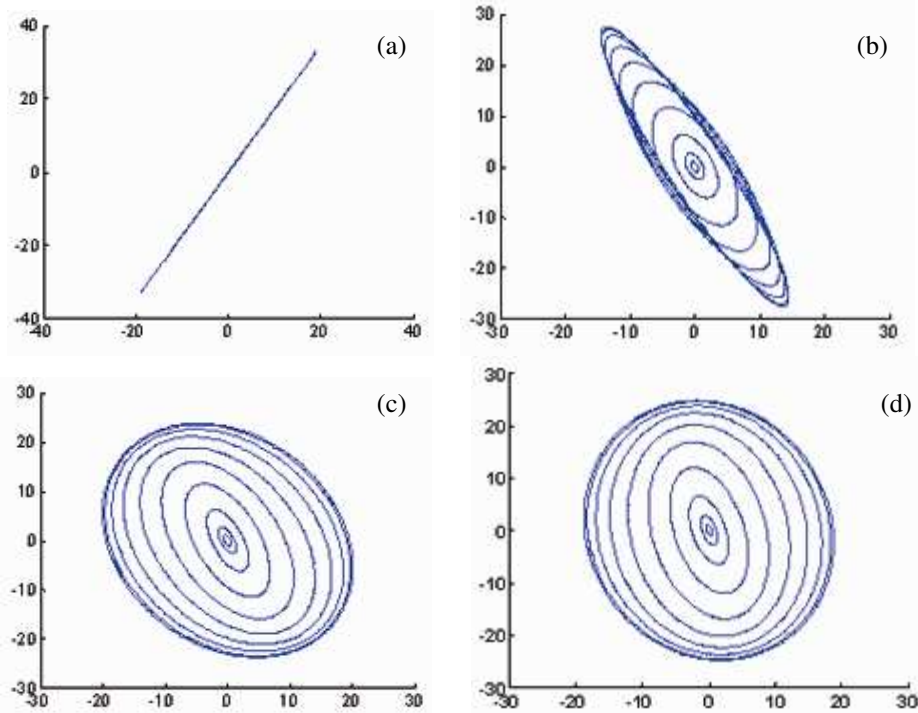


Fig. 5. Polarization states of the pulses at different locations: (a) after the polarizer (b) before the wave plates (c) after the 1/4 wave plate (d) after the 1/2 wave plate

Leblond and Sanchez [19]. This result illuminates the pulses' polarization states evolution directly and provides useful information to understand the mechanism of NPE mode locking.

#### 4. Conclusion

In this paper, a model of NPE fiber laser operating in the regime of self similar pulses is established by solving coupled GLE and using the transfer matrixes of the polarization components. Based on NPE mechanism, we simulate the pulses evolution in optical fiber and the polarization components in mode-locked fiber lasers. Self-similar regime is observed in a wide range by adjusting angles of wave plates. For self-similar pulses, it is found that the polarization states of pulses before polarizer are modulated to be nearly circular, and this feature contributes to the stable operation of the self-similar pulses in the laser.

This model of NPE mode-locked self-similar pulse fiber lasers, dealing with the polarization components as wave plates and polarizer rather than transmittance function, has some advantages comparing with the traditional ones. It allows us to set the parameters such as angles of wave plates to vary the pulse characteristics in simulations, and also provides more information about the polarization states of the pulse, which will contribute to further experimental research and help to study the dynamics of NPE mode-locked fiber lasers thoroughly and deeply.

#### Acknowledgments

This work was supported by the National Natural Science Foundation of China (60677013), the Specialized Research Fund for the Doctoral Program Foundation of Institute of Higher Education of China (20060055021), Chinese National Key Basic Research Special Fund (2006CB921703).