# A general performance model for MAC layer cooperative retransmission contention protocols

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## Abstract

Cooperative retransmission schemes can significantly improve transmission reliability and performance over high loss and time-varying links. However, analytically comparing performance between retransmission strategies is challenging and generally requires simplistic assumptions. This paper presents a general model for the performance of distributed, slot-based contention algorithms for opportunistic decode and forward retransmission algorithms. The model is independent of specific modulation or coding schemes and may be adapted to suit state-based transmission probability models. The model is validated through QualNet simulations.

## Keywords

general, cooperative, mac, model, performance, protocols, contention, retransmission, layer

## Disciplines

Engineering | Science and Technology Studies

## Publication Details

# A General Performance Model for MAC Layer Cooperative Retransmission Contention Protocols

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*Abstract*—**Cooperative retransmission schemes can significantly improve transmission reliability and performance over high loss and time-varying links. However, analytically comparing performance between retransmission strategies is challenging and generally requires simplistic assumptions. This paper presents a general model for the performance of distributed, slot-based contention algorithms for opportunistic decode and forward retransmission algorithms. The model is independent of specific modulation or coding schemes and may be adapted to suit state-based transmission probability models. The model is validated through QualNet simulations.**

*Index Terms*—**cooperative retransmission, IEEE 802.11, MAC, ARQ, DAFMAC, PRO**

## I. INTRODUCTION

Decode-and-forward cooperative relaying is a well-established technique for improving wireless network capacity [1]. Its effectiveness is strongly dependent on the choice of relay selection algorithm, as this determines the probability of retransmission success and also collisions between simultaneous retransmission attempts. Consequently, relay selection algorithm design remains an open area of research.

A direct comparison of the performance of cooperative algorithms is challenging because of the limitations and specificity of existing analytic models. Because of this, many published retransmission algorithms only evaluate cooperative performance against non-cooperative ARQ retransmission schemes. Further, evaluations frequently omit a detailed failure mode analysis or even assume no collisions during cooperative retransmissions [2]. Analytic evaluations typically use very simple and invalid assumptions, such as all relays having the same link quality [3].

There are two fundamentally different approaches to cooperative retransmission - firstly, either the source [4], [5] or the destination [6] may nominate a specific node to act as a relay; or secondly, a contending set of relays can use a distributed algorithm to attempt to select a single relay [3], [7]–[9]. Each approach has merit, but, as yet there is no single analytic method to comprehensively compare the performance of different techniques.

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This paper proposes a general model to analytically estimate the probability of successful retransmission in cooperative relaying schemes, including a detailed failure mode analysis. The model was designed to compare the behaviour of distributed, slot-based contention algorithms, and additionally is able to model all independent, slot-based retransmission schemes, including traditional 802.11 ARQ. The proposed model is shown to accurately predict the probability of success or failure of individual retransmission attempts, providing a quantitative measure of algorithm efficacy. The model is independent of the path-loss model or specific device hardware and only requires Received Signal Strength (RSS) and Packet Delivery Ratio (PDR) measurements which may be obtained from the MAC layer.

Retransmission performance is modelled by calculating the probability of each outcome, namely: retransmission success, or failure caused by data frame corruption, ACK frame corruption, retransmission collision or no relay availability. Distributed retransmission algorithms typically use a slot-based delay scheme to arbitrate between contending nodes. The probability of each outcome is derived as a function of the delay time-out probabilities at the MAC time-slot scale in Section II. Example time-out probabilities are derived for simple ARQ [10], DAFMAC [7] and PRO [3] in Section III. The analytic model is shown to accurately reproduce the results of a QualNet simulation in Section IV. This section also shows a Monte Carlo simulation converges to the analytic result, therefore the analytic result is the mean of the Monte Carlo simulation. Finally, Section V analytically compares the retransmission performance of randomly placed nodes.

## II. COOPERATIVE RETRANSMISSION MODEL

This section proposes a new model for performance analysis of retransmission algorithms that is applicable to any slot-based contention algorithm. Cooperative retransmission algorithms typically employ some extension to the distributed timer method originally proposed in [1] to autonomously select a relay. Nodes use channel state information available at the MAC layer (such as RSS and PDR) to estimate the probability of successful retransmission of a frame to its intended destination; nodes with a high likelihood of success will attempt to transmit first.

### A. Assumptions and Nomenclature

This model does not account for any retransmission algorithm computation complexity or control overhead. It also

assumes nodes external to the cooperative process do not interrupt, which is valid for a lightly loaded network, and will not favour one retransmission scheme over another.

$\mathcal{N}_n$ is the set of neighbour nodes with non-zero PDR to both source and destination. The participating set $\mathcal{N}_p$ defines the neighbouring nodes that have elected to act as relays. Finally, the contending set $\mathcal{N}_c$ is the set of participating relays which receive the source frame. Hence, $\mathcal{N}_c \subseteq \mathcal{N}_p \subseteq \mathcal{N}_n$.

Potential relay node $N_i$ may select a specific delay $t_i$ [3], [7], or it may randomly decide whether or not to contend for a given slot at the beginning of that slot [2]. The proposed model accommodates both strategies.

A relay with a pre-selected timer has three possible states; the timer expires before, during, or after a given time slot. Let the probabilities of being in each state be:

$$p_{ib}(t) \triangleq \Pr\{t_i < t\} \tag{1}$$

$$p_{it}(t) \triangleq \Pr\{t_i = t\} \tag{2}$$

$$p_{ia}(t) \triangleq \Pr\{t_i > t\} \tag{3}$$

respectively, for relay $N_i \in \mathcal{N}_c$ in time slot $t$. Therefore,

$$p_{ib}(t) + p_{it}(t) + p_{ia}(t) = 1. \tag{4}$$

$P_D(a, b)$ denotes the probability of a successful data frame transmission from node $a$ to $b$. It is assumed that frame transmission errors are i.i.d., although more complex transmission models may be used to simulate bursty losses.

Let $P_A(b, a)$ be the ACK transmission success probability from $b$ to $a$. Analysis in [5] suggests using $P_A(b, a) \approx 1$, which may be substituted into the model.

Cooperative relaying schemes frequently use RSS or historical PDR values to evaluate channel quality for potential relays. It is assumed that these values are known to the node, and remain valid during the observation period. Channels and device hardware are also assumed to be approximately reciprocal; $RSS_{a,b} \approx RSS_{b,a}$ and $P_D(b, a) \approx P_D(a, b)$.

The outcome probability of the contending node set during an attempted cooperative transmission is the sum of outcome probabilities during each time slot $t \in [0, T_{max} - 1]$, where $T_{max}$ is specific to each retransmission algorithm. Similarly, the outcome probability of a contending node set is the sum of individual node outcome probabilities.

Define all subsequent random variables as $X \sim \mathrm{U}[0, 1)$, which have the uniform distribution property of:

$$\Pr\{X \leq a \mid a \in [0, 1)\} = a. \tag{5}$$

### B. Probability of Successful Relaying

This section derives an expression for a successful frame retransmission by defining how a node can 'win' the distributed contention phase in one time-slot and then extending to the general result of the entire network for the whole period.

A relay wins contention if it is the only node in the set to transmit during a slot while all other node timers are non-zero. Consider the contending set $\mathcal{N}_c$; node $N_i$ wins contention if all other timers expire after it, hence the probability that $N_i$

wins contention in time slot $t$ is:

$$\Pr\{N_i \ wins, \ t \mid N_i \in \mathcal{N}_c\} = p_{it}(t) \prod_{\substack{N_j \in \mathcal{N}_c, \\ j \neq i}} p_{ja}(t)$$
$$= W_i(\mathcal{N}_c, t) \tag{6}$$

Values for $p_{it}(t)$ and $p_{ia}(t)$ depend on the retransmission algorithm used. Example values are derived in Section III. Extending (6), the probability that node $N_i$ wins contention in any time slot is:

$$\Pr\{N_i \ wins \mid N_i \in \mathcal{N}_c\} = \sum_{t=0}^{T_{max}-1} W_i(\mathcal{N}_c, t) \tag{7}$$

The probability that $N_i$ wins contention, retransmits the data frame to $N_d$ and $N_s$ receives the ACK is therefore:

$$\Pr\{N_i \ succeeds \mid N_i \in \mathcal{N}_c\} = \sum_{t=0}^{T_{max}-1} W_i(\mathcal{N}_c, t) P_D(i, d) P_A(d, s) \tag{8}$$

The probability of any contending node (from the contending set $\mathcal{N}_c$) successfully forwarding the frame is:

$$\Pr\{success \mid \mathcal{N}_c\} = \sum_{N_i \in \mathcal{N}_c} \sum_{t=0}^{T_{max}-1} W_i(\mathcal{N}_c, t) P_D(i, d) P_A(d, s) \tag{9}$$

Let $\mathcal{S}_c$ be the set of all possible combinations of contending relays and $\mathcal{N}_c$ be a specific contending set, such that $\mathcal{N}_c \in \mathcal{S}_c$. The probability of a specific set $\mathcal{N}_c$ having received the source frame and contending is:

$$\Pr\{\mathcal{N}_c\} = \prod_{N_i \in \mathcal{N}_c} P_D(s, i) \prod_{\substack{N_i \in \mathcal{N}_p, \\ N_i \notin \mathcal{N}_c}} (1 - P_D(s, i)) \tag{10}$$

A participating relay may or may not receive the source frame, so the set cardinality is $|\mathcal{S}_c| = 2^{|\mathcal{N}_p|}$. The sum of the probabilities of all contending relay combinations is unity:

$$\sum_{\mathcal{N}_c \in \mathcal{S}_c} \Pr\{\mathcal{N}_c\} = 1. \tag{11}$$

From (9) and (10), the probability that any node from any contending relay set successfully retransmits the frame to the destination and the ACK is successfully received is:

$$\Pr\{success\} =$$
$$\sum_{\mathcal{N}_c \in \mathcal{S}_c} \sum_{N_i \in \mathcal{N}_c} \sum_{t=0}^{T_{max}-1} W_i(\mathcal{N}_c, t) P_D(i, d) P_A(d, s) \Pr\{\mathcal{N}_c\} \tag{12}$$

If $\Pr\{success\} > P_D(s, d)$, then this cooperative retransmission scheme will result in a higher probability of frame delivery compared to the non-cooperative ARQ scheme.

### C. Probability of No Valid Relays

The probability that the contending set is empty (i.e. $|\mathcal{N}_c| = 0$) because no participating relay receives the source

frame is:

$$\Pr\{no\ relays\} = \prod_{N_i \in \mathcal{N}_p} (1 - P_D(s, i)) \quad (13)$$

## D. Probability of Collision

There is no collision in time slot $t$ if any node timer has expired before this slot, no node timers expire in this slot, or if only one node timer expires in this slot. Hence, the probability of collision is:

$$\Pr\{collision\} = 1 - \Pr\{before\} - \Pr\{none\} - \Pr\{one\} \quad (14)$$

where, for a given contending set $\mathcal{N}_c$ and time slot $t$:

$$\begin{aligned}\Pr\{&before,\ t \mid N_i \in \mathcal{N}_c\} \\ &= 1 - (1 - p_{1b}(t))(1 - p_{2b}(t)) \dots (1 - p_{|\mathcal{N}_c|b}(t)) \\ &= 1 - \prod_{i \in \mathcal{N}_c} (1 - p_{ib}(t)) \\ &= 1 - \prod_{i \in \mathcal{N}_c} (p_{it}(t) + p_{ia}(t)) \quad (15)\end{aligned}$$

$$\begin{aligned}\Pr\{none,\ t \mid N_i \in \mathcal{N}_c\} &= p_{1a}(t) p_{2a}(t) \dots p_{|\mathcal{N}_c|a}(t) \\ &= \prod_{N_i \in \mathcal{N}_c} p_{ia}(t) \quad (16)\end{aligned}$$

The probability of one relay winning contention is given by (7). The total collision probability for all contending sets is therefore:

$$\begin{aligned}\Pr\{collision\} = \sum_{\mathcal{N}_c \in \mathcal{S}_c} \sum_{t=0}^{T_{max}-1} &\left( \prod_{N_i \in \mathcal{N}_c} (p_{it}(t) + p_{ia}(t)) \right. \\ &\left. - \prod_{N_i \in \mathcal{N}_c} p_{ia}(t) - \sum_{N_i \in \mathcal{N}_c} W_i(\mathcal{N}_c, t) \right) \Pr\{\mathcal{N}_c\} \quad (17)\end{aligned}$$

## E. Probability of Data Retransmission Failure

The probability of any node winning contention then failing to successfully retransmit the data frame to the destination is:

$$\begin{aligned}\Pr\{D\ fail\} = \\ \sum_{\mathcal{N}_c \in \mathcal{S}_c} \sum_{N_i \in \mathcal{N}_c} \sum_{t=0}^{T_{max}-1} W_i(\mathcal{N}_c, t)(1 - P_D(i, d)) \Pr\{\mathcal{N}_c\}\end{aligned} \quad (18)$$

## F. Probability of ACK Retransmission Failure

The probability that the destination receives the data frame via retransmission, but the source fails to receive the ACK is:

$$\begin{aligned}\Pr\{A\ fail\} = \\ \sum_{\mathcal{N}_c \in \mathcal{S}_c} \sum_{N_i \in \mathcal{N}_c} \sum_{t=0}^{T_{max}-1} W_i(\mathcal{N}_c, t) P_D(i, d)(1 - P_A(d, s)) \Pr\{\mathcal{N}_c\}\end{aligned} \quad (19)$$

## III. EXAMPLE SLOT PROBABILITY DERIVATION

This section derives example slot time-out probabilities for the non-cooperative ARQ system, and the DAFMAC and PRO cooperative relaying schemes. Algorithm-specific values are denoted with the superscripts $A$, $D$ and $P$ respectively.

## A. ARQ Slot Probability Calculation

The basic Automatic Repeat Request (ARQ) algorithm is described in [10]. ARQ is a non-cooperative retransmission strategy where only the source node retransmits; hence $\mathcal{N}_p^A = \{N_s\}$. This is represented in the model by letting $P_D(s, i) = 1$ (i.e. a perfect channel) while $P_D(i, d)$ remains scenario-specific. $T_{max}^A$ is the upper bound of the contention window size from which $t$ is randomly selected:

$$t_i^A \sim \text{U}[0, T_{max}^A - 1] \quad (20)$$

The probability of a time-out in any given slot is:

$$p_{it}^A(t) = \tfrac{1}{T_{max}^A}, \quad \forall t \in [0, T_{max}^A - 1] \quad (21)$$

The bounds of $t$ are enforced using:

$$p_{it}^A(t) = \min\left(\max\left(\tfrac{1}{T_{max}^A}, 0\right), 1\right) \quad (22)$$

The probability of retransmission timer $t_i^A$ expiring before slot $t$ is the sum of slot probabilities less than $t$, such that:

$$p_{ib}^A(t) = \min\left(\max\left(\tfrac{t}{T_{max}^A}, 0\right), 1\right) \quad (23)$$

Using (4), the probability of expiring after slot $t$ is:

$$p_{ia}^A(t) = \min\left(\max\left(\tfrac{T_{max}^A - t - 1}{T_{max}^A}, 0\right), 1\right) \quad (24)$$

A similar method may be applied to retransmission algorithms where a single relay is preselected for cooperation, such as CoopMAC [4] or $\Delta$-MAC [5], with the exception that the relay must first receive the frame in order to contend.

## B. DAFMAC Slot Probability Calculation

Contending DAFMAC nodes estimate their cooperative eligibility using $RSS_{i,d}$. The participating set includes all neighbour nodes, such that $\mathcal{N}_p^D \triangleq \{\mathcal{N}_n\}$. For the contention delay algorithm defined in [7], it is assumed that there is no minimum RSS offset, there is a range of $RSS_{rng} = 16$ dBm between the lowest- and highest RSS values, and the contention period $T_{max}^D = 32$ time slots. The delay algorithm simplifies to:

$$t_i^D = \left\lfloor T_{max}^D - \tfrac{T_{max}^D}{RSS_{rng}} (RSS_{i,d} - RSS_{min} + X_i) \right\rfloor \quad (25)$$

with these assumptions. The timer delay is comprised of link quality and random components. Let the link quality component of delay $t_i$ be:

$$L_i = 32 - 2 (RSS_{i,d} - RSS_{min}). \quad (26)$$

From (2), DAFMAC timer $t_i^D$ expires in slot $t$ with probability:

$$\begin{aligned}p_{it}^D(t) &= \Pr\{\lfloor L_i - 2X_i \rfloor = t\} \\ &= \Pr\{L_i - 2X_i \geq t\} - \Pr\{L_i - 2X_i \geq t + 1\} \\ p_{it}^D(t) &= \Pr\left\{X_i \leq \tfrac{L_i - t}{2}\right\} - \Pr\left\{X_i \leq \tfrac{L_i - t - 1}{2}\right\} \quad (27)\end{aligned}$$

Using (5) and bounding the slot-time gives:

$$\begin{aligned}p_{it}^D(t) = &\min\left(\max\left(\tfrac{L_i - t}{2}, 0\right), 1\right) \\ &- \min\left(\max\left(\tfrac{L_i - t - 1}{2}, 0\right), 1\right)\end{aligned} \quad (28)$$

Similarly, from (3), the probability of $t_i^D$ expiring after $t$ is:

$$
\begin{aligned}
p_{ia}^D(t) &= \Pr\left\{\lfloor L_i - 2X_i \rfloor > t\right\} \\
&= \Pr\left\{L_i - 2X_i \geq t + 1\right\} \\
&= \Pr\left\{X_i \leq \frac{L_i - t - 1}{2}\right\} \\
p_{ia}^D(t) &= \min\left(\max\left(\frac{L_i - t - 1}{2}, 0\right), 1\right) \quad (29)
\end{aligned}
$$

### C. PRO Slot Probability Calculation

The PRO algorithm is described in [3]. PRO ranks neighbour nodes by $RSS_{i,d}$, with $RSS_{s,i}$ used to resolve a tie. Relays are added to the participating set until the cumulative joint retransmission probability reaches a set threshold (taken as 0.95 in this scenario). The participating set, $\mathcal{N}_p^P \subseteq \mathcal{N}_n$, is known to all neighbours from control transmissions. PRO uses a uniform random contention period where more highly ranked relays have a smaller contention window $T_{max}^P$. Let $T_{max}^P(i)$ be the contention window for node $N_i$:

$$
T_{max}^P(i) = 2^{\min\left(\left\lfloor \frac{i+9}{2} \right\rfloor, 10\right)} \quad (30)
$$

where $i$ is the ordered participating node index (from 1 to $|N_p|$). The contention delay for node $N_i$ is:

$$
t_i^P = \left\lfloor T_{max}^P(i) X_i \right\rfloor. \quad (31)
$$

Using (2), the probability of the PRO timer $t_i^P$ expiring in time slot $t$ is:

$$
\begin{aligned}
p_{it}^P(t) &= \Pr\left\{\lfloor T_{max}^P(i) X_i \rfloor = t\right\} \\
&= \Pr\left\{T_{max}^P(i) X_i < t+1\right\} - \Pr\left\{T_{max}^P(i) X_i < t\right\} \\
&= \Pr\left\{X_i < \frac{t+1}{T_{max}^P(i)}\right\} - \Pr\left\{X_i < \frac{t}{T_{max}^P(i)}\right\} \\
p_{it}^P(t) &= \min\left(\max\left(\frac{t+1}{T_{max}^P(i)}, 0\right), 1\right) \\
&\quad - \min\left(\max\left(\frac{t}{T_{max}^P(i)}, 0\right), 1\right)
\end{aligned} \quad (32)
$$

Similarly, from (3), the probability $t_i^P$ expires after $t$ is:

$$
\begin{aligned}
p_{ia}^P(t) &= 1 - \Pr\left\{t_i^P \leq t\right\} \\
&= 1 - \Pr\left\{\lfloor T_{max}^P(i) X_i \rfloor \leq t\right\} \\
&= 1 - \Pr\left\{T_{max}^P(i) X_i < t+1\right\} \\
&= 1 - \Pr\left\{X_i < \frac{t+1}{T_{max}^P(i)}\right\} \\
p_{ia}^P(t) &= 1 - \min\left(\max\left(\frac{t+1}{T_{max}^P(i)}, 0\right), 1\right) \quad (33)
\end{aligned}
$$

## IV. MODEL VALIDATION

This section validates the analytic cooperative retransmission model through QualNet simulations and Monte Carlo analysis. An example scenario is described and is used to demonstrate that the analytic model accurately reproduces an integrated PHY/MAC layer QualNet simulation [11]. The same scenario is then repeated to show a Monte Carlo analysis converges to the model result.

### A. Link Quality Relationship

The analytic retransmission model is specifically designed to be independent of the propagation or physical layer model.
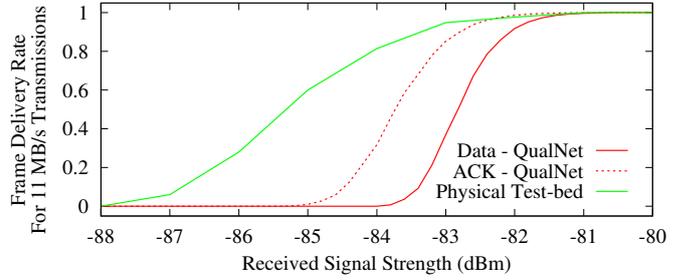


Fig. 1: The RSS to PDR relationship used by QualNet for IEEE 802.11b is more compressed when compared to the result obtained from a physical test-bed in [13].
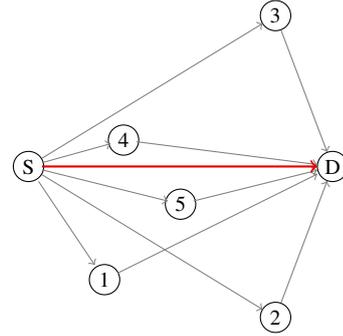


Fig. 2: Nodes source, destination and potential relays

For model verification, the RSS to PDR relationship was obtained from a QualNet simulation.

The simulated hardware parameters were based on the Senao IEEE 802.11b device [12]. Data frames contained a 1400 Byte payload and were transmitted at 11 MB/s. ACK frames were sent at the same transmission rate as data frames to illustrate the potential influence of ACK failures. The simulation used $10^5$ frame transmissions for each of $10^3$ seeds. The transmission reliability of data and ACK frames were measured individually. The resulting RSS to PDR relationships are shown in Figure 1.

### B. Scenario Configuration

The scenario contained between one and five relay candidates located as shown in Figure 2. This configuration is a pathological case for DAFMAC because relays 2, 3 and 5 have identical RSS values which will deliberately induce retransmission collisions. As such, this scenario should not be taken as a reflection of algorithm performance, rather it shows how the analytic retransmission model successfully predicts collisions. The RSS and PDR values for all links are given in Table I.

### C. QualNet Simulation Comparison

The QualNet simulation used between one and five relays to illustrate the changing behaviour as more relays are added to the retransmission process, which improve the probability of a relay being able to retransmit, but also increase the chance of

TABLE I: Scenario link RSS and transmission PDR values

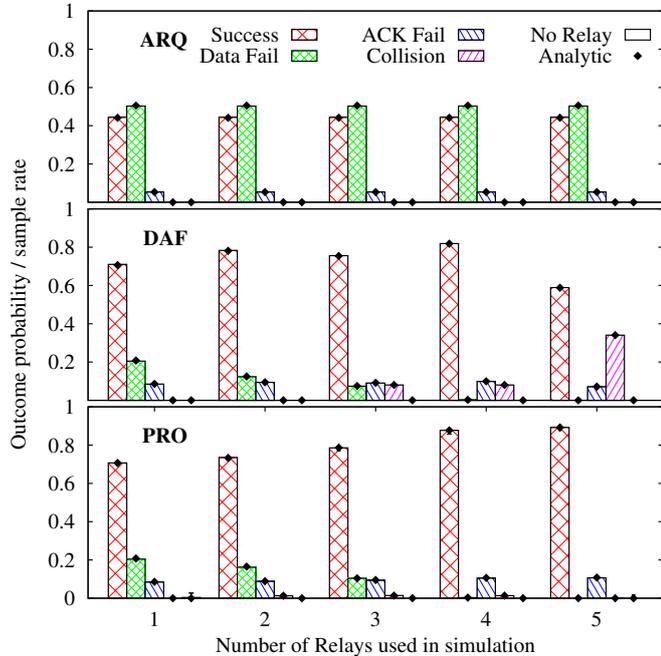| Node | $RSS_{s,i}$ (dBm) | $P_D(s,i)$ | $RSS_{i,d}$ (dBm) | $P_D(i,d)$ |
|------|------|------|------|------|
| $N_s$ | 0 | 1.0 | -83 | 0.5 |
| $N_1$ | -72 | 1.0 | -82 | 0.79 |
| $N_2$ | -83 | 0.40 | -78 | 1.0 |
| $N_3$ | -83 | 0.40 | -78 | 1.0 |
| $N_4$ | -71 | 1.0 | -81 | 0.99 |
| $N_5$ | -73 | 1.0 | -78 | 1.0 |



Fig. 3: The analytic model generates similar retransmission attempt outcome probabilities as a QualNet simulation, e.g. Pr{success}, Pr{collision}, etc.

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**Algorithm 1:** Simulation of the cooperative retransmission

**Input**: Neighbour relay set $\mathcal{N}_n$
**Input**: Relay RSS values $RSS(s,i)$ and $RSS(i,d)$
**Input**: Relay PDR values $P_D(s,i)$ and $P_D(i,d)$
**Input**: Destination to source ACK success probability $P_A(d,s)$
**Output**: The sample rate of contention outcomes

1  Determine participating set $\mathcal{N}_p$
2  $t_{min} \leftarrow T_{max}$
3  **for** $m$ **in** sample size $M$ **do**
4       $num_{best} \leftarrow 0$
5       **for** node $N_i$ **in** set $\mathcal{N}_p$ **do**
6           $t_i \leftarrow T_{max}$
7           **if** $X < P_D(i,s)$ **then**
8               Calculate $t_i$
9               **if** $t_i < t_{min}$ **then**
10                  $t_{min} \leftarrow t_i$
11                  $num_{best} \leftarrow 1$
12                  $r \leftarrow i$
13              **if** $t_i == t_{min}$ **then**
14                  $num_{best} \leftarrow num_{best} + 1$
15      **if** $t_{min} == T_{max}$ **then**
16          $x_{none} \leftarrow x_{none} + 1$
17      **else**
18          **if** $num_{best} > 1$ **then**
19              $x_{coll} \leftarrow x_{coll} + 1$
20          **else**
21              **if** $X < P_D(r,d)$ **then**
22                  **if** $X < P_A(d,s)$ **then**
23                      $x_{success} \leftarrow x_{success} + 1$
24                  **else**
25                      $x_{A\,fail} \leftarrow x_{A\,fail} + 1$
26              **else**
27                  $x_{D\,fail} \leftarrow x_{D\,fail} + 1$

---

collisions. Relays are always added to the system sequentially, so the set of three participating relays is $\mathcal{N}_3 = \{N_1, N_2, N_3\}$. Each relay set configuration was simulated using 1000 random seeds to provide a statistically meaningful result.

The analytic retransmission model produces results nearly identical to the QualNet simulation, as shown in Figure 3. The median QualNet result is presented as a histogram. The error bars represent the central 90% and is typically limited to within of 1% of the median. The diamond shaped point represents the corresponding analytically generated result.

*D. Monte Carlo Simulation Convergence*

This section shows that the Monte Carlo simulation converges to the predictions of the retransmission model.

The Monte Carlo simulation used the aforementioned network configuration and node parameters with all five possible relays. Frame transmission success was determined using a random variable $X \sim U[0,1)$, where a transmission is deemed successful if $X < P(a,b)$ and a failure otherwise. The full Monte Carlo algorithm is shown in Algorithm 1.

The simulation used $M \in [10^4, 10^{10}]$ samples to calculate the retransmission success rate. The variation between the analytic and simulation result for $M$ samples is:

$$\varepsilon_M(k) = \left| \Pr\{outcome\,k\} - \frac{1}{M}\sum_{m=1}^{M} x_k \right| \qquad (34)$$

where $x_k = 1$ if the retransmission outcome is $k$ and $x_k = 0$ otherwise. Figure 4 shows the simulation converges to the analytic result as $M \rightarrow \infty$ for each of the ARQ, DAFMAC and PRO models. Therefore, the proposed analytic retransmission model is the true mean result.

The Monte Carlo simulation was tested for convergence to the analytic result using a range of link parameters and node layout scenarios. However, the results presented here are limited to one input set by the available space.

V. EXAMPLE RETRANSMISSION PROTOCOL COMPARISON

This section presents a brief analytic comparison between the ARQ, DAFMAC and PRO retransmission algorithms in a more general scenario, obtained using the proposed model.
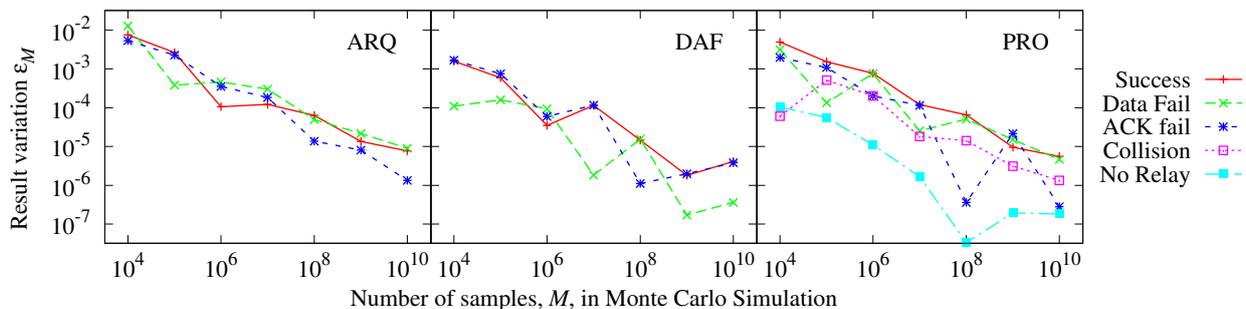
Fig. 4: The Monte Carlo simulation converges to the analytic result as the sample size increases.
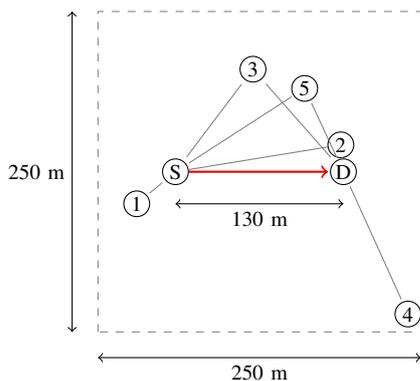


Fig. 5: Example scenario with fixed source and destination nodes with randomly placed neighbours
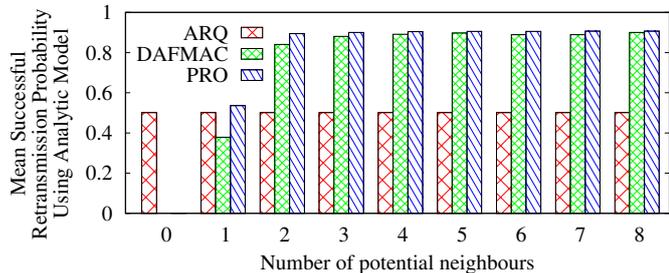


Fig. 6: The retransmission success of both DAFMAC and PRO is greater than ARQ with two or more neighbours in the area

Source and destination nodes were placed 130 m apart in the centre of a $250 \times 250$ m area, as shown in Figure 5. Using the aforementioned device parameters, the direct transmission probability was $P_D(s,d) \approx 0.5$. Between zero and eight neighbour nodes were randomly placed in the area and the retransmission probability calculated for ARQ, DAFMAC and PRO for 1000 node sets at each density.

The mean retransmission success probability from the random placements is shown in Figure 6. This scenario has a significant variation in performance from the random placements and the mean provides the best measure. Both DAFMAC and PRO significantly outperform ARQ when two or more neighbours are in the area, and the performance is bounded by the rate of ACK transmission failures. DAFMAC has a higher

collision rate than PRO and slightly trails in retransmission performance in this scenario.

## VI. CONCLUSION

This paper has proposed a general retransmission performance model. The proposed model is suitable for evaluating any independent, distributed, slot-based contention algorithm for MAC layer opportunistic retransmission. The slot time-out probabilities were derived for ARQ, DAFMAC and PRO retransmission schemes. The analytic model is validated by a direct comparison to a QualNet simulation and by showing the Monte Carlo simulation converges to the analytic result as the sample size increases. The proposed model is then used to perform a comparison between three retransmission strategies.

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