Analysis of contact mechanics in micro flexible rolling

Mingshuai Huo
University of Wollongong, mh317@uowmail.edu.au

Jingwei Zhao
University of Wollongong, jzhao@uow.edu.au

Haibo Xie
University of Wollongong, xie@uow.edu.au

Zhengyi Jiang
University of Wollongong, jiang@uow.edu.au

Publication Details

Research Online is the open access institutional repository for the University of Wollongong. For further information contact the UOW Library: research-pubs@uow.edu.au
Analysis of contact mechanics in micro flexible rolling

Abstract
Micro flexible rolling is a new microforming method by online controlling and adjusting the roll gap to make various strip thickness in the submillimeter range. The micro flexibly rolled strips can be divided into three zones of the thicker zone, the thinner zone and the transition zone after experiencing the upward and downward rolling processes. However, it is tough to achieve the final target thickness especially in the transition zone due to a number of issues relating to the contact mechanics such as the change in the central neutral point/zone, the touch at the edges of the work rolls, the elastic deformation of the work rolls, the roll bite arc modifications in real time and the tribological conditions. All of these factors have significant influences on flatness, profile and surface finish of the rolled products, as well as the rolling forces. In the current work, a new model has been developed in order to clarify the micro flexible rolling process. This model considers a non-circular contact arc which includes an elastic loading region at the start of the roll gap, a plastic reduction region with backward slip, a central flattened region without slip, the plastic reduction region with forward slip, and an elastic unloading region at the end of the roll gap. In this study, the effect of speed ratio (the ratio of the lifting speed and the rolling speed) on the dimensions and the rolling forces along the transition zones is investigated. The contact mechanics in the micro flexible rolling are systematically analysed. The simulation results are found to be in line with the experimental ones, which means that the developed model with high precision is suitable for the analysis of the micro flexible rolling process.

Publication Details

This journal article is available at Research Online: https://ro.uow.edu.au/eispapers1/2761
Analysis of contact mechanics in micro flexible rolling

Mingshuai Huo\textsuperscript{a,b}, Jingwei Zhao\textsuperscript{a}, Haibo Xie\textsuperscript{a}, Zhengyi Jiang\textsuperscript{a,b,*}

\textsuperscript{a}School of Mechanical, Materials, Mechatronic and Biomedical Engineering, University of Wollongong, NSW 2522, Australia
\textsuperscript{b}School of Materials and Metallurgy, University of Science and Technology Liaoning, Liaoning 114051, China

Abstract

Micro flexible rolling is a new microforming method by online controlling and adjusting the roll gap to make various strip thickness in the submillimeter range. The micro flexibly rolled strips can be divided into three zones of the thicker zone, the thinner zone and the transition zone after experiencing the upward and downward rolling processes. However, it is tough to achieve the final target thickness especially in the transition zone due to a number of issues relating to the contact mechanics such as the change in the central neutral point/zone, the touch at the edges of the work rolls, the elastic deformation of the work rolls, the roll bite arc modifications in real time and the tribological conditions. All of these factors have significant influences on flatness, profile and surface finish of the rolled products, as well as the rolling forces. In the current work, a new model has been developed in order to clarify the micro flexible rolling process. This model considers a non-circular contact arc which includes an elastic loading region at the start of the roll gap, a plastic reduction region with backward slip, a central flattened region without slip, the plastic reduction region with forward slip, and an elastic unloading region at the end of the roll gap. In this study, the effect of speed ratio (the ratio of the lifting speed and the rolling speed) on the dimensions and the rolling forces along the transition zones is investigated. The contact mechanics in the micro flexible rolling are systematically analysed. The simulation results are found to be in line with the experimental ones, which means that the developed model with high precision is suitable for the analysis of the micro flexible rolling process.

© 2018 The Authors. Published by Elsevier B.V.
Peer-review under responsibility of the scientific committee of the 17th International Conference on Metal Forming.

Keywords: Micro flexible rolling; Contact mechanics; Slab method modelling

* Corresponding author. Tel.: +61-2-4221-4545; fax: +61-2-4221-4545.
E-mail address: jiang@uow.edu.au
1. Introduction

Due to a large requirements of micro-electromechanical systems and micro-electromechanical systems, the research in materials with the range of submillimeter is getting increasing attracts in recent years [1, 2]. Micro flexible rolling is a novel microforming method by which micro-parts with periodic varying thickness in submillimetre range along the rolling direction can be fabricated through online controlling and adjusting of the roll gap. Conventional flexible rolling is commonly used to fabricate lightweight products [3] for applications in the automobile [4] or shipment field. Liu et al. [5] summarised the recent theoretical and practical development of conventional flexible rolling in processing, mechanics and products. A model was developed to analyse this process by considering the combined edge and flat rolling [6]. However, with the trend of minimisation, the theory and mechanics for the traditional flexible rolling cannot be directly applied in the micro flexible rolling process due to the size effects, which are well known as a unique characteristic with miniaturisation [7, 8]. Zhao et al. [9] analysed the size effects appeared in micro rolling processes and discussed the factors affecting materials deformation behaviour and final product quality. Qu et al. [10] investigated the springback phenomenon, which is caused by the size effects, in micro flexible rolling using 3D Voronoi tessellation technique. In addition, to clarify the contact mechanics in thin strip rolling, Jonhson et al. [11] studied the elastic stress in thin strip rolling process and proposed the onset yield of plastic occurs. Lu et al. [12] developed a model to simulate the elastic deformation length of the inlet zone in thin strip rolling process. Jiang et al. [13] developed a new algorithm to analyse the friction at the edge of roll bite condition which influences the work roll edge in a great content. Meanwhile, a rigid-plastic FEM method [14] which considered the influence of friction variation on the spread, forward slip and rolling pressure was studied.

Conventional theories for flexible rolling are based on a circular roll shape, which elide the elastic deformation of rolls to make the corresponding assumption. Up to now, however, there is barely literature reporting the contact mechanics about the micro flexible rolling especially in the transition zone which plays a key role in the deformation process. In the current work, theoretical analysis of micro flexible rolling is investigated and the deformation mechanics in the transition zone is discussed with consideration of the speed ratio (the ratio of the lifting speed and the rolling speed) and the elastic behaviour of the micro flexibly roll bite. The rolling force calculated by the developed model is compared with the experimental results.
2. Theory

Fig. 1 shows a schematic diagram of flexible rolling processes. Two kinds of rolling process, including downward rolling and upward rolling, classify the flexibly rolled strips into three various thickness zones which consists of the thicker zone, the thinner zone and the transition zone. It differs from conventional rolling that contact arc along the transition zone changes continuously, which is reflected not only in the dimensions but also in the contact directions.

Fig. 1. Schematic illustration of micro flexible rolling process.

2.1. Equilibrium of slab element

Slab method is chosen to analyse the contact mechanics in the transition zone of micro flexibly rolled strips. For this analysis, the plane-strain condition is assumed without front and back tensions. In addition, curling and peening effect are also ignored. Constant Coulomb’s friction behaviour is applied. With an infinitesimal time of $\Delta T$ the slab undergoes a small change at the slab plane with a slight increment of $\Delta j$, as shown in Fig. 2.

Fig. 2. Stress analysis of upward rolling process.

According to the stress analysis of the slab in the upward rolling process in Fig. 2, the following relationships can be obtained based on the element selected from the transition zone:

$$\tan \psi' = \frac{dj - \Delta j}{di} = \frac{dj - V_l \cdot dl}{di} = \frac{dj - V_l \cdot di}{di \cdot V_l} = \frac{dj - V_l}{V_l},$$

(1)
\[
\tan \psi' = \frac{dj}{di} - \gamma_1 \frac{V_j}{V_i}, \tag{2}
\]

where \(V_j\) is the vertical roll lifting speed and \(V_i\) is the rolling speed of the specimen at point \(i\). The value of \(\gamma_1\) is 1 in the upward rolling and -1 in the downward rolling.

Then the force balance equilibrium on this slab in the horizontal direction for the upward rolling process can be obtained as:

\[
2(\sigma + d\sigma)(j + dj) - 2\sigma \cdot (j + \Delta j) + \frac{2\gamma_2 q \cdot \cos \psi' di}{\cos \psi'} - \frac{2p \cdot \sin \psi' di}{\cos \psi'} = 0. \tag{3}
\]

Applying Eq. (2) into Eq. (3), the final equilibrium relationship can be achieved as follows:

\[
l \frac{d\sigma}{di} + (\sigma + p) \left(\frac{dl}{di} + \gamma_1 \frac{V_j}{V_i}\right) - 2\gamma_2 q = 0, \tag{4}
\]

where \(\gamma_2 q\) is the shear stress which is assumed by the well-known Coulomb’s model \(\gamma_2 \mu p\). Positive sign of \(\gamma_2\) is used at backward slip and negative sign is for the forward slip.

### 2.2. Roll shape

Fig. 3 shows the variation between the circular and non-circular rolls.

![Fig. 3. Comparison between circular and non-circular roll contact arcs.](image)

Due to the non-negligible effect from elastic deformation, the elastic half-space theory proposed and discussed by Johnson [15] is adopted in this study. The strip thickness at an arbitrary position for an elastic deformed roll is given by the following equation:

\[
l = l_o - \frac{i_a^2 - i^2}{R} + 2\delta(i), \tag{5}
\]

where \(i_a\) is the starting point of the contact arc at the entry of the roll bite, \(l_o - (i_a^2 - i^2)/R\) is corresponding to the thickness evolution along the roll bit arc, and \(\delta(i)\) is the vertical elastic deflection at arbitrary position \(i\) under the applied pressure distribution \(p\), which is given by:

\[
\delta(i) = -\frac{2(1 - v^2)}{\pi E_R} \int_{-i_a}^{i_a} p(t) \ln \left| \frac{t + i_a}{t - i} \right| dt, \tag{6}
\]

where \(t\) is a dummy variable of pressure distribution \(p\).

### 2.3. Elastic slip at entry and exit

The elastic strain of the strip in the vertical direction at the entry and exit is given by:
\[ \varepsilon_{s} = -\frac{(1 - v_{R}^{2})}{E_{s}}(p + \frac{v_{s}}{1 - v_{s}}\sigma), \]  

where \( p \) is the pressure, \( \sigma \) is the stress, and \( E_{s} \) is the elastic modulus.

After differentiation with respect to \( i \), the following expression can be obtained:

\[ \frac{dp}{di} = -\frac{v_{s}}{1 - v_{s}}\frac{d\sigma}{di} - \frac{E_{s}}{t}\frac{dl}{di}. \]  

Combining the equilibrium Eq. (4), then:

\[ \frac{dp}{di} = -\frac{E_{s}}{t(1 - v_{R}^{2})}\frac{dl}{di} - \frac{v_{s}}{1 - v_{s}}\frac{2\gamma_{2}q}{l} + \frac{v_{s}}{1 - v_{s}}(p + \sigma)(\frac{dl}{di} + \gamma_{1}\frac{v_{j}}{v_{i}}). \]  

2.4. Plastic slip

Tresca yield criterion is applied to the plastic deformation according to the vertical equilibrium acting on the slab of the strip.

\[ p - \sigma = Y_{s}, \]  

where \( Y_{s} \) is the uniaxial yield stress of the strip. Differentiate this expression and substitute it into Eq. (4) gives:

\[ \frac{dp}{di} = -(2p - Y_{s})\frac{dl}{di} + \frac{2\gamma_{2}q}{l} + 2\gamma_{2}q\frac{v_{j}}{v_{i}}. \]  

2.5. Plastic sticking region

For the plastic sticking region, Le et al. [16] proposed a model to simulate the deformation characteristic. Based on this study, the relationship between the roll shape and the shear stress is derived considering the variation of roll and strip strains in combination with the flow continuity of the strip. The detailed pressure gradient is given by:

\[ \frac{dp}{di} = -\frac{C_{1}E_{s}}{l(1 - v_{R}^{2})}\frac{dl}{di}, \]  

where \( C_{1} = \left(\frac{2 - 4v_{s}}{(1 - v_{s})} - \frac{E_{s}(1 - 2v_{s})}{E_{R}(1 - v_{R})}\right)^{-1} \). The shear stress \( q \) is given by:

\[ q = -\frac{C_{1}E_{s}}{2(1 - v_{R}^{2})}\frac{dl}{di}. \]  

2.6. Rolling force

After integrations of the separate pressure distribution from each zone mentioned before, the rolling force is calculated with the following expression:

\[ F = w\int_{i_{a}}^{i_{d}} pdi. \]  

2.7. Numerical implementation

The programming by MATLAB has been used for solving each deformation zone. A proper circular roll arc is initially assumed for the starting calculation of the pressure distribution in the first round. In this case, the entry point \( i_{a} \) can be determined by Eq. (5) with the given parameters. Then the neutral point \( i_{n} \) must be estimated at the beginning but it will be updated by using a relaxation method (Eq. (18)) in order to ensure the accuracy of the following iteration.
and improve the calculation speed. The initial boundary conditions are with \( p = 0 \) and \( \sigma = 0 \), and the interface of the roll-strip is divided into 5000 equal intervals. After simultaneously integrating the differential Eqs. (9) and (10), fourth order Runge-Kutta scheme is taken for the computation of the inlet elastic zone. When the Tresca yield criterion is satisfied, the calculation of the inlet elastic zone is stopped at \( i = i_b \). Meanwhile, the plastic sticking mechanics begins, which is calculated by combining Eqs. (10) and (14). Once the magnitude of the interfacial shear stress exceeds the Coulomb’s value \( |\mu|p| \), the plastic slip begins and the pressure distribution is obtained by Eq. (12). At each point, the finishing boundary condition in this zone is checked with \( \frac{dl}{di} \leq \frac{v_i}{v_j} \). For the exit elastic zone, the pressure gradient is calculated by Eqs. (9) and (10) again with a boundary condition with \( \sigma = 0 \).

\[
\begin{align*}
  p_{\text{new}} &= \varepsilon \cdot p_{\text{calculated}} + (1 - \varepsilon) \cdot p_{\text{previous}}. 
\end{align*}
\]  

(15)

If the exit stress cannot satisfy the boundary condition, the neutral point needs to be adjusted by using the dichotomy method. After this, the pressure distribution with a given roll shape is obtained and used for the following iteration with a relaxation method mentioned before. Then the roll shape is updated after the integration of the elastic deflection at each point. However, the final thickness at the exit always cannot obtain the real value. Thus, the roll shape is then updated with the same relaxation factor and the starting point of roll arc is shifted based on the difference in the exit thickness. The program should repeat the whole procedure until the convergence of the final exit thickness. Following this, the unit widths of roll load and roll torque can be calculated.

3. Experimental procedure

In order to verify the calculated results, the various micro flexible rolling tests were conducted with the same rolling parameters using a two-high micro flexible rolling mill (as shown in Fig. 4) in laboratory. Pure aluminium strip with a thickness of 108 µm was used as the initial material and then subjected to micro flexible rolling with three kinds of speed ratios (the ratio of the lifting speed and the rolling speed). 9Cr2Mo steel rolls with the radius and work roll’s barrel length of 12.5 mm and 40 mm, respectively, were used. The elastic modulus of the rolls is 236 GPa with a Poisson ratio of 0.3.

4. Results and discussion

The most special part of micro flexible rolling is the transition zone. Comparing with the constant roll arc length in the conventional flat rolling, the transition roll arc length increases during the downward rolling process, while it decreases gradually during the upward rolling process since the lifting speed in the upward rolling process makes the rolls predict different roll arc angles. Fig. 5 presents the results of rolling force and the profile of the transition zone obtained by both calculation and experiments under the same rolling parameters. It can be seen that the experimental results match the calculated ones very well. The rolling force is found to show an increasing trend with the increase
of rolling reduction along the transition zones due to different degrees of deformation rate. The slight difference between them should be caused by the real contact conditions, such as the work hardening evolution and the surface roughness transmission, which lead to the change of the friction coefficient, then affect the rolling force.

Fig. 5. Comparison of (a) rolling forces between experimental and calculated results and (b) profile of transition zone.

Another key factor affecting the micro flexible rolling process is the speed ratio since the variation in lifting speed will not only significantly change the force balance of each slab, but also make a difference in the roll arc length and distribution to a great extent as shown the shaded areas in Fig. 6. Accordingly, raw materials were flexibly rolled with four different kinds of speed ratios with 0.02, 0.04, 0.06 and 0.08. The results show that the flexibly rolled strips with the lowest speed ratio obtain the biggest roll arc length since the lifting speed of the roll eventually determines its position. In addition, it also influences the peak value of the pressure and the neutral point site. Due to the short roll arc from the 0.08 specimen, the lowest peak pressure value is therefore obtained.

Fig. 6. Comparison of rolling pressure gradient among four kinds of speed ratios: (a) 0.02, (b) 0.04, (c) 0.06 and (d) 0.08.
5. Conclusions

A calculation model is developed for micro flexible rolling of thin strips considering the contact mechanics in the interface. The model solves the pressure gradient in the deformed roll arc by dividing it into the elastic loading region at the entry, a plastic reduction region with backward slip, a central flattened region without slip, the other plastic reduction region with forward slip, and an elastic unloading region in the end of roll bite. The calculated rolling forces obtained in the transition zones are in good agreement with the experimental results. That means that the model is suitable for the analysis of micro flexible rolling process. In addition, the results with different speed ratio indicate that a higher speed ratio in the upward rolling process leads to a change in decreasing the roll arc length and a neutral point movement.

Acknowledgements

The authors kindly thank the grant from Australian Research Council (ARC, Grant No. FT120100432) and National Natural Science Foundation of China under Grant No. 51474127 for the financial support to the current research and the PhD scholarship from UOW.

References