2000

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Keywords

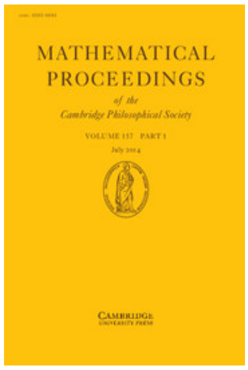
algebras, equivariance, c, imprimitivity, theorem, coactions, symmetric, induced

Disciplines

Engineering | Science and Technology Studies

Publication Details

Echterhoff, S. & Raeburn, I. F. (2000). Induced C*-algebras, coactions and equivariance in the symmetric imprimitivity theorem. Mathematical Proceedings of the Cambridge Philosophical Society, 128 (2), 327-342.



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Mathematical Proceedings of the Cambridge Philosophical Society / Volume 128 / Issue 02 / March 2000, pp 327 - 342
DOI: null, Published online: 08 September 2000

Math. Proc. Camb. Phil. Soc. (2000), **128**, 327
Printed in the United Kingdom

Induced C^*-algebras, coactions and equivariance in the symmetric imprimitivity theorem†

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(*Received* 30 *September* 1998)

Abstract

The symmetric imprimitivity theorem provides a Morita equivalence between two crossed products of induced C^*-algebras and includes as special cases many other important Morita equivalences such as Green's imprimitivity theorem. We show that the symmetric imprimitivity theorem is compatible with various inflated actions and coactions on the crossed products.

Introduction

Crossed products of C^*-algebras carry a variety of actions and coactions of locally compact groups. In several recent projects we have had to know that imprimitivity theorems and other Morita equivalences are equivariant, in the sense that the bimodules implementing the equivalences between crossed products carry actions or coactions compatible with those on the crossed products (see, for example, [**E1**, **ER1**, **ER2**, **KQR**]). Since constructing coactions on bimodules is technically very complicated, it is reasonable to ask if there is a general principle at work; in particular, does the symmetric imprimitivity theorem of [**R1**], which is the most general of the commonly used equivalences, have an equivariant version? Here we prove such a theorem for the most important case in which two subgroups act on opposite sides of a locally compact group.

To be more precise, we need to recall the set-up of [**R1**]. Suppose that K and H are closed subgroups of a locally compact group G; we think of K as acting by left multiplication on G and H as acting by right multiplication. Suppose also that $\alpha\colon K \to \operatorname{Aut} D$, $\beta\colon H \to \operatorname{Aut} D$ are commuting actions on a C^*-algebra D. The *induced C^*-algebra* $\operatorname{Ind}_H^G(D,\beta)$ is the C^*-subalgebra of $C_b(G,D)$ consisting of

† This research was supported by the Australian Research Council and Deutsche Forschungsgemeinschaft (SFB 478).

the functions which satisfy $g(sh) = \beta_{h^{-1}}(g(s))$ for $h \in H$, and for which the function $sH \mapsto \|g(s)\|$ vanishes at infinity on G/H. Similarly, $\mathrm{Ind}_K^G(D,\alpha)$ consists of functions $f \in C_b(G,D)$ such that $f(ks) = \alpha_k(f(s))$ for $k \in K$ and $Ks \mapsto \|f(s)\|$ vanishes at infinity on $K\backslash G$.

Let τ denote the action of K by left translation on $C_0(G)$ and σ the action of H by right translation. The diagonal actions $\tau\otimes\alpha$ and $\sigma\otimes\beta$ on $C_b(G,D) \subset M(C_0(G)\otimes D)$ restrict to actions on $\mathrm{Ind}_H^G(D,\beta)$ and $\mathrm{Ind}_K^G(D,\alpha)$, respectively, which satisfy

$$((\tau\otimes\alpha)_k(g))(s) = \alpha_k(g(k^{-1}s)) \quad \text{and} \quad ((\sigma\otimes\beta)_h(f))(s) = \beta_h(f(sh)). \qquad (0\cdot 1)$$

The symmetric imprimitivity theorem [**R1**, theorem 1·1] describes a Morita equivalence between the crossed products $\mathrm{Ind}_H^G(D,\beta) \rtimes_{\tau\otimes\alpha} K$ and $\mathrm{Ind}_K^G(D,\alpha) \rtimes_{\sigma\otimes\beta} H$.

If G is abelian, the dual actions $(\tau\otimes\alpha)^\wedge$ and $(\sigma\otimes\beta)^\wedge$ can be inflated to actions $\mathrm{Inf}\,(\tau\otimes\alpha)^\wedge$ and $\mathrm{Inf}\,(\sigma\otimes\beta)^\wedge$ of $\widehat{G}$ by composing with the restriction maps $\chi \mapsto \chi|_K$ and $\chi \mapsto \chi|_H$. Our first theorem says that the Morita equivalence of [**R1**] respects these actions of $\widehat{G}$:

Theorem 1. *Let α: $K \to \mathrm{Aut}\, D$ and β: $H \to \mathrm{Aut}\, D$ be commuting actions of closed subgroups K and H of a locally compact abelian group G. Then the inflated systems* $(\mathrm{Ind}_H^G(D,\beta) \rtimes_{\tau\otimes\alpha} K,\ \mathrm{Inf}\,(\tau\otimes\alpha)^\wedge)$ *and* $(\mathrm{Ind}_K^G(D,\alpha) \rtimes_{\sigma\otimes\beta} H,\ \mathrm{Inf}\,(\sigma\otimes\beta)^\wedge)$ *are Morita equivalent.*

To prove this, we construct a 'dual action' of $\widehat{G}$ on the imprimitivity bimodule X of [**R1**, theorem 1·1]. This will be easy to do once we have the formulas in front of us (see the beginning of Section 1). Things become much more complicated if G is not abelian, because we have to replace the dual actions by dual coactions. To state our main result we need to recall a few definitions.

A coaction of a locally compact group G on a C^*-algebra A is an injective and nondegenerate homomorphism δ: $A \to M(A\otimes C_r^*(G))$ satisfying

$$(\delta\otimes \mathrm{id}_G)\circ\delta = (\mathrm{id}_A \otimes \delta_G)\circ\delta \quad \text{and} \quad \delta(A)(1\otimes C_r^*(G)) \subset A\otimes C_r^*(G),$$

where δ_G is the usual comultiplication on $C_r^*(G)$. (In our use of reduced group algebras and spatial tensor products we follow the conventions of [**LPRS**] and [**PR**].) If α: $G \to \mathrm{Aut}\, A$ is an action, we denote the canonical embeddings in the crossed product by i_A: $A \to M(A\rtimes_\alpha G)$ and i_G: $G \to UM(A\rtimes_\alpha G)$; we also write u_G: $G \to UM(C^*(G))$ for the canonical embedding of G in its group algebra. Then $(i_A\otimes 1, i_G\otimes u_G)$ is a covariant homomorphism of (A,G,α) into $M\big((A\rtimes_\alpha G)\otimes C^*(G)\big)$. Composing the integrated form $(i_A\otimes 1)\rtimes(i_G\otimes u_G)$ with the regular representations of $A\rtimes_\alpha G$ and $C^*(G)$ gives a homomorphism which factors through a coaction

$$\widehat{\alpha}\colon A\rtimes_{\alpha,r} G \to M((A\rtimes_{\alpha,r} G)\otimes C_r^*(G)),$$

called the *dual coaction* of G on $A\rtimes_{\alpha,r} G$ (see [**PR**]).

If H is a closed subgroup of G, a theorem of Herz implies that the integrated forms of $\lambda^G|_H$ and λ^H have the same kernel in $C^*(H)$ ([**H**], see also [**LR**, lemma 3·1]). Thus $\lambda^G|_H$ factors through an injective and nondegenerate homomorphism C_H: $C_r^*(H) \to M(C_r^*(G))$. The arguments of [**PR**, example 2·4] for normal H carry over and show that if ε: $A \to M(A \otimes C_r^*(H))$ is a coaction of H, then

$$\mathrm{Inf}_H^G \varepsilon := (\mathrm{id}_A \otimes C_H) \circ \varepsilon \colon A \to M(A \otimes C_r^*(G))$$

is a coaction of G on A, called the *inflation* of ε to G.

We can now inflate the dual coactions on the crossed products in the symmetric imprimitivity theorem and our main theorem says that these inflated systems are Morita equivalent in the sense of [**B**, **ER1**].

THEOREM 2. *Suppose K and H are closed subgroups of a locally compact group G and α: $K \to \mathrm{Aut}\, D$, β: $H \to \mathrm{Aut}\, D$ are commuting actions on a C^*-algebra. Then the systems* $(\mathrm{Ind}_H^G(D,\beta) \rtimes_{\tau\otimes\alpha,r} K,\ \mathrm{Inf}_K^G(\tau\otimes\alpha)^\wedge)$ *and* $(\mathrm{Ind}_K^G(D,\alpha) \rtimes_{\sigma\otimes\beta,r} H,\ \mathrm{Inf}_H^G(\sigma\otimes\beta)^\wedge)$ *are Morita equivalent.*

We do not know whether this theorem has an analogue for full coactions on full crossed products. Certainly, it is crucial for our arguments that we are dealing with reduced crossed products: we use concrete realisations of these reduced crossed products to represent the imprimitivity bimodule X on Hilbert space as in [**ER1**] and construct the necessary coaction of G on X using this representation. To represent the crossed products, we start with a covariant representation of $(D, K \times H, \alpha \times \beta)$ and modify Blattner's construction of induced representations to give covariant representations of $(\mathrm{Ind}_H^G(D,\beta), K, \tau \otimes \alpha)$ and its sister. We then construct a representation of X which intertwines these induced representations (Theorem 1·4).

We begin in Section 1 by reviewing the construction of the bimodule X and immediately give a proof of Theorem 1. We then construct the representations of the bimodule X and discuss how these can be used to establish in a very concrete way that the symmetric imprimitivity theorem passes to reduced crossed products (cf. [**Kas**, **QS**]). We prove our main Theorem 2 in Section 2 and explain why one cannot expect a similar equivariance in the full generality of [**R1**]. We conclude with an application: we show that if the dual system $(A \times_\delta G, G, \widehat{\delta})$ is isomorphic to a system induced from a subgroup H, then the coaction δ is Morita equivalent to one inflated from a coaction of H (Corollary 3·2). This is a converse to a recent theorem of Quigg and Raeburn [**QR**], which says that the crossed product by an inflated coaction is isomorphic to an induced algebra.

1. *Induced representations*

Let K and H be closed subgroups of a locally compact group G, let α and β be commuting actions of K and H on D and consider the diagonal actions on the induced algebras as before. We view $A_0 := C_c(K, \mathrm{Ind}_H^G(D,\beta))$ and $B_0 := C_c(H, \mathrm{Ind}_K^G(D,\alpha))$ as dense $*$-subalgebras of the crossed products $A := \mathrm{Ind}_H^G(D,\beta) \rtimes_{\tau\otimes\alpha} K$ and $B := \mathrm{Ind}_K^G(D,\alpha) \times_{\sigma\otimes\beta} H$. In [**R1**, section 1], it is shown that $X_0 := C_c(G, D)$ is a $A_0 - B_0$ pre-imprimitivity bimodule with module actions and inner products defined by

$$\left.\begin{aligned} a \cdot x(s) &= \int_K a(k,s)\alpha_k(x(k^{-1}s))\Delta_K(k)^{\frac{1}{2}}\, dk \\ x \cdot b(s) &= \int_H \beta_{h^{-1}}(x(sh^{-1}))b(h, sh^{-1}))\Delta_H(h)^{-\frac{1}{2}}\, dh \\ {}_A\langle x, y\rangle(k,s) &= \Delta_K(k)^{-\frac{1}{2}} \int_H \beta_h(x(sh)\alpha_k(y(k^{-1}sh)^*))\, dh \\ \langle x, y\rangle_B(h,s) &= \Delta_H(h)^{-\frac{1}{2}} \int_K \alpha_k(x(k^{-1}s)^*\beta_h(y(k^{-1}sh)))\, dk, \end{aligned}\right\} \qquad (1·1)$$

for $a \in A_0$, $b \in B_0$ and $x, y \in X_0$. (The left action of H on G used in [**R1**] is given by $h \cdot s := sh^{-1}$.) The $A - B$ imprimitivity bimodule X of [**R1**, theorem 1·1] is the completion of X_0.

Proof of Theorem 1. Here G is abelian and we need an action γ of $\widehat{G}$ on X such that

(1) $\gamma_\chi(\langle x, y\rangle_B \cdot z) = \langle \gamma_\chi(x), \gamma_\chi(y)\rangle_B \cdot \gamma_\chi(z)$,
(2) ${}_A\langle \gamma_\chi(x), \gamma_\chi(y)\rangle = ((\tau \otimes \alpha)^\wedge)_{\chi|K}({}_A\langle x, y\rangle)$,
(3) $\langle \gamma_\chi(x), \gamma_\chi(y)\rangle_B = ((\sigma \otimes \beta)^\wedge)_{\chi|H}(\langle x, y\rangle_B)$

(see [**C**, **CMW**]). For $x \in X_0$, we define $\gamma_\chi(x)(s) := \overline{\chi(s)}x(s)$ and it follows immediately from the formulas (1·1) that (1), (2) and (3) hold for $a \in A_0$, $b \in B_0$ and $x, y \in X_0$. From (2) or (3) it follows that γ_χ is isometric for the norm on X_0 and hence extends to an isometry on the completion X; now (1) implies that γ_χ is an automorphism of X in the sense of [**C**]. Since $\chi \mapsto \gamma_\chi(x)\colon \widehat{G} \to X_0$ is continuous with respect to the inductive limit topology on X_0, it follows that $\gamma\colon \widehat{G} \to \operatorname{Aut} X$ is a strongly continuous action on X. The formulas (2) and (3) extend to $x, y \in X$ by continuity and hence (X, γ) is the required Morita equivalence.

Keeping our previous notation, we write $\alpha \times \beta$ for the action $(k, h) \mapsto \alpha_k\beta_h = \beta_h\alpha_k$ of $K \times H$ on D. Every covariant representation of $(D, K \times H, \alpha \times \beta)$ has the form $(\rho, U \times V)$, where U and V are commuting representations of K and H on $\mathscr{H}_\rho$ such that (ρ, U) and (ρ, V) are covariant representations of (D, K, α) and (D, H, β). Given such a representation $(\rho, U \times V)$, we shall construct representations of A and B on the underlying Hilbert spaces $\operatorname{Ind}_H^G \mathscr{H}_\rho$ and $\operatorname{Ind}_K^G \mathscr{H}_\rho$ of Blattner's induced representations $\operatorname{Ind}_H^G V$ and $\operatorname{Ind}_K^G U$. To make sure our conventions are clear, we recall the basic constructions of [**Bl**] and [**Ø**]. Throughout ds will denote left Haar measure and where one would naturally use right Haar measure in defining $\operatorname{Ind}_K^G \mathscr{H}_\rho$, we use the right-invariant Haar integral $f \mapsto \int_G f(s^{-1})\, ds$.

For $k \in K$ and $h \in H$ let

$$\mu_K(k) := \left(\frac{\Delta_G(k)}{\Delta_K(k)}\right)^{\frac{1}{2}} \quad \text{and} \quad \mu_H(h) := \left(\frac{\Delta_G(h)}{\Delta_H(h)}\right)^{\frac{1}{2}}$$

and let $T_K\colon C_c(G) \to C_c(K\backslash G)$ and $T_H\colon C_c(G) \to C_c(G/H)$ be the surjective linear maps such that

$$T_K\psi(Ks) = \int_K \psi(k^{-1}s)\, dk \quad \text{and} \quad T_H\varphi(sH) = \int_H \varphi(sh)\, dh.$$

Let $\mathscr{E}^H = \mathscr{E}^H_{(\rho,V)}$ and $\mathscr{E}^K = \mathscr{E}^K_{(\rho,U)}$ be the spaces of Bourbaki-measurable functions $\xi\colon G \to \mathscr{H}_\rho$ satisfying

$$\xi(sh^{-1}) = \mu_H(h)V_h\xi(s) \quad \text{for } h \in H \quad \text{or} \quad \xi(ks) = \mu_K(k)U_k\xi(s) \quad \text{for } k \in K. \tag{1·2}$$

We identify functions $\xi, \eta \in \mathscr{E}^H$ which agree locally almost everywhere. For $\xi, \eta \in \mathscr{E}^H$, there is a Radon measure $\nu_{\xi,\eta}$ on G/H such that

$$\nu_{\xi,\eta}(T_H\varphi) = \int_G \varphi(s)\langle \xi(s), \eta(s)\rangle\, ds,$$

and for $\xi, \eta \in \mathscr{E}^K$, a Radon measure $\mu_{\xi,\eta}$ on $K\backslash G$ such that

$$\mu_{\xi,\eta}(T_K\psi) = \int_G \psi(s^{-1})\langle \xi(s^{-1}), \eta(s^{-1})\rangle\, ds.$$

The induced Hilbert spaces $\mathrm{Ind}_H^G \mathscr{H}_\rho = \mathrm{Ind}_H^G(\mathscr{H}_\rho, V)$ and $\mathrm{Ind}_K^G \mathscr{H}_\rho = \mathrm{Ind}_K^G(\mathscr{H}_\rho, U)$ are defined by

$$\mathrm{Ind}_H^G \mathscr{H}_\rho := \{\xi \in \mathscr{E}^H \colon \mu_{\xi,\xi}(G/H) < \infty\} \quad \text{and} \quad \mathrm{Ind}_K^G \mathscr{H}_\rho := \{\xi \in \mathscr{E}^K \colon \mu_{\xi,\xi}(K\backslash G) < \infty\},$$

with inner products

$$\langle \xi, \eta\rangle := \nu_{\xi,\eta}(G/H) \quad \text{and} \quad \langle \xi, \eta\rangle := \mu_{\xi,\eta}(K\backslash G).$$

The induced representations $\mathrm{Ind}_H^G V$ and $\mathrm{Ind}_K^G U$ act on these spaces by

$$((\mathrm{Ind}_H^G V)_t \eta)(s) = \eta(t^{-1}s) \quad \text{and} \quad ((\mathrm{Ind}_K^G U)_t \xi)(s) = \xi(st).$$

We write $\mathscr{F}^H$ or $\mathscr{F}^H_{(\rho,V)}$ for the subspace of $\mathrm{Ind}_H^G \mathscr{H}_\rho$ consisting of the continuous functions which have compact support modulo H; [**Bl**, lemma 2b] says that $\mathscr{F}^H$ is dense in $\mathrm{Ind}_H^G \mathscr{H}_\rho$. We write $\mathscr{F}^K$ for the analogous dense subspace of $\mathrm{Ind}_K^G \mathscr{H}_\rho$. The inner products can be computed explicitly on these subspaces: for $\xi, \eta \in \mathscr{F}^H$, choose $\varphi \in C_c(G)$ such that $T_H\varphi \equiv 1$ on $\operatorname{supp}\xi \cap \operatorname{supp}\eta$ and then

$$\langle \xi, \eta\rangle = \int_G \varphi(s)\langle \xi(s), \eta(s)\rangle\, ds. \tag{1·3}$$

Similarly, if $\xi, \eta \in \mathscr{F}^K$ and $\psi \in C_c(G)$ satisfies $T_K\psi \equiv 1$ on $\operatorname{supp}\xi \cap \operatorname{supp}\eta$, then

$$\langle \xi, \eta\rangle = \int_G \psi(s^{-1})\langle \xi(s^{-1}), \eta(s^{-1})\rangle\, ds. \tag{1·4}$$

Lemma 1·1 ([**F**, lemma 2·3]). *If $\mathscr{D}$ is a linear subspace of $\mathrm{Ind}_H^G \mathscr{H}_\rho$ such that*

(1) *ξ is continuous for all $\xi \in \mathscr{D}$,*

(2) *$\mathscr{D}$ is invariant under pointwise multiplication by functions in $C_c(G/H)$ and*

(3) *$\{\xi(s) \colon \xi \in \mathscr{D}\}$ is dense in $\mathscr{H}_\rho$ for all $s \in G$,*

then $\mathscr{D}$ is dense in $\mathrm{Ind}_H^G \mathscr{H}_\rho$.

Proposition 1·2 (cf. [**LR**, proposition 2·3]). *Let $(\rho, U \times V)$ be a covariant representation of $(D, K \times H, \alpha \times \beta)$. For $g \in \mathrm{Ind}_H^G(D, \beta)$, $k \in K$ and $\eta \in \mathrm{Ind}_H^G \mathscr{H}_\rho$ define*

$$(\mathrm{Ind}_H^G \rho(g)\eta)(s) = \rho(g(s))\eta(s) \quad \text{and} \quad ((\tau \otimes U)_k \eta)(s) = U_k \eta(k^{-1}s).$$

Similarly, for $f \in \mathrm{Ind}_K^G(D, \alpha)$, $h \in H$ and $\xi \in \mathrm{Ind}_K^G \mathscr{H}_\rho$ define

$$(\mathrm{Ind}_K^G \rho(f)\xi)(s) = \rho(f(s))\xi(s) \quad \text{and} \quad ((\sigma \otimes V)_h \xi)(s) = V_h \xi(sh).$$

Then $(\mathrm{Ind}_H^G \rho, \tau \otimes U)$ is a covariant representation of $(\mathrm{Ind}_H^G(D, \beta), K, \tau \otimes \alpha)$ and $(\mathrm{Ind}_K^G \rho, \sigma \otimes V)$ is a covariant representation of $(\mathrm{Ind}_K^G(D, \alpha), H, \sigma \otimes \beta)$. Moreover, if ρ is faithful, then $\mathrm{Ind}_K^G \rho$ and $\mathrm{Ind}_H^G \rho$ are faithful.

Proof. We just prove the assertions about $(\mathrm{Ind}_H^G \rho, \tau \otimes U)$. As in the proof of [**LR**, proposition 2·3], one can check that $\mathrm{Ind}_H^G \rho$ is a well-defined representation of $\mathrm{Ind}_H^G(D, \beta)$, which is faithful if ρ is. To see that $\tau \otimes U$ is a unitary representation of K, just note that it differs from the restriction to K of Blattner's induced representation $\mathrm{Ind}_H^G V$ only by the presence of the unitary U_k; it is straightforward to check that this too is a unitary representation of K.

It remains to check the covariance condition. Let $k \in K$, $f \in \mathrm{Ind}_H^G(D,\beta)$ and $\xi \in \mathrm{Ind}_H^G \mathscr{H}_\rho$. Then, using the covariance of (ρ, U), we calculate

$$\begin{aligned}(\mathrm{Ind}_H^G \rho((\tau\otimes\alpha)_k(f))\xi)(s) &= \rho((\tau\otimes\alpha)_k(f)(s))\xi(s)\\ &= \rho(\alpha_k(f(k^{-1}s)))\xi(s)\\ &= U_k\rho(f(k^{-1}s))U_k^*\xi(s)\\ &= U_k\rho(f(k^{-1}s))((\tau\otimes U)_{k^{-1}}\xi)(k^{-1}s)\\ &= U_k(\mathrm{Ind}_H^G\rho(f)(\tau\otimes U)_{k^{-1}}\xi)(k^{-1}s)\\ &= ((\tau\otimes U)_k \mathrm{Ind}_H^G\rho(f)(\tau\otimes U)_{k^{-1}}\xi)(s).\end{aligned}$$

Thus $(\mathrm{Ind}_H^G\rho, \tau\otimes U)$ is covariant.

At various points we shall want to apply this construction to different covariant representations. Of particular importance will be one which yields spatial implementations of the reduced crossed products:

COROLLARY 1·3. *If $(\rho_1, U_1\times V_1)$ is a covariant representation of $(D, K\times H, \alpha\times\beta)$ with ρ_1 faithful and we apply the construction of Proposition 1·2 to*

$$(\rho, U\times V) := (\rho_1\otimes 1\otimes 1, (U_1\otimes\lambda^K\otimes 1)\times(V_1\otimes 1\otimes\lambda^H)),$$

then the representations $(\mathrm{Ind}_H^G\rho)\rtimes(\tau\otimes U)$ and $(\mathrm{Ind}_K^G\rho)\rtimes(\sigma\otimes V)$ factor through faithful representations of the reduced crossed products.

Proof. For any covariant representation $(\rho_2, U_2\times V_2)$ of $(D, K\times H, \alpha\times\beta)$, the map T of $\mathscr{F}^K_{(\rho_2,U_2)}\otimes L^2(H)$ into $\mathscr{F}^K_{(\rho_2\otimes 1, U_2\otimes 1)}$ defined by $T(\eta\otimes v)(s) := \eta(s)\otimes v$ intertwines $((\mathrm{Ind}_K^G\rho_2)\otimes 1, (\sigma\otimes V_2)\otimes\lambda^H)$ and $((\mathrm{Ind}_K^G\rho_2\otimes 1), \sigma\otimes(V_2\otimes\lambda^H))$. Since T is isometric and is surjective by Lemma 1·1, we deduce that the integrated forms of these two representations have the same kernel. Now consider $(\rho_2, U_2\times V_2) := (\rho_1\otimes 1_{L^2(K)}, (U_1\otimes\lambda^K)\times(V_1\otimes 1))$. Then ρ_1 faithful implies ρ_2 faithful, so $\mathrm{Ind}_K^G\rho_2$ is a faithful representation of $\mathrm{Ind}_K^G(D,\alpha)$. Since, for any covariant representation (π, W), $(\pi\otimes 1, W\otimes\lambda^H)$ is equivalent to the regular representation induced from π, we deduce that $((\mathrm{Ind}_K^G\rho_2)\otimes 1)\rtimes((\sigma\otimes V_2)\otimes\lambda^H)$ factors through a faithful representation of the reduced crossed product.

Now that we have represented the crossed products A and B on $\mathrm{Ind}_H^G\mathscr{H}_\rho$ and $\mathrm{Ind}_K^G\mathscr{H}_\rho$, we want to represent the bimodule ${}_AX_B$ as operators from $\mathrm{Ind}_K^G\mathscr{H}_\rho$ to $\mathrm{Ind}_H^G\mathscr{H}_\rho$. Recall from [**ER1**] that a representation π of an $A-B$ imprimitivity bimodule ${}_AX_B$ consists of representations $\pi_A\colon A\to B(\mathscr{H})$, $\pi_B\colon B\to B(\mathscr{K})$ and a linear map $\pi_X\colon X\to B(\mathscr{K},\mathscr{H})$ such that

(1) $\pi_X(x)\pi_X(y)^* = \pi_A({}_A\langle x,y\rangle)$,

(2) $\pi_X(x)^*\pi_X(y) = \pi_B(\langle x,y\rangle_B)$ and

(3) $\pi_X(a\cdot x\cdot b) = \pi_A(a)\pi_X(x)\pi_B(b)$.

THEOREM 1·4. *Let $(\rho, U\times V)$ be a covariant representation of $(D, K\times H, \alpha\times\beta)$ and let $\pi_A := (\mathrm{Ind}_H^G\rho)\rtimes(\tau\otimes U)$ and $\pi_B := (\mathrm{Ind}_K^G\rho)\rtimes(\sigma\otimes V)$ be the representations of*

$$A = \mathrm{Ind}_H^G(D,\beta)\rtimes_{\tau\otimes\alpha} K \quad\textit{and}\quad B = \mathrm{Ind}_K^G(D,\alpha)\rtimes_{\sigma\otimes\beta} H$$

constructed in Proposition 1·2. Then we can find $\pi_X\colon X\to B(\mathrm{Ind}_K^G\mathscr{H}_\rho, \mathrm{Ind}_H^G\mathscr{H}_\rho)$

such that (π_A, π_X, π_B) is an imprimitivity-bimodule representation of ${}_AX_B$ and such that

$$(\pi_X(x)\xi)(s) = \int_H V_h\rho(x(sh))\xi(sh)\Delta_G(sh)^{-\frac{1}{2}}\mu_H(h)\,dh \tag{1·5}$$

for $x \in X_0 = C_c(G, D)$ and $\xi \in \mathscr{F}^K$.

We divide the proof of Theorem 1·4 into three steps.

LEMMA 1·5. *For $x \in X_0$ and $\xi \in \mathscr{F}^K$, the function $\pi_X(x)\xi$ defined by (1·5) belongs to $\mathscr{F}^H$. Moreover, if for $\eta \in \mathscr{F}^H$ we define*

$$(\pi_{\tilde{X}}(x)\eta)(s) = \int_K U_k\rho(x(k^{-1}s)^*)\eta(k^{-1}s)\Delta_G(k^{-1}s)^{\frac{1}{2}}\mu_K(k)\,dk, \tag{1·6}$$

then $\pi_{\tilde{X}}(x)\eta \in \mathscr{F}^K$ and $\langle \pi_X(x)\xi, \eta\rangle = \langle \xi, \pi_{\tilde{X}}(x)\eta\rangle$.

Proof. It follows from (1·5) and the definition of $\mathscr{F}^K$ that $\pi_X(x)\xi$ has support in $(\operatorname{supp} x)\cdot H$ and satisfies $(\pi_X(x)\xi)(sl^{-1}) = \mu_H(l)V_l(\pi_X(x)\xi)(s)$ for $l \in H$. Thus $\pi_X(x)\xi \in \mathscr{F}^H$. Similarly, $\pi_{\tilde{X}}(x)\eta \in \mathscr{F}^K$ and $\operatorname{supp}\pi_{\tilde{X}}(x)\eta \subset K\cdot \operatorname{supp} x$.

To verify the equation $\langle \pi_X(x)\xi, \eta\rangle = \langle \xi, \pi_{\tilde{X}}(x)\eta\rangle$, suppose $\varphi, \psi \in C_c(G)$ satisfy $\int_H \varphi(sh)\,dh = 1$ on $(\operatorname{supp} x)\cdot H$ and $\int_K \psi(k^{-1}s)\,dk = 1$ on $K\cdot(\operatorname{supp} x)$. Then we can use (1·2) and (1·4) to compute:

$$\begin{aligned}
&\langle \pi_X(x)\xi, \eta\rangle \\
&\quad= \int_G \varphi(s)\big\langle (\pi_X(x)\xi)(s), \eta(s)\big\rangle\,ds \\
&\quad= \int_G\int_H \varphi(s)\big\langle V_h\rho(x(sh))\xi(sh)\Delta_G(sh)^{-\frac{1}{2}}\mu_H(h), \eta(s)\big\rangle\,dh\,ds \\
&\quad= \int_G\int_H \left(\int_K \psi(k^{-1}sh)\,dk\right)\varphi(s)\big\langle V_h\rho(x(sh))\xi(sh)\Delta_G(sh)^{-\frac{1}{2}}\mu_H(h), \eta(s)\big\rangle\,dh\,ds \\
&\quad= \int_G\int_H\int_K \Delta_G(h^{-1})\psi(s)\varphi(ksh^{-1}) \\
&\qquad\qquad\times\big\langle V_h\rho(x(ks))\xi(ks)\Delta_G(ks)^{-\frac{1}{2}}\mu_H(h), \eta(ksh^{-1})\big\rangle\,dk\,dh\,ds \\
&\quad= \int_G\int_H\int_K \Delta_G(h^{-1})\psi(s)\varphi(ksh^{-1}) \\
&\qquad\qquad\times\big\langle V_h\rho(x(ks))\mu_K(k)U_k\xi(s)\Delta_G(ks)^{-\frac{1}{2}}\mu_H(h), \mu_H(h)V_h\eta(ks)\big\rangle\,dk\,dh\,ds.
\end{aligned}$$

Taking inverses of all the variables, and gathering the modular functions in one function μ, this becomes

$$\int_G\int_H\int_K \mu(k,s,h)\psi(s^{-1})\varphi(k^{-1}s^{-1}h)\big\langle\rho(x(k^{-1}s^{-1}))U_{k^{-1}}\xi(s^{-1}), \eta(k^{-1}s^{-1})\big\rangle\,dk\,dh\,ds,$$

where

$$\begin{aligned}
\mu(k,s,h) &= \Delta_G(h)\mu_H(h^{-1})^2\Delta_G(ks)^{\frac{1}{2}}\mu_K(k^{-1})\Delta_H(h^{-1})\Delta_G(s^{-1})\Delta_K(k^{-1}) \\
&= \Delta_G(k^{-1}s^{-1})^{\frac{1}{2}}\mu_K(k).
\end{aligned}$$

Using Fubini's theorem, our integral becomes

$$\begin{aligned}
&\int_G\int_K\left(\int_H\varphi(k^{-1}s^{-1}h)\,dh\right)\psi(s^{-1})\\
&\qquad\qquad\times\big\langle\xi(s^{-1}),U_k\rho(x(k^{-1}s^{-1})^*)\eta(k^{-1}s^{-1})\Delta_G(k^{-1}s^{-1})\mu_K(k)\big\rangle\,dk\,ds\\
&\quad=\int_G\psi(s^{-1})\Big\langle\xi(s^{-1}),\int_K U_k\rho(x(k^{-1}s^{-1})^*)\eta(k^{-1}s^{-1})\Delta_G(k^{-1}s^{-1})\mu_K(k)\,dk\Big\rangle\,ds\\
&\quad=\langle\xi,\pi_{\tilde X}(x)\eta\rangle,
\end{aligned}$$

where the last equation follows from (1·3) and $\operatorname{supp}\pi_{\tilde X}(x)\eta\subset K\cdot\operatorname{supp}x$.

LEMMA 1·6. *For $x,y\in X_0$ let $\pi_X(y)\colon\mathscr{F}^K\to\mathscr{F}^H$ and $\pi_{\tilde X}(x)\colon\mathscr{F}^H\to\mathscr{F}^K$ be the linear maps given by* (1·5) *and* (1·6). *Then*

$$\pi_{\tilde X}(x)\pi_X(y)\xi=\pi_B(\langle x,y\rangle_B)\xi\quad\textit{and}\quad\pi_X(x)\pi_{\tilde X}(y)\eta=\pi_A({}_A\langle x,y\rangle)\eta$$

for $\xi\in\mathscr{F}^K$ and $\eta\in\mathscr{F}^H$.

Proof. We verify the first equation using (1·2) and the covariance of $(\rho,U\times V)$:

$$\begin{aligned}
&(\pi_{\tilde X}(x)\pi_X(y)\xi)(s)\\
&\quad=\int_K\int_H U_k\rho(x(k^{-1}s)^*)V_h\rho(y(k^{-1}sh))\xi(k^{-1}sh)\\
&\qquad\qquad\times\Delta_G(k^{-1}s)^{\frac12}\mu_K(k)\Delta_G(k^{-1}sh)^{-\frac12}\mu_H(h)\,dh\,dk\\
&\quad=\int_H\rho\left(\Delta_H(h)^{-\frac12}\int_K\alpha_k(x(k^{-1}s)^*\beta_h(y(k^{-1}sh)))\,dk\right)V_h\xi(sh)\,dh\\
&\quad=\int_H\rho(\langle x,y\rangle_B(h,s))V_h\xi(sh)\,dh\\
&\quad=(\pi_B(\langle x,y\rangle_B)\xi)(s).
\end{aligned}$$

The second equation follows from similar calculations.

Proof of Theorem 1·4. From Lemmas 1·5 and 1·6 we have

$$\begin{aligned}
\|\pi_X(x)\xi\|^2&=\langle\pi_X(x)\xi,\pi_X(x)\xi\rangle=\langle\pi_{\tilde X}(x)\pi_X(x)\xi,\xi\rangle\\
&=\langle\pi_B(\langle x,x\rangle_B)\xi,\xi\rangle\leqslant\|\langle x,x\rangle_B\|\|\xi\|^2,
\end{aligned}$$

for $x\in X_0$ and $\xi\in\mathscr{F}^K$. Thus $\pi_X(x)$ is bounded with $\|\pi(x)\|\leqslant\|x\|$ and extends uniquely to a bounded operator from $\operatorname{Ind}_K^G\mathscr{H}_\rho$ to $\operatorname{Ind}_H^G\mathscr{H}_\rho$. Since $\pi_X\colon X_0\to B(\operatorname{Ind}_K^G\mathscr{H}_\rho,\operatorname{Ind}_H^G\mathscr{H}_\rho)$ is norm-decreasing, it extends to all of X and Lemma 1·5 implies that $\pi_{\tilde X}(x)$ extends to give an adjoint. Thus Lemma 1·6 gives the equations $\pi_X(x)^*\pi_X(y)=\pi_B(\langle x,y\rangle_B)$ and $\pi_X(x)\pi_X(y)^*=\pi_A({}_A\langle x,y\rangle)$. Next we use the relations $a(k,sh)=\beta_{h^{-1}}(a(k,s))$ for $a\in A_0=C_c(K,\operatorname{Ind}_H^G(D,\beta))$, $\xi(k^{-1}s)=\mu_K(k^{-1})U_{k^{-1}}\xi(s)$ for $\xi\in\mathscr{F}^K$, and the covariance of $(\rho,U\times V)$, to compute:

$$\begin{aligned}
(\pi_X(a\cdot x)\xi)(s)&=\int_H\int_K V_h\rho(a(k,sh)\alpha_k(x(k^{-1}sh)))\xi(sh)\Delta_K(k)^{\frac12}\Delta_G(sh)^{-\frac12}\mu_H(h)\,dk\,dh\\
&=\int_H\int_K V_h\rho(\beta_{h^{-1}}(a(k,s))\alpha_k(x(k^{-1}sh)))U_k\xi(k^{-1}sh)\\
&\qquad\times\mu_K(k)\Delta_K(k)^{\frac12}\Delta_G(sh)^{-\frac12}\mu_H(h)\,dk\,dh
\end{aligned}$$

$$= \int_H \int_K \rho(a(k,s))V_h U_k \rho(x(k^{-1}sh))\xi(k^{-1}sh)\Delta_G(k^{-1}sh)^{-\frac{1}{2}}\mu_H(h)\,dk\,dh$$
$$= \int_K \rho(a(k,s))U_k \biggl(\int_H V_h \rho(x(k^{-1}sh))\xi(k^{-1}sh)\Delta_G(k^{-1}sh)^{-\frac{1}{2}}\mu_H(h)\,dh\biggr)dk$$
$$= (\pi_A(a)\pi_X(x)\xi)(s).$$

The equation $\pi_X(x \cdot b) = \pi_X(x)\pi_B(b)$ is proved similarly.

We now apply this Theorem to the particular covariant representations which we know give faithful realisations of the reduced crossed products (see Corollary 1·3).

COROLLARY 1·7 (cf. [**Kas**, Theorem 3·15], [**QS**, Theorem 4·2]). *Let $(\rho_1, U_1 \times V_1)$ be a covariant representation of $(D,\, K \times H,\, \alpha \times \beta)$ such that ρ_1 is faithful and let $\pi :=(\pi_A, \pi_X, \pi_B)$ be the representation of the imprimitivity bimodule ${}_AX_B$ which is obtained by applying Theorem* 1·4 *to $(\rho, U \times V) := (\rho_1 \otimes 1 \otimes 1, (U_1 \otimes \lambda^K \otimes 1) \times (V_1 \otimes 1 \otimes \lambda^H))$. If we identify the reduced crossed products $A_r := \mathrm{Ind}_H^G(D, \beta) \rtimes_{\tau\otimes\alpha,r} K$ and $B_r := \mathrm{Ind}_K^G(D, \alpha) \rtimes_{\sigma\otimes\beta,r} H$ with their images $\pi_A(A)$ and $\pi_B(B)$, as we can by Corollary* 1·3, *then $X_r := \pi_X(X)$ becomes an $A_r - B_r$ imprimitivity bimodule.*

Proof. Since the representation π_B is equivalent to the representation of B induced from π_A by [**ER1**, lemma 2·2], this follows from the Rieffel correspondence.

2. *Proof of the main theorem*

We aim to make the bimodule X_r into a Morita equivalence between the inflated systems by constructing a compatible coaction of G on X_r. To make this precise, we recall from [**ER1**] that the multiplier bimodule of an imprimitivity bimodule ${}_CY_D$ is an $M(C) - M(D)$ Hilbert bimodule $M(Y)$ which contains Y as an essential $C - D$ submodule. For our purposes, it is enough to know that if (π_C, π_Y, π_D) is a representation of ${}_CY_D$ on $(\mathscr{H}, \mathscr{K})$ such that π_D is faithful and nondegenerate, then π_Y extends uniquely to an isomorphism of $M(Y)$ onto

$$\{T \in B(\mathscr{K}, \mathscr{H})\colon T\pi_Y(Y) \cup \pi_Y(Y)T \subset \pi_Y(Y)\}$$

[**ER1**, proposition 2·4]. If δ_C, δ_D are coactions of G, then a Morita equivalence between (C, G, δ_C) and (D, G, δ_D) is an imprimitivity bimodule ${}_CY_D$ together with a linear map $\delta_Y\colon Y \to M(Y \otimes C_r^*(G))$ such that $(\delta_C, \delta_Y, \delta_D)$ is an imprimitivity-bimodule representation and

$$(\delta_Y \otimes \mathrm{id}_G) \circ \delta_Y = (\mathrm{id}_Y \otimes \delta_G) \circ \delta_Y$$

as maps into $M(Y \otimes C_r^*(G) \otimes C_r^*(G))$.

We use the realisation of X_r from Corollary 1·7; thus $A_r = \mathrm{Ind}_H^G(D, \beta) \rtimes_{\tau\otimes\alpha,r} K$ is identified with the subalgebra $\pi_A(A)$ of $B(\mathrm{Ind}_H^G\mathscr{H}_\rho)$, $B_r = \mathrm{Ind}_K^G(D, \alpha) \rtimes_{\sigma\otimes\beta,r} H$ is identified with $\pi_B(B) \subset B(\mathrm{Ind}_K^G\mathscr{H}_\rho)$, and $X_r := \pi_X(X) \subset B(\mathrm{Ind}_K^G\mathscr{H}_\rho, \mathrm{Ind}_H^G\mathscr{H}_\rho)$. The dual coaction $(\tau \otimes \alpha)^\wedge$ is the integrated form of $((\mathrm{Ind}_H^G\rho) \otimes 1, (\tau \otimes U) \otimes \lambda^K)$, in the sense that

$$(\tau \otimes \alpha)^\wedge(\pi_A(a)) = ((\mathrm{Ind}_H^G\rho) \otimes 1) \rtimes ((\tau \otimes U) \otimes \lambda^K)(a)$$

and its inflation to a coaction $\delta_{A_r}\colon A_r \to M(A_r \otimes C_r^*(G)) \subset B((\mathrm{Ind}_H^G\mathscr{H}_\rho) \otimes L^2(G))$ is given by

$$\delta_{A_r}(\pi_A(a)) = ((\mathrm{Ind}_H^G\rho) \otimes 1) \rtimes ((\tau \otimes U) \otimes \lambda^G|_K)(a).$$

To simplify our formulas, we write $\delta_A = \delta_{A_r} \circ \pi_A$, so that δ_A is the integrated form of the same representation viewed as a map on the full crossed product. There are similar formulas for δ_{B_r} and $\delta_B := \delta_{B_r} \circ \pi_B$.

We know from Theorem 1·4 that to represent X by operators on Hilbert space we just need a covariant representation of $(D, K \times H, \alpha \times \beta)$. To construct the coaction δ_{X_r}, we apply Theorem 1·4 to the covariant representation $(\rho\otimes 1, (U\otimes\lambda^G|_K)\times(V\otimes 1))$ and then twist the resulting representation of X by a version T of the canonical 'multiplicative unitary' W_G which plays a fundamental role in duality for nonabelian groups [**Kat**, **BS**]. In the calculations which follow, we view elements of, for example, $\mathscr{F}^H \odot C_c(G)$ as functions on $G \times G$. Since the spaces $(\mathrm{Ind}_H^G(\mathscr{H}_\rho, V)) \otimes L^2(G)$ and $\mathrm{Ind}_H^G(\mathscr{H}_\rho \otimes L^2(G), V \otimes 1)$ then have the same dense subspaces, we shall identify them without comment and write $\mathrm{Ind}_H^G \mathscr{H}_\rho \otimes L^2(G)$ for either.

LEMMA 2·1. *There is a linear map $\delta_X\colon X \to B(\mathrm{Ind}_K^G \mathscr{H}_\rho \otimes L^2(G), \mathrm{Ind}_H^G \mathscr{H}_\rho \otimes L^2(G))$ such that*

$$(\delta_X(x)\xi)(s,t) = \int_H (V_h\rho(x(sh)) \otimes 1)\xi(sh, h^{-1}s^{-1}t)\Delta_G(sh)^{-\frac{1}{2}}\mu_H(h)\, dh \qquad (2·1)$$

for $x \in X_0$ and $\xi \in \mathscr{F}^K \odot C_c(G)$ and such that $(\delta_A, \delta_X, \delta_B)$ is an imprimitivity-bimodule representation of ${}_AX_B$.

Proof. We apply Theorem 1·4 to the covariant pair $(\rho \otimes 1, (U \otimes \lambda^G|_K) \times (V \otimes 1))$ of $(D, K \times H, \alpha \times \beta)$ on $\mathscr{H}_\rho \otimes L^2(G)$, and obtain a representation $(\varepsilon_A, \varepsilon_X, \varepsilon_B)$ of ${}_AX_B$ as operators from $\mathrm{Ind}_K^G(\mathscr{H}_\rho \otimes L^2(G), U \otimes \lambda^G|_K)$ to $\mathrm{Ind}_H^G \mathscr{H}_\rho \otimes L^2(G)$. Under our identifications of Hilbert spaces, ε_A is precisely δ_A.

Now define T: $\mathrm{Ind}_K^G \mathscr{H}_\rho \otimes L^2(G) \to \mathrm{Ind}_K^G(\mathscr{H}_\rho \otimes L^2(G), U \otimes \lambda^G|_K)$ by $T\xi(s,t) = \xi(s, s^{-1}t)$; it is straightforward to check that T is unitary. For $b \in C_c(H, \mathrm{Ind}_K^G(D, \alpha))$ and $\xi \in \mathscr{F}^K \odot C_c(G)$, we have

$$\begin{aligned}
(\varepsilon_B(b)T\xi)(s,t) &= \int_H (\mathrm{Ind}_K^G(\rho \otimes 1)(b(h))(\sigma \otimes (V \otimes 1))_h T\xi)(s,t)\, dh \\
&= \int_H (\rho(b(h,s)) \otimes 1)(V_h \otimes 1)(T\xi(sh,t))\, dh \\
&= \int_H (\rho(b(h,s))V_h \otimes 1)(\xi(sh, h^{-1}s^{-1}t))\, dh \\
&= \int_H (\rho(b(h,s))V_h \otimes \lambda_h^G)(\xi(sh, s^{-1}t))\, dh \\
&= \int_H (((\mathrm{Ind}_K^G \rho) \otimes 1)(b(h))((\sigma \otimes V)_h \otimes \lambda_h^G)\xi)(s, s^{-1}t)\, dh \\
&= (\delta_B(b)\xi)(s, s^{-1}t) = (T\delta_B(b)\xi)(s,t).
\end{aligned}$$

Thus $\delta_B(b) = T^*\varepsilon_B(b)T$ for all $b \in B$. If we now set $\delta_X(x) := \varepsilon_X(x)T$ for $x \in X$, then it follows that $(\delta_A, \delta_X, \delta_B)$ is also an imprimitivity-bimodule representation of ${}_AX_B$. For $x \in X_0$ and $\xi \in \mathscr{F}^K \odot C_c(G)$ we use (1·5) to calculate

$$\begin{aligned}
(\varepsilon_X(x)T\xi)(s,t) &= \int_H (V_h\rho(x(sh)) \otimes 1)(T\xi)(sh,t)\Delta_G(sh)^{-\frac{1}{2}}\mu_H(h)\, dh \\
&= \int_H (V_h\rho(x(sh)) \otimes 1)\xi(sh, h^{-1}s^{-1}t)\Delta_G(sh)^{-\frac{1}{2}}\mu_H(h)\, dh,
\end{aligned}$$

which gives the desired formula for $\delta_X(x)$.

Tensoring with G gives a representation $(\pi_A\otimes\lambda^G, \pi_X\otimes\lambda^G, \pi_B\otimes\lambda^G)$ of the external tensor product ${}_{A\otimes C^*(G)}(X\otimes C^*(G))_{B\otimes C^*(G)}$ as operators from $\mathrm{Ind}_K^G\mathscr{H}_\rho\otimes L^2(G)$ to $\mathrm{Ind}_H^G\mathscr{H}_\rho\otimes L^2(G)$ and then $X_r\otimes C_r^*(G) = \pi_X\otimes\lambda^G(X\otimes C^*(G))$. Thus we can view $M(X_r\otimes C_r^*(G))$ as the set of operators

$$T\colon \mathrm{Ind}_K^G\mathscr{H}_\rho\otimes L^2(G)\to \mathrm{Ind}_H^G\mathscr{H}_\rho\otimes L^2(G)$$

such that $(A_r\otimes C_r^*(G))T$ and $T(B_r\otimes C_r^*(G))$ are contained in $X_r\otimes C_r^*(G)$. The next proposition completes the proof of Theorem 2.

PROPOSITION 2·2. *The map δ_X of Lemma 2·1 factors through a linear map $\delta_{X_r}\colon X_r\to M(X_r\otimes C_r^*(G))$ and (X_r, δ_{X_r}) is a Morita equivalence for the inflated dual coactions δ_{A_r} and δ_{B_r}.*

Proof. Let $x\in X_0$ and $z\in C_c(G)$ and define $x\diamond z\in C_c(G\times G, D)$ by $x\diamond z(s,r) = x(s)z(s^{-1}r)$. We claim that

$$\delta_X(x)(1\otimes\lambda^G(z)) = \pi_X\otimes\lambda^G(x\diamond z). \tag{2·2}$$

To see this, let $\xi\in\mathscr{F}^K\odot C_c(G)$. From (2·1) we have

$$\begin{aligned}
&(\delta_X(x)(1\otimes\lambda^G(z))\xi)(s,t)\\
&\quad=\int_H\int_G (V_h\rho(x(sh))\otimes 1)z(r)\xi(sh, r^{-1}h^{-1}s^{-1}t)\Delta_G(sh)^{-\frac12}\mu_H(h)\,dr\,dh\\
&\quad=\int_H\int_G (V_h\rho(x(sh))\otimes 1)z(h^{-1}s^{-1}r)\xi(sh, r^{-1}t)\Delta_G(sh)^{-\frac12}\mu_H(h)\,dr\,dh\\
&\quad=(\pi_X\otimes\lambda^G(x\diamond z)\xi)(s,t),
\end{aligned}$$

which gives (2·2). Standard approximation arguments show that the elements of the form $\pi_X\otimes\lambda^G(x\diamond z)$ span a dense subspace of $X_r\otimes C_r^*(G)$, so it follow from (2·2) that $\delta_X(x)(1\otimes\lambda^G(z))$ lies in $X_r\otimes C_r^*(G)$ for all $x\in X$ and $z\in C_c(G)$. From this and the nondegeneracy of δ_{B_r} we deduce that

$$\delta_X(X)(B_r\otimes C_r^*(G)) = \delta_X(X)\delta_{B_r}(B_r)(1\otimes C_r^*(G)) = \delta_X(X\cdot B)(1\otimes C_r^*(G))\subset X_r\otimes C_r^*(G).$$

A similar computation shows that $(1\otimes\lambda^G(z))\delta_X(x) = (\pi_X\otimes\lambda^G)(x\bullet z)$ belongs to $X_r\otimes C_r^*(G)$, where $x\bullet z(s,r) := \Delta_G(s^{-1})x(s)z(rs^{-1})\in C_c(G\times G, D)$; this implies that $(A_r\otimes C_r^*(G))\delta_X(X)\subset X_r\otimes C_r^*(G)$. Thus the range of π_X lies in $M(X_r\otimes C_r^*(G))$.

Since δ_A and δ_B factor through the faithful representations δ_{A_r} and δ_{B_r} of A_r and B_r, it follows from [**ER2**, lemma 2.7] that δ_X factors through an injective linear map $\delta_{X_r}\colon X_r\to M(X_r\otimes C_r^*(G))$ such that the triple $(\delta_{A_r}, \delta_{X_r}, \delta_{B_r})$ is an imprimitivity bimodule homomorphism from ${}_{A_r}X_{rB_r}$ to $M(X_r\otimes C_r^*(G))$. Hence it remains to verify the coaction identity

$$(\delta_{X_r}\otimes\mathrm{id}_G)\circ\delta_{X_r} = (\mathrm{id}_{X_r}\otimes\delta_G)\circ\delta_{X_r}.$$

In the following calculations we view $X_0\odot C_c(G)\odot C_c(G)$ as a subset of $C_c(G\times G\times G, D)$. For $y\in X_0$ and $g\in C_c(G\times G)$, define $y\triangleright g\in C_c(G\times G\times G, D)$ by

$$y\triangleright g(s,t,r) = y(s)g(s^{-1}t, s^{-1}r)$$

and for $z\in C_c(G)$ and $g\in C_c(G\times G)$, define $z\star g\in C_c(G\times G)$ by

$$z\star g(s,t) = \int_G z(r)g(r^{-1}s, r^{-1}t)\,dr;$$

we need the identities

$$\delta_G(\lambda^G(z))(\lambda^G \otimes \lambda^G)(g) = (\lambda^G \otimes \lambda^G)(z \star g) \tag{2·3}$$

and

$$\big(\pi_X \otimes (\delta_G \circ \lambda^G)(y \diamond z)\big)\big(1 \otimes (\lambda^G \otimes \lambda^G)(g)\big) = (\pi_X \otimes \lambda^G \otimes \lambda^G)(y \triangleright (z \star g)). \tag{2·4}$$

To prove (2·3), let $\xi \in L^2(G \times G)$. Then

$$\begin{aligned}(\delta_G(\lambda^G(z))(\lambda^G \otimes \lambda^G)(g)\xi)(s,t) &= \int_G z(r)((\lambda^G \otimes \lambda^G)(g)\xi)(r^{-1}s, r^{-1}t)\, dr \\ &= \int_G \int_G \int_G z(r)g(l,m)\xi(l^{-1}r^{-1}s, m^{-1}r^{-1}t)\, dr\, dl\, dm \\ &= \int_G \int_G \int_G z(r)g(r^{-1}l, r^{-1}m)\xi(l^{-1}s, m^{-1}t)\, dr\, dl\, dm \\ &= \int_G \int_G z \star g(l,m)\xi(l^{-1}s, m^{-1}t)\, dl\, dm \\ &= ((\lambda^G \otimes \lambda^G)(z \star g)\xi)(s,t),\end{aligned}$$

which is (2·3). For (2·4), we take $\xi \in \mathscr{F}^K \odot C_c(G)$ and use (1·5) to compute

$$\begin{aligned}&((\pi_X \otimes (\delta_G \circ \lambda^G))(y \diamond z)(1 \otimes (\lambda^G \otimes \lambda^G)(g))\xi)(s,t,n) \\ &\quad = \int_H \int_G \int_G \int_G (V_h \otimes 1 \otimes 1)(\rho \otimes 1 \otimes 1)(y \diamond z(sh, r)g(l,m)) \\ &\qquad\qquad \times \xi(sh, l^{-1}r^{-1}t, m^{-1}r^{-1}n)\Delta_G(sh)^{-\frac{1}{2}}\mu_H(h)\, dh\, dr\, dl\, dm \\ &\quad = \int_H \int_G \int_G \int_G (V_h \otimes 1 \otimes 1)(\rho \otimes 1 \otimes 1)(y(sh)z(h^{-1}s^{-1}r)g(l,m)) \\ &\qquad\qquad \times \xi(sh, l^{-1}r^{-1}t, m^{-1}r^{-1}n)\Delta_G(sh)^{-\frac{1}{2}}\mu_H(h)\, dh\, dr\, dl\, dm.\end{aligned}$$

Sending $l \mapsto r^{-1}l, m \mapsto r^{-1}m$ and $r \mapsto shr$ gives

$$\begin{aligned}&\int_H \int_G \int_G \int_G (V_h \otimes 1 \otimes 1)(\rho \otimes 1 \otimes 1)\big(y(sh)z(r)g(r^{-1}h^{-1}s^{-1}l, r^{-1}h^{-1}s^{-1}m)\big) \\ &\qquad \times \xi(sh, l^{-1}t, m^{-1}n)\Delta_G(sh)^{-\frac{1}{2}}\mu_H(h)\, dh\, dr\, dl\, dm \\ &= \int_H \int_G \int_G (V_h \otimes 1 \otimes 1)(\rho \otimes 1 \otimes 1)(y(sh)z \star g(h^{-1}s^{-1}l, h^{-1}s^{-1}m)) \\ &\qquad \times \xi(sh, l^{-1}t, m^{-1}n)\Delta_G(sh)^{-\frac{1}{2}}\mu_H(h)\, dh\, dl\, dm \\ &= \int_H \int_G \int_G (V_h \otimes 1 \otimes 1)(\rho \otimes 1 \otimes 1)(y \triangleright (z \star g)(sh, l, m)) \\ &\qquad \times \xi(sh, l^{-1}t, m^{-1}n)\Delta_G(sh)^{-\frac{1}{2}}\mu_H(h)\, dh\, dl\, dm \\ &= ((\pi_X \otimes \lambda^G \otimes \lambda^G)(y \triangleright (z \star g))\xi)(s,t,n),\end{aligned}$$

which proves (2·4).

Next recall from [**ER1**, proposition 1·3] that

$$M(X_r \otimes C_r^*(G) \otimes C_r^*(G)) \cong \mathscr{L}(B_r \otimes C_r^*(G) \otimes C_r^*(G), X_r \otimes C_r^*(G) \otimes C_r^*(G)),$$

so elements of $M(X_r \otimes C_r^*(G) \otimes C_r^*(G))$ are uniquely determined by their products with elements of $B_r \otimes C_r^*(G) \otimes C_r^*(G)$. Since δ_{B_r} is nondegenerate, the elements of the form

$$(\mathrm{id}_{B_r} \otimes \delta_G)((\delta_B(b) \cdot (1 \otimes \lambda^G(z))) \cdot (1 \otimes \lambda^G(v) \otimes \lambda^G(w))$$

for $b \in B_0$ and $z, v, w \in C_c(G)$ span a dense subset of $B_r \otimes C_r^*(G) \otimes C_r^*(G)$. We show that multiplying such an element by $(\delta_{X_r} \otimes \mathrm{id}_G) \circ \delta_{X_r}(\pi_X(x))$ or by $(\mathrm{id}_{X_r} \otimes \delta_G) \circ \delta_{X_r}(\pi_X(x))$ gives the same result.

On the one hand, we have

$$\begin{aligned}
&((\mathrm{id}_{X_r} \otimes \delta_G) \circ \delta_{X_r}(\pi_X(x))) \cdot ((\mathrm{id}_{B_r} \otimes \delta_G)((\delta_B(b) \cdot (1 \otimes \lambda^G(z))) \cdot (1 \otimes \lambda^G(v) \otimes \lambda^G(w))) \\
&\quad = (\mathrm{id}_{X_r} \otimes \delta_G)(\delta_X(x) \cdot \delta_B(b) \cdot (1 \otimes \lambda^G(z))) \cdot (1 \otimes \lambda^G(v) \otimes \lambda^G(w)) \\
&\quad = (\mathrm{id}_{X_r} \otimes \delta_G)(\delta_X(x \cdot b) \cdot (1 \otimes \lambda^G(z))) \cdot (1 \otimes \lambda^G(v) \otimes \lambda^G(w)) \\
&\quad = (\mathrm{id}_{X_r} \otimes \delta_G)(\pi_X \otimes \lambda^G((x \cdot b) \diamond z)) \cdot (1 \otimes \lambda^G(v) \otimes \lambda^G(w)) \quad \text{by (2·2)} \\
&\quad = (\pi_X \otimes (\delta_G \circ \lambda^G))((x \cdot b) \diamond z) \cdot (1 \otimes \lambda^G(v) \otimes \lambda^G(w)) \\
&\quad = (\pi_X \otimes \lambda^G \otimes \lambda^G)((x \cdot b) \triangleright (z \star (v \otimes w))),
\end{aligned}$$

where the last equation follows from (2·4). On the other hand, we have from (2·3) and the coaction identity for δ_{B_r} that

$$\left.\begin{aligned}
&(\mathrm{id}_{B_r} \otimes \delta_G)((\delta_B(b) \cdot (1 \otimes \lambda^G(z))) \cdot (1 \otimes \lambda^G(v) \otimes \lambda^G(w)) \\
&\qquad = ((\mathrm{id}_{B_r} \otimes \delta_G) \circ \delta_B(b)) \cdot ((1 \otimes \delta_G(\lambda^G(z))) \cdot (1 \otimes \lambda^G(v) \otimes \lambda^G(w))) \\
&\qquad = ((\delta_{B_r} \otimes \mathrm{id}_G) \circ \delta_B(b)) \cdot (1 \otimes (\lambda^G \otimes \lambda^G)(z \star (v \otimes w)))
\end{aligned}\right\} \quad (2·5)$$

and calculations like those which give (2·2) show that

$$(\delta_{X_r} \otimes \mathrm{id}_G)(\pi_X \otimes \lambda^G(y \diamond w)) \cdot (1 \otimes \lambda^G(v) \otimes 1) = (\pi_X \otimes \lambda^G \otimes \lambda^G)(y \triangleright (v \otimes w)). \quad (2·6)$$

Now we approximate $z \star (v \otimes w)$ in the inductive limit topology of $C_c(G) \odot C_c(G)$ by a finite sum $\sum_i v_i \otimes w_i$ and use (2·5) to compute

$$\begin{aligned}
&((\delta_{X_r} \otimes \mathrm{id}_G) \circ \delta_{X_r}(\pi_X(x))) \cdot ((\mathrm{id}_{B_r} \otimes \delta_G)((\delta_B(b) \cdot (1 \otimes \lambda^G(z))) \cdot (1 \otimes \lambda^G(v) \otimes \lambda^G(w))) \\
&\quad = ((\delta_{X_r} \otimes \mathrm{id}_G) \circ \delta_{X_r}(\pi_X(x))) \cdot ((\delta_{B_r} \otimes \mathrm{id}_G) \circ \delta_B(b)) \cdot (1 \otimes (\lambda^G \otimes \lambda^G)(z \star (v \otimes w))) \\
&\quad = ((\delta_{X_r} \otimes \mathrm{id}_G) \circ \delta_X(x \cdot b)) \cdot (1 \otimes (\lambda^G \otimes \lambda^G)(z \star (v \otimes w))) \\
&\quad \sim ((\delta_{X_r} \otimes \mathrm{id}_G) \circ \delta_X(x \cdot b)) \cdot \Big(1 \otimes (\lambda^G \otimes \lambda^G)\Big(\sum_i v_i \otimes w_i\Big)\Big) \\
&\quad = \sum_i ((\delta_{X_r} \otimes \mathrm{id}_G) \circ \delta_X(x \cdot b)) \cdot (1 \otimes \lambda^G(v_i) \otimes \lambda^G(w_i)) \\
&\quad = \sum_i ((\delta_{X_r} \otimes \mathrm{id}_G)(\delta_X(x \cdot b) \cdot (1 \otimes \lambda^G(w_i))) \cdot (1 \otimes \lambda^G(v_i) \otimes 1) \\
&\quad = \sum_i (\delta_{X_r} \otimes \mathrm{id}_G)(\pi_X \otimes \lambda^G((x \cdot b) \diamond w_i)) \cdot (1 \otimes \lambda^G(v_i) \otimes 1)
\end{aligned}$$

by (2·2). Now we use (2·6) to continue:

$$\begin{aligned}
&= \sum_i (\pi_X \otimes \lambda^G \otimes \lambda^G)((x \cdot b) \triangleright (v_i \otimes w_i)) \\
&= (\pi_X \otimes \lambda^G \otimes \lambda^G)\Big((x \cdot b) \triangleright \Big(\sum_i v_i \otimes w_i\Big)\Big) \\
&\sim (\pi_X \otimes \lambda^G \otimes \lambda^G)((x \cdot b) \triangleright (z \star (v \otimes w))).
\end{aligned}$$

This completes the proofs of Proposition 2·2 and Theorem 2.

Remark 2·3. It is important in Theorems 1 and 2 that we inflate the coactions all the way up to the group G: K and H could both lie in a smaller subgroup L, but the inflated coactions of L need not be Morita equivalent. For example, suppose $K = \{e\}$, so that we can take $L := H$. If we further take $D = \mathbb{C}$, then $\mathrm{Ind}_K^G\mathbb{C} = C_0(G)$, $\mathrm{Ind}_H^G\mathbb{C} = C_0(G/H)$ and the symmetric imprimitivity theorem says $C_0(G) \rtimes_\sigma H$ is Morita equivalent to $C_0(G/H)$. Since $K = \{e\}$, the inflated coaction of H on $C_0(G/H)$ is trivial and an H-equivariant version of the theorem would imply that $(C_0(G) \rtimes_\sigma H) \times_{\hat\sigma} H$ is Morita equivalent to $C_0(G/H) \otimes C_0(H)$; by the Rieffel correspondence, this equivalence would induce a homeomorphism on spectra. But the spectrum G of

$$(C_0(G) \rtimes_\sigma H) \times_{\hat\sigma} H \cong C_0(G) \otimes \mathcal{K}(L^2(H))$$

need not be homeomorphic to the spectrum $G/H \times H$ of $C_0(G/H) \otimes C_0(H)$, for example if $G = \mathbb{R}$ and $H = \mathbb{Z}$.

Remark 2·4. The symmetric imprimitivity theorem of [**R1**] concerns a locally compact space P which carries commuting free and proper actions of two groups K and H. The previous remark shows why we do not expect an equivariant version in this generality: so far at least, it does not make sense to talk about coactions of a space.

3. *When is a dual system induced?*

For our application, we shall need the one-sided version of our main theorem.

COROLLARY 3·1. *Let H be a closed subgroup of G, let β: $H \to \mathrm{Aut}\, D$ be an action of H and let τ: $G \to \mathrm{Aut}\,(\mathrm{Ind}_H^G(D,\beta))$ be the action of G by left translation. Then $\mathrm{Inf}_H^G\widehat{\beta}$ is Morita equivalent to $\hat\tau$.*

Proof. Take $K = G$ and $\alpha = \mathrm{id}$ in Theorem 2. The action $\tau\otimes\mathrm{id}$ of G on $\mathrm{Ind}_H^G(D,\beta)$ is precisely the action τ we consider here and $\mathrm{Inf}_G^G\hat\tau = \hat\tau$. The map Φ: $f \mapsto f(e)$ is an H-equivariant isomorphism of $\mathrm{Ind}_G^G(D,\mathrm{id})$ onto D, so the dual coactions $(\sigma \otimes \beta)^\wedge$ and $\widehat{\beta}$ are conjugate to each other. But then the same is true for their inflations to G and Theorem 2 provides a Morita equivalence between $\mathrm{Inf}_H^G\widehat{\beta}$ and $\hat\tau$.

If δ: $A \to M(A\otimes C_r^*(G))$ is a coaction, and A acts faithfully on $\mathcal{H}$, then the crossed product $A \times_\delta G$ is the closed linear span in $B(\mathcal{H} \otimes L^2(G))$ of

$$\{\delta(a)(1 \otimes M_f)\colon a \in A, f \in C_0(G)\},$$

where M denotes the action of $C_0(G)$ by pointwise multiplication on $L^2(G)$ (see [**LPRS**, lemma 2·5]). Up to isomorphism, $A \times_\delta G$ is independent of the choice of faithful representation of A. The dual action $\hat\delta$: $G \to \mathrm{Aut}\,(A \times_\delta G)$ is characterised by

$$\hat\delta_s(\delta(a)(1 \otimes M_f)) = \delta(a)(1 \otimes M_{\sigma_s(f)}).$$

If ε: $A \to M(A \otimes C_r^*(H))$ is a coaction of a subgroup H of G on A, then the inflated system $(A \times_{\mathrm{Inf}_H^G\varepsilon} G, G, (\mathrm{Inf}_H^G\varepsilon)^\wedge)$ and induced system $(\mathrm{Ind}_H^G(A \times_\varepsilon H, \hat\varepsilon), G, \tau)$ are isomorphic. (For normal H, this is the untwisted version of [**QR**, theorem 4·4]. The general version is proved in [**R2**, theorem 5] for full coactions; basically the same arguments carry over for the reduced coactions we consider here.) From our main theorem we can deduce the following converse, which characterises the coactions whose duals are induced. Since dual coactions are nondegenerate in the sense that $\delta(A)(1 \otimes C_r^*(G)) = A \otimes C_r^*(G)$, this is a necessary hypothesis.

Corollary 3·2. *Suppose that $\delta\colon A \to M(A \otimes C_r^*(G))$ is a nondegenerate coaction. If the dual system $(A \times_\delta G, G, \widehat{\delta})$ is isomorphic to the induced system $(\mathrm{Ind}_H^G(D,\beta), G, \tau)$, then δ is Morita equivalent to $\mathrm{Inf}_H^G \widehat{\beta}$.*

Proof. Assume that $\big(A \times_\delta G, G, \widehat{\delta}\big) \cong (\mathrm{Ind}_H^G(D,\beta), G, \tau)$. Then the double-dual coaction $\widehat{\widehat{\delta}}$ is isomorphic to $\widehat{\tau}$, which is Morita equivalent to $\mathrm{Inf}_H^G \widehat{\beta}$ by the above corollary. Thus $\widehat{\widehat{\delta}}$ is also Morita equivalent to $\mathrm{Inf}_H^G \widehat{\beta}$. Since δ is nondegenerate, it follows from Katayama's duality theorem, as formulated in [**EKR**, proposition 5·4], that δ is Morita equivalent to $\widehat{\widehat{\delta}}$. Thus δ is Morita equivalent to $\mathrm{Inf}_H^G \widehat{\beta}$.

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