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## Keywords

cold, tandem, schedules, mills, rolling, toward, design, optimum, heuristic

## Disciplines

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# Toward a heuristic optimum design of rolling schedules for tandem cold rolling mills

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## Abstract

Scheduling for tandem cold mills refers to the determination of inter-stand gauges, tensions and speeds of a specified product. Optimal schedules should result in maximized throughput and minimized operating cost. This paper presents a genetic algorithm based optimization procedure for the scheduling of tandem cold rolling mills. The optimization procedure initiates searching from a logical starting point — an empirical rolling schedule — and ends with an optimum cost. Cost functions are constructed to heuristically direct the genetic algorithm's searching, based on the consideration of power distribution, tension, strip flatness and rolling constraints. Numerical experiments have shown that the proposed method is more promising than those based on semi-empirical formulae. The results generated from a case study show that the proposed approach could significantly improve empirically derived settings for the tandem cold rolling mills. © 2000 Elsevier Science Ltd. All rights reserved.

*Keywords:* Rolling schedules; Evolutionary algorithms; Tandem cold rolling; Process optimization

## 1. Introduction

Automation systems for tandem cold rolling mills are continuously being improved due to today's stringent high throughput, quality and low scrap loss requirements for products. To consolidate competitive strengths in the global market, many steel companies are engaged in maximizing the reduction and consequently minimizing the cost of manufacture (Bryant, 1973; Yuen and Nguyen, 1996; Ozsoy et al., 1992). Rolling scheduling is an important aspect in the operation of tandem cold rolling mills. It defines stand reductions, tensions, rolling forces, roll torque, mill maximum speeds, and threading adjustments. The optimized scheduling should lead to improved thickness, surface finish and shape performance of the products.

In the last two decades, only a few papers have

addressed the rolling scheduling problem, especially for tandem cold rolling (Wang et al., 1998). An early work has led to the development of mill scheduling systems achieving correct output and satisfactory shape in a tandem cold rolling mill. The scheduling is described as a constrained two-point boundary-value problem that is solved using conjugate gradient and projection techniques. The cost functions defined for the optimization problem include the strip shape cost, tension cost and thermal-crown cost. Although the results generated from the optimized schedules are better than those from the original empirical schedules, power cost is not considered, while a uniform power distribution is desirable for the tandem cold rolling mills. Moreover, the calculation of the costs relies mostly on some linear equations, with linear coefficients given to the rolling parameters. Although the conjugate gradient methods are frequently used in practice, even when the cost function for the optimization problem is not convex, there are reasons to believe that such use leads to the computation of a

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local minimum (Polak, 1971; Luenberger, 1984; Nash and Ariela, 1996). The computing equipment available at that time also limits the calculation capacity. Ozsoy et al. employed a nonlinear programming method called the hill-climbing algorithm to optimize rolling schedules for a hot rolling process. The results show that although the optimization problem cannot be solved in a closed form because of the non-linearity of the defining equations and the amplitude constraints on the system variables, it can be solved numerically on a digital computer by nonlinear programming. However, the convergent behavior of the nonlinear programming method employed in (Ozsoy et al., 1992) is directly affected by the initial searching point used. Some other nonlinear programming methods, such as sequential quadratic programming, also have a number of disadvantages besides their added complexity (derivative calculations, etc.), such as the local minimum problem, nonguaranteed convergence, and expensive calculation cost. As an intelligent searching mechanism, genetic algorithms (GA) have potential, and seem to be flexible enough to overcome the abovementioned disadvantages.

This paper presents an investigation into an optimal scheduling for tandem cold rolling mills based on GAs. The scheduling process includes the following steps:

1. A rolling model is set up to establish the relationships among the rolling parameters.
2. The constraints in the GA-based optimization, i.e., the cost functions and validity checks, are defined. The cost functions include the power distribution cost function, the tension cost function and the optimum flatness condition. The validity checks include the roll force and torque limitations, work roll speed references, strip exit thickness, and threading conditions and tension force limitations. Once the effective cost functions have been properly constructed, the optimum scheduling problem is transformed into a nonlinear optimization problem.
3. The GAs are used to optimize the schedule. A total cost function is employed as the fitness function of the GA during the search for optimized rolling parameters.
4. The optimal rolling parameters generated from the GA optimization procedure are further checked against the practical rolling constraints, to ensure that the optimal rolling parameters would not result in unrealistic settings.

The paper is organized as follows: the basic principles of tandem cold rolling mill scheduling are described in Section 2; the GA-based optimization for scheduling is addressed in Section 3, followed by numerical experiments and conclusions.

## 2. Basic scheduling for tandem cold rolling mills

The basic procedure for the scheduling of tandem cold rolling mills is usually based on past experience, on trials or on rules of thumb. A typical scheduling procedure for the setup of tandem cold rolling mills is illustrated in Fig. 1.

As an illustration, the following steps describe such a semi-empirical scheduling procedure.

1. Based on the coil data and semi-empirical stand thickness-reduction patterns  $R'_i$  ( $1 \leq i \leq N$ );  $N$  is the number of stands), a reduction at each stand is allocated. The coil data includes strip material grade, strip width  $W$ , strip entry thickness  $H$  and strip exit gauge  $h$ . The stand thickness-reduction patterns are empirically generated on the basis of providing uniform power distribution and consistent flatness at each stand. The patterns vary with total mill reduction rate and last stand conditions (namely, shot-blast or bright rolling mode).
2. According to the empirical equations, inter-stand tension stresses (including front tension stress  $t_f$  and back tension stress  $t_b$ ) are determined. These tension stresses depend on the strip width, nominal yield stress deviation of the coil material, and the mill exit thickness at each stand, generated in Step 1.
3. Based on the Bland–Ford–Hill force formula, the roll force  $P_i$  ( $1 \leq i \leq N$ ) at each stand is calculated. The formula is a function of variables including the deformed work roll radius  $R'_w$ , the average yield stress  $\bar{k}$ , the stress state coefficient  $Q_p$ , the front and back tensions determined in Step 2, the strip width, strip entry and exit gauges at each stand, and the friction coefficient at each stand.
4. The forward slip ratio  $f_i$  ( $1 \leq i \leq N$ ) at each stand is calculated from the Bland–Ford forward slip formula, hence determining the location of the neutral point of roll bite.
5. Based on the torque formula derived by Bryant, the

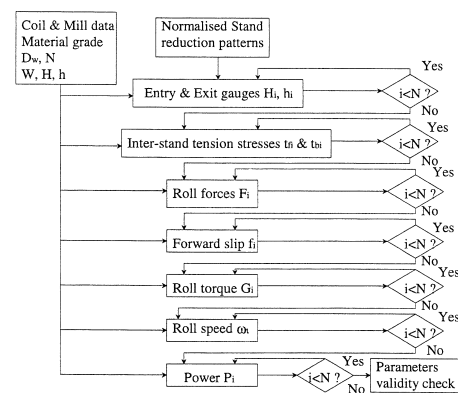


Fig. 1. A typical rolling scheduling procedure for the setup of tandem cold rolling mills.

roll torque  $G_i$  ( $1 \leq i \leq N$ ) for each stand is determined. The torque formula is a function of the work roll diameter  $D_w$ , the angle of contact, the yield stresses at the roll bite entry  $k(H)$  and exit  $k(h)$ , and the front and back tension stresses.

6. According to the constant mass flow principle, the roll speed  $\omega_i$  ( $1 \leq i \leq N$ ) at each stand is calculated. The forward slip factor and the motor droop are considered in the calculation.
7. The power  $P_{wi}$  ( $1 \leq i \leq N$ ) at each stand is estimated, based on the results of the roll speed and the roll torque in Steps 5 and 6.
8. The validity of the rolling parameters is checked to ensure that none of the rolling parameters has exceeded the mill capability; for instance, the limitations of the physical capability of the rolling mill, and electrical requirements.

After the schedule has been determined, the exit strip thickness and speed of each stand, the inter-stand tension stresses, etc., are employed for further calculation of the corresponding actuator references. On the basis of these references, the rolling mill will then be preset to roll the required product.

### 3. Optimized scheduling

A schedule is a sequence of elements, where every element is considered as the aggregate of all the important system parameters, described as a combination of their respective cost functions. An optimal sequence is arrived at wherever there exists a minimum mismatch between succeeding sequence elements. Optimum rolling scheduling deals with the selection of rolling mill schedule parameters, such as the reductions and motor speeds, which lead to the desired thickness, surface finish and shape for a given product at a minimum cost. In fact, these key schedule parameters fall at corresponding intervals in the data space as described in Section 3.3.1. The objective of the optimal scheduling is to find the best presetting for the parameter combination. The optimization is essentially a nonlinear data-sampling process. Usually, the effective cost functions are first defined, and then optimization techniques are employed to search for the optimal solution. In this paper, an optimization methodology based on GAs is employed to solve the mill-scheduling problem.

#### 3.1. Cost functions

During optimization, one of the constraints should be a constant mass flow at each of the stands. At the same time, the optimization should meet the requirements of all the constraints, and minimize the cost of

the rolling operation. The cost functions include the power distribution cost function, the tension cost function and the perfect shape condition.

##### 3.1.1. Power distribution cost

One of the most important objectives in the scheduling is to provide a uniform power distribution and consistent flatness at each stand. Hence, the power distribution is a suitable measurement of how well the current schedule is meeting this requirement. The power distribution cost function is defined as follows:

$$\text{Cost\_Power} = K_1 \sum_{i=1}^{N-1} \sum_{\substack{j=2 \\ j \neq i}}^N (P_{wi} - P_{wj})^2 \quad (1)$$

where  $K_1$  is the weighting constant, and  $N$  is the total number of stands. The power at each stand is calculated according to the following equation:

$$P_{wi} = \omega_i^* G_i \quad (2)$$

where  $\omega_i^*$  and  $G_i$  are the work roll rotational speed and torque at stand  $i$ , respectively.

According to the constant mass flow guideline,  $Wh_i v_i = WH_i V_i = \text{constant}$ , where  $W$  is the strip width,  $H_i$  and  $h_i$  are the input and output strip thicknesses, and  $V_i$  and  $v_i$  are the input and output strip velocities, respectively. By considering the forward slip factor (Eq. (3)) and the motor droop (Eq. (4)), the work roll speed can be described as Eq. (5):

$$f_i = v_i / \omega_i R_i \quad (3)$$

$$\omega_i^* = \omega_i (1 + d_i G_i / G_{\text{base } i}) \quad (4)$$

$$\omega_i^* = \frac{h_5 v_5 f_5}{h_i f_i R_i} (1 + d_i G_i / G_{\text{base } i}) \quad (5)$$

where  $h_5$  and  $v_5$  are the strip thickness and speed at the exit from stand 5. The roll torque formula can now be derived as follows:

$$G_i = RPC_p + R(t_f H - t_b h) / 2.0 \quad (6)$$

where

$$C_p = \frac{L \left[ \frac{1}{3}(k(h) - t_b) + \frac{2}{3}(k(H) - t_f) \right]}{\left[ \frac{K(H) - t_f}{E} \right] + \left[ \frac{K(h) - t_b}{E} \right]} \quad (7)$$

$$L = \sqrt{R'(H - h_m)} \quad (8)$$

$$h_m = h \left[ 1 - (1 - v^2)(k(h) - t_s) / E \right] \quad (9)$$

Here,  $t_f$  and  $t_b$  are the front and back tension stresses at the stand, respectively;  $k(H)$  and  $k(h)$  are the

yield stresses at entry and exit points of roll bite, respectively;  $\nu$  is Poisson's ratio of the strip;  $R'$  is the deformed work roll radius which is described by Eq. (15) according to Hitchcock's equation; and  $P$  is the roll force that can be obtained based on the Bland–Ford–Hill force calculation formula (Bland and Ford, 1948, 1952), namely

$$P = Wk(\bar{r})\sqrt{\frac{R'}{H}}Q_p n_t \quad (10)$$

where  $k(\bar{r})$  is the average yield stress evaluated at the mean reduction bar  $\bar{r}$ , defined as Eq. (12):

$$k(\bar{r}) = a_1(\bar{r} + a_2)^{a_3} \quad (11)$$

$$\bar{r} = 1 - 0.4H/H_0 - 0.6h/H_0 \quad (12)$$

where  $H_0$  is the annealed strip thickness. According to Hill's experimental results,

$$Q_p = 1.08 + 1.79u\frac{H-h}{H}\sqrt{\frac{R'}{H}} - 1.02\frac{H-h}{H} \quad (13)$$

$$n_t = 1 - \frac{T_s u_t + t_s(1 - u_t)}{k(\bar{r})} \quad (14)$$

$$R' = R \left[ 1 + \frac{cP}{W(H-h)} \right] \quad (15)$$

$$c = 16(1 - \nu_{\text{roll}}^2)/(\pi E_{\text{roll}}) \quad (16)$$

where  $u_t$  is a constant;  $\nu_{\text{roll}}$  is Poisson's ratio of the work roll;  $E_{\text{roll}}$  is Young's modulus of the work roll;  $R$  is the work roll radius; and  $u$  is the friction coefficient.

### 3.1.2. Tension cost function

The tension between stands should be kept midway between its lower limit  $t_{f \min}^i$  and upper limit  $t_{f \max}^i$ . The lower limit is specified as the maximum value of the measured noise under mill conditions, while the upper limit is assigned on the basis of a consideration of strip skidding and tearing situations. Normally, the tension cost function is described as follows:

$$\text{Cost\_Tension} = K_2 \sum_{i=1}^N \left[ t_{fi} - \left( \frac{t_{f \min}^i + t_{f \max}^i}{2} \right) \right]^2 \quad (17)$$

where  $t_{fi}$  is the tension between two adjacent stands (the  $i$ th and  $(i + 1)$ th stands),  $K_2$  is the weighting constant of the tension cost function, and  $N$  is the total number of stands. When  $N = 5$ , the tension  $t_{f5}$  means the tension between the fifth stand and the coiler of the mill.

### 3.1.3. Perfect shape condition

Good rolling scheduling should lead to uniformity of the tension distribution in the strip width direction, or the minimal possibility of the strip buckling during rolling. The optimum shape condition is obtained when the strip is uniformly rolled across the strip width (ignoring the insignificant lateral spread in cold rolling). This means that the deformed roll profile should perfectly match the incoming transverse strip thickness profile geometrically. For example, the input profile of the incoming strip may be described as follows:

$$H(x) = H_c + P_1 x^2 + Q_1 x^4 \quad (18)$$

The reason to choose the model with  $x^2$  and  $x^4$  in Eq. (18) is that both the strip to be rolled and the strip after rolling are usually symmetrical about the centerline. So neither  $x$  nor  $x^3$  is considered in the model. The parameters  $H_c$ ,  $P_1$ , and  $Q_1$  are determined by the measured strip thickness profile, where  $H(x)$  is the entry thickness of the strip at a location of a distance  $x$  from the strip centerline,  $H_c$  is the entry thickness of the strip at the strip centerline, and  $P_1$  and  $Q_1$  are constants. The output profile is shown in Fig. 2 and can be described as follows:

$$h(x) = h_c + P_2 x^2 + Q_2 x^4 \quad (19)$$

where  $h(x)$  is the exit thickness of the rolled strip at a location of a distance  $x$  away from the strip centerline,  $h_c$  is the exit thickness of the strip at its centerline, and  $P_2$  and  $Q_2$  are constants.

It can be shown that the condition for a perfect shape requires:

$$P_2 = P_1(1 - r) \quad (20)$$

$$Q_2 = Q_1(1 - r) \quad (21)$$

where  $r$  is the thickness reduction.

Thus, if the profile of the rolls exactly matches the incoming transverse gauge profile of the strip geometrically, a perfect exit shape will be obtained (Sabatini and Yeomans, 1968).

So the cost function for perfect shape can be described as follows:

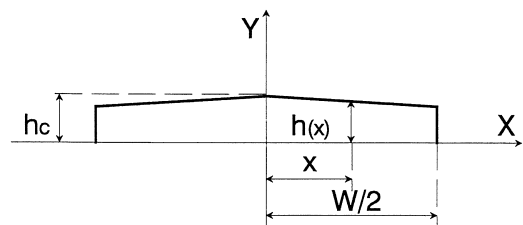


Fig. 2. The transverse profile of a rolled strip.

$$\text{Cost\_Shape} = K_3 \sum_{i=1}^N 2 \int_0^{W/2} [y_w(x) - C_w(x) - h(x)]^2 dx \quad (22)$$

where  $K_3$  is the weighting constant, and  $y_w(x)$  is the deformed roll profile, which is a function of the roll force and the roll bending force if used. It is calculated by summing the deflections due to bending, shear and work roll flattening.  $C_w(x)$  is the total work roll crown, including the machined crown, the thermal crown and the roll wear. For mills with roll-bending jacks, the deformed roll profile could be geometrically adjusted by changing the jack force, which would be reflected in the term  $y_w(x)$ . The deformed work roll profile is obtained by calculating the roll deflections due to bending, shear, the effect of Poisson’s ratio, the bending moment, the interference between the work and backup rolls, and the interference between the work roll and the strip, which are described in the following sections.

**3.1.3.1. The deflection due to bending.** Beam theory for the bending and shear components has been widely employed to calculate the roll deflections. A typical roll deflection model under the effect of point loads is shown in Fig. 3.

The roll deflection of the beam under the effect of bending at a position  $x$  can be described as follows:

$$y(x) = \begin{cases} [q(z) - p(z)](L - z)^2 [3(L - x) - L + z] / (6EI) & x \leq z \\ [q(z) - p(z)] [(x - z)^3 - (L - y)^2 (3x - y - 2L)] / (6EI) & x > z \end{cases} \quad (23)$$

where  $E$  is Young’s modulus,  $I$  is the second moment of area, and  $p(z)$  and  $q(z)$  are point loads.

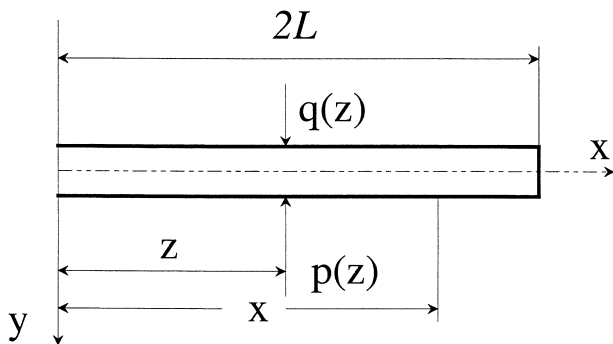


Fig. 3. A deflection of the neutral axis due to point loads.

**3.1.3.2. The deflection due to shear.** According to O’connor and Weinstein (1972), the deflections of the neutral axis for short stubby beams due to shear are given by:

$$y(x) = \begin{cases} 4(L - z)[q(z) - p(z)] / (3AG) & x \leq z \\ 4(L - x)[q(z) - p(z)] / (3AG) & x > z \end{cases} \quad (24)$$

where  $A$  is the cross-sectional area and  $G$  is the shear modulus of the beam.

**3.1.3.3. The deflection due to a bending moment.** If there is a bending moment, the deflection of the neutral axis can be expressed as:

$$y(x) = M[(L - x)^2 + \nu R(x)^2] / (2EI) \quad (25)$$

where  $\nu$  is Poisson’s ratio, and  $M$  is the bending moment (Fig. 4) (O’connor and Weinstein, 1972).

**3.1.3.4. Deflection due to the effect of Poisson’s ratio.** The deflection of the roll neutral axis due to the effect of Poisson’s ratio on the movement of the surfaces is given by:

$$y(x) = \begin{cases} 0 & x \leq z \\ \nu R^2 [q(z) - p(z)] (x - z) / (2EI) & x > z \end{cases} \quad (26)$$

where  $R$  is the work roll radius.

**3.1.3.5. Interference between the work and backup rolls.** Based on the assumption of two infinitely long elastic cylinders in contact (Cresdee et al., 1991), the interference under inter-roll pressure  $q(x)$  can be described as follows:

$$y_{wb}(x) = q(x)(c_w + c_b) \ln \left\{ e^{2/3} (D_w + D_b) / [2q(x) \times (c_w + c_b)] \right\} \quad (27)$$

where  $y_{wb}(x)$  is the interference between the work and backup rolls; subscripts w and b refer to the work roll and the backup roll, respectively; and  $c_w$  and  $c_b$  are

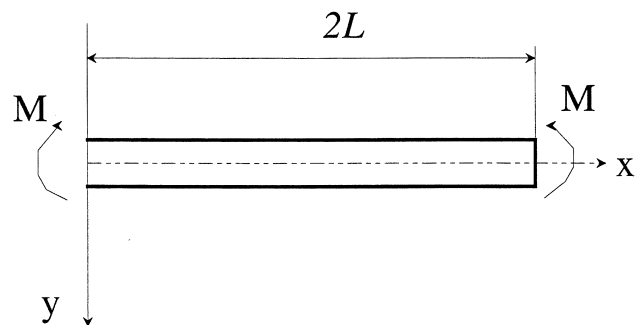


Fig. 4. The deflection due to a bending moment.

determined by the following equation:

$$c_w = c_b = (1 - \nu^2)/(\pi E) \quad (28)$$

where  $\nu$  is Poisson's ratio.

Based on the contours of the work roll and the backup roll, the interference between the work and backup rolls can be calculated as follows:

$$y_{wb}(x) = y_{wb}(0) + y_b(x) - y_w(x) - C_b - C_w \quad (29)$$

where  $y_{wb}(0)$  is the total roll interference at the strip center,  $y_b(x)$  is the total backup roll-axis deflection, and  $c_b(x)$  is the total backup roll crown including the ground crown, the thermal crown and the roll wear.  $y_w(x)$  is the total work roll-axis deflections, and  $C_w(x)$  is the total work roll crown, including the ground crown, the thermal crown and the roll wear.

**3.1.3.6. Work roll flattening.** Work roll flattening at the contact area of the work roll and strip can be described as follows:

$$y_{ws}(x) = [b_1 + b_2 p(x)] y_H(x) \quad 0 < x \leq W \quad (30)$$

where  $W$  is the strip half-width, and  $y_H(x)$  is given by

$$y_H(x) = 2p(x)c_w \ln \left[ e^{2/3} D_w / (2c_w p(x)) \right] \quad (31)$$

Here,  $p(x)$  is the total roll gap pressure, and  $b_1$  and  $b_2$  are constants determined by experiments. For mild steel,  $b_1$  and  $b_2$  are estimated at 32.92 and 0.86 mm<sup>2</sup>/kN, respectively. When rolling tinplate, the embedding of the strip can be sufficient to result in work roll contact beyond the edges of the strip. The interference between two work rolls,  $y_{ww}(x)$ , can be calculated as follows:

$$y_{ww}(x) = y_{ws}(0) - h(0) + 2[y_w(x) - C_w(x)] \quad W < x \leq l \quad (32)$$

where  $l$  is the work roll half-width, and  $h(0)$  is the exit strip thickness at the strip center.

**3.1.3.7. Total roll deflections.** The total roll deflections are obtained by adding the deflections given by Eqs. (23), (24), (26), (30) and (25) if a bending moment exists.

### 3.1.4. Total cost function

In the above cost functions (Eqs. (1), (17) and (22)), the power is a function of the roll torque and speed. The roll torque is related to the roll force, which is in turn a function of the entry and exit strip tensions, and the input and output strip thicknesses. So the optimization problem addressed here involves combinations of numerous variables. Finally, the complete

cost function is described as follows:

$$\begin{aligned} \text{Cost} = & K_1 \sum_{\substack{i, j=1 \\ i \neq j}}^N (P_{wi} - P_{wj})^2 + K_2 \sum_{i=1}^N \left[ t_{fi} \right. \\ & \left. - \left( \frac{t_{f \min}^i + t_{f \max}^i}{2} \right) \right]^2 + K_3 \sum_{i=1}^N 2 \int_0^W [y_w(x) \\ & - C_w(x) - h(x)]^2 dx \end{aligned} \quad (33)$$

where  $K_1$  is the weighting constant for power distribution cost defined in Eq. (1),  $K_2$  is the weighting constant for tension cost described in Eq. (17), and  $K_3$  is the weighting constant for strip shape cost in Eq. (22). Determination of  $K_1$ ,  $K_2$  and  $K_3$  is based on their significance for individual rolling conditions. A large value of  $K_1$  means that dominant consideration is given to uniform power distributions for maximizing the throughput of the rolling mill. If a thin strip is rolled,  $K_2$  should be assigned a large value to make sure that neither skidding nor tearing occurs. Strip shape is always an important factor to be considered for final product quality. So  $K_3$  should be assigned as large a value as possible, based on  $K_1$  and  $K_2$  satisfying the condition  $K_1 + K_2 + K_3 = 1$ .

## 3.2. Constraints for validity checks

### 3.2.1. Roll force and roll torque

The total roll force and roll torque are limited to the corresponding maximum values due to the mechanical design limits imposed by manufacturers of the rolling mill and electrical drive motors. This constraint can be described as follows:

$$P_i \leq P_{\max} \quad (34)$$

$$G_i \leq G_{\max} \quad (35)$$

### 3.2.2. Strip exit thickness

The exit strip thickness is kept within its upper and lower limits as follows:

$$h_{\min} \leq h_i \leq h_{\max} \quad (36)$$

The bounds  $h_{\min}$  and  $h_{\max}$  are determined on the basis of the validity check requirements of the rolling mill. Usually, they are physically determined by the mechanical design limits of the mill. The validity checks are used to ensure that operator adjustments do not result in an infeasible set of screw positions on adjacent stands.



### 3.2.3. Tension force

The tension stress at each stand should not exceed its upper and lower limits.

$$t_{f \min}^i \leq t_{fi} \leq t_{f \max}^i \quad (37)$$

The  $t_{f \min}^i$  is based on a lower limit on the tension, set by measurement noise, so that the strip does not loop between two neighboring stands. The  $t_{f \max}^i$  is set by strip tearing and skidding considerations. The  $t_{f \max}^i$  is usually assigned a value of about one-third of the yield stress of the strip.

After the construction of the cost functions and constraints, the schedule-optimization problem may be described as: starting from an initial searching point  $\{h_1, h_2, h_3, h_4, h_5\}$ , find a satisfactory combination of the strip exit gauges at minimum cost without violating the constraint limitations.

## 3.3. Genetic-algorithm-based optimization

### 3.3.1. Genetic algorithms

Genetic algorithms were first proposed by Holland as heuristic searching mechanisms for intelligent systems. Though their use has been growing since the early 1970s (Holland, 1969, 1975), only recently has their commercial potential been demonstrated (Goldberg, 1989; Davis, 1991; Fogel, 1995). The main operations in a GA include parent selection, reproduction, crossover, and mutation. The goal of these operations is to generate meaningful offspring. A typical step-by-step procedure for a GA is described as follows:

1. Define a fitness function for the optimization problem, e.g. Eq. (33) in this paper.
2. Encode the variables into binary codes, e.g.  $h_1, h_2, h_3, h_4$ , and then combine the individual binary code for each variable together into a binary chain, as a combined single variable.
3. Initialize a population as the population for the first generation.
4. Evaluate each individual in the population.
5. Implement the genetic operations: reproduction, crossover, and mutation.
6. Rank the population.
7. Delete the lowest-ranked genomes in the population, and keep high-ranked individuals.
8. Repeat Steps 5–7 until the evolutionary termination criterion is satisfied.

#### 3.3.1.1. Genetic encoding of rolling scheduling problem.

In the optimization of the rolling schedule, as the exit thickness at the last stand is fixed, for a five-stand tandem rolling mill, only the exit gauges at the first four stands  $h_1, h_2, h_3, h_4$  need to be encoded. For such a

multiple variable encoding problem, each variable should be encoded first, and then linked together as a chain. For example, suppose that a steel strip with an initial gauge of 2.5 mm is to be rolled to 0.5 mm, with the following exit gauges based on the empirical reduction patterns:

$$h_1 = 1.874, \quad h_2 = 1.214, \quad h_3 = 0.853, \quad h_4 = 0.623,$$

$$h_5 = 0.50$$

If the average of two adjacent exit gauges is taken as the boundary between the possible value fields of these two exit gauges, the possible value fields for the exit gauges are as follows:

$$1.54 \leq h_1 \leq 2.5, \quad 1.03 \leq h_2 \leq 1.54, \quad 0.74 \leq h_3 \leq 1.03,$$

$$0.56 \leq h_4 \leq 0.74, \quad h_5 = 0.50$$

If the granularity for the exit gauge at each stand is 0.01, the total number of binary bits representing  $h_1$  should be 7, since  $2^6 \leq (2.5 - 1.54)/0.01 \leq 2^7$ ; 6 bits for representing  $h_2$  since  $2^5 \leq (1.54 - 1.03)/0.01 \leq 2^6$ ; 5 bits for representing  $h_3$  since  $2^4 \leq (1.03 - 0.74)/0.01 \leq 2^5$ ; and 5 bits for representing  $h_4$  since  $2^4 \leq (0.74 - 0.56)/0.01 \leq 2^5$ . The binary bits representing each exit gauge are finally linked together as a binary chain. So the total bit number of the final chain is 23.

The initial population is usually chosen at random, or by a heuristic approach. In order to get the best solution, as many points in the search space as possible should be searched. Thus, some variations are introduced into the new population by genetic operations, which include reproduction, crossover and mutation.

**3.3.1.2. Operation of the genetic algorithms.** To maximize the possibility of an improvement being achieved by a new population, reproduction is employed to select some parent binary strings (also called chromosomes) with high fitness values. This means that if a binary structure represents a point in a high-performance area of the search space, it may lead to further exploration in this area of the search space. The selected populations are then duplicated, and go on to the next generation. The most important genetic operation is crossover. Under the crossover operator, two binary structures in the new population exchange portions of their binary bits. This is implemented by randomly choosing a crossover point, and exchanging the segments to the right of the crossover point. Also, multiple crossover points could be randomly assigned to get higher evolutionary efficiency. To avoid convergence to a local minimum area during searching, mutation is employed to make the search jump outside the local minimum area. This is implemented by changing the binary value 1 to 0 or 0 to 1 at a randomly

Table 1  
Comparison of an empirical rolling schedule and the optimized schedules

Rolling parameters	Empirical schedule	Optimized schedule 1	Optimized schedule 2	Optimized schedule 3
		$K_1 = 0.4, K_2 = 0.2, K_3 = 0.4$	$K_1 = 0.35, K_2 = 0.3, K_3 = 0.35$	$K_1 = K_2 = K_3 = 0.3333$
Exit gauge (mm)				
Stand 1	1.874	1.715	1.778	1.792
Stand 2	1.214	1.223	1.248	1.257
Stand 3	0.853	0.894	0.915	0.925
Stand 4	0.623	0.644	0.662	0.668
Stand 5	0.500	0.500	0.500	0.500
Cost of power	1.0	0.6673	0.8980	0.9714
Cost of tension	1.0	1.0516	0.6880	0.6069
Cost of shape	1.0	0.9963	0.9981	0.9986
Total cost	1.0	0.8758	0.8700	0.8590

selected bit of a binary representation. The whole genetic operation is a systematic binary structure recombination process.

**3.3.1.3. Evaluation.** A fitness function is used to measure the performance of each binary string. How close a particular binary string is to the final optimization destination is judged by its corresponding fitness value, and whether the rolling parameters resulting from it violate the constraints described in Section 3.2. In this paper, the total cost function (Eq. (33)) is employed as the fitness function. All the binary strings are ranked on the basis of how small their fitness values are. The higher a string is ranked, the smaller its fitness value is. The top 20 binary strings will be selected as the candidates for the next generation. The whole evolutionary process ceases once the termination criterion is met. The termination criterion in this research is the total number of trials, 5000. Numerical experiments show that this termination criterion is suitable for the research addressed here.

The general genetic model has been constructed in C/C++. Numerical calculations were carried out on Pentium 200, Pentium II 266 and 400 PCs. The whole optimization process took 33 s on the Pentium II 400 MHz PC, 43 s on the Pentium II 266 MHz PC and 99 s on a Pentium 200 MHz PC. The final optimization results are reported in the following section.

### 3.3.2. Optimization parameters and results

**3.3.2.1. Optimization parameters.** The total number of trials is 5000 for this optimization procedure. The initial population size is assigned to 20, which means the number of initial randomly generated binary strings is 20. The binary structure length is 23. The crossover and mutation rates are set to 0.65 and 0.005, respectively. The numerical calculation is done under the following three conditions:

1. The coefficients of the cost function are assigned to 0.4 for  $K_1$ , 0.2 for  $K_2$ , and 0.4 for  $K_3$ .
2. The coefficients of the cost function are assigned to 0.35 for  $K_1$ , 0.3 for  $K_2$ , and 0.35 for  $K_3$ .
3. The coefficients of the cost function are assigned to 0.3333 for  $K_1$ , 0.3333 for  $K_2$ , and 0.3333 for  $K_3$ .

In the numerical calculation, the assumed rolling condition is: the work roll diameter is 500 mm, the backup roll diameter is 1300 mm, the roll face width is 1426 mm, the strip width is 960 mm, the strip entry gauge is 2.5 mm, and the strip exit gauge is 0.5 mm.

**3.3.2.2. Optimization results.** The optimized results are listed in Table 1 in comparison with the empirically based rolling schedule. The distributions of roll force, power, work roll speed, tension force and torque at each of the five stands are shown in Figs. 5–9 for the purposes of comparison between the schedules under the empirical method and the GA-based optimized approach. The data describing the costs of power, tension and shape, and the total cost in Table 1 have been normalized.

As defined in Eq. (2), the power is the product of the work roll rotational speed and torque. Fig. 7 shows the work roll speeds for all the five stands under different schedules, including the semi-empirical

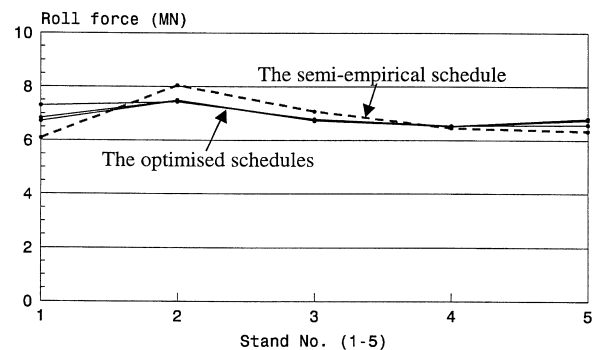


Fig. 5. Roll forces for different schedules.

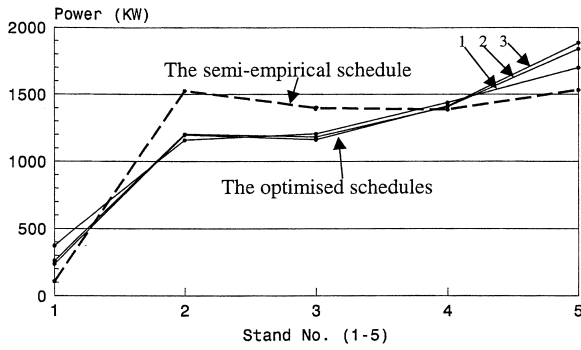


Fig. 6. Power distributions for different schedules.

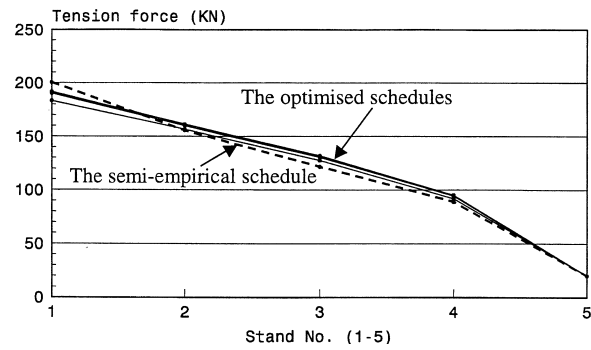


Fig. 8. Tension forces for different schedules.

schedule, with three optimized schedules. Figs. 6 and 9 demonstrate the power distribution and torque for each stand under the different schedules. From Fig. 6, conclusions can be drawn that the power distributions under optimized schedules are more uniform than those under the semi-empirical schedule. The roll force shown in Fig. 5 is an important factor during rolling, in addition to the results for the power, the tension and the strip flatness. Although the tension distributions under the different schedules in Fig. 8 are not obviously similar, the tension distributions under the three optimized schedules are likely to be kept midway between the lower and upper limits for each neighboring pair of mill stands, which is also numerically demonstrated in Table 1.

3.3.3. Discussion of the results

As illustrated in Table 1, the optimized schedules result in a lower cost of power, more uniform distributions of roll forces at the different stands, and better shape. The tension forces are more likely to be kept midway between the upper and lower limits. For example, the power distribution cost value generated by the GA approach is reduced by about 33.4% in comparison with the schedule based on the semi-empirical formula when the weight coefficient of the power cost function is assigned to a value of 0.4. The

cost of power increases from 0.6673 to 0.9714 when the weight coefficient is assigned a smaller value from 0.4 to 0.3333. The cost of tension is significantly reduced when the weight coefficient of the tension cost function is changed from 0.2 to 0.3 and then to 0.3333. The cost of tension in the optimized schedule 1 is slightly greater than that in the empirical schedule. This is the result of a small weight coefficient of the tension cost function, the value 0.2, which means that less attention is paid to the tension distribution. With the change of the weight coefficient of the shape cost function from 0.3333 to 0.4, the shape cost is slightly decreased from 0.9986 to 0.9963. Although the shape cost is slightly reduced, this is an important move towards better flatness. The optimized schedules shown in Table 1 demonstrate that the weight coefficient can be assigned a certain value, based on the different significance considerations given to the shape, power and tension distributions. Generally, the weight coefficients of the power, tension and shape costs should be based on the significance given to the shape, power and tension distributions, respectively.

The results shown in Table 1 demonstrate that the optimized schedule 3 in Table 1 is the best of all four schedules, including the empirical schedule. The total cost is the lowest when an equal importance factor, i.e.

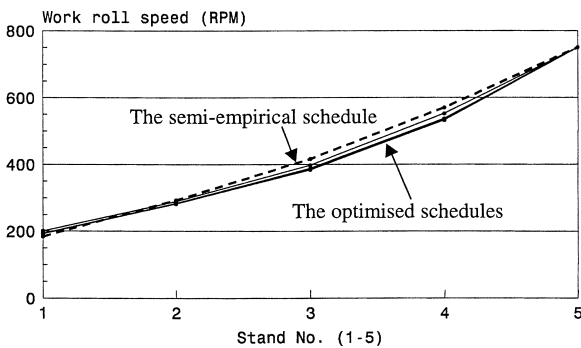


Fig. 7. Work roll speeds for different schedules.

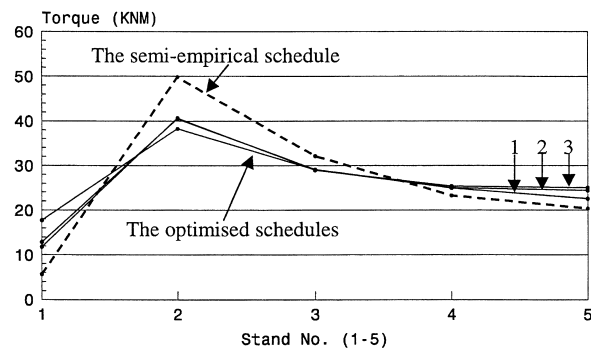


Fig. 9. Torque at each stand for different schedules.

weight coefficient, is considered. For industrial situations, the three weighting coefficients can be assigned on the basis of the current practical situation.

#### 4. Conclusions

An optimal rolling scheduling method is introduced here, which aims at achieving uniform power distribution, maximum safe level of strip tension, and good flatness. The results generated from the case study show that the proposed GA-based optimization approach has the potential to significantly improve empirically derived scheduling parameters for tandem cold rolling mills. With the GA-based optimization procedure, a considerable cost reduction can be obtained. The numerical evaluation also shows that the GA-based evolutionary searching is efficient, and has the potential to handle on-line scheduling. The power distribution of the tandem cold rolling mill is more uniform than those generated from the experience-based scheduling. Moreover, the shape achieved by the optimized schedules is likely to be improved. The tension under the optimized schedules 2 and 3 in Table 1 is more likely to be kept midway between the upper and lower limits. The numerical experimental results show that the proposed approach is an efficient way of solving rolling scheduling problems.

Further work will focus on the on-line adaptation capability due to changes in rolling conditions such as threading, tailing out, and the passing of a weld through the mill, and of strip properties such as entry thickness, hardness, etc.

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