University of Wollongong

Research Online

Faculty of Engineering and Information
Sciences - Papers: Part A

Faculty of Engineering and Information
Sciences

1-1-2014

Limiting force profile and laterally loaded rigid piles in sand

Hongyu Qin
Griffith University

Wei Dong Guo
University of Wollongong, wdguo@uow.edu.au

Follow this and additional works at: <https://ro.uow.edu.au/eispapers>

Part of the [Engineering Commons](#), and the [Science and Technology Studies Commons](#)

Research Online is the open access institutional repository for the University of Wollongong. For further information contact the UOW Library: research-pubs@uow.edu.au

Limiting force profile and laterally loaded rigid piles in sand

Abstract

In this paper, closed-form solutions were used to analyze the response of eight laterally loaded rigid piles obtained from field test, centrifuge test, and 1g laboratory model tests, allowing the modulus of subgrade reaction and the profile of limiting lateral resistance force per unit length (LFP, p_u profile) to be back calculated. The study shows that (1) with a constant modulus of subgrade reaction, a linear increasing LFP and elasto-plastic $p \sim y$ curves, the response of laterally loaded rigid piles can be well predicted in the framework of load transfer approach; (2) the gradient of the LFP is 0.64 ~ 3.0 times that of the Barton LFP; and (3) the gradient of the LFP and modulus of subgrade reaction increases approximately linear with the confining pressure in the model tests.

Keywords

sand, rigid, loaded, piles, laterally, limiting, profile, force

Disciplines

Engineering | Science and Technology Studies

Publication Details

Qin, H. & Guo, W. Dong. (2014). Limiting force profile and laterally loaded rigid piles in sand. *Applied Mechanics and Materials*, 553 452-457.

Limiting Force Profile and Laterally Loaded Rigid Piles in Sand

Hongyu Qin^{1, a} and Wei Dong Guo^{2, b}

¹School of Engineering, Griffith University, Gold Coast campus, QLD 4222, Australia

²School of Civil, Mining and Environmental Engineering, University of Wollongong, NSW 2522, Australia

^ah.qin@griffith.edu.au, ^bwdguo@uow.edu.au

Keywords: Lateral loading, rigid piles, limiting force profile, ultimate soil resistance, ultimate lateral capacity, modulus of subgrade reaction

Abstract. In this paper, closed-form solutions were used to analyze the response of eight laterally loaded rigid piles obtained from field test, centrifuge test, and 1g laboratory model tests, allowing the modulus of subgrade reaction and the profile of limiting lateral resistance force per unit length (LFP, p_u profile) to be back calculated. The study shows that (1) with a constant modulus of subgrade reaction, a linear increasing LFP and elasto-plastic $p \sim y$ curves, the response of laterally loaded rigid piles can be well predicted in the framework of load transfer approach; (2) the gradient of the LFP is 0.64 ~ 3.0 times that of the Barton LFP; and (3) the gradient of the LFP and modulus of subgrade reaction increases approximately linear with the confining pressure in the model tests.

Introduction

Pile foundations are frequently used to support structures, such as electrical transmission towers, bridge abutments and offshore wind turbines. These structures usually experience lateral static and cyclic loading generated by wind, wave, and current. A number of approaches have been developed to investigate the behavior of piles subjected to lateral loading [1-7]. Recent study reveals that response of a laterally loaded pile is dominated by the profile of limiting lateral resistance force per unit length (LFP) mobilized along the pile and depends on the pile-soil relative rigidity [6]. Several methods have been developed for predicting lateral capacity of rigid piles based on an assumed profile of soil resistance per unit length along a pile. However, they generally offer different predicted ultimate lateral capacity [7]. Alternatively, an LFP may be deduced from the analysis of test results using closed-form solutions. Such study has been conducted for 52 laterally loaded flexible piles tested in clay and sand [6].

In this paper, elasto-plastic solutions were used to assess the measured response of piles from field, centrifuge and 1g laboratory model tests. The results allow the effect of pile flexibility, modulus of subgrade reaction distribution and confining pressures to be investigated. The deduced limiting force profiles were compared with the existing LFP for sands.

Elasto-plastic Solutions for Laterally Loaded Rigid Piles

A pile is defined as rigid if the pile-soil relative stiffness, E_p/G_s exceeds a critical ratio, $(E_p/G_s)_c$, where $(E_p/G_s)_c = 0.052(l/r_0)^4$ [7] and E_p is Young's modulus of an equivalent solid cylindrical pile of diameter d , G_s is the soil shear modulus, l is the pile embedded length, and r_0 is the outer radius of the pile. The elasto-plastic solutions were developed for laterally loaded rigid piles using a load transfer model [7]. As shown in Fig. 1(a), the pile head is free with no constraints. The pile soil interaction is characterized by a series of springs distributed along the shaft. The spring has an elastic-plastic $p \sim y$ (u) curve at each depth, where p is the soil lateral resistance per unit length, u is the pile displacement. The lateral resistance, p is proportional to the local pile displacement, u at that depth and the modulus of subgrade reaction, kd , i.e. $p = kdu$, where k is the gradient of the $p \sim u$ curve and d is pile outer diameter. The gradient k may be written as $k_0 z^m$, with $m = 0$ and 1 being referred to as constant k and

Gibson k hereafter. Where the soil resistance reaches the limiting p_u , relative slip takes place along the pile-soil interface and extends to a depth z_0 , which is called *pre-tip yield state*. With increasing load the pile-soil relative slip may also initiate from the pile tip ($z = l$) and expand upwards to another depth z_1 (see Fig. 1(c)). The two plastic zones tend to merge at which the pile reaches the ultimate state, i.e. *yield at rotation point* ($z_0 = z_1 = z_r$). It is assumed that the p_u varies linearly with depth z and is described by $p_u = A_r dz$, where $A_r d$ is the gradient of the LFP. The solutions allow the nonlinear responses (e.g. load, displacement, rotation and maximum bending moment) to be readily estimated, using the two parameters k and A_r . Conversely, the two parameters can be deduced from the measured results of pile tests. The solutions are presented in explicit expressions in Table 1 for pre-tip yield state and have been implemented into a spreadsheet program to conduct investigation.

T_t = lateral load; e = eccentricity; u_0 = pile displacement at ground surface;
 ω = angle of rotation (in radian); z = depth from ground surface;
 l = embedded length; z_0 = depth of slip; z_r = depth of rotation point;
 p = soil resistance per unit length; p_u = ultimate soil resistance per unit length;
 A_r = gradient of limiting force profile; d = outer diameter of the pile;
 u = pile displacement; u^* = local threshold u above which pile soil relative slip is initiated;
 k, k_0 = modulus of subgrade reaction, $k = k_0 z^m$, $m = 0$, and 1 for constant and Gibson k .

Figure 1 Schematic analysis for a rigid pile [7] (a) pile-soil system, (b) load transfer model
 (c) p_u (LFP) profiles, (d) pile displacement characteristic

Table 1 Solutions for pre-tip and tip yield state [7]

$u = \omega z + u_0$ and $z_r/l = -u_0/\omega l$	
$p = kdu$, $p_u = A_r dz$, kd is the modulus of subgrade reaction, k is written as $k_0 z^m$.	
Gibson k ($m = 1$)	Constant k ($m = 0$)
$\frac{T_t}{A_r dl^2} = \frac{1}{6} \frac{1 + 2\bar{z}_0 + 3\bar{z}_0^2}{(2 + \bar{z}_0)(2\bar{e} + \bar{z}_0) + 3}$ (G-1)	$\frac{T_t}{A_r dl^2} = \frac{\bar{z}_0}{2(2 + 3\bar{e} + \bar{z}_0)}$ (C-1)
$\frac{u_0 k_0}{A_r} = \frac{3 + 2(2 + \bar{z}_0^3)\bar{e} + \bar{z}_0^4}{[(2 + \bar{z}_0)(2\bar{e} + \bar{z}_0) + 3](1 - \bar{z}_0)^2}$ (G-2)	$\frac{u_0 k}{A_r l} = \frac{(2 + 3\bar{e})\bar{z}_0}{(2 + 3\bar{e} + \bar{z}_0)(1 - \bar{z}_0)^2}$ (C-2)
$\omega \frac{k_0 l}{A_r} = \frac{-2(2 + 3\bar{e})}{[(2 + \bar{z}_0)(2\bar{e} + \bar{z}_0) + 3](1 - \bar{z}_0)^2}$ (G-3)	$\omega \frac{k}{A_r} = \bar{z}_0 \frac{\bar{z}_0^2 + 3(\bar{z}_0 - 2)\bar{e} - 3}{[2 + 3\bar{e} + \bar{z}_0](1 - \bar{z}_0)^2}$ (C-3)
Note: T_t , u , u_0 , ω , z , z_0 , z_r , e and l are defined in Fig. 1. $\bar{z}_0 = z_0/l$, $\bar{e} = e/l$.	

Analysis of Test Results

Field Test A full-scale 0.75 diameter pier was tested in a deposit of cohesionless soil at Waiouru, New Zealand [8]. The pier was cast in a pre-bored hole to an embedded depth of 1.97 m. The lateral load was applied at 5.40 m above the ground surface. The effective unit weight of the silty sand is 8.2 kN/m³. The internal friction angle was estimated as 30° from penetrometer tests. The measured load $T_t \sim$ horizontal displacement of the pier at the ground surface u_0 and load $T_t \sim$ angle of rotation ω curves are plotted in Fig. 2. By using the solutions, the best match between the estimated and the measured responses were obtained and are shown in Fig 2, allowing $A_r = 222.5$ kN/m³, $k_0 = 38$ MN/m⁴, $k = 34$ MN/m³ uniquely determined from the two measured curves. They are summarized in Table 2. The comparison also indicates that the solution with a constant k offers a better estimation against measured piles responses than that based on Gibson k for the same A_r .

Figure 2 Predicted and measured response of pile [8]

Centrifuge Tests Three centrifuge model tests were conducted on laterally loaded piles in uniform fine-grained dry sand at an acceleration of 50 g [9]. The pile was instrumented with five pairs of strain gauges to measure the bending moments along the pile length. Among them, a prototype steel pipe pile (pile P2 [9]) was 9.05 m in length, 1.224 m in outer diameter and 17.25 mm in wall thickness with bending rigidity $E_p I_p = 2495.0$ MN·m². The sand was compacted to medium dense to dense state with a relative density, D_r of 60%. The average unit weight and effective internal frictional angle was 16.3 kN/m³ and 36°, respectively. Load was applied at an eccentricity, e of 1.25 m. The measured load $T_t \sim$ pile head displacement u_t curve is plotted in Fig. 3(a). The distribution of

bending moment, shear force and pile displacement with depth at $T_t = 1304$ kN are plotted in Fig. 3(b) to 3(d).

With the recommended $k_0 d = 11$ MN/m³ [9] and $E_p I_p = 2495.0$ MN·m², the dimensionless length ηl of the pile is calculated as 3, in which $\eta = (k_0 d / E_p I_p)^{1/5} = 0.34/\text{m}$. The pile is neither short ($\eta l \leq 2$) nor long ($\eta l \geq 2$) [1], but of medium length. Although the pile is not strictly rigid, back calculations were carried out to explore the flexibility effect on the pile response. With $A_r = 280$ kN/m³, $k_0 = 3$ MN/m⁴, $k = 10$ MN/m³, the predicted $T_t \sim u_t$ curve and distributions of bending moment and shear force at $T_t = 1304$ kN compare well with the experimental data in Fig. 3. The predictions generally bracket the measured pile response using the Gibson k and constant k , with a marginally better match using the Gibson k . However, the solutions overestimate the pile displacement around pile tip in Fig. 3(b), showing that the tested pile maybe relatively flexible.

Figure 3 Predicted and measured response of pile P2 [9]

Model Tests Model tests were conducted on a pile subjected to lateral loading in air-dried coarse angular sand [10]. The pile has a diameter of 168.3 mm and was installed to an embedment depth of 1.2 m. The sand has a unit weight of 16.24 kN/m³ and internal friction angle of 32°. The model ground was prepared in a steel chamber lined with air bladder and varying confining pressure was applied to the sand. Six tests were conducted at a confining pressure p_c of 0, 20, 40, 60, 80 and 100 kPa, respectively. The lateral load was applied at 0.195 m above the model ground surface for all the tests. The measured load $T_t \sim$ pile displacement at the loading point u_t curves are plotted in Fig. 4(a). The distribution of bending moment and pile displacement with depth under certain lateral loads are plotted in Fig. 4(b) to 4(d). Back calculations were made for all tests and the deduced parameters A_r , k and k_0 are presented in Table 2. The predicted curves of $T_t \sim u_t$ and distributions of bending moment and displacements are also plotted in Fig. 4. The analyses show the following features:

1. Taking the same value of A_r , the solutions with a constant k gives better match with the measured pile responses as shown in Fig. 4(a). Therefore, except for tests with $p_c = 0$ kPa and 20 kPa, the other four tests were analyzed using the solutions with constant k only.
2. The solutions well capture the displacements for all the piles, but generally overestimating the displacements at ground surface by 15 ~ 20% in Fig. 4(c) and 4(d).
3. Fig. 4(b) shows the distributions of bending moment down the pile were well predicted, especially for confining pressure from 40 kPa to 100 kPa, while the maximum bending

moment M_{max} was overpredicted by $\sim 30\%$ for tests under 0 kPa and 20 kPa. The predicted M_{max} occurs at a depth of 0.36 m below the ground surface for all the piles, while measured depth of maximum bending moment is 0.41 m [10]. Overall, the depth at which the maximum bending moment occurs can be crudely taken as one third of the embedded depth.

4. The deduced A_r and k increases as the confining pressure p_c increases from 0 kPa to 100 kPa, as shown in Fig. 4(e). This is expected because higher confining pressure offers larger soil resistance, resulting in greater capacity for piles sustained lateral loading. They can be estimated using $A_r = 15.49p_c + 5.71$, $k = 0.68p_c + 9.19$, where A_r is in kN/m^3 , k in MN/m^3 , and p_c in kPa.
5. The current estimated ultimate lateral load capacities are 2.87, 5.74, 14.36, 23.50, 36.55 and 39.16 kN for the tests with $p_c = 0, 20, 40, 60, 80$ and 100 kPa, respectively. These values are much larger than the measured ultimate loading capacity of 1.34, 2.93, 4.98, 7.83, 10.33 and 13.55 kN given by [10]. This is owing to that, in their study, the ultimate loading capacity was obtained as the load at 6.25 mm pile displacement at the loading point; whereas the ultimate load capacity in the current study is determined at yield at rotation point [7].

Table 2 Summary of pile properties and parameters

Tests	l (m)	d (m)	e (m)	γ'_s (kN/m^3)	ϕ'_s ($^\circ$)	A_r (kN/m^3)	N_g	k_0 (Gibson) (MN/m^4)	k (constant) (MN/m^3)
Field	1.97	0.75	5.40	8.2	30	222.5	3.0	38.0	34.0
Centrifuge	9.05	1.224	1.25	16.3	36	280.0	1.16	3.0	10.0
Model (0kPa)	1.2	0.168	0.195	16.24	32	110	0.64	22.5	12.5
Model (20kPa)	1.2	0.168	0.195	16.24		220		42.5	24.5
Model (40kPa)	1.2	0.168	0.195	16.24		550			32.5
Model (60kPa)	1.2	0.168	0.195	16.24		900			48.5
Model (80kPa)	1.2	0.168	0.195	16.24		1400			56.0
Model (100kPa)	1.2	0.168	0.195	16.24		1500			86.0

Figure 4 Predicted and measured response of six model piles [10]

Comparison of LFP To facilitate comparison, a non-dimensional parameter N_g is defined as $N_g = A_r / \gamma'_s K_p^2$, where γ'_s is the effective unit weight of the soil (dry unit weight above water table,

and buoyant unit weight below), $K_p = \tan^2(45^\circ + \phi'_s/2)$, is the coefficient of passive earth pressure, ϕ'_s is the effective frictional angle. The N_g was calculated for each pile test and tabulated in Table 2. The deduced LFPs are compared with the Broms LFP of $p_u = 3\gamma'_s K_p dz$ (i.e. $N_g = 3/K_p$) [1] and Barton LFP of $p_u = \gamma'_s K_p^2 dz$ (i.e. $N_g = 1$) [2], respectively. Results in Table 2 show $N_g = 3.0, 1.16$ and 0.64 for the field test, centrifuge test and model test at $p_c = 0$ kPa. Thus the N_g for rigid pile is on average 60% higher than the average of the Barton LFP, which is consistent with that obtained from the analysis of 51 pile tests results [11]. The analysis of 20 laterally loaded flexible piles in sand gives $N_g = 0.4 \sim 2.8$, but for p_u varying with $z^{1.7}$ to reflect the impact of pile flexibility [6].

Conclusion

The responses of laterally loaded piles were back calculated using closed-form solutions against the results from field test, centrifuge test and 1g laboratory model tests. The results allow the effect of pile flexibility, modulus of subgrade reaction distribution and confining pressures to be investigated. The study shows (1) with a constant modulus of subgrade reaction, a linear increasing LFP and elasto-plastic $p \sim y$ curve, the response of laterally loaded rigid piles can be well predicted in the framework of load transfer approach; (2) the gradient of the LFP is $0.64 \sim 3.0$ times that of the Barton LFP; (3) the gradient of the LFP and modulus of subgrade reaction increases approximately linearly with the confining pressure in the model tests.

References

- [1] B. B. Broms, Lateral resistance of piles in cohesiveless soils, *Journal of Soil Mechanics and Foundation Engineering*, ASCE, 90 (1964) 123-156.
- [2] Y. O. Barton, Laterally loaded model piles in sand: centrifuge tests and finite element analysis, Ph D theiss, University of Cambridge, 1982.
- [3] L. C. Reese, W. R. Cox, and F. D. Koop, Analysis of laterally loaded piles in sand, *Proc., 6th Annual Offshore Technology Conf.*, Houston, 1974 473-485.
- [4] H. G. Poulos, Behaviour of laterally loaded piles: I-Single Piles, *Journal of Soil Mechanics and Foundation Engineering*, ASCE, 97(1971) 711-731.
- [5] W.G. K. Fleming, A. J. Weltman, M.F. Randolph, and W.K. Elson, *Piling Engineering*. Taylor and Francis, London and New York, 2009.
- [6] W. D. Guo, Simple model for nonlinear response of 52 laterally loaded piles, *Journal of Geotechnical and Geoenvironmental Engineering*, ASCE, 139(2013) 234-252.
- [7] W. D. Guo, Laterally loaded rigid piles in cohesionless soil, *Canadian Geotechnical Journal*, 45(2008) 676-697.
- [8] M. J. Pender, and T. Matuschka, Interpretation of lateral tests on rigid poles in cohesionless soils. *Proc. 5th Australia-New Zealand conference on Geomechanics*, Sydney, 1988 542-546.
- [9] M. Georgiadis, C. Anagnostopoulos, and S. Saflekou, Centrifuge testing of laterally loaded piles in sand, *Canadian Geotechnical Journal*, 29(1992) 208-216.
- [10] M. H. El Naggar, and J. Q. Wei, Response of tapered piles subjected to lateral loading, *Canadian Geotechnical Journal*, 36(1999) 52-71.
- [11] H. Y. Qin, and W. D. Guo, Nonlinear response of laterally loaded rigid piles in sand, submitted to *Journal of Geomechanics and Engineering* (2013).