Flux pinning behaviour of different types of Bi2212 single crystals

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FLUX PINNING BEHAVIOUR OF DIFFERENT TYPES OF Bi2212 SINGLE CRYSTALS

TEMPERATURE SCALING OF ACTIVATION ENERGY

A thesis submitted in fulfillment of the requirements for the award of the degree

Honours Master of Engineering

from

University of Wollongong

by

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1999
ACKNOWLEDGEMENT

I would firstly like to express my most sincere gratitude to my supervisors, Professor H. K. Liu and Professor S. X. Dou for their academic guidance, financial support and continuous encouragement throughout the project. I would also like to express my sincere thanks to Josip Hovart for his great assistance during the thesis writing and helpful comments on the manuscript. I am also grateful to X. L. Wang and M. Ionescue who helped me in many ways, and also to Dr. G. D. Gu, Dept. of Physics, University of NSW, who provided me with iron doped Bi2212 single crystals.

Many thanks go to all my friends and all the members at the Institute for Superconducting & Electronic Materials, and to all the technicians at the Department of Materials Engineering. Thanks also go to Mrs. B.M. Allen, Mrs. R. Cambareri and Mrs. J. De Mestre for their help in official matters.
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Abstract

A study of the magnetic relaxation of a pure Bi2212 crystal, two lead-doped Bi2212 crystals and an iron doped Bi2212 crystal at various temperatures was carried out. The crystals in these measurements were fabricated by the self flux growth technique. The high quality of the samples with a high degree of preferred orientation of the crystallographic planes was observed from X-ray diffraction pattern (XRD) and electron back scattering pattern (EBSP) techniques. From XRD, the c-axis parameters of the pure single crystal, the lead doped single crystal and iron doped single crystal were found to be \((30.7959 \pm 0.0733)\text{Å}\), \((30.6666 \pm 0.1037)\text{Å}\) and \((30.8005 \pm 0.0573)\text{Å}\), respectively. The critical temperature \(T_c\) of the samples Bi2212, Bi(Pb1)2212, Bi(Pb2)2212 and Bi(Fe)2212 were found to be 88.5K, 67K, 82K and 72K, respectively.

The hysteresis loop and magnetic relaxation measurements of the samples were made using a QD PPMS magnetometer. A strong anomalous peak effect in the hysteresis loops is seen for the both lead doped samples over a wide range of temperatures, from 20K to \(T_c\). This indicates the presence of strong pinning centres in these samples. The anomalous peak of the iron doped crystal was obtained between 23K and 35K. The hysteresis loops and zero field critical current density \(J_{c0}\) of the iron doped sample were found to be strongly field and temperature dependent. This indicates the presence of weak pinning centres in this sample. A smaller magnetic relaxation rate for the lead doped samples was observed compared to pure and iron doped samples.
The current dependence of the activation energy $U(J)$ was obtained from the measurements of the magnetisation decay at different temperatures and constant field, using a method introduced by Maley. However, $U$ depends on temperature, as well. Therefore, a proper temperature scaling of $U$ had to be found to obtain a smooth $U(J)$. It was found that $(1 - (T/T_c)^2$ gives the best fit for the experimentally measured activation energies $U(M_{in})$ for approximately $20K < T < 30K$. The main outcome of this research is that the same temperature scaling of the activation energy was obtained for all of the samples. This is a remarkable result, because the samples have different flux pinning.
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CHAPTER ONE: INTRODUCTION

The discovery of High Tc Superconductivity has initiated an unprecedented level of world wide activities in both theoretical and experimental studies of these materials. The main features of high Tc superconductors (HTSC) are: high critical temperature $T_c$ ($T_c > 77K$), high upper critical field ($H_{c2} > 200T$), extremely short coherence length and large anisotropy. The first two properties make them good candidates for the technical application of these materials but the last two are disadvantageous compared to low temperature conventional superconductors. The large anisotropy and the extremely short coherence length together with thermal fluctuation are the main causes for depinning of magnetic vortices from their pinning sites, which is the main cause of energy dissipation. This depinning flux line produces a finite voltage in these materials and therefore, dissipation. The dissipation is one of the serious problems in bringing them into technical applications.

There are various method for studying the dissipation. One of them is magnetic relaxation, which has become a fascinating and widely accepted phenomenon to study dissipative effects in high $T_c$ superconductors, helping to better understand the magnetic phase diagram and pinning mechanisms of HTSC. In magnetic relaxation, a change of magnetic moment with time is measured. The basic mechanism for the magnetic relaxation is that a configuration of vortices is in a non-equilibrium state because of flux...
pinning and it will move to a state with minimum energy by thermal excitation of vortices over the pinning barriers.

The phenomenon of magnetic relaxation was first suggested by P.W. Anderson (1964) to explain the experimental results of Kim (1962)[11]. Anderson and Kim (1964) predicted a logarithmic decay of magnetic moment with time [12]. Beaseley at el (1969), while elucidating experimentally the mechanism of Anderson's thermal activation flux creep process, observed a non linear U-J relationship, leading to non-logarithmic decay of magnetisation [52]. The full impact of the non linear U(J) dependence have become apparent only after development of vortex glass theory by Fisher and the collective pinning theory by Feigel’mann. A result of these theories is that the activation barrier diverges as the current J approaches to zero.

Feigelmann, in his collective pinning theory showed that at an applied current J<<J_c, the activation barrier U(J) between different metastable states grows with a power law U(J) = U_0(Jc/J)^\alpha, where U_0 is the characteristic energy scale and the exponent \alpha depends on the dimensionality, on the particular regime of the flux creep. In three dimensions \alpha = 1/7 in the weak field, low temperature regime where creep is dominated by the motion of the individual flux line. For \alpha = 3/2, the collective creep of small bundles takes place ( at higher fields) and for \alpha = 7/9, collective creep of large bundles takes place ( at very large fields). For two dimensional collective creep process, \alpha = 9/8. Using this power law, they obtained a famous interpolation formula, from where different relaxation laws were observed.

The temperature and field dependence of the activation energy is very puzzling. Maley at el (1990) were able to define a universal curve for the activation energy as a function of magnetisation measured at different temperatures. Here they were able to present
various values of $U(J)$ on a smooth curve by adding a physically acceptable constant $C=18$ to $U(J)$ measured at different temperatures.

It has been observed that the Maley’s approach to elucidate the explicit dependence of $U_c$ on $M$ by using a single value of $C$ for each field and temperature does not result in a smooth curve over a wide range of temperatures. Therefore, a scaling approach has been introduced to study the temperature dependence of the activation energy. It has been suggested by various groups that the smooth $U(J)$ dependence measured over a wide temperature range can be obtained by appropriate temperature scaling of $U$. There are different scaling laws for the activation energy and they are given by different groups such as M.Tinkham, P.J.Kung et al., Hai-hu Wen et al. and so on. We have tried all of all the suggested scaling laws, for examples $U_{\text{eff}}(J,T) = \left(1 - \frac{T}{T_c}\right)^{1.5} U_{\text{eff}}(J,0)$, $U_{\text{eff}}(J,T) = 1 - \left(\frac{T}{T_{irr}}\right)^2 U_{\text{eff}}(J,0)$ and $U_{\text{eff}}(J,T) = (1 - T_{N}^2)^{e}(1 + T_{n})^{f} U_{\text{eff}}(J,0)$. However, only $1 - \left(\frac{T}{T_c}\right)^2$ worked well for all of our samples, with the choice of the physically meaningful value of constant $C = 18$. The scaling here is the same as that used by P. J. Kung et al. except that the irreversible temperature $T_{irr}$ in their scaling is replaced by the critical temperature $T_c$. One of interesting things in this scaling is that it can be used over wide range of temperatures and for samples with very different flux pinning.
2. Introduction:
Superconductivity has been one of the most significant discoveries in solid state physics. Superconductivity was discovered by Kamerlingh Onnes in 1911 while he was liquefying Helium. He found that the DC resistance of his lead sample dropped to zero when it was cooled to a temperature of 4.2K, the liquefying temperature for He. Superconducting material has two unique properties. The first one is the zero d-c electrical resistance found by Onnes and the second is the perfect diamagnetism found by Meissner and Ossenfeld where magnetic fields are expelled from the interior of the superconducting samples.

2.1 Zero resistivity:
At the critical temperature $T_c$, the resistivity $\rho$ of the superconductor drops discontinuously to zero (Fig. 2.1)[1]. For low $T_c$ materials, such as Niobium (Nb), the width of the resistive transition is $\Delta T \leq 10^{-2}K$.

Fig: 2.1: Resistive transition for a superconductor
2.2 Perfect diamagnetism (Meissner effects)

In superconductors, magnetic flux is expelled completely from the sample as the sample is cooled through its critical temperature \( T_c \)[1]. This is illustrated in the figure 2.2. This implies that the total induction field \( B = H + 4\pi M \) is zero inside the superconductor and the susceptibility \( \chi = -\frac{M}{H} = -\frac{1}{4\pi} \), indicating perfect diamagnetic properties. Here \( M \) is the magnetisation of the superconductor and \( H \) is the applied field [Fig. 2.3]. The superconductors with perfect Meissner effect are called pure (Type I) superconductors. Here the Meissner current flows within layers at the surface of the superconductor. It produces a field of the same magnitude as the external field. The net field \( B \) inside the superconductor is therefore zero.

Fig. 2.2: Reversible flux expulsion for the superconductor
(i) normal state \( (T > T_c) \) (ii) Superconducting state \( (T < T_c) \)
This phenomenon is not always true for all superconductors. There are some superconductors called impure (Type II) superconductors, where there are normal conducting regions inside the bulk of the superconductor. These superconductors do not show the complete Meissner effect for an applied field greater than $H_{c1}$ (lower critical field) because the applied field penetrates these superconductors in the form of filaments above this lower critical field. The material containing the filaments is in normal state. Therefore, they have a mixed state between the lower critical field $H_{c1}$ and upper critical field $H_{c2}$ [1]. This is shown in figures 2.4 and 2.5.
2.3 Specific Heat (Electronic):
There is a discontinuity in the electronic specific heat [See figure 2.6] at \( T_c \) [1]. The activated form of \( C_v \) at \( T << T_c \) varies as \( T_c \exp(-1.76T_c/T) \). From thermodynamics, we can obtain the following relation

\[
[C_v]_N - [C_v]_S = -T_c/4 \pi [dH_c/dT]^2
\]

where \([C_v]_N\) is the specific heat in the normal state and \([C_v]_S\) is the specific heat in the superconducting state. This gives a jump of the order \(-T_c/4 \pi [dH_c/dT]^2\) at \( T_c \).

The entropy difference between normal and superconducting states

\[
S_N - S_S = -T_c/4 \pi H_c[dH_c/dT] > 0 \quad \text{and} \quad dH_c/dT < 0.
\]

This gives us evidence that the superconducting state is more ordered than the normal state.

Fig. 2.6: Specific heat jump at \( T_c [1] \)
2.4 Isotope Effect:
It has been observed that the lattice vibrations play an essential role in bringing about the superconductivity [1]. The transition temperature $T_c$ varies according to

$$T_c \sim \frac{1}{M^\alpha} \quad 2.4.1$$

where $M$ is the atomic mass of the isotopes. Thus, $T_c$ is larger for lighter isotopes. The value for $\alpha$ lies between 0.45 to 0.5 for many superconductors.
3.1 The London Theory:

London brothers F. and H. London, developed a phenomenological theory to describe an electromagnetic behaviour of superconductors [2]. They postulated two equations. One equation describes a relationship between super and normal current densities and the electric field $E$.

$$\frac{dJ_s}{dt} = \frac{n_se^2}{m} E \quad (J_s = -en_s v_s) \quad 3.1.1$$

$$J_n = \sigma_n E \quad (J_n = -en_n v_n) \quad 3.1.2$$

In the second equation, they have given a relationship between super current and the magnetic field $B$. This is considered as the most important equation of their two postulations because this leads to the Meissner effect.

$$\nabla \times J_s = -\frac{n_se^2}{mc} B \quad 3.1.3$$

By taking the curl of both sides of the maxwell equation $\nabla \times B = \frac{4\pi}{c} J$, we get

$$\nabla \times \nabla \times B = \frac{4\pi}{c} \nabla \times J_s \quad 3.1.4$$

By combining equations (3.1.3) and (3.1.4), we have

$$\nabla^2 B = \frac{4\pi n_se^2}{mc^2} B = \frac{1}{\lambda_L^2} B \quad 3.1.5$$
where $\lambda_L = \left( \frac{mc^2}{4\pi m_s e^2} \right)^{\frac{1}{2}}$ is called the London penetration depth.

Using boundary condition, equation (3.1.5) can be solved to obtain the very famous equation

$$B(x) = B(0)e^{-\frac{x}{\lambda_i}}$$  \hspace{1cm} 3.1.6

This equation gives an exponential decay of magnetic field from surface to the interior of the superconductor. The magnetic field vanishes in the bulk of the material and one obtains perfect diamagnetism, as required for the complete Meissner effect.

The temperature dependence of the London penetration depth can be obtained from the Gorter and Casimir model and it is equal to

$$\lambda(T) = \frac{\lambda(0)}{\left[ 1 - \left( \frac{T}{T_c} \right)^4 \right]^{\frac{1}{2}}}$$  \hspace{1cm} 3.1.7

This temperature dependent penetration depth is found to be very close to the experimental results for pure superconductors. From this equation, we see for $T = T_c$, $\lambda = \infty$ (infinite) so that no flux is excluded at $T_c$. As $T$ drops infinitesimally below $T_c$, $\lambda$ decreases rapidly, thereby establishing the Meissner effect in the bulk materials for all temperatures less that $T_c$.

### 3.2. Pippard theory:

Pippard, on the basis of numerous experimental results, found that the penetration depth $\lambda$ increases appreciably if a sufficient amount of impurity is introduced into the
materials [3]. In this case, the London equation has to be modified to obtain an accurate result for the penetration depth of impure materials as the London theory is defined locally where the current density and electromagnetic potential are at the same point in space. For the impure superconductors, Pippard corrected the London local relations by introducing a non local relation. In the non local relation, the currents \( J(r) \) at a given point \( r \) in space are taken from the average field strengths over a region of extent \( \xi_0 \), where \( \xi_0 \) is the coherence length and it represents the size of the pair bound state for carriers. In other words, the current \( J(r) \) at a point \( r \) depends on the vector potential within a distance \( \xi_0 \) of the point \( r \).

One of the most compelling arguments for this generalisation is that the penetration depth increases appreciably if a sufficient amount of impurity is introduced in to the materials.

Here Pippard modified Chamber’s non local expression for the current density and electric field strength in the normal metal with London’s equation [4],

\[
J_s(r) = -\frac{3n_s(T)}{4\pi^2 \xi_0 cm} \int \frac{R[R.A(r')] - R}{R^4} e^{-\frac{R}{\xi}} d^3 r'
\]

where \( R = r - r' \) is the average area for the field strength. The effective coherence length

\[
\frac{1}{\xi} = \frac{1}{\xi_0} + \frac{1}{\alpha l}
\]

where \( \alpha = 1 \) and \( l \) is the mean field path of electrons in the normal metals (this comes from impurity). For pure materials, Pippard’s equation reduces to London’s equation since \( A(r) \) varies slowly over \( \xi_0 \) and it becomes

\[
J_s(r) = -\frac{n_s(T)e^2}{cm}
\]
For impure superconductors, Pippard's equation leads to an extra factor $\xi/\xi_0 < 1$ multiplying London’s relation.

### 3.3 Ginzburg-Landau (GL) Phenomenology:

The first macroscopic theory was introduced by two genius scientists Ginzburg and Landau [5]. It was based on Landau’s general phenomenology of second order phase transition in terms of the order parameter expansion and symmetry of a free energy functional. The theory in its original form was valid only close to the critical temperature $(T_c-T)/T_c \ll 1$ where the order parameter was small and slowly varying. Later, it was extended to arbitrary temperatures but close to the critical field $H_{c2}$.

They phrased the theory in terms of an order parameter $\Psi = |\Psi_0|\exp(i\theta)$, where $\theta$ is the phase factor and where they defined a local density of condensed electron by $|\Psi|^2 = n_s/n$ for $T < T_c$, where $n_s$ is super current density and $n$ the total number of electrons per unit volume.

For $T$ close to $T_c$, where superconducting carriers are in very small number, Ginsburg and Landau defined $f_{sh}(\Psi(r), T)$ as the difference of free energy per unit volume between the superconducting and normal conducting phases, by expanding it as a power series in $|\Psi|^2$.

For an isotropic and homogenous system in a magnetic field, the free energy function becomes

$$F_{sh} = \int |\Psi|^2 f_{sh} \, dr$$

3.3.1
with $f_{sh} =$

$$
\alpha(T)\Psi(x)\Psi^*(x) + b(T) / 2|\Psi\Psi^*|^2 + (1 / 2)m^*\left[-ih\Delta / 2\pi\right] - e^*\Lambda(x) / c|^2 + H^2(x) / 8\pi
$$

where $\alpha(T)$ and $b(T)$ are experimentally measurable quantities and $H$ is the macroscopic magnetic field.

A relation for the current density $J_s$ can be obtained using the G-L free energy function:

$$
J_s = -ieh / 4\pi m^*\left(\Psi^*\Delta\Psi - \Psi\Delta\Psi^*\right) - \left\{ e^* / m^* c\right\}(\Psi^*\Psi)\Lambda
$$

where $\nabla \times H = 4\pi J_s / c$. The first term in the above equation refers to a paramagnetic current and the second term refers to a diamagnetic current.

GL phenomenology gives two very important length scales $\lambda(T)$ and $\xi(T)$ in terms of dimensionless quantity $\kappa = \lambda(T) / \xi(T)$ called the Ginzburg-Landau parameter, where the temperature dependent coherence length $\xi(T)$ and penetration depth $\lambda(T)$ are given as

$$
\xi(T) = \xi(0)(T - T_c / T_c)^{-1/2}
$$

$$
\lambda(T) = \lambda(0)(T - T_c / T_c)^{-1/2}
$$

These are the two fundamental lengths that control the spatial variation of the order parameter and electromagnetic response of the order parameter.

The ratio of $\lambda / \xi = \kappa$ distinguishes between the type I ($\kappa \leq 1 / \sqrt{2}$ e.g. Al, Pb) (Fig 3.3a) and type II ($\kappa \geq 1 / \sqrt{2}$ e.g Nb$_3$Sn) superconductors (Fig. 3.3b). Typically, for a conventional type I superconductor $\lambda \sim 10^2 \AA$, $\xi \sim 10^4\AA(\kappa = 10^{-2} \leq 1 / \sqrt{2} )$ and for type II superconductor, $\lambda \sim \xi \sim 10^2 \AA(\kappa = 1)$. 
The G-L equation is also used to obtain a relation for the normal and superconducting interface energy $\sigma_{ns} \sim H_c^2 / 8 \pi (\xi - \lambda)$ and this also gives a clear distinction for type I and type II superconductors. For the type II superconductor, the interfacial energy is negative and the material favours the entering of flux lines while for the type I superconductor, the interfacial energy is positive and the material favours a complete expulsion of flux lines.

![Fig. 3.3a](image1)

![Fig. 3.3b](image2)

Fig. 3.3a: Penetration depth $\lambda$ and coherence length $\xi$ for type I superconductor and for type II superconductor (fig 3.3b) [1]

### 3.4 BCS theory of Superconductivity:

The theory given by Bardeen, Cooper and Schriefer in 1957 is the most successful microscopic theory of superconductivity [6,7]. This theory is strickly valid for the low temperature superconductors (some experiments in HTSC confirm the BCS theory but there are still unsolved questions). The main idea of this theory is electron pairing i.e. the pair-wise occupation of time-reversed one-electron states $(K \uparrow, -K \downarrow)$ of conduction electrons in the superconductor, interacting via virtual
phonon exchange. The central idea of the formation of cooper pairs is as follows.

When an electron moves among the positive ions of lattice, it attracts them. The ions move very slowly due to their large mass. By the time the ions respond the electron, the electron advances a long distance, leaving behind in effect, a positive charge. Another electron traversing the same path finds that positive charge. Due to the slow response of the ions, there appears an effective attractive interaction between electrons.

![Diagram of a cooper pair with opposite momenta -k and +k and exchanging phonon between them.](image)

Fig. 3.4 A cooper pair with opposite momenta -k and +k and exchanging phonon between them.

The reduced BCS Hamiltonian, as obtained by Bardeen, Cooper and Schriefer from the famous Frohlich Hamiltonian, is given as:

$$
H_{red}^{BCS} = \sum_{\kappa} \epsilon_{\kappa} C_{\kappa}^{+} C_{\kappa} - V_0 \sum_{\kappa} C_{\kappa \uparrow}^{+} C_{-\kappa \downarrow}^{+} C_{\kappa \uparrow} C_{-\kappa \downarrow} 
$$

3.4.1

where $\epsilon_{\kappa} = \frac{\hbar^2 k_{F}^2}{2m}$ is the kinetic energy, $C_{\kappa \uparrow}^{+} C_{-\kappa \downarrow}^{+}$ creates a cooper pair and the conjugate of this annihilates a pair. The second term in the BCS hamiltonian represents an interaction with attractive potential $V_0$.

The BCS wave function

$$
|\Psi_{BCS}\rangle = \prod_{\kappa} \left[ u_{\kappa} C_{\kappa \uparrow}^{+} C_{-\kappa \downarrow}^{+} + v_{\kappa} C_{\kappa \uparrow} C_{-\kappa \downarrow} \right] |0\rangle,
$$

3.4.2
with $|u_k|^2 + |v_k|^2 = 1$, where $|u_k|^2$ is the probability that $(\kappa^{\uparrow}, -\kappa^{\downarrow})$ state is empty and $|v_k|^2$ is the probability that $(\kappa^{\uparrow}, -\kappa^{\downarrow})$ state is occupied. $|0\rangle$ is the state with no electrons. The main point of the BCS wave function is that if $\kappa^{\uparrow}$ is occupied, then $-\kappa^{\downarrow}$ is also occupied.

The important results obtained from BCS theory are summarised below.

i. The critical gap ratio $\frac{2\Delta(0)}{K_B T_c} = 3.52$ agrees very well with experimental results.

Here $\Delta(0)$ is the binding energy per electron in the pairing process and $K_B$ is the Boltzmann's constant.

For example:

<table>
<thead>
<tr>
<th>Element</th>
<th>Critical Temperature (Tc)</th>
<th>Critical gap ratio $\frac{2\Delta(0)}{K_B T_c} = 3.52$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Al</td>
<td>1.2</td>
<td>3.4</td>
</tr>
<tr>
<td>Sn</td>
<td>3.75</td>
<td>3.5</td>
</tr>
<tr>
<td>Cd</td>
<td>0.56</td>
<td>3.3</td>
</tr>
</tbody>
</table>

The deviation from the BCS value may be due to isotopic impurity.

ii. The discontinuity of the Specific Heat $\Delta C$ at $T_c$ is $\frac{\Delta C}{K_B T_c} = 1.43$.

iii. The coherence length is obtained as $\xi_0 = 0.18 \hbar v_F / K_B T_c$. 
iv. The critical field \( \frac{H_c^2}{8\pi} = 2\Delta^2(0)N(0) \), as calculated from BCS ground state Energy
\( W_0 = \langle \Psi_0 | H_{\text{red}} | \Psi_0 \rangle \), is in reasonably good agreement with the experimental results.

Here, \( \Delta(0)N(0) \) is the energy difference between the superconducting ground state and the normal ground state.

In fact, all the properties mentioned in this section for low temperature superconductor can be derived from BCS theory.

### 3.5 Magnetic flux quantisation:

When the magnetic field penetrates into the interior of a type II superconductor, for a field \( H > H_{\text{ci}} \), a vortex structure is formed about each quantised unit of flux line \( \Phi_0 = 2.07 \times 10^{-7} \) gauss/cm². In order to describe a vortex line, we use the following three equations.

(i). Current density in the superconductor

\[
J_e = \frac{e\hbar}{m} |\Psi|^2 \nabla \vartheta - \frac{2e^2}{m} |\Psi|^2 A
\]

3.5.1

Here \( \Psi = \Psi_0 e^{i\vartheta} \) is the superconducting order parameter, \( \nabla \vartheta \) is the phase difference, \( A \) the vector potential, \( e \) the electron charge and \( \hbar \) is Plank constant.

(ii) Maxwell equation

\[
\nabla \times B = \mu_0 J_e
\]

3.5.2

where \( B \) is an applied field.

(iii) Supercurrent velocity
\[ J_e = 2e|\Psi|^2 v_s \] 3.5.3

where \( v_s \) is the supercurrent velocity.

From equations (3.5.1), (3.5.2) and (3.5.3), we get

\[ 2v_s = \frac{\hbar \nabla \theta}{m} - \frac{2e}{m} A \] 3.5.4

\[ \hbar \nabla \theta = 2mv_s + 2eA \] 3.5.5

\[ \hbar \nabla \theta = \frac{m}{en_s\mu_0} \nabla \times B + 2eA \] 3.5.6

![Figure 5.1. Supercurrent encircling a flux tube](image)

In order to calculate quantisation of vortex line, we integrate equation (3.5.5) over enclosed counter line \( L \) [Fig.5.1].

\[ \oint_{L_i} \hbar \nabla \theta \cdot dl = 2m \oint_{L_i} v_s \cdot dl + 2e \oint_{L_i} A \cdot dl \] 3.5.7

The left hand side of the equation (3.5.7) contributes a phase difference of \( 2\pi n \) on encircling suppcurrent once around the line \( L \). The current and the field exist near the
line and are screened from the region at large distance from the line. For large distance, \( v_x \) is zero.

Then equation 3.5.7 becomes

\[
2e \oint_l A \cdot dl = \hbar 2\pi n
\]

3.5.8

We use Stokes theorem to change this line integral to surface integral

\[
2e \oint_s \nabla \times A \cdot ds = \hbar 2\pi n
\]

3.5.9

\[
2e \oint_l B \cdot ds = \hbar 2\pi n
\]

3.5.10

\[
2e \Phi = \hbar 2\pi n
\]

3.5.11

\[
\Phi = \frac{\hbar}{2e} n = n\Phi_0
\]

Where \( \Phi \) is the total magnetic flux through the hole (normal core) and the region surrounding the hole where field penetrates into the superconductor.

In order to see the structure of the vortex line, we take curl on the both sides of 3.5.6

\[
h \nabla \times \nabla \theta = \frac{m}{en_s \mu_0} \nabla \times \nabla \times B + 2e \nabla \times A = 0
\]

3.5.12

\[
\lambda^2 \nabla^2 B - B = 0 , \text{ where } \lambda^2 = \frac{m}{2e^2 n_s \mu_0} \text{ is the penetration depth. The radial part of the equation is}
\]

\[
\lambda^2 \frac{1}{r} \frac{d}{dr} \left( r \frac{dB}{dr} \right) - B = 0
\]

3.5.13
The solution for B is \( \frac{n \Phi_0}{2 \pi \lambda^2} K_0 \left( \frac{r}{\lambda} \right) \), where \( K_0 \) is the Bessel function of the order zero; this is the magnetic field for individual vortex lines.

Using this value in equation (3.5.2), we get the current density

\[
J_e = -\frac{n \Phi_0}{2 \pi \lambda^3 \mu_0} K_1 \left( \frac{r}{\lambda} \right)
\]

For a large distance, equation (3.5.14) can be solved for \( v_s \)

\[
v_s \approx e^{-\left( \frac{r}{\lambda} \right)}
\]

The supercurrent velocity and magnetic field B die exponentially at large distance. The sharp decay is due to the coupling between supercurrent and magnetic field. We see from equation (3.5.8) that \( v_s \) tends to diverge for the small value of “r”. To get rid of this divergence, we have to introduce a core whose radius is \( r = \xi \). This determines the radius of a vortex line. The picture of the vortex line in the superconductor can be summarised as below.

![Diagram](image)

Fig. 3.5.2: The supercurrent velocity \( v_s \), magnetic field B around the flux line
3.6 Josephson effect

A S-I-S junction is a structure consisting of two superconducting layers divided by a thin insulating layer (Fig. 3.6).

For an S-I-S junction, with a thin insulating barrier (<10Å), Josephson, in 1962, predicted that a supercurrent $I_s$ can flow across the junction in the absence of any applied electric field [8]. It is shown in the figure 3.6. This is called DC Josephson effect. He showed that the tunneling of the supercurrent is determined by the change in phase of the order parameter $\Psi(r)$ across the junction. Furthermore, he was able to show that the tunneling current depends, in sensitive way, upon any magnetic field present in the junction. The tunneling current in the presence of a magnetic field will have the form

$$I = I_0 \frac{\sin \frac{\pi \Phi}{\Phi_0}}{\frac{\pi \Phi}{\Phi_0}}$$

where $\Phi$ is the total magnetic flux in the junction and $\Phi_0$ is the quantisation of flux.

Similarly, Josephson predicted that a dc electric field applied across the junction S-I-S induces an oscillatory supercurrent with a frequency $\hbar \omega / 2\pi = 2eV$. This is called the AC Josephson effect.
3.7 Critical state models:

In the critical state model, the critical current density is defined as the current density which produces a Lorentz force \( F_L \) on the vortices that is in equilibrium with the pinning force \( F_p \). This was first introduced by Bean (1962) for absolute zero temperature \( T = 0 \) [9-10]. For a finite temperature, the Bean critical state model has been modified by Anderson and Kim in 1964 [11,12]. We will present here both of these two models.

3.7.1 Bean Critical - State Model:

The basic premise of this model is that in a type II superconductor with homogenously distributed defects, the hysteretic magnetisation \( M \) is related to the critical current density \( J_c \) through a modified Ampere's law \( \nabla \times H = \left( \frac{4\pi}{c} \right) J_c \) where \( H \) is the magnetic induction inside the superconductor. Here the gradient in \( H \) (or \( \nabla \times H \) ) is determined by flux pinning. Thus, the hysteretic magnetisation can be used to derive \( J_c \).

For a slab of thickness \( d \), \( J_c = \frac{20\Delta M}{d} \) (\( H/\) slab surface)

For a cylinder of radius \( R \), \( J_c = \frac{15\Delta M}{R} \) (\( H/\) c axis)

For a flat sample with the lateral dimensions \( a \) and \( b \), \( J_c = \frac{20\Delta M}{a - a^2 / 3b} \) (\( H/\) ab plane)

For a slab of thickness \( d \) with external field \( H \) parallel to the slab plane, Bean pictured the penetration of flux from both side of the sample, which is shown in Fig.3.7.
Fig. 3.7.1: An illustration for the field penetration inside the sample (slab) of thickness d. Here $H^*$ is the field at full penetration.

It is clear from figure (3.7) that, as the field increased, the flux fronts penetrate further into the sample, and at $H = H^* = \frac{2\pi}{c} d J_c$ (field at full penetration), they reach the centre.

### 3.7.2 Anderson and Kim theory:

The critical state model states that at $T = 0$, the fluxoids in the bulk of a type II superconductor can move only if the Lorentz force $F_L$ acting on the fluxoids exceeds the maximum pinning force $F_p$. Inside the sample, the fluxoids can relax into a preferable arrangement when there is a balance between the Lorentz force and the pinning force.

$$F_L = \frac{1}{c} J \times B \equiv \frac{1}{c} \nabla \times B \times B = F_p$$

where $J = \nabla \times B$. 
Anderson suggested that at temperatures greater than zero, fluxoids can move with the help of thermal activation over a potential barrier $U$, even if the Lorentz force $F_L$ is smaller than the pinning force $F_P$. The motion of the fluxoids with the thermal activation is called flux creep (classical flux creep). Anderson further suggested that the rate at which the flux bundles jump over the pinning barrier can be described by the usual Arrhenius expression [11,12].

$$\nu = \nu_0 e^{-\frac{U(J)}{KT}}, \quad 3.7.2.1$$

where $\nu$ is the attempt frequency, at which flux bundles jump and $U$ is an effective activation energy. At finite temperature the rate of thermally activated motion is strongly enhanced by the driving force $F$, and therefore the activation barrier $U$ must be a decreasing function of $J$.

To account for $U$ as a decreasing function of $J$, Anderson and Kim in 1964 defined a linear relationship, i.e.

$$U = U_0 [1-J/J_{c0}] \quad 3.7.2.2$$

The first term $U_0$ is the effective activation barrier due to thermal activation in absence of the driving force. The second term is a modulation term, to decrease the height of $U_0$ due to applied force.

Anderson and Kim used equations (3.7.2.1) and (3.7.2.2), where they obtained logarithmic classical creep rate equation [12]:

$$J = J_{c0} [1-(KT/U)] \ln(t/t_0)].$$
3.8 Lawrence and Doniach Model:

Lawrence and Doniach have developed a model to take into account layered structures of high Tc superconductor based on the modified version of Ginzburg-Landau theory [13]. In this model, the thickness of the superconducting layer $d$ is comparable to the coherence length $\xi(T)$, leading to a homogenous current $J_s$ (Josephson current) and field distribution in the $z$-direction within layers.

The GL free energy has been modified in two aspects:

(a). The effective mass in this model is $m^* = 2m$, $M^* = 2M$ for the electron moving parallel and perpendicular to the plane respectively.

(b). The order parameter $\Psi(r)$ with coordinates $r = (\rho,z)$ of GL theory is replaced by $\Psi_n(\rho) = |\Psi_n(\rho)| \exp[i\varphi_n(\rho)]$, where $n$ is the number of superconducting layers. The kinetic energy term $|\nabla_z \Psi|^2$ is also described in the form $\frac{\partial \Psi}{\partial z} \approx \frac{\partial}{\partial z} (\Psi_{n+1} - \Psi_n)$.

One of the interesting results in this modified G-L model is that it gives an understanding of Josephson intercoupling of the vortices from obtained Josephson current as $J_z(\rho,z_n) \approx J_{co} \sin(\varphi_{n+1} - \varphi_n)$ where $J_{co}$ is the critical Josephson current density.

3.9 Elastic continuum model:

The Abrikosov vortex lattice can be considered within the elastic continuum model if the vortex line varies slowly within the distance $a_0$ [14-18]. Furthermore, these vortices have long range interaction which can be treated with non-local elastic moduli $C_{ij}$. The elastic free energy with the non local moduli in Fourier transform representation is
\[ F_{el} = \frac{1}{2} \int \frac{d^2 k}{(2\pi)} u_\alpha(k) \Phi_{\alpha\beta}(k) u_\beta(-k) \] with \( \alpha, \beta = x, y, z \),

where

\[
\Phi_{\alpha\beta}(k) = \left[ c_{11}(k) - c_{66} \right] k_\alpha k_\beta + \delta_{\alpha,\beta} \left[ c_{66} k_\perp^2 + c_{44}(k) k_z^2 \right]
\]

with \( k_\perp^2 = k_x^2 + k_y^2 \). The moduli \( c_{11}(k) \), \( c_{44}(k) \) and \( c_{66}(k) \) are temperature and field dependent. Here \( c_{11}(k) \), \( c_{44}(k) \) and \( c_{66}(k) \) are respectively compression modulus, tilt modulus and shearing modulus. The shearing modulus \( c_{66}(k) \) determines the stability of the flux lines within a single plane. For small fields, the shearing modulus can be used to determine the melting temperature \( T_m \) of the two dimension (2D) vortex lattice of strongly anisotropic high Tc superconductors (i.e. \( c_{66}(k) = 0 \) indicates the vortex liquid phase). The tilt modulus \( c_{44}(k) \), for small fields, depends strongly on the anisotropic axis of high Tc superconductor. For example, an estimate of the Josephson and magnetic coupling field contributions to the interlayer coupling of high Tc superconductor at low temperature can be obtained from \( c_{44}(k) \) for small fields (a straight vortex line has no Josephson coupling) \[14\] and the compression modulus \( c_{11}(k) \) determines the softness of the vortex lattice.
CHAPTER FOUR: HIGH TEMPERATURE SUPERCONDUCTORS ( HTSC )

4.1 Introduction:

The discovery of High $T_c$ superconductors is of revolutionary significance in the history of superconductivity (see fig. 4.1) and it was started in the work by Bednorz and Muller in identifying La$_{2-x}$(Ba,Sr)$_x$CuO$_4$ with $T_c = 34K$ in 1986 [19]. Since then, this discovery has stimulated an enormous experimental and theoretical effort in understanding of these superconducting materials. Before this, the highest critical temperature was 23.3 K with the upper critical field $H_{c2} \approx 40$ T (tesla) at 4.2K. Because of their low $T_c$, Nb$_3$Ge and other intermetallic alloys are grouped as low temperature superconductors.

Now we have many Cu-O$_2$ based superconductors with transition temperature $T_c$ greater than the liquid nitrogen boiling temperature, 77K. Among them, YBa$_2$Cu$_3$O$_{7-\delta}$ (with $T_c = 90K$ )[20], Bi$_2$Sr$_2$CaCu$_2$O$_8$ ($T_c = 80 - 90K$)[22,85], Bi$_2$Sr$_2$Ca$_2$Cu$_3$O$_{10}$ ($T_c =105 - 110$)[22] and Tl$_2$Ba$_2$Sr$_2$Ca$_2$Cu$_3$O$_{10}$ ($T_c = 110 - 127K$)[23] are considered as the most important superconductors. There is no doubt that the main motivation to search for high $T_c$ superconductivity derives from their great technological application. All the above mentioned superconductors possess one common feature, that is, their superconducting properties arise from the in two dimensional CuO$_2$ planes.
Fig. 4.1: A history of the improvement of $T_c$. Here $\bigcirc$ stands for measured $T_c$[84].

4.2 Technical application of high $T_c$ superconductors

Because high $T_c$ superconductors give a zero resistance above liquid nitrogen boiling temperature 77K and have high $H_{c2}$ (~200 T at 4.2K), it is possible to use these
materials for a superconducting magnet which may be applied to magnetically levitated train and nuclear fusion, energy storage, electricity transmission, flux transformer etc[83]. For the electronic application, since Josephson effects have also been observed for high $T_c$ superconducting junctions, they might be applied to a microprocessor, superconducting quantum interference devices (SQUID) etc.

4.3 A few most important high $T_c$ superconductors:

4.3.1 YBa$_2$Cu$_3$O$_{6+x}$ (1-2-3)

This is the first high $T_c$ superconductor with $T_c$ above 77K (90K) and this was discovered by Wu [20]. This discovery initiated an unprecedented level of world-wide activity due to its great possible technological potential of superconductivity in industry. The structure of YBa$_2$Cu$_3$O$_{6+x}$ is orthorhombic. The orthorhombic structure contains double CuO$_2$ layers oriented in the (001) plane. It has a square pyramidal oxygen environment around copper with the apices directed above and below the CuO$_2$ double layer. Yttrium cations reside between the Cu-O sheets of double layer in 8-fold coordination with oxygen. Ba ions are found above and the below the double Cu-O sheets. The BaO/CuO$_2$/Y/CuO$_2$/Ba) slabs are interconnected by a sheet of Cu and O atoms with variable composition CuO$_x$.

4.3.2 Bi$_2$Sr$_2$Ca$_{n-1}$Cu$_n$O$_{2n+4}$

The Bi-Sr-Ca-Cu-O high $T_c$ superconductor with $T_c = 20K$ was first discovered by Michel et al [21] and then a family of this compound was discovered by Maeda et al
with $T_c = 85K \ (n = 2)$ and $T_c = 105K \ (n = 3)$ \[22\]. The subcell structure of Bi-based oxide superconductors are given in figure 4.3.2. The homologous series of bismuth compounds, $\text{Bi}_2\text{Sr}_2\text{Ca}_{n-1}\text{Cu}_n\text{O}_{2n+4+\delta} \ (n = 1, 2, 3)$ suggests a possible existence of other structures with more copper layers.

4.3.2.1 $\text{Bi}_2\text{Sr}_2\text{CuO}_6 \ (2-2-0-1, \ n = 1)$

The Bi 2201 structure consists of corner-linked CuO$_4$ plane sandwiched between two Sr-O$_2$ layers. Each copper atom has one oxygen atom positioned above and below the CuO$_2$ plane, thus forming an axially elongated CuO$_6$ octahedron. The Strontium has nine nearest oxygen atoms [Fig. 4.3.2].

4.3.2.2 $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_8 \ (2-2-1-2, \ n = 2)$

The topological structure of Bi 2212 is similar to that of Bi 2201 except Cu-O sheets in Bi 2201 are replaced by CuO$_2$/Ca/CuO$_2$ layers. Each Cu atom in Bi2212 has two additional oxygen atoms positioned above or below the CuO$_2$ sheet thus forming a square pyramids [Fig. 4.3.2].

4.3.2.3 $\text{Bi}_2\text{Sr}_2\text{Ca}_2\text{Cu}_3\text{O}_{12} \ (2-2-2-3, \ n = 3)$

The topological structure of Bi 2223 is similar to that of 2212, however here an additional CuO$_2$ and Ca layers are inserted within the CuO$_2$/Ca/CuO$_2$ sandwich of Bi 2212, yielding a CuO$_2$/Ca/CuO$_2$/Ca/CuO$_2$ sandwich. It is reported that pure Bi 2223 phase is difficult to synthesise because it is always in phases with 2212 [Fig. 4.3.2].
4.3.4 Tl\textsubscript{2}Ba\textsubscript{2}Ca\textsubscript{n-1}Cu\textsubscript{n}O\textsubscript{2n+4+δ}

The Tl\textsubscript{2}Ba\textsubscript{2}Ca\textsubscript{n-1}Cu\textsubscript{n}O\textsubscript{2n+4+δ} structure was first discovered by Sheng and Hermann [23]. The structure of the Thallium based oxide superconductor is similar to that of the Bi-based superconductor with three different new superconducting phases. The \( T_c \) dependence on the number of copper layers for this family is summarised below.

<table>
<thead>
<tr>
<th>Ideal formula</th>
<th>Acronym</th>
<th>( T_c )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tl\textsubscript{2}Ba\textsubscript{2}CuO\textsubscript{6}</td>
<td>2201 (n = 1)</td>
<td>90K[23]</td>
</tr>
<tr>
<td>Tl\textsubscript{2}Ba\textsubscript{2}CaCu\textsubscript{2}O\textsubscript{8}</td>
<td>2212 (n = 2)</td>
<td>110K[23]</td>
</tr>
<tr>
<td>Tl\textsubscript{2}Ba\textsubscript{2}Ca\textsubscript{2}Cu\textsubscript{3}O\textsubscript{10}</td>
<td>2223 (n = 3)</td>
<td>122K[23]</td>
</tr>
</tbody>
</table>
Fig. 4.3.2: Crystal structures of the series of compounds for $\text{Bi}_2\text{Sr}_{2-n}\text{Ca}_n\text{Cu}_{n+4}O_{2n+4}$ for (1) $n = 1$, (2) $n = 2$ and (3) $n = 3$. 
4.4 A comparison study between High Tc superconductivity (HTSC) and Low temperature superconductivity (LTSC)

<table>
<thead>
<tr>
<th>Low Tc superconductor</th>
<th>High Tc superconductor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Resistivity transition width ($\Delta T$) is</td>
<td>Resistivity transition width ($\Delta T$) is</td>
</tr>
<tr>
<td>$10^{-2}$K.</td>
<td>0.5K.</td>
</tr>
<tr>
<td>Vortex lattice</td>
<td>Vortex glass</td>
</tr>
<tr>
<td>The Ginzburg-Landau parameter</td>
<td>The Ginzburg-Landau parameter</td>
</tr>
<tr>
<td>$\kappa = \frac{\lambda}{\xi} \approx 1$ (for a type II superconductor)</td>
<td>$\kappa = \frac{\lambda}{\xi} \approx 10^2$ (for a HTSC superconductor)</td>
</tr>
<tr>
<td>The upper critical field $H_{c2}$ is less than</td>
<td>The upper critical field $H_{c2}$ is more than</td>
</tr>
<tr>
<td>40 Tesla at 4.2K[83]</td>
<td>200 Tesla at 4.2K[83].</td>
</tr>
</tbody>
</table>

4.5. Vortex

4.5.1 Types of vortices:

In high temperature superconductors, due to their high critical temperatures and large anisotropy, a magnetic vortex matter can have mainly four different configurations. These are three dimensional (3D) vortex solid [Fig. 4.5.1a], three dimensional (3D) vortex liquid [Fig. 4.5.1b, 1c], two dimensional (2D) vortex solid (or 2D pancake vortex) [Fig. 4.5.1d] and two dimensional (2D) vortex liquid [24-26]. It is also observed that the 3D vortex liquid state can have configurations: entangled [Fig. 4.5.1c] and disentangled vortex liquid states [Fig. 4.5.1b].
4.5.2 Thermal fluctuation of the vortices:

In High Tc superconductors, due to their high $T_c$ and small coherence length $\xi \propto h v_F / T_c$, the superconducting order parameter $\Psi(r) = |\Psi(r)| \exp(i\phi(r))$ is subjected to a large thermal fluctuation [26]. Here, $v_F$ is the Fermi velocity.

This thermal fluctuation depins the vortices even for the external current $J$ less than the critical current $J_c$ and produces dissipation in the system. This phenomenon in high Tc superconductors is illustrated through the following diagrams:

- **3D vortex solid lattice**
- **3D disentangled liquid**
- **3D entangled liquid**
- **2D pancake vortices coupled through the Josephson vortices**

Fig. 4.5. Four different configurations of vortices.
superconductor is known as the classical flux creep phenomenon. Here the creep refers to a small but finite directed motion of the vortex lines.

4.5.3 Quantum fluctuation of the vortices:

According to the classical picture, the decay rate of a vortex density gradient due to thermal fluctuation should be zero at zero temperature. However, in a number of recent experiments the low temperature relaxation rate has been found not to extrapolated to zero, suggesting the existence of vortex motion via quantum tunnelling [27]. This phenomenon of vortex motion is known as quantum creep. In this creep phenomenon, the quantum fluctuation also acts on the order parameter [26].

4.6 Pinning centres:

4.6.1. Types of pinning centres:

There are mainly two types of pinning centres for the vortices which are introduced as either point defects or extended defect.

The defect which has its size comparable to the coherence length is called the point defect [28]. In high Tc superconductors, oxygen vacancies are the source of point defects. These point defects have small pinning energy and generally a single defect can not pin a single vortex line (except at very low field, where it can pin a single vortex line, described by a single vortex pinning theory). In order to pin the vortex lines, collective pinning by large number of these point defects is required. This is described by the collective pinning theories.
Extended defects such as dislocation, grain boundaries and columnar defects have large normal core [28]. These defects can pin vortices by themselves because they have a large pinning energy. They can even pin vortex bundles. These defects are very important in technical applications of the high T_c superconductors because they localise the vortices.

4.6.2: Disorder of vortex lattice:

Any reversible type II superconductors in a field up to \( H < H_{c1} \), show the Meissner effect, and they go to normal state as the field goes above \( H_{c2} \). At \( H_{c1} \), the magnetic flux starts to penetrate in the form of vortex lines. As \( H \) increases further, flux lines are compressed close together, and in the field close to \( H_{c2} \), they are arranged in the regular lattice form (called Abrikosov lattice), due to a long range interaction between the neighbouring vortices [Fig. 4.6.2a] [29].

![Vortex lattice](image1)

![Vortex glass (disordered)](image2)

Fig. 4.6.2: Vortex lattice (a) and vortex glass (b).
But, in the presence of large concentration of randomly distributed point defects (which act as pinning centres), the translational long range order of the vortex lattice is not retained in the crystal due to distortion of the vortex lattice. This distorted vortex lattice is called a vortex glass (Fig.4.6.2b) [30-33].

Similarly, the distorted vortex lattice with extended defects is called a Bose glass [34]. The vortex lines can be pinned strongly by these defects.

4.6.3 Surface barriers and Geometrical barriers:

It was considered that the common source of magnetic hysteresis in any type II superconductor is the bulk pinning due to inhomogeneities and material defects such as extended defects, point defects. However, there are plenty of experimental and theoretical results where it is mentioned that, besides bulk pinning, there are two other mechanisms which make an important contribution to the magnetic hysteresis. They are called Bean-Livingston (BL) surface barriers and geometrical barriers [35-42]. The BL surface barrier, which affects the vortex entry (and exit from) in a superconductor, results from the competition between two forces. The first force arises from the attractive interaction between a vortex and its mirror image anti-vortex which acts to move the interior vortex toward the surface. The second force arises from the repulsive interaction between a vortex and surface shielding currents which tend to push the vortex inward. The barrier acts to prevent the first flux penetration $H_p$ when $H_p < H_{c1}$ (lower critical field) [37-42]. Therefore, the surface barrier is defined by the surface of the sample. The geometrical barrier arises from a competition between the elongation energy of a vortex penetrating into the sample through corners of the sample and the Lorentz force [35-36]. The geometrical barrier is determined by the shape of the sample.
4.7 Vortex - glass theory:

The vortex glass theory is used to investigate the effect of disorder introduced by randomly distributed point defects on the magnetic vortex phase diagram and also to investigate the effect of thermal activation on the magnetic vortex phase diagram [30-33]. In this theory, the vortex glass transition line $B_c(T)$ is added to the mean field transition line $B_{c2}(T)$ in low temperature superconductor and the Abrikosov vortex lattice phase is replaced by the vortex glass phase. The introduced disorder leads to pinning of vortex lines and therefore produces a highly non-linear current-voltage characteristics. This non-linear behaviour of the vortex system has been considered in the study of the vortex glass phase as a function of temperature and magnetic field.

![Log E vs Log J graph](image)

Fig. 4.7: Current-voltage characteristic of high Tc superconductors [26].
For fields $B < B_G$, the current-voltage characteristics can be expressed as

$$E(J) \propto \exp\left(-\frac{J}{J_c}\right)^{-\mu},$$

with the exponent $0 < \mu < 1$. This is in contrast to the Anderson and Kim model and its extension by considering a distribution of activation energies, where there is a finite ohmic resistance even for the zero external current density $J$.

At the phase transition, $B = B_G$, Fisher obtained a power law for the $J$-$E$ response, $E = J^\beta$, where the exponent $\beta > 2.5$ is predicted. Above the transition ($B > B_G$), the system is in liquid phase and the resistance $\rho(T)$ is expected to reach a constant non-zero value at low currents. At higher current densities, viscous flux flow will occur, which is characterised by a $J$-$E$ response very close to linear voltage. The crossover between the low-current regime and the power-law regime occurs around the current density $J^-$ at $T < T_g$ and $J^+$ at $T > T_g$ [24]. The crossover current vanishes according to $J^- \propto (T_g - T)^{2\nu}$ and $J^+ \propto (T - T_g)^{2\nu}$ at $T = T_g$. The first experimental evidence to a continuous transition into a vortex glass phase was performed by Koch, where he carried out his experiments on YBCO films [30-31]. The exponent $\nu$ is 1.7 for YBCO films. The plot shown in Fig. 4.7 gives all the relevant information on the dynamic response into a vortex glass phase.

### 4.8 Magnetic phase diagram:

The vortex matter structure in high Tc superconductor is a complicated function of temperature, magnetic field, and material disorder. This results in a rich H-T phase diagram in high Tc superconductor, where we have numerous phase transitions and crossovers [24,26,43-46]. The H-T phase diagram for the BSCCO crystal contains a
larger portion of liquid phase than that for YBCO. We can see from figure 4.8 that there are four different phases: three dimensional vortex solid (3D), three dimensional vortex liquid (3D), two dimensional pancake vortices (2D) and two dimensional pancake liquid (2D).

For fields $B > B_{cr}$ (critical magnetic field) and $T > T_m$ (melting temperature), we see a vortex liquid consisting of fairly independent pancake vortices and for the same field $B > B_{cr}$ but with temperature $T_{dc} < T < T_m$, ($T_{dc}$ decoupling temperature) we see 2D pancake vortices. Similarly, for fields $B < B_{cr}$ and $T > T_m$ we see a 3D vortex liquid and with the same fields $B < B_{cr}$ but with $T < T_m$ we see a 3D vortex solid. The 3D liquid vortices close to the melting lines are pinned since the pinning barrier against the vortex motion is still large and they are called disentangled vortices. But close to the upper critical field $H_{c2}$ the 3D liquid vortices are unpinned and these unpinned vortices are called entangled vortices.
CHAPTER FIVE:

MAGNETIC RELAXATION:

A study of the magnetic relaxation in high Tc superconductors (HTSC) is very important because it helps us to understand the physical properties of the HTSC, such as magnetic phase diagram, pinning mechanism and thermodynamic properties[47-51]. It is also mentioned that, from the technical point of view, magnetic relaxation modifies the current-voltage characteristics of the high Tc superconductors, determines the temperature and time dependence of the current density and dictates limits to the stability of high Tc superconductor devices, such as persistent mode magnets [47,50].

The basic mechanism for the magnetic relaxation is that a configuration of vortices is in a non-equilibrium state because of the flux pinning and it will try to relax to a state with a minimum energy. This relaxation process will lead to a redistribution of current loops in the superconductor and hence to a change of the magnetic moment with time. The change of magnetic moment with time is studied in the magnetic relaxation measurement. The measured magnetic relaxation can also be thought as being caused by the spontaneous motion of magnetic vortices out of their pinning centres. Such motion usually arises from thermal fluctuations and quantum fluctuations over activation energy barriers.

The phenomenon of the magnetic relaxation was first suggested by P.W. Anderson in 1964 to explain the experimental data of Kim (1964) [11,12]. He defined a thermally activated flux creep model to explain the relaxation phenomenon. According to Anderson’s flux creep model, the motion is thermally activated, and the rate with which flux lines jump over pinning barriers can be described by an Arrhenius-type expression.
\[ \gamma = \gamma_0 \exp \left( - \frac{U(J)}{kT} \right), \]

where \( \gamma_0 \) is an attempt frequency and \( U(J) \) is the effective activation energy. Writing this in terms of time, since frequency is inversely proportional to time, the hopping time of the flux line is

\[ t = t_0 \exp \left( \frac{U(J)}{kT} \right) \]

The hopping process is assisted by the driving force \( F = \left( \frac{1}{c} \right) J \times B \). Therefore, \( U \) must be a decreasing function of \( J \),

\[ U = U_0 \left( 1 - \frac{J}{J_{c0}} \right), \]

Where \( U_0 \) is the barrier height in the absence of the driving force and \( J_{c0} \) is the critical current density. Using equations (5.2) and (5.3), we obtain the Anderson classical flux creep equation, which is logarithmic with time

\[ J = J_{c0} \left[ 1 - \frac{(kT/U_0) \ln(t/t_0)}{2} \right] \]

Beasely, Labuch and Webb in 1969, while elucidating experimentally the mechanism of Anderson’s thermally activated flux-creep process, observed a non-linear \( U-J \) relationship leading to non-logarithmic decay of magnetisation [52]. They made their experimental investigation and theoretical calculation for solid superconducting cylinders. In the experimental investigation, they made their observation with a Superconducting Quantum Interference Device (SQUID) to observe flux configuration change. In the theoretical calculation, they used a nonlinear magnetic diffusion equation to describe the equation of motion for a flux creep process:

\[ \frac{dB}{dt} = -\frac{dD}{dx}, \]
where \( D = B \alpha \gamma_0 \exp[-\frac{U_{\text{eff}}}{K T}] \) is the flux flow density. Here \( B \) is the local flux density and \( \alpha \) is the hop distance for flux bundles.

Their results can be summarised as below:

1. A logarithmic time dependence of the creep process prevails in the critical state, due to exhaustion of the excess driving force over the pinning barrier. The linear approximation \( U = U_0 - F V X \) made by Anderson and Kim is justified in the limit \( K T \ll U \), where \( V \) is flux bundle and \( X \) is pinning length.

2. A non-logarithmic time dependence of the creep process prevails in the sub-critical state. They further suggested that, on departing from the critical state to the subcritical state, the creep rate becomes exponential by decreasing temperature and applied field. When the thermal activation takes place near the top of an energy barrier, \( U \) becomes non linear function of \( F(\sim |V|B|) \), and therefore the parameters \( U_0 \) and \( V X \) used by Anderson-Kim are not trivially related to the height and width of the activation barrier.

The full impact of the non-linear \( U(J) \) dependence has become apparent only after development of vortex glass theory by Fisher [30-32] and collective pinning theory by Feigel'mann [26, 53-54]. The results of these theories is that the barrier diverg as the current \( J \) approaches zero.

Feigel'mann, in his collective pinning theory, showed that at an applied current \( J \ll J_c \), the activation barrier \( U(J) \) between different metastable states grows with a power law

\[
U(J) = U_0(J_c/J)^\alpha
\]

(5.7)

where \( U_0 \) is the characteristic energy scale and the exponent \( \alpha \) depends on the dimensionality, on the particular regime of the flux creep. In three dimensions \( \alpha = 1/7 \) in the weak field, low temperature regime where creep is dominated by the motion of the individual flux line. For \( \alpha = 3/2 \), the collective creep of small bundles takes place.
(at higher fields) and for $\alpha = 7/9$, collective creep of large bundles takes place (at very large fields). For two-dimensional collective creep process, $\alpha = 9/8$.

If current relaxation is due to some thermal activation process, then Geshkenbein proposed a relation for the activation barrier $U_\alpha$ and the persistent current $J(t)$ [55] and it is given by

$$U_\alpha(J) = T \ln(t/t_0),$$

where $t_0$ is the attempt time.

Using equations (5.7) and (5.8), they obtained the current relaxation law,

$$J(t) = J_c[(U_0/T)\ln(t/t_0)]^{-1/\alpha}$$

Blatter et al in 1994 have used simple electrodynamic equation to obtain the classical theory of the flux creep equation[26]. They used two Maxwell equations:

$$\nabla \times B = \frac{4\pi}{c} J,$$  

5.10

where $B$ is the magnetic induction and $J$ is the transport current and

$$\nabla \times E = -\frac{1}{c} \frac{dB}{dt},$$

5.11

where $E$ is the electric field and $c$ is the velocity of light.

Equation (5.10) gives a relation between current density and vortex density gradient as a result of pinning and equation (5.11) determines the decay of the current density.

These authors mentioned in their paper that the vortices in the magnetisation measurement moved due to the driving forces that arose from a density gradient of the fluxoids with a relation $\nabla \times B = (\frac{4\pi}{c^2})J$. This gradient of the vortices arise because of flux pinning and it produces a current which decays due to thermal activation of the
vortices. The electric field $E$ (i.e. dissipation) generated by vortex motion is given by

$$E = \frac{1}{c} B \times v$$

where $v$ is the velocity of the vortices.

With a field $B$ parallel to the $z$ axis and electric field $E$ parallel to the $y$ axis (or current $J$ parallel to $y$ axis), we have a vortex velocity in the direction of the $x$ axis [Fig. 5].

Fig. 5: Applied field $B//z$-axis, $E//y$-axis and $v//x$-axis

$$E_y = \frac{1}{c} B_z v_x = \frac{1}{c} BV$$

Using equations (5.11) and (5.12), we get

$$\frac{dB}{dt} = \frac{d}{dx} (VB)$$

With equation 5.10, equation 5.13 becomes

$$\frac{dJ}{dt} = \frac{c}{4\pi} \frac{d^2}{dx^2} (VB)$$
In Anderson and Kim model, the rate at which a flux bundle jumps over the pinning barrier is given by

\[ V = V_0 e^{-\frac{U(J)}{K T}} \]  \hspace{1cm} 5.15

With equation (5.15), equation (5.14) becomes

\[ \frac{dJ}{dt} \approx \frac{J_c}{\tau_0} \exp\left(\frac{U(J)}{K T}\right) \] where \( t_0 = \frac{K T \tau_0}{J_c \left| \frac{dU}{dJ} \right|} \) is the attempt time.

The above equation can be solved with logarithmic accuracy (Geshkenbein and Larkin).

\[ U(J(t)) = K BT \ln \left[ 1 + \frac{t}{t_0} \right] \]  \hspace{1cm} 5.16

As we mentioned before, in the Anderson and Kim model, the thermally activated motion will be strongly assisted by the Lorentz force (i.e. current density), therefore the activation energy should be a decreasing function of the current density.

To account for the fact that \( U \) is a decreasing function of the Lorentz force, \( U \) is defined as

\[ U(J) = U_c \left[ 1 - \frac{J}{J_c} \right]^\alpha \]  \hspace{1cm} 5.17

with the condition that for \( J = J_c \), the activation energy should vanish.

Now, substituting (5.17) into (5.16), we get

\[ J(t) = J_c \left[ 1 - \left( \frac{K T}{U_c} \ln \left( 1 + \frac{t}{t_0} \right) \right)^\frac{1}{\alpha} \right] \rightarrow J_c \]  \hspace{1cm} 5.18

For \( \alpha = 1 \), the above relation agrees well with the Anderson and Kim formula.

This relation is very good for conventional type II superconductor, when \( J \rightarrow J_c \).
However, for high Tc superconductor where the values of $J \ll J_c$ are reached already for laboratory times, the collective pinning theory and vortex glass theory predict an activation barrier which should diverge as $J \rightarrow 0$.

\[ U(J) = U_c \left( \frac{J_c}{J} \right)^{-1/\mu} \]  \hspace{1cm} 5.19

Inserting equation (5.19) into equation (5.16), we get a non-purely-logarithmic (a power law) time dependence current density

\[ J(t) = J_c \left[ \frac{KT}{U_c} \ln \left( \frac{t}{t_0} \right) \right]^{-1/\mu} J \ll J_c \]  \hspace{1cm} 5.20

Equations 5.18 and 5.20 can be combined to obtain one interpolation formula.

\[ J(t) = J_c \left[ 1 + \frac{\mu KT}{U_c} \ln \left( 1 + \frac{t}{t_0} \right) \right]^{-1/\mu} \text{ where } \alpha = 1 \]  \hspace{1cm} 5.21

and the activation barrier is

\[ U(J) \simeq \frac{U_c}{\mu} \left[ \left( \frac{J_c}{J} \right)^{\mu} - 1 \right] \]  \hspace{1cm} 5.22

Within single vortex pinning regime, the exponent $\frac{1}{\mu} = 7$, such that for

\[ \left( \frac{\mu KT}{U_c} \right) \ln \left( 1 + \frac{t}{t_0} \right) \ll 1 \text{ the interpolation formula becomes} \]

\[ J(t) \approx J_c \left( 1 + \frac{t}{t_0} \right)^{-KT/\mu U_c} \text{ with logarithmic potential} \]

\[ U(J) = U_c \ln \left( \frac{J_c}{J} \right) \]  \hspace{1cm} 5.23
CHAPTER SIX:

EXPERIMENTAL METHODS

6.1 Fabrication of Bi-2212 single crystals:

The following are the standard accepted techniques for the growth of Bi2212 single crystal [36].

i. Self-flux growth,

ii. Alkali-halide flux growth,

iii. Travelling solvent floating zone (TSFZ), and

vi. Seeding.

But the most widely used methods are the self flux growth method and travelling solvent floating zone method. In this section I would like to concentrate only on the self flux growth method because I have used this for the preparation of the Bi-2212 single crystal.

6.1.1 Preparation of high quality powders with solid state reaction:

The high purity (99.9%) powders of Bi$_2$O$_3$, SrCO$_3$, CaCO$_3$ and CuO in a stoichiometric ratio of the cations Bi:Sr:Ca:Cu = 2:2:1:2 were used in the preparation of the superconducting powders. In order to obtain high electric conducting performance of the sample, the ceramic powder precursor must have 100% superconducting phase purity, optimum particle size and low carbon content are required. There are a number
of different techniques used for the preparation of Bi2212 superconducting powders. However, here we have used a solid state reaction in the preparation of the powders. In the first phase of the solid state reaction, a mixture of the appropriate amounts of carbonates SrCO$_3$:CaCO$_3$ in 2:1 ratio) was placed in an alumina mortar and the mixture was calcined at 1250°C for 10 hours in flowing air. After that the appropriate amount of oxides (Bi$_2$O$_3$:CuO) were added into the calcined mixture and was calcined in flowing air at 750°C for 6 hours. In order to improve the reaction rate and to facilitate solid state diffusion in solid state phase, high temperatures are generally needed. The mechanical grinding steps are important to homogenise the sample and ensure complete reaction. Taking this into account, the calcined powders were ball milled in a zirconia container with zirconia balls for half an hour and pelletised by uniaxial pressing in a zirconia die. Forming the materials into a pellet minimises the surface area and helps to limit the reaction with containers.

The pellet was calcinated in the following three steps with different heat treatments, with constant temperature sweep at 350°C per hour between the temperature steps and different atmospheres. In the first calcinated step, the temperature of the pellet was raised to 750°C in flowing air and kept there for 10 hours. The pellet was then crushed into fine powder and pressed again to form a pellet. In the second step, this pellet was treated to 790°C with the same rate in an oxygen atmosphere and kept for 20 hours. This pellet was again crushed to fine powder and pressed again to form a pellet. In the third step, this pellet was heated to 830°C with oxygen atmosphere and kept there for 20 hours. With this process high phase purity was obtained for the composition with stoichiometric ratio of the cations Bi: Sr : Ca : Cu = 2:2:1:2.
6.1.2 Flux growth methods:

The chemical composition of the powders is very important in order to get the best result for the critical temperature. The crystal size and quality depend strongly upon the different starting compositions [57]. The $T_c$ dependence on the stoichiometric values of the nominal composition of the powder based on the formula $\text{Bi}_{2+x} \text{Sr}_{2+y} \text{Ca}_{1+z} \text{Cu}_{2+w} \text{O}_{8+\delta}$, is given in the figure 6.1.1 [58].

The high purity (99.9%) powders of $\text{Bi}_2\text{O}_3$, $\text{SrCO}_3$, $\text{CaCO}_3$ and CuO were mixed well according to atomic ratios $\text{Bi}:\text{Sr}:\text{Ca}:\text{Cu}=4:2:2:2$. The mixed powder composition was loaded into a high purity Alumina crucible $\text{Al}_2\text{O}_3$. Figure 6.1.2 shows a schematic diagram of a cross section of the configuration of a Muffle furnace for the crystal growth. The temperature of the furnace was raised to 1050°C, which is above the melting temperature of the powder sample, and held at this temperature for 4 hours to reach a thermodynamic equilibrium. The temperature was then lowered to 980°C (just above onset of solidification) at a cooling rate of 400°C per hour and then slowly lowered down to 840°C at a rate of 0.8°C per hour. It was then cooled down to 550°C rapidly (200°C per hour) and held at this temperature for approximately 8 hours. The crucible was then taken to air for quenching. A temperature profile for the crystal growth is shown in figure 6.1.3. The crucible was then crushed mechanically to obtain a matrix of crystals. The single crystals were extracted from the matrix by cleaving exposed crystal with an adhesive tape. The adhesive tape was removed from the crystal by dissolving it in hexane.
Figure 6.1.1. The $T_c$ dependence on the concentration of the $\text{Bi}_2\text{O}_3$, $\text{Sr CO}_3$, $\text{CaCO}_3$ and $\text{CuO}$. The top of each above figures represents the constant composition projection plane and the constant composition line on which the measurement was carried out [58].
Figure 6.1.2: A configuration of the Muffle furnace with sample
6.1.3 Fabrication of Bi-(Pb)-Sr-Ca-Cu-O single crystals:

The lead doped Bi-based oxide superconductors were grown by using the Self Flux Method. High purity (99.9%) powders of Bi₂O₃, PbO, SrCO₃, CaCO₃ and CuO were mixed well according to atomic ratios Bi:Pb:Sr:Ca:Cu = 4:2:4:2:2 [59]. About 50 grams of the mixture was put into an Al₂O₃ crucible. The experimental arrangement for this growth is similar to that for the pure Bi2212 single crystal. Due to significant instability...
of Pb at high temperatures, because of the low melting point and high pressure of PbO, there are some modifications on the above mentioned procedures.

The first modification is that the maximum temperature and holding time is lower and shorter than that for undoped crystal, in order to prevent the significant loss of Pb through evaporation at high temperatures. The second modification is that a faster cooling rate of 500 °C per hour (1-5 °C for pure Bi 2212) was employed. After the crystal growth, the crucible was crushed mechanically and crystal were obtained by cleaving from the matrix of the grown bulk crystal. The as grown sample with $T_c = 84$ K was annealed in air at a temperature $550^\circ$C and then quenched to obtain a sample with $T_c = 67$K.

**6.1.4 Fabrication of Bi-(Fe)-Sr-Ca -Cu -O single crystals:**

The samples were obtained from Dr. G. D. Gu, Department of Physics, University of New South Wales. The samples were prepared by the floating zone method (FZM). The experiments were performed in an infrared radiation furnace. This furnace was equipped with two ellipsoidal mirrors and two halogen lamps. The lamps were placed in the focus of the mirrors. The detailed procedures for the preparation and the compositions are presented in the paper “Growth and superconductivity of Bi$_{2.1}$Sr$_{1.9}$Ca$_{1.0}$($Cu_{1-y}Fe_y$)$_2$O$_x$” [85]. The nominal iron (Fe) content in the sample is 0.02 (i.e. $y=0.02$). The analysed single crystal composition by electron probe micro-analysis (EPMA) is Bi$_{2.08}$Sr$_{1.79}$Ca$_{0.98}$(Cu$_{0.984}$Fe$_{0.016}$)$_2$O$_x$. 
6.2 Sample characterisation:
X-ray diffraction pattern (XRD) technique and electron back scattering pattern (EBSP) technique, connected with scanning electron microscope (SEM) were used for studying the quality of the grown sample. Fig. 6.2.1 shows the X-ray diffraction pattern of the three different samples, Bi2212 (top), Bi(Fe)2212 (middle), and Bi(Pb)2212 (bottom). It is seen from that figure 6.2.1 that the peaks corresponding to (00l) Miller indices (l = 2, 6, 8, 10, 12) are very strong. These strong diffraction peaks show a high degree of preferred orientation of the crystallographic planes. The same properties were also observed through the electron back scattering pattern (Fig 6.2.2a) for Bi2212 pure single crystal, where the same orientation pattern appeared at the various points of the crystal (figure 6.2.2b).

It is also seen from fig. 6.1 that the peak positions in the lead doped and iron doped sample were shifted by a small amount from that of the pure single crystal. This can be seen more clearly from Table 6.2 [Bi2212, Bi(Fe)2212, Bi(Pb)2212]. The volume of the unit cell for the Bi2212 as observed from the XRD was \(917.6539 \pm 2.2659 \text{Å}^3\) with \(a = 5.1280 \pm 0.0021\text{Å}, b = 5.8109 \pm 0.0030\text{Å}\) and \(c = 30.7959 \pm 0.0733\text{Å}\), for Bi(Fe)2212 was \(917.7840 \pm 1.8123\text{Å}^3\) with \(a = 5.1280 \pm 0.0021\text{Å}, b = 5.8108 \pm 0.0030\text{Å}\) and \(c = 30.8005 \pm 0.0573\text{Å}\), and for Bi(Pb)2212 was \(917.6539 \pm 2.2659\text{Å}^3\) with \(a = 5.1280 \pm 0.0021\text{Å}, b = 5.8109 \pm 0.0030\text{Å}\) and \(c = 30.7959 \pm 0.0733\text{Å}\). The anisotropy axis was found to be larger for the Fe doped sample than that for the pure single crystal whereas it is found to decrease for the Pb doped sample.
Table 6.1 Peak positions for Pure Bi2212, iron doped Bi2212 and lead doped Bi2212.

<table>
<thead>
<tr>
<th>Miller indices (HKL)</th>
<th>Bi2212 Peak position angle 2(\theta)</th>
<th>Bi(Fe)2212 Peak position angle 2(\theta)</th>
<th>Bi(Pb)2212 Peak position angle 2(\theta)</th>
</tr>
</thead>
<tbody>
<tr>
<td>002</td>
<td>5.833</td>
<td>5.809</td>
<td>5.901</td>
</tr>
<tr>
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<td>47.183</td>
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<tr>
<td>0020</td>
<td>60.019</td>
<td>60.011</td>
<td>60.293</td>
</tr>
</tbody>
</table>
6.2.1. X-ray diffraction spectra for the samples (a) Bi2212 pure single crystal (b) Bi(Pb)2212 single crystal and (c) Bi2212(Fe)2212 single crystal.
Fig. 6.2.2: (a) SEM Image of Bi2212 single crystal (b) EBSP pattern of crystallographic planes observed at each point indicated by "+" in fig. a. The observed patterns were identical and Fig. b represents only one of them. Here, [001] and [501] are the respective poles.
6.3 A.C. Susceptibility measurement

This measurement is a simple and the most commonly used method for determining the critical temperature of the high Tc superconductors [65]. The measurements were performed with the Quantum Design’s Physical Property Measurement System (PPMS) with sensitivity of up to $10^8$ emu at 10KHz. In this measurement, the sample is placed in a system of coils, consisting of a primary and two secondary coils. The primary coil produces an excitation field ($H_e$) of maximum amplitude 15 Oe and frequency 10 KHz. The sample is placed in one of the secondary pick-up coils, where the magnetic moment of the sample varies with varying field produced by the primary coil. The voltage of the pick-up coil varies as:

$$V = -\frac{d\Phi}{dt}$$

where $\Phi = B.A = H_e A_c + 4\pi M A_s$, where $A_c$ and $A_s$ are the effective cross-sectional areas of the coil and the sample respectively.

Therefore, this voltage is proportional to the change of magnetisation of the sample, but also to $H_e$. In order to eliminate $H_e$, a compensating secondary coil is connected in opposition to the pick-up coil, so that

$$V = -4\pi A_s \frac{dM}{dt} = -4\pi A_s \frac{dM}{dH} \frac{dH}{dt}$$

Therefore, the measured susceptibility $\chi$ is given by

$$\chi = \frac{-V}{4\pi A_s \frac{dH}{dt}}$$

When the sample is normal, the ac magnetic field produced in the coil extends throughout the sample. As the sample becomes superconducting, the ac field is excluded from the interior by superconducting shielding currents that produce the field in the direction opposite to the applied field. This drastic change in the field profile gives a
very significant change in the in-phase component of the voltage across the coil and this change can be detected by a Lock-In Amplifier. The temperature at which this change occurs is the critical temperature of the sample (T_c). With this procedure, T_c of pure Bi2212 single crystal, lead doped Bi2212(Pb1), another lead doped Bi2212(Pb2) and iron doped Bi2212 was measured as 88.5K, 67K, 82K and 70K respectively [Fig. 6.3.2].

Fig. 6.3.1: A simple schematic of a mutual inductance technique of measuring the transition temperature. The sample is placed in one of the coils.
Fig. 6.3.2. The real part of the susceptibility $\chi$ as a function of temperature for Bi2212, Bi(Pb1)2212, Bi(Pb2)2212 and Bi(Fe)2212 single crystals.

6.4 DC Magnetisation Measurement.

The magnetisation measurements of the samples were performed using the Quantum Design’s Physical Property Measurement System (PPMS) Extraction Magnetometer with sensitivity of $2.5 \times 10^{-5}$ emu. In the measurements, magnetised samples were moved through the detection coils (as mentioned above) which induced a voltage in the detection coil set:

$$V = -4\pi N S \frac{dM}{dt}.$$
The amplitude of this signal is proportional to the magnetic moment and speed of the sample during the extraction.

\[
M = \frac{m}{\Omega} = -\frac{1}{4\pi \Lambda_s} \int V dt
\]

where \( m \) is the magnetic moment and \( \Omega \) is the volume of the sample.

Four rectangular samples with different \( T_c \) and different pinning centres were measured. The samples were Bi2212 (pure), Bi(Pb)2212 (sample Pb1), Bi(Pb)2212 (sample Pb2) and Bi(Fe)2212 single crystals. In the relaxation measurement, the sample magnetisation has to be homogeneous because magnetometer measures an averaged magnetic moment of the sample. With inhomogeneous magnetisation, the decay rates vary locally (depending on the local flux gradients). Therefore, it is highly questionable what such relaxation measurement represent. The sharp drop of the transition temperature of the four samples indicates their high qualities. The hysteresis loops of the samples were recorded at different temperatures, for which the relaxation would be measured. The irreversible component of the magnetisation \( M_{irr}(H) \) was obtained from the hysteresis loop, since \( M_{irr} = M_{eq} - M \) where \( M_{eq} \) was calculated from ascending \( M^+ \) and descending \( M^- \) branches of the hysteresis loop using the Bean relation \( M_{eq} = (M^+ + M^-)/2 \). The importance of choosing the \( M_{irr} \) is that it is proportional to the critical current density \( J_c \) and also it removes the moment of the sample holder. The irreversibility field \( H_{irr} \) of the sample from the hysteresis loop was estimated. Here \( H_{irr} \) is defined as the field above which the ascending and descending branches of the magnetisation coincide. The first full penetration field from the hysteresis loop was also measured. After the hysteresis measurement, the samples were cooled in zero field from above the critical temperature \( T_c \) to the measurement temperature \( T \). The magnetic fields \( H \) smaller than
$H_{\text{in}}$ but several times larger than the first penetration field were employed (to ensure full penetration). The decay of magnetisation was measured with time.

6.5 Annealing method:

It is well known that oxygen off-stoichiometry plays a major role in determining the critical temperature $T_c$ of the sample. It is not always possible to grow a single crystal with a perfect homogeneous oxygen-stoichiometry. The oxygen inhomogeneity is always observed in the as grown single crystals using the flux growth method. Therefore, in order to get a desired value of the critical temperature $T_c$, the annealing of the sample at different temperatures with different atmospheres is very important, because it helps oxygen redistribution in the sample. The annealing effects on Bi2212 single crystal were studied. It was found that the temperature and atmosphere of the sample treatment are very important. For Bi2212 single crystal, the best annealing temperature and atmosphere was $550^\circ\text{C} - 600^\circ\text{C}$ and 7.5 % oxygen in nitrogen [ Fig. 6.5]
As Grown Sample = Circle
Annealed at 600 °C for 5.15 hours
with 7.5% of O_2 = square Solid
Annealed at 400 °C at 15 hours
with 7.5% of O_2 = Open triangle

Fig. 6.5. Annealing effect on T_c for Bi2212 single crystal.
EXPERIMENTAL RESULTS

7.1 Magnetic hysteresis:

Magnetic hysteresis loops were obtained from the dc magnetisation measurement, where the magnetic field was changed at the rate of 2.1 Oe per second. The measurements of hysteresis loop can give critical current density as a function of field and temperature, using Bean’s formula, as well as the irreversibility field $H_{irr}$ and the equilibrium magnetisation $M_{eq}$ [9-10]. All of these are the essential tools for the physical characterisation of the sample. We have measured four different samples with different critical temperatures $T_c$. They are pure Bi2212 single crystal (Bi2212), lead doped Bi2212 single crystal (Bi(Pb1)2212), another lead doped Bi2212 crystal (Bi(Pb2)2212) and iron doped Bi(Fe)2212 single crystal, with critical temperatures 88.5K, 67K, 82K and 72K, respectively. The samples were prepared by the flux growth technique. The crystals (Bi2212), Bi(Pb1)2212, Bi(Pb2)2212 and Bi(Fe)2212 have dimensions of $0.195\text{cm} \times 0.21\text{cm} \times 0.0166\text{cm}$, $0.12\text{cm} \times 0.13\text{cm} \times 0.0098\text{cm}$, $0.12\text{cm} \times 0.13\text{cm} \times 0.0098\text{cm}$ and $0.145\text{cm} \times 0.18\text{cm} \times 0.0076\text{cm}$, respectively. The measurements were performed using a QD PPMS magnetometer, with sensitivity of $10^{-5}$ emu. One of the reasons to measure the hysteresis loop, is to obtain $M_{eq}$, using $M_{eq} = (M^- + M^+)/2$, where $M^-$ and $M^+$ are the branches of the hysteresis loop corresponding to decreasing and increasing field respectively. The true irreversible component of magnetisation $M_{irr}$ was then obtained by using the relation $M_{irr} = M - M_{eq}$, where $M$ is the measured
magnetisation [61]. From the hysteresis loop, we have also estimated the zero field critical current density $J_{c0}$ using the Bean’s critical state model, which for our samples can be written as:

$$J_c = \frac{20\Delta M}{a\left(1 - \frac{a}{3b}\right)},$$  \hspace{1cm} 7.1.1

where $J_c$ is the magnetisation current density measured in A/cm$^2$, $\Delta M = M^- - M^+$ is the width of hysteresis loop measured in emu/cm$^3$ and $a$ and $b$ are the dimensions of the sample ($b > a$), measured in cm. The magnetisation was obtained by dividing the measured moment by the crystal volume. The irreversibility field $H_{irr}$ at different temperatures for all four crystals was also estimated from the measured magnetisation loop. A comparison between different hysteresis loops is mainly given by $J_{c0}$ and $H_{irr}$.

### 7.1.1 Bi2212 pure single crystal ($T_c = 88.5$)

Fig. 7.1.1a represents the hysteresis loop for pure single crystal measured at temperatures 22K, 26K and 28K. The estimated zero-field critical current density $J_{c0}$ calculated at different temperatures is shown in fig. 7.1.2b. The $J_{c0}$ at temperature 22K is approximately 142,900 A/cm$^2$, whereas at 26K it is approximately 32,480 A/cm$^2$. The irreversibility field, $H_{irr}$, at 26K is 7977 Oe and is 5333Oe at 28K (Fig. 7.1.2c). This indicates a large reduction in the hysteresis loop with increasing temperatures. One of the reasons for the reduction of the hysteresis loop is due to a large anisotropy of this sample. It is well established that because of the highly anisotropic nature of layered Bi2212, the magnetic vortices are two dimensional objects, often described as pancake vortices [26, 28, 61-63]. At low temperature, the pancake vortices are coupled through
Josephson strings and are pinned, but as the temperature increases the two dimensional vortices are easily depinned due to thermal fluctuation of the vortices. Fig. 7.1.2d shows the calculated $M_{eq}$ from the hysteresis loop.

**7.1.2 Bi(Pb1)2212 single crystal ($T_c = 67K$)**

This is an overdoped crystal, which was annealed in air in order to increase the oxygen content. After annealing, it was quenched in air to introduce additional defects. The hysteresis loops for this sample are shown in fig. 7.1.2a and they are measured at 20K, 25K, 30K and 40K. This sample has a wide hysteresis loop and also a broad secondary peak effect (anomalous peak effect). This is because of the presence of strong pinning centres which were introduced there by lead doping [64]. A decrease on the c-axis of this sample, which was calculated from XRD (chapter 6), also indicates enhancement in the coupling of pancake vortices. This sample, therefore, shows a weaker field and temperature dependence of $J_c$ than that of pure single crystal [64]. This can be seen more clearly from fig. 7.1.2b, where $J_{c0}$ is presented at various temperatures, and also from the figure 7.1.2c, where $H_{in}$ is presented as a function of reduced temperatures. One of the interesting things in this sample is the appearance of anomalous peak effect over a wide range of temperature, from 20K up to $T_c$ (Fig. 7.1.3b)[65-67]. Fig 7.1.2c shows the $M_{eq}$ of this sample.

**7.1.3 Bi(Pb2)2212 single crystal ($T_c = 82K$)**

Fig. 7.1.3a shows the magnetisation hysteresis loops for sample Bi(Pb2)2212 measured at 20K, 25K, 30K, and 40K. From fig. 7.1.2b and fig. 7.1.2c, it is clear that this sample has weaker field and temperature dependence than the pure single crystal, but stronger than Bi(Pb1)2212 with $T_c = 67K$. This may be due to the presence of less pinning
centres than Bi(Pb1)2212, because some defects were introduced in the Bi(Pb1)2212 by quenching in air. Similar to Bi(Pb1)2212, this sample also has anomalous peak for 20K<T<Tc. Fig. 7.1.3c represents Meq, as calculated from the hysteresis loops.

7.1.4 Bi(Fe)2212 single crystal (Tc = 72K)

The magnetic hysteresis loops for this sample are shown in figure 7.1.4a and also Meq is shown in figure 7.1.4b. The estimated Jc0 and Hjn for this sample are shown in fig.7.1.2b and fig 7.1.2c, respectively. They show a strong field and temperature dependence of Jc0 for this sample. For example, the value of Jc0 at 20K is 63900 A/cm² whereas at 24K it is 19900 A/cm², Hjn is 2400 Oe at 23K, whereas it is 343Oe at 30K. The anomalous peak effect of this sample starts at 20K and completely disappears at 35K. This sample also has a large anisotropy, compared with other samples. One of the reasons for this strong field and temperature dependency is that the iron, a ferromagnetic particle, in this sample introduces many normal large conducting regions by breaking cooper pairs, even on CuO2 planes [68,86]. These ferromagnetic particles due to their strong coupling reduce Tc and therefore critical current Jc significantly.
Fig. 7.1.1a: Magnetic hysteresis loop for Bi2212 pure single crystal \([T_c = 88.5K]\) measured at temperatures 22K, 26K, 28K with applied field parallel to c axis.

Fig. 7.1.1b: Equilibrium magnetic moment \(M_{eq}\) (squares) and hysteresis loop (line) for the Bi2212 single crystal plotted at temperature 28K.
Fig. 7.1.2a: Magnetic hysteresis loop for Bi(Pb1)2212 single crystal \([T_c=67K]\) measured at temperatures 20K, 25K, 30K and 40K with \(H//c\) axis.

Fig. 7.1.2b: Zero field critical current density \(J_{c0}\) as a function of reduced temperature \(t=T/T_c\) for the samples Bi2212(A), Bi(Pb1)2212(B), Bi(Pb2)2212(C) and Bi(Fe)2212(D)
Fig. 7.1.2c: Irreversible field $H_{irr}$ as a function of reduced temperature $t = T/T_c$ for the samples Bi2212(A), Bi(Pb1)2212(B), Bi(Pb2)2212(C) and Bi(Fe)2212(D).

Fig. 7.1.2d: Equilibrium magnetic moment $M_{eq}$ (circle) and hysteresis loop (line) for the Bi(Pb1)2212 single crystal plotted at temperature 25K.
Fig. 7.1.3a: Magnetic hysteresis loop for Bi(Pb2)2212 single crystal \([T_c= 82K]\) measured at temperatures 20K, 25K, 30K and 40K with H//c axis.

Fig. 7.1.3b: Magnetic hysteresis loop for Bi(Pb2)2212 single crystal \([T_c= 82K]\) measured at temperatures 72K, 73K, and 75K with H//c axis.
Fig. 7.1.3c: Equilibrium magnetic moment $M_{eq}$ (square) and hysteresis loop (line) for the Bi(Pb2)2212 single crystal plotted at temperature 25K.
Fig. 7.1.4a: Magnetic hysteresis loop for Bi(Fe)2212 single crystal \([T_c=72K]\) measured at temperatures 30K, 35K, and 40K with \(H//c\) axis.

Fig. 7.1.4b: Equilibrium magnetic moment \(M_{eq}\) (square) and hysteresis loop (line) for the Bi(Fe)2212 single crystal plotted at temperature 23K.
7.2 Magnetic relaxation:

Magnetic relaxation arises from the decay of persistent currents with time. The persistent currents come from the irreversible component of the magnetisation $M_{irr}$. Otherwise, in the relaxation measurement, we would include the moment of the sample holder.

The magnitude of the persistent current is observed to decrease logarithmically/non-logarithmically, depending upon the field and temperature. For high $T_c$ superconductors, the relaxation is more severe than in conventional superconductors because of higher thermal energy and lower pinning energy. In this section, the relaxation of the magnetic moment of the samples Bi2212, Bi(Pb1)2212, Bi(Pb2)2212 and Bi(Fe)2212 single crystals was measured at different fields and temperatures. Measurements were performed using a QD PPMS magnetometer. They are described as below.

7.2.1 Bi2212 single crystal

Figures 7.2.1a and 7.2.1b show $M_{irr}$ vs $t$ for sample Bi2212 plotted in linear-log and log-log scales respectively. A logarithmic decay of magnetic moment with time is indicated in fig. 7.2.1a, with a transition to a slower approximately logarithmic decay for $t > 1000s$. On the other hand, fig. 7.1.1b indicates a power law decay with a slower decay for $t > 1000s$ seen from these two figures. The decay rate at 26K is larger than that at 22K. This is because the vortices get released from pinning centres easier at 26K than at 22K [69]. Therefore, the relaxation is faster at higher temperatures than at low temperatures.

7.2.2 Bi(Pb1)2212 single crystal

Figures 7.2.2a and 7.2.2b represent a decay of magnetic moment $M_{irr}$ as a function of time ($t$) for Bi(Pb1)2212 plotted in linear-log and log-log scales respectively. Fig. 7.2.2a indicates a logarithmic decay of magnetic moment with time, with a transition to a
slower approximately logarithmic decay for \( t > 1000 \) whereas, fig. 7.2.2b indicates a power law decay with a slower decay for \( t > 1000 \) seen from these two figures. The decay rate at this sample is lower than that at pure single crystal. This is because the vortices in this sample are pinned stronger than that at pure sample [64]. It is also seen that the decay rate at 30K is larger than that at 20K.

### 7.2.3 Bi(Pb2)2212 single crystal

The linear-log and log-log plot of \( M_{irr} \) with time are shown in figure 7.2.3a and 7.2.3b respectively. A logarithmic decay of magnetic moment with time is indicated in fig. 7.2.3a, with a transition to a slower approximately logarithmic decay for \( t > 1000 \). On the other hand, fig. 7.1.3b indicates a power law decay with a slower decay for \( t > 1000 \). The relaxation rate at this sample is lower than that of pure single crystal.

### 7.2.4 Bi(Fe)2212 single crystal

Figs 7.2.4a and 7.2.4b show the relaxation rate plotted in linear-log and log-log scale and they are measured at different temperatures. The decay rate of this sample is different from that of other samples. Here, it is observed that the decay rate is decreased with increasing temperatures. This relaxation behaviour may be contributable to the fact that the decay rate was faster at the initial stage of relaxation at higher temperatures. However, the points for \( t<100 \) were not plotted because of the uncertainty in determining the zero time at which the field was applied\(^1\). Previous sample shows that

---

\(^1\) Our instrument shows time measured from the moment the instrument installed. We obtained the time zero as the time at which the first was measured after application of the field. However there is always a decay between the application of the field and the first point, which depends on the particular condition of the experiment (value of initial moment, decay rate, previous experiment). Therefore, there is uncertainty in the measuring time of the order of 10s. Plotting such measurements with log t scale would give much distorted and therefore misleading results. Because of this we chose not to use the experimental points for \( t<100 \).
there is a transition to a slower relaxation rate for \( t > 1000s \), presumably because the vortex excitation over stronger pinning centres than for \( t \leq 1000s \) dominates the magnetic relaxation experiments. This leads to a condition that for \( T < 26K \) in figure 7.4.2a and b, there is still a fast relaxation over the wider pinning centres, with a transition to a slower relaxation for \( t \geq 1000s \). However, for \( T > 26K \), this transition occurs at shorter times \( t \leq 100s \) because of large thermal excitation and therefore faster initial relaxation. These points were not plotted in the figure 7.2.4a and b because of uncertainty in determining \( t=0 \) for each of the relaxation. The other samples had much stronger pinning energy than Bi(Fe)2212 and these transition was observed for all measured temperatures.
7.2.1a: Magnetic moment decay vs time (linear-log scale) of Bi2212 pure single crystal measured at temperatures 22K, 26K and 28K.

7.2.1b: Magnetic moment decay vs time (log-log scales) of Bi2212 pure single crystal measured at temperatures 22K, 26K and 28K.
7.2.2a: Magnetic moment decay vs time (linear-log scales) of Bi(Pb1)2212 single crystal measured at temperatures 20K, 22.5, 25K, 27.5 and 30K.

7.2.2b: Magnetic moment decay vs time (log-log scales) of Bi(Pb1)2212 single crystal measured at temperatures 20K, 22.5, 25K, 27.5 and 30K.
7.2.3a Magnetic moment decay vs time (linear-log scales) of Bi(Pb2)2212 single crystal measured at temperatures 20K, 25K, and 30K.

7.2.3b Magnetic moment decay vs time (log-log scales) of Bi(Pb2)2212 single crystal measured at temperatures 20K, 25K, and 30K.
Fig. 7.2.4a  Magnetic moment decay vs time (linear-log scales) of Bi(Fe)2212 single crystal measured at temperatures 22K, 24K, 26K, 28K, 30K and 33K.

Fig. 7.2.4b  Magnetic moment decay vs time (log-log scales) of Bi(Fe)2212 single crystal measured at temperatures 22K, 24K, 26K, 28K, 30K and 33K.
8.1 Non linear model for $U_{\text{eff}}(J, T)$:

A linear approximation of the dependence of pinning energy on magnetic moment (i.e., $J$), as given by Anderson and Kim, leads to the well-known logarithmic decay of persistent current, with a simple relationship between activation energy $U_0$ and measurable logarithmic creep rate [11, 12]:

$$U_0 = \kappa T \left[ \frac{1}{M_0} \ln \left( \frac{\partial M}{\partial \ln t} \right)^{-1} \right]$$

(8.1)

Here $M_0$ is the initial value of the magnetic moment. The temperature dependence of $U_0$ is quite a puzzle. There are different models and theories that have been developed to describe the temperature dependence of the activation energies. Maley et al [70], were able to extend the accessible region of $J$ ($M_{\text{tr}} \approx J$) by recording the decay of magnetisation from its critical state $M_{\text{tr}} \approx J$ for various temperatures. In order to elucidate the explicit dependence of $U_c$ on $M$ or $J$ for various temperatures, Maley pointed out a suitable form of the rate equation for thermally activated flux motion, which can be written as

$$\frac{U_c}{k} = -T \ln \left| \frac{d(M - M_{\text{eq}})}{dt} \right| + \ln \left( \frac{B\omega}{\pi d} \right)$$

(8.2)

Where $k$ is the Boltzmann constant, $\omega$ is the attempt frequency and $d$ is the sample thickness.
They have obtained the equation (8.2) from Beaseley's flux diffusion equation [52]

\[
\frac{dB}{dt} = \nabla \left[ \frac{B_{\alpha} \exp\left( - \frac{U_{\text{eff}}(J)}{kT} \right)}{\pi d} \right]
\]

8.3

The differential equation for the averaged magnetic induction is

\[
\frac{d\langle B \rangle}{dt} = 4\pi \frac{dM_{\text{irr}}}{dt} = \left[ \frac{4B_{\alpha} \omega}{d} \right] \exp\left[ - \frac{U_{\text{eff}}(J)}{kT} \right]
\]

8.4

and

\[
\frac{dM_{\text{irr}}}{dt} = \left[ \frac{B_{\alpha} \omega}{d\pi} \right] \exp\left[ - \frac{U_{\text{eff}}(J)}{kT} \right]
\]

8.5

Taking the logarithm of both sides of equation (8.5), it becomes

\[
\ln \left[ \frac{dM_{\text{irr}}}{dt} \right] = \ln \left[ \frac{B_{\alpha} \omega}{d\pi} \right] - \frac{U_{\text{eff}}(J)}{kT}
\]

8.6

and

\[
\frac{U_{\text{eff}}(J)}{k} = -T \ln \left[ \frac{dM_{\text{irr}}}{dt} \right] + T \ln \left[ \frac{B_{\alpha} \omega}{\pi d} \right]
\]

8.9

where \( \ln \left( \frac{B_{\alpha} \omega}{\pi d} \right) = C = \text{constant} \)

8.10

Adding a constant term \( C = 18 \) to each of the relaxation data for different temperatures, a smooth dependence of \( U_{e} \) on \( M \) was obtained. This value of constant \( C \) gives physically acceptable value for \( \omega \alpha \approx 25 \text{cm s}^{-1} \), consistent with hopping distance \( \approx 1000 \text{Å} \) and attempt frequency of \( 2.5 \times 10^{7} \text{Hz} \) [70]. All these are measurable values, comparable to those commonly found in the literature.

At present, numerous research work on temperature dependence of the activation energy is being carried out by various groups using a non-logarithmic time decay of the persistent current \( J \) (or \( M_{in} \)) as predicted by Feigelman et al [71-81], in the theory of flux creep based on the collective pinning model.
In their model, a power law was assumed for the $U_0(J)$ relationship.

\[ U(J) = U_0 \left( \frac{J_0}{J} \right)^\mu \]  \hspace{1cm} 8.11

where $U_0$ is the characteristic energy scale and the exponent $\mu$ depends on the dimensionality of the flux line lattice and on the particular creep regime. They obtained non-logarithmic form for the current relaxation:

\[ J(t) = J_0 \left[ \left( \frac{U_0}{kT} \right) \ln \left( \frac{t}{\tau} \right) \right]^\frac{-1}{\mu}. \]  \hspace{1cm} 8.12

where $\tau$ is the attempt time.

In the case of three dimensional systems of the flux line lattice, $\mu = 7/9$, $3/2$ and $1/7$ corresponding to flux creep regime caused by large vortex bundles, small vortex bundles and a single vortex bundle, respectively. In the case of the two dimensional system, $\mu = 9/8$ corresponds to the creep of pancake vortices.

The Maley's approach to elucidate the explicit dependence of $U_c$ on $M$ by using a single value of $C$ for each field does not result in a smooth curve over a wide range of temperatures, especially for the higher temperatures in high $T_c$ superconductors. Therefore, a scaling approach has been introduced to study the temperature dependence of activation energy. It has been presented by various groups that the smooth $U(J)$ dependence measured over a wide temperature range can be obtained by appropriate temperature scaling of $U$ [71,72, 76-80]. This is because, the relaxation of magnetic moment or activation energy in high $T_c$ superconductor exhibits scaling behaviour. The use of scaling in the collective pinning theory has been widely accepted to determine the dimensionality of the flux lines and also the nature of the flux creep regimes. There are different scaling laws for the activation energy.
M. Tinkham [71], in 1988, mentioned that the height of the activation barrier $U/kT$ can be scaled by $g(t) = 1 - t^2(1-t^2)^{-1/2}$, where $t = \frac{T}{T_c}$. He proposed that the value of $g(t)$ for high $T_c$ superconductors is $(1-t)^{\frac{3}{2}}$, which is similar to the Ginzburg-Landau temperature dependence of $U$. This has been supported experimentally by H. Darhmaoui and J.Jung [80] in YBCO.

P.J. Kung [77] et al proposed that, keeping the applied field constant, the activation energy $U_{\text{eff}}(J,t)$ is usually expressed in the following scaling form to separate its thermal dependence:

$$U_{\text{eff}}(J,T) = G(T) U_{\text{eff}}(J,0) \quad 8.13$$

Here, $G(T)$ is the temperature dependence function, which is chosen as

$$G(T) = \left[1 - \left(\frac{T}{T_{ir}}\right)^2\right]^m \quad 8.14$$

where $1 \leq m \leq 2$ and $T_{ir}$ is the irreversibility temperature. They studied this scaling property in Y-Ba-Cu-O.

Hai-hu Wen et al [82] demonstrated the temperature dependent activation energy in the following form:

$$U_c(T_N) = U_c(0)(1 - T_N^e)^f(1 + T_N^2)^f \quad 8.15$$

where $U_c(0)$ is the activation energy at zero temperature and $T_N = \frac{T}{T_c}$. They obtained, for the three dimensional vortex, the exponent $e = f = 1$ and for two dimensional vortex, the exponent $e = 3/2$ and $f = 1/2$. They studied this scaling properties in TlBaCaCuO high $T_c$ superconductor.
In the present study, the same temperature scaling as that employed by P. J. Kung was used except that $T_{\text{irr}}$ was replaced by $T_c$. This scaling was made for all measured samples. Figures 8.1.1b, 8.1.2b, 8.1.3b and 8.1.4b show a log-log plot of effective activation energy $U_{\text{eff}}$ vs irreversible magnetic moment $M_{\text{irr}}$ of these samples with temperature scaling $G(T) = 1 - \left( \frac{T}{T_c} \right)^2$, where a universal curve for $U_{\text{eff}}$ was obtained.

All other scaling laws were tried too: $1 - \left( \frac{T}{T_{\text{irr}}} \right)^{1.5}$, $1 - \left( \frac{T}{T_{\text{irr}}} \right)^2$ and $(1 - \frac{T_{\text{irr}}}{T})^c (1 + T)^f$. However, only $1 - \left( \frac{T}{T_c} \right)^2$ scaling worked well for all samples, with the choice of physically meaningful value of constant $C$. If the Maley approach was used without temperature scaling, a smooth $U(M_{\text{irr}})$ for all the samples could be obtained, however the value of $C$ was very large. For example, the value of $C$ for iron doped sample was 36. This large value of $C$ would give hopping distance of a few billions of centimetres, which is meaningless. Other scaling laws would not give smooth $U(M_{\text{irr}})$ at all for our samples.

For Bi2212 pure single crystal, Fig.8.1.1b shows a good tendency of fitting $U(M_{\text{irr}})$ on a smooth curve with temperature scaling. But Fig. 8.1.1a does not show this for the same sample, since the temperature scaling was not used here. The sample Bi(Pb1)2212 shows very good smooth $U(M_{\text{irr}})$ curve with temperature scaling over the range of temperatures, from 20K to 30K [Fig 8.1.2b]. Similarly, Bi(Pb2)2212 also shows good fitting (Fig 8.1.3b). This may be due to the presence of strong pinning centres in the lead doped samples. Fig. 8.1.4b shows $U(M_{\text{irr}})$ plotted on a logarithmic scale for Bi(Fe)2212, measured at temperatures 22K, 24K, 26K, 28K, 30K and 33K. This sample has an upward curvature, in contrast to the other samples, which have a downward curvature.
This is because $U(M_{\text{irr}})$ measured at a particular temperature for large times after the application of $H$ (i.e., for small $M_{\text{irr}}$) tends to deviate upwards from the universal smooth $U(M_{\text{irr}})$. This is especially so for the highest temperatures. Other samples do not show this effect in the measured time windows, because they have much stronger pinning than the iron doped samples.
Fig. 8.1.1a: The dependence of activation energy $U$ on the magnetic moment $M_{irr}$ at temperatures 22K, 26K, 28K for the sample Bi2212 (pure single crystal). $U$ was calculated from the relaxation using the Maley procedure ($C=18$).

Fig. 8.1.1b: The dependence of activation energy $U$ on the magnetic moment $M_{irr}$ at temperatures 22K, 26K, 28K for the sample Bi2212 pure single crystal. $U$ was obtained from the temperature scaling in the Maley procedure ($C=18$).
Fig. 8.1.2a: The dependence of activation energy $U$ on the magnetic moment $M_{irr}$ at temperatures 20K, 22.5K, 25K, 27.5K and 30K for the sample Bi(Pb1)2212. $U$ was calculated from the relaxation using Maley procedure (C=18).

Fig. 8.1.2b: The dependence of activation energy $U$ on the magnetic moment $M_{irr}$ at temperatures 20K, 22.5K, 25K, 27.5K and 30K for the sample Bi(Pb1)2212(sample 1). $U$ was obtained from the temperature scaling in the Maley procedure (C=18).
Fig. 8.1.3a: The dependence of activation energy $U$ on the magnetic moment $M_{\text{ irr}}$ at temperatures 20K, 26K, and 30K for the sample Bi(Pb2)2212. $U$ was calculated from the relaxation using Maley procedure ($C=18$).

Fig. 8.1.3b: The dependence of activation energy $U$ on the magnetic moment $M_{\text{ irr}}$ at temperatures 20K, 25K, and 30K for the sample Bi(Pb)2212 (sample 2). $U$ was obtained from the temperature scaling in the Maley procedure ($C=18$).
Fig. 8.1.4a: The dependence of activation energy $U$ on the magnetic moment $M_{\text{irr}}$ at temperatures 22K, 24K, 26K, 28K and 30K for the sample Bi(Fe)2212. $U$ was calculated from the relaxation using Maley procedure (C=18).

Fig. 8.1.4b: The dependence of activation energy $U$ on the magnetic moment $M_{\text{irr}}$ at temperatures 22K, 24K, 26K, 28K and 30K for the sample Bi(Fe)2212. $U$ was obtained from the temperature scaling in the Maley procedure (C=18).
CHAPTER NINE: CONCLUSION

The magnetic relaxation of a pure Bi2212, two lead-doped Bi2212 (sample 1 and sample 2) and iron doped Bi2212 samples, with $T_c$ 88.5K, 67K, 82K and 72K, respectively, was measured. The samples were prepared using the self flux growth technique. The quality of the samples was studied with the help of X-ray diffraction pattern (XRD) and electron back scattering pattern (EBSP) techniques. The high quality of the samples was evidenced by a high degree of preferred orientation of the Crystallographic planes. From XRD, it is observed that the crystalline anisotropy of the lead doped sample Bi(Pb)2212 ($30.6666 \pm 0.1037 \text{Å}$) was smaller than that pure single crystal ($30.7959 \pm 0.0733 \text{Å}$) whereas for iron doped sample ($30.8005 \pm 0.0573 \text{Å}$) the anisotropy was larger. We found a strong anomalous peak effect in the hysteresis loops of the lead doped samples over a wide range of temperature, from 20K to $T_c$. This indicates strong pinning centres in this sample. For iron doped sample, the anomalous peak was obtained between 23K and 35K. Hysteresis loops and zero field critical current density $J_{c0}$ of this sample was strongly field and temperature dependent. This indicates a presence of weak pinning centres in this sample. The magnetic relaxation rate for the lead doped sample was smaller than that for pure single crystal and for Fe doped crystal. This also supports for the presence of strong pinning centres in the lead doped sample.

The main result of this thesis is that the same temperature scaling of the activation energy was obtained for all of the samples measured, even though they exhibit greatly different flux pinning. This interesting finding will be a subject of future research work. It was found that the scaling $1 - (T/T_c)^2$ gave the best fit for $U(M_{in})$, where $U(M_{in})$ was
obtained from the measurement of the magnetic relaxation at different temperatures. This scaling law was obtained by Tinkham using Ginzburg-Landau temperature dependence of the intrinsic superconducting parameters (i.e., the coherence length, London penetration depth, and fluxon lattice spacing) at low temperature, together with Yeshurun and Malozemoff's expression for the activation energy[71]. This indicates that the temperature dependence of $U_{\text{eff}}$ obtained within the time window used in our measurements is defined by the temperature dependence of these basic superconducting parameters. The success of this fit may be taken to imply that the temperature dependent basic parameters determine the scale of the effective activation energy.

Other scaling laws employed elsewhere could not give a good fit for all the samples measured. Therefore we show that $1-(T/T_c)^2$ scaling can be successfully used to study $U(M_{jn})$ of Bi2212 for a variety of single crystals with very different flux pinning. This was a remarkable result, because one would expect different types of magnetic relaxation in crystals with such a large difference in pinning.
References:


59. X. L. Wang, Ph. D thesis in preparation, University of Wollongong, Australia.


