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Mean reversion and structural breaks in the Australian dollar real exchange rate

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Keywords
Mean, reversion, structural, breaks, Australian, dollar, real, exchange, rate

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Key words: Real exchange rate, purchasing power parity, unit-root, structural breaks.

JEL Classification: F13, F31, F41
Mean Reversion and Structural Breaks In The Australian Dollar Real Exchange Rate

Introduction

Real exchange rate (RER) – the ratio of price of tradables to price of nontradables - measures the external competitiveness of an economy. The policy issue of 'overvaluation/undervaluation' and the resultant existence and magnitude of distortions is discussed in terms of the RER movements. Since RER is a price that ensures internal and external equilibrium simultaneously, it plays a pivotal role in macroeconomic adjustment. RER misalignment has adverse welfare and efficiency costs on small, open economies like Australia.

Testing for mean reversion in RER is one way of testing the purchasing power parity (PPP) theory. The result of the unit-root test is crucial since it will indicate whether the long-run PPP holds on a continuous basis. Short run deviations from PPP are significant, while in the long-run, the deviations from PPP are eliminated slowly over time. To highlight this point, let us define $s_t$ be the Australian dollar price of a unit of foreign currency, $p_t$ the Australian price level, $p_t^*$ the foreign price level and $q_t$ the RER, with all variables expressed in natural logarithms. Thus the RER, $q_t$ can be expressed as follows:

$$q_t = s_t + p_t^* - p_t$$

In the absolute sense, the nominal exchange rate ($s_t$) is proportional to the relative price ratio ($p_t/p_t^*$) thus rendering the RER ($q_t$) to remain constant over time. If $q_t$ changes over time and follows a stationary ARMA (p,q) process, then deviations from PPP are transient and will be
eliminated over time. It is common knowledge that short-run deviations from PPP are perfectly consistent with efficiently functioning financial markets. However, if the movement in $q_t$ follows a non-stationary ARMA process, then the deviations will not be eliminated over time resulting in the failure of PPP in the long-run.

Empirical examinations of the theory in the 1960s lend some support of PPP condition over long periods of time. Since then empirical evidence on the validity of PPP and hence mean reversion of RERs has been mixed such that the validity of PPP has been in doubt. With the collapse of the Bretton Woods system in the early 1970s, it was generally assumed that the exchange rate would move quickly in line with changes in relative price levels. Dornbusch’s (1976) ‘overshooting’ proposition provided some theoretical justification for the transient deviations from PPP. Empirical tests of the mid-1980s tended to reject PPP except in countries with high inflation (Frenkel, 1981). Such a view was criticised as being too simplistic since the time series properties of exchange rates and relative prices are ignored. Increasing evidence in favour of mean reversion of RERs in industrialised countries in the post-float period have been found from studies that employ the panel unit-root test (MacDonald, 1996; Papell, 1997; Papell and Theodoridis, 1998; 2001 among others). Critics argue that the evidence of mean reversion of RERs is suspect given the low power of the unit-root tests employed and the tests suffer from serious size distortions. Some empirical studies show that the behaviour of the exchange rate is in fact non-linear in nature (Granger and Terasvirta, 1993; Micheal et al., 1997; Sarno, 2000a, b; Taylor and Peel, 2000; Baum et al., 2001; Liew et al., 2004) where the exchange rate adjustment is shown to vary non-linearly with respect to the size of deviation from equilibrium level that can be characterised as a smooth transition autoregressive (STAR) process.
Given the conundrum of results, the central objective of this paper is test for mean reversion of RER of Australia in the presence of endogenously determined structural breaks in the post-float period (since December 1983). It is common knowledge that the traditional unit-root tests (like Dickey-Fuller (DF), Augmented Dickey-Fuller (ADF) and Phillips-Perron (PP)) and tests accounting for a single structural break have low power when multiple structural breaks are ignored. To the best of my knowledge, it is the first study that employs RER data of Australia and tests the null hypothesis of unit-root in the presence of multiple endogenous structural breaks. Allowing for structural breaks or regime shifts is particularly important considering the nature of the post-float experience for Australia. The changes in exchange rate policy that have occurred give rise to the possibility of multiple structural breaks in the data.

The structure of the paper is as follows: In Section II we provide a succinct critique of the previous studies on testing for unit-roots of RER of Australia only. In Section III we conduct the Lee and Strazicich (LS, 2003) minimum Lagrange Multiplier (LM) unit-root test to determine structural breaks endogenously. The LS unit-root test with two structural breaks endogenously determines the location of two breaks in level and trend and tests the null of a unit-root. The LS unit-root test with two structural breaks is invariant to the magnitude of the breaks and the alternative of the minimum LM unit-root test with two structural breaks unambiguously implies trend stationarity. The results are discussed in Section IV and Section V concludes with a summary of the findings.

2. Past studies of unit-root of RER of Australia

Past studies on testing for unit-root of RER of Australia are sparse. A majority of these studies have used the traditional unit-root tests (DF, ADF, KPSS and others) which suffer

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1 The examples of policies with break consequences include frequent devaluations, deregulation of both real and financial sectors and policy regime shifts, abrupt exogenous changes like the H1N1, SARS pandemic etc. This can lead to huge forecasting errors and unreliability of the model in general.
from power deficiency when structural breaks are ignored. Two studies (Chowdhury, 2007; Henry and Olekalns, 2002) have incorporated a single endogenous structural break while testing for unit-root with opposing results. So far empirical results are overwhelming in favour of rejection of the mean reversion hypothesis as can be seen from the discussion below.

In earlier empirical research, the Australian RER was characterised as a unit-root process (Blundell-Wignall and Gregory (1990), Blundell-Wignall, Fahrer and Heath (1993) and Gruen and Wilkinson (1994)). Gruen and Kortian (1996:10) “estimate the real exchange rate models over the post-float period; a sample so short that tests of non-stationarity generates ambiguous results”. Tests on a longer sample of Australia’s trade-weighted RER suggest it is stationary, possibly around a trend (Gruen and Shuetrim 1994:353). Tarditi (1996), using Reserve Bank of Australia (RBA) quarterly data from 1973:4 to 1995:2, found the trade-weighted RER to be stationary around a trend by using the ADF test and Kwiatkowski et al. (KPSS) (1992) test. A notable feature of this study is that RER was found to be stationary on the basis of ADF and KPSS unit-root test for the entire sample period while for the post-float period RER was nonstationary which was contradicted by the KPSS test.

Chand (2001) used RBA quarterly data from 1981:3 to 2000:4 to quantify the extent to which the Australian trade-weighted RER was misaligned relative to its long-run equilibrium value. Chand (2001:12) wrote “The time series properties of the data were examined. The Dickey-Fuller test was unable to reject the null hypothesis of stationarity for all of the variables.” It is very unusual to find “the null hypothesis of stationarity” for the DF
test as is mentioned in this paper. Results reported in Table 1 page 19 reveal that all the variables are in fact nonstationary.

By employing the ADF test and quarterly data from 1973:1 to 1995:3, Bagchi et al. (2004) finds the RER of Australia to be integrated of order 1. Bagchi et al. (2004:80) defined the bilateral RER \((q) = \frac{e \text{CPI}^{US}}{\text{CPI}^{Aus}}\), where, \(e\) = nominal exchange rate and \(\text{CPI}^{US}, \text{CPI}^{Aus}\) represent the consumer price indices of the US and Australia respectively. This definition of RER is extremely restrictive and does not capture the overarching influence of relative prices and bilateral exchange rates of the trading partners. Hence, the result obtained by Bagchi et al. (2004) can be suspect.

These unit-root tests were carried out while trying to establish the fundamental determinants of the RER of Australia. It seems that the choice of a particular test method and the length of the sample period can influence the result to a large extent. Further, none of these studies took into account the presence of structural break in the data and the profound influence it can have on the dynamic time series properties of the data. Some researchers (Henry and Olekalns, 2002 and Chowdhury, 2007) enter this debate by taking issue with the unit-root testing procedure by including the influence of structural change in the Australian economy. These studies assess whether the presence of a single structural break has any perceptible influence on the result.

Henry and Olekalns (2002) used Zivot and Andrews (ZA, 1992) and Perron (1997) unit-root tests, which are robust to a one-off structural break, failed to find evidence of mean reversion in RER of Australia. The testing power of Perron (1997) and ZA (1992) tests are almost the same. On the other hand, Perron (1997) model is more comprehensive than ZA (1992) model as the former includes both \(t\) and \(DT\), while the latter includes \(t\) only. Overall

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2 The usual (conventional) null hypothesis of DF unit-root test is nonstationarity. It is only in the KPSS unit-root test that the null hypothesis is one of stationarity.
the unit-root tests suggest that the trade-weighted RER is non-stationary over the period 1973:1–1999:1. The break dates given by ZA (1992) and Vogelsang (1997) models place the break in first quarter of 1984. The innovational outlier tests place the break at 1984:3, while the Additive Outlier (AO) test dates the break at 1989:2. It is worth noting that trade-weighted RER data has been calculated by the authors by using the Jones and Wilkinson (1990) index of RER with no mention of various trade-weights being used nor the number of trading partners in their calculation. The source of data is also not mentioned. Thus, the RER measure on page 653 may not be an accurate and comprehensive measure of RER over the sample period.


Using the Shrestha-Chowdhury (2005) general-to-specific search procedure, Chowdhury (2007) found Perron’s (1997) AO model was the optimal model. His findings show that three indices ((Trade-weighted index (TWI), Export-weighted index (EWI) and Import-weighted index (IWI)) are stationary while G7-GDP weighted index (G7-GDPWI) is nonstationary. The endogenous structural break dates for these variables are 1990:3 (for TWI); 1991:3 (for EWI); 1989:2 (for IWI) and 1982:4 (for G7-GDPWI) respectively. Chowdhury’s (2007) result is totally different from the result obtained by Henry and Olekalns (2002) and thus requires further scrutiny. In addition, both these studies (Chowdhury, 2007; Henry and
Olekalns, 2002) report the single break date while testing for unit-root but do not report the statistical significance of the break date.

It is important to note that unit-root tests in the above studies, which either do not allow for a break under the null such as ZA (1992) or model the break as an Innovational Outlier (IO) as Perron (1997)\(^3\), suffer from severe spurious rejections in finite samples when a break is present under the null hypothesis (LS, 2001, 2003). Because the spurious rejections are not present in the case of a known break point, LS (2001) identify the inaccurate estimation of the break date as source of the spurious rejections. Furthermore, LS (2001) found that the asymptotic null distributions of the DF-type endogenous break test statistics are affected by nuisance parameters indicating the magnitude and location of the break.

This shallow evidence, based on weak test procedures, in the Australian literature highlights the difficulties of detecting robust evidence in favour of, or against, the PPP hypothesis. A summary of past results is given in Table 1 for a ready reference. Therefore, further research is warranted to determine if PPP provides a valid representation of the long-run equilibrium relation between the exchange rate and relative prices in Australia by exploring the possibility of including multiple structural breaks. The next section is devoted to this particular aspect.

[Insert Table 1 here]

3. Time-series properties of RER in the presence of structural breaks\(^4\)

3.1 Data and data source

In order to eliminate the uncertainty that exists in the literature about the time series properties of the Australian RER, we consider here quarterly data from 1984:1 to the latest

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\(^3\) ZA (1992) and Perron (1997) unit-root tests are derived from Perron (1989) test where the ADF test regression is augmented with dummy variables accounting for the break.

\(^4\) The examples of policies with break consequences include frequent devaluations, deregulation of both real and financial sectors and policy regime shifts, abrupt exogenous changes like the H1N1, SARS pandemic etc. This can lead to huge forecasting errors and unreliability of the model in general.
available data until 2009:2. The sample period considered here is the post-float period of the Australian dollar. The choice of the sample period is premised on the criterion of evaluating the effect of free floating RER and the ensuing reforms in the Australian economy that included the removal of controls on a range of interest rates (both borrowing and lending rates), the opening up of banking to foreign banks, the streamlining of Reserve Bank regulation of bank reserves and improved prudential arrangements. In contrast, the sample period of all earlier studies encompasses two broad exchange rate regimes where the dynamics of the RER are unlikely to be identical in the two periods. Since the float in December 1983, the RBA intervened occasionally for “smoothing” and “testing” purposes. Smoothing operations are undertaken by the RBA for elimination of perceived excessive volatility in the forex market while testing is conducted to evaluate how strong the market’s sentiment is in either direction. Thus, the current system is one of limited intervention. Edison et al. (1999) studied the effects of intervention by the RBA on the level and volatility of the Australian dollar and found the effects to be modest on both the level and the volatility of the Australian dollar.

Data on trade-weighted real exchange rate index (RER) is extracted from Reserve Bank of Australia (RBA) Table F.15 Real Exchange Rates Measures. It is this trade-weighted RER of RBA that forms the basis of all earlier studies except in studies by Henry and Olekalns (2002) and Bagchi et al. (2004) where the authors have calculated their own RER indices.

### 3.2 Stationarity of Data: Unit-root tests in the presence of structural breaks

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3After the collapse of the Bretton Woods system in February 1973, Australia pursued an exchange rate policy that combined some flexibility as well as some control by the Reserve Bank of Australia (RBA). The RBA pegged the Australian dollar with a basket of currencies of its trading partners. The Australian dollar was completely floated from December 1983.
We performed the LS (2003) minimum Lagrange Multiplier (LM) unit-root tests to determine structural breaks endogenously. The LS unit-root test with two structural breaks endogenously determines the location of two breaks in level and trend and tests the null of a unit-root. The LS unit-root test with two structural breaks is invariant to the magnitude of the breaks. LS noted that the alternative of the minimum LM unit-root test with two structural breaks unambiguously implies trend stationarity; however, it could be true that the series can possess unit-root with structural breaks.

3.3 Lee and Strazicich (2003) (LS) unit-root test

LS propose a minimum Lagrange multiplier (LM) unit-root test in which the alternative hypothesis unambiguously implies trend stationarity. Consider the DGP as follows:

\[ \Delta y_t = \delta \Delta Z_t + \phi \delta_{t-1} + u_t \]

where \( \delta_{t} = y_t - \psi_x - Z_t \delta \) ( \( t = 2, \ldots, T \)) and \( Z_t \) is a vector of exogenous variables defined by the data generating process; \( \delta \) is the vector of coefficients in the regression of \( \Delta y_t \) on \( \Delta Z_t \), respectively with \( \Delta \) the difference operator; and \( \psi_x = y_1 - Z_1 \delta \), with \( y_1 \) and \( Z_1 \) the first observations of \( y_t \) and \( Z_t \), respectively.

Model B of Perron (1989) is omitted from further discussion by LS (2003), as it is commonly held that most economic time-series can be adequately described by model A or C. Equivalent to Perron’s (1989) Model C, which allows for a shift in intercept and change in trend slope under the null hypothesis and is described as \( Z_t = [1, t, D_t, DT_t] \)’, where \( DT_t = t - T_B \) for \( t > T_B + 1 \), and zero otherwise. It is important to note here that testing the regression \( \Delta y_t = \delta \Delta Z_t + \phi \delta_{t-1} + u_t \) involves using \( \Delta Z_t \) instead of \( Z_t \). \( \Delta Z_t \) is described by \( [1, B_t, D_t] \)’, where \( B_t = \Delta D_t \) and \( D_t = \Delta DT_t \). Thus, \( B_t \) and \( D_t \) correspond to a change in the intercept and
trend under the alternative and to a one period jump and (permanent) change in drift under
the null hypothesis, respectively.

The unit-root null hypothesis is described by $\phi = 0$ and the LM $t$-test is given by $\tilde{t}$ ;
where $\tilde{t} = t$-statistic for the null hypothesis $\phi = 0$.

The augmented terms $\Delta \tilde{S}_{t-j} , \; j = 1,...,k$, terms are included to correct for serial
correlation. The value of $k$ is determined by the general to specific search procedure. To
dermogenously determine the location of the break $(T_B)$, the LM unit-root searches for all
possible break points for the minimum (the most negative) unit-root $t$ -test statistic as follows:

$$\inf \; \tilde{t} (\hat{\lambda}) = \inf \tilde{t} (\hat{\lambda}) ; \text{ where } \hat{\lambda} = \frac{T_B}{T} .$$

The two-break LM unit-root test statistic can be estimated analogously by regression
according to the LM (score) principle. Here, Model A of Perron (1989) allows for two shifts
in level while Model C includes two changes in level and trend. Critical values of the
endogenous two-break LM unit-root test $(T = 100)$ is reported in Table 2 by LS (2003: 1084).
LS (2003: 1087) conclude “In summary, the two-break minimum LM unit-root test provides
a remedy for a limitation of the two-break minimum LP test that includes the possibility of a
unit-root with break(s) in the alternative hypothesis. Using the two-break minimum LM unit-
root test, rejection of the null hypothesis unambiguously implies trend stationarity.”

Unit-root tests for one (LS1) and two breaks (LS2) were conducted with RATS 7.2
with the following codes (LS_UROOT.SRC, and LS_UROOT_run.prg). We estimated two
models: LS-Break Model and LS-Crash Model. The LS-Break Model captures the change
that is gradual whereas LS-Crash Model picks up the change that is rapid. In this paper we
have reported the results of both models in Table 2 which are contradictory to each other.
Here we find that the result of the unit-root test is contingent upon the way the breaks are
modelled. It is worth highlighting here that it is up to the researcher to choose the “best
model”, which, in our view, should be based on economic theory and reality. Based on our judgement, we think the LS Trend Break model is the optimal model to discuss.

The $\tilde{\tau}$ statistic of the coefficient of $S_{(t-1)}$ in the LM test can then be examined to test the null of a unit-root. We use the critical values in Table 2 of LS (2003) for the two-break LM test. On the basis of LS1 unit-root test we find $\text{LnRER}$ to be stationary. Given a loss of power from ignoring one structural break, it is logical to expect a similar loss of power from ignoring two or more breaks in the one-break test LS1. By applying the LS2 unit-root test we found that $\text{LnRER}$ is also stationary since the calculated $\tilde{\tau}$ statistic exceeds the critical values. Rejection of the unit-root null provides evidence of mean reversion and hence PPP.

[Insert Table 2 here]

3.4 Endogenously Determined Structural Break Dates

The estimated single structural break date as determined by the LS1 Break Model corresponds to 2003:2 for $\text{LnRER}$, The break date is statistically significant at 5 per cent level of significance. By considering the two breaks LS2 Trend Break Model, the corresponding break dates for $\text{LnRER}$ are 1988:2 and 2002:4. The structural break dates are all statistically significant. The first break date of $\text{LnRER}$ coincided with the abandonment of the “check-list” approach in favour of “discretionary” approach to monetary policy by RBA in 1988:2. This structural break may also be capturing the effect of the stock market crash of October 1987, and the onset of recession at the end of the 1980s culminating into the “recession that we had to have” in 1990. The behaviour of the Australian $RER$ shows periods of instability. One such period was centred around June 1986, the other between March 1998 and June 1999. After a sustained period of depreciation, appreciations of the RER occurred during 1986-1989 so that the break date for the RER is picked up in 1988:2 followed by the meltdown in 2001 and again a recovery in early 2002. The second break date is found to be in 2002:04

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6 The catchphrase of the then treasurer, Paul Keating.
which is due to the sudden appreciation of the Australian dollar. Between January 2002 and July 2008, the Australian dollar appreciated sharply from 51 US cents to 97 US cents which was largely driven by increased demand for Australian exports.

4. Summary and Conclusion
In this paper we investigate evidence of mean reversion in the Australian dollar RER. Conventional unit-root tests fail to provide evidence of stationarity of the RER series that would have supported the PPP assumption. These results might be spurious since they do not account for structural breaks in the data. To overcome the loss of power in conventional unit-root tests, we performed the LS (2003) minimum Lagrange Multiplier (LM) unit-root tests to determine structural breaks endogenously. The LS unit-root test with two structural breaks endogenously determines the location of two breaks in level and trend and tests the null of a unit-root. They also show that the two-break LM unit root test statistic which is estimated by the regression according to the LM principle will not spuriously reject the null hypothesis of a unit root.

Based on our result, we were able to reject the unit-root null hypothesis and find evidence of mean reversion and hence PPP with structural break points. This result is consistent with the result obtained by Chowdhury (2007) although the break dates are different. This is a startling result reversing results of past works that failed to reject that the data are nonstationary. The corresponding break dates for $LnRER$ are 1988:2 and 2002:4 respectively; and the structural break dates are all statistically significant. The estimated break dates mostly correspond to the period of RER instability (1986-1989) and the recovery of the Australian dollar driven by the resources boom (2001-2002).
Table 1. Summary of Past Results of Unit-root in RER

<table>
<thead>
<tr>
<th>Author(s)</th>
<th>Finding</th>
<th>Data Source</th>
<th>Sample Period</th>
<th>Test Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>Blundell-Wignall &amp; Gregory (1990)</td>
<td>NS</td>
<td>Authors’ calculation with OECD data</td>
<td>1970:1 to 1988:4</td>
<td>ADF</td>
</tr>
</tbody>
</table>

Note: S = Stationary; NS = Non-stationary; @=Assume no break under the null hypothesis of unit root.
Table 2. Unit-Root Tests in the Absence and Presence of Endogenous Structural Breaks

<table>
<thead>
<tr>
<th>Variable: LnRER</th>
<th>LS-Break Model Result</th>
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</thead>
<tbody>
<tr>
<td>Test</td>
<td>$\hat{\tau}$</td>
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<tr>
<td>ADF</td>
<td>-2.425</td>
</tr>
<tr>
<td>LS1</td>
<td>-3.568*</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Variable: LnRER</th>
<th>LS-Crash Model Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test</td>
<td>$\hat{\tau}$</td>
</tr>
<tr>
<td>LS1</td>
<td>-2.334</td>
</tr>
<tr>
<td>LS2</td>
<td>-2.714</td>
</tr>
</tbody>
</table>

Note:
1. NC = Not calculated; S = Stationary, NS = Nonstationary.
2. $\hat{\tau}$ = $t$-statistic for the null hypothesis $\phi = 0$.
3. ADF Test critical values at 1, 5 and 10 per cent level are -4.054, -3.456 and -3.153 respectively.
4. Critical values of the endogenous two-break LM unit-root test at 10%, 5% and 1% level of significance are -3.504, -3.842 and -4.545 respectively from Table 2 Lee and Strazicich (2003:1084).
5. (*), (**) and (***) refer to significant at 10, 5 and 1 per cent level of significance respectively.
References


Appendix A

A Brief Review of Unit-root Tests with Endogenous Structural Break

Traditional (First Generation Models) tests for unit-roots (such as Dickey-Fuller, Augmented Dickey-Fuller and Phillips-Perron) have low power in the presence of structural break. Perron (1989) demonstrated that, in the presence of a structural break in time-series, many perceived non-stationary series were in fact stationary. Perron (1989) re-examined Nelson and Plosser (1982) data and found that 11 of the 14 important US macroeconomic variables were stationary when known exogenous structural break is included. Perron (1989) allows for a one time structural change occurring at a time $T_B$ ($1 < T_B < T$), where $T$ is the number of observations.

The following models were developed by Perron (1989) for three different cases. Notations used in equations A1- A16 are the same as in the papers quoted.

Null Hypothesis:

Model (A)  
$y_t = \mu + dD(TB) + y_{t-1} + \epsilon_t$  \hspace{1cm} (A 1)

Model (B)  
$y_t = \mu + (\mu_2 - \mu_1)DU + y_{t-1} + \epsilon_t$  \hspace{1cm} (A 2)

Model (C)  
$y_t = \mu_1 + y_{t-1} + dD(TB)_t + (\mu_2 - \mu_1)DU + \epsilon_t$  \hspace{1cm} (A 3)

where $D(TB)_t = 1$ if $t = T_B$, 0 otherwise.

Alternative Hypothesis:

Model (A)  
$y_t = \mu + \beta_t + (\mu_2 - \mu_1)DU + \epsilon_t$  \hspace{1cm} (A 4)

Model (B)  
$y_t = \mu + \beta_t t + (\beta_2 - \beta_1)DT^- + \epsilon_t$  \hspace{1cm} (A 5)

Model (C)  
$y_t = \mu_1 + \beta_t t + (\mu_2 - \mu_1)DU + (\beta_2 - \beta_1)DT^- + \epsilon_t$  \hspace{1cm} (A 6)

where $DT^- = t - T_B$, if $t > T_B$, and 0 otherwise.

Model A permits an exogenous change in the level of the series whereas Model B permits an exogenous change in the rate of growth. Model C allows change in both. Perron (1989) models include one known structural break. These models cannot be applied where such breaks are unknown. Therefore, this procedure is criticised for assuming known break date which raises the problem of pre-testing and data mining regarding the choice of the

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7 The discussion that follows is for reference only and may be omitted.

8 However, subsequent studies using endogenous breaks have countered this finding with Zivot and Andrews (1992) concluding that 7 of these 11 variables are in fact non-stationary.
break date (Maddala and Kim 2003). Further, the choice of the break date can be viewed as being correlated with the data.

**Second Generation Models**

**Unit-Root Tests in the Presence of a Single Endogenous Structural Break**

Despite the limitations of Perron (1989) models, they form the foundation of subsequent studies that we are going to discuss hereafter. Zivot and Andrews (1992), Perron and Vogelsang (1992), and Perron (1997) among others have developed unit-root test methods which include one endogenously determined structural break. Here we review these models briefly and detailed discussions are found in the cited works.

Zivot and Andrews (ZA) (1992) models are as follows:

**Model with Intercept**

\[ y_t = \hat{\mu} + \hat{\theta} DU_t(\hat{\lambda}) + \hat{\beta} t + \hat{\alpha} y_{t-1} + \sum_{j=1}^{k} \hat{\gamma}_j \Delta y_{t-j} + \hat{\epsilon}_t \]  

(A 7)

**Model with Trend**

\[ y_t = \hat{\mu} + \hat{\beta} t + \hat{\gamma} DT_t^*(\hat{\lambda}) + \hat{\alpha} y_{t-1} + \sum_{j=1}^{k} \hat{\gamma}_j \Delta y_{t-j} + \hat{\epsilon}_t \]  

(A 8)

**Model with Both Intercept and Trend**

\[ y_t = \hat{\mu} + \hat{\theta} DU_t(\hat{\lambda}) + \hat{\beta} t + \hat{\gamma} DT_t^*(\hat{\lambda}) + \hat{\alpha} y_{t-1} + \sum_{j=1}^{k} \hat{\gamma}_j \Delta y_{t-j} + \hat{\epsilon}_t \]  

(A 9)

where, \( DU_t(\hat{\lambda}) = 1 \) if \( t > T\hat{\lambda} \), 0 otherwise;

\( DT_t^*(\hat{\lambda}) = t - T\hat{\lambda} \) if \( t > T\hat{\lambda} \), 0 otherwise.

The above models are based on the Perron (1989) models. However, these modified models do not include \( DT_b \).

On the other hand, Perron and Vogelsang (PV) (1992) include \( DT_b \) but exclude \( t \) in their models. PV (1992) models are given below:

**Innovational Outlier Model (IOM)**

\[ y_t = \mu + \delta DU_t + \delta D(T_b)_t + \alpha y_{t-1} + \sum_{i=1}^{k} c_i \Delta y_{t-i} + \epsilon_t \]  

(A 10)

**Additive Outlier Model (AOM) – Two Steps**

\[ y_t = \mu + \delta DU_t + \bar{y}_t \]  

(A 11)

and

\[ \bar{y}_t = \sum_{i=0}^{k} w_i D(T_b)_{i,t} + \alpha \bar{y}_{t-1} + \sum_{i=1}^{k} c_i \Delta \bar{y}_{t-i} + \epsilon_t \]  

(A 12)
\( \tilde{y} \) in the above equations represents a detrended series \( y \).

Perron (1997) includes both \( t \) (time trend) and \( DT_b \) (time at which structural change occurs) in his Innovational Outlier (IO1 and IO2) and Additive Outlier (AO) models.

**Innovational Outlier Model allowing one time change in intercept only (IO1):**

\[
y_t = \mu + \theta DU_t + \beta t + \delta DT_b + \alpha y_{t-1} + \sum_{i=1}^{k} c_i \Delta y_{t-i} + e_t
\]

**(A 13)**

**Innovational Outlier Model allowing one time change in both intercept and slope (IO2):**

\[
y_t = \mu + \theta DU_t + \beta t + \gamma DT_b + \delta D(T_b) + \alpha y_{t-1} + \sum_{i=1}^{k} c_i \Delta y_{t-i} + e_t
\]

**(A 14)**

**Additive Outlier Model allowing one time change in slope (AO):**

\[
y_t = \mu + \beta t + \gamma DT^*_t + \tilde{y}_t
\]

**(A 15)**

where \( DT^*_t = 1(t > T_b)(t - T_b) \)

\[
\tilde{y}_t = \alpha y_{t-1} + \sum_{i=1}^{k} c_i \Delta \tilde{y}_{t-i} + e_t
\]

**(A 16)**

The Innovational Outlier models represent the change that is gradual whereas Additive Outlier model represents the change that is rapid. All the models considered above report their asymptotic critical values.

Regarding the power of tests, the PV (1992) model is robust. The testing power of Perron (1997) and ZA models (1992) are almost the same. On the other hand, Perron (1997) model is more comprehensive than ZA (1992) model as the former includes both \( t \) and \( DT_b \) while the latter includes \( t \) only.

More recently, additional test methods have been proposed for unit-root test allowing for multiple structural breaks in the data series (Lumsdaine and Papell (LP) 1997; Bai and Perron (BP) 2003; Lee and Strazicich (LS) 2003). One important issue common to the ZA and LP (and other similar) endogenous break tests is that they assume no break(s) under the unit-root null and derive their critical values accordingly. Thus, the alternative hypothesis would be “structural breaks are present,” which includes the possibility of a unit-root with break(s). Thus, rejection of the null does not necessarily imply rejection of a unit-root per se, but would imply rejection of a unit-root without breaks.
Third Generation Models


LS propose a minimum Lagrange multiplier (LM) unit-root test in which the alternative hypothesis unambiguously implies trend stationarity. Consider the DGP as follows:

\[ \Delta y_t = \delta \Delta Z_t + \phi \tilde{S}_{t-1} + u_t \]  

(A17)

where \( \tilde{S}_t = y_t - \tilde{\psi}_x - Z_t \tilde{\delta} \) \((t=2,..,T)\) and \( Z_t \) is a vector of exogenous variables defined by the data generating process; \( \tilde{\delta} \) is the vector of coefficients in the regression of \( \Delta y_t \) on \( \Delta Z_t \) respectively with \( \Delta \) the difference operator; and \( \tilde{\psi}_x = y_1 - Z_1 \tilde{\delta} \), with \( y_1 \) and \( Z_1 \) the first observations of \( y \) and \( Z \) respectively.

Model B of Perron (1989) is omitted from further discussion by LS (2003), as it is commonly held that most economic time-series can be adequately described by model A or C. Equivalent to Perron’s (1989) Model C, which allows for a shift in intercept and change in trend slope under the null hypothesis and is described as \( Z_t = [1, t, D_t, DT_t]' \), where \( DT_t = t - T_B \) for \( t > T_B + 1 \), and zero otherwise. It is important to note here that testing regression (1) involves using \( \Delta Z_t \) instead of \( Z_t \). \( \Delta Z_t \) is described by \( [1, B_t, D_t]' \) where \( B_t = \Delta D_t \) and \( D_t = \Delta DT_t \). Thus, \( B_t \) and \( D_t \) correspond to a change in the intercept and trend under the alternative and to a one period jump and (permanent) change in drift under the null hypothesis, respectively.

The unit-root null hypothesis is described in (A17) by \( \phi = 0 \) and the LM \( t \)-test is given by \( \tilde{\tau} \); where \( \tilde{\tau} = t \)-statistic for the null hypothesis \( \phi = 0 \).

The augmented terms \( \Delta \tilde{S}_{t-j}, j = 1,...,k \), terms are included to correct for serial correlation. The value of \( k \) is determined by the general to specific search procedure. General to specific procedure begins with the maximum number of lagged first differenced terms max \( k = 8 \) and then examine the last term to see if it is significantly different from zero. If insignificant, the maximum lagged term is dropped and then estimated at \( k = 7 \) terms and so on, till the maximum is found or \( k = 0 \). To endogenously determine the location of the break \( (T_B) \), the LM unit-root searches for all possible break points for the minimum (the most negative) unit-root \( t \)-test statistic as follows:
\[ \text{Inf} \bar{\tau}(\tilde{\lambda}) = \text{Inf} \underline{\tau}(\lambda) \; \text{where} \; \lambda = T_h / T. \quad (A18) \]

The two-break LM unit-root test statistic can be estimated analogously by regression according to the LM (score) principle from equation A17. Here, Model A of Perron (1989) allows for two shifts in level while Model C includes two changes in level and trend. Critical values of the endogenous two-break LM unit-root test \((T = 100)\) is reported in Table 3 by LS (2003: 1084). LS (2003: 1087) conclude “In summary, the two-break minimum LM unit-root test provides a remedy for a limitation of the two-break minimum LP test that includes the possibility of a unit-root with break(s) in the alternative hypothesis. Using the two-break minimum LM unit-root test, rejection of the null hypothesis unambiguously implies trend stationarity.