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Modeling for optical feedback laser diode operating in period-one oscillation and its application

Bin Liu
University of Wollongong

Yuxi Ruan
University of Wollongong, yr776@uowmail.edu.au

Yanguang Yu
University of Wollongong, yanguang@uow.edu.au

Qinghua Guo
University of Wollongong, qguo@uow.edu.au

Jiangtao Xi
University of Wollongong, jianhtao@uow.edu.au

See next page for additional authors

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Authors
Bin Liu, Yuxi Ruan, Yanguang Yu, Qinghua Guo, Jiangtao Xi, and Jun Tong

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Modeling for optical feedback laser diode operating in period-one oscillation and its application

BIN LIU, YUXI RUAN, YANGUANG YU,* QINGHUA GUO, JIANGTAO XI, AND JUN TONG

School of Electrical, Computer and Telecommunications Engineering, University of Wollongong, Wollongong 2522, NSW, Australia
*yanguang@uow.edu.au

Abstract: An optical feedback laser diode (OFLD) operating in period-one oscillation (POO) with a moving external target is investigated by exploring its potential sensing capability. First, the modeling of an OFLD-POO sensing system is presented. An analytical expression is derived for OFLD-POO sensing signal, from which a new displacement measurement method is developed. The proposed sensing model is verified by the well-known Lang-Kobayashi equations used to describe the dynamic behavior of a laser with optical feedback. Then, an experimental OFLD-POO system is built in order to demonstrate an application example for displacement sensing. The measurement results show that the OFLD-POO sensing system can achieve displacement measurement with large measurement range, high sensitivity, and resolution.

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1. Introduction

Optical feedback interferometry (OFI) or Self-mixing interferometry (SMI) is a promising non-contact sensing technology, which uses the self-mixing effect that occurs when a fraction of the light back-reflected or back-scattered by an external target re enters the laser cavity [1,2]. In this case, the steady-state intensity of the lasing light is modulated by the external optical feedback. A typical OFI system consists of a laser diode (LD), a photodiode (PD) packaged at the rear of LD, a lens and an external target [3–6]. We call the laser diode in this case as optical feedback laser diode (OFLD) and the modulated intensity as OFLD signal. As a preferred, minimum part-count scheme useful for engineering implementation, various OFLD-based applications have been developed in the industrial and laboratory environment, such as measurement of displacement [7,8], vibration [9], velocity [10], alpha factor [11,12], Young’s modulus [13], strain [14], acoustic emission [15], acoustic field imaging [16], etc.

Most of the OFLD-based applications are based on the stable OFI mathematic model, which is derived from the steady-state solution of the Lang and Kobayashi (L-K) equations [17], or from the classical three-mirror model [2], by assuming that the OFLD system operates in stable mode, i.e., both the electric field and carrier density in an LD with a stationary external cavity can reach a constant state after a transient period. However, many works [18–20] have demonstrated that an OFLD system with a stationary external target may be unstable under some certain operation conditions, such as in periodic, quasi-periodic or chaotic oscillation state. We studied OFLD signals with a moving target in different conditions and our recent works [21,22] shows that such sensing system may produce different types of OFLD signals which are classified into three regions defined in [21], i.e. stable, semi-stable and unstable region under different feedback level factors C. C is defined as: 

\[ C = \kappa \tau \sqrt{1 + \alpha^2} / \tau_m, \]

where \( \kappa \) is the optical feedback strength, \( \tau \) and \( \tau_m \) is the laser round-trip time in the external and internal cavity respectively, and \( \alpha \) is the linewidth enhancement factor [2]. In stable region, for a stationary target, the laser intensity will be constant after the
transient state, whereas in semi-stable and unstable region, relaxation oscillation will be undamped and the laser may enter period-one, quasiperiodic or chaotic oscillation under different feedback strength. The work [21,22] discovers that an OFLD can easily and sometimes inevitably enter the period-one oscillation, e.g. for the case when the feedback level factor C is around 2.5, which is quite commonly reported for the moderate feedback regime in the literature. In this case, the OFLD signals (laser intensity) generated by an OFLD system cannot be described by the existing OFI model. The laser intensity is modulated in a quite complicated form and contains very high-frequency components. In order to differentiate the conventional OFLD signals, we name the laser intensity signals generated by the OFLD system operating in period-one oscillation (POO) as OFLD-POO signals. The work in [23] also depicts some high-frequency components in the sawtooth-like OFLD signals, but the LD used in [23] still operates in conventional self-mixing conditions rather than in POO state. Usually, the PD packaged inside an LD is used to detect the laser intensity, but it has limited bandwidth less than 1GHz. However, the frequency components contained in an OFLD-POO signal can even higher than several GHz [21,22]. Hence, using existing OFI configuration for detection of OFLD-POO signal, many frequency components are missing. The work in [22] has experimentally revealed the features of such signals and indicated the possible sensing application but it did not make analytic expression on the signal waveform. As an OFLD-POO signal contains rich information associated with the laser and its external target, it is of great interest and significance to investigate the signal features by modelling the signal waveform, from which we can develop a suitable signal processing algorithm to retrieve the useful information contained in the signals.

In this work, an OFLD operating in POO with a moving external target is investigated by exploring its potential sensing capability. Starting from the L-K equations, we firstly derive an approximate analytical model to describe the OFLD-POO signals. The numerical simulations for original L-K equations are then conducted to verify the derived analytical expression. Finally, a displacement sensing method by simultaneously using the time-domain OFLD-POO waveform and relaxation oscillation (RO) frequency contained in this signal is proposed. From both simulation and experiment, the feasibility of proposed displacement sensing method has been verified.

2. Modelling and analysis

2.1 Theoretical modelling

The behavior of an LD with optical feedback can be described by the well-known L-K equations [17], as shown in Eqs. (1)-(3). Here, $E(t)$ is the amplitude of the electric field, $\phi(t)$ is the phase of the electric field, and $N(t)$ is the carrier density. $E^2(t)$ is usually considered as the laser intensity or power, denoted by $P(t) = E^2(t)$.

$$\frac{dE(t)}{dt} = \frac{1}{2} \left[ G[N(t),E(t)] - \frac{1}{\tau_p} \right] E(t) + \frac{\kappa}{\tau_m} \cdot E(t-\tau) \cdot \cos \left[ \omega_\phi \tau + \phi(t) - \phi(t-\tau) \right].$$

$$\frac{d\phi(t)}{dt} = \frac{1}{2} \alpha \left[ G[N(t),E(t)] - \frac{1}{\tau_p} \right] - \frac{\kappa}{\tau_m} \cdot \frac{E(t-\tau)}{E(t)} \cdot \sin \left[ \omega_\phi \tau + \phi(t) - \phi(t-\tau) \right].$$

$$\frac{dN(t)}{dt} = I - \frac{N(t)}{\tau_s} - G[N(t),E(t)]E^2(t).$$

where, $G[N(t),E(t)] = g_N \left[ N(t) - N_0 \right] \left[ 1 - \epsilon TE^2(t) \right]$ is the modal gain per unit time. In the following derivation, the non-linear gain is ignored [18]. There are 3 controllable parameters
for a specific OFLD, i.e. injection current ($I$), feedback strength ($\kappa$) and external cavity laser round-trip time ($\tau$). The physical meanings and typical values of the other symbols appearing in Eqs. (1)-(3) are listed in Table 1, which are adopted from [19,24].

### Table 1. Meanings of the symbols in the L-K equations

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Physical Meaning</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega_0$</td>
<td>laser angular frequency without feedback</td>
<td>$2.42 \times 10^{15}$ rads$^{-2}$</td>
</tr>
<tr>
<td>$G_N$</td>
<td>modal gain coefficient</td>
<td>$8.1 \times 10^{-13}$ m$^3$s$^{-1}$</td>
</tr>
<tr>
<td>$N_0$</td>
<td>carrier density at transparency</td>
<td>$1.1 \times 10^{24}$ m$^{-3}$</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>nonlinear gain compression coefficient</td>
<td>$2.5 \times 10^{-7}$ m$^3$</td>
</tr>
<tr>
<td>$\Gamma$</td>
<td>confinement factor</td>
<td>0</td>
</tr>
<tr>
<td>$\tau_p$</td>
<td>photon life time</td>
<td>$2.0 \times 10^{-12}$ s</td>
</tr>
<tr>
<td>$\tau_s$</td>
<td>carrier life time</td>
<td>$2.0 \times 10^{-9}$ s</td>
</tr>
<tr>
<td>$\tau_{in}$</td>
<td>internal cavity round-trip time</td>
<td>$8.0 \times 10^{-12}$ s</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>line-width enhancement factor</td>
<td>3</td>
</tr>
</tbody>
</table>

Starting from the L-K equations, we aim to get an approximate analytical expression for OFLD-POO signal (i.e., $E(t)$). Based on the numerical simulation of L-K equations and the work in [18,20], when an OFLD operates in POO, $E(t)$, $N(t)$ and $\phi(t)$ can be written in the form of below:

$$E(t) = \bar{E} + \Delta E \cos(\omega t + \theta).$$

$$N(t) = \bar{N} + \Delta N \cos(\omega t + \theta).$$

$$\phi(t) = (\bar{\phi} - \omega_0) t + (\Delta \phi / 2) \cos \omega t.$$  \hspace{1cm} (4) \hspace{1cm} (5) \hspace{1cm} (6)

where, $\bar{E}$, $\bar{N}$, and $\bar{\omega}$ are respectively the mean values of $E(t)$, $N(t)$ and optical frequency, $\Delta E$, $\Delta N$ and $\Delta \phi / 2$ are the modulation amplitudes, $\omega_0$ is the RO angular frequency when the OFLD is in POO, $\theta$ and $\theta_0$ are the initial phase of $E(t)$ and $N(t)$ respectively. We will need to express the mean values and the modulation amplitudes as well as the initial phases using the parameters related to the LD and its external cavity listed in Table 1.

Regarding $\bar{E}$ and $\bar{N}$, they can be determined by solving Eqs. (1)-(3) through setting $dE(t) / dt = 0, dN(t) / dt = 0, d\phi(t) / dt = \omega_0 - \omega_0$ [18], as shown below:

$$\bar{E} = \frac{1}{\frac{\bar{N}}{G_N} - \frac{\bar{N}}{G_N} \frac{\tau_p}{\tau_s}}.$$

$$\bar{N} = N_0 \left[ 1 + \frac{2k \cos(\omega_0 \tau)}{G_N \tau_p \tau_{in} G_N} \right].$$

$$\omega_0 = \omega_0 - \frac{k \sqrt{1 + \alpha^2}}{\tau_{in}} \sin(\omega_0 \tau + \arctan \alpha).$$  \hspace{1cm} (7) \hspace{1cm} (8) \hspace{1cm} (9)

where $\omega_0$ is the optical frequency when the OFLD is stable. Next, we look into the analytical expressions for $\Delta E$, $\Delta N$, $\Delta \phi$, $\theta$, and $\theta_0$. With a similar treatment as done in [21], we set $\omega_0 \tau = (2m + 1) \cdot \pi$. Note that, in the case of $\omega_0 \tau = 2m \cdot \pi$, the OFLD will not be in POO [25].
Substituting $E(t)$ and $N(t)$ in Eq. (3) by Eq. (4) and Eq. (5), and neglecting the terms with $\Delta E \cdot \Delta N$, $\Delta E^2$ and $\Delta N^2$ (as $\Delta E / E << 1$ and $\Delta N / N << 1$) [18], we have:

$$2G_\gamma (\bar{N} - N_0)E\Delta E \cos(\omega t + \theta)$$

$$= I - \bar{N} / \tau_e - G_\gamma \cdot (\bar{N} - N_0) \cdot \bar{E}^2$$

$$+ \Delta N \sqrt{\omega^2 + \left(1 / \tau_e + G_\gamma \bar{E}^2 \right)^2} \cos \{ \omega t + \theta_e - \arctan \left[ \left(1 / \tau_e + G_\gamma \bar{E}^2 \right) / \omega \right] - \pi / 2 \}. \quad (10)$$

As Eq. (10) is an identical equation, letting the corresponding terms in both sides are equal, we have:

$$2G_\gamma (\bar{N} - N_0)E\Delta E = \Delta N \sqrt{\omega^2 + \left(1 / \tau_e + G_\gamma \bar{E}^2 \right)^2} \cos \{ \omega t + \theta_e - \arctan \left[ \left(1 / \tau_e + G_\gamma \bar{E}^2 \right) / \omega \right] - \pi / 2 \}. \quad (11)$$

$$\theta_e = \theta_e - \arctan \left[ \left(1 / \tau_e + G_\gamma \bar{E}^2 \right) / \omega \right] - \pi / 2. \quad (12)$$

$$I - \bar{N} / \tau_e - G_\gamma \cdot (\bar{N} - N_0) \cdot \bar{E}^2 = 0. \quad (13)$$

Interestingly, it can be found that Eq. (13) is the same as Eq. (7). In a similar way, substituting $E(t)$, $N(t)$ and $\phi(t)$ in Eq. (1) by Eqs. (4), (5) and (6), we can get:

$$- \omega \Delta E \sin(\omega t + \theta)$$

$$= \left\{ \frac{1}{2} \left[ G_\gamma \cdot (\bar{N} + \Delta N \cos(\omega t + \theta_e) - N_0) - \frac{1}{\tau_p} \right] \right\} [\bar{E} + \Delta E \cos(\omega t + \theta_e)]$$

$$+ \frac{\kappa}{\tau_m} \left[ \Delta E \cos(\omega t + \theta_e) \right] \cdot [\omega t + \Delta \phi \cos(\omega t)]. \quad (14)$$

After expansion and reorganization of Eq. (14), we have:

$$- \omega \Delta E \sin(\omega t + \theta) - \frac{1}{2} \left[ G_\gamma \cdot (\bar{N} - N_0) - 1 / \tau_p \right] \bar{E} + \frac{\kappa}{\tau_m} \cdot \bar{E} \cos \omega t \cdot J_0(\Delta \phi)$$

$$+ \frac{1}{2} G_\gamma \bar{E} \Delta N \cos(\omega t + \theta_e) - I \bar{E} - \left[ G_\gamma \cdot (N_e - N_0) - 1 / \tau_p \right] \Delta E \cos(\omega t + \theta_e)$$

$$- 2 \frac{\kappa}{\tau_m} \cdot \bar{E} \sin \omega t \cdot J_1(\Delta \phi) \cos \omega t. \quad (15)$$

where $J_0(\Delta \phi)$ and $J_1(\Delta \phi)$ are the 0th-order and 1st-order Bessel function respectively. Note that higher harmonic components of $\omega t$ are neglected [18]. Then, we have:

$$\frac{1}{2} \left[ G_\gamma \cdot (\bar{N} - N_0) - 1 / \tau_p \right] + \frac{\kappa}{\tau_m} \cos \omega t \cdot \Delta E \cos(\omega t + \theta_e) = 0. \quad (16)$$

$$\omega \Delta E \sin(\omega t + \theta_e) = 2 \frac{\kappa}{\tau_m} \cdot \bar{E} \sin \omega t \cdot J_1(\Delta \phi) \cos \omega t$$

$$- \frac{1}{2} \left[ G_\gamma \cdot (\bar{N} - N_0) - 1 / \tau_p \right] \Delta E \cos(\omega t + \theta_e) - \frac{1}{2} G_\gamma \bar{E} \Delta N \cos(\omega t + \theta_e). \quad (17)$$

After expansion and reorganization of Eq. (17), we have:
\[ \omega \Delta E \cos \theta \sin \omega t + \omega \Delta E \sin \theta \cos \omega t = \]
\[ \left\{ \frac{1}{2} \left[ G_N \cdot (\bar{N} - N_0) - 1/ \tau_p \right] \cdot \Delta E \sin \theta + \frac{1}{2} G_N \cdot \bar{E} \Delta N \sin \theta \right\} \sin \omega t \]
\[ + \left\{ - \frac{1}{2} \left[ G_N \cdot (\bar{N} - N_0) - 1/ \tau_p \right] \cdot \Delta E \cos \theta - \frac{1}{2} G_N \cdot \bar{E} \Delta N \cos \theta + \frac{2 \kappa}{\tau_{in}} \bar{E} \sin \omega \tau \cdot J_\phi (\Delta \phi) \right\} \cos \omega t. \]

(18)

As Eq. (18) is an identical equation as well, letting the corresponding terms in both sides are equal, we have:

\[ \omega \Delta E \cos \theta = \frac{1}{2} \left[ G_N \cdot (\bar{N} - N_0) - 1/ \tau_p \right] \cdot \Delta E \sin \theta + \frac{1}{2} G_N \cdot \bar{E} \Delta N \sin \theta. \]

(19)

Inserting Eq. (12) into Eq. (19) yields:

\[ \cot \theta = \frac{1}{\omega_r} \cdot \frac{\omega^2 - G_N^2 \bar{E}^2 (N_r - N_0) - (1/ \tau_x + G_N \bar{E}^2) (\kappa / \tau_{in}) J_\phi (\Delta \phi) \cos \omega \tau}{1/ \tau_x + G_N \bar{E}^2 + (\kappa / \tau_{in}) J_\phi (\Delta \phi) \cos \omega \tau}. \]

(20)

Similarly, substituting \( E(t) \), \( N(t) \) and \( \phi(t) \) in Eq. (2) with Eqs. (4), (5) and (6), we get:

\[ \omega \tau_0 = - \alpha \frac{K}{\tau_{in}} \cos \delta \tau \cdot J_\phi (\Delta \phi) - \frac{\kappa}{\tau_{in}} \sin \delta \tau J_\phi (\Delta \phi). \]

(21)

\[ \frac{1}{2} \alpha G_N \cdot \Delta N \cos \theta - \frac{2 K}{\tau_{in}} \cos \delta \tau J_\phi (\Delta \phi) = 0. \]

(22)

\[ \Delta \phi \omega = \alpha G_N \cdot \Delta N \sin \theta. \]

(23)

Combining Eqs. (16) and (8), we have:

\[ \cos \delta \tau \cdot J_\phi (\Delta \phi) = \cos \omega \tau. \]

(24)

With the Taylor expansion of the Bessel functions, we have the approximations:

\[ J_\phi (\Delta \phi) = 1 - \Delta \phi^2 / 4, \] and \( J_\phi (\Delta \phi) = \Delta \phi / 2 - \Delta \phi^3 / 16. \) Solving Eqs. (20), (22), (23) and (24), we get:

\[ \cot \theta = \frac{1}{\omega_r} \cdot \frac{\omega^2 - G_N^2 \bar{E}^2 (N_r - N_0) - (1/ \tau_x + G_N \bar{E}^2) \frac{K}{\tau_{in}} \cos \omega \tau}{G_N \bar{E}^2 + 1/ \tau_x + \frac{K}{\tau_{in}} \cos \omega \tau}. \]

(25)

\[ \Delta \phi = \frac{2}{\omega \tau_x \cot \theta - 2 \kappa \cos \omega \tau} \sqrt{\frac{\omega \tau_{in} \cot \theta - 2 \kappa \cos \omega \tau}{\omega \tau_x \cot \theta - \kappa \cos \omega \tau}}. \]

(26)

\[ \Delta N = \frac{2 \omega \tau_{in} \cot \theta - 2 \kappa \cos \omega \tau}{\omega \tau_x \cot \theta - \kappa \cos \omega \tau} \sqrt{\frac{\omega^2 + (G_N \bar{E}^2 + 1/ \tau_x)^2}{G_N (\bar{N} - N_0) \bar{E}}} \].

(27)

Inserting Eq. (27) into Eq. (11) leads to

\[ \Delta E = \frac{\omega}{\alpha G_N \sin \theta} \sqrt{\frac{\omega \tau_x \cot \theta - 2 \kappa \cos \omega \tau}{\omega \tau_{in} \cot \theta \cos \omega \tau} \sqrt{\frac{\omega^2 + (G_N \bar{E}^2 + 1/ \tau_x)^2}{G_N (\bar{N} - N_0) \bar{E}}}}. \]

(28)
Since $\omega_r \gg 1/\tau_r + G_N E^2$ [20], Eq. (28) can be simplified as:

$$\Delta E = \frac{\omega_r^2}{\alpha G_N \sin \theta_r} \sqrt{\frac{\omega_r \tau_m \cot \theta_r - 2\kappa \cos \omega_r \tau}{\omega_r \tau_m \cot \theta_r - \kappa \cos \omega_j \tau}} \cdot \frac{1}{G_N (\bar{N} - N_0) E}. \quad (29)$$

Now, we can have the expression for $E^2(t)$ or $P(t) = E^2(t)$ as below:

$$P(t) = E^2(t) = \bar{E}^2 + 2\bar{E}\Delta E \cos(\omega_j t + \theta_j) + \Delta E^2 \cos^2(\omega_j t + \theta_j). \quad (30)$$

From Eq. (8), we have $G_N (\bar{N} - N_0) = 1/\tau_p - 2(\kappa / \tau_m) \cos \omega_r \tau$. In the case of $\kappa < 0.01$, neglecting the terms with $\Delta E^2$, Eq. (30) can be approximated as follows:

$$P(t) = \bar{E}^2 + 2\frac{\tau_p \omega_r^2}{\alpha G_N \sin \theta_r} \sqrt{\frac{\omega_r \tau_m \cot \theta_r - 2\kappa \cos \omega_r \tau}{\omega_r \tau_m \cot \theta_r - \kappa \cos \omega_j \tau}} \left(1 + \frac{2\kappa \tau_r}{\tau_m} \cos \omega_j \tau\right) \cos(\omega_j t + \theta_j). \quad (31)$$

where $\bar{E}^2$ is shown in Eq. (7). Inserting Eq. (8) into Eq. (7), in the case of $\kappa < 0.01$, it can be rewritten as [1]:

$$\bar{E}^2 = E_{s_0}^2 + 2(E_{s_0}^2 + \frac{1}{G_N \tau_s}) \frac{\kappa \tau_r}{\tau_m} \cos(\omega_j \tau). \quad (32)$$

where $E_{s_0}^2$ is the laser intensity of the LD without optical feedback, which is determined by the injection current. Inserting Eqs. (7) and (8) into Eq. (25), we have

$$\cot \theta_r = \frac{\tau_r (\omega_r^2 - \omega_{s_0}^2) - \frac{\kappa}{\tau_m} \cos \omega_j \tau}{\omega_r}. \quad (33)$$

where $\omega_{s_0}$ is the RO frequency of the LD without optical feedback, which is determined by the injection current. Inserting Eqs. (32) and (33), into Eq. (31) and considering the case of $\kappa < 0.01$, we obtain:

$$P(t) = E_{s_0}^2 + 2(E_{s_0}^2 + \frac{1}{G_N \tau_s}) \frac{\kappa \tau_r}{\tau_m} \cos(\omega_j \tau)$$

$$+ 2\frac{\tau_p \omega_r^2}{\alpha G_N \sin \theta_r} \sqrt{\omega_r^2 + \left[\tau_r (\omega_r^2 - \omega_{s_0}^2) - \frac{\kappa}{\tau_m} \cos \omega_j \tau\right]^2 \cos(\omega_j t + \theta_j). \quad (34)$$

Therefore, Eq. (34) is the laser intensity when the OFLD system is in period-one oscillation. When the external cavity is moving, it will cause a varying feedback phase $\phi_e = 2 \omega_e L / c$, where $c$ is the vacuum light speed [2,11–13] as well as $\omega_e$ [24]. We re-write Eq. (34) as below to include displacement sensing:

$$\Delta P(L,t) = P(t) - E_{s_0}^2 = \Delta P_{OFLD}(L) + \Delta P_{OFLD-POO-EP}(L) \cos(\omega_j (L) \cdot t + \theta_j). \quad (35)$$

where,

$$\Delta P_{OFLD}(L) = 2(E_{s_0}^2 + \frac{1}{G_N \tau_s}) \frac{\kappa \tau_r}{\tau_m} \cos(\phi_e (L)). \quad (36)$$
\[ \Delta P_{\text{OFLD-POO-EP}}(L) = 2 \frac{\tau_p \omega_r(L)}{\alpha G_N} \sqrt{\omega_r^2(L) + [\tau_p \omega_r^2(L) - \omega_r^2]} - \frac{K}{\tau_{in}} \cos[\phi_r(L)] \]  

Equation (36) is the conventional OFLD signal reported in the literature [2,11,12]. This signal is in the form of sinusoidal (at weak feedback case) or sawtooth-like (moderate or above feedback level) waveform when the external cavity length \(L\) changes. Each periodical variation (called a fringe) corresponds to a displacement with half laser wavelength \(\lambda_0 / 2\) [2]. For a varying external target, the external cavity length can be expressed as \(L=L_0 + \Delta L\), where \(L_0\) is the initial external cavity length, and \(\Delta L\) is the displacement.

We are more interested in the last term appeared in Eq. (35). Since this term also contains the information of displacement \(\Delta L\), we can make use of it to achieve displacement sensing. The RO frequency of an OFLD has been investigated and an approximate analytical expression for RO frequency has been derived for an OFLD in steady state [20]. The analysis on the RO frequency and its application for measuring linewidth enhancement factor of the LD has been discussed in [26]. When the OFLD operates in POO, the expression is invalid and the RO frequency is different from that of the OFLD in steady state. In this case, the frequency of POO can be treated as the RO frequency, i.e. \(\omega_r\) contained in Eqs. (35) and (37), as in our previous work [24], which shows that \(\omega_r\) changes with displacement \(\Delta L\) in the form of a saw-tooth waveform with a period of \(\lambda_0 / 2\) when the OFLD system is in period-one oscillation. Thus, we have \(\omega_r(\Delta L) = \omega_r(\Delta L + \lambda_0 / 2)\). Denoting the maximum and minimum value of \(\omega_r\) within a period by \(\omega_{r_{\text{max}}}\) and \(\omega_{r_{\text{min}}}\) respectively, the expression for \(\omega_r\) within one period \((0 < \Delta L < \lambda_0 / 2)\) can be expressed as:

\[ \omega_r(\Delta L + N \frac{\lambda_0}{2}) = \omega_r + \omega_{r_{\text{offset}}} + \frac{2(\omega_{r_{\text{max}}} - \omega_{r_{\text{min}}})}{\lambda_0} \Delta L. \]  

(38)

where \(N\) is an integer, \(\omega_r\) is the angular RO frequency of the solitary laser diode which is determined by injection current density, \(\omega_{r_{\text{offset}}}\) is the offset value which is determined by the initial external cavity length and feedback level, and \(\omega_{r_{\text{max}}}\) and \(\omega_{r_{\text{min}}}\) are constant when the operation conditions of an LD are fixed. Equation (37) deserves a close examination. Since both \(\omega_r\) and \(\cos[\phi_r(L)]\) are periodical with same period of \(\lambda_0 / 2\) for a varying external cavity length, we have:

\[ \Delta P_{\text{OFLD-POO-EP}}(L + N \frac{\lambda_0}{2}) = 2 \frac{\tau_p \omega_r(L + N \frac{\lambda_0}{2})}{\alpha G_N} \sqrt{\omega_r^2(L + N \frac{\lambda_0}{2}) + [\tau_p \omega_r^2(L + N \frac{\lambda_0}{2}) - \omega_r^2]} - \frac{K}{\tau_{in}} \cos[\phi_r(L + N \frac{\lambda_0}{2})]. \]

\[ \Delta P_{\text{OFLD-POO-EP}}(L) \]

(39)

It can be seen that \(\Delta P_{\text{OFLD-POO-EP}}(L)\) also periodically change with displacement \(\Delta L\) by \(\lambda_0 / 2\). Figure 1(a) shows an example of the relationship between the angular RO frequency and displacement of external target when \(L_0=24\text{cm}, I=1.3I_{th}\) and \(C=4.5\), where \(I_{th}\) is the threshold injection current. Figure 1(b) shows the waveform of \(\Delta P_{\text{OFLD-POO-EP}}(L)\) for a varying external cavity with the same operation conditions.
In the following, we only consider the last term in Eq. (35), which is called OFLD-POO signal denoted by $\Delta P_{\text{OFLD-POO}}(L,t)$, and re-write it as below:

$$\Delta P_{\text{OFLD-POO}}(L,t) = \Delta P_{\text{OFLD-POO-\Ep}}(L) \cos[\omega_{r}(L) t + \theta_{r}].$$

(40)

Now, we can describe that the amplitude and frequency of an OFLD-POO signal is modulated by the displacement to be measured. Both the envelope ($\Delta P_{\text{OFLD-POO-\Ep}}(L)$) and frequency of the signal contain displacement information. So we can explore a new sensing approach by using this signal. We denote the maximum and minimum values of $\Delta P_{\text{OFLD-POO-\Ep}}$ as $\Delta P_{\text{OFLD-POO-\Ep-max}}$ and $\Delta P_{\text{OFLD-POO-\Ep-min}}$, respectively. Based on the simulation and the work in [24], $\omega_{r,\text{offset}}$ and $\omega_{r,\text{max}} - \omega_{r,\text{min}}$ is much smaller than $\omega_{r,0}$ and we have $\omega_{r} = \omega_{r,0}$. In the case of $\kappa < 0.01$, we get:

$$\Delta P_{\text{OFLD-POO-\Ep-min}} = 2 \frac{\omega_{r,0}^{2}}{\alpha G_{N}}.$$

(41)

$$\Delta P_{\text{OFLD-POO-\Ep-max}} = 2 \frac{\omega_{r,0}^{2}}{\alpha G_{N}} \sqrt{1 + [2 \tau_{r}(\omega_{r,\text{offset}} + \omega_{r,\text{max}} - \omega_{r,\text{min}})]^{2}}.$$

(42)

Thus the peak-peak variation in $\Delta P_{\text{OFLD-POO-\Ep}}(L)$ (denoted by $\Delta P_{\text{OFLD-POO-\Ep-pp}}$) is expressed as:

$$\Delta P_{\text{OFLD-POO-\Ep-pp}} = 2 \frac{\omega_{r,0}^{2}}{\alpha G_{N}} - \Delta P_{\text{OFLD}}.$$

(43)

Making a comparison on the magnitude of $\Delta P_{\text{OFLD-POO-\Ep}}$ and $\Delta P_{\text{OFLD}}$ as below:

Fig. 1. (a) Relationship between RO frequency and displacement of external cavity, (b) waveform of $\Delta P_{\text{OFLD-POO-\Ep}}$ for a change of external cavity length when $L_{s} = 24 cm$, $I = 1.3I_{th}$ and $C = 4.5$. 
\[
\frac{\Delta P_{\text{OFLD-POO-Ep-pp}}}{\Delta P_{\text{OFLD-pp}}} = \frac{2 \tau \omega^2}{\alpha G_N} \left( \sqrt{1 + \left[ 2 \tau \left( \omega_{\text{r-off}} + \omega_{\text{r-max}} - \omega_{\text{r-min}} \right) \right]^2} - 1 \right)
\]

\[
= \frac{\tau_s}{2 \alpha \tau_p} \left( \sqrt{1 + \left[ 2 \tau_s \left( \omega_{\text{r-off}} + \omega_{\text{r-max}} - \omega_{\text{r-min}} \right) \right]^2} - 1 \right).
\]

(44)

Based on the simulation and the work in [24], \( \omega_{\text{r-off}} \) and \( \omega_{\text{r-max}} - \omega_{\text{r-min}} \) are usually in the order of \( 10^8 \text{rad/s} \). E.g., in the case in Fig. 1, \( \omega_{\text{r-off}} + \omega_{\text{r-max}} - \omega_{\text{r-min}} = 6 \times 10^8 \text{ rad/s} \). Substituting the typical values in Table 1 into Eq. (44), i.e., \( \tau_{\text{in}} = 8.0 \times 10^{-12} \text{s} \), \( \tau_p = 2.0 \times 10^{-12} \text{s} \), \( \alpha = 3 \), \( \tau_s = 2.0 \times 10^{-9} \text{s} \) and \( \kappa = 0.007 \) in the case in Fig. 1, we get \( \frac{\Delta P_{\text{OFLD-POO-Ep-pp}}}{\Delta P_{\text{OFLD-pp}}} = 154 \). This reveals that the envelope is much more sensitive to the displacement compared to using conventional OFLD signal, which motivates the use of the envelop for displacement detection.

Since the frequency of the OFLD-POO signal is also modulated by the displacement, we can detect a micro-displacement within \( \lambda_0 / 2 \) through frequency variation. This enables us to achieve very high sensing resolution for displacement. Regarding this point, our work in [24] has demonstrated that the sensing resolution can be as high as \( \lambda_0 / 1280 \) when the spectrum analyzer has a resolution bandwidth of 62.5 KHz. If we use envelop for large range displacement by fringe-counting with a resolution of \( \lambda_0 / 2 \) and cooperate it with frequency sensing, a new displacement sensor can be designed with large measurement range, high sensitivity and resolution. The restriction on the speed of the target movement (denoted by \( v \)) depends on the value of the RO frequency. Each OFLD-POO fringe should at least cover a few RO periods. Thus we can evaluate that the speed meet: \( v < \lambda_0 f_{\text{RO}} / 2K \), where \( K \) is number of the RO periods within one OFLD-POO fringe. Based on our simulation, to get a clear envelop of OFLD-POO signal, we need to have \( K > 20 \). Theoretically, the dynamic measurement range is the maximum measurable displacement that guarantees OFLD always operating in POO during the displacement measurement. This limits the displacement measurement range. The work in [27] has investigated how POO region change with external target position in a certain injection current, from which we can estimate the dynamic displacement range is around several centimeters.

2.2 Verification for the proposed sensing model

Figure 2 shows the simulation results by numerically solving the original L-K Eqs. (1)-(3) and the approximate analytical expression of Eq. (40) when the external cavity length has a linear displacement of \( 3\lambda_0 \), the initial external cavity length \( L_0 = 24\, \text{cm} \), and the injection current \( I = 1.3 I_a \). Note that \( \omega f = 23\pi \) in this case, which satisfies the requirement for Eq. (40). Figure 2(a) is the displacement; Figs. 2(b) and 2(c) show the numerically simulation results of L-K equations with \( C = 2.8 \) and \( C = 4.5 \); Figs. 2(d) and 2(e) are the simulation results of the Eq. (40) with the same operation conditions as those in Figs. 2(b) and 2(e). For the simulation of Eq. (40), we firstly obtain the relationship between \( \omega f \) and \( \Delta L \) by numerically solving the L-K equations as shown in Fig. 1(a). From Fig. 2, it can be concluded that the simulation results from the L-K equations and the approximate analytical expression match closely, demonstrating that the approximate expression is valid when the OFLD is in period-one oscillation. Note that, in Figs. 2(b) and 2(d), there are regions between the fringes where the OFLD is stable. Therefore, it is essential to ensure that the OFLD system is always in period-one oscillation with the change of external cavity length when using RO frequency.
for displacement sensing, as shown in Figs. 2(c) and 2(e). To guarantee the robustness of the so-called OFLD-POO regime, we need to choose suitable injection current the initial external cavity length. The method of how to choose them is detailed in our previous work [24].

3. Displacement sensing

As discussed in Section 2, the displacement of the external target can be retrieved by using both time-domain waveform (OFLD-POO signal) and the RO frequency contained in the signal. Let us express this displacement to be measured as below:

\[
\Delta L = \Delta L_0 + \Delta L_f.
\]  

(45)

where \(\Delta L_0\) is measured by fringe counting with \(\lambda_0 / 2\) resolution by using the envelop in OFLD-POO signal, and \(\Delta L_f\) is measured by the RO frequency for the fractional displacement within \(\lambda_0 / 2\). As an example, Fig. 3 shows an OFLD-POO signal obtained by Eq. (40) as in Fig. 2(e) and the corresponding displacement information is contained in it.

It can be seen that an OFLD-POO signal contains a few complete fringes and two incomplete fringes (or called fractional fringes) at the beginning and end part of the waveform. The measurement of the fractional fringe enable a displacement measurement with resolution higher than \(\lambda_0 / 2\). As described in Fig. 1, there is a certain linear relationship
between $\omega_1$ and the displacement within each fringe. As in Eq. (38), the maximum and minimum RO frequency within $\lambda_0/2$ are $\omega_{\text{max}}$ and $\omega_{\text{min}}$. We denote the RO angular frequency at the start and end point in a fractional fringe by $\omega_{\text{r}_1}$ and $\omega_{\text{r}_2}$ respectively. Then the displacement corresponding to fractional fringes can be obtained as:

$$\Delta L_f = \frac{\lambda_0}{2} \cdot \frac{\omega_{\text{r}_2} - \omega_{\text{r}_1}}{\omega_{\text{max}} - \omega_{\text{min}}}.$$  \hspace{1cm} (46)$$

where, for the first fractional fringe, $\omega_{\text{r}_2} = \omega_{\text{min}}$, and $\omega_{\text{r}_1}$ is the RO angular frequency at the start point of the displacement, for the last fractional fringe, $\omega_{\text{r}_1} = \omega_{\text{min}}$ and $\omega_{\text{r}_2}$ is the RO angular frequency at the end point of the displacement. Because $\omega_{\text{max}}$ and $\omega_{\text{min}}$ are fixed for a LD with certain operation conditions, the displacement corresponding to fractional fringes can be determined by only measuring the RO frequency of the OFLD when displacement starts and ends. Note that the first and last fringes are always considered as fractional fringes.

A detailed procedure of retrieving the displacement from an OFLD-POO signal is presented as below. Assuming that the external target has a linear movement with $\Delta L=3\lambda_0$ as shown in Fig. 4(a), the corresponding OFLD-POO signal is shown in Fig. 4(b) which can be obtained by Eq. (40). The upper envelope of the OFLD-POO signal can be retrieved by upper-peak detection shown in Fig. 4(c). Then, applying differentiation on Fig. 4(c), we can get pulse train shown in Fig. 4(d). By counting the number of pulses, we can obtain $\Delta L=(N - 1) \cdot \lambda_0 / 2$ (here $N$ is the pulse train number). It can be seen from Fig. 4(d) that there are 6 pulses. Thus $\Delta L = 2.5 \lambda_0$. Then we need to obtain the fractional displacement determined by the first fractional fringe and last fractional fringe denoted by $\Delta L_{f_1}$ and $\Delta L_{f_2}$ as shown in Fig. 4(b). In order to get $\Delta L_{f_1}$ and $\Delta L_{f_2}$, the RO angular frequency at the start and end of the displacement should be obtained firstly. Before the displacement starts and after the displacement ends, for the stationary target, we get the laser intensity and then the RO angular frequency by applying FFT on the intensity as done in Fig. 1(a). Using this method, we get that the RO angular frequencies at these two time points are the same, which are $1.399 \times 10^{10}$ rad/s. According to Fig. 1(a), $\omega_{\text{max}} = 1.436 \times 10^{10}$ rad/s and $\omega_{\text{min}} = 1.394 \times 10^{10}$ rad/s. Hence, based on Eq. (42), for $\Delta L_{f_1}$, $\omega_{\text{r}_2} = 1.436 \times 10^{10}$ rad/s, and $\omega_{\text{r}_1} = 1.399 \times 10^{10}$ rad/s, whereas for $\Delta L_{f_2}$, $\omega_{\text{r}_2} = 1.399 \times 10^{10}$ rad/s, and $\omega_{\text{r}_1} = 1.394 \times 10^{10}$ rad/s. Accordingly, we can get $\Delta L_{f_1} = 0.441 \lambda_0$, and $\Delta L_{f_2} = 0.059 \lambda_0$. Thus, $\Delta L = \Delta L_{f_1} + \Delta L_{f_1} + \Delta L_{f_2} = 3 \lambda_0$, which is the same as the pre-set value.
In the following, an experimental system is built and an application example of displacement sensing by using the proposed method is presented. Figure 5 shows the experimental system. The laser source is a single mode laser diode (Hitachi, HL8325G) with a wavelength of 830 nm, maximum output power of 40 mW, and threshold current of 35 mA. The laser diode is driven and temperature-stabilized by an LD controller (Thorlabs, ITC4001). The temperature of the LD is stabilized at 21 ± 0.01 °C. A mirror is used as the external cavity which is affixed on the surface of a piezoelectric transducer (PZT) (PI, P-841.20) in order to provide sufficient optical feedback to the LD. The PZT actuator with an integrated displacement sensor is used to generate the displacement, and it has a displacement resolution of 9 nm with a traveling range up to 30 μm, which is driven by a PZT controller (PI, E-625). A high-speed digital oscilloscope (Tektronix DSA 70804) with sampling rate 25 GS/s is used to observe OFLD-POO signals. An attenuator is used to adjust the optical feedback level of the OFLD system. As the OFLD-POO signals contain high-frequency components, high-speed detection devices are needed. A beam splitter (BS) with splitting ratio of 50:50 is used to direct a part of the self-mixing light into the external fast PD, with a bandwidth of 9.5GHz, through the fiber port coupler. The detected OFLD/OFLD-POO signals are then collected and recorded by the digital oscilloscope.

To make the system operate in period-one oscillation, the initial external cavity length and injection current should be properly chosen, which is referred to the work in [24]. We choose $I = 50 mA$ and $L_0 = 18 cm$ in this work. Before staring the measurement, we firstly need to
get the relationship between RO frequency ($\omega_f$) and external target displacement ($\Delta L$) when the external cavity length changing by $\lambda_n/2$.

We apply 30 nm displacement on the PZT each time, then use the Tektronix DSA 70804 oscilloscope to record the laser intensity waveform and thus obtain the corresponding peak frequency (denoted by $f_{RO} = \omega_f / 2\pi$) by using the FFT function provided by the oscilloscope [24]. Figure 6 shows an example of the laser intensity waveform and the corresponding spectrum. Thus, in each half wavelength, we will get about 13 points. Figure 7 illustrates the experimental results for the relationship between $f_{RO}$ and displacement $\Delta L$.

![Fig. 6. Experimental results of laser intensity when $I = 50 mA$, $L_o = 18 cm$, and $\Delta L = 300 nm$.](image)

![Fig. 7. RO frequency variation for different displacements when $I = 50 mA$, and $L_o = 18 cm$.](image)

Then we apply a dynamic linear displacement on the PZT by applying a controlling voltage through the PZT driver. Figure 8 (a) is the PZT controlling signal applied on the PZT, Fig. 8(b) is the corresponding OFLD-POO signal. The displacement generated by the PZT (PI, P-841.20) can be read from its internal integrated sensor which indicates that the displacement is $\Delta L = 2304 nm$.

![Fig. 8. (a) PZT controlling signal, (b) corresponding OFLD-POO signal.](image)
The experimental OFLD-POO signal in Fig. 8(b) is processed according to the procedure described in Fig. 4 to get the displacement. 5 complete fringes can be found which corresponds to $\Delta L = 5\lambda_0/2$. In order to obtain the displacement of the first and last fractional fringes, we need to obtain the RO frequency at the start and end points. By using the method in Fig. 6, the RO frequency at these two points are measured as 2.634 GHz and 2.595 GHz respectively. From Fig. 7, it can be seen the variation range of $f_{RO}$ in each period is from 2.579 GHz to 2.645 GHz. Thus, we can get $\Delta L_{f1}=0.083\lambda_0$ and $\Delta L_{f2}=0.121\lambda_0$ based on Eq. (46). So $\Delta L = \Delta L_{f1} + \Delta L_{f2} = 2.704\lambda_0$ (2244 nm for $\lambda_0=830$ nm), which matches the value from the integrated sensor in the PZT. We then repeat the experiments under the same laser operation conditions for different displacements. The results are shown in Table 2, from which, it can be found that the results from these two methods are consistent and the average difference is about 60 nm. The difference between them may come from the accuracy of the PZT integrated sensor, laser wavelength shift induced by external optical feedback, and the accuracy of the measured RO frequencies.

Table 2. Displacement results by PZT sensor and OFLD-POO

<table>
<thead>
<tr>
<th>PZT sensor (nm)</th>
<th>OFLD-POO (nm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>550</td>
<td>602</td>
</tr>
<tr>
<td>1000</td>
<td>950</td>
</tr>
<tr>
<td>2304</td>
<td>2244</td>
</tr>
<tr>
<td>3400</td>
<td>3466</td>
</tr>
<tr>
<td>4506</td>
<td>4438</td>
</tr>
</tbody>
</table>

The resolution of this method is determined by the measured RO frequency and its variation range within $\Delta_0/2$. In this experiment, the frequency resolution of the oscilloscope in the experiments is 62.5 KHz and the variation of $f_{RO}$ in each period is measured as 66 MHz. Hence, in theory, if a perfect linearity can be guaranteed, the proposed experimental system can achieve a resolution of $\lambda_0/2112$ for the displacement measurement, which is 0.39 nm for $\lambda_0 = 830$ nm. Therefore, the proposed method can be used for displacement measurement with high resolution.

4. Conclusion

Precision displacement measurement and positioning have become increasingly important in many areas, e.g. precision manufacturing, semiconductor industry, biotechnology, etc. With the progress of these fields, there is great demand for development of displacement sensing technique with high sensitivity, resolution and relatively simple configuration. In this work, a sensing system with an OFLD operating in POO is implemented for measuring displacement of a target. An analytical expression is presented for describing the sensing signals generated by the OFLD-POO system. It can be seen that both the amplitude and frequency of an OFLD-POO sensing signal are modulated by the displacement to be measured. Hence, we developed a new algorithm for measuring the displacement by simultaneously using the time-domain OFLD-POO signal waveform and RO frequency, which enables us to achieve displacement sensing with large measurement range, high sensitivity and resolution.

References