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An intertemporal optimisation model with imperfect competition is solved to generate a dynamic Phillips curve. The predicted wage inertia and impact effects on employment accord with empirical evidence on the effects of monetary policy. The employment dynamics allow an alternative interpretation of the NAIRU relative to the 'natural rate of unemployment' (NRU).

Keywords

Phillips, curve, NAIRU, reinterpretation

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The Phillips Curve and the NAIRU: A Reinterpretation

INCOMPLETE DRAFT

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ABSTRACT

This paper criticises the use of partial equilibrium analysis in new Keynesian explanations of wage and price stickiness, specifying the Phillips curve as a structural equation (rather than the outcome of the monetary transmission process), and using comparative static *ad hoc* models to characterise the dynamic trade-off.

An intertemporal optimisation model with imperfect competition is solved to generate a dynamic Phillips curve. The predicted wage inertia and impact effects on employment accord with empirical evidence on the effects of monetary policy. The employment dynamics allow an alternative interpretation of the NAIRU relative to the ‘natural rate of unemployment’ (NRU).

Keywords: Phillips curve, monetary policy, wage inflation, NAIRU.

JEL Classifications: E24, E52, E31, E12.

1. Introduction

The analysis of the short run tradeoff between inflation and unemployment, and the associated long run NAIRU, has generated a lot of debate over the decades. The reasons for this interest appear to be twofold. The first is the need for an understanding of the effects of monetary policy on the business cycle movements in wages, prices and real economic activity. Romer (2006, p.264) summarises early empirical studies as indicating "...the real wage in the United States and other countries is approximately acyclical or moderately procyclical". However more recent studies show real wages tend to move procyclically, that is, the money wage rises by relatively more than the prices of goods and services when unemployment is falling. Wickens (2008, pp.205-6) presents stylized facts about these adjustments. He argues they change at varying rates over different stages of the business cycle. For example, wages rise by more than prices late in the expansion phase of the cycle, meaning the real wage increases and profits decrease. His other conclusions include rigidities appear to be temporary so that prices and wages change in unsynchronized ways, but only around two or three times a year.

Romer argues these observations should not be interpreted as moving up along a short run labour supply schedule because the estimated elasticities of labour supply due to cyclical variation in wages are too high (1.0 to 1.4), even for temporary changes. Moreover the short run movements in wages do not appear to be temporary. This evidence implies there are other factors operating in these adjustments and a key candidate is the new Keynesian explanation of 'sticky' wages and prices. The three famous derivations are Taylor's (1979) exogenously predetermined prices with overlapping contracts, Calvo's (1983) disaggregated random price changes and Rotemberg's (1982) endogenously determined optimal price changes.

The second reason for the interest in the Phillips curve is its continuing inability to explain the well understood effects of monetary policy on real output and inflation. Mankiw (2001, p.XX) argues that although "... the new Keynesian Phillips curve has many virtues, it has also one striking vice. It is completely at odds with facts. In particular it cannot come even close to explaining the dynamic effects of monetary policy on inflation and unemployment. This harsh conclusion shows up several places in the recent literature but judging from the continued popularity of this model, I think it's fair to say that its fundamental inconsistency with the facts is not widely appreciated."

Mankiw identifies three interrelated problems arising from the forward looking pricing behaviour in the new Keynesian models. Whilst prices exhibit Calvo style persistence, the forward looking behaviour means the inflation rate will jump to equilibrate the model. This causes the model to typically predict that a credible monetary policy contraction will lead to a pre-emptive reduction in inflation, which increases real balances, output and employment. This contradicts the consensus view that a monetary contraction will increase unemployment. Second, the rapid adjustment of the inflation rate is inconsistent with the data which show a high degree of inflation persistence. Finally, the new Keynesian models tend to force monetary policy shocks to have lagged impulse response effects on unemployment. The inflation and unemployment responses need to be consistent with the observed effects of monetary policy on unemployment with delayed and gradual effects on inflation.

This damning evaluation of the new Keynesian Phillips curve has prompted a large response in applied empirical work to better measure the relationships between inflation and unemployment. The theoretical developments have focussed on modifying the forward looking behaviour of the new Keynesian models, adding backward looking characteristics, focusing on the error correction restrictions and identifying core inflation, to name a few.

A common characteristic of these developments is the modelling of price adjustments by Taylor style overlapping contracts or Calvo-Rotemberg minimisation of quadratic costs. This paper makes four interrelated criticisms of this modelling, with the first disputing the common use of partial equilibrium analysis. Whilst the original sticky price models were encompassing, many subsequent works just use the reduced form solution for inflation. The second criticises the use of comparative statics to characterise dynamic relationships. The empirically driven discrete time analysis over one, two or three lags (and/or leads) is really comparative static, not dynamic, analysis. The third disagrees with the use of *ad hoc* models to explain the possible tradeoff, with many models based on IS, output gaps and Phillips curve equations. The final, related problem is the misunderstanding of the Phillips curve relationship. The short run tradeoff is the outcome of the monetary transmission process and it is therefore inappropriate to specify this as the basis of a model's structure. A surprising number of articles take the Phillips curve relationship as a structural equation, which

incorrectly prescribes causation from inflation to unemployment, and *vice versa*.¹

This confusion is at the centre of the new Keynesian Phillips curve inability to correctly model the persistence of inflation and the intertemporal effects of monetary policy. Mankiw (2001) argues that central bankers and monetary economists agree that contractionary monetary policy will not initially affect inflation. His simulations show there will be no change to inflation for around two quarters and then it will gradually build up with maximum disinflation after five quarters and maximum overall effect after nine quarters. In contrast, unemployment will increase by more and possibly quicker, depending on the model (the new Keynesian model predicts a reduction in unemployment).²

It is now well known that Hume (1752) argued “... it is only in the interval or immediate situation, between the acquisition of money and the rise in prices, that the quantity of gold or silver is favourable to industry. ... The farmer or gardener, finding that their commodities are taken off, apply themselves with alacrity to the raising of more. It is easy to trace the money in its progress through the whole commonwealth; where we will find that it must first quicken the diligence of every individual, before it increases the price of labour.”³ The sequencing of inflation first and then employment is clear, even back then and notice the effect on employment is argued to be widespread, on “every individual”.

Whilst Phillips (1958) is credited with being the first to analyse the (nonlinear) inverse relationship between the rates of wage inflation and unemployment, Fisher (1926) published a statistical analysis of the relationship (arguing the causation ran from inflation to unemployment). Whilst both variables are endogenous, these arguments are consistent with focussing on the impact effect of a monetary policy shock on unemployment. This is in contrast to the rational expectations hypothesis whereby the jump variable prices are assumed to adjust instantaneously, with quantities adjusting relatively slowly over time.

The following section will demonstrate these aspects with a Ramsey style intertemporal optimisation model. The continuous time model will be solved to generate a Phillips curve for the shadow real wage inflation and employment. The main result is the dynamic specification

¹ Indeed, putting inflation on the vertical axis of the Phillips curve diagram may be interpreted by some as implying causation from unemployment to inflation.

² This is the opposite to what the rational expectations hypothesis predicts.

³ Reproduced from Mankiw (2001) .

introduces inertia in the shadow wage adjustment process and increases the impact of monetary policy on employment, which accords with empirical evidence.

2. The Model

Consider a two sector economy of households and firms. Households provide labour services, n to firms in exchange for real wages, w . They also receive real interest income, r from the real capital, k they own, which is used by firms to produce real output, y at price, p (measured in commodity goods). Firms are assumed to maximise profits and employ workers in imperfectly competitive labour markets. We assume zero population growth for the period of interest which may be interpreted as monthly over two (or more) years, giving a time frame of around thirty periods. The firms and households sectors will now be explained.

2.1 Firms

Firms employ labour, n and real capital, k to produce real goods, y paying real wage, w and real rent, r for these factors to households. The production function with variable technological progress, $A(t)$ is given by:⁴

$$y = Af(k, n). \quad (1)$$

Output is used for household real consumption, c or saved as additional capital assets, \dot{k} . The labour market is assumed to be imperfectly competition, whilst the capital market is assumed to be characterised by perfect competitive.⁵

The representative firm's profit will be:

⁴ The marginal products are $f'_n = \partial y / \partial n = \alpha y / n$ and $f'_k = \partial y / \partial k = (1 - \alpha) y / k$. The Inada conditions are satisfied.

⁵ The model can easily be extended to include imperfectly competitive capital markets. However our focus is on the labour market.

$$\pi = Af(k, n) - (\gamma w)n - rk .$$

where $\gamma \geq 1$ represents the degree of monopoly power of workers.⁶ It is the markup by workers and is a function of the elasticity of labour supply, ε_n^s . A higher elasticity represents less market power and lower markup, $\gamma = 1 + \frac{1}{\varepsilon_n^s}$, which in turn decreases the cost of labour.

The profit maximising level of employment is:

$$\frac{\partial \pi}{\partial n} = Af'_n(k, n) - \gamma w = 0$$

so that:

$$\gamma w = Af'_n(k, n) . \quad (2)$$

The profit maximising marginal product of labour is larger than what it would be for perfect competition and so the level of employment, n will be lower. The value, $\gamma w - Af'_n(k, n)$ is the marginal cost of labour.

Now let there be adjustment costs in employing labour and accumulating capital, denoted by $\tilde{f}(k, n)$ where $\tilde{f} < f$ is due to resources used in employing labour and capital. (NOTE that this will be expanded to include Calvo (1983) and Rotemberg (1982) style minimisation of quadratic loss – see comments in Section 2.3).

The firm's net cash flow is:

$$A\tilde{f}(k, n) - \gamma wn - rk . \quad (3)$$

The present value of the firm (which can be considered in terms of its share value), V is given by discounting the dividends paid (in the form of net cash flows) by the real rate of return, r :

$$V = \int_0^\infty e^{-\int_0^t r_s ds} [A\tilde{f}(k, n) - \gamma wn - rk] dt . \quad (4)$$

The Hamiltonian, H is:

⁶ The parameter, γ can be interpreted as an index of human capital. Alternatively it can be a measure of efficiency wage, although this paper will only consider Walrasian markets (albeit imperfectly competitive with adjustment costs).

$$H = e^{-\int_0^\infty r_s ds} \left[A\tilde{f}(k, n) - \gamma wn - rk \right] + \mu \dot{n} + \zeta \dot{k} \quad (5)$$

where μ and ζ are the respective shadow prices of additional employment, \dot{n} and capital, \dot{k} (the current values of employment and capital, measured in units of current output). Focusing initially on employment, the first order conditions for maximisation are, $\dot{\mu} = -\frac{\partial H}{\partial n}$ with transversality condition, $\lim_{t \rightarrow \infty} [\mu(t) \cdot n(t)] = 0$.⁷ Setting the present value of the shadow wage, ω conveniently as, $\mu = \omega e^{-\int_0^\infty r_s ds}$, the solution is:

$$\begin{aligned} \frac{\partial \omega e^{-\int_0^\infty r_s ds}}{\partial t} &= -A\tilde{f}'_n(k, n) e^{-\int_0^\infty r_s ds} + \gamma \omega e^{-\int_0^\infty r_s ds} \\ \therefore (\dot{\omega} - r\omega) e^{-\int_0^\infty r_s ds} &= -A\tilde{f}'_n(k, n) e^{-\int_0^\infty r_s ds} + \gamma \omega e^{-\int_0^\infty r_s ds} \end{aligned}$$

which solves to:

$$\dot{\omega} = r\omega - A\tilde{f}'_n(k, n) + \gamma \omega. \quad (6)$$

Note the difference between the shadow real wage, ω and the real market wage, w . This is the important result of this paper where the right side of (6) represents the marginal cost of employment, $r\omega + \gamma \omega - A\tilde{f}'_n(k, n)$. Importantly, there is a positive relationship between shadow wage inflation, $\dot{\omega}$ and employment, n . Assuming a fixed labour force, l then $n = l - l\hat{u}$ for the rate of unemployment, \hat{u} . Substituting for n in (6) gives:

$$\dot{\omega} = r\omega - A\tilde{f}'_n[k, (l - l\hat{u})] + \gamma \omega \quad (7)$$

where $\partial \tilde{f}'_n[k, (l - l\hat{u})] / \partial \hat{u} > 0$. So (7) is a Phillips curve with inverse relationship between $\dot{\omega}$ and \hat{u} as shown in Figure 1.

⁷ This is the standard requirement that the value of the shadow price, $\mu(t)$ must go to zero as $n(t)$ grows infinitely large.

Insert Figure 1

Since this is the growth in the real wage, then $\dot{\omega} = \dot{W} - \pi$ with \dot{W} the growth in market unit labour costs and π the inflation rate. This is the form of the accelerationist specification used in empirical works including Gruen, Pagan and Thompson (2000). Note also that the shadow real wage, ω can adjust as a jump variable and differ from the real market wage, w . However after a jump in ω , equations (6) and (7) demonstrate the shadow wage inflation will exhibit inertia for $0 < r < 1$. The fact that the real shadow wage can jump will cause employment to change, whilst the real wage will adjust relatively slowly. This is empirically required from the discussion in the introduction in Section 1.

The solution for the variable coefficient first order differential equation gives the shadow wage as the sum of the net present values of the future marginal products of unemployment:

$$\omega = \int_0^T A\tilde{f}'_n \left[k, (l - l\hat{u}) - \gamma w \right] e^{-\int_0^t (r_s) ds} dt. \quad (8)$$

In steady state there will be a constant real wage inflation, $\dot{\omega} = \bar{\omega}$. Solving (7) gives the steady state shadow wage:

$$\omega^* = \bar{\omega} + \frac{A\tilde{f}'_n \left[k^*, (l - l\hat{u}^*) \right] - \gamma w^*}{r^*} \quad (9)$$

with the steady state values indicated by an asterisk. This is another important result because \hat{u}^* describes the NAIRU (since $\bar{\omega}$ is non-accelerating). The NAIRU is also not identified because there is an infinite number of potential combinations of capital, k^* , the real rate of interest, r^* and the real market wage, w^* . Moreover the consequences of the feasible set of values for the steady state wage also depend on the different possible rates of (constant) wage inflation, $\bar{\omega}$. It therefore becomes necessary to pin down the values of k^* , r^* , w^* and $\bar{\omega}$. The first two will now be considered (and the latter two will be determined in the following subsection).

The first order condition for maximising the Hamiltonian defined previously for capital

accumulation, $H = e^{-\int_0^\infty r_s ds} [A\tilde{f}(k, n) - \gamma wn - rk] + \mu\dot{n} + \zeta\dot{k}$ is given by the Euler equation,

$\dot{\zeta} = -\frac{\partial H}{\partial k}$.⁸ The present value of the shadow price, $\zeta = qe^{-\int_0^\infty r_s ds}$ steady state solution is:

$$q^* = \frac{\tilde{f}'_k(k^*, n^*)}{r^*} \quad (10)$$

where the shadow price is Tobin's q , defined as the marginal market value of capital relative to its replacement cost. When $q > 1$, there is positive investment, ($\dot{k} > 0$), when $q < 1$ there is disinvestment ($\dot{k} < 0$) and when $q = 1$, then $\dot{k} = 0$. The equation of motion is, $\dot{q} = rq - A\tilde{f}'_k(k, n)$ such that the saddlepath solution, SS, is shown in the phase plane diagram.⁹

This determines the steady state values of k^* and r^* required for the wage relationship, (9). Note that for $\tilde{f} < f$ implying $\tilde{f}'_k < f'_k$ then k^* will be lower than the steady state value of capital in the absence of transactions costs. Relationship (10) also allows the analysis of a reduction in the real rate of interest as shown in Figure 2. The subsequent increase in Tobin's q and the ensuing saddlepath dynamics will increase capital to the new steady state value, k^{**} .

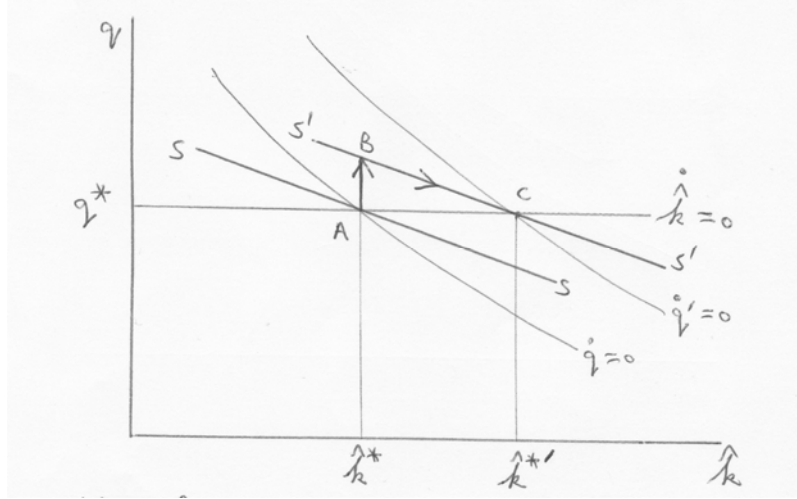
Given the specific steady state values of k^* and r^* , it only remains to include real money balances, and therefore inflation, into the model.¹⁰ This will allow us to analyse the effects of a monetary expansion on the real and nominal interest rate.

⁸ The transversality condition is $\lim_{t \rightarrow \infty} [\lambda(t) \cdot k(t)] = 0$.

⁹ The $\dot{q} = 0$ schedule has a negative slope since $\partial q / \partial k = \tilde{f}''_k(k, l - l\hat{u}) / r < 0$, and is globally unstable because $\partial \dot{q} / \partial q = r > 0$. When $\dot{k} = 0$, the $q^* = 1$ schedule is horizontal. Since $\partial \dot{k} / \partial k = q - 1$, then $q < 1$, means $\partial \dot{k} / \partial k < 0$ and $q > 1$ means $\partial \dot{k} / \partial k > 0$. The directions of arrows indicate the globally unstable, locally stable SS saddlepath.

¹⁰ The paper wishes to focus on wage inflation rather than price inflation, π .

Figure 2



2.1 Households

Households provide labour services, n to firms in exchange for a real wage, w . They also receive real interest income, r from the real capital, k they own, which is used by firms to produce real output, y . Households purchase real consumption goods, c and save by accumulating capital assets, \dot{k} and real money balances, $m = M/P$. Again we assume zero population growth and the relevant time frame of interest is around thirty monthly periods.

Each householder maximises total individual utility, U for the constant rate of time preference, $\rho > 0$:

$$U = \int_0^{\infty} u[c(t), m(t)] e^{-\rho t} dt \quad (11)$$

which is the sum of the present value of future utilities derived from consumption, $c(t)$ and real money holdings, $m(t)$. Dropping the time subscript wherever possible (in order to keep the notation as simple), assume the standard utility concavity conditions, $u'_c(c, m) > 0$ and $u''_{cc}(c, m) < 0$ plus $u'_m(c, m) > 0$ and $u''_{mm}(c, m) < 0$.¹¹ These standard conditions mean that householders are risk averse and therefore prefer to smooth consumption and money balances over time. Assume the instantaneous felicity function satisfies the conditions:

¹¹ The definitions apply, $u'_x = \partial u / \partial x$ and $u''_{xx} = \partial^2 u / \partial x^2$. We assume there is no covariance in utilities gained from consumption and money, such that $u''_{cm} = \partial^2 u / \partial c \partial m = 0$.

$$\lim_{c, m \rightarrow 0} u'(c, m) \rightarrow \infty \quad \text{and} \quad \lim_{c, m \rightarrow \infty} u'(c, m) \rightarrow 0.$$

The householder's budget constraint is defined as:

$$\begin{aligned} \dot{k} &= \gamma wn + rk - c - \frac{\partial M / \partial t}{P} \\ &= \gamma wn + rk - c - \dot{m} - \pi m \end{aligned} \tag{12}$$

since $\frac{\partial M / \partial t}{P} = \frac{\partial m}{\partial t} + \frac{\partial P}{\partial t} m$. Constraint (12) shows that householder's income, $\omega n + rk$ is used to fund consumption and increase real money balances (which can be considered as the consumption of services obtained from the money balances). Define household wealth as, $a = k + m$, differentiate with respect to time and substitute in (12):¹²

$$\dot{a} = \gamma wn + ra - c - (r + \pi)m. \tag{13}$$

The term, $\dot{m} + \pi m$ represents the opportunity cost of interest foregone by the householder holding money balances instead of saving in the form of capital accumulation. The cost is the nominal interest rate, $r + \pi$ times real money balances held by the householder.

Each householder chooses the time path of consumption which maximises the integral sum of future discounted instantaneous felicity function (11) subject to the budget constraint (13) for positive initial real wealth, $a(0) > 0$. This is another dynamic optimisation problem involving, \dot{a} and the Hamiltonian:

$$J = u(c, m)e^{-\rho t} + \lambda [\gamma wn + rk - c - (r + \pi)m]$$

in present value terms. The first term on the right hand side is the current flow of the present value of the householder's utility from consumption of goods and money services. The second

¹² The standard no-Ponzi condition, $\lim_{t \rightarrow \infty} a(t)e^{-\int_0^t r_s ds} \geq 0$ is applied to restrict the present value of householder's real capital assets to be asymptotically non-negative.

term, $\lambda \dot{a}$ represents the value of wealth accumulation, where λ is the present value shadow price which values (in utility units) marginal income associated with asset accumulation at time, t in terms of utility at time zero. The first order condition for a maximum is:

$$\begin{aligned}\frac{\partial J}{\partial c} &= \frac{\partial u(c, m)}{\partial c} e^{-\rho t} - \lambda = 0 \\ \therefore \lambda &= u'_c(c, m) e^{-\rho t}\end{aligned}\tag{14}$$

for fixed parameter ρ . The shadow price of capital asset accumulation at each instant in time must be equal to the present value of the marginal utility of consumption. The household maximises when the marginal gain from the extra consumption equals the marginal loss from foregoing some \dot{a} (measured by its shadow price).

The Euler equation is:

$$\dot{\lambda} = -\frac{\partial J}{\partial k} = -r\lambda\tag{15}$$

and transforming gives, $\frac{\dot{\lambda}}{\lambda} = -r$. This shows the real return, r from holding the capital asset, k is the negative of the shadow price of capital gain per unit of asset, $\dot{\lambda}/\lambda$. Equations (13) and (15) are first order differential equations in a and its shadow price, λ . However, there is only one initial condition, $a(0) = a_0 > 0$ for the two equations. This means that there are a family of (one parameter) solutions. We need another condition to obtain a single unique solution. This is the transversality condition:

$$\lim_{t \rightarrow \infty} [\lambda(t) \cdot k(t)] = 0\tag{16}$$

which states that as $a(t)$ grows infinitely large, its value (in terms of its shadow price) must go to zero. Solving for the utility maximising intertemporal consumption, take Napierian logs of (14):

$$\begin{aligned}\lambda &= u'_c(c, m) e^{-\rho t} \\ \therefore \ln \lambda &= -\rho t + \ln u'_c(c, m)\end{aligned}$$

Differentiating with respect to time t , $\frac{\partial \ln \lambda}{\partial t} = \frac{\dot{\lambda}}{\lambda} = -\rho + \frac{\partial u'_c / \partial t}{u'_c}$ and eliminating $\dot{\lambda} / \lambda$ using (15) gives:

$$\therefore r = \rho - \frac{\partial u'_c / \partial t}{u'_c}. \quad (17)$$

This shows that households select consumption which equates the rate of return to saving, r to the rate of time preference, ρ , net of the rate of decrease in the marginal utility, u'_c due to the increase in consumption, c . The right hand side of (17) can be rearranged,

$$\frac{\partial u'_c / \partial t}{u'_c} = \frac{\frac{1}{\partial t} \left(\frac{\partial u'_c}{\partial c} \right) \partial c}{u'_c} = \frac{\left(\frac{\partial u'_c}{\partial c} \right)}{u'_c} \times \frac{c}{c} \times \frac{\partial c}{\partial t} = \frac{u''_c c}{u'_c c} \dot{c}, \text{ which gives:}$$

$$r = \rho + \left(\frac{-u''_c c}{u'_c} \right) \frac{\dot{c}}{c}. \quad (18)$$

If $\dot{c} > 0$ then current consumption is low relative to future consumption. Agents wishing to smooth consumption will try to bring some future consumption forward to the present. The term, $\frac{-u''_c c}{u'_c}$ is the elasticity of marginal utility, u' with respect to c (ie. the curvature-concavity of the utility function, $u(c)$). This measure of elasticity multiplied by the growth in consumption, \dot{c}/c gives the compensation households require at the margin in terms of the rate of return, r exceeding the discount rate, ρ . The larger the elasticity (or the larger is \dot{c}/c) then the larger r must be in relation to ρ . Rearranging (18) gives the well known result:

$$\frac{\dot{c}}{c} = \phi(r - \rho) \quad (19)$$

where ϕ is the elasticity of intertemporal substitution.¹³ An increase in the return to asset accumulation, r (ie. saving) means that households will willingly forego more current

¹³ The required proportionate change in u'_c relative to the proportionate change in c , is given by

$$\varepsilon = -\frac{\partial u'_c / u'_c}{\partial c / c} = -\frac{\partial u'_c}{\partial c} \cdot \frac{c}{u'_c} = -\frac{u''_c c}{u'_c}.$$

consumption for future consumption, thus increasing \dot{c} . This is also true for a reduction in the discount rate, ρ .

In order to determine the path of consumption, integrate the evolution of the shadow price in (15):

$$\lambda(t) = \lambda(0) e^{-\int_0^t r_s ds}.$$

(Note the reverse process which determines the initial shadow price, $\lambda(0) = \lambda(t) e^{\int_0^t r_s ds}$).

Substituting for $\lambda(t)$ in the transversality condition (16):

$$\lim_{t \rightarrow \infty} \left[a(t) e^{-\int_0^t r_s ds} \right] = 0. \quad (20)$$

Rearranging the budget constraint (13):

$$\dot{a} = ra + [\gamma wn - c - (r + \pi)m]$$

shows that this is a first order differential equation in \dot{a} with a variable coefficient, r_s .

Solving:

$$a(t) = -e^{-\int_0^t r_s ds} \left[\int_0^T \gamma w(t) e^{\int_t^T r_s ds} dt - \int_0^T c(t) e^{\int_t^T r_s ds} dt - \int_0^T [r(t) + \pi(t)] m(t) e^{\int_t^T r_s ds} dt - b \right]$$

where $e^{-\int_0^t r_s ds}$ is the integrating factor and b is the constant of integration (which we will ignore). This integrating factor effectively discounts to time zero:

$$a_0 = -\int_0^T \gamma w(t) e^{-\int_0^t r_s ds} dt + \int_0^T c(t) e^{-\int_0^t r_s ds} dt + \int_0^T [r(t) + \pi(t)] m(t) e^{-\int_0^t r_s ds} dt$$

such that:
$$\int_0^T c(t) e^{-\int_0^t r_s ds} dt = [a_0 - (r_0 + \pi_0)m_0] + h_0. \quad (21)$$

It is clear that $a_0 - (r_0 + \pi_0)m_0 = k_0$ represents non-human real wealth and

$h_0 = \int_0^T \gamma w(t) e^{-\int_0^t r_s ds} dt$ represents the sum of human wealth, measured as the net present value of labour income, w .

Integrating (19) from time zero to t gives:

$$c(t) = c(0) e^{\phi \int_0^t r_s ds}. \quad (22)$$

Substituting for $c(t)$ in (21) derives, $c(0) \int_0^T e^{(\phi-1) \int_0^t r_s ds} dt = k_0 + h_0$, which can be simplified to current consumption:

$$c(0) = \beta (k_0 + h_0) \quad (23)$$

with $\beta = \left[\int_0^T e^{(\phi-1) \int_0^t r_s ds} dt \right]^{-1}$ representing the marginal propensity to consume out of total wealth. A decrease in the expected path of interest rates (given wealth) has two effects on β . The first is, a lower interest rate decreases the cost of current consumption relative to future consumption. This intertemporal substitution effect causes households to bring future consumption forward to the present. The second, is a wealth effect which will decrease consumption. Given the relatively short planning time frame of this paper, we would expect the positive effect to dominate.

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We can also solve the Hamiltonian for m to give, $u'_m/u'_c = r + \pi$. However from (22) and (23), money is ‘super-neutral’ in steady state in this model and money has no more interest other than to provide π to link the nominal and real interest rates, $r + \pi$.

We can also add the disutility of employment into the utility function, $u(c, m, n)$ with $u'_n < 0$, such that householders balance the disutility from extra employment with the additional consumption of goods and services from real money balances. The Hamiltonian maximisation will derive the \dot{n}/n function which provides the labour supply details. The solution is:

$$r = \rho - \left(\frac{u''_{cc} c}{u'_c} \right) \frac{\dot{c}}{c} - \left(\frac{u''_{cn} n}{u'_c} \right) \frac{\dot{n}}{n}$$

where $\varphi = \varphi = -u'_c / u''_{cn} n$ is the cross elasticity of intertemporal consumption and employment. We can assume they are complements such that, $u'_{cn} < 0$. A reduction in current employment, which increases the growth in employment \dot{n}/n , adds to the rate of time preference as households will have to work harder in the future.

This closes the model and we are able to consider the dynamic adjustments of the real wage and employment to an expansionary monetary policy (which increases real money balances and reduces the nominal interest rate).

2.3 Monetary Policy

Explain how expansionary monetary policy reduces the nominal and real interest rates, which immediately increase Tobin's q via equation (10), increase investment, \dot{k} and capital. Consumption will also increase via the discussion on equation (23). The increase in aggregate demand, shown by the householder's budget constraint (12), $c + \dot{k} = \gamma wn + rk - (\dot{m} + \pi m)$ will increase incomes and output, $\gamma wn + rk = y$. The shadow wage inflation, \dot{w} will increase via (7), which will increase employment, n . However note that the real market wage, w will not rise by much. This delay in the adjustment of the real wage has been specified in terms of the costs of adjustment and the divergence of it from the shadow price of labour (which is treated as a jump variable). These higher shadow price returns to the firm is what drives the increase in employment. We have previously considered this in general terms $\tilde{f}(k, n)$ where $\tilde{f} < f$ is due to resources used in employing labour and capital. However it is better to include this via a Calvo (1983) and Rotemberg (1982) minimisation of quadratic costs of adjustment, $\alpha \dot{w}^2$ in the Hamiltonian (5):

$$H = e^{-\int_0^\infty r_s ds} \left[A \tilde{f}(k, n) - \alpha \dot{w}^2 + \gamma wn - rk \right] + \mu \dot{n} + \zeta \dot{k}. \quad (5)$$

The distinction between the shadow and real wage adjustments drive the model.

This analysis overcomes Mankiw's (2001) three criticisms arising from the forward looking pricing behaviour in the new Keynesian models. The first is the discussion above which clearly describes a short run Phillip curve tradeoff. Second, the adjustment of the real wage is consistent with the data which show a high degree of inflation persistence. Finally, unlike the new Keynesian models, this model predicts that monetary policy shocks will have relatively fast impacts on unemployment. Overall the model explains inflation and employment responses which are more consistent with the observed effects of monetary policy on inflation and unemployment.

This needs to be expanded.

3. The NAIRU

In addition to this, the model also shows there are more than one possible NAIRU's. Equation (9) shows that for a constant real wage inflation, $\dot{\omega} = \bar{\omega}$:

$$\omega = \bar{\omega} + \frac{A\tilde{f}'_n[k, (1 - l\hat{u})] - \gamma w}{r}. \quad (9)$$

This is an important result because \hat{u} describes the NAIRU (since $\bar{\omega}$ is non-accelerating). The NAIRU is also not identified because there is possibly more than one combination of k , r and w . Whilst k^* will be determined when $q = 1$, it is unclear whether w reaches steady state with the costs of adjustment in the time frame of interest. Moreover the consequences of the feasible set of values for the steady state shadow wage also depend on the different possible rates of (constant) wage inflation, $\bar{\omega}$.

This possibility of a non-unique NAIRU drives a wedge between it and the natural rate of unemployment (NRU).¹⁴ Friedman's (1968, p.8) defined the natural rate of unemployment as:

¹⁴ Debelle and Vickery (1998) and Clark and Laxton (1997) use the concept of a nonlinear Phillips curve to show the NAIRU will be less than the natural rate of unemployment.

“... the level that would be ground out by the Walrasian system of general equilibrium equations, provided there is embedded in them the actual structural characteristics of the labour and commodity markets, including market imperfections, stochastic variability in demands and supplies, the cost of gathering information about job vacancies and labor availabilities, the costs of mobility, and so on.”

The name was borrowed from Wicksell’s ‘natural rate of interest’ and it has generated considerable debate, which continues today. Hahn and Solow (1997, pp.137-8) claim the term:

“... is surely one of the most successful examples of persuasive definition in economics. The phrase begs for the uncritical acceptance of the notion that its achievement should be the goal of policy. (Would anyone think of aiming for an unnatural level of employment?)”

Friedman’s structural characteristics of the labour market are captured by the imperfect competition parameter, γ and the costs of adjustment of employment (and capital) in the production function, $\tilde{f}(k, n) < f(k, n)$. However, the model presented here will only settle at the natural rate of unemployment, when the shadow wage is zero.

The other important aspect of the NAIRU is the consideration of shifts in it due to hysteresis effects. Inspection of equation (9):

$$\omega = \bar{\omega} + \frac{A\tilde{f}'_n[k, (l - l\hat{u})] - \gamma w}{r}.$$

shows that hysteresis can be interpreted as changes in l . The approach here has more theoretical underpinning than including hysteresis effects via the random walk specification by many empirical researchers:¹⁵

¹⁵ The term hysteresis appears to have different interpretations that are used by economists. It comes from the physical sciences which describes the delay a ‘plastic’ medium will take in forming a new shape, for example the movement of a sticky substance. Ball and Mankiw (2002, p.119) interpret this as a “failure of an object to return to its original value after being changed by an external force, even after the force is removed.”

$$u_t^* = u_{t-1}^* + v_t \sim N(0, \sigma_v^2).$$

Indeed using a random walk specification is complex in relation to interpreting its time series properties. It is well known a random walk is stationary in mean (in that with repeated substitutions from time zero to time, t , $E(u_t^*) = u_0^*$ which is constant) but non-stationary in variance. This can be seen by:

$$\begin{aligned} \text{Var}(u_1^*) &= \text{Var}(u_0^*) + \text{Var}(v_1) \\ &= \sigma_v^2 \\ \text{Var}(u_2^*) &= \text{Var}(u_1^*) + \text{Var}(v_2) \\ &= 2\sigma_v^2 \\ &\vdots \\ \text{Var}(u_t^*) &= t\sigma_v^2 \end{aligned}$$

which is called stochastic non-stationarity. However, since the unemployment rate is bounded, $0 \leq u_t^* \leq 1$, then the probability space will sum to unity. Over time, the distribution will become uniform in this range so that all possible realisations in the interval will become equally likely. Since the variance is bounded, u_t^* will be I(0) stationary. This important point does not seem to be realised in some of the empirical literature.

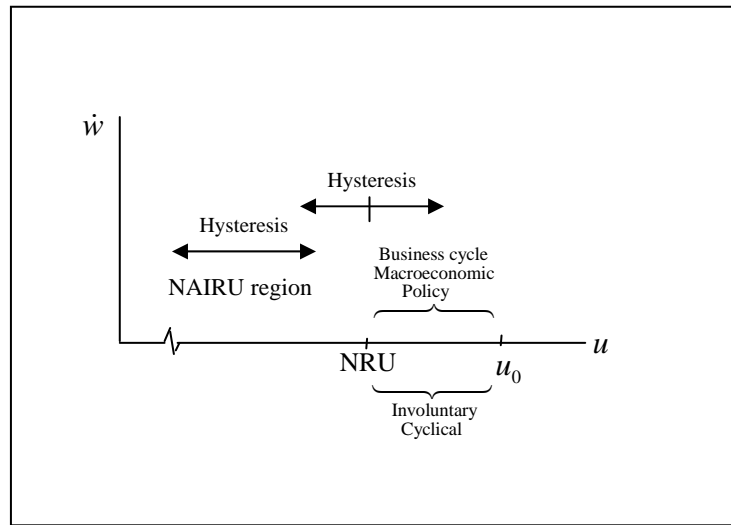
Whilst we are considering some time series properties of estimation, note that equation (7):

$$\dot{\omega} = r\omega - A\tilde{f}_n' [k, (l - l\hat{u})] + \gamma w.$$

describes an error correction process for the shadow real wage.

Finally, a schematic representation of some of these concepts is included in Figure 3 to add clarity.

Figure 3



4. Conclusion

This paper makes four criticisms of the popular approaches to modelling the Phillips curve and the NAIRU. The first disputes the use of partial equilibrium analysis in new Keynesian explanations of wage and price stickiness. The second argues the Phillips curve is the outcome of the monetary transmission process and it is therefore inappropriate to specify this trade-off as the basis of the model's structure. The third criticises the use of comparative statics to characterise a dynamic relationship, whilst the fourth disagrees with using *ad hoc* models to explain the possible trade-off.

A Walrasian two sector model, based on intertemporal microeconomic optimisation, is solved to generate a Phillips curve for real wage inflation and employment. (This contrasts with the new Keynesian models which are criticised for showing that anti-inflationary monetary policy increases employment.) The dynamic specification introduces inertia in the wage adjustment process and increases the impact effects on employment. These predictions accord with the consensus view and with empirical evidence on the effects of monetary policy. Furthermore, the employment dynamics allow another interpretation of the NAIRU relative to the 'natural rate of unemployment' (NRU).

Change from Abstract and add more.

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