1-1-2013

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Abstract

The upper bound method of plasticity limit analysis is used to estimate the ultimate capacity of laterally loaded long piles. Based on a generic limiting force (or the ultimate soil resistance per unit length) profile, both freehead and fixed-head piles are considered. The current solutions can be reduced to available solutions for some special cases. Design charts may be constructed to estimate the lateral capacity of piles in terms of plastic moment and the parameters describing the limiting force profile.

Keywords

lateral, analysis, limit, bound, pile, upper, capacity

Disciplines

Engineering | Science and Technology Studies

Publication Details

Qin, H. Y., Oh, E. Y. N., Guo, W. D. & Dai, P. F. (2013). Upper bound limit analysis of lateral pile capacity. International Conference on State of the Art Pile Foundation and Pile Cast Histories (PILE 2013)

UPPER BOUND LIMIT ANALYSIS OF LATERAL PILE CAPACITY

H. Y. Qin[1], E. Y. N. Oh[2], W. D. Guo[3] and P.F. Dai[4]

ABSTRACT: The upper bound method of plasticity limit analysis is used to estimate the ultimate capacity of laterally loaded long piles. Based on a generic limiting force (or the ultimate soil resistance per unit length) profile, both free-head and fixed-head piles are considered. The current solutions can be reduced to available solutions for some special cases. Design charts may be constructed to estimate the lateral capacity of piles in terms of plastic moment and the parameters describing the limiting force profile.

Keywords: Lateral capacity, piles, plasticity, theoretical analysis.

INTRODUCTION

Estimation of the ultimate lateral capacity is an important component in the analysis and design of pile foundations subjected to lateral loadings and soil movements. Broms (1964a, b) identified that the failure mode of a laterally loaded pile relies on the pile length, stiffness of the pile section, pile head fixity conditions, and load-deformation characteristics of the soil. The ultimate lateral resistance per unit length that soils can exert against the pile is a key quantity in the calculation of ultimate lateral capacity. Different approaches have been adopted to determine the ultimate soil resistance for both undrained and drained soils, ranging from experimental measurements (Matlock, 1970, Reese et al. 1974, Barton, 1982, Guo and Qin, 2010), simplified wedge-type failure analysis (Reese, 1958, Ashour and Norries, 2000) to rigorous plasticity limit analysis (Randolph and Houlsby, 1984, Murff and Hamilton, 1993, Martin and Randolph, 2006). With a given ultimate soil resistance variation with depth or limiting force profile, the ultimate lateral capacity of a pile can be calculated from limit equilibrium analysis of the lateral force and moments by treating the soil as rigid plastic material and considering the failure mechanism. The analysis procedures have been described by Brinch Hansen (1961), Broms (1964a, 1964b), Poulos and Davis (1980), Viggiani (1981), Fleming et al. (2009).

The upper bound method of plasticity limit analysis was elaborated by Drucker and Prager (1952) and Chen (1975). It has been used to tackle a variety of geotechnical problems, ranging from estimating the bearing capacity of shallow foundations, assessing the limiting resistance of penetrometer, to evaluating the stability of slopes and tunnels. The upper bound theorem states that if in any admissible failure mechanism, the rate of work done by external forces exceeds the internal rate of energy dissipation, equating the rate of external work to internal rate of energy dissipation for any such mechanism gives an upper bound to the true limit load (Chen, 1975). To apply the approach, a valid mechanism of collapse must be assumed. The failure mechanism comprising a complete velocity field must satisfy the kinematic boundary conditions. The rate of internal energy dissipation associated with the failure mechanism and the rate of work due to externally applied load are calculated. The principle of virtual work is then applied to determine the least upper bound collapse load by optimizing the failure mechanism. Among the previous studies, Murff and Hamilton (1993) developed an upper bound method to estimate the collapse load of a laterally loaded pile under undrained conditions. They suggested a three-dimensional collapse mechanism that comprises a conical wedge near the surface with flow horizontally around the pile below the wedge.

In this paper, the upper bound plasticity method is used to evaluate the ultimate lateral capacity of a long pile. Following the methodology of Murff and Hamilton (1993), a wedge failure mechanism in conjuction with a generic limiting force profile proposed by Guo (2006) is postulated in the determination of the ultimate lateral capacity of a free-head or fixed-head long pile.

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UPPER BOUND MECHANISM FORMULATION

Failure Mechanism for a laterally loaded long pile

Figure 1 schematically depicts the failure mechanism assumed for a long pile. The pile is subjected to a lateral load, T_t at pile head. The embedded pile length is sufficiently longer than its critical length so that the whole pile can not rotate as a rigid body with 'kicking out' of the pile tip. For an unrestrained or free-head pile, it is assumed that the pile fails by development of a plastic hinge at some depth down the pile. Assuming a uniform pile cross section, a plastic hinge with a moment of M_p will develop at the depth of maximum bending moment that has no shear force, i.e. at point A in Fig. 1. If the pile head is perfectly fixed from rotation, another plastic hinge occurs at the pile cap level at point B as shown in Fig. (1b). It is further assumed that the reduced pile segment rotates about the plastic hinge, A under the lateral loading and has a virtual lateral velocity, v_0 at the pile head. The lateral velocity, v at any depth along the pile is assumed decreasing linearly from $\boldsymbol{v_0}$ to 0 at point A and can be expressed as

$$v = v_0 (1 - \frac{z}{l}) \tag{1}$$

where z is the depth measured from pile head, l is the depth where plastic hinge forms. This mechanism was originally proposed by Murff and Hamilton (1993).

Generic limiting force profile

It is assumed that the lateral soil resistance is fully developed at the ultimate state. The ultimate soil resistance is described by the generic limiting force profile (LFP) proposed by Guo (2006)

$$p_u = A_r (z + \alpha_0)^n \tag{2}$$

where p_u = ultimate soil resistance or limiting force per unit length; A_r = $s_u N_g d^{1-n}$ (cohesive soil) and $\gamma_s' N_g d^{2-n}$ (cohesionless soil), gradient of the limiting force profile; d = the outer diameter of the pile; α_0 = an equivalent depth to consider the resistance at the ground surface, and n (<3) = the power governing the shape of the limiting force profile shown in Figure 2, the values of n = 0.7 and 1.7 are generally sufficient accurate for piles in clay and sand; z = depth below the ground level; s_u = average undrained shear strength of cohesive soil; γ_s' = effective unit weight of overburden soil (i.e. dry weight above water table and buoyant weight below; N_g = gradient to correlate clay strength or sand weight with the ultimate resistance p_u. The magnitude of the three input parameters α_0, N_g, and n are independent of load levels over the entire loading regime. Guidelines for determining the values of the parameters are discussed by Guo (2006, 2012, 2013).

The generic limiting force profile (LFP) becomes that suggested for sand by Broms (1964b), and Barton (1982), and that for clay by Matlock (1970), and Reese *et al*. (1975), by choosing an appropriate set of N_g, α_0 and n. For example, selecting $N_g = 3K_p$, $\alpha_0 = 0$, n=1, K_p = the coefficient of passive earth pressure, the limiting force profile becomes the Broms' (1964b) LFP for sand, while giving $\alpha_0 = 2d/N_g$, $N_g = \gamma_s' d/s_u + 0.5$, and n = 1, it reduces to Matlock's (1970) LFP for soft clay.

Upper Bound Limit Analysis

The energy dissipation rate along a unit pile length due to soil resistance is

$$d\dot{E}_1 = v p_u dz = A_r v_0 (1 - \frac{z}{l})(z + \alpha_0)^n dz \tag{3}$$

The total energy dissipation above the center of rotation, point A, of the mechanism is then

$$\dot{E}_1 = \int_0^l d\dot{E}_1 dz = v_0 A_r \left[\frac{(l+\alpha_0)^{n+1} - \alpha_0^{n+1}}{n+1} - \frac{1}{l} \left(\frac{(l+\alpha_0)^{n+2} - \alpha_0^{n+2}}{n+2} - \frac{\alpha_0 (l+\alpha_0)^{n+1} - \alpha_0^{n+2}}{n+1} \right) \right] \tag{4}$$

The energy dissipation due to the plastic moment, M_p at point A and B is

$$\dot{E}_2 = M_p \dot{\theta} = M_p \frac{v_0}{l} \quad \text{for free-head} \tag{5}$$

$$\dot{E}_2 = 2M_p \dot{\theta} = 2M_p \frac{v_0}{l} \quad \text{for fixed-head} \tag{6}$$

The external rate of work arising from the pile head load is the product of the load T_t and the pile velocity v_0

$$\dot{W} = T_t v_0 \tag{7}$$

Finally, the external rate of work is set equal to the total internal energy dissipation rate

$$\dot{W} = \dot{E}_1 + \dot{E}_2 \tag{8}$$

and an estimate of the lateral load T_t can be solved for. It will be noted that the virtual velocity v_0 will be cancelled. The best solution, i.e. the largest load, is found by minimizing the load T_t with respect to the optimization parameter l. It is presented below for the free-head and fixed-head piles, respectively.

ULTIMATE LATERAL CAPACITY

Free-head piles

Substituting equations (4), (5) and (7) into equation (8) leads to

$$T_t v_0 = M_p \frac{v_0}{l} + v_0 A_r \left[\begin{array}{l} \dfrac{(l+\alpha_0)^{n+1} - \alpha_0^{n+1}}{n+1} \\ -\dfrac{1}{l}\left(\dfrac{(l+\alpha_0)^{n+2} - \alpha_0^{n+2}}{n+2} - \dfrac{\alpha_0 (l+\alpha_0)^{n+1} - \alpha_0^{n+2}}{n+1} \right) \end{array} \right] \tag{9}$$

Cancelling the virtual velocity v_0 gives the solution

$$T_t = \frac{M_p}{l} + A_r \left[\begin{array}{l} \dfrac{(l+\alpha_0)^{n+1} - \alpha_0^{n+1}}{n+1} \\ -\dfrac{1}{l}\left(\dfrac{(l+\alpha_0)^{n+2} - \alpha_0^{n+2}}{n+2} - \dfrac{\alpha_0 (l+\alpha_0)^{n+1} - \alpha_0^{n+2}}{n+1} \right) \end{array} \right] \tag{10}$$

The least upper bound solution or ultimate lateral load is determined by minimizing the solution with respect to the depth l. From

$$\frac{\partial T_t}{\partial l} = 0$$

Thus

$$\frac{\partial T_t}{\partial l} = -\frac{M_p}{l^2} + A_r \left[(l+\alpha_0)^n - \frac{1}{l}\left((l+\alpha_0)^{n+1} - \alpha_0 (l+\alpha_0)^n \right) \right]$$

$$+ \frac{A_r}{l^2}\left(\frac{(l+\alpha_0)^{n+2} - \alpha_0^{n+2}}{n+2} - \frac{\alpha_0 (l+\alpha_0)^{n+1} - \alpha_0^{n+2}}{n+1} \right) = 0 \tag{11}$$

After simplification

$$\frac{M_p}{A_r} = l^2 (l+\alpha_0)^n - l\left((l+\alpha_0)^{n+1} - \alpha_0 (l+\alpha_0)^n \right) + \frac{(l+\alpha_0)^{n+2} - \alpha_0^{n+2}}{n+2} - \frac{\alpha_0 (l+\alpha_0)^{n+1} - \alpha_0^{n+2}}{n+1} \tag{12}$$

From equation (10)

$$\frac{T_t l}{A_r} - \frac{M_p}{A_r} = l \frac{(l+\alpha_0)^{n+1} - \alpha_0^{n+1}}{n+1} - \left(\frac{(l+\alpha_0)^{n+2} - \alpha_0^{n+2}}{n+2} - \frac{\alpha_0 (l+\alpha_0)^{n+1} - \alpha_0^{n+2}}{n+1} \right) \tag{13}$$

Substituting equation (12) into equation (13) gives

$$\frac{T_t}{A_r} = \frac{(l+\alpha_0)^{n+1} - \alpha_0^{n+1}}{n+1} \tag{14}$$

Therefore

$$l = \left[\alpha_0^{n+1} + (n+1)\frac{T_t}{A_r} \right]^{\frac{1}{n+1}} - \alpha_0 \tag{15}$$

From (13)

$$\frac{M_p}{A_r} = \frac{(l+\alpha_0)^{n+2} - \alpha_0^{n+2}}{n+2} - \frac{\alpha_0 (l+\alpha_0)^{n+1} - \alpha_0^{n+2}}{n+1} \tag{16}$$

Alternatively, the ultimate lateral capacity can be calculated implicitly by

$$\frac{M_p}{A_r} = \frac{1}{n+2}\left[\alpha_0^{n+1} + (n+1)\frac{T_t}{A_r} \right]^{\frac{n+2}{n+1}} - \left(\frac{\alpha_0^{n+2}}{n+2} + \alpha_0 \frac{T_t}{A_r} \right) \tag{17}$$

The influence of the loading eccentricity may be considered by replacing the plastic moment M_p with M_p-M_0, where M_0=$T_t e$, e is the eccentricity. Consequently

$$\frac{M_p}{A_r} = \frac{1}{n+2}\left[\alpha_0^{n+1} + (n+1)\frac{T_t}{A_r} \right]^{\frac{n+2}{n+1}} - \left(\frac{\alpha_0^{n+2}}{n+2} + \alpha_0 \frac{T_t}{A_r} \right) + \frac{T_t e}{A_r} \tag{18}$$

Equations (15) and (18) derived here using the upper bound method are identical to the elastic-plastic solutions developed by Guo (2006).

Fixed-head piles

For the case of a fixed-head pile, the energy dissipation due to the plastic moment M_p at point A and B is calculated from Equation (6). Following the same derivations as for the free-head piles, the ultimate lateral capacity for fixed-head piles can be easily determined. The equations for calculating the ultimate lateral load and the optimization depth are identical to equations (14) and (15), whereas equations (16'), (17') and (18') correspond to equations (16), (17) and (18), respectively.

$$\frac{2M_p}{A_r} = \frac{(l+\alpha_0)^{n+2} - \alpha_0^{n+2}}{n+2} - \frac{\alpha_0 (l+\alpha_0)^{n+1} - \alpha_0^{n+2}}{n+1} \tag{16'}$$

$$\frac{2M_p}{A_r} = \frac{1}{n+2}\left[\alpha_0^{n+1} + (n+1)\frac{T_t}{A_r} \right]^{\frac{n+2}{n+1}} - \left(\frac{\alpha_0^{n+2}}{n+2} + \alpha_0 \frac{T_t}{A_r} \right) \tag{17'}$$

$$\frac{2M_p}{A_r} = \frac{1}{n+2}\left[\alpha_0^{n+1} + (n+1)\frac{T_t}{A_r} \right]^{\frac{n+2}{n+1}} - \left(\frac{\alpha_0^{n+2}}{n+2} + \alpha_0 \frac{T_t}{A_r} \right) + \frac{T_t e}{A_r} \tag{18'}$$

It is noted that the difference between the solutions for fixed-head piles and free-head piles is a factor of 2 on the value of the plastic moment, M_p in eq. (A-16), (A-17) and (A-18). They could be used to produce charts for estimating the lateral load capacity in terms of plasticity moment, M_p and the parameters A_r, α_0 and n for the limiting force profile.

DISCUSSIONS

Some Extreme Cases

Due to the versatility and flexibility of the generic limiting force profile, the current solutions can be reduced to available solutions for some special cases. They are summarized in Table 1 and 2 for a uniform and a linearly increasing distribution of ultimate soil resistance. For instance, setting n = 1, $\alpha_0 = 0$, and $A_r = 3\gamma_s^{'} K_p d$, for a fixed-head pile with eccentricity *e*, after simplification, equation (15) and (18′) reduce to

$$l = \sqrt{2\frac{T_t}{A_r}} = 0.82\sqrt{\frac{T_t}{\gamma_s^{'} K_p d}} \quad (19)$$

$$2M_p = T_t(\frac{2}{3}l + e) \quad (20)$$

Alternatively

$$T_t = \frac{2M_p}{e + 0.54\sqrt{\frac{T_t}{\gamma_s^{'} K_p d}}} \quad (21)$$

These results were identical to those presented by Brooms (1964b) for a long restrained pile.

Design Charts

Adopting the generic limiting force profile, the expressions derived may be used to produce charts for estimating the lateral capacity of a pile in terms of plastic moment and the parameters describing the limiting force profile, similar to those presented by Broms (1964). Such charts have been constructed by Guo (2006, 2012, 2013), in which the predicted maximum bending moment were compared with measured test data of 52 laterally loaded piles in clay and sands. Notice that in Guo's presentation, the load and bending moment are normalized by the reciprocal of characteristic length to provide a consistent presentation, but in essence the ultimate lateral capacity is independent of the characteristic length but A_r, α_0 and *n* for the limiting force profile.

CONCLUSIONS

An upper-bound solution of limit analysis was developed to estimate the ultimate lateral capacity of a long pile based on a kinematically admissible wedge failure mechanism and a generic limiting force profile. The proposed solutions can be reduced to a number of available solutions. The expressions derived can be used to construct design charts or implemented into a spreadsheet program to conduct the estimation.

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Table 1 Pile capacity with uniform distribution of ultimate soil resistance
(n = 0 and $\alpha_0 = 0$)

Solutions	$e = 0$	$e \neq 0$
	Free-head	
Current solutions	$l = \sqrt{\frac{2M_p}{p_u}} = \frac{T_t}{p_u}$ $T_t = \sqrt{2M_p p_u} = \frac{2M_p}{l}$ $M_p = \frac{1}{2}T_t l$	$l = \sqrt{\frac{2(M_p - M_0)}{p_u}} = \frac{T_t}{p_u}$ $T_t = \sqrt{2(M_p - M_0)p_u} = \frac{2(M_p - M_0)}{l}$ $M_p = \frac{1}{2}T_t l + M_0 = T_t(\frac{l}{2} + e)$
Previous solutions	$\frac{T_t}{s_u d^2} = \sqrt{2N_p \frac{M_p}{s_u d^3}}$ (N_p is a resistance factor) Klar and Randolph (2008)	$l = \sqrt{\frac{2(M_p - M_0)}{p}}$ $T_t = \sqrt{2(M_p - M_0)p}$ (p is the lateral resistance of the soils) Murff (1998) Chiou *et al* (2009)
	Fixed –head	
Current solutions	$l = 2\sqrt{\frac{M_p}{p_u}} = \frac{T_t}{p_u}$ $T_t = 2\sqrt{M_p p_u} = \frac{4M_p}{l}$ $M_p = \frac{1}{4}T_t l$	$l = \sqrt{\frac{2(2M_p - M_0)}{p_u}} = \frac{T_t}{p_u}$ $T_t = \sqrt{2(2M_p - M_0)p_u} = \frac{2(2M_p - M_0)}{l}$ $M_p = \frac{1}{2}(\frac{1}{2}T_t l + M_0) = \frac{T_t}{2}(\frac{l}{2} + e)$

Table 2 Pile capacity with linear increasing distribution of ultimate soil resistance
(n = 1 and $\alpha_0 = 0$)

Solutions	$e = 0$	$e \neq 0$
Free-head		
Current solutions	$l = \sqrt[3]{\frac{3M_p}{A_r}} = \sqrt{2\frac{T_t}{A_r}}$ $T_t = \sqrt[3]{\frac{9}{8}A_r M_p^2} = \frac{3}{2}\frac{M_p}{l}$ $M_p = \frac{2}{3}T_t l$	$l = \sqrt[3]{\frac{3(M_p - M_0)}{A_r}} = \sqrt{2\frac{T_t}{A_r}}$ $T_t = \sqrt[3]{\frac{9}{8}A_r (M_p - M_0)^2} = \frac{3}{2}\frac{(M_p - M_0)}{l}$ $M_p = \frac{2}{3}T_t l + M_0 = T_t(\frac{2}{3}l + e)$
Previous solutions	$\frac{T_t}{nd^3} = 1.04\left(\frac{M_p}{nd^4}\right)^{\frac{2}{3}}$ $(A_r = nd)$ Randolph (2004)	$M_p = T_t(\frac{2}{3}l + e)$ $l = 0.82\sqrt{\frac{T_t}{\gamma_s' d K_p}}$ $(A_r = 3K_p \gamma_s' d)$ Broms (1964b), Chai (2002) and Guo (2008)
Fixed-head		
Current solutions	$l = \sqrt[3]{\frac{6M_p}{A_r}} = \sqrt{2\frac{T_t}{A_r}}$ $T_t = \sqrt[3]{\frac{9}{2}A_r M_p^2} = 3\frac{M_p}{l}$ $M_p = \frac{1}{3}T_t l$	$l = \sqrt[3]{\frac{3(2M_p - M_0)}{A_r}} = \sqrt{2\frac{T_t}{A_r}}$ $T_t = \sqrt[3]{\frac{9}{8}A_r (2M_p - M_0)^2} = \frac{3}{2}\frac{(2M_p - M_0)}{l}$ $M_p = \frac{1}{2}(\frac{2}{3}T_t l + M_0) = \frac{T_t}{2}(\frac{2}{3}l + e)$
Previous solutions	$\frac{T_t}{nd^3} = 1.65\left(\frac{M_p}{nd^4}\right)^{\frac{2}{3}}$ $(A_r = nd)$ (Randolph 2004) $\frac{l}{d} = \sqrt[3]{\frac{2M_p}{K_p \gamma_s' d^4}}$ $(A_r = 3K_p \gamma_s' d)$ $\frac{T_t}{K_p \gamma_s' d^3} = \frac{3}{2}\left(\frac{l}{d}\right)^2$ Song *et al* (2005)	$M_p = \frac{T_t}{2}(\frac{2}{3}l + e)$ $l = 0.82\sqrt{\frac{T_t}{\gamma_s' d K_p}}$ $(A_r = 3K_p \gamma_s' d)$ Broms (1964b)

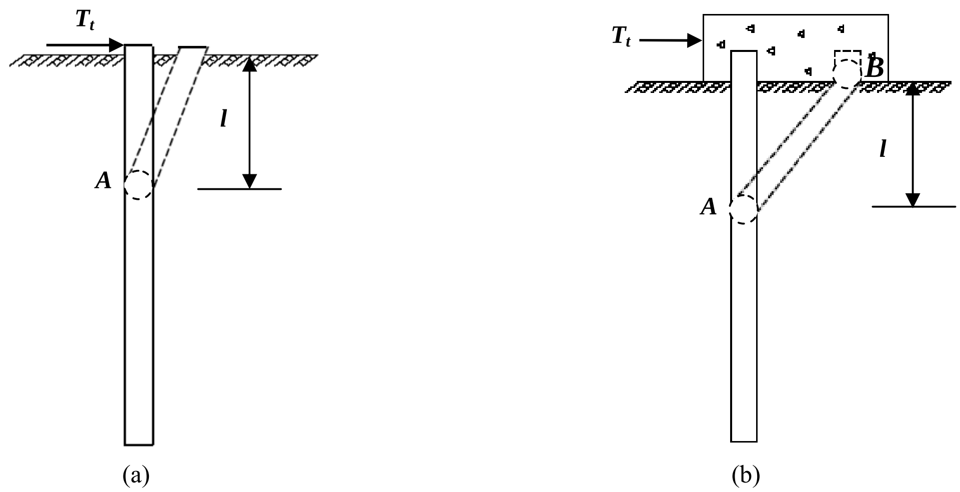

Figure 1 Pile collapse mechanism (a) free-head pile (b) fixed-head pile

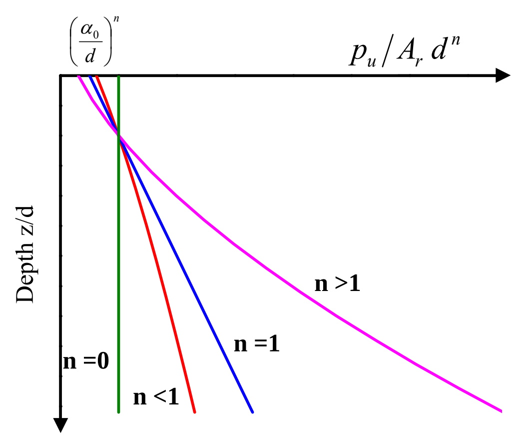

Figure 2 Schematic generic limiting force profiles