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Abstract

There are two competing pairing mechanisms for the superconductivity of doped Weyl semimetals, i.e., the internode Bardeen-Cooper-Schrieffer (BCS) pairing and the intranode Fulde-Ferrell-Larkin-Ovchinnikov (FFLO) pairing. To understand the edge excitations at the interface between the Weyl semimetal and the superconducting Weyl semimetal (WSM/WSM) mediated by two different pairings, we study the energy dispersions and the density of states under a strong magnetic field. It is found that only the chiral zeroth Landau level exhibits a significant difference for the two pairings; the excitation spectra of higher Landau levels are insensitive to the way of pairings. In the vicinity of interface in the hybrid of WSM/WSM, the spatial distributions of transverse current and the transverse conductance are independent of pairing mechanism. The pairing independence in the macroscopic conductance can be understood with the quantum effect of phase-coherent electron-hole states at the WSM/WSM interface, which is responsible for the magnetically induced edge states supported by the Weyl Landau levels.

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Transverse conductance in a three-dimensional Weyl semimetal and Weyl superconductor hybrid under a strong magnetic field

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I. INTRODUCTION

The discovery of Weyl semimetals (WSM) offers new opportunities to explore various exotic quantum phenomena associated with the linear dispersion and the nontrivial Berry phases in their band structure [1–3]. The WSM is characterized by a pair of band crossing points, termed the Weyl nodes. The low-energy excitations in the WSM have a linear dispersion around nodes [3–5]. The WSMs present a set of novel features which are robust [6–10]: The pairs of Weyl nodes are protected by Chern numbers ±1, the Fermi arcs exist in the surface band structure, and the WSMs display the chiral magnetotransport phenomena. The unconventional superconductivity has been predicted to occur in doped WSM with an inversion symmetry and a topologically nontrivial Fermi surface [11–13]. UPt3 has been discovered to exhibit the superconducting phase [14], a possible BCS state with the superconducting transition temperature about 0.5 K. It has been considered as a superconducting Weyl semimetal (SWSM) with the nodal points [15]. Besides UPt3, the unconventional superconductivity is also discovered by hard point contact on the WSM TaAs crystals [16,17]. Experimentally, the superconducting states in the WSM have also been studied in other SWSMs, such as MoTe2, WTe2, TaP, and TaIrTe4 [18–23].

The property of superconducting states in the SWSM is related to the details of the superconducting pairings. Theoretically, two competing pairing mechanisms are possible to realize the SWSMs [24–30], i.e., internode BCS pairing [31] (the Bardeen-Cooper-Schrieffer pairing mechanism) and intranode FFLO pairing [32,33] (the Fulde-Ferrell-Larkin-Ovchinnikov pairing mechanism). The BCS superconducting state is formed by a zero-momentum Cooper pair and the FFLO pairing states are formed by a finite-momentum Cooper pair of paired fermions with opposite momenta. The FFLO superconducting state is formed by a finite-momenta Cooper pair and has a spatially nonuniform Cooper-pair correlation function. The requirement of unconventional pairing symmetries in the SWSM has currently been studied intensively. The studies show that the SWSM pairing states are topologically nontrivial with gapless nodes in the energy dispersion while the FFLO paired state is topologically trivial with a full nodeless gap. To understand the interplay between superconductivity and nontrivial topology in electronic structures, many works have been devoted to investigate which kind of pairing state is preferred in the doped WSM. Based on the assumption of only one kind of pairings, the ground state energies are calculated for the BCS pairing and the FFLO pairing, respectively. In Refs. [24,25], it was argued that the FFLO state has a lower energy. Later, by using the standard mean-field theory [28] and the Bogoliubov-de Gennes (BdG) equations [29], it was demonstrated that the energy of the BCS paired state is lower than that of the FFLO state. To date, there is no conclusive result on preferred pairing in the doped WSM. The SWSM hybrid structures are proposed to search for distinguishable signatures with respect to these two distinct pairings. The Andreev reflections (AR) [34] at the interface of a WSM/SWSM hybrid structure [35] and a WSM/normal-superconductor hybrid structure [36] have been reported. The mixed pairing in the AR has been also investigated [37]. A four terminal transport Josephson junction of WSM has been
proposed to probe the nodal BCS and the FFLO states in the superconducting phase of the inversion-symmetric doped WSM [38]. These previous studies mainly concentrated on the AR characteristics of longitudinal transport. There has been little discussion of the transverse conducting properties associated with the edge excitations.

Although some distinct signatures have been exhibited in the longitudinal ARs, the feature of chiral anomalous transports [39–51] originated from carrier’s chirality in WSM/SWSM hybrid structure has not been studied. On the basis of the gapless nature of the doped WSMs associated with the topological phases, it is interesting to know how the superconducting phases originated from the bulk Weyl nodes are manifested by topologically protected surface states, in particular, the anomalous transports associated with these two possible pairings. Because of the fine experimental control over the hybrid structures, the edge states can be manipulated by using magnetic fields [52–55]. The WSM/SWSM hybrid structure under a magnetic field can be used to analyze the relevant signatures of topological edge excitations for the nodal BCS and the FFLO superconducting pairing states. Correspondingly, the transverse (anomalous) electromagnetic response can be investigated. Topological features can be manifested by the edge excitations, where the edge channels consist of a coherent superposition of electrons and Andreev reflected holes associated with two distinct pairings. The transverse conductance embodies the general topological properties of the magnetic helicity, which is topologically well defined and invariant when all the available states have been summed over. The similar setups have been widely used in studies of the edge excitations, such as the two-dimensional electron gas in hybrid structures [56–62], graphene [63–65], and topological insulator [66,67]. The previous studies suggested that hybrid structures of superconducting topological materials can support the neutral Majorana states [66,68–70].

In this work, to understand the interplay between the nontrivial microscopic feature and physical effects traceable to the edge excitations, we investigate the edge excitations associated with the intranode and the internode electron-hole conversion. Our studies show that for a finite system with a limiting surface the Weyl Landau levels (LLs) can support the magnetic helicity, which is topologically well defined and invariant when all the available states have been summed over. The similar setups have been widely used in studies of the edge excitations, such as the two-dimensional electron gas in hybrid structures [56–62], graphene [63–65], and topological insulator [66,67]. The previous studies suggested that hybrid structures of superconducting topological materials can support the neutral Majorana states [66,68–70].

The paper is organized as follows. In Sec. II the BdG Hamiltonian of the WSM/SWSM hybrid structure is given, according to the specific requirements concerning the electron-hole conversion with the BCS and FFLO pairings. We solve the BdG equations in the presence of magnetic field. The LL spectra and wave functions for WSM/SWSM hybrid structure in a magnetic field are obtained. The corresponding DOS near the Fermi level is analyzed for the BCS and FFLO pairings, separately. In Sec. III, we calculate the transverse current distributions in the vicinity of the interface between the WSM and the SWSM numerically. The interplay between the electron-hole conversion and the cyclotron motion at the interface of the WSM/SWSM hybrid structure is discussed. The transverse conductance parallel to the interface for the BCS and FFLO pairings is presented in Sec. IV. Using a four-terminal SWSM/SWSM hybrid structure, the contributions from the various edge channels to the transverse conductance are analytically calculated. We summarize the results in Sec. V.

II. THE SPECTRUM PROPERTIES OF WSM/SWSM HYBRID STRUCTURE

A. The BdG equation in the presence of a magnetic field

We consider a WSM/SWSM hybrid structure along the $x$ axis with the interface at $x = 0$. The WSM ($x < 0$) and the SWSM ($x > 0$) are assumed to extend to infinity in the $y$ and $z$ directions. For the sake of simplicity, in the following calculations, we consider a pair of Weyl nodes with the opposite chirality, localized at $K_{\pm} = (0, 0, \pm \hbar k)$ in momentum space, respectively. We assume the same Fermi velocity and distance between two nodes in the WSM and the SWSM. A uniform magnetic field, $\mathbf{B} = B\hat{e}_z$, is applied parallel to the $z$ axis. The Meissner effect is assumed to be held for the SWSM, so that the magnetic field is excluded from the SWSM region.
In addition, we also ignore the inhomogeneity of the magnetic field in the vicinity of the interface. Denoting the excitation energy of the electron above the Fermi energy \( E_F \) by \( E \), the BdG equation for the excitations of quasiparticles is given by

\[
\mathcal{H}_{\text{BdG}} \Psi = E \Psi, \tag{1}
\]

where \( \Psi = (u_{+\uparrow}, u_{+\downarrow}, u_{-\uparrow}, u_{-\downarrow}, v_{-\uparrow}, v_{-\downarrow}, v_{+\uparrow}, v_{+\downarrow})^T \) in the Nambu representation \( \tilde{\Psi} = (\tilde{\Psi}_{+\uparrow}, \tilde{\Psi}_{+\downarrow}, \tilde{\Psi}_{-\uparrow}, \tilde{\Psi}_{-\downarrow}, -\tilde{\Psi}_{-\uparrow}, -\tilde{\Psi}_{-\downarrow}, -\tilde{\Psi}_{+\uparrow}, -\tilde{\Psi}_{+\downarrow})^T \). \( T \) stands for transpose, the subscripts \( \pm \) are for the Weyl nodes \( K_\pm \), and \( \uparrow (\downarrow) \) is for the spin-up (down). The BdG Hamiltonian takes a form \([24,35,37]\)

\[
\mathcal{H}_{\text{BdG}} = \begin{pmatrix} \mathcal{H}(\pi, K) - E_F & \Delta(r, K) \Theta(x) \\ \Delta^*(r, K) \Theta(x) & -T \mathcal{H}(\pi, K) T^{-1} \end{pmatrix}, \tag{2}
\]

where

\[
\mathcal{H}(\pi, K) = v_F[\tau_0(\sigma_x \pi_x + \sigma_y \pi_y - \sigma_z H K) + \tau_+ \sigma_z \pi_z] \tag{3}
\]

is the Hamiltonian for electronlike carriers and

\[
\Delta(r, K) = v_0 \sigma_0 \Delta_B + \sum_{\kappa=\pm} \tau_\kappa \sigma_0 \Delta_P e^{i2K \cdot r} \tag{4}
\]

is the superconducting pairing potential. Here \( \Delta_B \) and \( \Delta_P \) are the BCS and the FFLO order parameters, \( \pi = p + (e/c)A \) is the mechanical momentum, \( A = \Theta(x)(0, B x, 0) \) is the vector potential with the Heaviside step function \( \Theta(x) \), \( \Theta(x) = 1 \) for \( x > 0 \) and \( 0 \) for \( x < 0 \), \(-e\) is the electron charge, \( v_F \) is the Fermi velocity, \( \sigma_x, \sigma_y, \sigma_z \) are the Pauli matrices for the spin, \( \tau_+ \) and \( \tau_- \) are the Pauli matrices for the chirality in a space spanned by two Weyl points, \( \tau_\kappa = (\tau_\kappa \pm i \tau_\kappa) / 2 \), \( \sigma_0 \) and \( \tau_0 \) are 2x2 identity matrices, \( T = i \tau_+ \sigma_K \) is the time reversal operation, and \( K \) is the complex conjugation operation.

The BdG Hamiltonian in Eq. (2) conserves the inversion symmetry \( \mathcal{P} = I_{2 \times 2} \otimes I \mathcal{P} \) with \( \mathcal{P} = \tau_+ \sigma_+ \) but breaks time-reversal symmetry \( \mathcal{T} = I_{2 \times 2} \otimes \mathcal{T} \), where \( I_{2 \times 2} \) is the 2x2 identity matrix for the electron and hole and \( \mathcal{P} \) and \( \mathcal{U} \) are a Pauli matrix independent of \( p \) and \( r \). We can show that \( \mathcal{P} \mathcal{H}_{\text{BdG}}(\pi, r) \mathcal{P}^{-1} = -\mathcal{H}_{\text{BdG}}(-\pi, -r) \) and \( \mathcal{U} \mathcal{H}_{\text{BdG}}(\pi, K) \mathcal{U}^{-1} = -\mathcal{H}_{\text{BdG}}(-\pi, K) \). \([71,72]\) The change that \( K \to -K \) implies that the time-reversal symmetry is not preserved. The BdG Hamiltonian is invariant under charge-conjugation (or particle-hole) symmetry with \( \mathcal{C} \mathcal{H}_{\text{BdG}}(p^* + (e/c)A, K) \mathcal{C}^{-1} = -\mathcal{H}_{\text{BdG}}(-p + (e/c)A, K) \) \(([\mathcal{C}, \mathcal{H}_{\text{BdG}}(\pi, K)] = 0, \{ \cdots \} \) is an anticommuting relation), where \( \mathcal{C} = \lambda_+ \tau_+ \sigma_+ K \) is charge-conjugation (particle-hole) operator and \( \lambda_+ \) is a Pauli matrix for the electron-hole degree of freedom. The chiral symmetry is represented by operator \( \mathcal{S} = \mathcal{T} \mathcal{C} \) \([71,72]\), a combination of particle-hole and time-reversal symmetry. It is found that \( \mathcal{S} \mathcal{H}_{\text{BdG}}(\pi, K) \mathcal{S}^{-1} = -\mathcal{H}_{\text{BdG}}(-p + (e/c)A, -K) \). The change of Weyl nodes under the \( \mathcal{S} \) reflects the fact that the Weyl superconductor has the chiral symmetry with a pair of nodes of opposite chirality.

**B. The wave functions of the quasiparticle excitations**

The wave functions of the quasiparticle excitations in the WSM and SWSM regions can be expressed as a linear combination of eigenstates of BdG equations. Considering the nonzero magnetic field in the WSM and the gap function in the SWSM, we will solve the BdG equation in the WSM and the SWSM separately.

**1. Energy spectrum of a WSM**

In the WSM region \((x < 0)\), \( \Delta(r, K) = 0 \). In the presence of a magnetic field, the electron energy spectrum is in the form of unequally-spaced LLs,

\[
E_{n,\eta,\kappa}/E_0 = -\eta(E_F/E_0 + \kappa_\eta \frac{\hbar v_F k^x}{E_0})
+ (1 - \delta_{n,0}) \text{sgn}(n) \sqrt{2|n| + \left( \frac{\hbar v_F k^x}{E_0} \right)^2}, \tag{5}
\]

where \( n \) is a real integer number, \( E_0 = \hbar v_F / k_B \) with \( k_B = \sqrt{\epsilon_\parallel (\epsilon E)} \), \( k^x_\eta = k_\eta - \kappa K \) with \( \kappa = \pm 1 \) for the chirality of two Weyl nodes, \( \eta = \pm 1 \) are for the electronlike (+) and holelike (−) branches. The eigenstates of Eq. (1) take the form \( \Phi^W_{n,\eta,\kappa}(X_k, z), x, \pm X_k \) \( e^{i \kappa \lambda y z}, y, z, z \), where \( \lambda = \pm 1 \) are for spin subbands, \( X_k = -k_\eta k_B^2 \) is the guiding-center coordinate for cyclotron motion, and \( \pm X_k \) correspond to the cyclotron motions of electrons and holes, respectively. Functions \( \Phi^W_{n,\eta,\kappa}(X_k, z), x, \pm X_k \) are the 8x1 column matrices with a parabolic cylinder function of variable \( x \pm X_k \). The asymptotic limit of the wave function vanishes, \( \Psi^W(x \to -\infty) = 0 \). We give the analytical expressions of \( \Phi^W_{n,\eta,\kappa}(X_k, z), x, \pm X_k \) in Appendix A.

**2. Energy spectrum of a SWSM**

In the SWSM, the magnetic field is absent and the solutions of the BdG equation depend on the pairing. In the following, we solve the BdG equation with respect to the BCS pairing and the FFLO pairing, respectively.

(a) BCS pairing. In Eq. (4), \( \Delta_B = 0 \) and \( \Delta_P = 0 \). With the same notation specifying indices \( \kappa, \eta, \lambda \), the eigenvalues of the BdG equation (1) are found as

\[
E_{n,\eta,\kappa}^{(B)}/E_0 = \eta \lambda \left( \frac{\Delta_0}{E_0} \right)^2 + \left( \frac{E_F}{E_0} \right)^2 + \left( \frac{\hbar v_F k^x}{E_0} \right)^2 - \Xi_{\lambda,\kappa}(k_\eta, E_F). \tag{6}
\]

where the superscript “B” indicates the BCS pairing,

\[
\Xi_{\lambda,\kappa}(k_\eta, E_F)
= 2\lambda \sqrt{\left( \frac{\Delta_0}{E_0} \right)^2 + \left( \frac{\hbar v_F k^x}{E_0} \right)^2 \left( \frac{E_F}{E_0} \right)^2 + \left( \frac{\hbar v_F k^x}{E_0} \right)^2}, \tag{7}
\]

with \( k^x = \sqrt{\frac{\hbar v_F}{E_0}} \left( k^{(\kappa,\lambda,\eta)}_x \right)^2 + k^z_\eta + k^z_\eta^2 \) and \( k^{(\kappa,\lambda,\eta)}_x = \frac{\eta}{\hbar v_F} \sqrt{\Delta_\kappa(k_\eta, k_z, E) + 2\lambda \sqrt{\Sigma_\kappa(k_z, E)}}. \tag{8} \)

Here \( \Delta_\kappa(k_\eta, k_z, E) = -\Delta^2_0 + E^2_F + E^2 - (\hbar v_F)^2(k^2_\eta + k^2_z) \) and \( \Sigma_\kappa(k_z, E) = \Delta_0^2(\frac{\hbar v_F k^x}{E_0})^2 - E^2_F + E^2 F^2 \). The eigenstates characterized by the indices \( (\kappa, \lambda, \eta) \) take the
form $\Phi^{S(B)}_{k,n}(k_y, k_z, E) e^{ik'_{x,y,z}x+i k_y y+i k_z z}$. Equation (8) shows that $k'_{x,(\lambda,\eta)}$ is purely imaginary for $\Sigma_k(k_y, E) > 0$, where $k'_{x,(\lambda,\eta)} = \eta(\lambda k'_{x} + i k''_{x})$ for $\Sigma_k(k_y, E) \leq 0$, where $k'_{x} = (1/\sqrt{2\hbar v_F}) \sqrt{\Lambda^2_k + 4|\Sigma_k| + \Delta_k}$ and $k''_{x} = (1/\sqrt{2\hbar v_F}) \sqrt{\Lambda^2_k + 4|\Sigma_k| - \Delta_k}$. The imaginary part in $k'_{x,(\lambda,\eta)=+1}$ guarantees the asymptotic convergence $\Psi^S(x \to +\infty) = 0$ for the wave function in region SWSM. The analytical function expressions of $\Phi^{S(B)}_{k,n}(k_y, k_z, E)$ are given in Appendix A.

(b) FFLO pairing. In Eq. (4), $\Delta_0 = 0$ and $\Delta_F = \Delta_0$. The eigenvalues $E_{k,n}(\lambda,\eta)$ of BdG equation have the same form as that given in Eq. (6) but with $\Sigma_{\lambda,\eta}(k_y, E_F) = 2\lambda E_F / E_0 (\hbar v_F k_n^2 / E_0)$ and $k'_{x,(\lambda,\eta)} = \eta(\lambda k'_{x} + i k''_{x})$, where $k'_{x} = (1/\sqrt{2\hbar v_F}) \sqrt{\Lambda^2_k + 4(\Delta^2_F - E^2)E_F^2 + \Delta_k}$ and $k''_{x} = (1/\sqrt{2\hbar v_F}) \sqrt{\Lambda^2_k + 4(\Delta^2_F - E^2)E_F^2 - \Delta_k}$. The superscript “F” indicates the FFLO pairing. The eigenstates take the form $\Phi^{F(S)}_{k,n}(k_y, k_z, E) e^{ik'_{x,y,z}x+i k_y y+i k_z z}$. The imaginary part of $k'_{x,(\lambda,\eta)=+1}$ is nonzero so that the asymptotic convergence $\Psi^F(x \to +\infty) = 0$ is guaranteed. The analytical function expressions of $\Phi^{F(S)}_{k,n}(k_y, k_z, E)$ are also given in Appendix A.

3. The wave functions of the quasiparticle excitations

In writing the wave function in a WSM/SWSM hybrid structure, the electron-to-hole conversion with the BCS and the FFLO pairings should be considered. Let us first analyze the electron-to-hole conversion with the BCS and the FFLO pairings. Due to the nature of the interface pairing for the BCS states and the intranode pairing for the FFLO states, the ARs related to the electron-to-hole conversion can occur at different Weyl nodes. Specifically, when an electron in the state at the node $k\text{e}_\lambda$ moves toward the interface of a WSM/SWSM from the WSM side, it forms a Cooper pair with another electron in the state at the node $k\text{e}_\lambda$ with $(k' - \lambda)k - k_\lambda$. It results in a hole in the state at the node $k\text{e}_\lambda$ with $k' - (k' + \lambda)k$. The pairing state has a momentum $\Delta k = (k' + \lambda)k \neq 0$. Therefore, the Andreev reflected hole is in the state at the same node as that of the incident electron for the FFLO pairing $(k' = k)$, while in the state at the node with an opposite chirality for the BCS pairing $(k' = -k)$. Because the AR can occur for precipitating electrons in the states at both nodes, the holes for the AR are of the equal rights for two nodes in the WSM. As a consequence, there exist not only two possible electron reflection processes but also two possible electron-to-hole conversion processes at the WSM/SWSM interface. The difference between the BCS and the FFLO pairings is microscopic, i.e., the “converted” hole is in a Weyl node different from its precipitating electron for the BCS pairing while within the same Weyl node as its precipitating electron for the FFLO pairing.

Taking into account all possible electron-to-hole conversions at the WSM/SWSM interface, the wave functions of the quasiparticle excitations with the energy $E$ in the regions $x < 0$ and $x > 0$ can be expressed in the form of

$$\Psi_e(x, y, z) = A \sum_{k_{eta}}^{|E|=1} \Phi^{W}_{k_{eta},\lambda=-1}(E, x_k, k_z, x \pm x_k, \lambda)$$

$$\times e^{ik'_{x,y,z}x+i k_y y+i k_z z}$$

and

$$\Psi^{(B/F)}_>(x, y, z) = A \sum_{k_{eta}} d_{k_{eta},\lambda=1}^{|E|=1} \Phi^{S(B/F)}_{k_{eta},\lambda}(E, k_y, k_z)$$

$$\times e^{ik'_{x,y,z}x+i k_y y+i k_z z},$$

respectively, where $A = (L_x L_z)^{-1/2}$, the superscripts “B” and “F” of the wave functions in $\Psi^{(B/F)}_e$ refer to the type of pairing in the WSM. The required normalization condition is held by $\int_{-\infty}^{+\infty} \int_{L_x}^{L_y} \int_{L_z}^{L_z} d\psi \psi^+(r) \psi(r) = 1$, where $\psi(x, y, z) = \Psi_e(-x) + \Psi^{(B/F)}_e(x)$ and the integration is taken over the entire WSM/SWSM stretching region. The wave functions in Eqs. (9) and (10) promise the coherent superpositions of an electron and a hole in the WSM region and those of evanescent excitations in the SWSM region. They are constrained in the vicinity of interface so that the wave functions are asymptotically convergent.

C. The excitation spectra in relation to the BCS and FFLO pairings

In order to investigate the coherent superposition of electron and hole states, which is accompanied by the electron-to-hole conversion, the energies $E$ of electron and hole excitations are restricted within the range of $|E| \leq \Delta_0$, where $\Delta_0$ is the magnitude of the pair potential. Electron and hole cannot enter the SWSM region, with the constraint of energies $|E| \leq \Delta_0$, unless a Cooper pair is formed in the SWSM.

The boundary condition requires $\Psi_e(x = 0) = \Psi^{(B/F)}_e(x = 0)$ at $x = 0$. This leads to a set of linear homogeneous equations for the coefficients $c_{k_{eta}}^{S(B)}$ and $d_{k_{eta}}^{S(B)}$ in Eqs. (9) and (10). Vanishing the determinant coefficient, we can obtain the excitation spectrum. For the excitation energy $E$ within the range of $\Delta_0$, the WSM/SWSM hybrid structure under a magnetic field, the quantum number $n$ in $E_{n_{eta},k}^e$ is no longer an integer number but depends on $E$ and $k_n^2$, which is different from the pure WSM under a magnetic field. We, therefore, rewrite the quantum number $n$ as a function of $E$ and $k_n^2$ as $n_{eta,k} = [(E + \eta E_F)^2 - (\hbar v_F k_n^2)^2]/2$.

For simplicity, we have considered an ideal interface in the calculations. The discrete excitation spectra are demonstrated in Figs. 1(a) and 2(a) for the BCS pairing and Figs. 1(b) and 2(b) for the FFLO pairing, where the yellow, green, cyan, magenta, and blue curve surfaces correspond to the Weyl LLs $N = -2, -1, 0, 1,$ and 2, respectively. The pair potential has been taken as $\Delta_0 = 4 E_0$ and $K = 5 I_B$ for demonstration purposes, where the length is scaled in units of $I_B$ and the energy in $E_0$. In order to illustrate the impact of Fermi energy on the excitation spectra, two values of Fermi energy, $E_F = 0$ (Fig. 1) and $0.4 E_0$ (Fig. 2), are chosen in the calculations.

In Figs. 1 and 2, an auxiliary plane in gray color has been added in the range of $x < 0$ to facilitate the comparison of the two excitations. In Figs. 3 and 4, the auxiliary plane in gray color has been added in the range of $x > 0$ to facilitate the comparison of the two excitations. In Figs. 5 and 6, the auxiliary plane in gray color has been added in the range of $x < 0$ to facilitate the comparison of the two excitations. In Figs. 7 and 8, the auxiliary plane in gray color has been added in the range of $x > 0$ to facilitate the comparison of the two excitations. In Figs. 9 and 10, the auxiliary plane in gray color has been added in the range of $x < 0$ to facilitate the comparison of the two excitations. In Figs. 11 and 12, the auxiliary plane in gray color has been added in the range of $x > 0$ to facilitate the comparison of the two excitations.

\[ E_N(E_F, X_{k_y}, k_z) = E_N(E_F, X_{k_y}, -k_z) \]

and (b) the FFLO pairing.

\[ \Delta E_{N=0}^\text{bulk}(E_F, k_z) = 2|E_F - (K \pm k_z)| E_{\text{bulk}} \]

for the FFLO pairing, while (c) and (d) the FFLO pairing.

\[ \Delta E_{N=0}^\text{bulk}(E_F, k_z) = 2|E_F - (K \pm k_z)| E_{\text{bulk}} \]

for the FFLO pairing. Although the deformation arises in the spectra with change of \( E_F \), the particle-hole symmetry is still intact. To exhibit the particle-hole symmetry for the BCS and FFLO pairings, we define an asymptotical difference for \( X_{k_y} \rightarrow \infty \).

\[ \frac{\partial E_N}{\partial k_z} \]

relevant to the edge excitations.

\[ \frac{\partial E}{\partial k_z} \]

Energy and the Weyl LLs are shifted as the Fermi energy changes. Physically, the coherence among those electron and hole LL states within the energy gap \( |E| \leq \Delta_0 \) causes the spectra to be dependent on the Fermi energy. Intuitively, the change of the Fermi energy results in a deformation of those LLs within interval of \( \Delta_0 \). Figure 2 is the energy dispersions with \( E_F = 0.4 E_0 \). It is found that the spectra of the electronlike levels shift downward by an amount \( E_F \), while the holelike energy levels shift upward by the same amount. As a consequence, the \( N = 0 \) LL for the FFLO pairing becomes \( X_{k_y} \) dependent when \( E_F \neq 0 \) (Figs. 1(b) and 2(b)). Although the deformation arises in the spectra with change of \( E_F \), the particle-hole symmetry is still intact.

The edge states emerge due to the interplay between the effect of magnetic field and the coherent superposition of quasiparticle states in the hybrid. In comparison with the quantized LLs, \( E_{\alpha,\eta,\kappa} \) shown in Eq. (5), with the numerical plots in Figs. 1(a) and 2(a) and \( k_y, \pm K = \pm E_F \) as shown by the white dotted lines in Figs. 1(a) and 2(a) for the BCS pairing, while by \( (K \pm k_y)|/\tan(1.3 X_{k_y} l_b) = 0 \) for the FFLO pairing [as shown by the white dotted lines in Figs. 1(b) and 2(b)]. The zero-energy line is independent of \( X_{k_y} \) when \( E_F = 0 \) but depends on the \( X_{k_y} \) if \( E_F \neq 0 \) for the FFLO pairing.

D. The edge states and their wave-function profiles

The edge states emerge due to the interplay between the effect of magnetic field and the coherent superposition of quasiparticle states in the hybrid. In comparison with the quantized LLs, \( E_{\alpha,\eta,\kappa} \) shown in Eq. (5), with the numerical plots in Figs. 1(a) and 2(a) and \( k_y, \pm K = \pm E_F \) as shown by the white dotted lines in Figs. 1(a) and 2(a) for the BCS pairing, while by \( (K \pm k_y)|/\tan(1.3 X_{k_y} l_b) = 0 \) for the FFLO pairing [as shown by the white dotted lines in Figs. 1(b) and 2(b)]. The zero-energy line is independent of \( X_{k_y} \) when \( E_F = 0 \) but depends on the \( X_{k_y} \) if \( E_F \neq 0 \) for the FFLO pairing.
excitations. Figure 3 shows the wave-function profiles with the BCS pairing [Figs. 3(a) and 3(b)] and the FFLO pairing [Figs. 3(c) and 3(d)] in a WSM/SWSM hybrid structure, where we choose $X_{k_y} = 0$, $E_F = 0.4E_0$, and $\Delta_0 = 4E_0$ in the calculations.

Figure 3 shows the wave functions of edge excitations for the BCS and the FFLO pairings differents features.

To demonstrate the different real space distribution of edge states compared to bulk states under a magnetic field, we also calculate the wave-function profiles with $X_{k_y} = -10l_B$, which correspond to the bulk states. Figures 4(a) and 4(b) are for the BCS pairing, while Figs. 4(c) and 4(d) are for the FFLO pairing. It is shown that different from those with $X_{k_y} = 0$, the wave-function profiles with $X_{k_y} = -10l_B$ characterize the holonomic distribution of states under a magnetic field and are localized inside of the WSM. The wave functions of the bulk states are the same for two distinct pairings.

E. The density of states for the LL spectra

The DOS for $|E| \leq \Delta_0$ reveals some interesting features. As a demonstration, we select some specific values of $E_F$ near the first three LLs. The DOSs with $E_F = 0.0E_0$, $0.8E_0$, $1.3E_0$, $1.5E_0$, $1.8E_0$, and $2.1E_0$ are shown in Figs. 5(a)–5(f), respectively, where the value $\Delta_0 = 0.5E_0$ has been taken for the demonstration. The black curves are for the BCS and the red curves are for the FFLO pairings. For $E_F = 0.0E_0$, there is only $N = 0$ LL in the energy window $[-\Delta_0, \Delta_0]$. Figure 5(a) shows that the DOS has a peak at $E = 0$ for the BCS pairing while the DOS is constant for the FFLO pairing. The appearance reflects the difference of the zeroth LL DOS for the two pairings. When the Fermi energy is increased to $0.8E_0$ [Fig. 5(b)], the $N = \pm 1$ LLs are present in the energy window $[-\Delta_0, \Delta_0]$. The peak at $E = 0$ is reduced for the BCS pairing and two additional peaks appear at $E = \pm 0.5E_0$ for both the BCS and the FFLO pairings. Further, the peaks correspond to the $N = \pm 1$ LLs being squeezed to $\pm(\sqrt{2}E_F - 1.3E_0) \approx \pm 0.14E_0$ for $E_F = 1.3E_0$ [Fig. 5(c)] and $\mp(1.5E_0 - \sqrt{2}E_F) \approx \mp 0.084E_0$ for $E_F = 1.5E_0$ [Fig. 5(d)]. The nonzero value around $E = 0$ resulting from the $N = \pm 1$ LLs crossing the zero-energy reference plane (the auxiliary gray plane in Figs. 1 and 2). Similarly, for $E_F = 1.8E_0$ and $2.1E_0$, the $N = \pm 2$ LLs access the energy window $[-\Delta_0, \Delta_0]$ and two more additional peaks appear at $\pm(2E_F - 1.8E_0) \approx \pm 0.2E_0$ and $\mp(2.1E_F - 2E_0) \approx \mp 0.1E_0$.  

![Figure 3](image-url)  

**FIG. 3.** The edge excitation profiles for the spectra of $N = 0$, $\pm 1$, $\pm 2$. (a) and (b) are for the BCS pairing, (c) and (d) are for the FFLO pairing. $X_{k_y} = 0$, $E_F = 0.4E_0$, and $\Delta_0 = 4E_0$ are used in the calculations.

![Figure 4](image-url)  

**FIG. 4.** The bulk excitation profiles for the spectra of $N = 0$, $\pm 1$, $\pm 2$. (a) and (b) are for the BCS pairing, (c) and (d) are for the FFLO pairing. $X_{k_y} = -10l_B$, $E_F = 0.4E_0$, and $\Delta_0 = 4E_0$ are used in the calculations.

![Figure 5](image-url)  

**FIG. 5.** The DOS for the BCS and the FFLO superconducting pairings with $E_F = 0.0E_0$, $0.8E_0$, $1.3E_0$, $1.5E_0$, $1.8E_0$, and $2.1E_0$. $\Delta_0 = 0.5E_0$.  

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Figure 5 shows that when $E_F$ is within the range between the $N = 0$ and 1 LLs, the DOSs for the energy around Fermi energy exhibits different features for two pairings. Such a difference between the DOS for the two pairings becomes smaller at the higher LLs. The feature of the DOS is consistent with what was seen in the last subsection: Only the zeroth LL in the excitation spectra is relevant to the way of pairing. The property exposed in $E_F$-dependent DOS can be used to tune the states in the WSM/SWSM hybrid structure by varying the Fermi energy.

III. TRANSVERSE CURRENT DISTRIBUTION IN THE VICINITY OF THE INTERFACE BETWEEN WSM AND SWSM

To see the influence of the pairing and the effect of a magnetic field on the electronic transport properties, we investigate the spatial current distributions in the area adjacent to the interface of a WSM/SWSM hybrid structure. In the following, we first derive the analytic formula of the current density. Then, we calculate the spatial distributions of transverse charge current in the WSM/SWSM hybrid structure with two superconducting pairings numerically.

A. The derivation of the transverse current density

From the time-dependent BdG equation, $i\hbar \partial_t \Psi = \mathcal{H}_{\text{BdG}} \Psi$, the continuity equation for the probability current of electronlike and holelike quasiparticles can be derived [73],

$$\partial_t \rho^p = \langle \bar{\rho} \partial_t \rho \rangle_{\Omega}$$

where $\rho^p = (\rho^p_{\Omega,\sigma}, \rho^h_{\Omega,\sigma})^T$ is the probability density of quasiparticles, $\mathcal{J}^p = (\mathcal{J}^p_{\Omega,\sigma}, \mathcal{J}^p_{\Omega,\sigma})^T$ is the probability current density, $S_{\Omega}^p = (S_{\Omega}^+, -S_{\Omega}^{-})$, and $\Omega$ stands for the quasiparticle state of energy $E_N(x_k, y_k)$. $\rho^p_{\Omega,\sigma} = \sum_{x_k = \pm} u_{\Omega,\sigma}^+ u_{\Omega,\sigma}$, $\rho^h_{\Omega,\sigma} = \sum_{x_k = \pm} v_{\Omega,\sigma}^+ v_{\Omega,\sigma}$, and $S_{\Omega} = (2/\hbar) \sum_{x_k = \pm} \text{Im} (\Delta u_{\Omega,\sigma}^+ v_{\Omega,\sigma})$.

$\mathcal{J}^p_{\Omega,\sigma}$ and $\mathcal{J}^p_{\Omega,\sigma}$ are the probability current of electronlike and holelike quasiparticles, respectively, written as,

$$\mathcal{J}^p_{\Omega,\sigma}(x, y, z) = v_F \sum_{x_k = \pm} [u_{\Omega,\sigma}^+(\sigma, \sigma, \kappa \sigma) u_{\Omega,\sigma}]$$

and

$$\mathcal{J}^p_{\Omega,\sigma}(x, y, z) = -v_F \sum_{x_k = \pm} [v_{\Omega,\sigma}^+(\sigma, \sigma, \kappa \sigma) v_{\Omega,\sigma}]$$

Specific to our WSM/SWSM hybrid structure where the magnetic field is in the $z$ direction, we focus on the $y$ component of probability current density. $\mathcal{J}^p_{\Omega,\sigma}(x) = \int dy dz J^p_{\Omega,\sigma}(x, y, z)$ and $\mathcal{J}^p_{\Omega,\sigma}(x) = \int dy dz J^p_{\Omega,\sigma}(x, y, z)$. The charge current density can be obtained using the probability current density multiplied by the corresponding charge. According to Datta and Bagwell [73], the charge current density can be further expressed as the sum of a BdG quasiparticle current (BdGC) and a vacuum current (VAC) [73] $J^p_{\Omega,\sigma} = J^p_{\text{BdGC,} \sigma} + J^p_{\text{VAC,} \sigma}$, where

$$J^p_{\text{BdGC,} \sigma}(x) = (-e) \sum_{\Omega} [J^p_{\Omega,\sigma}(x) - J^p_{\Omega,\sigma}(x)] f_{\Omega}$$

respectively; $f_{\Omega} = [e^{\beta (E_F - E_{\Omega})} + 1]^{-1}$ is the Fermi-Dirac distribution. The transverse ($y$ component of) charge current distribution is then rewritten in the form

$$J^p_{\sigma}(x) = (-e) \sum_{\Omega} [f_{\Omega} J^p_{\Omega,\sigma}(x) + (1 - f_{\Omega}) J^p_{\Omega,\sigma}(x)].$$

B. The spatial distributions of transverse charge current in the WSM/SWSM hybrid structure

With these formulas and the wave functions obtained in Sec. II, we shall calculate the spatial distribution of transverse charge current at zero temperature for two different pairings, respectively. Based on the parameters from experiments: $B \sim 1$ T [74], $v_F \sim 10^7$ m/s [12,13], and $T_c \sim 9$ K for the proximity effect of a superconductivity of a WSM caused extrinsically [55,74,75] and intrinsically [18,76], the parameters used in our calculations are given as: $E_0 = \hbar v_F / l_B \approx 2.5$ meV, $\delta E_F = 0.01 E_0$, and $\Delta_0 = 0.5 E_0$.

The distributions of transverse current, for the BCS and the FFLO pairings, as a function of Fermi energy [in units of $J_0 = eE_0 / (2\pi \hbar \Omega)$] are shown in Figs. 6(a) and 6(b), respectively. It is found that the transverse current is mainly concentrated in the vicinity of the interface. The transverse current density behaves quite differently in the WSM and the SWSM regions. In the WSM side, the transverse current density oscillates away from the interface and tends to vanishing asymptotically. The node number of oscillations in the spatial distributions of current depends on the Weyl LL indices and increases with an increase of Fermi energy. In the SWSM side, the current density displays a periodic oscillation, which is a result of magnetic quantization of the energy in the WSM.
and the supercurrent flowing in the SWSM. The same spatial pattern of the current distribution under the BCS and the FFLO pairings indicates that the transverse current is not affected by the nature of pairings. The spatial distributions of transverse current depend mainly on the consequence that the phase-coherent electron-hole states are produced, which are responsible for the magnetically induced edge states supported by the Weyl LLs.

In Sec. II, we have shown that the energy dispersion of the zeroth LL and the DOS of the quasiparticle excitations are dependent on pairing; the results of the spatial distributions of transverse current indicate that those differences arisen from the zeroth LL disappear when the sum is taken over all possible states for a given energy. Only those effects related to the edge excitations associated with the Weyl LLs are survived. As a consequence, the transverse current distribution does not depend on the details of pairing. From a semiclassical point of view, the cyclotron motion with the trajectory in a collision at the interface is not affected by the microscopic pairing mechanism.

IV. THE TRANSVERSE CONDUCTANCE IN A SWSM/WSM/SWSM HYBRID STRUCTURE

A. Calculation of the transverse conductance from the current distribution

The total transverse current can be obtained from the spatial distributions, \( I_y^Q = I_{BAGC,y}^Q + I_{VAC,y}^Q = \int dx J_y^Q(x) \). To study the transverse conductance, we consider a SWSM/WSM/SWSM hybrid to calculate the transverse current with different chemical potentials in two SWSMs. The WSM layer is assumed to be wide enough so that edge states of the two interfaces are not overlapped. Under a magnetic field, the distributions of charge current are constrained mainly within the areas adjacent to the interfaces of WSM/SWSM. The directions of transverse current near the left interface are opposite to those near the right interface. Under a finite bias, the chemical potentials in two SWSM differ by an amount, \( \delta E_F = E_{FR} - E_{FL} \neq 0 \). A net transverse current is generated under the bias. In the present setup, the bias voltage \( V = \delta E_F / (e - e) \) is in the \( x \) direction, while the transverse current is in the \( y \) direction. The transverse current is given by \( I_H = \sum_{i=L,R} \int dx J_{y,i}^Q(x) \), where the integration is taken along the secant line perpendicular to the interfaces over the whole SWSM/WSM/SWSM structure. Because \( J_{y}^Q(x) = J_{BAGC,y}^Q(x) + J_{VAC,y}^Q(x) \), \( I_H \) can be written in the form \( I_H = \sum_{i=L,R} (I_{BAGC,y}^Q + I_{VAC,y}^Q) \), where the subscripts \( R \) and \( L \) stand for the integrated current of the right SWSM/WSM hybrid and the left WSM/SWSM hybrid, respectively.

The transverse conductance is defined by \( \sigma_{AR} = I_H / V \). We show the transverse conductance \( \sigma_{AR} \) in Fig. 7(a) in a unit \( \sigma_0 = e^2 / (\pi h \ell_B) \). Figure 7(a) shows that the conductance is proportional to the Fermi energy \( E_F \) when \( E_F \) is between the \( N = 0 \) and 1 LLs. When the \( E_F \) is between the \( N = 1 \) and 2 LLs, one more LL contributes and the slope of the conductance becomes steeper. Figure 7(a) indicates that the BCS and the FFLO pairings give the same \( E_F \)-dependent \( \sigma_{AR} \). The pairing independence of the transverse conductance can be understood as follows. The transverse conductance contains two contributions, the intrinsic chiral anomalous Hall conductance \([45,77]\) and the Hall-like conductance. The intrinsic chiral anomalous Hall conductance, \( \sigma_{IAC}^{\text{int}} = e^2 K / (2\pi^2 \hbar^2) \) (when the chemical potential is exactly at the Dirac point), originates from the chirality of the zeroth LL in the two Weyl node points and is independent of magnetic field. The Hall-like conductance induced by the magnetic field is a direct physical consequence of the topology of the Weyl Landau band structure. In the WSM/SWSM hybrid structures, the intrinsic chiral anomalous Hall current can be generated in both the WSM \([78,79]\) and the WSM \([80]\). The intrinsic part of currents in the two flanks of the interface have the same magnitude but opposite directions, so that the intrinsic chiral anomalous Hall currents formatted in the area adjacent to the interface offset each other. As a result, the intrinsic chiral anomalous Hall conductance is absent in the transverse conductance \([81]\). However, the Hall-like conductance associates

FIG. 7. (a) Transverse conductances \( \sigma_{AR} \) versus \( E_F \) for the BCS and the FFLO superconducting pairing. The black circles for the BCS-CD and the red squares for the FFLO-CD are calculated by the current distribution (CD). The magenta diamonds for the BCS-LB and the blue down triangles for the FFLO-LB are calculated by the Landauer-Büttiker formula (LB). The green dashed curve is the transverse conductivity for an infinite WSM without superconducting electrodes and the cyan solid curve is the double result of this green dashed curve. (b) The derivation of the conductances with respect to the Fermi energy. (c) Current distribution \( I(k_z) \) as a function of \( k_z \) around \( k_z = 5/\ell_B \). \( E_F = 0.4E_0 \), \( \Delta_0 = 0.5E_0 \), \( V = -\delta E_F/e = -0.01(E_0/e) \), \( \sigma_0 = e^2 / (\pi h \ell_B) \), and \( I_0 = eE_0 / (2\pi h) \).
with the magnetic field [82]. We have assumed no magnetic field in the SWSM, the Hall-like current then presents in the WSM only. As a consequence, the contribution induced by the magnetic field survives. Physically, the electron and Andreev reflected hole in the WSM side are governed by the same law of motion and characteristic properties connected with the topological structure of the WSM. The difference of momenta read

\[ I_L = I_{UL} - I_{LD} = -\frac{e}{h} R_{AL}(\mu_L - \mu_D), \]

\[ I_R = I_{DR} - I_{RU} = -\frac{e}{h} R_{AR}(\mu_R - \mu_U), \]

\[ I_U = I_{DU} + I_{RU} - I_{UD} - I_{UL} = -\frac{e}{h} [M(\mu_U - \mu_D) - R_{UL}(\mu_L - \mu_D)], \]

\[ I_D = I_{UD} - I_{LD} + I_{UL} - I_{DR} = -\frac{e}{h} [M(\mu_U - \mu_D) - R_{AR}(\mu_R - \mu_U)], \]

respectively, where \( R_{AL} \) and \( R_{AR} \) are the AR coefficients at the left and the right interfaces of WSM/SWSMs.

For the BCS pairing, the number of edge channels \( M \) is given by

\[ M = (1/2\pi) \int \frac{dk_z}{2\pi} \sum_{N,k_z} M_N(X_{k_z}, k^2_z), \]

\[ M = (1/2\pi) \int \frac{dk_z}{2\pi} \sum_{N,k_z} 1. \]

The integration and the sum in Eqs. (20) and (21) are taken over all the allowed channels specified by the energies \( E_N(X_{k_z}, k_z) \). The reflection probabilities of electronlike and holole like quasiparticles at the left and the right interfaces of WSM/SWSMs are given by

\[ R_{N,X_{k_z},k_z,AL}^{(e)} = T_{N,X_{k_z},k_z,LD}^{(ee)} + T_{N,X_{k_z},k_z,LD}^{(eh)}, \]

\[ R_{N,X_{k_z},k_z,AR}^{(e)} = T_{N,X_{k_z},k_z,RU}^{(be)} + T_{N,X_{k_z},k_z,LU}^{(eh)}, \]

\[ R_{N,X_{k_z},k_z,AL}^{(h)} = T_{N,X_{k_z},k_z,LD}^{(eh)}, \]

\[ R_{N,X_{k_z},k_z,AR}^{(h)} = T_{N,X_{k_z},k_z,RU}^{(eh)}, \]

where \( T_{N,X_{k_z},k_z,ij}^{(ab)} \) is the transmission probability of the mode \((N, X_{k_z}, k_z)\) from the type \(b\) quasiparticles in terminal \(j\) to the type \(a\) quasiparticles in terminal \(i\), and \(a, b\) represent the electronlike (e) and the holole like (h) quasiparticles, and \(i, j\) (\(= L, R, U, D\)) note the left, right, up, and down terminals, respectively. The particle conservation is ensured by the relations among these transmission probabilities,

\[ T_{N,X_{k_z},k_z,LD}^{(ee)} + T_{N,X_{k_z},k_z,LD}^{(eh)} = 1, \]

\[ T_{N,X_{k_z},k_z,RU}^{(be)} + T_{N,X_{k_z},k_z,LU}^{(eh)} = 1, \]

\[ T_{N,X_{k_z},k_z,LD}^{(eh)} + T_{N,X_{k_z},k_z,LD}^{(eh)} = 1, \]

and

\[ T_{N,X_{k_z},k_z,RU}^{(eh)} + T_{N,X_{k_z},k_z,RU}^{(eh)} = 1. \]

Therefore, the AR coefficients are found as

\[ R_{AL} = \int \frac{dk_z}{2\pi} \sum_{N,k_z} (T_{N,X_{k_z},k_z,LD}^{(ee)} + T_{N,X_{k_z},k_z,LD}^{(eh)}). \]
and

\[
R_{AR} = \int \frac{dk_z}{2\pi} \sum_{N,k_z} (T^{(he)}_{N,k_z} + T^{(eh)}_{N,k_z}) \cdot (R_{U} + R_{RU}).
\]  

(23)

The derivation of these probabilities related to the incoming edge excitations is presented in Appendix B.

By requiring \( I_L = I_R = 0 \), we have \( \mu_D = \mu_L \) and \( \mu_U = \mu_R \). The charge current flowing between terminals \( U \) and \( D \) is found as \( I_U = -I_D = I_H = -\epsilon(h^2/4\pi)(\mu_U - \mu_R) \). The transverse conductance is defined as \( \sigma_{AR} = I_H / (|\mu_R - \mu_L|/(-e)) \). Hence, \( \sigma_{AR} \) is found as

\[
\sigma_{AR} = \frac{l_B}{2} \sum_{N,k_z} \int d{k_z} \left\{ \frac{1}{E_F} \sum_{N} M_N \left( x_k, k_z^2 \right) (\text{FFLO}), \frac{1}{E_F} \sum_{N} M_N \left( x_k, k_z^2 \right) (\text{BCS}) \right\} .
\]  

(24)

To calculate \( \partial \sigma_{AR} / \partial E_F \), we employ the excitation spectra obtained in Sec. II C with the energy \( |E| < \Delta_0 \) in the calculations of \( M_N \left( x_k, k_z \right) \). We show the results with a subscript “LB” in Fig. 7(a), where the same parameters as those given in Sec. III B are used. It is found that the results calculated from the edge-channel description are the same as those obtained by using the current distribution in Sec. III. The results unambiguously confirm the visualization that the contribution of edge excitations associated with the LL structures to the transverse conductance is significant.

It is interesting to note that \( d\sigma_{AR}/dE_F \) exhibits the quantized signatures in the LL intervals. The slope in each interval between the adjacent LLs can be understood from the quantization of 3D electronic and holelike quasiparticle motion. Figure 7(b) shows that the plateaus appear with a finite value \( 2\sigma_0 / E_0 \) for \( N = 0 \) LL, \( 8\sigma_0 / E_0 \) (tendency) for \( N = 1 \) LL, and \( 14\sigma_0 / E_0 \) (tendency) for \( N = 2 \) LL. This reflects the nature of the magnetic quantization of a 3D system.

Now let us analyze the contributions from the zeroth LL and see why the BCS and FFLO pairings give the same transverse conductance when \( E_F / \sqrt{\Delta_0} \) is independent for the FFLO pairing with a relation \( (K \pm k_z)B + E_F / \sqrt{\Delta_0} \tan h(1.3X_k l_B) = 0 \). These characteristics have been reflected in the DOS. To demonstrate the peculiarity of edge excitations, which relate to the interplay between the effect of magnetic field and the coherent superposition of quasiparticle states in the hybrid, we calculate the wave-function profiles of edge states and bulk states. It is found that the wave-function profiles of the edge excitations are different for two distinct pairings. But, the wave-function profiles of the bulk states are the same for two distinct pairings.

From the energy dispersion, we can obtain the mode-dependent velocities associated with the LL mode of quasiparticle excitations. It is found that the mode-dependent velocities in the \( y \) direction are nonzero in the vicinity of the WSM/WSWM interface and vanish for \( X_k \rightarrow \pm \infty \). This implies that the edge states evolve into the \( k_z \)-independence LLs in the region far from the interface. We have presented an exact expression for the transverse current and evaluated the spatial distributions of charge current numerically. It is shown that the distribution of transverse current is irrelevant to the nature of pairing. By using a WSM/WSM/WSWM hybrid, we calculate the total transverse conductance. The results show that the total transverse conductance is independent of the specific pairing mechanism. The intrinsic chiral anomalous Hall current is absent in the transverse conductance due to its destruction in the two flanks of the interface in the WSM/WSWM hybrid. To provide a quantitative understanding of such a
pairing irrelevancy, we analyze the contribution to the transverse conductance from various edge channels in a four-terminal SWSM/WSM/SWSM hybrid structure. The calculations with the edge-channel descriptions reproduce the results obtained by the spatial distributions. The pairing independence of the transverse conductance reflects that it depends on the processes producing the phase-coherent electron-hole states at the SWM/SWSM interface, which are responsible for the magnetically induced edge states supported by the Weyl LLs and the carriers’ cyclotron motions under magnetic field.

Our result provides possible explanations for the experimentally observed breakdown of chiral anomaly in the WSM hybrid structures. The WSM/SWSM hybrid is a better platform to probe these effects because the surface states are gapless due to the Weyl chirality preserved in two sides of interface. The predicted irrelevance of pairings in the transverse conductance could be investigated in conductivity of interface. The predicted irrelevance of pairings in the transverse conductance could be investigated in conductivity of interface. The predicted irrelevance of pairings in the transverse conductance could be investigated in conductivity of interface. The predicted irrelevance of pairings in the transverse conductance could be investigated in conductivity of interface. The predicted irrelevance of pairings in the transverse conductance could be investigated in conductivity of interface. The predicted irrelevance of pairings in the transverse conductance could be investigated in conductivity of interface. The predicted irrelevance of pairings in the transverse conductance could be investigated in conductivity of interface.

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APPENDIX A: THE EIGENSTATES OF BDG EQUATIONS IN THE WSM AND THE SWSM

The BdG equation for the WSM and the SWSM in the presence of magnetic field with the BCS and the FFLO pairing potentials can be solved analytically. We present the eigenstates and eigenvalues below.

In the WSM \((x < 0)\), we take \(\Delta_0 = 0\) in Eq. (2). The eigenvalues are given in Eq. (5). The eigenstates have the form

\[
\Phi^{W}_{k,\nu,\lambda}(n, k_y, k_z, r) = (1/\sqrt{L_xL_yL_z})e^{ik_y y + ik_z z} \Phi^{W}_{k,\nu,\lambda}(n, k_y, k_z, x)
\]

where \(k_y = k_z - \nu K, D_n(x)\) is a parabolic cylindrical function, and \(X_{k_z} = -k_z i_0^2\). In these expressions, lengths are measured in units of the magnetic length \(l_B\), and energies are measured in units of \(E_0 = \hbar v_F/l_B\).

In the SWSM \((x > 0)\), we solve the BdG equation, respectively, for the BCS pairing and the FFLO pairing. For a BCS pairing, we have \(\Delta_B = \Delta_0\) and \(\Delta_F = 0\) in Eq. (4). The eigenvalues of BdG equations are given in Eq. (6). The eigenstates have the form

\[
\Phi^{S(B)}_{\kappa,\eta,\lambda}(x, k_y, k_z, k_x) = (1/\sqrt{L_yL_z})e^{ik_y y + ik_z z} \Phi^{S(B)}_{\kappa,\eta,\lambda}(k_x, k_y, k_z)
\]

where

\[
\Phi^{S(B)}_{\kappa,\eta,\lambda}(k_x, k_y, k_z) = \begin{pmatrix}
E_F k_x^2 + (E_F E_{k_x} - E_{F_k} - \Delta_0^2 k_z^2)^2 + \sqrt{\Delta_0^2 k_z^2 + E^2_F k_z^2}(k_x + i k_y) \\
E^2_F k_x^2 + \sqrt{\Delta_0^2 k_z^2 + E^2_F k_z^2}(k_x + i k_y) \\
0 \\
0 \\
( - E^2_F k_x^2 + \beta \sqrt{\Delta_0^2 k_z^2 + E^2_F k_z^2}) \Delta_0 \\
0 \\
\Delta_0 (E_F + k_z^2) (k_x + i k_y) \\
0
\end{pmatrix}
\]
and

\[
\Phi_{-\eta, \lambda}(k_x, k_y, k_z) = \begin{pmatrix}
E_F k^{-2} - (E_F E_{\eta\lambda} - E_F^2 - \Delta^2_0) k_z^{-2} + (E_{\eta\lambda} - E_F - k_z^2) \beta \sqrt{\Delta_0^2 k_z^{-2} + E_F^2 k^{-2}} \\
(E_{\eta\lambda} E_F - E_F^2 + \beta \sqrt{\Delta_0^2 k_z^{-2} + E_F^2 k^{-2}})(k_x + i k_y) \\
0 \\
0 \\
(E_{\eta\lambda} k_z^{-2} - k_z^{-2} + \beta \sqrt{\Delta_0^2 k_z^{-2} + E_F^2 k^{-2}}) \Delta_0 \\
\Delta_0 (E_F - k_z^2)(k_x + i k_y)
\end{pmatrix}.
\]

\[ (A6) \]

For a FFLO pairing, we have \( \Delta_B = 0 \) and \( \Delta_F = \Delta_0 \) in Eq. (4). The eigenvalues are presented in Sec. II B 2 b. The eigenstates are given by \( \Psi_{k_x, k_y, k_z}(x, y, z, k_x, k_y, k_z) = (1/\sqrt{L_y L_z}) e^{i k_x x + i k_y y + i k_z z} \Phi_{-\eta, \lambda}(k_x, k_y, k_z) \) with

\[
\Phi_{+\eta, \lambda}(k_x, k_y, k_z) = \begin{pmatrix}
\text{sgn}(E_F)k^+ + (\text{sgn}(E_F)E_{\eta\lambda} - |E_F|)k^+_z + (E_{\eta\lambda} - E_F + k^+_z) \beta k^+
\\
\text{sgn}(E_F)E_{\eta\lambda} - |E_F| + \beta k^+ (k_x + i k_y)
\\
0
\\
0
\\
0
\\
0
\\
0
\\
\Delta_0 \text{sgn}(E_F)(k_x + i k_y)e^{-i 2k_z}
\end{pmatrix}
\]

\[ (A7) \]

and

\[
\Phi_{-\eta, \lambda}(k_x, k_y, k_z) = \begin{pmatrix}
\text{sgn}(E_F)k^{-2} - (\text{sgn}(E_F)E_{\eta\lambda} - |E_F|)k_z^{-2} + (E_{\eta\lambda} - E_F - k_z^2) \beta k^-
\\
(\text{sgn}(E_F)E_{\eta\lambda} - |E_F| + \beta k^-)(k_x + i k_y)
\\
(-\text{sgn}(E_F)k_z^{-2} + \beta k^-) \Delta_0 e^{i 2k_z}
\\
\Delta_0 \text{sgn}(E_F)(k_x + i k_y)e^{i 2k_z}
\end{pmatrix}
\]

\[ (A8) \]

**APPENDIX B: LANDAUER-BÜTTIKER APPROACH**

In this appendix, we assume that the population of the edge states are in equilibrium, so that we can use Landauer-Büttiker formula for the superconducting terminals. We show the diagrammatic sketch of four-terminal hybrid structure in Fig. 8, where two superconducting terminals are taken as the left and right leads. We use notation \( I_{N,X_i,k,\alpha}^{(x)} \) to define the current flows along the route \( \alpha \), where \( \alpha = (ij) \) represents the route from the lead \( i \) to the lead \( j \), \( i, j = L, R, U, D \), \( \chi \) labels the nature of quasiparticle [electron \((e)\) and hole \((h)\)], and \((N, X_i, k_x, k_y)\) identifies the state for the quasiparticle excitations. According to the Landauer-Büttiker scattering matrix approach, the current \( I_{N,X_i,k,\alpha}^{(x)} \) can be expressed in terms of the transmission coefficients \( T_{N,X_i,k,\alpha}^{(x)} \). It should be emphasized that the contributions of electron partners are taken in the states of \( N \geq 0 \) LLs and those of the hole partners are taken in the states of \( N < 0 \) LLs. For the difference of chemical potentials \( \mu_i \) among the leads \( L, R, U, \) and \( D \), we have

\[
I_{N,X_i,k,\alpha}^{(x)} = -\frac{e}{h} \left[ \left(1 - T_{N,X_i,k,LD}^{(x)}(\mu_L - \mu_R) \right) - T_{N,X_i,k,LD}^{(x)}(\mu_R - \mu_L) \right],
\]

\[ (B1) \]

\[
I_{N,X_i,k,\alpha}^{(x)} = -\frac{e}{h} \left[ \left(1 - T_{N,X_i,k,LD}^{(x)}(\mu_D - \mu_R) \right) - T_{N,X_i,k,LD}^{(x)}(\mu_R - \mu_D) \right],
\]

\[ (B2) \]

\[
I_{N,X_i,k,LD}^{(x)} = -\frac{e}{h} \left[ \left(1 - T_{N,X_i,k,LD}^{(x)}(\mu_D - \mu_R) \right) + T_{N,X_i,k,LD}^{(x)}(\mu_R - \mu_D) \right],
\]

\[ (B3) \]

\[
I_{N,X_i,k,LU}^{(x)} = -\frac{e}{h} \left[ \left(1 - T_{N,X_i,k,LU}^{(x)}(\mu_U - \mu_R) \right) - T_{N,X_i,k,LU}^{(x)}(\mu_R - \mu_U) \right],
\]

\[ (B4) \]
The currents flowing through the route $\alpha$ ($UL, LD, UD, RU, DU$, and $DR$) are given by

$$I_\alpha = \sum_{N,k_y} \int \frac{dk_z}{2\pi} I_{N,Xy,k_z}^{(x)}.$$

Substituting Eqs. (B11)–(B16) into Eq. (B17), we found

$$I_{UL} = -\frac{e}{h} R_{AL}(\mu_L - \mu_R),$$

$$I_{LD} = -\frac{e}{h} R_{AL}(\mu_D - \mu_R),$$

$$I_{RU} = -\frac{e}{h} R_{AR}(\mu_U - \mu_R),$$

$$I_{DR} = -\frac{e}{h} R_{AR}(\mu_R - \mu_R),$$

$$I_{UD} = -\frac{e}{h} (M - R_{AL})(\mu_D - \mu_R),$$

$$I_{DU} = -\frac{e}{h} (M - R_{AR})(\mu_U - \mu_R),$$

where

$$R_{AL} = \sum_{N,k_y} \int \frac{dk_z}{2\pi} R_{N,Xy,k_z}^{(x)},$$

$$R_{AR} = \sum_{N,k_y} \int \frac{dk_z}{2\pi} R_{N,Xy,k_z}^{(x)},$$

and

$$M = \int \frac{dk_z}{2\pi} \sum_{N,k_y} \left(-\delta_{N0}(E_F + \kappa h v_F k_z^*) + (1 - \delta_{N0})\right).$$

for BCS pairing superconducting states while

$$M = \int \frac{dk_z}{2\pi} \sum_{N,k_y} 1$$

for FFLO pairing superconducting states.

The total currents flowing through lead $i$ ($= L, R, U, D$) are obtained by

$$I_L = I_{UL} - I_{LD},$$

$$I_R = I_{DR} - I_{RU},$$

$$I_U = I_{DU} + I_{RU} - I_{UL} - I_{UD},$$

and

$$I_D = I_{UD} + I_{LD} - I_{DR} - I_{DU}.$$