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Fei Xia

University of Wollongong, fx999@uowmail.edu.au

Bin Liu

University of Wollongong, bl987@uowmail.edu.au

Lingzhi Cao

Zhengzhou University of Light Industry

Yanguang Yu

University of Wollongong, yanguang@uow.edu.au

Jiangtao Xi

University of Wollongong, jiangtao@uow.edu.au

See next page for additional authors

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Experimental study on simultaneously measuring Young's modulus and internal fraction using self-mixing system

Abstract

Material parameters such as Young's modulus and internal friction are important for estimation of material performance. This paper presents an experimental study for measuring material related parameters using a selfmixing interferometric (SMI) configuration. An SMI system consists of a laser diode (LD), a lens and an external target to be measured. When a part of the lasing light back-reflected or back-scattered by the external target re-enters the LD internal cavity, both optical frequency and intensity of the lasing light can be modulated. This modulated laser intensity is often referred as SMI signal. Generally, the target related movement or its surface information can be retrieved from this SMI signal. In this paper, an SMI system is implemented. A tested sample is used as the target to form the external cavity of the LD. The tested sample is stimulated in vibration. Continuous wavelet transform (CWT) is utilized to retrieve the vibration information of the tested sample from an SMI signal. We are able to obtain both Young's modulus and internal friction from a piece of an experimental SMI signal. This work provides a novel, simple non-destructive solution for simultaneous measurement of Young's modulus and internal friction.

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Authors

Fei Xia, Bin Liu, Lingzhi Cao, Yanguang Yu, Jiangtao Xi, Qinghua Guo, and Jun Tong

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Experimental study on simultaneously measuring Young's modulus and internal friction using self-mixing system

Fei Xia^a, Bin Liu^a, Lingzhi Cao^{*b}, Yanguang Yu^a, Jiangtao Xi^a, Qinghua Guo^a, Jun Tong^a

^aSchool of Electrical, Computer and Telecommunications Engineering, University of Wollongong, Wollongong, NSW, 2522, Australia; ^bSchool of Electrical and Information Engineering, Zhengzhou University of Light Industry, Zhengzhou, Henan, 450000, China

ABSTRACT

Material parameters such as Young's modulus and internal friction are important for estimation of material performance. This paper presents an experimental study for measuring material related parameters using a self-mixing interferometric (SMI) configuration. An SMI system consists of a laser diode (LD), a lens and an external target to be measured. When a part of the lasing light back-reflected or back-scattered by the external target re-enters the LD internal cavity, both optical frequency and intensity of the lasing light can be modulated. This modulated laser intensity is often referred as SMI signal. Generally, the target related movement or its surface information can be retrieved from this SMI signal. In this paper, an SMI system is implemented. A tested sample is used as the target to form the external cavity of the LD. The tested sample is stimulated in vibration. Continuous wavelet transform (CWT) is utilized to retrieve the vibration information of the tested sample from an SMI signal. We are able to obtain both Young's modulus and internal friction from a piece of an experimental SMI signal. This work provides a novel, simple non-destructive solution for simultaneous measurement of Young's modulus and internal friction.

Keywords: self-mixing interferometry, laser diode, Young's modulus, internal friction, wavelet transform

1. INTRODUCTION

Material related parameters such as Young's modulus and internal friction are important for mechanical and material engineering. These parameters play key roles in the material performances. It has been a great interest to measure the value of these parameters. Traditional methods including tensile test [1], flexure test [2], and others [3, 4] are static methods which are often conducted based on the direct measurement of the stresses and strains of the tested sample loaded by a known force. As a result, the tested sample is often damaged after the test, as a relatively high stress must be applied to reach a measurable deformation. Therefore, more interests have been focusing on the dynamic methods, which are non-destructive and more accurate with a simpler system.

In recent years, the impulse excitation technique (IET), a non-destructive technique to determine Young's modulus and internal friction of the material has attracted great attention. The detection system used for IET is normally microphone, accelerometer and so on. Self-mixing interferometry (SMI), an emerging sensing technique, which is non-destructive and non-contact has been developed for high accuracy sensing applications, such as displacement [5], velocity [6] and distance [7] measurement and so on. It is also suitable for the material related parameters measurement. Figure 1 shows a basic SMI system, which consists of a LD and a target to form the external cavity of the LD. When a part of the lasing light emitted by the LD back-reflects or back-scatters by the external target and re-enters the LD internal cavity, both optical frequency and the intensity of the lasing light can be modulated. This modulated laser intensity contains the vibration information of the external target.

Figure 1. Basic SMI system

This paper proposes an intact system design for Young's modulus and internal friction measurement based on SMI technique with IET. An experimental SMI system is built, where a tested sample for the experiments is used as the external target. With a light strike on the tested sample, the tested sample is stimulated into damping vibration. This damping vibration signal is captured by the SMI system and a corresponding SMI signal is generated. By applying wavelet transform onto the SMI signal, both fundamental resonant frequency and the magnitude decay of the damping vibration from the tested sample can be obtained, from which we can calculate material related parameters Young's modulus and internal friction.

2. MEASUREMENT PRINCIPLE

The widely accepted SMI model is derived from the Lang Kobayashi equations [8, 9]. The SMI model is relisted below to investigate the relationship between Young's modulus and internal friction and the SMI signal.

$$\phi_0 = 4\pi L / \lambda \quad (1)$$

$$\phi_s = \phi_0 - C \sin(\phi_s + \arctan \alpha) \quad (2)$$

$$G(t) = \cos(\phi_s) \quad (3)$$

$$P(t) = P_0 [1 + mG(t)] \quad (4)$$

where the physical meaning of each parameter is described as following in the Table 1.

Table 1. Meanings of the symbols in the L-K equations

Symbols	Physical meaning
ϕ_0	light phase without external optical feedback
L	Distance between the LD front facet and the external target
λ	Laser wavelength
ϕ_s	Light phase with external optical feedback
C	Optical feedback factor
α	Linewidth enhancement factor
$G(t)$	Interferometric function
$P(t)$	Laser power emitted by the LD with external optical feedback
P_0	Laser power emitted by the LD without external optical feedback
m	Modulation index

Generally, we treat $G(t)$ as the SMI signal as it can be normalized from the laser power $P(t)$. In our system, the external target is the tested sample, so that $L = L_0 + y(t)$, where L_0 is the initial external cavity length, that is the distance between the LD and the tested sample in static state, $y(t)$ is the damping vibration of the tested sample. Based on the equation of motion [10], the damping vibration $y(t)$ can be written as

$$y(t) = A_0 e^{-kt} \cos(2\pi f_{RO} t) \quad (5)$$

Where A_0 is the maximum amplitude of the damping vibration, k is the damping factor, f_{RO} describes the resonant frequency of the tested sample. The Young's modulus E of the tested sample can be calculated as [11]

$$E = 0.9465 \cdot \frac{mf_{RO}^2 L^3}{bh^3} \cdot T \quad (6)$$

where $T = 1 + 6.585 \left(\frac{h}{L} \right)^2$, when $L/h \geq 20$ [12]. m , L , b , and h are the mass, length, width, and height of the tested sample, which can be determined from the dimension of the tested sample. The remaining value needs to be determined is f_{RO} , which can be found in equation (5). The internal friction Q^{-1} of the tested sample can be calculated as [13]

$$Q^{-1} = \frac{k}{\pi f_{RO}} \quad (7)$$

where the values need to be determined are k and f_{RO} , which can be found in equation (5). The Young's modulus E and the internal friction Q^{-1} can be calculated by knowing information in the damping vibration of the tested sample $y(t)$, which can be determined from the SMI signal $G(t)$.

The simulation is conducted to investigate the relationship between the SMI signal $G(t)$ and the damping vibration of the tested sample $y(t)$. For the SMI part, we assume the laser wavelength $\lambda = 7.85 \times 10^{-7} m$, the initial external cavity length $L_0 = 0.1 m$, the optical feedback $C = 3$ and the linewidth enhancement factor $\alpha = 6$. The sampling frequency is $100 kHz$. For the IET part, we assume the maximum amplitude of the damping vibration $A_0 = 4.317 \times 10^{-6} m$, the damping factor $k = 4.6$ and the resonant frequency $f_{RO} = 444 Hz$. We can write the damping vibration as $y(t) = 4.317 \times 10^{-6} e^{-4.6t} \cos(2\pi(444t))$. The simulated damping vibration and its corresponding SMI signal $G(t)$ are shown in Figure 2. From Figure 2 we can observe that, the fundamental period of SMI signal $G(t)$ is the same as the period of the damping vibration signal $y(t)$. The fringe numbers on the SMI signal $G(t)$ indicates the magnitude of the damping vibration in each vibration period, where the fringe numbers decrease as the damping vibration magnitude decreases with time. It can be found that the information of the damping vibration $y(t)$, which are the resonant frequency f_{RO} and the damping factor k , are carried in the SMI signal $G(t)$.

Figure 2. Simulated damping vibration and its corresponding SMI signal

The resonant frequency f_{RO} are the dominating frequency in the SMI signal and the damping factor k are indicated as the high frequency decay in the SMI signal $G(t)$. Both information can be found in the frequency

domain of the SMI signal. To achieve this, the continuous wavelet transform (CWT) can be applied onto the SMI signal $G(t)$.

3. EXPERIMENT STUDY

3.1 Experimental setup

The experiment work is conducted on a brass bar with the mass $m = 29.84g$, length $L = 138.10mm$, width $b = 12.01mm$, and height $h = 2.08mm$. The overall experiment setup is shown in figure 3. The experiment setup consists of two parts: IET part and SMI part. In the IET part, it consists of a tested sample, a stimulate tool, and the support. The stimulate tool is used to strike the tested sample and a damping vibration can be generated. In the SMI part, it consists of a LD (Thorlabs, RLD65MZZT2), a photodiode (PD) packaged in the rear of the LD, a temperature controller (Thorlabs, TED200C), a LD controller (Thorlabs, LDC200C), a trans-impedance amplifier, an oscilloscope, a DAQ card (NI USB-6361) and a personal computer. A SMI signal is generated corresponding to the damping vibration and picked up by the PD. The temperature controller and LD controller are used to ensure a stable condition of the LD. The trans-impedance amplifier is to transfer the current signal from the PD into a voltage signal and have it amplified. A DAQ card is used to sample the signal. The sampled SMI signal is then displayed on a personal computer. The amplified signal can be also displayed on an oscilloscope. The SMI signal is normally a saw tooth-like waveform, by doing signal processing onto the SMI signal, we can retrieve the material related parameters such as Young's modulus and internal friction.

Figure 3. Overall experiment setup

3.2 Experimental results

Figure 4 shows the experimental SMI signal from the system. The laser current and the temperature are set to be 14mA and 25°C. The sampling rate is 200kHz and the data length of a SMI signal is 1s.

Figure 4. Experimental SMI signal. (a), (b) zoomed in parts of (c) SMI signal

In figure 4, the top two figures (a) and (b) are the zoomed in parts of the SMI signal. It is clear to see that the fringe numbers decrease with the time. The continuous wavelet transform (CWT) is applied onto the SMI signal to retrieve the material related parameters. The CWT is a method to investigate the frequency domain of a signal, especially when frequency content varies over time [14]. The CWT of a SMI signal can be written as

$$W(a,b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{+\infty} G(t) \psi\left(\frac{t-b}{a}\right) dt \quad (8)$$

where $W(a,b)$ is the coefficient of the transform, a represents the scale wavelet parameter, b represents the shifting wavelet parameter and $\psi\left(\frac{t-b}{a}\right)$ is the scaled and shifted version of mother wavelet $\psi(t)$. $W(a,b)$ is normalized by $\frac{1}{\sqrt{a}}$ to ensure all the wavelets have the same energy at every scale. Scale a is inversely proportional to the frequency component of the analyzed signal. There are different mother wavelets, the choosing of one of them depends on the nature of the analysed signal. The Morlet complex wavelet is chosen to analyse our SMI signals because it shows good results in analysing sinusoidal-like signal [15] and an SMI signal waveform is sinusoidal-like or sawtooth-like.. The Morlet wavelet is defined in the time domain as

$$\psi(t) = e^{i\omega_e t} e^{-\frac{t^2}{2}} \quad (9)$$

In the frequency domain

$$\Psi(\omega) = \int_{-\infty}^{+\infty} \psi(t) e^{-j\omega t} dt = \sqrt{2\pi} e^{-\frac{(\omega-\omega_e)^2}{2}} \quad (10)$$

Where ω_e is the central angular frequency of the wavelet. The admissibility condition for a wavelet is

$$\int_0^{+\infty} \frac{|\Psi(\omega)|^2}{|\omega|} d\omega < +\infty \quad (11)$$

It requires the Fourier transform of the wavelet should be zero at zero frequency, i.e. $|\Psi(\omega)| = 0$ when $\omega = 0$.

Using the admissibility condition in equation (11), the following can be obtained

$$\Psi(0) = \sqrt{2\pi} e^{-\frac{\omega_e^2}{2}} \quad (12)$$

Figure 5 shows the value of $\Psi(0)$ at different ω_e for Morlet wavelet. For $\omega_e \geq 5$, the admissibility condition is fulfilled. $\omega_e = 6$ is chosen for the simulation.

Figure 5. Admissibility condition for the Morlet wavelet

In wavelet analysis, the way to relate scale parameter to frequency is to determine the central frequency of the wavelet, $f_e = \omega_e/(2\pi)$, where we have set the value for $\omega_e = 6$. So $f_e = \omega_e/(2\pi) \approx 0.9549\text{Hz}$. The frequency

content (denoted by f_a) is $f_a = \frac{f_e}{a} = \frac{0.9549}{a}$. Due to the dyadic nature of wavelets [16], it is computationally

efficient to choose scale parameter a in terms of a power of 2. $a = a_0 \times 2^{jd}$, $j = 0, 1, 2, \dots, J-1$, where a_0 is the smallest resolvable scale corresponding to the Nyquist frequency which is half of the sampling frequency

$f_{\max} = \frac{f_s}{2} = \frac{100\text{kHz}}{2} = 50\text{kHz}$ (the sampling frequency f_s was set as 100kHz for the sampling rate is 200kHz

). Then we have $a_0 = \frac{f_0}{f_{\max}} = \frac{0.9549}{50\text{kHz}}$. d is the increment in scale and it is chosen to be $1/16$ to balance the

resolution and computing time. The total scale J can be set based on the frequency range to be analysed, where we already have the maximum frequency $f_{\max} = 50\text{kHz}$, and we set the minimum frequency to be $f_{\min} = 1\text{Hz}$.

Therefore, we will analyze the frequency range between 1Hz and $5 \times 10^4\text{Hz}$ which determines the number of

octaves $N = \text{floor} \left(\log_2 \left(\frac{f_{\max}}{f_{\min}} \right) \right) = 15$. The total number of scales are $J + 1 = \frac{N}{d} + 1 = 241$.

Compute the Morlet wavelet CWT of the experimental SMI signal $G(t)$, that is $W(a, b)$ by equation (8). The scalogram of the real part of $W(a, b)$ is plotted to visualize the spatial and temporal components of experimental SMI signal $G(t)$, as shown in Figure 6

Figure 6. Scalogram of the CWT of experimental SMI signal

In Figure 6, the horizontal axis represents the time index, the vertical axis represents the frequency index (note that the vertical axis is logarithmic) and the colour means the magnitude (from 0.02 to 0.16). It can be seen that the yellow colour located at frequency around 500 Hz appears in the time period from $t = 500$ ms to about $t = 900$ ms, which is the dominating frequency contained in the experimental SMI signal $G(t)$. It is the same as the frequency of the target vibration. It also can be found that a yellow line in decreasing trend from $t = 0$ ms to about $t = 500$ ms with frequency varying from 8 kHz to 1 kHz. This decreasing trend line indicates the decreasing of the fringe numbers in each vibration period in the experimental SMI signal $G(t)$.

Firstly, we determine the value of the dominating frequency by extracting the frequency component with maximum magnitude from the scalogram in Figure 6. The frequency component with maximum magnitude is $f = 424.8$ Hz, which is shown in Figure 7.

Figure 7. Real part $W(a, b)$ plotted against the frequency, at Hz, denoting the maximum frequency is 443.6 Hz

Then, we determine the value of the slope of the decreasing trend. In order to do so, we need to locate the high frequency component from the scalogram in Figure 6. That can be achieved by extracting the frequency component with maximum magnitude at each time instance. Figure 8 (a) shows the frequency component with maximum magnitude at each time instance, Figure 8 (b) is the curve-fitting result of Figure 8 (a). From the curve-fitting result, it can be seen that most of the points fall on the curve, which shows that the curve can represent the decent trend of the frequency. The decent slope of the curve is -2.30×10^{-5} . The sampling rates are 200 kHz so the measured damping factor is $k = -4.6$.

Figure 8. (a) Frequency with maximum amplitude against time. (b) curve-fitting result after excluding other points

The measured resonant frequency is $f = 424.8 \text{ Hz}$ and damping factor is $k = -4.6$. Based on the equations (6) and (7), the Young's modulus and internal friction can be calculated. The experiments are repeated 10 times with the brass sample. The average measured value and the standard deviation (σ) is shown in Table 2.

Table 2. Measurement results of brass sample

Resonant frequency f (Hz)	Damping factor k	Young's modulus E (GPa)	Internal friction Q^{-1}	Standard deviation σ
424.8	-4.6	120.9	3.4×10^{-3}	0.03

The standard deviation σ is obtained by $\sigma = \sqrt{\frac{1}{N} \sum_{i=1}^N (x_i - \mu)^2}$, where $N = 20$, x_i refers to each measured value, and μ is the mean value over the measured 10 values. The measured value of Young's modulus for brass sample is also very close value between 102 GPa and 125 GPa reported in the literature [17].

4. CONCLUSION

This paper proposes an intact system design for simultaneous measurement of Young's modulus and internal friction. An experimental system has been built and SMI signals have been captured. By applying wavelet transform onto the SMI signal, the material related parameters, such as resonant frequency and damping factor are retrieved simultaneously, so that to achieve the measurements of Young's modulus and internal friction of the tested sample. The experimental results prove the feasibility of using SMI technique for non-destructive and real-time measurement of material related parameters.

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