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Shear strength of a vegetated soil incorporating both root reinforcement and suction

Muditha Pallewattha

University of Wollongong, map804@uowmail.edu.au

Buddhima Indraratna

University of Wollongong, indra@uow.edu.au

Ana Heitor

University of Wollongong, aheitor@uow.edu.au

Cholachat Rujikiatkamjorn

University of Wollongong, cholacha@uow.edu.au

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Muditha Pallewattha

BSc Eng (Hons), PhD (Wollongong)

Research Associate, Centre for Geomechanics and Railway Engineering, Faculty of
Engineering and Information Sciences, University of Wollongong, Wollongong, NSW 2522,
Australia

Buddhima Indraratna[†]

PhD (Alberta), DIC, FIEAust., FASCE, FTSE, FGS

Distinguished Professor of Civil Engineering, Director, Centre for Geomechanics and
Railway Engineering, Faculty of Engineering and Information Sciences, University of
Wollongong, Wollongong, NSW 2522, Australia

Ana Heitor

PhD (Wollongong), MIEAust

Senior Lecturer, Centre for Geomechanics and Railway Engineering, Faculty of
Engineering and Information Sciences, University of Wollongong, Wollongong, NSW 2522,
Australia

Cholachat Rujikiatkamjorn

PhD (Wollongong), MIEAust, CPEng

Associate Professor, Centre for Geomechanics and Railway Engineering, Faculty of
Engineering and Information Sciences, University of Wollongong, Wollongong, NSW 2522,
Australia

[†]Author for correspondence:

Distinguished Prof. Buddhima Indraratna
Centre for Geomechanics and Railway Engineering,
Faculty of Engineering, University of Wollongong
Wollongong, NSW 2522
AUSTRALIA
Ph: +61 2 4221 3046
Email: indra@uow.edu.au

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Abstract

Shear strength of the root permeated soil increases due to the mechanical effects of root reinforcement and most of the past studies have been conducted to capture this effect under saturated soil conditions. However, the soil adjacent to the tree roots is usually in an unsaturated condition and this leads to alterations in root-soil interaction mechanisms and associated shear strength of the root permeated soil system. In this paper, the increment in shear strength is studied considering both the effect of suction and root reinforcement patterns. A number of direct shear tests were conducted for different suction levels in root-permeated and unreinforced soil specimens. The results indicate that the shear strength behaviour of the soil-root system is governed by the level of suction and root failure patterns and a new mathematical model incorporating the effect of both parameters is proposed.

Keywords: Soil, Tree roots, Matric Suction, Shear strength

Introduction

The current demand for rail transport in Australia has been driving the research into more cost-effective and reliable soil improvement methods over the last decades. This is particularly important for controlling settlement and instability for example in locations where there is a soft soil subgrade beneath the track. In this regard, the green corridor concept is an ideal natural proposition in lieu of traditional soil treatment methods to improve the soil strength. Previous studies conducted by Molz and Remzon (1970), Fredlund and Hung (2001), Indraratna et al. (2006), Potter (2006), Ng et al (2013, 2016) and Liang and Knappett (2017) have shown promising results that support the potential use of natural vegetation for stabilizing the soil.

The main mechanisms associated with the stabilization of soil using tree roots are the extraction moisture from the ground derived from the evapo-transpiration cycle processes and mechanical strengthening. The former natural phenomenon leads to an increase in soil suction over larger periods of time (e.g. Cameron 2001, Fatahi et al. 2014, Indraratna et al. 2008) which contributes to an increase in the shear strength of the soil. Potter (2006) conducted field trials in South Australia and reported that the variations in total suction beneath railway track in non-vegetated and vegetated conditions were distinct, with the latter typically exhibiting larger values of suction. The tree root system also provides significant mechanical strengthening by means of anchoring and reinforcement (e.g. Waldron and Dakessian 1981, Gray and Leiser 1982, Docker and Hubble 2001, Meijer et al, 2016). Docker and Hubble (2009) conducted extensive field testing to evaluate the mechanical strengthening parameters under saturated soil conditions, and they proposed a semi-empirical solution for a root reinforcement system. While most of the past studies investigated either the role of mechanical strengthening or the soil suction, the overall shear strength of the root system in ground is governed by the coupled effect of both these factors. In this paper, a novel mathematical model is proposed to predict the shear strength behaviour of a root permeated soil considering the role of the root failure patterns and suction.

Root Reinforcement models

The effect of mechanical strengthening generated through root reinforcement can be assessed as an increment to the shear strength ($\Delta\tau$) considering saturated soil conditions (Wu et al. 1979, Waldron and Dakessian 1981, Docker and Hubble 2009) as follows,

$$\tau = c + \sigma_N \tan\phi + \Delta\tau \quad (1)$$

where, τ is the shear strength of soil, c is the apparent cohesion of the soil, σ_N is the applied normal stress and ϕ is the friction angle of the soil.

Gray and Leiser (1982) modified the simple root model (Eq. 1) for an inclined root and mobilised tensile strength of the root was defined by;

$$t_r = T_R \frac{A_R}{A} \quad (2)$$

In Eq. 2, t_r is the mobilized tensile strength of the soil per unit area of soil, T_R is the tensile strength developed in root and $\frac{A_R}{A}$ is the root area ratio (RAR), with A_R representing the area of the root along the shear plane and A the area of the shear plane.

Docker and Hubble (2009) developed an empirical solution after conducting a number of in-situ shear tests for real tree roots in saturated soil and ΔS (increment of shear strength) was then represented as a function of root area ratio (RAR) as shown in Eq. 3, as follows,.

$$\Delta S = \alpha \cdot (RAR) + \beta \quad (3)$$

where α and β are empirical constants which vary for different tree species.

There are a number of parameters that govern root reinforcement level, i.e. the mechanical properties of the root (e.g. tensile strength of the root), the root density in the soil matrix or root area ratio (RAR), the failure patterns of the root domain and the root orientation. While the past

models have considered some of these aspects, none to the Authors knowledge have incorporated all of them. In this study, a novel mathematical model incorporating the effect of all these factors is proposed, in addition to the consideration of the soil suction on its shear strength.

Materials and Testing Program

Soil type and tree species

The tree species *Eucalyptus botryoides* was selected because it is a commonly available native plant in the state of NSW (Australia) and has a uniform root distribution. The soil selected for this study is a gravelly sand available locally (Wollongong, Australia). The particle size distribution of this sand shown in Figure 1 represents 15% fine gravel, 80% sand, 5% silt and clay size fraction. All specimens were prepared in wooden boxes and compacted to a dry unit weight of 13.2 kN/m^3 and water content of 18%. The specimens were compacted in three layers and then all the boxes were scanned using CT technique using a medical grade X-ray CT scanner (Toshiba Asteion S4) to verify the uniformity of the compacted soil. The reconstruction function adopted in this study enabled the correction of image artefacts that could result from the absence of lower energy X-rays. The X-ray tube voltage and current were 135 kV and 200 mA, respectively. The X-ray beam was 3 mm wide (i.e., slice thickness), the exposure time was 1s. An example of the CT-scan micrographs of different boxes is shown in Figure 2 and it can be observed that the different specimens were compacted relatively uniformly.

Direct shear tests

A series of direct shear tests were conducted for root-permeated and unreinforced soil specimens. A large shear box apparatus (300×300×200mm) equipped with an electrical motor, a vertical and horizontal dial gauges (accuracy of 0.01mm) and proving ring (accuracy of 0.035kN) was used (Figure 3). Recently germinated small plants (approximately 15-20 mm tall) of *Eucalyptus*

botryoides with undistorted root systems were then planted in the boxes. The plants were carefully selected to ensure they had approximately the same growth rate so that all specimens displayed a similar final root system. The plants were allowed to grow under the same environmental conditions for 10 months and then were transferred to the shear box apparatus for testing with minimal disturbance. A suction sensor (MPS-2 with accuracy of 0.5kPa) was installed inside the shear box near the shear zone (see Figure 3). This sensor measures the water potential of soil and other porous materials and it is ideal for permanent burial in the soil and continuous reading with a data logger owing to its low power requirements. While all the boxes were prepared at approximately the same level of initial suction, to achieve the target suction values the soil was first wetted, dried (typically over a few days) and left for equilibration prior to the start of the shearing stage. A vertical sand pile with the diameter of 15 mm was also installed in the shear box specimens to examine the width of the shear zone as shown in Figure 3.

A total of number of 30 direct shear tests were conducted (15 soil only and 15 root-permeated) and the summary of their respective test conditions (e.g. vertical stress, initial matric suction and moisture content) is given in Table 1. Direct shear tests were conducted for three different values of normal stress (10kPa, 20kPa, 30kPa) and five initial suction levels varying from 0 kPa (fully saturated) to 200 kPa at 50kPa intervals to mimic typical site conditions of a railway tracks in the state of New South Wales. A confining pressure up to 30kPa was adopted to simulate real field conditions based on past in situ measurements (Indraratna et al., 2011), i.e. confining stress relevant to a typical rail track and at the root depth. The required suction values inside the boxes were obtained by varying the initial water content in the soil by drying. A displacement rate of 2.5mm/min was selected and the tests were conducted at constant water content. In essence, the constant water content type tests were adopted because they can mimic real field conditions, where shearing is likely to take place under relatively constant water content conditions. Further, as it is a relatively quick test, the amount of water losses through evaporation and root uptake could be

minimised. Nevertheless, throughout the test the plant was kept alive and suction was monitored during the test. Additional specimens having no roots were prepared at the selected suction levels and tested at the same normal stresses so that the effect of tree roots could be evaluated. The tests were conducted at different soil suction levels, and as the test duration was small, the role of evapotranspiration could not be properly captured. The process of evapotranspiration in reality is of cyclic nature occurring over a much longer time than that of the test period.

Upon shearing, the specimens were exhumed to quantitatively evaluate the root failure patterns, including root mapping (i.e. root orientation and root area ratio).

Root Tensile strength

Additional tests were performed to evaluate the tensile strength of intact roots. The tests were conducted using an INSTRON system. Subsequently, a direct shear test was conducted on vertical root to evaluate the bond strength between the soil and the root. Additional direct shear tests with a vertical root (12mm diameter) fixed to the upper and lower plates of the shear box were also conducted to evaluate and quantify the elongation of the roots inside the soil.

Results and Discussion

Direct shear tests

Figure 4 shows the direct shear test results conducted on fully saturated specimens under the normal stresses of 10kPa, 20kPa and 30kPa. The shear stress and vertical displacement results with horizontal displacement are shown in Figures 4a and 4b, respectively. As expected the root-permeated soil specimens show greater shear strength by 5-8kPa compared to the unreinforced specimens. These results are generally in acceptable agreement with the data reported in past studies (e.g. Docker and Hubble, 2009) and confirm the important contribution of physical reinforcement provided by the roots. The yield point of root-permeated specimens occurs at a slightly higher

horizontal displacement (4-8mm) as shown in Figure 4a, and this suggests some enhanced ductility. Figure 4b shows that the deformation behaviour is mainly compressive and is less influenced by the presence of roots.

The variation of shear stress, vertical displacement and degree of saturation with respect to the horizontal displacement of specimens (with and without roots) tested at different suctions under 20kPa normal stress is shown in Figure 5. Similar to the results obtained under fully saturated condition, root-permeated specimens attain yield at larger horizontal displacements, exhibit a higher peak shear stresses. The specimens tested at the higher suction, e.g. initial suction of 200kPa display a slight tendency for strain softening. Furthermore, the difference between peak stress of root permeated specimens and those without roots is also dependent of the level of suction, e.g. for the specimens subjected to suction of 0 and 50kPa, the difference in peak stress is approximately 10kPa, whereas for values of suction larger than 100kPa, the difference exceeds 15kPa. This clearly indicates that there is additional interaction between the roots and the soil suction apart from the physical reinforcement effect. While the moisture exchange during shearing is likely to remain unchanged owing to the relatively short duration, the roots are active and that alone seems to cause a difference in the soil behaviour, e.g. changing from predominately strain hardening to slightly strain softening. For all specimens, there is a slight reduction in vertical displacement for the root-permeated samples. However, there is a substantial reduction under higher levels of suction, e.g. for lower suctions (<50kPa) vertical displacement exceeds 1mm whereas for larger suction of 200kPa it is nearly half (approximately 0.65mm). Furthermore the difference in compressive displacement reduces (Figure 5b). Figure 5(c) shows the variation of degree of saturation computed based on the measurements of vertical displacements. It can be observed that changes in the degree of saturation were marginal, and this is partly the reason for very small changes in suction measured during shearing, i.e. typically within ± 3 kPa. Similar trends were observed for the other two levels of vertical stress considered, i.e. 10 and 30kPa.

Figure 6 shows a summary of the results obtained for all specimens represented in terms of the variation of peak shear stress with suction, degree of saturation and volumetric water content. In Figure 6(a), the variation of peak shear stress with matric suction is plotted, which shows quasi-linear trends. This is likely due to the different failure patterns of the roots identified in root mapping and elaborated in more detail in the next section. Figure 6b illustrates the relationship between degree of saturation and the measured peak shear stress. While the variation of the degree of saturation during shearing is marginal, the initial value of degree of saturation governs the peak shear stress; in particular there is a sharp increase when the degree of saturation is smaller than 0.75. This is not surprising as past studies on the behaviour of compacted soils demonstrate that both the suction and the degree of saturation control shear strength (e.g. Tarantino and Tombolato, 2005). Furthermore, as expected, the relationship between the volumetric water content and peak shear stress shows a similar trend to that of degree of saturation. It is interesting to note that while a clear bifurcation is observed in the degree of saturation and volumetric water content relationships with the peak stress for all specimens, such a trend is not observed in the case of suction. This is likely associated with the water retention characteristics of the soil.

Root mapping

After the shear tests, the root systems of the specimens were exhumed to evaluate the root failure patterns and associated incidence. Three different root failure patterns were identified in the laboratory (Figure 7). The roots that fail under the tension (Figure 7a) are likely to be fixed in the soil body at two ends, and as shearing progresses they undergo gradual stretching accompanied by an increase in tensile stress. Rupture of the root is expected to occur when this value exceeds the tensile strength of the root. While Docker and Hubble (2001) suggest that most roots within the soil matrix fail in tension during shearing, this failure mechanism seems to be more prominent under saturated soil conditions. In unsaturated conditions, alternative failure patterns such as pure slipping and slipping with the soil annulus can be observed (Figures 7b and 7c). For the roots that are pulled

out and experience pure slipping, the bond strength at the root-soil interface governs the shear resistance. Other roots are pulled out with a soil annulus, but the number and size of soil annuli observed for the different specimens seems to be independent of both the level of initial soil suction and the vertical stress, on the average at approximately 3-4 roots in each specimen. This is likely associated with the fibrous root growth around the main roots, which is equivalent for all the specimens tested. Thus, the number and size of the soil annuli were assumed to be constant in this study, and considered as a relevant model parameter that is governed by the root growth. Table 1 shows a summary of the results obtained from the direct shear tests and root mapping of different specimens, including number of root failures in distinctly different modes, root area ratios and peak shear stresses. The number of broken roots increased with the increase in matric suction and normal vertical stress (Table 1). This is due to the increase in the bond strength between root - soil interface ($R_{r,i}$), thus, the mobilised tensile stress of the root reaches to the upper bound value (i.e. tensile strength of the root) before it reaches the bond strength. For instance, the mobilised shear stress in root-permeated specimens at 200 kPa matric suction starts to exhibit a mainly strain softening behaviour, because more roots are broken (Figure 5).

Theoretical model development

The following section describes a mathematical formulation to explain the root failure mechanisms predominantly inspired by the current laboratory observations in tandem with the basic theories of soil reinforcement in unsaturated soil. While its theoretical components represented by mathematical equations cannot be explicitly calibrated with the measured experimental data, the proposed model serves primarily as a conceptual development to complement the experimental observations.

Based on the experimental observations reported in Figure 5, the total shear strength of the root-permeated soil can be interpreted based on three different components, i.e. (i) shear strength of the soil in a saturated condition, (ii) the increment in shear strength caused by an increase in suction, and (iii) the physical reinforcement provided by the roots (Eq. 4). For instance, for saturated conditions (suction = 0kPa) the specimens permeated with roots showed a greater strength (peak stress) compared to those of soil alone. In contrast, for soil only specimens those tested at large suction values showed a greater peak shear strength.

$$\tau_{Total} = \tau_s + \tau_U + \Delta\tau_T \quad (4)$$

where, τ_{Total} is the total shear strength, τ_s is the shear strength of the saturated soil, τ_U is the shear strength increased by soil suction caused by drying out or desiccation (without the role of tree roots) and $\Delta\tau_T$ is the additional effect on shear strength due to the presence of tree roots (i.e. both mechanical reinforcement effect and suction generated by root water uptake). As both τ_U and $\Delta\tau_T$ are dependent on the soil suction, they can be simply considered in tandem as the role of suction and root reinforcement.

Based on the observed shear stress results reported in previous studies (e.g. Docker and Hubble 2009, Indraratna et al. 2006) the total increase in shear strength due to root effect $\Delta\tau_T$ may be postulated by:

$$\Delta\tau_T = \Delta\tau_R + \Delta\tau_S \quad (5)$$

where, $\Delta\tau_R$ is the increase in shear strength only due to root reinforcement (including soil-root interface shear strength), and $\Delta\tau_S$ is an increase in shear strength due to increase of soil suction only, as derived by moisture extraction induced by the evapo-transpiration process. The detailed formulation for the quantification of the additional shear strength associated with the soil-root

interface is shown in Appendix A. As stated in Eq. 5, $\Delta\tau_R + \Delta\tau_S$ is equal to $\Delta\tau_T$, however $\Delta\tau_S$ which is the increase in shear strength due to suction due to evapotranspiration cannot be captured due to the short duration of tests. It is postulated that the suction increment caused by evapotranspiration can then be added directly to the unsaturated term of the shear strength, i.e. τ_U in Equation 4. Table 2 shows the values of the τ_T variable obtained during the direct shear tests.

Root failure patterns

Waldron (1977) argues that the shear resistance ($\Delta\tau_1$) increases with the stretching of roots having the same diameter as follows:

$$\Delta\tau_1 = \alpha_r T_N \delta \quad (7)$$

where $\alpha_r = A_r/A_S$, and A_S is the total shearing cross section whereas A_r is the total area of the root across the shearing plane, T_N is the maximum tensile stress developed in the root for a given displacement and δ is parameter related to the orientation of the root and can be expressed as follows:

$$\delta = (\cos\beta_i + \sin\beta_i \tan\phi) \quad (8)$$

where, β is the angle of the deformed root to the horizontal plane, ϕ is the friction angle at a given suction level and " i " denotes the data related to the ' i^{th} ' root. This model can be applied to the slipping, stretching or breaking roots having the same diameter.

In this study, Waldron (1977) model was extended to estimate the tree root contribution to the shear strength increment ($\Delta\tau_T$) at a given displacement, by considering the three failure patterns observed in the experiments (Figure 7). Equations 9-11 can then be used for vertical roots with different

diameters, because the root area and the tensile properties, or the bonding properties of the roots are taken into account for each root separately.

where Δf_{Resist}^1 , Δf_{Resist}^2 and Δf_{Resist}^3 are the resisting forces due to the shear displacement as the roots stretch, slip, and are pulled out with a soil annulus, respectively, n_1 is the number of roots broken during shearing, T_i is tensile stress generated by the i^{th} stretching root, n_2 is the number of roots which slip without breaking, R_{ri} is the bond stress of the i^{th} slipping root developed between the root and soil, A_{ri} is the circumferential area of roots undergoing frictional displacement against shear displacement, n_3 is the number of roots that slip with a soil annulus, R_{si} is the bond strength between the root and soil i^{th} root pulled out with the soil annulus and lef_f and def_f are the effective length and circumferential diameter of the soil annulus that slips, respectively.

Equations 9-11 were developed in terms of resisting forces rather than shear stresses, by conveniently considering the derivation of forces along each root separately. However, the shear stress can be computed for the direct shear test results by dividing the total resisting forces on the shear plane by the total effective area, as per Gray and Leiser (1982) suggestions for a simple root

$$\Delta f_{Resist}^1 = \sum_{i=0}^{n_1} A_{ri}(T_i(\cos\beta_i + \sin\beta_i \tan\phi)) \quad (9)$$

$$\Delta f_{Resist}^2 = \sum_{i=0}^{n_2} A_{ri}(R_{ri}(\cos\beta_i + \sin\beta_i \tan\phi)) \quad (10)$$

$$\Delta f_{Resist}^3 = \sum_{i=0}^{n_3} lef_f \cdot \pi def_f \cdot (R_{si}(\cos\beta_i + \sin\beta_i \tan\phi)) \quad (11)$$

model, as follows:

$$t_r = T_R \frac{A_R}{A} \quad (12)$$

where t_r is the mobilised tensile strength of the soil per unit area of soil and T_R is the tensile strength developed in a root, and $\frac{A_R}{A}$ is the root area ratio (RAR).

In Eq. 13, the total resisting force (F_{Resist}) can now be computed based on the type of root failure mechanism and the shear strength of the unreinforced soil (τ_{unsat}), total area of shear plane (A_T) and the root cross sectional area $A_{root}(z)$ as given by:

$$\begin{aligned}
 F_{Resist} = & \left(\tau_{unsat} (A_T - A_{root}(z)) \right) + \sum_{i=0}^{n_1} A_{ri} (T_i (\cos \beta_i + \sin \beta_i \tan \phi) \\
 & + \sum_{i=0}^{n_2} A_{ri} (R_{ri} (\cos \beta_i + \sin \beta_i \tan \phi) \\
 & + \sum_{i=0}^{n_3} l_{eff} \pi d_{eff} (R_{si} (\cos \beta_i + \sin \beta_i \tan \phi)
 \end{aligned} \tag{13}$$

The first term on the right hand side of the Eq. 13 ($\tau_{unsat} (A_T - A_{root}(z))$) represents the basic shear stress within soil domain, whereas the other terms represent the resisting force generated by the proposed three root failure patterns. In Eq.13, each root is assumed to undergo only one failure mode during whole shearing process. Furthermore it is assumed that there no elongation occurring in roots either at pure slipping or while slipping with soil annulus. The upper bound of the generated tensile stress is indeed the tensile capacity of the root, and it is a material property which depends on the cell structure of the root.

The root-soil interface friction ($B_r(\psi)$) and the soil- interface friction ($B_s(\psi)$) depend on the developed suction in the soil. Calculations and experimental procedures to obtain $B_r(\psi)$, $B_s(\psi)$ and T_i are shown in Appendix A.

To evaluate the increase in shear strength derived by the presence of roots without carrying out numerous laboratory tests to map the different root failures a theoretical model for a tree with known parameters like RAR and tensile properties was developed and main features are described herein. A rational test was implemented to check the possible number of root failures in different situations using a MATLAB sub-routine. The parameters S1, S2 and S3 were introduced in the model as shown in Eq. 14 to examine the possibility of the failure pattern.

$$\begin{aligned} \Delta S_T = & \sum_{i=0}^{n1} S1i A_{ri} (T_i (\cos \beta i + \sin \beta i \tan \phi)) / A_T \\ & + \sum_{i=0}^{n2} S2i A_{ri} (R_{ri} (\cos \beta i + \sin \beta i \tan \phi)) / A_T \\ & + \sum_{i=0}^{n3} S3i l_{eff} \pi d_{eff} (R_{Si} (\cos \beta i + \sin \beta i \tan \phi)) / A_T \end{aligned} \quad (14)$$

Eq. 14 can be re-written considering a root in a three-dimensional space as shown in Figure 8, as follows,

$$\begin{aligned} \Delta S_T = & \sum_{i=0}^{n1} S1i A_{ri} (T_i \cos \alpha_{1i} (\cos \alpha_2 i + \sin \alpha_2 i \tan \phi)) / A_T \\ & + \sum_{i=0}^{n2} S2i A_{ri} \cos \alpha_{1i} R_{ri} (\cos \alpha_2 i + \sin \alpha_2 i \tan \phi) / A_T \\ & + \sum_{i=0}^{n3} S3i l_{eff} \pi d_{eff} \cos \alpha_{1i} R_{Si} (\cos \alpha_2 i + \sin \alpha_2 i \tan \phi) / A_T \end{aligned} \quad (15)$$

where α_1 and α_2 in Eq. 15 represents the inclined angles of the root in the horizontal and vertical planes (Figure 8).

S1, S2 and S3 have the values of 1 and 0 according the following conditions. If the tensile stress (T_i) is greater than the tensile strength (T_S) of the root, then the root is already broken and it has no contribution to the increase of the shear strength. For this condition we have S1=0, S2=0 and S3=0. If the tensile stress (T_i) is less than or equal to the tensile strength (T_S) of the root then the following two conditions have to followed. For the roots with T_i less than equal to the adhesive stress (R_{ri}) of the root, then they undergo stretching, and S1=1, S2=0 and S3=0 for this condition. For roots having a generated tensile stress (T_i) greater than the adhesive stress (R_{ri}), they slip out and for this case, S1=0, S2=1 and S3=0. A summary flowchart displaying the conditions mentioned earlier is shown in Figure 9. While the consideration of the role of ground stratigraphy is important, that aspect could not be considered within the scope of the current study and it is therefore a limitation.

Figures 10 and 11 shows the comparison between the experimental and predicted results. As mentioned earlier, the model was programmed using a MATLAB sub-routine. The root system parameters (e.g. root geometry, tensile strength, Young's modulus, root orientation) listed in Table 3 are the most suitable for the root system relevant to the time of the tree growth and were determined based on the root mapping data. The experimental ΔS value in Figures 10 and 11 is the increase in shear strength due to root reinforcement at the yield stress. Adhesive strength derivations are given in Appendix A, whereby the adhesive coefficient for roots pulled out with soil annulus was taken as equal to the term $\tan \phi$, where ' ϕ ' is the friction angle of the soil at a given suction level. The model predictions are in good agreement with the experimental results as shown in Figures 10 and 11. Figure 10a reports the experimental results of the additional shear strength mobilised by the roots in the suction-normal stress plane, whereas in Figure 10b the model predictions are presented for comparison. It can be observed that there is an acceptable agreement

between the measured and predicted values. This is illustrated in greater clarity in Figure 11, where the relationship between increments of shear strength is plotted with the initial suction values, and different data series represent the adopted normal stress levels (e.g. 10, 20 and 30kPa). It can be observed that the proposed conceptual model can predict the experimental trends to an acceptable accuracy. While only values at yield are represented herein, the model can still be used for any level of horizontal displacement. In the case of specimens prepared at an initial suction of 200kPa, roots showed a clear yielding peak point at a shear displacement of 11mm (Figure 5a). Therefore, the increase in shear strength for all the events was calculated at a shear displacement of 11mm, supported by the comparison shown in Figure 10 and 11. These results verify beyond any doubt that the mechanical effect of root reinforcement is clearly a function of both the matric suction and the applied normal stress.

Conclusion

From a number of direct shear tests it was observed that the shear strength of the root permeated soil specimens is influenced by the presence of tree roots and matric suction. Laboratory direct shear testing was conducted under five different suction levels and for three different applied normal stresses for mimicking typical rail track conditions. The results indicate that for the root-permeated specimens, both the influence of suction and the root reinforcement effect plays an important role in the overall shear strength of soil. Tests conducted at null suction (saturated conditions) enabled the evaluation of the role of root reinforcement and normal stress (improvement approximately of 5-8 kPa), whereas other specimens prepared at different levels of suction showed that there was an inter-dependency between the root reinforcement and suction surrounding the roots (improvement approximately of 10-15kPa for suction levels of 50kPa and over 100kPa).

Inspired by the experimental observations, a novel conceptual mathematical model was formulated on the basis of root geometry (e.g. length and diameter), root mapping and failure patterns observed in exhumed specimens upon shearing. The predictions based on the model were in acceptable agreement with the experimental results. In this regard, the increment in the shear strength of a root-permeated soil can be quantified considering the individual contributions of suction variation ($\Delta\tau_S$), root reinforcement effect ($\Delta\tau_R$) for a given pattern of failure of the root-soil system during shear displacement. Finally, this study shows that the stress-strain behaviour of a given soil-root system as well as the level of reinforcement offered by the presence of vegetation, is influenced by changes in hydraulic regimes (soil suction), which are typically governed by the evapo-transpiration processes (plant) and interactive climatic action with the soil. This indicates that this variation should be considered when evaluating long term performance of the root-soil systems, particularly in sites where large variations in suction are expected.

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AAPENDIX A

Calculation of soil-root interface shear strength

Hamid and Miller (2009) suggested the interface friction for an unsaturated vertical shear interface to be calculated as follows,

$$\tau_f = c' + (\sigma_n - u_a)\tan\delta' + (u_a - u_w) \left[\tan\delta' \left(\frac{\theta - \theta_r}{\theta_s - \theta_r} \right) \right] \quad (\text{A.1})$$

where, τ_f is the interface frictional stress with respect to net normal stress and δ' is the interface friction angle with respect to net normal stress. In vegetated soil system, τ_f and δ' should be defined with respect to soil-root interface. Equation A.1 and A.2 represent the normal and upward forces acting on the small element of root (Figure 13a).

$$R_n = V\cos\alpha + H\sin\alpha \quad (\text{A.2})$$

$$R_u = V\sin\alpha - H\cos\alpha \quad (\text{A.3})$$

where R_n is the force component normal to root and R_u is the upward force component. V is the vertical load applied on root element and H is the horizontal force acting on the root element (Figure 13a). The resisting force to the upward movement of the root R_r can be illustrated as shown in equation A.4.

$$R_r = c' + (V\cos\alpha + H\sin\alpha)\tan\delta' + (u_a - u_w) \left[\tan\delta' \left(\frac{\theta - \theta_r}{\theta_s - \theta_r} \right) \right] - (V\sin\alpha - H\cos\alpha) \quad (\text{A.4})$$

‘H’ in the equation A.4 directly proportional to the lateral earth pressure and ‘V’ is directly proportional to the applied normal stress. Therefore, the bond strength is influenced by the soil

suction and normal stress. The $\tan\delta'$ in equation A.4 can be defined as root-soil bond coefficient which is a material property and it is represented as 'tan λ ' in further references. Then the equation A.4 yields to the equation A.5 as follows.

$$R_r = c' + [(\sigma_N - u_a)\cos\alpha + (k_0((\sigma_N - u_a) + \gamma h)\sin\alpha)]\tan\lambda + (u_a - u_w) \left[\tan\delta' \left(\frac{\theta - \theta_r}{\theta_s - \theta_r} \right) \right] - ((\sigma_N - u_a)\sin\alpha - (k_0((\sigma_N - u_a) + \gamma h)\sin\alpha)\cos\alpha) \quad (\text{A.5})$$

Calculation of bond coefficient (tan λ)

Bond coefficient was calculated using a 18mm vertical root and it was sheared under pure slipping as explained in a previous section (Figure 7b). In Figure 12b Δd is the shear displacement, Δt is the thickness of the shear zone, L is the length of the root, $L1$, $L2$ are the length of the root which lie above and below of the shear zone and $\Delta L1$ is the moved length of the root $\Delta L1$ can be calculated using Eq. A.6, because the $\Delta L1$ cannot be measured directly. The difference between the results of soil and root permeated tests were used for calculation of bond coefficient. The mobilized total energy by the root element from the pure slipping can be considered as equal to the area difference between soil only test and root permeated soil, since there was only a pure slipping.

$$\Delta L1 = L2 - (L - L1 - \sqrt{(\Delta d^2 + \Delta t^2)}) \quad (\text{A.6})$$

The mobilised energy by a small root element due to slipping can be illustrated as follows,

$$E = R_r 2\pi r \delta z \quad (\text{A.7})$$

where, E is the mobilised energy due to slipping, r is the radius of the root and z is the vertical measurement of the root. By integrating Eq. A.7 along the area of the root then,.

$$E = R_r \pi r \left(L2 - (L - L1 - \sqrt{(\Delta d^2 + \Delta t^2)}) \right)^2 \quad (\text{A.8})$$

The bond coefficient was calculated using the Eq. A.9 which is based on the combination of Eqns. A.5, A.6 and A.8.

$$\tan \lambda = \left[\frac{\Delta A}{\pi r \left(L2 - (L - L1 - \sqrt{(\Delta d^2 + \Delta t^2)}) \right)^2} \right] - (u_a - u_w) \left[\tan \delta' \left(\frac{\theta - \theta_r}{\theta_s - \theta_r} \right) \right] + \frac{\left((\sigma_N - u_a) \sin \alpha - (k_0 ((\sigma_N - u_a) + \gamma h) \sin \alpha) \cos \alpha \right) - c'}{[(\sigma_N - u_a) \cos \alpha + (k_0 ((\sigma_N - u_a) + \gamma h) \sin \alpha)]} \quad (\text{A.9})$$

where ΔA is the area difference between the graphs of direct shear test results which were conducted with vertical root and soil only test with the same test condition. The bond coefficient value was computed to be 0.21 to 0.22 for the three test conducted using vertical root.

Calculation of generated tensile strength

When there is no slipping (root is fixed at the bottom), the $\Delta L1$ value in the Eq. A.6, changes into the elongation of the root. The strain (ϵ) of the root is at each horizontal displacement can be calculated using Eq. A.10.

$$\epsilon = \frac{L2 - (L - L1 - \sqrt{(\Delta d(t)^2 + \Delta t^2)})}{L} \quad (\text{A.10})$$

The mobilised tensile energy in the root (E(t)) can be expressed as follows,

$$E(t) = \frac{1}{2} T_i \varepsilon \quad (\text{A.11})$$

where, T_i is the mobilised tensile stress. Therefore the mobilised tensile stress at each horizontal displacement can be calculated based on the difference between root permeated and soil only specimens. Furthermore it can be expressed using Young's modulus for the tree roots as shown in Eq. A.12.

$$E(t) = \frac{1}{2} T_i \varepsilon = \frac{1}{2} k e^2 \quad (\text{A.12})$$

where k is the Young's modulus of root and e is the elongation of root. Different k values calculated using tensile test results are shown in Table 3. Therefore the mobilised tensile stress at any shear displacement for an inclined root can be calculated using Eq. A.13.

$$T_i = k \left[\frac{L_2}{\sin \beta_2} + \left(L - \frac{L_1}{\sin \beta_2} - \sqrt{((\Delta t \cdot \tan \beta + \Delta d)^2 + \Delta t^2)} \right) \right] \quad (\text{A.13})$$

where, β_2 is the final inclination of the deformed root with respect to the horizontal plane and can be calculated using Eq. A.14. Both angles β_2 and α are illustrated using Figure 13c.

$$\tan \beta_2 = \frac{\Delta t}{\Delta t / \tan \alpha + \Delta d} \quad (\text{A.14})$$

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Table 1: Number of root failures in different modes, root area ratios and peak shear stress of root permeated soil and peak shear stress of equivalent soil only specimen.

	Vertical stress	Matric Suction/Moisture content	τ - Peak	Soil only τ - Peak	RAR	No. of broken roots	No. of roots with soil lump
Sample No	(kPa)	(kPa) /	(kPa)		%		
1	10	0/0.43	11	8.2	0.58	2	3
2	20		13	11.5	0.56	2	3
3	30		16.2	14.8	0.54	2	3
4	10	50/0.36	17	15.2	0.55	2	3
5	20		23	18.3	0.55	2	3
6	30		28	22.2	0.52	2	3
7	10	100/0.35	28.6	25.8	0.54	4	3
8	20		36.9	30.5	0.52	4	4
9	30		49.8	35.6	0.52	4	3
10	10	150/0.34	41.5	33.5	0.52	7	3
11	20		45	38.2	0.52	7	4
12	30		53.8	43.4	0.52	8	4
13	10	200/0.32	46.5	41.1	0.53	10	4
14	20		58.3	45.5	0.51	12	3
15	30		64.8	50.6	0.51	12	3

Table 2 Values of the parameters

Sample no	Suction ($u_a - u_w$)	σ (kPa)	τ (kPa)	τ_T (kPa)
1	0kPa	$\sigma_1 = 10$	$\tau_1 = 8.2$	$\tau_{T1} = 11$
2		$\sigma_2 = 20$	$\tau_2 = 11.5$	$\tau_{T2} = 13$
3		$\sigma_3 = 30$	$\tau_3 = 14.8$	$\tau_{T3} = 16.2$
4	50kPa	$\sigma_1 = 10$	$\tau_1 = 14.5$	$\tau_{T1} = 18$
5		$\sigma_2 = 20$	$\tau_2 = 18.3$	$\tau_{T2} = 23$
6		$\sigma_3 = 30$	$\tau_3 = 22.2$	$\tau_{T3} = 28$
7	100kPa	$\sigma_1 = 10$	$\tau_1 = 24.8$	$\tau_{T1} = 29.6$
8		$\sigma_2 = 20$	$\tau_2 = 30.5$	$\tau_{T2} = 36.9$
9		$\sigma_3 = 30$	$\tau_3 = 35.6$	$\tau_{T3} = 49.8$
10	150kPa	$\sigma_1 = 10$	$\tau_1 = 33.5$	$\tau_{T1} = 41.5$
11		$\sigma_2 = 20$	$\tau_2 = 38.2$	$\tau_{T2} = 45$
12		$\sigma_3 = 30$	$\tau_3 = 43.4$	$\tau_{T3} = 53.8$
13	200kPa	$\sigma_1 = 10$	$\tau_1 = 33.5$	$\tau_{T1} = 46.5$
14		$\sigma_2 = 20$	$\tau_2 = 38.2$	$\tau_{T2} = 58.3$
15		$\sigma_3 = 30$	$\tau_3 = 43.4$	$\tau_{T3} = 64.8$

Table 3 Input data file for MATLAB simulation.

Diameter (m)	Youngs Modulus kN/m ² /mm	Length (m)	Thickness of the shear zone (m)	L1 (m)	L2 (m)	Δd (m)	Moved length (m)	Tensile Strength (kN/m ²)	Orientation		Mean soil height (m)
									$\alpha 1$	$\alpha 2$	
0.003	10000	0.15	0.02	0.0985	0.0985	0.033	0.018588	10000	0	30	0.098
0.003	10000	0.15	0.02	0.097	0.097	0.033	0.017224	10000	20	45	0.142
0.002	10100	0.135	0.02	0.096	0.096	0.033	0.0164	10000	40	60	0.14
0.004	10000	0.16	0.02	0.096	0.096	0.033	0.0164	10000	60	30	0.098
0.003	10000	0.15	0.02	0.975	0.096	0.033	0.0164	10000	80	45	0.142
0.004	10000	0.16	0.02	0.0985	0.096	0.033	0.0164	10000	100	60	0.14
0.0025	10100	0.14	0.02	0.096	0.097	0.033	0.0164	10000	120	30	0.098
0.003	10000	0.15	0.02	0.096	0.096	0.033	0.0164	10000	140	45	0.142
0.004	10000	0.16	0.02	0.0985	0.096	0.033	0.018588	10000	160	60	0.14
0.003	10000	0.15	0.02	0.096	0.096	0.033	0.0164	10000	180	30	0.098
0.0025	10100	0.14	0.02	0.096	0.096	0.033	0.0164	10000	200	45	0.142
0.003	10000	0.15	0.02	0.0985	0.0985	0.033	0.018588	10000	220	60	0.14
0.002	10100	0.135	0.02	0.096	0.096	0.033	0.0164	10000	240	30	0.098
0.003	10000	0.15	0.02	0.097	0.097	0.033	0.017224	10000	260	45	0.142