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Keywords

design, concrete, beams, optimization, dynamic, response, problem, continuous, geometric, reinforced

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Geometric Design Optimization for Dynamic Response Problem of Continuous Reinforced Concrete Beams

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Abstract

This paper presents a computer-aided design method for conceptual geometric layout optimization of multi-span reinforced concrete (RC) beams for any type of dynamic responses. A key feature of the paper is the development of a new cost optimization method for geometric layout problems that take both cost parameters and dynamic responses into account to achieve an optimum design along with acceptable dynamic performance of reinforced concrete (RC) beams. This method takes advantage of employing a new optimization formulation that considerably simplifies the preliminary layout design computations. It focuses on minimizing the structural cost subject to constraints on eigenfrequencies, modal shapes and the static and dynamic equilibrium. Then, the proposed structural optimization problem is solved employing an Ant Colony Optimization (ACO) algorithm. In order to verify the suitability of the methodology and illustrate the performance of the algorithm, solved examples are presented.

CE Database subject headings: Optimization models; Geometry; Costs; Reinforced Concrete; Continuous Beams; Dynamic Loads; Algorithms.

Author Keywords: cost optimization; layout design; dynamic responses; reinforced concrete; multi-span beam; ACO.

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Introduction

In the computer-aided conceptual design of structural systems, it is often desired to find the general geometric layout of the system that most naturally and efficiently supports the expected design loads. Such a design is sometimes done by optimizing the overall layout of the structural system as well as the topology of structural elements comprising the structure. Layout optimization is probably the most difficult class of problems in structural optimization. On the other hand, it is also a very important one, because it results in much higher material savings than cross-section optimization (Rozvany 1992). The literature of geometric layout and topological optimization are vast, and their topics are diverse. A critical review of established methods of structural topology optimization was carried out by Rozvany (2009). One of the most challenging and difficult parts of applying the geometric layout optimization method to new areas is to develop reasonable choices and combinations of objective functions and constraints. For example, structures that have been optimized with respect to a particular load system may not be optimized when subjected to another load system. Therefore, various load cases must be taken into consideration when formulating the optimization problem.

Improving the vibration characteristics of a structure by changing the structural topology has long been a subject that has attracted the minds of engineers. One of the first applications of the layout or topology optimization methods for the design of machines and structures subjected to dynamic loads was in eigenvalue optimization for free vibrations. One method for optimizing the overall stiffness of a structure without reference to any specific loading is to maximize a linear combination of the first N vibrational eigenvalues (Pedersen 2000).

In case of topology optimization of structures under dynamic loadings, Rong et al. (2000) considered the extension and modification of the Evolutionary Structural Optimization method to control the structural random dynamic responses. Jog (2002) proposed two measures for the minimization of vibrations of structures subjected to periodic loading. Kang et al. (2006) carried out a review of optimization of structures subjected to transient loads. Yoon (2010) used model reduction schemes methods to calculate accuracy for topology optimization in the frequency domain. Barbarosie and Toader (2010a; 2010b) proposed a methodology for shape and topology optimization algorithm for periodic problems. Moreover, in recent decades, in dynamic optimization, the sensitivity optimization of eigenvalue and frequency response problems has vastly been considered in the literature to investigate the variation of the responses to that of the design variables (Choi and Kim 2005).

A great majority of the published studies, in geometric or topology layout optimization of structures for dynamic responses, deal with the weight minimization of structures by taking the cross-sectional areas and nodal coordinates as design variables (Abachizadeh and Tahani 2009; Achtziger and Kočvara 2007; Barbarosie and Toader 2010a; Barbarosie and Toader 2010b; Jog 2002; Kang et al. 2006; Rong et al. 2000; Soh and Yang 1996; Yoon 2010). Even though employing the above-mentioned methods made it possible to obtain the optimal dynamic response of a structure by minimizing the total weight, it is necessary to obtain an optimal cost for a truly optimal design; because the weight minimization of structures does not necessarily lead to the cost minimization, particularly for reinforced concrete (RC) and composite structures, as different materials are involved (Reddy et al. 1993).

In a comprehensive computer-aided structural optimization process, selecting an appropriate preliminary geometric layout design of structures is of great importance, as it influences all the subsequent stages of the design procedure. The aim of this computational effort is to determine the preliminary geometry so that the desired behavior is achieved at the lowest possible cost. Rafiq et al. (2003) proposed an evolutionary approach for conceptual design that permits the manipulation of both structural and architectural design aspects. Reddy et al. (1993) developed an expert system for optimum design of concrete structures in the phase of preliminary (conceptual) design.

This paper discusses a preliminary geometric design method to determine the optimum spans of a vibrating continuous beam for controlling dynamic responses, while a given set of design conditions, including the cost of the involved material, loading and support conditions, are given. That is, the optimization problem is formulated such that it considers both the involved cost elements and the dynamic responses. For this purpose, a methodology is employed that links the cross-sectional action effects of the RC beam to the involving cost parameters (Sharafi et al. 2012). Then, based on the proposed objective function and the relevant constraints, an Ant Colony Optimization (ACO) algorithm is presented to solve the proposed optimization problem.

Statement of the problem

When optimizing the cost and geometric layout and/or topology for complex structures under dynamic conditions a lot of variables need to be taken into account that result in a large computation time. As a result, most of the cost optimization methods developed for multi-member structures are confined to structures with predefined shapes , and may not always be applicable to topology design of large-scale problems due to difficulties in computational cost and convergence properties.

In a classical geometric layout optimization problem, a general goal is to minimize the structural weight while satisfying the requirements of structural performance and the related constraints. Therefore, the layout optimization problem of a continuous beam under dynamics response constraints can be stated as follows:

$$\begin{cases} Minimize \quad W(A_e, L_e) = \sum_{e=1}^{m} \rho A_e L_e \\ s.t. \quad \begin{cases} |\delta_i| \leq \delta_a\,, & (i = 1, 2, \dots, n) \\ |\sigma_e| \leq \sigma_a, & (e = 1, 2, \dots, m) \\ \omega_N \in \{allowable\ frequencies\} \\ A_{min} \leq A_e \leq A_{max} \\ L_{min} \leq L_e \leq L_{max} \\ [M]\{\ddot{u}(x,t)\} + [C]\{\dot{u}(x,t)\} + [K]\{u(x,t)\} = \{-F(x,t)\} \end{cases} \end{cases} \tag{1}$$

in which W represents the total weight of the structure, ρ, m, n are the density, number of members and number of control sections respectively. A_e is the cross-sectional area and l_e is the length of eth element. δ_a, σ_a and ω_N denote the allowable displacement, allowable stress and the fundamental natural frequency (eigenvalue) of the structure respectively. $[M]$, $[C]$, $[K]$, $\{u(x,t)\}$ and $\{F(x,t)\}$ are the system inertia or mass matrix, damping matrix, stiffness matrix, vector of displacements and force vector respectively. The design variables of the Eq. (1) are the cross-section and the lengths of beams. Displacements, stresses and the natural frequencies, as the responses of the structure, form the state variables. In fact, according to Eq. (1), in such a classic formulation for the layout optimization of a vibrant system, the first three constraints, as behavioral constraints, act on the state variables; while the fourth and fifth constraints, as design constraints, act on the design variables. Mostly, the constraints on natural frequencies are taken into account when the free vibration of the structure is studied. The last constraint is an equilibrium one, which is common in almost all dynamic optimization problems of this kind. In case of eigenfrequency optimization the objective function is swapped with the frequency constraints.

The optimization formulation, presented by Eq. (1) for a structure under dynamic conditions, leads to a layout design that optimizes the weight of the structure. For reinforced concrete structures, such a formulation might not reach an optimum design, as different materials are involved in the structure. Furthermore, for preliminary stage of design, say topology and layout optimization of RC structures, it seems unnecessary to present a realistic and robust definition of the lifecycle cost. Therefore, in this case, the initial cost that is represented by the volume of the structural material is used for the objective function of optimization problems. For cost optimization of general reinforced concrete structures, a basic objective function can be represented by Eq. (2).

$$Cost = \sum_{e=1}^{m} c_c A_{c_e} + c_{sl} A_{sl_e} + c_{sv} A_{sv_e} + c_f P_{f_e} \quad (2)$$

in which c_c, c_{sl}, c_{sv} and c_f are the unit costs of concrete, longitudinal steel, shear steel and formwork respectively and A_c, A_{sl}, A_{sv} and P_f are their corresponding quantities.

In design optimization, design variables are the quantities that are modified by the optimizer during the search for an improved design. For cost optimization of layout design of a concrete structure under dynamic conditions, Eq. (1) needs to be formulated in a way that involves the abovementioned cost components. In order to achieve such a formulation, there is a need for some revisions to design and/or state variables, which consequently result in new objective functions and set of constraints.

A New Space for the Optimization Problem

For topology or layout optimization of large RC structures, where the shape or the number or lengths of structural members are not predefined, using Eq. (2) leads to a significant number of design variables and constraints. Moreover, in order to determine cross-sectional parameters, say and A_c, A_{sl}, A_{sv} and P_f, each step of the cost optimization for layout design of a structure includes both analysis and design processes. In this case, besides the classic cross-sectional variables, the layout characteristics of a structure, say the lengths of spans in a multi-span structure, would be variables of the problem and the designer has to repeat the design procedure to achieve the optimal cross-sectional variables that are usually functions of other layout variables.

Sharafi et al. (2012) presented a methodology, to use structural analysis outputs, such as internal actions of a member, as design variables instead of cross-sectional properties, and represented the cost as a function of design action effects based on Australian Standards for concrete structures, AS3600 (2009). Therefore, having the design action effects in critical sections, all the cross-sectional parameters and consequently, the total cost based on Eq. (2) can be calculated as shown in Eq. (3).

$$
\begin{cases}
\Delta Cost = c_1 \Delta M_{u_i}^{+} + c_2\, \Delta M_{u_i}^{-} + c_3\, \Delta V_{u_i} \\
\begin{cases}
c_1 = \frac{1}{3} c_c K_4 + \frac{1}{2} c_{sl} K_1 + \frac{2}{3} c_f K_7 \\
c_2 = \frac{1}{3} c_c K_3 + \frac{1}{2} c_{sl} K_1 + \frac{2}{3} c_f K_6 \\
c_3 = \frac{1}{3} c_c K_5 + c_{sv} K_2 + \frac{2}{3} c_f K_8
\end{cases} \\
\begin{cases}
K_1 = (f_{yl}\, d(1 - 0.5\gamma k_u))^{-1} \\
K_2 = \left(f_{yv}\, d\right)^{-1} \Delta V_u \\
K_3 = \left[f_{yl} \frac{A_{sc}}{b} (1 - 0.5\gamma k_u)\right]^{-1} \\
K_4 = \left[f_{yl} \frac{A_{st}}{b} (1 - 0.5\gamma k_u)\right]^{-1} \\
K_5 = \left[f_{yl} \frac{A_{st}}{b} (1 - 0.5\gamma k_u)\right]^{-1} \\
K_6 = 2\left[f_{yl} A_{sc} (1 - 0.5\gamma k_u)\right]^{-1} \\
K_7 = 2\left[f_{yl} A_{st} (1 - 0.5\gamma k_u)\right]^{-1} \\
K_8 = \left[\frac{2}{\frac{f_{yv} A_{sv}}{s} + \beta b (f'_c)^{\frac{1}{2}}} + \frac{1}{\beta d (f'_c)^{\frac{1}{2}}}\right]
\end{cases}
\end{cases}
\tag{3}
$$

where f_{yl} is the yield strength of the longitudinal reinforcement steel, f_{yv} is the yield strength of the shear reinforcement steel, f'_c is the characteristic compressive cylinder strength of concrete at 28 days, and β is a coefficient based on the AS3600 (2009) to represent the effects of arching, size and axial forces on the beam section shear capacity. Other parameters are shown in Fig. 1. Eq. (3) denotes how cost variation, in an iterative procedure, can be presented by a weighted sum of the variations of action effects. Such a formulation for cost function is capable of being easily employed in topology and geometric layout optimization of RC structures (Sharafi et al. 2012).

Dynamic Response of Multi-span Beams

Consider a typical continuous beam as an assembly of *M* uniform Euler-Bernoulli beam segments that are serially connected at *M-1* nodes; see Fig. 2. For the *k*th beam segment, S_k, which is bounded by Nodes *k-1* and *k*, the transverse displacement is governed by Eq. (4).

$$\rho_k \frac{\partial^2}{\partial t^2} u_k(x,t) + EI_k \frac{\partial^4}{\partial x^4} u_k(x,t) + d_v \frac{\partial}{\partial t} u_k(x,t) = -F(x,t) \qquad : for \;\; 0 \leq \; x \leq L_k \tag{4}$$

where ρ_k and EI_k are the linear density and bending stiffness of the segment respectively, d_v is viscous damping coefficient, and *F(x, t)* is the external force. The transverse displacement *u(x, t)* of each beam is approximated by a *p*-term modal series:

$$u_k(x,t) \approx \sum_{j=1}^{p} \varphi_{k_j}(x) q_{k_j}(t) = [\varphi_k(x)]^T \{q_k(t)\} \tag{5}$$

$[q_k(t)]$ is the unknown generalized coordinates' vector, and $[\varphi_{k_j}]$ is the eigenfunctions matrix of the beam segment. After imposing nodal constraints and boundary conditions, Eq. (5), define an eigenvalue problem for the continuous beam, the solutions of which characterize the vibration of the beam in specific patterns or modes. Such a formulation forms the equilibrium constraints of the above-mentioned structural optimization. Employing such a superposition method, cause the equilibrium equations to be transformed into a form in which the step-by-step solution is less costly than the traditional methods. Having the transverse displacements, the bending moments and shearing forces, as the required action effects, can be determined along the beam sections, using Eq. (6).

$$\begin{cases} M(x,t) = EI \frac{\partial^2}{\partial x^2} u(x,t) \\ V(x,t) = EI \frac{\partial^3}{\partial x^3} u(x,t) \end{cases} \tag{6}$$

The effectiveness of the method depends on the number of modes that must be considered. If only a few modes may require to be considered, the mode superposition procedure can be quite effective for the layout optimization problem. Although the computational cost is reduced by using only a few modes for expansion, in terms of sensitivity analysis of the structures, sometimes higher modes are needed to accurately express the sensitivity coefficients of the lower eigenmodes (Choi and Kim 2005). In the case of free vibration analysis, natural vibration is a way to describe the behavior of the structure. In this case, the structure vibrates based on the characteristics of its material property and geometric shape. In this study, for free vibration analysis of multi-span beams, it is assumed that all eigenvalues and eigenvectors are simple, and they are not repeated. When the repeated eigenvalues are due to symmetry of the structure, then the symmetry reduction techniques of Kosaka and Swan (1999) can regularize the problem.

Formulation for the Geometric Layout Optimization Problem

Consider a multi-span RC beam with M spans and a total constant length of L, subject to an arbitrary dynamic loading system $F(x,t)$ as shown in Fig. 2. The aim is to re-design the layout by determining the optimum span lengths. The matrix differential equations for determining the dynamic responses of the beam can be solved by employing an iterative method of approximation of solution, say Rung-Kutta method (Lin and Trethewey 1990; Sinha et al. 1993).

Now, in order to re-analyze the rectangular RC beam to achieve the optimum criteria, the cost can be considered the sum of cost functions of all selected sections in the structure, which in turn are the functions of the action effects on the sections.

$$C = \sum_{i=1}^{NS} C_i = \sum_{i=1}^{NS} (c_{1i} M_{ui}^{+} + c_{2i} M_{ui}^{-} + c_{3i} V_{ui}) \tag{7}$$

where C is the total cost of the structure and NS is the number of selected sections to control the cost. The control sections can be selected according to the importance or effectiveness. That is, for example, at least three crucial sections along the beam are considered to design a RC beam: two ends and one in the middle. Depending on the loading system, constraints and boundary conditions larger number of cross-sections might be taken into account.

The strength constraints on each selected section i in the structure under a load case may be written as

$$\begin{cases} \emptyset_f \, M_{ui}^{+} \ \geq M_i^{*^{+}} \\ \emptyset_f \, M_{ui}^{-} \ \geq M_i^{*^{-}} \\ \emptyset_s \, V_{ui} \ \geq |V_i^{*}| \end{cases} \qquad for\ i=\{1,2,\ldots NS\} \tag{8}$$

in which, $M_i^{*^{+}}$, $M_i^{*^{-}}$ and $|V_i^{*}|$ are positive and negative flexure and shear action effects of Section i, and $\emptyset_f$ and $\emptyset_s$ are strength reduction factors in flexure and shear respectively. The serviceability requirements limit the maximum deflection u_{max} on the entire member to Δ_{max} under the serviceability load case. So, for all sections:

$$\left| \delta_i \right| \leq \{\delta_u\} \qquad for\ i=\{1,2,\ldots NS\} \tag{9}$$

Other constraints for durability, fire resistance, minimum cover and minimum flexural strength, can be easily added to the problem as well, based on the relevant design codes.

Now, the general formulation of the geometric layout optimization problem can be presented as:

$$
\begin{cases}
\text{Minimize} \quad Cost(l_1, l_2, \ldots, l_N) = \sum_{i=1}^{NS}(c_{1i}M_{ui}^{+} + c_{2i}M_{ui}^{-} + c_{3i}V_{ui}) \\
s.t \begin{cases}
\{\delta\} \leq \{\delta_u\} & for\ all\ control\ sections \\
\emptyset_f\{M_u^{+}\} \geq \{M^{*^{+}}\} & for\ all\ control\ sections \\
\emptyset_f\{M_u^{-}\} \geq \{M^{*^{-}}\} & for\ all\ control\ sections \\
\emptyset_s\{V_u\} \geq |V^{*}| & for\ all\ control\ sections \\
\omega_N \in \{allowable\ frequencies\} \\
L_{e_{min}} \leq L_e \leq L_{e_{max}} \quad (e = 1, 2, \ldots, M) \\
[M]\{\ddot{u}(x,t)\} + [C]\{\dot{u}(x,t)\} + [K]\{u(x,t)\} = \{-F(x,t)\} \\
other\ constraints\ based\ on\ standards
\end{cases}
\end{cases}
\tag{10}
$$

Compared to Eq. (1), in Eq. (10), the span lengths are the only design variables, and cross-sectional action effects of selected sections are state variables. That is, the behavioral constraints are imposed on sections' bending moments, shear forces and displacements of control sections, rather than the entire structure. Moreover, removing the designed section characteristics, such as the sections area or reinforcement details, from the design variables, helps the iterative optimization procedure not deal with the design parameters.

Using the Euler-Bernoulli beam element for finite element analysis of structures, the dimension of the above behavioral constraints vectors will be *2N*; where *N* is the number of spans. The dimension of other variable constraint vectors, say $\{M_{ui}^{+}\}$, $\{M_{ui}^{-}\}$, is *NS*. In practice one does not solve the behavioral constraint equation for all modes of the eigenvalue problem, and only the first up to 10 modes usually are considered in determining the dynamic response of a structure (Bendsøe and Sigmund 2003). In symmetric structures, where there are techniques that simply determining the natural frequencies (Chen and Feng 2012), one can use a higher number of modes for an iterative optimization procedure due to the simplicity of computations. Furthermore, the spans lengths or the nodal coordinates of the multi-span beam are considered as design variables, and the optimal solutions may be found by using appropriate methods of mathematical programming. In this process, side constraints are usually given for the nodal coordinates to prevent existence of short or long members to satisfy architectural considerations. Therefore, the length of each span can be constrained between L_{min} and L_{max}.

Ant Colony Algorithm for Solving the Optimization Problem

An amazing behavioral pattern displayed by certain ant species is the ability to find what scientists call the shortest path. Such an ability has inspired computer scientists to develop algorithms for the solution of optimization problems. Biologists have shown experimentally that this is possible by exploiting communication based only on pheromones, an odorous chemical substance that ants may deposit and smell. Ant colony optimization (ACO) is a method developed by Dorigo and his associates in 1990's based on the cooperative behavior of real ant colonies to find the shortest path from their nest to a food source (Dorigo et al. 2006; Rao 2009). Various types of ant-based algorithms have found vast implementations in civil engineering and structural optimization (Christodoulou 2010; Lee 2012; Sharafi et al. 2012). The present ACO algorithm for geometric layout optimization of multi-span beams is described in a pseudo-code, as shown in Fig. 3. The procedure comprises three main phases: initializing data, constructing ant solution and updating pheromone.

The construction graph for the algorithm is formed as a multilayered graph as shown in Fig. 4, where the number of layers equals the number of design variables, that is spans' lengths, and the number of nodes in a particular layer equals the number of discrete probable values permitted for the corresponding design variable. Thus each node on the graph is associated with a permissible discrete value of a design variable. For discretizing the domain and forming the construction graph, a graph with M layers is constructed. Knowing that each span length is bounded in $[L_{e_{max}}, L_{e_{min}}]$, the permissible values for each span length, which are represented by the nodes on the graph, can be discretized with intervals (accuracy) equal to ε. That is, each L_e rests in the set of $\{L_{min}\ , L_{min} + \varepsilon,\ L_{min} + 2\varepsilon\ , \ldots, L_{max} - \varepsilon\ , L_{max}\}$. Each member of this set corresponds to a node on the graph. That is, the number of nodes for each layer is $(L_{e_{max}} - L_{e_{min}})/\varepsilon$. Obviously, the smaller ε is, the more accurate the results will be and the more running time the algorithm needs.

The heuristic values are specified according to the designers' preferences. For example in this case, if there are any preferences for certain span lengths, say due to architectural constraints, higher values are assigned to the heuristic arrays corresponding to those span lengths. Such an assignment will cause the desired lengths to be more likely to be chosen by ants for the corresponding spans. Then by choosing appropriate values for α and β the degree of influence of the heuristic values in comparison with the pheromone trail is determined. Moreover, in order to save time, some criteria like the symmetry of the beam are considered when the heuristic matrix is formed. Using such a heuristic matrix or defining the initial pheromone matrix using the above-mentioned structural rules helps the algorithm converge sooner.

The initial magnitude of pheromone on the construction graph must be set. A good heuristic to initialize the pheromone trails is to set them to a value slightly higher than the expected amount of pheromone deposited by the ants in one iteration. A rough estimate of the value can be obtained by setting, $\forall(i,j)\tau_{ij} = \tau_0 = N/C^{nn}$ where N is the number of ants, and C^{nn} is the cost of a tour generated by the nearest neighbor heuristic (Kaveh and Sharafi 2008a; 2008b; 2009). As boundary conditions and constraints, plus any loads can affect the length of spans, the entries of heuristic matrix and initial pheromone matrix might be organized based on such parameters. Thus, each value (*i,j*) of choice information matrix, which is obtained by multiplying the corresponding arrays of heuristic matrix by those of pheromone matrix, shows the tendency or desirability of the ant located on Node i of the construction graph to choose Edge j to move towards Node $i+1$.

Now, N artificial ants are located on the home node construct their solution by selecting only one node in each layer in accordance with the random proportional rule given by Eq. (11). Each value (i,j) of pheromone matrix and heuristic matrix shows the tendency of the ant located on Node i of a layer to choose the Node j on the next layer as the next node. In each iteration, there are as many as $(L_{max} - L_{min})/\varepsilon$ possible options (nodes) to be selected.

$$p_{ij}^{k} = \begin{cases} \dfrac{[\tau_{ij}]^{\alpha}[\eta_{ij}]^{\beta}}{\sum_{l \in N_i^k}[\tau_{il}]^{\alpha}[\eta_{il}]^{\beta}} & \text{if } j \in N_i^k \\ 0 & \text{if } j \notin N_i^k \end{cases} \tag{11}$$

This relationship forms the basis of the Ant System (Ant System) algorithm and shows that if Ant k is positioned on Node i, it will move to the next Node j with the probability of p_{ij}^{k}. In this relationship, τ_{ij} is the magnitude of pheromone on the trails and η_{ij} is the heuristic value. α and β are two parameters which determine the relative influence of the pheromone trail and the heuristic information. N_i^k is the feasible neighborhood of Ant k when being at Node i, that is, the set of edges that Ant k is allowed to choose as its next destination, and is decided depending on the problem in hand. The nodes selected along the path visited by an ant represent a candidate solution.

Using the information obtained in the phase of constructing the solution, the trail presenting the best cost up to this stage is selected as the best so-far iteration. Then pheromone trails of construction graph are updated. This act is done by first lowering the pheromone value on all arcs (pheromone evaporation), and then adding pheromone on the edges the ants have crossed (pheromone depositing). Pheromone evaporation is done by lowering the pheromone value on all edges by a constant factor ρ according to Eq. (12):

$$\tau_{ij} \leftarrow (1-\rho)\tau_{ij} \quad \forall (i,j) \in A \tag{12}$$

Then, once the path is complete, the ant deposits some pheromone on the path based on the rule given by Eq. (13).

$$\tau_{ij} \leftarrow \tau_{ij} + \sum_{k=1}^{m} \Delta\tau_{ij}^{k} \;,\; \forall (i,j) \in A \tag{13}$$

where $\Delta\tau_{ij}{}^{k}$ is the pheromone added to τ_{ij} by Ant *k,* and defined as follows:

$$\Delta\tau_{ij}^{k} = \begin{cases} 1/C^{k} & \text{if edge } (i,j) \text{ belongs to } T^{k} \\ 0 & \text{otherwise} \end{cases} \tag{14}$$

As such, in each iteration, all the ants, in parallel, start from the home node and end at the destination node by randomly selecting a node in each layer. The optimization process is terminated if no better solution is found in a pre-specified number of successive iterations.

Design Examples

In order to evaluate the effects of geometric layout on cost, and to demonstrate the robustness of the proposed approach, a general multi-span beam under different loading conditions is examined. The geometric optimum layout of the beam is designed under two loading system; first a static Uniformly Distributed Load (UDL), and second a moving point load along the beam. Then the results are compared with those of obtained from an eigenvalue optimization method, which is presented by Achtziger and Kočvara 2007. Since the eigenvalue optimization method does not necessarily lead to cost optimization, this method is only considered as a benchmark and a starting point for the algorithm. That is, first, the topology is optimized using the eigenvalue optimization method, and the cost is determined. Then, the topology design obtained from the present method is compared with that obtained from the eigenvalue optimization method, in terms of cost, to determine the cost saving.

In order to ensure that the obtained solution is global or near global optimum, many runs were made in parallel. Since each run is fully independent of the others, the program can be run in parallel so that the total execution time will be practically the same as required for a single run. All computations were performed on P9700 @2.80 GHz computer running MATLAB R2009b.

Consider a multi-span continuous RC beam with a total length of L, as shown in Fig. 5. The material properties and relative cost factors are: f'_c=25 MPa, f_{yl}=500 MPa, f_{yv} =250 MPa, c_c=1, c_{sl}=75, c_{sv} =64, c_f=0.45. The aim is to determine the optimum spans' ratios, L_1 /L through L_M /L, under a number of different loading systems, so that the cost is minimized. The methodology is examined for the different number of spans ($M \in \{3,4,\ldots,10\}$). For each single beam three control sections (C_{i1} trough C_{i3}) are selected, which leads to $3M$ control sections for each problem. In practice, the number of control sections is completely dependent on the number of critical ones, and depending on the loading system, one may choose more sections to increase the accuracy of results.

The boundary conditions and support constraints are set based on the fact that all the supports act like perfect hinges. An average viscous damping ratio of ξ=0.03 is considered for the whole structure (Adams and Askenazi 1999), and for approximation of solution of the matrix differential equations the Runge-Kutta algorithm is employed.

Every initial design based on preliminary judgment of the designer and/or using approximate charts or formulas, which meet the design code requirements, can be used as the initial design and as the starting point of optimization process. For this problem, the case of design for maximum fundamental eigenvalue for free vibration was considered as an initial design and a benchmark to compare the results with. For this purpose, the layout of the beam is optimized in order to maximize the fundamental eigenfrequency for all cases and the results are considered the starting point for the optimization problem. The results show that the case of maximum fundamental eigenfrequency, for such a continuous beam, leads to identical spans for all cases. That is, the continuous beams with all the spans are identical have the maximum fundamental eigenfrequency among the other possible layout designs. Having the initial design and using Eq. (3) the values of K_1 to K_8, and then, the values of c_1, c_2 and c_3 for Sections A to O are obtained. Having the necessary coefficients, the cost function can be defined.

Two different loading systems are considered and the optimum layout designs are obtained for each one independently. The first one is a uniformly distributed static load, and the second case is a point wise load of 100 kN, moving along the beam at a speed of 2 m/sec from $x=0$ to $x=L$. As mentioned above, for all cases, the designs with all spans are equal to L/M, and the corresponding costs are considered as the initial design, and the starting point for the algorithm.

For the initial design, that is, the layout with the maximum eigenvalue, the spans ratios, which result in the maximum fundamental eigenvalue, is taken. For the case one and static loading, the equilibrium constraint in Eq. (3) is turned to the static condition $[K]\{u(x)\}=\{F_{st}\}$. In this example, the behavioral constraint on natural frequency is not required, neither for the static loading nor for the dynamic one. In the second case, the dynamic load is formulated as:

$$F(x,t)= -\delta(x-vt)P \tag{15}$$

In which δ is a Dirac delta function, v is the constant speed of the load motion, which is 2 m/s in this case and P is weight of the moving object, which is 100 kN.

In the phase of the construction graph, the span lengths, as the problem domain, are discretized and for each span a number of possible lengths are considered. For this purpose the maximum and the minimum possible span lengths and the problem accuracy (ε) are defined and the construction graph is produced (see Fig. 4). Then ants are located on the starting and ending nodes to make their trails in parallel. The reason why two groups of ants are employed is to give all spans the same probability of being selected as the optimum lengths. Then, based on the random proportional rule, given by Eq. (11), and the pheromone update, given by Eqs. (12) through (14), ants find the optimum spans lengths. In each iteration, the beam needs to be reanalyzed according to the new span lengths to determine the costs and the best so far iteration. In the proposed ACO algorithm for this instance, 10 parallel runs were made to achieve the optimum solution. The solution goes on until the termination criterion is reached. The termination criterion for this problem is defined as the number of iterations, when the improvement in the solution quality was less than 0.02% after ten consecutive iterations.

The results, that is, the optimum spans' ratios and the achieved cost saving, compared to the initial design, for each case are shown in Table 1.

As shown in Table 1, the proposed methodology provides cost savings between 4 to 7 percent compared to the benchmark designs. The benchmark designs were carried out only based on the eigenvalue optimization method without considering the cost elements. Such an achievement is obtained by optimizing the preliminary layout design of multi-span beams based on the involved cost elements, which mainly prevents the final cost optimization procedure from reaching a sub-optimal solution. Employing other cost optimization methods without considering the effect of cost elements on the preliminary layout design or employing other layout optimization methods without considering the cost elements would result in a suboptimal solution. In the proposed methodology, the preliminary layout design is optimized such that the cost optimization procedure starts from an optimal point, and is more likely to get to a global optimization compared to other methods.

Concluding Remarks

The preliminary layout design of a structure affects the entire design process, construction, and consequently the total cost. Without an optimum layout plan, the optimum design of structures will lead to suboptimal solutions. Moreover, an optimum layout design can be effective when the involving cost elements are taken into account. This paper presents a heuristic methodology for geometric layout optimization of multi-span continues RC beams under dynamic loading systems. The proposed formulation has the distinction of taking the involving cost elements into account, and the ability of being employed for different boundary conditions, support constraints and loading systems with no limitations. The solved examples demonstrate the robustness of the approach and show that the methodology with the help of the proposed ACO algorithm can be easily carried out under any dynamic conditions to simplify the computer-aided preliminary layout design.

Table 1: Results for Different Number of Spans

Loading System 1: Static UDL											
Number of spans	Optimum Spans Ratios										Cost Saving Compared to the Benchmark
	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8	S_9	S_{10}	
M=3	0.31	0.38	0.31	-	-	-	-	-	-	-	6.83%
M=4	0.23	0.27	0.27	0.23	-	-	-	-	-	-	5.53%
M=5	0.18	0.22	0.20	0.22	0.18	-	-	-	-	-	6.05%
M=6	0.15	0.18	0.17	0.17	0.18	0.15	-	-	-	-	7.15%
M=7	0.12	0.16	0.15	0.14	0.15	0.16	0.12	-	-	-	6.98%
M=8	0.11	0.14	0.12	0.13	0.13	0.12	0.14	0.11	-	-	5.45%
M=9	0.09	0.13	0.11	0.12	0.10	0.12	0.11	0.13	0.09	-	4.01%
M=10	0.08	0.11	0.11	0.10	0.10	0.10	0.10	0.11	0.11	0.08	5.52%

Loading System 2 : Point wise Moving Load											
Number of spans	Optimum Spans Ratios										Cost Saving Compared to the Benchmark
	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8	S_9	S_{10}	
M=3	0.34	0.32	0.34	-	-	-	-	-	-	-	6.18%
M=4	0.26	0.24	0.24	0.26	-	-	-	-	-	-	6.53%
M=5	0.22	0.19	0.18	0.19	0.22	-	-	-	-	-	4.57%
M=6	0.18	0.16	0.16	0.16	0.16	0.18	-	-	-	-	6.18%
M=7	0.16	0.14	0.14	0.12	0.14	0.14	0.16	-	-	-	4.77%
M=8	0.14	0.12	0.12	0.12	0.12	0.12	0.12	0.14	-	-	5.55%
M=9	0.12	0.12	0.11	0.10	0.10	0.10	0.11	0.12	0.12	-	5.86%
M=10	0.12	0.10	0.10	0.09	0.09	0.09	0.09	0.10	0.10	0.12	4.36%

Notation

A_e = cross-sectional area of elements
A_{sc} = cross-sectional area of compression reinforcement
A_{st} = cross-sectional area of tension reinforcement
A_{sv} = cross-sectional area of shear reinforcement
b = width of a cross-section
c_c = unit cost of concrete
c_{sl} = unit cost of longitudinal steel
c_{sv} = unit cost of shear steel
c_f = unit cost of formwork
d_c = cover to reinforcing steel or tendons
d = effective depth of a cross-section
d_v = viscous damping coefficient
E = modulus of elasticity
f_{yl} = yield strength of longitudinal reinforcing steel
f_{yv} = yield strength of shear reinforcing steel

f'_c = characteristic compressive cylinder strength of concrete at 28 days
h = overall depth of a cross-section in bending plane
k_u = neutral axis parameter, being the ratio, at ultimate strength, of depth to neutral axis from the extreme compressive fiber, to d
L = center-to-center distance between the supports of a flexural member
L_e = length of elements
M^* = applied bending moment
M^+_u = ultimate strength in positive bending at a cross-section of a member
M^-_u = ultimate strength in negative bending at a cross-section of a member
P_f = formwork area
V^* = applied shear force
V_u = ultimate shear strength
W = total weight of the structure
β = a fixity factor
γ = ratio, under design bending or combined bending and compression, of the depth of the assumed rectangular compressive stress block to k_u
δ_a = allowable displacement
η_{ij} = heuristic value
ρ = density of concrete, in kilograms per cubic meter
σ_a = allowable stress
τ_{ij}= magnitude of pheromone on trails
$\emptyset_f$ = strength reduction factors in flexure
$\emptyset_s$ = strength reduction factors in shear
ω_N = fundamental natural frequency (eigenvalue) of the structure

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