Three dimensional sensing by digital video fringe projection

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Three Dimensional Sensing by Digital Video Fringe Projection

Matthew J. Baker
B.E. Telecommunications (Hons),
University of Wollongong

A Thesis presented for the degree of
Doctor of Philosophy

School of Electrical, Computer and Telecommunications Engineering,
University of Wollongong,
Australia,
April 2008

Thesis supervisors: A/Prof. Jiangtao Xi and Prof. Joe F. Chicharo
This thesis is dedicated to my family and friends.
Abstract

Fast, high precision and automated optical noncontact surface profile and shape measurement has been an extensively studied research area due to the diversity of potential application which extends to a variety of fields including but not limited to industrial monitoring, computer vision, virtual reality and medicine. Among others, structured light Fringe Projection approaches have proven to be one of the most promising techniques. Traditionally, the typical approach to Fringe Projection 3D sensing involves generating fringe images via interferometric procedures, however, more recent developments in the area of digital display have seen researchers adopting Digital Video Projection (DVP) technology for the task of fringe manufacture. The ongoing and extensive exploitation of DVP for Fringe Projection 3D sensing is derived from a number of key incentives the projection technology presents relative to the more traditional forms of projection. More specifically, DVP allows for the ability to accurately control various attributes of the projected fringe image at high speed in software, along with the capabilities to develop multi-channel techniques via colour projection. Furthermore, considering the typical DVP source is capable of projecting a standard 24 bit bitmap computer generated image, when interfaced to a personal computer, DVP makes for a very affordable projection source. However, despite the aforementioned incentives, in contrast to the more traditional methods of generating fringe images, the digitally projected fringe signal presents a number of shortcomings which ultimately hinder the effective application of the technology for Fringe Projection 3D sensing.

This thesis aims to improve the effectiveness of the deployment of DVP technology for Fringe Projection 3D sensing approaches. The proposed initiative is facilitated through extensive analysis of the application of DVP technology for fringe processing, and furthermore by the proposal of new digital fringe calibration procedures.

Firstly, this work demonstrates a comprehensive survey of current Fringe Projection 3D sensing approaches including an introductory review of the rudimentary notion of projecting fringes for 3D data acquisition. The survey also provides a thorough description of the evolution of the three major forms of fringe processing i.e. Fringe Phase Stepping,
Fourier Fringe analysis and Direct Detection.

The limitations of DVP for Fringe Projection are demonstrated through the development of a novel fringe phase emulation approach. The phase emulation approach is subsequently employed to establish empirical insight into the application of DVP technology for Fringe Projection. More specifically, the preliminary empirical analysis is used to test the veracity of the application of the two chief DVP technologies (Liquid Crystal Display, LCD and Digital Light Processing, DLP, Texas Instruments) for Fringe Projection. Through this study the camera / projector non-linear intensity response is shown to be the single most significant shortcoming inherent to DVP based Fringe Projection implementations.

Following the findings of the preliminary empirical analysis the influence of the Display Gamma attributes of the projection system is extensively investigated. The harmonic structure of a typical digitally projected fringe signal is examined and an approximate analysis framework proposed. The framework is subsequently utilised to form a set of equations defining the true $\gamma$ sensitivity of a range of highly exploited fringe processing techniques. The approximate analysis is later verified and the practical significance of the findings demonstrated. Through this study the true nature of the Display Gamma related phase measuring residual error is revealed.

With the aid of a verified framework, investigations into additional Display Gamma related Fringe Projection phenomena is undertaken. More specifically, the optimisation of digitally projected fringes by fringe parameter manipulation is demonstrated. The temporal nature of digitally projected fringe images is studied for the well exploited single shot Fourier Transform Profilometry technique and the digital fringe harmonic dependence on the projector optical modulation transfer function is revealed. Subsequently, the elimination of Display Gamma related Fringe Projection phase measuring residual error for phase stepping techniques by projector defocus optimisation is shown.

Finally, a novel digital fringe calibration approach ideal for minimum shot fringe processing techniques is proposed. The calibration procedure is centered on the application of Artificial Neural Networks (ANNs) to correct the non-linear intensity distortion associated with the camera / projector system. Unlike previously proposed gamma correction techniques, the neural fringe calibration technique requires no additional data acquisi-
tion with effective calibration requiring no more than a single cross-section of a reference fringe. The neural network fringe calibration approach is also shown to significantly outperform simple filter based techniques of similar computational complexity. Given the reduced data requirements for the neural approach its application for multi-channel fringe calibration is also considered.
Certification

I, Matthew John Baker, declare that this thesis, submitted in fulfilment of the requirements for the award of Doctor of Philosophy, in the School of Electrical, Computer and Telecommunications Engineering, University of Wollongong, is wholly my own work unless otherwise referenced or acknowledged. The document has not been submitted for qualifications at any other academic institution.

Matthew John Baker
17th February 2008
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doubtedly never be able to repay the latter debts, however, i should be ok for the financial
debs.... eventually.

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without a doubt my greatest accomplishment and although he cannot read many of the
words contained within this document watching him grow and learn how to has truly
been an inspiration for the completion of this work.
List of Publications

Journal publications:


2. Matthew J. Baker, Joe Chicharo and Jiangtao Xi, “Accuracy Limitations in Profilometric Metrology Schemes using Digital Structured Light”, *Accepted subject to revisions, IEEE Transactions on Instrumentation and Measurement*

3. Matthew J. Baker, Jiangtao Xi and Joe Chicharo “The Implications of Display Gamma for Fringe Projection 3D sensing using Digital Video Projection”, *To be submitted to Applied Optics*

Conference publications:


# Table of Contents

Abstract iii  
Certification vi  
Acknowledgements vii  
List of Publications ix  
Table of Contents xi  
List of Abbreviations xv  
List of Figures xvii  
List of Tables xxii  

## 1 Preliminaries

1.1 Introduction .......................... 1  
1.2 Three-Dimensional Sensing .................. 2  
  1.2.1 Photogrammetry Methods .......... 3  
  1.2.2 Time-of-Flight Techniques ....... 6  
  1.2.3 Interferometric Techniques ....... 6  
  1.2.4 Structured Light Techniques ...... 9  
  1.2.5 Applications .................. 14  
1.3 Motivation for this Research .......... 16  
1.4 Approach and Contributions of this Thesis .................. 18  
  1.4.1 Overview of Structured Light 3D Sensing by Fringe Projection .... 18  
  1.4.2 Creating Structured Light .......... 19  
  1.4.3 Fringe Projection and Display Gamma .......... 19  
  1.4.4 Additional Display Gamma Phenomena ........ 20  
  1.4.5 Digital Fringe Calibration using Neural Networks ........ 20  
1.5 Summary of Contributions in Order of Presentation .......... 21  

## 2 Overview of Structured Light 3D Sensing by Fringe Projection 23  
2.1 Introduction ......................... 23  
2.2 Optical Triangulation ................ 23  
  2.2.1 Optical Arrangement ........ 26
# CONTENTS

2.2.2 Extracting Height ........................................ 27
2.3 Projecting Fringes ........................................ 27
2.4 Fringe Processing Techniques .............................. 30
  2.4.1 Phase Measuring or Shifting ......................... 30
  2.4.2 Fourier Transform Fringe Processing ............... 36
  2.4.3 Direct Phase Detection .............................. 46
2.5 Phase Unwrapping ........................................ 49
  2.5.1 Phase Unwrapping Principle ......................... 49
  2.5.2 Path-Dependent Unwrapping ......................... 52
  2.5.3 Path-Independent Unwrapping ....................... 54
2.6 Calibration ................................................ 56
2.7 Summary ................................................... 57

3 Creating Structured Light Fringe Images ................. 58
  3.1 Introduction .............................................. 58
  3.2 Projection Technologies ................................. 58
    3.2.1 Analog Vs. Digital .................................. 59
    3.2.2 Analog Fringe Projection .......................... 59
    3.2.3 Digital Fringe Projection (DFP) .................. 60
  3.3 Digital Video Projection Technology ................... 61
    3.3.1 Liquid Crystal Display (LCD) ..................... 62
    3.3.2 Digital Light Processing (DLP) .................. 65
  3.4 Digital Video Projection Signal Characteristics ....... 67
    3.4.1 Finite Projection ................................... 67
    3.4.2 Projected Signal Geometric Structure ............ 72
  3.5 Colour Projection ........................................ 74
    3.5.1 Colour Theory ...................................... 76
    3.5.2 Coupling of Colour Channels ...................... 80
  3.6 DLP Vs. LCD ............................................. 82
    3.6.1 Empirical Procedure ............................... 82
    3.6.2 Results and Discussion ............................ 85
  3.7 Summary .................................................. 91

4 Fringe Projection and Display Gamma ...................... 92
  4.1 Introduction .............................................. 92
  4.2 Modeling a $\gamma$ Distorted Fringe .................... 94
    4.2.1 Harmonic Structure of $\gamma$ Distorted Fringe 95
  4.3 Impact of Harmonics for Fringe Processing Phase Estimation 99
    4.3.1 Analytical Procedure ................................ 100
    4.3.2 Traditional Phase Measuring Algorithm .......... 101
    4.3.3 $90^\circ$ 3 Step .................................... 104
    4.3.4 $90^\circ$ 3 Step with Phase Offset ................. 107
    4.3.5 $2+1$ ............................................. 108
    4.3.6 $3+3$ ............................................. 109
    4.3.7 Double Three Step ................................ 111
    4.3.8 Fourier Transform Profilometry (FTP) ............ 115
  4.4 Display Gamma Measurement Error ....................... 117
4.4.1 Magnitude of Measurement Error ........................................ 117
4.4.2 Relating Gamma and System Accuracy ................................. 117
4.4.3 Frequency Dependence ..................................................... 117
4.5 Simulation ........................................................................ 119
4.5.1 Simulating an Appropriate Test Surface ............................... 119
4.5.2 Traditional Phase Measuring Algorithm ............................... 122
4.5.3 90° 3 Step .................................................................. 125
4.5.4 90° 3 Step with Phase Offset .......................................... 125
4.5.5 2+1 ........................................................................... 127
4.5.6 3+3 ........................................................................... 129
4.5.7 Double Three Step .......................................................... 131
4.5.8 Fourier Transform Profilometry (FTP) ................................. 132
4.5.9 Magnitude of Measurement Error ...................................... 134
4.6 Experimentation Evaluation .................................................. 135
4.7 Summary ........................................................................... 138
5 Additional Display Gamma Phenomena .................................... 140
5.1 Introduction ...................................................................... 140
5.2 Fringe Offset and Contrast Parameter Manipulation ............... 140
5.2.1 Empirical Verification ...................................................... 143
5.3 Temporal Gamma ............................................................... 145
5.3.1 FTP Temporal $\gamma$ / Harmonic Error Analysis ................. 146
5.3.2 Gamma Compensated Analysis ......................................... 148
5.3.3 Simulation ................................................................... 149
5.3.4 Example FTP Reconstruction ......................................... 150
5.3.5 Gauging the Magnitude of the Reference Ripple .................. 151
5.3.6 Empirical Example FTP Reconstruction ............................. 154
5.4 Fringe Image Formation ...................................................... 155
5.4.1 Modeling the Optical Modulation Transfer Function .......... 155
5.4.2 Elimination of $\gamma$ Non-linear Luminance Effects for Stepping Techniques .......................................................... 156
5.4.3 Attenuating Higher Order Harmonics ................................. 158
5.4.4 Simulation ................................................................... 159
5.4.5 Empirical Verification ...................................................... 161
5.4.6 Display Gamma Phase Residual Frequency Dependence .... 162
5.5 Summary ........................................................................... 163
6 Digital Fringe Calibration using Neural Networks ..................... 165
6.1 Introduction ...................................................................... 165
6.2 Neural Computing .............................................................. 167
6.3 Proposed Neural Network Fringe Calibration ......................... 169
6.3.1 Neural Network Design .................................................. 171
6.3.2 Noise Removal ............................................................. 172
6.4 Simulation ....................................................................... 173
6.5 Performance Comparison with existing approaches ................ 176
6.5.1 Simple Filter Based Technique ........................................ 176
6.5.2 Double Three-Step ......................................................... 179
6.6 Experimental Verification .................................................. 180
6.7 Multi-channel Digital Fringe Calibration using Neural Networks ........ 183
  6.7.1 Principle Digital Multi-channel Fringe Profilometry .................. 184
  6.7.2 Error Analysis .......................................................... 185
  6.7.3 Proposed Multi-channel Fringe Calibration Technique ............... 187
  6.7.4 Simulation ............................................................. 188
  6.7.5 Experimentation Verification ......................................... 188
6.8 Summary ........................................................................... 191

7 Conclusions and Suggestions for Further Research ......................... 193
  7.1 Conclusions ..................................................................... 193
  7.2 Suggestions for Further Research ........................................ 197

A Finite Projection Empirical Verification ...................................... 200

B Physical Fringe Projection Arrangement ...................................... 204
  B.1 Projection ........................................................................ 205
  B.2 Acquisition ....................................................................... 205
  B.3 Software Interface ........................................................... 206

References ............................................................................... 209
List of Abbreviations

3D    Three-Dimensional
2DFTP Two-Dimensional Fourier Transform Profilometry
Am-Si Amorphous Silicon
ANN   Artificial Neural Network
AOM   Acousto-Optic Modulator
CCD   Charged Couple Device
DC    Direct Component
DFP   Digital Fringe Projection
DFT   Discrete Fourier Transform
DLP   Digital Light Processing
DMD   Digital Micromirror Device
DPD   Direct Phase Detection
DVP   Digital Video Projection
FIR   Finite Impulse Response
FTP   Fourier Transform Profilometry
I3PSP Improved Three Step Phase Stepping Profilometry
IFTP  Improved Fourier Transform Profilometry
IIR   Infinite Impulse Response
LCD   Liquid Crystal Display
LCOS  Liquid Crystal on Silicon
MEMS  MicrcoElectroMechanical System
MFTP  Modified Fourier Transform Profilometry
MMP   Modulation Measurement Profilometry
OPD   Optical Path Difference
<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>PC</td>
<td>Personal Computer</td>
</tr>
<tr>
<td>PLL</td>
<td>Phase Locked Loop</td>
</tr>
<tr>
<td>PMP</td>
<td>Phase Measuring Profilometry</td>
</tr>
<tr>
<td>Poly-Si</td>
<td>Polycrystalline Silicon</td>
</tr>
<tr>
<td>PSD</td>
<td>Position Sensitive Detector</td>
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<tr>
<td>PSI</td>
<td>Phase Shifting Interferometry</td>
</tr>
<tr>
<td>PSP</td>
<td>Phase Stepping Profilometry</td>
</tr>
<tr>
<td>SLM</td>
<td>Spatial Light Modulator</td>
</tr>
<tr>
<td>SNR</td>
<td>Signal to Noise Ratio</td>
</tr>
<tr>
<td>SPD</td>
<td>Spatial Phase Detection</td>
</tr>
<tr>
<td>TFT</td>
<td>Thin Film Transistor</td>
</tr>
<tr>
<td>TI</td>
<td>Texas Instruments</td>
</tr>
<tr>
<td>TN</td>
<td>Twisted Nematic</td>
</tr>
</tbody>
</table>
# List of Figures

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1</td>
<td>Simple Telecentric Optical Triangulation</td>
<td>24</td>
</tr>
<tr>
<td>2.2</td>
<td>Simple Full Field Optical Triangulation via the projection of a Structured</td>
<td>25</td>
</tr>
<tr>
<td></td>
<td>Light Pattern</td>
<td></td>
</tr>
<tr>
<td>2.3</td>
<td>Typical Crossed Optical Axes arrangement</td>
<td>26</td>
</tr>
<tr>
<td>2.4</td>
<td>Diverging Illuminance:- Typical Crossed Optical Axes arrangement</td>
<td>28</td>
</tr>
<tr>
<td>2.5</td>
<td>Example Projecting Fringes</td>
<td>29</td>
</tr>
<tr>
<td>2.6</td>
<td>Example phase modulated sinusoidal Fourier Spectra</td>
<td>38</td>
</tr>
<tr>
<td>2.7</td>
<td>Example Baseband Fourier Spectra $Q(f, y)$</td>
<td>39</td>
</tr>
<tr>
<td>2.8</td>
<td>Example: General Fourier Spectra $G(f, y)$ for a Projected Fringe</td>
<td>41</td>
</tr>
<tr>
<td>2.9</td>
<td>Typical Direct Phase Detection process</td>
<td>48</td>
</tr>
<tr>
<td>2.10</td>
<td>Wrapped and Unwrapped Phase Map</td>
<td>50</td>
</tr>
<tr>
<td>2.11</td>
<td>Cross-sections of Wrapped and Unwrapped Phase Maps of Figures (2.10)</td>
<td>50</td>
</tr>
<tr>
<td></td>
<td>(a) and (b)</td>
<td></td>
</tr>
<tr>
<td>2.12</td>
<td>Independent Fringe Paths</td>
<td>53</td>
</tr>
<tr>
<td>3.1</td>
<td>Generalised schematic of the major optical and electrical components of a</td>
<td>63</td>
</tr>
<tr>
<td></td>
<td>LCD</td>
<td></td>
</tr>
<tr>
<td>3.2</td>
<td>(a) Single Panel LCD. (b) Three panel Poly-Si LCD.</td>
<td>64</td>
</tr>
<tr>
<td>3.3</td>
<td>(a) Two Digital Micromirror Device (DMD) Pixels. (b) Digital Micromirror-</td>
<td>66</td>
</tr>
<tr>
<td></td>
<td>Device (DMD) functionality. (c) Example of a single chip DLP projection</td>
<td></td>
</tr>
<tr>
<td></td>
<td>system.</td>
<td></td>
</tr>
<tr>
<td>3.4</td>
<td>Screen Door Effect.</td>
<td>69</td>
</tr>
<tr>
<td>3.5</td>
<td>Finite Projection Characteristic</td>
<td>70</td>
</tr>
<tr>
<td>3.6</td>
<td>Projected Pixel Size Variation in the $x$ direction</td>
<td>73</td>
</tr>
<tr>
<td>Figure</td>
<td>Description</td>
<td>Page</td>
</tr>
<tr>
<td>--------</td>
<td>-----------------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>3.7</td>
<td>Camera / Projector Intensity Response for various $\gamma$</td>
<td>74</td>
</tr>
<tr>
<td>3.8</td>
<td>Distorted Fringe Cross-section</td>
<td>75</td>
</tr>
<tr>
<td>3.9</td>
<td>Electromagnetic Spectrum</td>
<td>77</td>
</tr>
<tr>
<td>3.10</td>
<td>Spectral sensitivity curves for the short, medium and long cones</td>
<td>77</td>
</tr>
<tr>
<td>3.11</td>
<td>Colour Matching Functions</td>
<td>78</td>
</tr>
<tr>
<td>3.12</td>
<td>CIE Colour Matching Functions</td>
<td>79</td>
</tr>
<tr>
<td>3.13</td>
<td>CCD Filter response</td>
<td>81</td>
</tr>
<tr>
<td>3.14</td>
<td>Arbitrary Phase Distribution $\phi(x)$</td>
<td>84</td>
</tr>
<tr>
<td>3.15</td>
<td>Greyscale LCD DLP sample cross-sections for both low and high frequency cases</td>
<td>86</td>
</tr>
<tr>
<td>3.16</td>
<td>Multi-channel LCD DLP sample cross-sections for both low and high frequency cases</td>
<td>89</td>
</tr>
<tr>
<td>4.1</td>
<td>$e_n$, $1 &lt; \gamma &lt; 3$, for normalised fringe parameters $a = b$</td>
<td>97</td>
</tr>
<tr>
<td>4.2</td>
<td>Power ratio $p$, 2nd order harmonic to fundamental Vs. $\gamma$, for normalised fringe parameters $a = b$</td>
<td>99</td>
</tr>
<tr>
<td>4.3</td>
<td>Traditional 3 Step PMP 2nd order harmonic phase measuring residual for constant phase modulation with $p = 0.082$ corresponding to $\gamma = 2.2$ for fringe offset and contrast parameters $a = b$</td>
<td>103</td>
</tr>
<tr>
<td>4.4</td>
<td>$90^\circ$ 3 Step 2nd order harmonic phase measuring residual for constant phase modulation with $p = 0.082$ corresponding to $\gamma = 2.2$ for fringe offset and contrast parameters $a = b$</td>
<td>105</td>
</tr>
<tr>
<td>4.5</td>
<td>Maximum phase measuring error, $\delta_{\text{max}}$, for the $90^\circ$ 3 Step for various $p$ corresponding to $1 &lt; \gamma &lt; 3.$</td>
<td>106</td>
</tr>
<tr>
<td>4.6</td>
<td>Minimum phase measuring error, $\delta_{\text{min}}$, for the $90^\circ$ 3 Step for various $p$ corresponding to $1 &lt; \gamma &lt; 3.$</td>
<td>106</td>
</tr>
<tr>
<td>4.7</td>
<td>$90^\circ$ 3 Step with Phase Offset 2nd order harmonic phase measuring residual for constant phase modulation with $p = 0.082$ corresponding to $\gamma = 2.2$ for fringe offset and contrast parameters $a = b$</td>
<td>108</td>
</tr>
<tr>
<td>4.8</td>
<td>2+1 2nd order harmonic phase measuring residual for constant phase modulation with $p = 0.082$ corresponding to $\gamma = 2.2$ for fringe offset and contrast parameters $a = b$</td>
<td>110</td>
</tr>
</tbody>
</table>
4.9 3+3 2nd order harmonic phase measuring residual for constant phase modulation with \( p = 0.082 \) corresponding to \( \gamma = 2.2 \) for fringe offset and contrast parameters \( a = b \) ........................................... 112

4.10 \( \varepsilon_{\text{max}}(x, y) \) as a function of \( p \) for the 3+3 2nd order harmonic phase measuring residual ........................................... 112

4.11 Double Three Step 2nd order harmonic phase measuring residual for constant phase modulation with \( p = 0.082 \) corresponding to \( \gamma = 2.2 \) for fringe offset and contrast parameters \( a = b \) ....................... 114

4.12 FTP 2nd order harmonic phase measuring residual for constant phase modulation with \( p = 0.082 \) corresponding to \( \gamma = 2.2 \) for fringe offset and contrast parameters \( a = b \) ........................................... 116

4.13 Maximum Absolute Measurement Error, \( \epsilon \) Vs. \( \gamma \) for fringe parameters \( a = b \) .......................... 118

4.14 Simulated Phase Distribution \( \phi(x, y) \) ........................................... 121

4.15 Simulated Fringes, \( \gamma = 2.2 \) ........................................... 122

4.16 3 Step PMP Gamma distortion Simulation ........................................... 123

4.17 4 Step PMP Gamma distortion Simulation ........................................... 124

4.18 \( 90^\circ \) 3 Step Gamma distortion Simulation ........................................... 126

4.19 \( 90^\circ \) 3 Step with Phase Offset Gamma distortion Simulation ....................... 127

4.20 2+1 Gamma distortion Simulation ........................................... 128

4.21 \( 90^\circ \) Residual Functions ........................................... 129

4.22 3+3 Gamma distortion Simulation ........................................... 130

4.23 3+3 2nd order harmonic Simulation ........................................... 131

4.24 Double Three Step Gamma distortion Simulation ........................................... 132

4.25 Fourier Transform Profilometry Gamma distortion Simulation ....................... 133

4.26 Various Fringe Processing technique and corresponding \( \gamma \) sensitivity, \( \epsilon \) (mm) ........................................... 134

4.27 Experimental Reconstructions ........................................... 136

5.1 3 Step PMP Maximum absolute error, \( a = b, a = 0.6, b = 0.4 \) and \( a = 0.8, b = 0.2 \) ........................................... 142
5.2 Percentage Improvement in system accuracy, for $a = 0.6$, $b = 0.4$ and $a = 0.8$, $b = 0.2$ relative to $a = b$ Vs. $\gamma$ ........................................ 142
5.3 Fringe parameter manipulation experimental results, 3-Step PMP reconstruction .......................................................... 144
5.4 Cross-section of reconstructed diffuse surface for fringe parameters, $a = b$, $a = 0.6$, $b = 0.4$ and $a = 0.8$, $b = 0.2$ ...................... 145
5.5 Maximum absolute profile measurement error: $\gamma \pm 5\%$ ............. 150
5.6 Temporal Variation in $p$ .......................................................... 151
5.7 Simulated Reconstruction .......................................................... 152
5.8 Temporal Reference Plane Residual Function, for $p_0 = 0.1084$ i.e $\gamma_0 = 2.47$ and $p = 0.0753$ i.e. $\gamma = 2.13$ ......................... 153
5.9 Empirical FTP reconstruction ..................................................... 154
5.10 Simulated Reconstructions ....................................................... 159
5.11 Empirical Reconstructions ....................................................... 161
5.12 Cross-sections of Empirical Reconstructions ................................ 162

6.1 Simple Neuron ............................................................... 168
6.2 Proposed Multilayer Signal Mapping Calibration Neural Network for arbitrary $a \in n = 0, 1, 2, 3...N$ ........................................ 170
6.3 Simulated phase distribution, reference / deformed fringe and fringe cross-section .......................................................... 175
6.4 Simulated reconstructed surfaces for 3-Step PMP and FTP with and without neural network calibration ................................. 177
6.5 Cross section of reconstructed surface for simulated fringe images for both calibrated and non-calibrated scenarios ...................... 178
6.6 Cross-sections of Reconstructed Surface for Non-calibrated, Neural Calibration and Filtering Calibration, 10 Tap and 100 Tap ........ 180
6.7 Cross section of reconstructed surface for neural calibration and Double Three Step scenario ................................................... 181
6.8 Experimental object and fringe patterns ...................................... 182
6.9 Experimental reconstructions for 3 Step PMP and FTP techniques with and without neural network calibration .......................... 183
6.10 Cross section of Experimental reconstructions for both 3 Step PMP and FTP techniques .................................................. 184
6.11 Simulated reconstructed surface with and without the proposed fringe calibration .................................................. 189
6.12 Multi-channel Experimental Results ............................................. 190
6.13 Cross-section of reconstructed diffuse surface seen in Figure 6.12(a) ... 192

A.1 Finite Projection Experimental Verification .............................. 202
B.1 Physical Fringe Projection Arrangement ................................. 204
B.2 Software Interface Screen Shot .............................................. 207
## List of Tables

<table>
<thead>
<tr>
<th>Table</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.1</td>
<td>Greyscale reconstruction average phase errors and standard deviations</td>
<td>85</td>
</tr>
<tr>
<td>3.2</td>
<td>Colour reconstruction average phase errors and standard deviations</td>
<td>88</td>
</tr>
<tr>
<td>4.1</td>
<td>Absolute Relative Mean Error $\epsilon_2$ for various $\gamma$</td>
<td>97</td>
</tr>
<tr>
<td>4.2</td>
<td>Measured Absolute Maximum Measurement Error ($\epsilon_{\text{max}}$) and Estimated Absolute Maximum Measurement Error ($\epsilon_{\text{max}}$)</td>
<td>137</td>
</tr>
<tr>
<td>5.1</td>
<td>Various $\beta$ and the corresponding gain coefficients $T(kf_0)$ for $k = 1, 2, 3$.</td>
<td>158</td>
</tr>
<tr>
<td>5.2</td>
<td>Mean Error ($\bar{\epsilon}$), Standard Deviation ($\bar{\sigma}$) and Maximum Absolute Error ($\epsilon$) in mm for the PMP 3 and 4 Step algorithms for both the Focused and Defocused cases with $\gamma = 3$.</td>
<td>160</td>
</tr>
<tr>
<td>6.1</td>
<td>Calibrated and Non-Calibrated Absolute Mean Profile Reconstruction Errors and Standard Deviations.</td>
<td>176</td>
</tr>
<tr>
<td>6.2</td>
<td>Absolute Mean Errors ($\bar{\epsilon}$) and Standard Deviations ($\bar{\sigma}$) for Neural and Filter Calibration</td>
<td>179</td>
</tr>
<tr>
<td>6.3</td>
<td>Absolute Mean Errors ($\bar{\epsilon}$) and Standard Deviations ($\bar{\sigma}$) for Neural and Double Three Step Technique</td>
<td>181</td>
</tr>
<tr>
<td>6.4</td>
<td>Calibrated and Non-calibrated mean absolute reconstruction errors and standard deviations.</td>
<td>188</td>
</tr>
<tr>
<td>B.1</td>
<td>Projector Specifications</td>
<td>205</td>
</tr>
</tbody>
</table>