2018

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Jiawei Zhang  
University of Wollongong, jz004@uowmail.edu.au

Tanju Yildirim  
Shenzhen University, ty370@uowmail.edu.au

Gursel Alici  
University of Wollongong, gursel@uow.edu.au

Shiwu Zhang  
University of Science and Technology of China, zhangs@uow.edu.au

Weihua Li  
University of Wollongong, weihuali@uow.edu.au

Publication Details
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The nonlinear vibration analysis of a magneto-rheological elastomer (MRE) sandwich plate is conducted experimentally. Experiments have been performed in order to construct the frequency-response curves in the vicinity of the fundamental natural frequency of an MRE sandwich plate (plate A) in either the absence or presence of a localised external magnetic field at 3 different geometrical locations, for both small and medium magnetic fields. Furthermore, experiments have also been conducted on a pure aluminium plate (plate B) with an equal thickness to the MRE sandwich plate (plate A) in order to examine the influence of the MRE layer on the nonlinear dynamics of the system. An electrodynamic shaker was used to directly force each system and the displacement at the centre of the plate was measured. Meanwhile, permanent magnets were used to apply a localised magnetic field for the experiments where the MRE sandwich plate was subject to an external magnetic field. It was observed all the MRE systems displayed strong hardening-type nonlinear behaviour, however, with increasing magnetic field this behaviour transitioned to a weak hardening-type nonlinearity.

Disciplines
Engineering | Science and Technology Studies

Publication Details

This journal article is available at Research Online: https://ro.uow.edu.au/eispapers1/1871
Experimental nonlinear vibrations of an MRE sandwich plate

Jiawei Zhang1a, Tanju Yildirim1b, Gursel Alici1,3c, Shiwu Zhang1d and Weihua Li1*

1School of Mechanical, Materials and Mechatronics Engineering, University of Wollongong, NSW 2522, Australia
2College of Chemistry and Environmental Engineering, Shenzhen University, Shenzhen 518060, Guangdong, China
3ARC Centre of Excellence for Electromaterials Science, University of Wollongong, Innovation Campus, NSW 2522, Australia
4Department of Precision Machinery and Precision Instrumentation, University of Science and Technology of China, Hefei, Anhui 230027, China

(Received October 4, 2017, Revised January 18, 2018, Accepted February 5, 2018)

Abstract. The nonlinear vibration analysis of a magneto-rheological elastomer (MRE) sandwich plate is conducted experimentally. Experiments have been performed in order to construct the frequency-response curves in the vicinity of the fundamental natural frequency of an MRE sandwich plate (plate A) in either the absence or presence of a localised external magnetic field at 3 different geometrical locations, for both small and medium magnetic fields. Furthermore, experiments have also been conducted on a pure aluminium plate (plate B) with an equal thickness to the MRE sandwich plate (plate A) in order to examine the influence of the MRE layer on the nonlinear dynamics of the system. An electrodynamic shaker was used to directly force each system and the displacement at the centre of the plate was measured. Meanwhile, permanent magnets were used to apply a localised magnetic field for the experiments where the MRE sandwich plate was subject to an external magnetic field. It was observed that the MRE systems displayed strong hardening-type nonlinear behaviour, however, with increasing magnetic field this behaviour transitioned to a weak hardening-type nonlinearity.

Keywords: experiments; sandwich plate; magneto-rheological elastomer; frequency-response; nonlinear dynamics

1. Introduction

Magneto-rheological elastomers (MREs) can be found in many industrial applications (Carlson and Jolly 2000, Chen et al. 2008, Li et al. 2010). MREs are a composite material where magnetic iron particles are suspended in a rubber matrix; with this composition, MREs possess the inherent ability of a controllable shear modulus and material damping when polarised by an external magnetic field. The controllable properties of MREs make them ideal in suppressing unwanted resonance vibration in automotive suspensions (Du et al. 2011) and resonance-based vibration absorbers in civil structures (Deng and Gong 2007, Jung et al. 2011, Karavasilis et al. 2011, Sun et al. 2014, Sun et al. 2015).

There are two main classes in which MREs are used as the core element to mitigate and control vibration (York et al. 2011). The first class are MRE sandwich beams (i.e., one-dimensional systems) (Ni et al. 2011, Korobko et al. 2012, Nayak et al. 2013). The second class, MRE plates are another common engineering element which are used in many engineering systems, for example in automotive, mechanical and civil engineering applications (Moradi et al. 2015). However, the literature on the dynamical behaviour of the second class is limited when compared to the first class.

For the first class, Dyniewicz et al. (2015) implemented semi-active control into a partially filled MRE sandwich beam where an analytical solution was first derived to implement a control strategy and the control algorithm effectiveness was experimentally verified. Hu et al. (2011) investigated the dynamical response of an MRE sandwich beam under non-homogenous magnetic fields experimentally, and the results showed a reduction of 13.9% for the linear transverse vibration amplitude. Sun et al. (2003) investigated the controllable properties of an MRE sandwich beam and the experimental results showed that the sandwich beam can limit unwanted transverse vibrations at the first fundamental mode of the sandwich beam. Zhou and Wang (2006) numerically investigated the dynamic response of a simply supported MRE sandwich beam with a changing magnetic field. It was shown that a 40% shift in the anti-resonance frequencies occurred, however, only a slight shift in the resonant frequencies.

For the second class, Ying et al. (2014) numerically investigated the dynamical response of an MRE sandwich plate when subjected to a stochastic support excitation. It was shown that with an increasing localised magnetic field, the MRE layer can suppress the dynamical response of the system. More recently, Aguib et al. (2014) investigated the linear dynamical response of a clamped-free MRE sandwich plate with different magnetic
fields and iron particle concentrations both numerically and experimentally.

This paper investigates the nonlinear vibrations of an MRE sandwich plate experimentally. In particular, the nonlinear dynamical behaviour of a composite MRE sandwich plate in both the absence or presence of a localised external magnetic field, covering different surface areas of the sandwich plate and this is compared to an equal thickness aluminium plate. It is shown that the MRE-layer has substantial effects on the nonlinear dynamics of the system in the absence or presence of an external magnetic field. A substantial qualitative and quantitative change in the nonlinear dynamical response of a thin MRE sandwich plate was observed.

2. Experimental setup

2.1 System description

The system shown in Fig. 1 (a) is a fully-clamped MRE sandwich plate. The bilayer MRE sandwich plate (Plate A) is composed of a 0.55 mm aluminium plate (HI) and a 0.25 mm MRE layer (H2). The sandwich plate has an equal width and length (W), and thickness (H) of 368 and 0.8 mm, respectively. The aluminium plate has mechanical properties with Young's modulus (E) and density (ρ) of 69.5 GPa and 2700 kg/m³, respectively. A 0.8 mm thick aluminium plate with an equal height and length of 368 mm was also fabricated (plate B).

![Diagram](image1)

Plate A
MRE Layer
W = 368
H2 = 0.25
Aluminium Plate
W = 368 mm
H1 = 0.55 mm

(a) MRE sandwich Plate (Plate A)

Plate B
Aluminium Plate
W = 368 mm
H = 0.8 mm

(b) Aluminium Plate (Plate B)

Fig. 1 Problem scenario of the fully clamped MRE plate

2.2 MRE Fabrication

The MRE layer has been bonded to an aluminium plate in order to create a sandwich plate. The MRE applied in this experiment consisted of 80% iron particles (Carbonyl iron, C2518, Sigma-Aldrich Pty Ltd.) with 5 μm mean particle size, 10% silicone rubber (Selleys Pty. Ltd) and 10% silicone oil. The MRE was fabricated and allowed to cure on a 0.55 mm aluminium plate under a press for 5 days (plate A). A magnetic sweep test conducted at a constant angular frequency of 5 rad/s was used with a parallel plate rheometer (MCR 301, Anton Paar Companies, Germany) to test the mechanical properties of the fabricated MRE. It is shown in Fig. 2 that with an increasing magnetic field the storage modulus and loss modulus increased with an increasing magnetic field. The storage modulus is the ability of a viscoelastic material to store deformation energy and the loss modulus represents the ability of a viscoelastic material to dissipate deformation energy (Jung et al. 2009, Eem et al. 2012)

2.3 Experimental setup and data recording system

In this section, the experimental setup and data recording system including interaction of system components is described in detail and how the localised magnetic field was applied to the MRE sandwich plate (plate A) is also described.

![Graph](image2)

(a) Storage modulus

(b) Loss modulus

Fig. 2 Rheometer reading of the MRE Sample
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(a) Photograph of experimental setup

(b) Flowchart of experimental setup

(c) Case 1,2 and 3 for a localised magnetic field

Fig. 3 Experimental system setup

Fig. 3 shows the experimental setup used to acquire and process data to produce the frequency-response curves for a fully-clamped MRE sandwich plate in the absence or presence of an external localised magnetic field. For the experiments where a magnetic field was applied, the notation Case 1, Case 2 and Case 3 have been used and these cases are shown in Fig. 3(c). An electrodynamic shaker (Sinocera Piezotronics modal shaker JZK-5) with a force sensor (CLD Y303) attached to a stinger was used to excite each system and the displacement at the centre of the plate was measured using a laser displacement sensor (Microepsilon opto NCDT1700ILD1700-20), for various forcing amplitudes in the absence or presence of an external localised magnetic field. The recorded data signals were processed via a data acquisition board (DAQ National instruments USB-6259) which also controlled the output voltage of the power amplifier (Sinocera YE5872H) connected to the electrodynamic shaker. The closed loop control was based on the force sensor and sufficient settling time was allowed between data points for the system to reach steady-state before a measurement was taken and a 5 kHz sampling rate was used for sampling experimental data. The interaction of system components is shown in Fig. 3 (b).

Each system was excited by the stinger tip located at 100 mm up from the bottom and 15 mm from a clamped edge. LABVIEW software was used to control the force sensor to keep a constant forcing amplitude of $F = \sin(2\pi\Omega t)$, where $F$ is the forcing amplitude (N), $\Omega$ is the excitation frequency (Hz) and $t$ is time (seconds). For the experiments involving the MRE sandwich plate (plate A) in the presence of a localised magnetic field. There are two different magnetic fluxes were tested, one above and one below 50 mT. As the system transitioned between strong
and weak hardening-type nonlinear behaviour, the magnetic fields were tested using a gauss-meter before a test was conducted.

3. Experimental results

In this section, the MRE sandwich plate (plate A) has been transversely excited by the stinger attached to the electrodynamic shaker (see Fig. 3(a)) when the plate is either in the absence or presence of an externally applied magnetic field at different localised geometrical coordinates. The corresponding displacement at the centre of the plate has been measured in order to construct the frequency-response curves near the first fundamental natural frequency for the out-of-plane motion of each system. The frequency-response curves of an equal thickness aluminium plate (plate B) have also been constructed in order to compare the effect of an MRE-layer on the nonlinear dynamical response of the system. Experiments have been performed for both the forward and reverse frequency sweeps at constant forcing amplitudes when nonlinear dynamical behaviour was observed. A non-dimensional frequency has been introduced as the excitation frequency divided by the first fundamental natural frequency of the plate (Ω/ω₁,₁). Similarly, a non-dimensional amplitude has also been introduced as the motion amplitude divided by the thickness of the plate (Displacement/h). The frequency-response curves, time traces, fast Fourier transforms (FFTs), phase-plane diagrams, probability density functions (PDFs) and autocorrelation functions have also been plotted.

3.1 Fundamental response of an MRE sandwich plate in a medium external magnetic field

In this section, the nonlinear dynamical response of an MRE sandwich plate when excited near the first fundamental natural frequency for the out-of-plane motion ω₁,₁ and in the presence of an external magnetic field above 50 mT has been plotted and results are discussed.

Fig. 4 shows the frequency-response curves of an MRE sandwich plate (plate A) in the presence of a localised magnetic field and an aluminium plate (plate B) with an equal thickness (0.8 mm). It was observed the MRE sandwich plate in the presence of an 80 mT magnetic field for Case 3 (see Fig. 3(e)) showed a semi-linear response with a gradual incline to a maximum out-of-plane motion amplitude followed by a continuous decline. The dynamics of an equal thickness aluminium plate (plate B), however, showed substantially different dynamical behaviour. A strong softening-type nonlinearity was observed with the frequency-response curve leaning towards the left. For the aluminium plate (plate B), the system reaches a maximum motion amplitude at Ω/ω₁,₁ = 0.9 (the system displayed 10% nonlinearity from Ω/ω₁,₁ = 1) followed by a discontinuous point, these points theoretically correspond to limit point bifurcations and experimentally occur as jump up and down points. Multiple solutions exist at certain excitation frequencies (Ω) which are the result of combining the forward and reverse frequency sweeps. An unstable solution also exists between the forward and reverse frequency sweeps, however, the unstable points are not able to be obtained experimentally. This nonlinear behaviour is due to geometrical nonlinearity as well as the excitation and geometrical imperfection. Large deformations about the initial equilibrium state and in-plane stretching along the plate also caused this nonlinear dynamical behaviour (Pasquali et al. 2014, Tang et al. 2014, Yuda et al. 2015). Moreover, it was observed the type of nonlinearity between the two systems are substantially different with the MRE sandwich plates (plate A) frequency-response curve leaning towards the right and the aluminium plate (plate B) leaning slightly towards the left.

The frequency-response curves of the MRE sandwich plate (plate A) in the absence or presence of a localised magnetic field with a 5 N forcing amplitude is shown in Fig. 5. It was observed the MRE sandwich plate displayed a strong hardening-type nonlinear behaviour which transitioned to a weak hardening-type behaviour when the localised magnetic field was applied. At the first fundamental natural frequency for the out-of-plane motion amplitude, the MRE sandwich plate in the absence of an external magnetic field displayed the largest out-of-plane motion amplitude with the frequency-response curve leaning towards the right reaching a maximum non-dimensional motion amplitude of 1.063 at Ω/ω₁,₁ = 1.021 (which corresponds to 45.25 Hz) followed by a discontinuous jump down point. With a localised magnetic field at different geometries the nonlinear dynamical behaviour substantially changed. It was observed for Case 1 with a 62 mT localised magnetic field, the system displayed a strong hardening-type nonlinear behaviour with a reduced out-of-plane motion amplitude compared to the MRE sandwich plate in the absence of an external magnetic field. Both cases displayed a weak internal resonance before the maximum motion amplitude which was not observed for any of the cases. For a localised magnetic field applied for Case 2, the system displayed a weak hardening-type nonlinear behaviour with a discontinuous bifurcation point.

![Fig. 4 Frequency-response curves in the vicinity of the first fundamental natural frequency with F = 5 N](image-url)
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Fig. 5 Frequency-response curves of an MRE sandwich plate (Plate A) in the vicinity of the first fundamental natural frequency with $F = 5$ N

Fig. 6 Frequency-response curves of MRE sandwich plate (Plate A) in the vicinity of the first fundamental natural frequency with $F = 1$ N

Different magnetic field locations above 50 mT influence the nonlinear dynamical response from strong to weak hardening-type behaviour and with a larger flux area a linear response was observed (Case 3). This was due to the changing stiffness, damping and stretching of the elastomer.

The linear response of the system in Fig. 5 are shown in Fig. 6. A linear response was observed for each of these systems when $F = 1$ N, the linear natural frequencies for the MRE sandwich plate (plate A) in the absence of a magnetic field and the presence of a magnetic field for cases 1, 2 and 3 were experimentally obtained as 44.33, 47.24, 41.98 and 43.79 Hz, respectively.

3.2 Fundamental response of an MRE sandwich plate in a low external magnetic field

In this section, experiments have been performed for the nonlinear forced dynamical response of an MRE sandwich plate in the presence of a low external magnetic field (i.e., below 50 mT), for cases 1, 2 and 3. The system was transversely forced in the vicinity of its first natural frequency and the corresponding frequency-response curves were experimentally obtained.

Fig. 7 Frequency-response curves of MRE sandwich plate (Plate A) in the vicinity of the first fundamental natural frequency with $F = 5$ N

The frequency-response curves of the MRE sandwich plate (plate A) in the absence or presence of an external magnetic field below 50 mT are shown in Fig. 7. For the different cases when the localised magnetic field was applied to the MRE-layer it was observed that all systems displayed strong hardening-type nonlinear behaviour. Comparing Figs. 5 and 7, the out-of-plane motion amplitude is larger with a reduced magnetic field for all cases and all systems display hardening behaviour with their corresponding frequency-response curves leaning towards the right; it was also observed that with reduced magnetic field the nonlinearity increases (i.e., the percentage increase from $\Omega/\omega_{n1} = 1$). Moreover, for Case 3 with a 48 mT localised magnetic field applied it was observed the system displayed two internal resonances one below and one slightly above $\Omega/\omega_{n1} = 1$ before reaching a maximum motion amplitude of 0.6589 at $\Omega/\omega_{n1} = 1.026$. For this point in the parameter space the time trace, phase-plane diagram, FFT, PDF and autocorrelation are shown in Figs. 8(a)-8(e), respectively, illustrating a slightly non-symmetric periodic motion about the initial equilibrium position.

Fig. 9 shows the frequency-response curves of an MRE sandwich plate when transversely forced by a 1 N forcing amplitude (F) and in the presence of a low magnetic field. For cases 1, 2 and 3, it was observed each system displayed linear dynamical behaviour and the experimentally obtained fundamental natural frequencies were 41.68, 45.07 and 43.79 Hz, respectively.

Comparing Figs. 5 and 7 (i.e., the systems subject to medium and low magnetic fields, respectively), when the MRE is in the presence of a low magnetic field a hardening-type behaviour was observed with clear discontinuous bifurcation points, for all cases; however, when the MRE-layer is in the presence of a medium external magnetic field (i.e., above 50 mT) the nonlinear dynamical response for cases 2 and 3 displayed weak hardening behaviour and a linear response, respectively, which resulted in a much lower out-of-plane motion amplitude. This is due to the increased damping on the system when the MRE-layer is exposed to larger magnetic fields. Moreover, comparing the linear responses of the MRE in the presence of low and medium magnetic fields (i.e., Figs. 6 and 9), it was
Fig. 8 Motion characteristic of the system in Fig. 7 for MRE sandwich plate (plate A) for Case 3 with a 48 mT localised magnetic field at $\Omega/\omega_{1,1} = 1.026$.

observed the out-of-plane motion amplitude was larger in the presence of a medium magnetic field due to the magnetic coupling pulling the MRE.

3.3 Nonlinear dynamical response of an MRE sandwich plate in the absence of an external magnetic field

In this section, the nonlinear dynamical response of an MRE sandwich plate (plate A) in the absence of an external magnetic field at various forcing amplitudes have been experimentally obtained and plotted.
The frequency-response curves of the MRE sandwich plate (plate A) in the absence of an external magnetic field for various forcing amplitudes are shown in Fig. 10. A linear response was observed for a 1 N forcing excitation, however, with increased forcing the system displayed hardening behaviour from 2 N onwards with increasing nonlinearity. The nonlinear behaviour is caused by large deformations about the initial equilibrium position (for this case, larger than the thickness of plate A). For the maximum out-of-plane motion amplitude at $F = 5$ N ($\Omega/\omega_{1,1} = 1.021$), the time trace, phase-plane diagram, FFT, PDF and autocorrelation have been experimentally obtained and plotted in Figs. 11(a)-11(e), respectively. For this point in the parameter space the system illustrates a non-symmetric period-1 motion about the initial equilibrium state due to low damping in the elastomer and stretching of the elastomer at large deformations.

![Figure 9](image9.png)

**Fig. 9** Frequency-response curves of MRE sandwich plate (Plate A) in the vicinity of the first fundamental natural frequency with $F = 1$ N

![Figure 10](image10.png)

**Fig. 10** Frequency-response curve of an MRE sandwich plate (plate A) in the absence of an external magnetic field at various forcing amplitudes

![Figure 12](image12.png)

**Fig. 12** Frequency-response curve of an aluminium plate (plate B) at various forcing amplitudes

The frequency-response curves of an aluminium plate (plate B) with a point excitation applied for various forcing amplitudes are shown in Fig. 12. A linear response was observed for a 1 N forcing, however, with larger forcing and larger deformation about the equilibrium state the system displayed strong softening behaviour from 2 N onwards. The softening behaviour was due to geometric imperfection in the aluminium plate (plate B) which substantially changed the nonlinear dynamical response of the system compared to the MRE sandwich plate (plate A). Comparing Figs. 10 and 12, while the motion amplitude was larger for the MRE sandwich plate (plate A) the MRE-layer substantially reduces the nonlinearity of the system (i.e. the percentage increase or decrease from $\Omega/\omega_{1,1} = 1$) as the MRE sandwich plate showed 2% nonlinearity with $F = 5$ N compared to the aluminium plate which displayed 10% nonlinearity. Moreover, the amount of nonlinearity is substantially larger between 3 N and 5 N for the aluminium plate (plate B) a change from $\Omega/\omega_{1,1} = 0.9713$ to 0.9, respectively. The corresponding shift in the MRE sandwich plate from 3 N to 5 N was $\Omega/\omega_{1,1} = 1.011$ to 1.021, this difference was due to the additional damping of the elastomer and in-plane stretching of the elastomer at large deformations.

### 4. Conclusions

Experimental investigations have been performed to assess the nonlinear dynamical response of a fully-clamped MRE sandwich plate in the absence or presence of a localised magnetic field at 3 different geometries. Experiments have been conducted when the MRE sandwich plate was excited near the first fundamental resonant frequency when: (i) in the absence of an external magnetic field, (ii) in the presence of a medium localised magnetic field at 3 different geometrical locations, (iii) in the presence of a small localised magnetic field at 3 different geometrical locations. Moreover, experiments have been conducted at various forcing amplitudes to show the transitions from linear to nonlinear dynamical behaviour.
Experiments (i-iii) were compared to an equal thickness aluminium plate to assess the qualitative and quantitative differences for the nonlinear forced dynamical response of each system.

A non-symmetric response was observed for all systems due to the nonlinearity caused by large deformations about the initial equilibrium state. For all the MRE systems, a strong hardening-type nonlinearity was observed, however, with the effect of a magnetic field applied at different locations this behaviour transitioned to weak hardening-type behaviour, with a substantial magnetic field a linear response was observed. In the case of a pure aluminium plate, a strong softening-type nonlinearity was observed due to geometrical nonlinearity and geometrical imperfection (with limit point bifurcations occurring below the fundamental resonant frequency). Due to damping and in-plane stretching of the elastomer a period-1 motion was observed for the MRE sandwich plate in the absence of a magnetic field (particularly when passing through the initial position), however, with an increased magnetic field the damping increases and this effect was substantially reduced. For certain combinations of forcing and applied magnetic
fields. Internal resonances were observed and in some cases two internal resonances were observed, one above and one below the fundamental resonant frequency.

Acknowledgments

This work is partially supported by an ARC Discovery Grant (DP150102636).

References


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