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This paper deals with a widespread type of investment in personal health that is not adequately explained by the economic literature. The analysis of people's choice of intensity of engagement in health enhancing activities is made within an integrative, stochastic, micro-dynamic optimisation framework in which people's utility is accumulated along a health-dependent random lifespan with direct and indirect mutual effects among exercise, health, consumption, utility, happiness, productivity and survival. Distinction is made between exercise's length and exercise's vigour in analysing the effect of exercising on health and rest. A link between health and utility is introduced: health improves (declines) as utility surpasses (falls below) a threshold due to a greater level of happiness (unhappiness).

Keywords
Rational, exercising, lifetime, choice, link, between, health, happiness

Disciplines
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Rational Exercising:
A Lifetime Choice with a Link between Health and Happiness

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Abstract

This paper deals with a widespread type of investment in personal health that is not adequately explained by the economic literature. The analysis of people's choice of intensity of engagement in health enhancing activities is made within an integrative, stochastic, micro-dynamic optimisation framework in which people's utility is accumulated along a health-dependent random lifespan with direct and indirect mutual effects among exercise, health, consumption, utility, happiness, productivity and survival. Distinction is made between exercise's length and exercise's vigour in analysing the effect of exercising on health and rest. A link between health and utility is introduced: health improves (declines) as utility surpasses (falls below) a threshold due to a greater level of happiness (unhappiness).

1. Links between happiness and health

With $H(t) \geq 0$ denoting the individual's mental and physical health at t, $0 \leq \ell(t) < 1$ the fraction of time she allocates to exercising at t (length), $v(t) \geq 0$ the intensity (vigour) of the exercise at t; and $c(t) \geq 0$ the individual's spending on the consumption of goods at *t* (instantaneous consumption), the following assumptions are made.

Assumption 1 (*utility and happiness*). At every instance, utility is derived from consumption $u(t) = u(c(t))$ with $u_c > 0$ and $u_{cc} < 0$. However, there exists a threshold utility level $\underline{u} > 0$ below which the individual is unhappy (frustrated). As the instantaneous utility rises above this threshold the happier the individual. That is, the individual's instantaneous happiness is given by $h(t) = u(c(t)) - \underline{u}$.

Assumption 2 (*happiness and health*). Happiness contributes to the individual's health, whereas frustration deteriorates the individual's health. With symmetry and fixed marginal effect ($\alpha > 0$) assumed for simplicity, the contribution of happiness and unhappiness to health is: $\alpha[u(c(t)) - \underline{u}]$.

Assumption 3 (*health and exercising*). The contribution of the exercise's length and vigour to the individual's current health is given by $g(\ell(t), v(t))$ exhibiting positive, but diminishing, marginal contributions and a complementary relationship between duration and intensity: $g_\ell > 0, g_v > 0, g_{\ell\ell} < 0, g_{vv} < 0, g_{\ell v} > 0$.

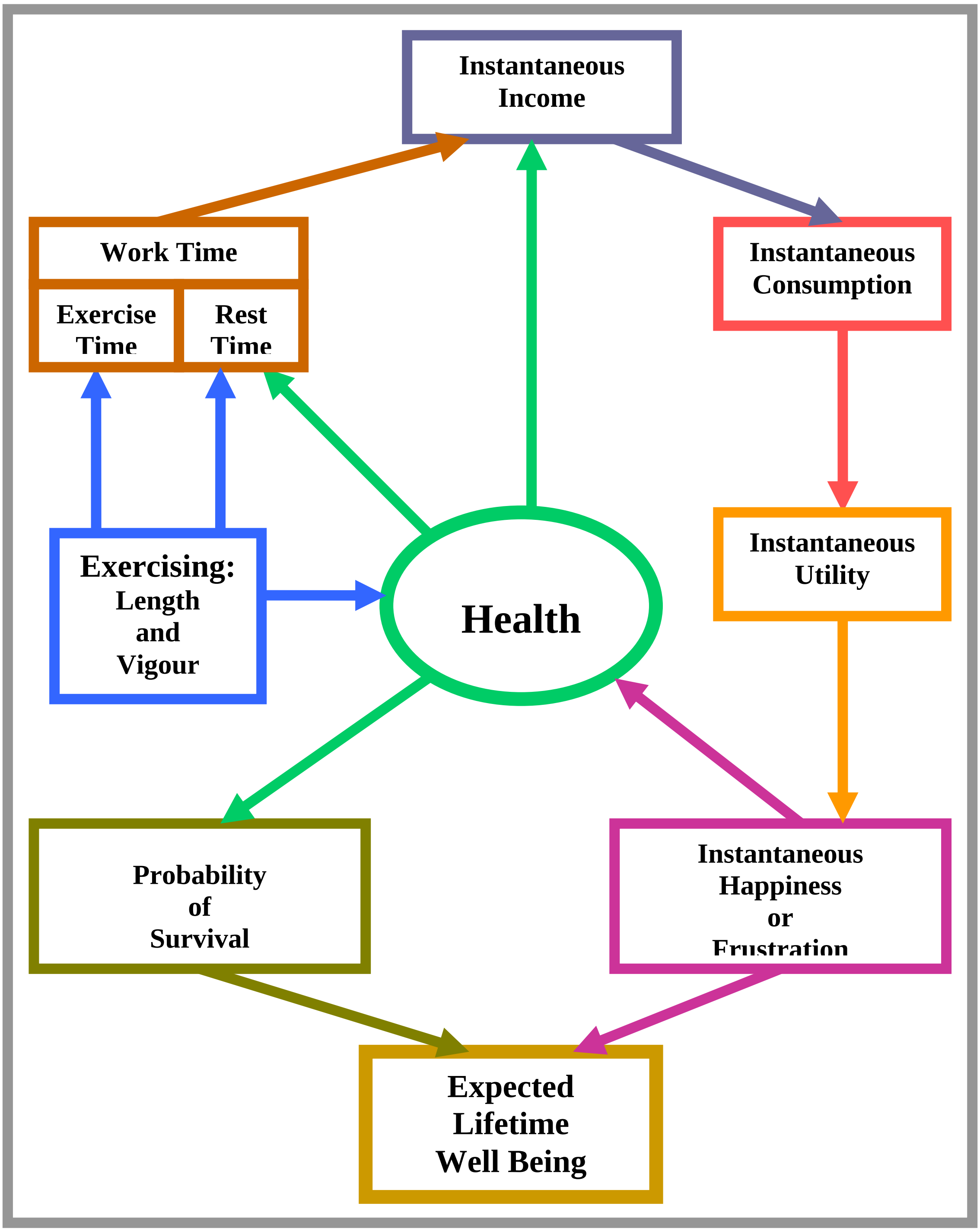
Assumption 4 (*health depreciation*). In the absence of exercising and happiness, or frustration, health deteriorates at a constant rate $0 < \delta < 1$.

Assumption 5 (*health and mortality*). The individual's life expectancy is random. The likelihood of dying at *t* is given by a probability density function exhibiting $0 \leq p \leq 1$, $p(H = 0) = 1$, $p_H < 0$, $p_{HH} > 0$ (e.g., $p(t) = 1/[1 + \gamma H(t)]$).

Assumption 6 (resting and working time). The rest required for recuperation from the exercise increasingly rises with the exercise's length and vigour but decreases with the individual's health. The fraction of time spent on recuperation is measured by the function $R(\ell(t), v(t), H(t))$ exhibiting $R(0, v(t), H(t)) = 0$, $R(\ell(t), 0, H(t)) = 0$, $R(\ell(t), v(t), 0) = 0$ and $R_\ell > 0, R_{\ell\ell} \geq 0, R_v > 0, R_{vv} \geq 0, R_{\ell v} > 0, R_H < 0, R_{HH} \geq 0$.

Assumption 7 (*health, income and consumption*). The individual's income at t is $[1 - \ell(t) - R(\ell(t), v(t), H(t))]y(H(t))$, where $[1 - \ell(t) - R(\ell(t), v(t), H(t))]$ is the fraction of time allocated to work and $y(H(t))$ the income in the case of full-time work (i.e., $\ell(t) = 0$) and exhibiting $y'(H) > 0$ and $y''(H) < 0$. The only cost of exercising is foregone income and at every instance t consumption is equal to income.

The nexus of relationships between exercising, health, income, consumption, utility, survival and expected lifetime well being is illustrated in the following flowchart.


Instantaneous
Income
Work Time
Exercise
Time
Rest
Time
Instantaneous
Consumption
Instantaneous
Utility
Exercising:
Length
and
Vigour
Health
Probability
of
Survival
Instantaneous
Happiness
or
Frustration
Expected
Lifetime
Well Being

2. Choice

On major issues, such as personal health and its maintenance, people make decisions in a farsighted, rational manner. They consider trajectories of the exercising length and vigour and choose those that maximise their expected lifetime well being (*ELTWB*) subject to their inter-temporal budget constraint, law of motion of health, and initial and terminal conditions of health. With $\rho \geq 0$ indicating the individual's time-preference rate, the individual's choice of trajectory of the intensity of physical activities is expressed as:

$$\max_{\{\ell,v\}} \left\{ ELTWB = \int_0^\infty p(H(t)) \int_0^t e^{-\rho\tau}[u(c(\tau)) - \underline{u}]d\tau dt \right\} \quad (1)$$

subject to:

$$\dot{H}(t) = g(\ell(t), v(t)) + \alpha[u(c(t)) - \underline{u}] - \delta H(t) \quad \text{(health law of motion)} \quad (2)$$

$$c(t) = [1 - \ell(t) - R(\ell(t), v(t), H(t))]y(H(t)) \quad \text{(budget constraint)} \quad (3)$$

$$H(t = 0) = H_0 > 0 \quad \text{(initial health)} \quad (4)$$

ELTWB belongs to a class of expected lifetime utility functions developed and applied by Levy (2002a, 2002b, 2009a and 2009b) for analysing the intensity and prevalence of substance abuse, overweight/obesity and HIV/AIDS as well as decisions on proximity of residence to polluting facility. Integrating by parts,

$$ELTWB = \int_0^\infty e^{-\rho t}\Phi(H(t))[u(c(t)) - \underline{u}]dt \quad (5)$$

where $\Phi(H(t))$ is equal to 1 minus the cumulative density function associated with $p(H(t))$ and hence indicates the probability of living beyond t and exhibiting $\Phi_H > 0$ and $\Phi_{HH} < 0$.

The current value Hamiltonian associated with this problem is

$$\mathrm{H}(t) = \Phi(H(t))[u(c(t)) - \underline{u}] + \lambda(t)\{g(\ell(t), v(t)) + \alpha[u(c(t)) - \underline{u}] - \delta H(t)\}. \quad (6)$$

In addition to the health law of motion, the necessary set of the necessary conditions includes:

$$\dot{\lambda} = \rho\lambda - \frac{\partial \mathrm{H}}{\partial H} = (\rho + \delta)\lambda - \Phi_H[u - \underline{u}] - [\Phi + \lambda\alpha]u_c y_H[1 - \ell - R - R_H] \quad (7)$$

$$\frac{\partial \mathrm{H}}{\partial \ell} = -[\Phi + \lambda\alpha]u_c[1 + R_\ell]y + \lambda g_\ell = 0 \quad (8)$$

$$\frac{\partial \mathrm{H}}{\partial v} = -[\Phi + \lambda\alpha]u_c R_v y + \lambda g_v = 0 \quad (9)$$

and the transversality condition $\lim_{t\to\infty} e^{-\rho t}\lambda(t)H(t) = 0$.

From the optimality conditions, the ratio of the marginal contributions of the exercise's length and vigour should be equal to the ratio of the foregone marginal incomes generated by their diversion of time from work:

$$\frac{g_\ell}{g_v} = \frac{1+R_\ell}{R_v} \qquad (10)$$

The optimality condition (9) implies:

$$[\Phi + \lambda\alpha]u_c y = \lambda g_v / R_v \qquad (11)$$

By substituting this equality into the adjoint equation:

$$\dot{\lambda} = (\rho+\delta)\lambda - \Phi_H[u - \underline{u}] - \lambda(g_v / R_v)[1 - \ell - R - R_H] \qquad (12)$$

In steady state, the shadow price of health:

$$\lambda_{ss} = \frac{\Phi_H[u_{ss} - \underline{u}]}{(\rho+\delta) - (g_v / R_v)[1 - \ell - R - R_H]} \qquad (13)$$

and from the heath motion equation:

$$u_{ss} - \underline{u} = [\delta H_{ss} - g_{ss}]/\alpha \qquad (14)$$

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