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# Analytical and numerical investigations into belt conveyor transfers

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# **ANALYTICAL AND NUMERICAL INVESTIGATIONS INTO BELT CONVEYOR TRANSFERS**

by

**Shams Tamjeed Huque**

Bachelor of Engineering (Mechanical), Graduate Certificate in Business

A thesis submitted in fulfilment of the  
requirements for the award of the degree

**Doctor of Philosophy**

from

**University of Wollongong  
Faculty of Engineering, School of Mechanical,  
Materials & Mechatronic Engineering**

**December 2004**

# Abstract

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The mining industry is an immense field with granular flows (e.g. coal) occurring in numerous areas. Accordingly there are a significant number of problems that arise, with a great number requiring solutions that are difficult to achieve by conventional industrial means. The modelling of granular flow using the numerical technique known as Distinct Element Method (DEM) has great potential in industry, particularly for solving transfer point problems. The advantage of DEM for transfer applications is that an entire system can be simulated using the single numerical technique, as opposed to the existing situation where a myriad of design techniques are required (e.g. analytical solution for one component and graphical solution for another). DEM involves solving the equations of motion for the trajectory/rotation/orientation of each particle and modelling each collision between particles and between particles and boundary objects.

The research presented a comprehensive overview of all of the available analytical processes available to design chute system components, such as material trajectory calculations, impact plate models, and gravity flow chute aspects. To the author's knowledge, this was the first such review in the literature. A detailed comparison between the most common analytical design methods was conducted, recommendations for which method to use were established, and areas of weakness and further study were identified. It was found that: most areas apart from the prediction of the initial material discharge and trajectory were lacking in design method; often the few available design methods for chute components, such as impact plates and gravity flow chutes, were lengthy and often difficult to implement.

A computer code was developed during the course of the research to simulate bulk material using the Distinct Element Method (DEM). A background into DEM and its application to modelling material flow at transfer points was presented. One major drawback found in the recent transfer studies was the lack of quantification of the velocity distributions obtained using the DEM against existing analytical design theories. Contour coloured particulate simulations have also been recently produced by a number of companies (e.g. Overland Conveyor Company Inc.) however the flow

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regimes observed from the relevant simulation screen captures were not adequately scrutinised. All the DEM mathematical formulation and numerical methods utilised for the current work were comprehensively described and relevant computational aspects were also detailed, such as the coding of a pre-processor and post-processor allowing animations of the DEM particles. A series of tests was conducted to gauge the validity of the computer code, and this produced satisfactory results.

The DEM code was also applied to simulate two separate transfers originally designed by The Gulf Group using their EasyFlow<sup>TM</sup> technology, and currently in operation in industry in Lithgow, Australia. By observing animation screen captures the current research confirmed the advantage of maintaining particle speed through the system when using curved chute elements. Quantitative DEM velocity data were compared to the velocities predicted by the most favourable analytical methods. It was found that DEM generally produced velocity regimes close to those of the analytical techniques. However it also provided the additional benefit of providing data on stream characteristics such as impact forces and velocities in the vicinity of the hood and spoon elements, which are difficult to examine in detail using analytical methods. An analysis of the micro dynamics of individual particles also identified that there are differing scales of contact during the flow through a chute. Although the analytical methods do not allow closer scrutiny of the flowing stream at the micro scale, they have the advantage of providing much faster solutions and are good for chute designs for free flowing material transfers.

# Disclaimer

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I, Shams T. Huque, declare that this thesis, submitted in fulfilment of the requirements for the award of Doctor of Philosophy, in the School of Mechanical, Materials & Mechatronic Engineering, Faculty of Engineering, University of Wollongong, is wholly my own work unless otherwise referenced or acknowledged, as defined by the University's policy on plagiarism, and that I may have received assistance from others on style, presentation and further formatting aspects. The document has not been submitted for qualifications at any other academic institution.

Shams T. Huque

16 December 2004

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# Nomenclature

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The author attempted to use symbols as close to common interpretations as possible in the thesis (for example,  $g$  is frequently used to represent gravitational acceleration and is therefore used similarly here). However due to this and the number of symbols required, some overlapping did occur. Therefore in the following nomenclature the symbol  $\{\clubsuit\}$  represents the interpretation as used in Chapters Two and Three, and  $\{\spadesuit\}$  represents the interpretation as used in Chapters Four and Five.

## ARABIC LETTERS

$a$	$\{\clubsuit\}$ Acceleration along the tangent $\{= \ddot{s} = \dot{v}\}$ ( $\text{ms}^{-2}$ ); $\{\spadesuit\}$ Index allowing for differing loading and unloading paths {NFD model}
$A$	Total cross-sectional area of bulk solid in flowing stream ( $\text{m}^2$ )
$A_0$	Initial cross-sectional area of the flowing stream at the point of entry of the chute ( $\text{m}^2$ )
$A_{1,2}$	Cross-sectional areas {rectangular portion, circular segment} of bulk solid in flowing stream ( $\text{m}^2$ )
$A_a$	Cross-sectional area of material stream at exit to 'flow-round' zone ( $\text{m}^2$ )
$A_b$	Area of trapezoidal {3 idler system} or triangular {2 idler system} area ( $\text{m}^2$ )
$A_{BC}$	Non-dimensional cross-sectional area factor
$ac$	Y-axis intercept of the perpendicular to the chord between successive points on the arc
$A_i$	Cross-sectional area of free-falling stream ( $\text{m}^2$ )
$am$	Gradient of the perpendicular to the chord between successive points on the arc
$A_p$	Cross-sectional area of material stream at entrance to 'flow-round' zone ( $\text{m}^2$ )
$A_s$	Area of segment ( $\text{m}^2$ )
$A_T$	Total area of material on the belt in the troughed portion ( $\text{m}^2$ )

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$a_w$	Proportionality factor for air drag
$A(\kappa)$	Function that describes cross-sectional area of flow stream on impact plate ( $m^2$ )
$b$	{♣} Width of belt (m); {♠} Fixed parameter, often set to 1/3 to agree with Mindlin's frictional sphere theory {TFD model}
$B$	Width of rectangular chute (m)
$B_0$	Width of entry for converging chute (m)
$b_s$	Mean width of material stream on the belt prior to discharge (m)
$b_t$	Thickness of belt (m)
$bw_2$	Width of material on flattened belt {troughed belts only} (m)
$c$	{♣} Cohesive stress ( $kNm^2$ ); {♠} Y-intercept of straight line
$C$	Constant of integration
$C_{1,2\&3}$	Constants used during calculation of the load cross-sectional area
$C_{grav}$	Distance from belt surface to centre of mass (m)
$C_l$	Inverse velocity Coulomb drag coefficient
$C_s$	Intergranular stress constant ( $s^2m^{-2}$ )
$D$	Horizontal distance from discharge point to impact point (m)
$D_{base}$	Base particle diameter (m)
$d_{ij}$	Sum of contacting sphere radii (m)
$D_{max}$	Maximum particle diameter (m)
$D_{min}$	Minimum particle diameter (m)
$D_{mono}$	Mono-sized particle (m)
$dn$	Displacement between particles (m)
$D_{var}$	Variance between particle sizes (m)
$dx$	Horizontal displacement difference between particles (m)
$dy$	Vertical displacement difference between particles (m)
$E$	Young's modulus ( $Nm^{-2}$ )
$E_{ij}$	Equivalent elastic modulus ( $Nm^{-2}$ )
$E_T$	Total energy of a particle (J)
$E_{1,2}$	Parameters in Equation (2.116)
$f_0$	Friction value of motion at the initial point of the chute
$F_D$	Drag force (N)
$F_n$	Normal force in Distinct Element Model (N)

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$F_N$	Normal force in gravity flow chute theory (N)
$F_n^{max}$	Maximum force ever experienced by the contact (N)
$F_t$	Tangential force in Distinct Element Model (N)
$F_t^*$	Value of the tangential force $F_t$ whenever the magnitude changes from increasing to decreasing, or vice versa (N)
$F_v$	Velocity dependent drag force (N)
$f_\varphi$	Friction value of motion at any angle $\varphi$ around chute
$F_\mu$	Coulomb frictional drag force (N)
$F\_mag_t$	Magnitude of tangential force (N)
$F\_x_t$	Horizontal component of tangential force (N)
$F\_x_{t,u}$	Horizontal component of unit vector (N)
$F\_y_t$	Vertical component of tangential force (N)
$F\_y_{t,u}$	Vertical component of unit vector (N)
$g$	Acceleration due to gravity ( $\text{ms}^{-2}$ )
$G$	Shear {or rigidity} modulus ( $\text{Nm}^{-2}$ )
$G_{ij}$	Equivalent shear modulus ( $\text{Nm}^{-2}$ )
$h$	Material drop height (m)
$H$	Flowing stream thickness (m)
$H_0$	Initial stream thickness (m)
$H_{1,2}$	Stream thickness {rectangular portion, circular segment} (m)
$h_a$	Thickness of material stream at exit of 'flow-round' zone (m)
$h_b$	Thickness of material on belt prior to discharge (m)
$h_p$	Thickness of material stream entering 'flow-round' zone (m)
$h_\varphi$	Stream thickness at any angle $\varphi$ around curved chute (m)
$I$	Moment of inertia ( $\text{kgm}^2$ )
$K$	Constant of proportionality usually between 1.11 – 1.42
$k_{EO}$	Effective linear pressure gradient down the wall surface at zero velocity
$k_i$	Number of particles in contact with particle $i$
$k_{max}$	Largest inter-particle spring stiffness ( $\text{Nm}^{-1}$ )
$K_n$	Some normal stiffness coefficient ( $\text{Nm}^{-1}$ )
$K_{n1}$	Normal stiffness coefficients for the (loading stage) ( $\text{Nm}^{-1}$ )
$K_{n2}$	Normal stiffness coefficients for the (unloading stage) ( $\text{Nm}^{-1}$ )

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$K_t$	Some tangential stiffness coefficient ( $\text{Nm}^{-1}$ )
$K_t^0$	Initial tangential stiffness ( $\text{Nm}^{-1}$ )
$K_T$	Effective incremental tangential stiffness ( $\text{Nm}^{-1}$ )
$k_v$	Coefficient relating lateral pressure at the chute wall to the average normal pressure during flow
$L$	Distance between periodic boundaries (m)
$L_{BC}$	Contact perimeter of material burden on discharging belt (m)
$m$	Particle mass (kg) / gradient of straight line
$\dot{m}$	Mass flow rate of material ( $\text{kgs}^{-1}$ )
$m_{ij}$	Effective mass of particles $i$ and $j$ acting in series (kg)
$m_{min}$	Mass of smallest particle in system (kg)
$n$	Parameter that is a function of the total number of particles in the system
$N$	Number of particle in system
$N_{grid}$	User defined term that specifies the maximum number of particles to be allowed in one cell
$n_s$	Number of time steps between searches
$P_n$	Pressure in normal direction (kPa)
$Q_m$	Flowrate ( $\text{th}^{-1}$ )
$r$	Non-dimensional parameter representing ratio between outside and central idler contact
$R$	{♣} Pulley radius; radius of curvature of curved chute (m); {♠} Radius of sphere (m)
$R_0$	Radius of the conveying stream midpoint at the start of the chute (m)
$r_1$	Radius of interior sphere in Verlet neighbour list (m)
$r_2$	Radius of exterior sphere in Verlet neighbour list (m)
$R_b$	Distance from centre of discharge pulley to outer surface of belt (m)
$R_c$	Radius of curvature of discharge trajectory (m)
$R_e$	Distance from discharge pulley centre to material centre of mass (m)
$R_{fz}$	Radius of the ‘flow-round’ zone (m)
$R_{ij}$	Relative contact curvature (m)
$R_m$	Distance from centre of pulley to top of material upon belt (m)
$R_{min}$	Radius of smallest sized particle in the system (m)
$R_p$	Radius of curved impact plate (m)

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$s$	Displacement along tangent (m)
$S$	{♣} Distance between end of ‘flow-round zone’ and bottom of the plate (m); {♠} An empirically determined model parameter
$S_{flowround}$	Portion of curved impact plate in contact with material stream (m)
$S_p$	Length of impact plate {flat or curved} (m)
$s_v$	Vertical fall distance (m)
$t$	Time (s)
$U_{max}$	Maximum particle velocity ( $\text{ms}^{-1}$ )
$v$	Velocity {= $\dot{s}$ } ( $\text{ms}^{-1}$ )
$v_0$	{♣} Initial velocity of the flowing stream at the point of entry of the chute ( $\text{ms}^{-1}$ ); {♠} Relative velocity of approach ( $\text{ms}^{-1}$ )
$v_{0,S}$	Velocity of stream parallel to chute surface after impact ( $\text{ms}^{-1}$ )
$v_1^*$	Velocity of stream before impact ( $\text{ms}^{-1}$ )
$v_2^*$	Velocity of stream after the first deflection ( $\text{ms}^{-1}$ )
$v_3^*$	Velocity of stream after second deflection ( $\text{ms}^{-1}$ )
$v_4^*$	Velocity of stream after impact for a single deflection ( $\text{ms}^{-1}$ )
$v_a$	Exit velocity of material leaving ‘flow-round’ zone ( $\text{ms}^{-1}$ )
$v_b$	Conveyor belt velocity ( $\text{ms}^{-1}$ )
$v_c$	Critical velocity ( $\text{ms}^{-1}$ )
$v_d$	Discharge velocity ( $\text{ms}^{-1}$ )
$v_e$	Exit velocity from bottom of flat impact plate ( $\text{ms}^{-1}$ )
$v_{\theta}$	Vertical component of bulk solid discharging velocity ( $\text{ms}^{-1}$ )
$v_i$	Velocity of impact with the curved chute ( $\text{ms}^{-1}$ )
$v_l$	Velocity of stream before impact ( $\text{ms}^{-1}$ )
$v_p$	Material velocity at entrance to ‘flow-round’ zone ( $\text{ms}^{-1}$ )
$v_t$	Tangential velocity; velocity of load stream centre ( $\text{ms}^{-1}$ )
$v(\kappa)$	Velocity of stream at angle $\kappa$ in ‘flow-round’ zone ( $\text{ms}^{-1}$ )
$v(\psi)$	Discharge velocity at angle $\psi$ ( $\text{ms}^{-1}$ )
$v_{\infty}$	Terminal velocity ( $\text{ms}^{-1}$ )
$x$	General x-coordinate (m)
$\dot{x}$	Velocity in x-direction ( $\text{ms}^{-1}$ )
$\ddot{x}$	Acceleration in x-direction ( $\text{ms}^{-2}$ )

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$x_1$	First x-coordinate of line / arc (m)
$x_{1,2,3,4}$	Four x-coordinates representing a boundary (m)
$x_2$	Second x-coordinate of line / arc (m)
$x_3$	Third x-coordinate of line / arc (m)
$x_4$	Fourth x-coordinate of line / arc (m)
$X_c$	X-coordinate of arc centre (m)
$x_h$	Height of material bed on belt (m)
$X_{len}$	Width of calculation space (m)
$y$	General y-coordinate (m)
$\dot{y}$	Velocity in y-direction ( $\text{ms}^{-1}$ )
$\ddot{y}$	Acceleration in y-direction ( $\text{ms}^{-2}$ )
$y_1$	First y-coordinate of line / arc (m)
$y_{1,2,3,4}$	Four y-coordinates representing a boundary (m)
$y_2$	Second y-coordinate of line / arc (m)
$y_3$	Third y-coordinate of line / arc (m)
$y_4$	Fourth y-coordinate of line / arc (m)
$Y_c$	Y-coordinate of arc centre (m)
$Y_{len}$	Height of calculation space (m)
$y(x)$	Function that describes the trajectory of free fall (m)
$z_{1,2,3,4}$	Four z-coordinates representing a boundary (m)

## GREEK LETTERS

$\alpha$	Angle of convergence for chute side walls ( $^\circ$ )
$\alpha_b$	Conveyor belt inclination angle before discharge ( $^\circ$ )
$\alpha_d$	Bulk solid stream discharge angle measured from the vertical ( $^\circ$ )
$\alpha_r$	Angle at which material starts to slip on discharge pulley ( $^\circ$ )
$\beta$	Impact plate inclination angle ( $^\circ$ )
$\beta_i$	Angle of idler roll ( $^\circ$ )
$\beta_v$	Viscous drag coefficient ( $\text{s}^{-1}$ )
$\Delta m$	Elementary mass of bulk solid (kg)

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$\delta_n$	Normal overlap {relative displacement of the centres of the two spheres} (m)
$\dot{\delta}_n$	Rate of change of the distance between centres of the colliding particles ( $\text{ms}^{-1}$ )
$\delta_{n0}$	Residual displacement after complete unloading {the value where the unloading curve goes to zero} (m)
$\delta_r$	Residual tangential displacement (m)
$\Delta r_x$	Horizontal component of change in relative position vector (m)
$\Delta r_y$	Vertical component of change in relative position vector (m)
$\delta_t$	Tangential overlap between particles (m)
$\Delta t$	Time step (s)
$\Delta t_c$	Critical time step (s)
$\Delta \delta_t$	Incremental tangential displacement (m)
$\Delta \delta x_t$	Horizontal component of relative surface displacement vector (m)
$\Delta \delta y_t$	Vertical component of relative surface displacement vector (m)
$\varepsilon$	Coefficient of restitution
$\phi$	{♣} Wall friction angle used in gravity flow chute work $\{= \tan^{-1} \mu\}$ (°); {♠} Angle from horizontal {line} / angle from horizontal of the perpendicular to the chord between successive points {arc} (°)
$\Phi$	Poisson's ratio ( $\nu$ ) dependent parameter for Rayleigh Wave speed critical time step determination
$\Phi_{ij}$	Angle of the particle with reference to the arc during contact (°)
$\phi_w$	Kinematic angle of wall friction between material and conveyor belt (°)
$\gamma$	Specific weight of the material being conveyed $\{= \rho g\}$ ( $\text{kNm}^{-3}$ )
$\gamma_1$	Start angle of an arc (°)
$\gamma_2$	Finish angle of an arc (°)
$\gamma_n$	Damping constant
$\varphi$	Chute slope angle for Korzen's work $\{= 90 - \theta\}$ (°)
$\varphi_0$	Angle of chute to horizontal at impact (°)
$\kappa$	Angle of impact to horizontal {for flat plates}; angle the tangent to the end of the plate makes with the horizontal {for curved plates} (°)

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$\lambda$	Angle of surcharge of material ( $^{\circ}$ )
$\lambda_{bottom}$	Angle tangent to end of curved plate makes with the vertical ( $^{\circ}$ )
$\mu$	{♣} Coefficient of internal friction used in flat impact plate model {= $\tan\zeta$ }; coefficient of wall friction used in gravity flow chute work {= $\tan\phi$ }; {♠} Coefficient of friction
$\mu_E$	Equivalent coefficient of friction
$\mu_k$	Kinematic friction coefficient between material and belt {= $\tan\phi_w$ }
$\mu_s$	Static friction coefficient
$\nu$	Poisson's ratio
$\theta$	{♣} Chute slope angle for Roberts' work {= $dy/dx$ } ( $^{\circ}$ ); {♠} General rotation (radians)
$\dot{\theta}$	Angular velocity ( $\text{rads}^{-1}$ )
$\ddot{\theta}$	Angular acceleration ( $\text{rads}^{-2}$ )
$\theta_1$	Angle of incoming stream relative to chute surface ( $^{\circ}$ )
$\theta_2$	Angle after impact of material stream relative to chute surface ( $^{\circ}$ )
$\theta_3$	Angle of incoming stream relative to chute surface {for double deflection of material stream} ( $^{\circ}$ )
$\theta_a$	Angle from horizontal made by incoming material stream to impact plate ( $^{\circ}$ )
$\theta_c$	Corrected angle of entry of material on a curved impact plate ( $^{\circ}$ )
$\theta_{co}$	Optimum cutoff angle for curved chute ( $^{\circ}$ )
$\theta_f$	Limiting angle for maintenance of 'fast' flow ( $^{\circ}$ )
$\theta_i$	Instantaneous angle of impact ( $^{\circ}$ )
$\theta_s$	Angle opposite arc length $S_{flowround}$ ( $^{\circ}$ )
$\rho$	{♣} Bulk density ( $\text{kgm}^{-3}$ ); {♠} Particle density ( $\text{kgm}^{-3}$ )
$\sigma_1$	Normal stress corresponding to conditions on the belt prior to discharge (kPa)
$\sigma_a$	Adhesive stress (kPa)
$\tau$	Shear stress (kPa)
$\omega$	Angular velocity
$\xi$	{♣} Percentage admissible relative deviation for the estimation of the k-th value of $v_a$ {impact plate model}; tolerated relative deviation for the

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	estimation of the k-th value of $v(\varphi)$ {gravity flow chute model}; {♣}
	Percentage overlap or overlap ratio of two contacting particles
$\psi$	Wrap angle around discharge pulley (°)
$\zeta$	Effective angle of internal friction (°)

## SUBSCRIPTS

$i$	Particle number $i$
$j$	Particle / boundary number $j$
$\parallel$	Denotes parallel component
$\perp$	Denotes perpendicular component
$old$	Denotes previous time step

## SUPERSCRIPTS

$N$	Time $t^N$
$N+1$	Time $t^{N+1}$
$N-1$	Time $t^{N-1}$
$N+1/2$	Time $t^{N+1/2}$
$N-1/2$	Time $t^{N-1/2}$
line	Represents line
arc	Represents arc

## VECTOR QUANTITIES

$\mathbf{F}_n$	Normal contact force
$\mathbf{F}_t$	Tangential contact force
$\mathbf{g}$	Gravitational vector
$\mathbf{i}$	Denotes x-direction
$\mathbf{j}$	Denotes y-direction

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$\mathbf{k}$	Denotes z-direction
$\hat{\mathbf{k}}_{ij}$	Unit vector in normal direction between particles
$\mathbf{r}$	Position vector for a particle
$\mathbf{r}_{ij}$	Relative position vector between two particles
$\mathbf{R}$	Radius vector
$\hat{\mathbf{t}}_{ij}$	Unit vector in the direction of the virgin loading
$\mathbf{T}_{ij}$	Torque
$\mathbf{v}$	Velocity vector for a particle
$\dot{\mathbf{x}}$	Velocity vector in x-direction
$\dot{\mathbf{y}}$	Velocity vector in y-direction
$\Delta\mathbf{r}_{ij}$	Change in the relative position vector during the last time step
$\Delta\delta_{\tau}$	Relative surface displacement vector