2000

Analytical and experimental modelling of coupled water and air flow through rock joints

Ranjith Gamage
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ANALYTICAL AND EXPERIMENTAL MODELLING OF
COUPLED WATER AND AIR FLOW THROUGH ROCK JOINTS

A thesis submitted in fulfilment of the requirements of the degree of

DOCTOR OF PHILOSOPHY

from

UNIVERSITY OF WOLLONGONG

by

RANJITH PATHEGAMA GAMAGE

B.Sc.Eng. (Hons)

Faculty of Engineering
July 2000
AFFIRMATION

I, Ranjith Pathegama Gamage, declare that this thesis, submitted in fulfilment of the requirements for the award of Doctor of Philosophy, in the Faculty of Engineering, University of Wollongong, is wholly my own work unless otherwise referenced or acknowledged. The document has not been submitted for qualifications at any other academic institution.

Ranjith Pathegama Gamage
ACKNOWLEDGEMENTS

I have many people to thank for the incredible experience I have had at University of Wollongong, but here I will name a few. I would like to express my sincere gratitude to my supervisors Professor Buddhima Indraratna, Faculty of Engineering, University of Wollongong and for Dr. Winton Gale, Strata Control Technology, Wollongong who provided me an opportunity to start this research project. Professor Indraratna's constant support, positive encouragement on writing technical papers, guidance, editorial assistance and constructive criticism throughout this study are invaluable. Dr. Winton Gale has also provided me excellent technical and practical guidance, particularly during the experimental phase.

I would also like to extend my sincere thanks to Associate Professor Naj Aziz for his advice and assistance during the stay of Wollongong. In addition, my many thanks are due to Professor R.N. Singh, Associate Professor M. Sivakumar for their input on the project.

I am particularly grateful for the financial support given to me through Overseas Postgraduate Research Scholarship (OPRS) and University Postgraduate Award (UPA) by University of Wollongong throughout this research. This challenging research was also made possible because of the financial and technical support provided by Strata Control Technology (SCT), for which I am greatly indebted.

This research work would not have been possible without generous support of Alan Grant particularly during experimental phase. His invaluable experience in constructing laboratory equipment accelerated my experimental work. Also, Ian Bridge, Ian Laird.
Bob Rowlan and Richard Webb provided me generous assistance during various stages of the research. I thank you all for your good humour and support. Many thanks are also extended to Pam Burnham, Elaine Rhodes and Jenny Robertson for their continuous support on administrative work.

I also wish to thank Professor Robin Chowdhury and other faculty members in Civil Engineering discipline for their continuous support and encouragement through my study at Wollongong. Special thanks to Jeff Price for proof reading the thesis. The assistance given by my fellow postgraduate students, Daniela Ionescu, Mark Locke, Bruce Blunden, Ashi Dey, Stuart Chambers, Asadul Haque, Phillip Flentje and Wayan Redana are also acknowledged.

Last but not the least, I would like to thank my wife Nirdosha for the scarsifies she had to endure during my studies, her patience, for her encouragement and advice. To my mother and father. You have always been a source of inspiration throughout my career. My mother and father prayed for me to succeed. I also like to thank my brothers, sisters and my friends for their continuous support for this achievement.
ABSTRACT

Two-phase flow is of considerable importance in many applications such as chemical engineering, petroleum recovery, underground storage plants and mining engineering. Most commonly encountered two-phase flows are gas-liquid, gas-solid, liquid-liquid and liquid-solid. Out of these, the most complex flow is the gas-liquid phase, because of the complex interaction between fluids including compressibility and solubility characteristics. In fields such as, chemical and biomedical engineering, a vast number of studies have been carried out in order to understand the complex nature of two-phase flows. However, no generalized solution method for water-gas has been formulated in the rock mechanics literature because of the large number of geo-hydraulics variables involved, including the deformation characteristics of each phase and their influence on one another.

In this research study, the characterisation of two-phase flow in a fractured rock mass was investigated, in order to understand the coupled flow-deformation mechanisms under various stress conditions. A comprehensive mathematical model to predict the quantity of each flow component in a single joint was developed. A joint with two parallel walls filled with layers of water and air was analysed. Effects of mechanical deformation of the joint, compressibility of fluids, the solubility of air in water and the phase change between fluids have been taken into account to develop analytical expressions, which describe the behaviour of the air-water interface. A state-of-the-art two-phase high pressure triaxial equipment was developed in order to calibrate the model. Prior to testing, all the fractured specimens were mapped using the digital co-ordinate profilometer to estimate the roughness of the fractured surface. Tests were
conducted on fractured hard rock specimens for different boundary conditions including confining pressures with inlet water and air pressures.

Accurate determination of flow structures is very important in developing mathematical models for multiphase flow analysis. An indirect method based on fluid flow parameters was employed for identifying flow patterns within rock joints. Using the plot of liquid superficial velocity against gas superficial velocity, a clear margin to identify bubble flow pattern from annular flow was observed. For a water saturated specimen, at a relatively low inlet air pressure, the expected flow pattern within the rock joint is bubble flow. At intermediate air pressures, the flow regime is best described by annular flow, whereas at elevated air pressures, a complex flow pattern may develop. Although these plots do not show the clear margins of all different flow regimes, they distinguish clearly, the bubble flow pattern from annular or complex flow regimes.

Findings of this study also show that two-phase flow rate follows a linear relationship against inlet fluid pressures when inlet air pressure \( (p_a) = \) inlet water pressure \( (p_w) \). At relatively low inlet fluid pressures, flow rate linearly varies with the fluid pressures. However, the linear relationship between the flow rate and the fluid pressure vanishes once the inlet fluid pressure exceeds a certain value. The non-linearity may probably be due to the formation of non-parallel laminar or turbulent flow at rough joint surfaces. According to the analysis of flow type (i.e., turbulent or laminar flow), flow can be best described by laminar flow. Non-parallel laminar flow occurs at elevated fluid pressures, whereas the development of turbulent flow within rock fractures is very remote.
From the comparison of single-phase flow with two-phase flow, it is evident that the two-phase flow rate is much lower than that of single-phase flow. The significant reduction of two-phase flow rate is attributed to the influence of one phase on the other. As an example, at 0.2MPa inlet fluid pressure, the individual components of water and airflow rates of two-phase flow have decreased by 50% and 95% from the respective single-phase flow rates. These findings also confirm that two-phase flows follow Darcy's law for the considered range of confining pressure and inlet air and water pressures. Therefore, Darcy's law can be extended to model unsaturated flow through jointed rocks by introducing the factor, 'relative permeability' \( (K_r) \).

For fully saturated water flow through rock joints, various studies have shown that the flow rates decrease with the increase in confining pressure due to the closure of apertures. Similar to single-phase flow, two-phase flow is also influenced by the confining pressures in a similar way. However, beyond a confining pressure of 6MPa, the rate of change of flow becomes marginal. For example, the flow rates of both phases decrease by as much as 80% when the confining pressure exceeds 6MPa, which is mainly due to the reduction of effective aperture by joint deformation. Beyond this point, any change of flow rate is mainly due to interaction between the fluids including solubility, compressibility and change of fluid properties. Depending on the magnitude and orientation of the joint network relative to the direction of axial loading, two-phase flow rate may increase or decrease.

From the mathematical model, it is shown that the joint deformation contributes most to the change in phase levels of air and water layers, and the effects of air solubility and compressibility components are relatively small. Nevertheless, at significantly elevated
confining pressures, where the joint apertures have reached their residual values, the effects of compressibility and solubility of air in water become increasingly more pronounced. The measured flow rate of the water phase was found to be almost equal to the calculated flow rate. However, for the flow rate of air, a slight deviation from theory was encountered, which was probably due to some discontinuous air pockets trapped within the rock matrix.

It is evident that the flow through a fractured sample is not always stratified. Nevertheless, the computed water and air phase heights, i.e. $h_w(t)$ and $h_a(t)$, introduced in the model give a realistic prediction of flow volumes, as verified by the laboratory measurements. The study confirms that the analytical model can accurately predict the flow rates of both air and water phases in a single joint, for given applied stress conditions. The developed theory can be employed to predict water and gas flows through fractured rock mass in underground works and in oil recovery process in petroleum engineering. Particularly in nuclear waste storage plants, the common unsaturated flow through fractured tight rocks can be simulated using the developed theory in order to estimate radioactive contamination with groundwater. The measured relative permeability values can be incorporated in numerical models to model actual flow behaviour in fractured rock mass.
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<td>$r$</td>
<td>Discontinuities, which terminate in the rock exposure</td>
</tr>
<tr>
<td>$x$</td>
<td>Discontinuities, which extend to the outside rock exposure</td>
</tr>
<tr>
<td>$d$</td>
<td>Discontinuities, which terminate against the other discontinuities in the rock exposure</td>
</tr>
<tr>
<td>$T_x$</td>
<td>Termination index</td>
</tr>
<tr>
<td>$l$</td>
<td>Discontinuity length</td>
</tr>
<tr>
<td>$l_i$</td>
<td>Individual discontinuity length</td>
</tr>
<tr>
<td>$n$</td>
<td>Number of discontinuities</td>
</tr>
<tr>
<td>$d$</td>
<td>Measured distance along the tape or scanline</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>The angle between the scanline and the discontinuity</td>
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<tr>
<td>$x$</td>
<td>The mean discontinuity spacing</td>
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<tr>
<td>$z$</td>
<td>Standard normal variable</td>
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<td>$\varepsilon$</td>
<td>Proportionate error</td>
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<tr>
<td>$E$</td>
<td>Young’s modulus</td>
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<td>$G$</td>
<td>Shear modulus</td>
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<tr>
<td>$K$</td>
<td>Bulk modulus</td>
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<tr>
<td>$\nu$</td>
<td>Poissons ratio</td>
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<tr>
<td>$\Delta a$</td>
<td>Axial metric deformation</td>
</tr>
<tr>
<td>$\Delta d$</td>
<td>Diametric deformation</td>
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<tr>
<td>$L$</td>
<td>Length of the specimen</td>
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<tr>
<td>$D$</td>
<td>Diameter of the specimen</td>
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<td>$P$</td>
<td>Total load on the plate</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
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<tr>
<td>$W_a$</td>
<td>Average deflection of the plate</td>
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<tr>
<td>$R$</td>
<td>Radius of the rigid plate</td>
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<td>$V_1$</td>
<td>Volume of the pore fluid at the top of the sample</td>
</tr>
<tr>
<td>$V_2$</td>
<td>Volume of the pore fluid at the bottom of the sample</td>
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<tr>
<td>$k$</td>
<td>Coefficient of permeability</td>
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<td>$q$</td>
<td>Flow rate</td>
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<td>$A$</td>
<td>Area</td>
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<td>$\rho$</td>
<td>Density of fluid</td>
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<td>$\mu$</td>
<td>Dynamic viscosity of fluid</td>
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<td>$f$</td>
<td>Obstruction factor to gas due soil particles</td>
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<td>$p$</td>
<td>Pressure</td>
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<td>$N^T$</td>
<td>Total flux</td>
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<td>Gas constant</td>
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<td>$S$</td>
<td>Degree of saturation</td>
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<td>$g$</td>
<td>Acceleration due to gravity</td>
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<td>Fracture width</td>
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<td>Dip angle of a discontinuity</td>
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<td>Dp direction</td>
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<td>Mean</td>
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<td>Standard deviation</td>
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<td>$m$</td>
<td>Median</td>
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<td>$K_f$</td>
<td>Fracture transmissivity</td>
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<td>$h$</td>
<td>Hydraulic head</td>
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<td>$Q_s(w)$</td>
<td>Constant point source</td>
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\( \delta \)  
Kroneker delta

\( E(\hat{n}) \)  
Probability density function

\( \hat{n} \)  
Unit normal vector

\( \sigma_n \)  
Normal stress

\( \delta_n \)  
Normal displacements

\( \delta_s \)  
Shear displacements

\( v_0 \)  
Closure of the joint when the hydraulic aperture becomes zero

\( k_0 \)  
Initial fracture permeability at initial normal stress \((\sigma_0)\)

\( \sigma_{ch} \)  
Confining healing pressure

\( P_f \)  
Effective modulus of the asperities

\( f \)  
Auto correlation distance

\( a_{02} \)  
Half aperture at the reference pressure

\( J \)  
Hydraulic gradient

\( b_i \)  
Segment aperture

\( v \)  
Average velocity

\( dh/dx \)  
Hydraulic gradient

\( b \)  
Width of the fracture

\( r_o \)  
Outer radius

\( r_w \)  
The well radius

\( H_0 \)  
Depth of water table to the center of the tunnel

\( \gamma_w \)  
Unit weight of rock

\( \gamma \)  
Surface tension between wetting and non wetting phases

\( a_f \)  
Fracture aperture

\( \sigma_1 \)  
Major principle stresses

\( \sigma_3 \)  
Minor principle stresses
\( \sigma_2 \) Intermediate stress

\( u \) Pore pressure

\( T \) The torque

\( \sigma_e \) External stresses

\( \sigma_i \) Internal radial confining stresses

\( F \) Axial force

\( k_f \) Fracture permeability

\( k_m \) Matrix permeability

\( z \) Elevation head

\( p_i \) Inlet pressure of gas

\( p_e \) Exit pressure of gas

\( e \) Joint aperture

\( dp/dx \) Pressure gradient

\( a \) Maximum amplitude

\( E \) Mechanical aperture

\( f(x) \) Amplitude of asperity height at distance \( x \), along the length \( L \)

\( F \) Relative roughness

\( \lambda \) Pressure drop coefficient

\( R_e \) Reynolds number

\( A \) Area of the fluid

\( S_{wa} \) Perimeter of the interface

\( S_{jw} \) Perimeter of bottom joint wall

\( F_{wa} \) Shear stress acting on the water-air interface

\( F_{jw} \) Shear stress acting on the wall

\( \sigma_1 \) Vertical stress applied on the discontinuity
\( \sigma_3 \) Horizontal stress applied on the discontinuity

\( M \) Total mass rate

\( Q_a \) Volumetric flow rate of air-phase

\( Q_w \) Volumetric flow rate of water-phase

\( R \) Fraction of the total mass flow across the interface

\( \eta \) Fraction of the force taken by air phase

\( \beta \) Orientation of the joint

\( S_{ja} \) Perimeter of the bottom joint wall

\( F_{ja} \) Shear stress acting on the wall

\( f_a \) Friction factor between wall and air

\( F_T(x, y)_h \) Top joint surface profile

\( F_B(x, y)_t \) Bottom joint surface profile

\( F_I(x, y)_0 \) Interface position

\( h_w(r) \) Water-phase height

\( h_a(r) \) Air-phase height

\( \xi_{wc} \) Change of phase level due to compressibility of water

\( \xi_{ac} \) Change of phase level due to compressibility of air

\( \xi_{ad} \) Change of phase level due to solubility of air in water

\( V_{dt} \) Volume of dissolved air in water

\( M_d \) Mass of dissolved air in water

\( W_a \) Molecular mass of air

\( C_a \) Compressibility coefficient of air

\( C_w \) Compressibility coefficient of water

\( \rho_a(t) \) Final density of air

\( \rho_a(0) \) Initial density of air
Initial mass of water
Final mass of dissolved air in water
Final volume of water
Aperture at time, $t = 0$
Aperture at time, $t$
Normal stiffness of discontinuity
Shear stiffness of discontinuity
Water pressure within the discontinuity
Tangential displacement of discontinuity
Water pressure
Air pressure
Hydraulic gradient
Partial differential operator
Relative permeability
Kinematic viscosity
Single-phase permeability
Second moment of inertia
Point contact permeability factor
Bulk modulus of water
Joint permeability factor
Joint normal displacement
Fluid pressure along the vertical boundaries
Fluid pressure along the horizontal boundaries
Depth of the water table
Hydraulic gradient in Y-direction
<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>BEM</td>
<td>Boundary Element Method</td>
</tr>
<tr>
<td>DEM</td>
<td>Discrete Element Method</td>
</tr>
<tr>
<td>FEM</td>
<td>Finite Element Method</td>
</tr>
<tr>
<td>FDM</td>
<td>Finite Difference Method</td>
</tr>
<tr>
<td>FLAC</td>
<td>Fast Lagrangian Analysis of Continua</td>
</tr>
<tr>
<td>UDEC</td>
<td>Universal Distinct Element Code</td>
</tr>
<tr>
<td>JCS</td>
<td>Joint Compressive Strength</td>
</tr>
<tr>
<td>JRC</td>
<td>Joint Roughness Coefficient</td>
</tr>
<tr>
<td>TPHPTA</td>
<td>Two-Phase High Pressure Triaxial Apparatus</td>
</tr>
<tr>
<td>LVDT</td>
<td>Linear Voltage Differential Transformer</td>
</tr>
</tbody>
</table>
CHAPTER 1

INTRODUCTION

1.1 RATIONALE

Fluid flow analysis plays a major role in various geotechnical applications (e.g. mining and petroleum industry and nuclear waste storage plants), and the understanding of flow mechanisms is essential for the development of a hydro-mechanical flow model suitable for underground excavations in rock. Accurate prediction of mine water inflow to a tunnel is one of the essential tasks of underground works during the design and construction stages. Under the ever increasing environmental and regulatory controls, the evaluation of the quantity and quality of total inflow to excavations, and the procedures for discharging polluted mine water are significant factors in the development and operational stages of underground mining. Moreover, in nuclear waste storage plants, special attention should be given to prevent any radioactive contamination of groundwater. The accurate prediction of inflow volumes is expected to minimize most environmental hazards and damage to mine equipment, as well as to reduce the time delay associated with dewatering. In petroleum engineering, understanding multiphase flow behavior is imperative particularly during the secondary stage, in order to enhance the oil recovery process.

In a comprehensive study of flow, one has to consider an array of geo-hydrological factors including the type of flow (i.e. saturated or unsaturated) and joint geometrical and material properties. If the rock mass is saturated with a single fluid, then a single-
phase flow analysis must be carried out. Fully saturated flow analysis techniques have been well established during the past four decades. Depending on the availability of geological data and the required accuracy of flow estimation corresponding to the availability of time, numerical techniques and computer resources, the most appropriate flow approach can be selected for a particular medium (i.e. discrete, continuum or combination of both). The use of a discrete method is more realistic, if fluid flow takes place mainly through a network of fractures. In contrast, a continuum approach can be employed for relatively high porosity rocks (e.g. sandstone and limestone), or if the fracture density is extremely high, in which case, the media is assumed to behave as a porous medium.

In the past, various fluid flow models based on single-phase flow (i.e. empirical, analytical and numerical) have been employed to analyse flow through a rock mass (Goodman et al., 1965; Sharp, 1970; Maini, 1971; Neuman, 1973; Crouch & Starfield, 1983; and Crotty & Wardle, 1985; Zhang and Franklin, 1993; Beer & Poulsen, 1994). It is an established fact that rock joints are usually unsaturated, and they conduct both gas (CO$_2$, CH$_4$, and air) and water together. Therefore, the single-phase flow analysis is often unrealistic in most circumstances. However, less attempts have been made on multiphase flow analysis in rock media because of the complex nature of interaction between fluids under ground stresses. In view of this, the present study aims to shed light on the relatively complicated two-phase flow system, and to provide a comprehensive mathematical model to compute the quantities of each fluid phase travelling in a given joint domain. This study mainly consists of two stages: (a) development of the mathematical model, and (b) laboratory testing to verify the mathematical model, using a two-phase, high-pressure triaxial cell.
1.2 PURPOSE OF TWO-PHASE FLOW ANALYSIS THROUGH A ROCK MASS

Rock fractures are usually unsaturated and they carry water, gas and solid. On some occasions (e.g. petroleum recovery process), fractures may be partially saturated with three phases, such as oil-water-gas. Two-phase flow through jointed rock media has gained increasing interest in both industry and academia due to the important applications in two-phase flow transportation through joint networks. Importance of such flows extends from petroleum engineering, mining engineering and nuclear engineering to medical applications and food manufacturing. Two-phase flow through rock fractures can be in the form of water-gas, water-solid, solid-gas, water-oil and oil-gas. Out of these, the most complex is the gas-liquid phase, because of the compressibility and solubility characteristics. For gas-water flow, the complex feature is the existence of a deformable interface whose shape and distribution are of critical importance in determining the flow characteristics. From the published data, no work on flow pattern of multiphase flow through rock joints can be found in the literature. However, some work on two-phase flow through artificial and natural rock joints has been conducted for a given flow pattern.

A number of numerical, analytical and experimental models for single-phase flow (i.e. either water or gas) through a single joint can be found in the literature (Louis, 1976; Brown, 1987). These single-phase models have often been used to simulate the fluid flow (usually either two or three phase flow) in many underground applications because of the lack of knowledge on unsaturated flow through rock. At present, due to the lack of proper understanding of multi-phase flows (water-gas), it is difficult to predict the
risk of groundwater inundation and outbursts in complex hydro-geological environments prevailing in underground mines. These potential outbursts become worst when fractures contain two-phase flows, because of the decreasing desorption rate due to presence of water. As an example, in coal mining, the existence of gas pockets (e.g. CO$_2$ and CH$_4$) within a complex hydro-geological regimes present obvious safety hazards. Mining coal reserves under these circumstances always carries the risk of groundwater inundation and gas outbursts, and sites affected by unexpected inflows have been reported in many parts of the world including Australia (e.g. Bulli and Westcliff Collieries, New South Wales: Lama and Bodziony, 1996). The hazardous nature of inundation, carbon dioxide concentration and the catastrophic consequences of methane explosions are well known. The polluted (acidic) mine water may contaminate with the existing groundwater table or may be discharged to the surface water, stream or river, causing serious threats to the environment. Moreover, in nuclear wastage storage plants, the probable radioactive contamination of the existing groundwater table can be minimised using proper design of underground storage plants in fractured rocks.

1.3 SPECIFIC OBJECTIVES OF THE STUDY

(a) Thorough investigation of existing fluid flow models for analysing flow in fractured rock mass based on single-phase flow analysis, and limited attempts on two-phase flow through rock fractures,

(b) The development of a state-of-the-art two-phase high pressure triaxial equipment to calibrate the mathematical model, and to
investigate fluid flow parameters including relative permeability for natural rock fractures,

(c) Effects of joint aperture and joint roughness on fluid flow through a single joint,

(d) The development of a comprehensive mathematical model to describe two-phase flow in rock joints,

(e) Comprehensive laboratory study on single-phase and two-phase flow through fractured rock specimens,

(f) Numerical modelling on fluid flow through rock mass with respect to water ingress to an underground cavity.

The obtained knowledge on two-phase flow through a single rock fracture gives a better understanding of unsaturated flow through a rock mass. The measured relative permeability values can be incorporated in numerical models to model actual flow behaviour in fractured rock mass.

1.4 OUTLINE OF THESIS

The thesis is divided into five major parts and a brief description of each part is outlined below.

Chapter 1 introduces and outlines the topic of the research. Chapter 2 is devoted to discuss the mechanisms of formation of discontinuities and their classification systems. The measurements and effects of physical properties of intact rocks and discontinuities
(length, orientation, shape and infill materials) on permeability are also highlighted. Field investigation of discontinuities in various sites in Australia is also included.

A critical review of literature on fluid flow through rock masses is given in Chapter 3, introducing the numerical and analytical approaches employed for fluid flow estimations. The applicability of fluid flow models for various conditions is highlighted. Limited attempts of modelling two-phase flow through rock fractures are also discussed. Various designs of triaxial apparatus used in the past for tests on soil and rocks are described in the chapter.

Chapter 4 begins with the illustration of various triaxial apparatus and their application in soil and rock property measurement. Chapter 4 also presents a novel design of a state-of-the-art, two-phase, high-pressure triaxial equipment which is capable of measuring absolute permeability, relative permeability and the deformation response of fractured specimens under various boundary conditions, including axial stress, confining pressure and inlet fluid pressures.

Apart from inlet fluid pressures and ground stresses, fluid flow through rock fractures is predominantly governed by joint aperture and its variability along the joints, which is described in Chapter 5. Joint roughness is an influencing factor on fluid flow when the apertures are small, and they are subjected to normal and shear deformation. This is elaborated in Chapter 6.

The development of a mathematical model for two-phase flow in a single rock joint is described in Chapter 7. Assuming the fluid flow pattern within rock fracture to be
stratified two-phase flow, governing flow equations were developed based on mass, momentum and energy equations.

The use of two-phase, high-pressure triaxial equipment to study the behavior of two-phase flow through real rock fractures is described in Chapter 8. Naturally and artificially (using Brazilian test) fractured rock specimens with different joint roughness coefficients were selected for testing. More significantly, Chapter 8 describes the comparison of laboratory test data with the model data. Chapter 9 presents fluid flow through large fracture network based on Universal Distinct Element Code (UDEC). Prediction of total water ingress to underground cavity was carried out for different boundary conditions including the effect of variable ground stresses and joint geometrical parameters.

In Chapter 10, the conclusions from this research in relation to stratified two-phase flow through real rock fractures are summarised. For the benefit of future studies in multiphase flow, a number of recommendations with respect to analytical, numerical and experimental are made.
CHAPTER 2

CHARACTERIZATION OF A JOINTED ROCK MASS

2.1 INTRODUCTION

This chapter describes rock mass classification systems, physical and mechanical properties of discontinuities and intact rocks, which are important to understand before any fluid flow through jointed rock media could be analysed. The understanding of physical properties of geological structures, their formation and mineral contents are also essential in the design and construction stages of underground excavations.

Various attempts have been made in the past to classify rock masses for important engineering applications with regard to the design of mines, highway tunnels, slopes, dam foundations and underground nuclear waste storage plants. Classification of rocks can be based on their mineral composition and their origin (usually employed by geologists) or types for engineering applications (used in rock mechanics). Figure 2.1 distinguishes these two classifications systems. The geological classification provides mineralogical and chemical data, but it does not directly provide deformation characteristics of rocks, which are imperative in design. Under the first category, depending on the geological origin, rocks are usually classified into three major groups, (a) sedimentary rocks, (b) igneous rocks and (c) metamorphic rock. Figure 2.2 shows the common rock types under these principal categories.
Figure 2.1. Principal classification systems for rocks (after Santosh, 1991).

Figure 2.2. Most common rock types (after Fowler, 1990; Santosh, 1991).
2.2 ROCK CLASSIFICATION SYSTEMS

2.2.1 Geological classification

(a) Sedimentary rocks

Sedimentary rocks are formed by sedimentation process followed by cementing of small rock particles. Sedimentary rocks are usually formed under the following different stages, (a) weathering, (b) transportation, (c) deposition and (d) consolidation (Tucker, 1993). A simplified geological process is shown in Figure 2.3. Sedimentary rocks are sometime referred to as stratified rocks, because sediments are often deposited and consolidated as layers. These layers interfaces are usually identified as 'bedding planes', along which movements can occur under stress changes. In underground coal mines, planes of weakness also occur as shale and sandstone deposits that are inter-layered.

As described by Santosh (1991), from a geotechnical engineering point of view, one may classify sedimentary rocks as (a) limestone group, (b) sandstone group, (c) shale and (d) salt group. However, the most appropriate geological method of classification is based on the type of sediments: (a) chemical, (b) clastic and (c) organic rocks. Limestone and dolomite may contain several minerals, such as CaCO₃, SiO₂ and MgCO₃. These rocks may be loosely packed or densely packed, and further sub-divided to chalk, shaley and argillaceous materials.
(b) **Metamorphic rocks**

The existing rocks may be subjected to high pressure and temperature changes resulting in the alteration of the existing rocks either structurally, mineralogical or texturally.

![Simplified geological processes](image)

Figure 2.3. Simplified geological processes (after Santosh, 1991; Fowler, 1990).

This complex process is referred to as metamorphism, and the altered rocks are called metamorphic rocks. As an example, gneiss and quartz are the result of metamorphism process of granite and sandstones, respectively. Metamorphism may be divided into following sub groups depending on the influence of temperature, pressure and chemical agents. They are: (a) plutonic, (b) thermal, and (c) clastic metamorphism. Figure 2.2 describes the formation of different type of metamorphic rocks from various rocks.
When the metamorphism process takes place in a large area of rocks, the modified rocks are referred to as regional metamorphic rocks (e.g., gneiss, quartz). Usually, gneiss is a banded rock, which permits relatively easy split through layers. The metamorphic rocks contain minerals such as quartz, feldspar, mica, chlorite and talc. The mineral quantities and the mineral type of these rocks entirely depend on the influence of metamorphism. During the metamorphism process, fractures may develop within the rock mass and their extent depends on the temperature and pressure change. Based on the mineral grain shape, size and arrangement, metamorphic rocks have different structures such as banded, slaty and foliated. As an example, when sedimentary rocks undergo metamorphism, the existing bedding planes in the sedimentary rocks may also be seen in the newly formed metamorphic rock. However, the bond between these layers is stronger than in sedimentary rocks. It is important to note that during excavation or blasting process, fractures may easily develop through bedding layers and such discontinuities provide paths to fluid flow.

(c) Igneous rocks

Igneous rock can be classified in different ways, and one such common classification is based on the grain size, i.e. (a) coarse-grained plutonic rock, and (b) fine-grained volcanic rocks. Plutonic rocks are formed from intrusions of magma, and hardened by cooling process, whereas volcanic rocks are derived from extrusions, on the surface solidified by rapid cooling. Granite and diorite are examples of plutonic rocks, whereas basalt and picrite are volcanic rocks. The grain size of granite is typically larger than 2mm, whereas basalt is fine grained, smaller than 0.06mm. The principal chemical composition of different metamorphic rocks can be found in the literature (Nockolds et al. 1978; Smith and Erlank. 1982). The chemical composition of typical igneous rock is given in Table 2.1. Depending on chemical composition and the grain size, some
igneous rocks such as basalt display as columnar fractures with hexagonal cross-section. Formation of discontinuities in different rocks including basalt will be discussed in Section 2.3.

Table 2.1. Chemical composition of some igneous rocks

<table>
<thead>
<tr>
<th>Chemical composition</th>
<th>Granite</th>
<th>Diorite</th>
<th>Basalt</th>
<th>Andesite</th>
</tr>
</thead>
<tbody>
<tr>
<td>Al₂O₃</td>
<td>14.4</td>
<td>15.7</td>
<td>14.1</td>
<td>17.7</td>
</tr>
<tr>
<td>SiO₂</td>
<td>70.4</td>
<td>66.9</td>
<td>50.8</td>
<td>54.9</td>
</tr>
<tr>
<td>Na₂O</td>
<td>3.2</td>
<td>3.8</td>
<td>2.2</td>
<td>3.7</td>
</tr>
<tr>
<td>K₂O</td>
<td>5.0</td>
<td>3.1</td>
<td>0.8</td>
<td>1.1</td>
</tr>
<tr>
<td>CaO</td>
<td>2.0</td>
<td>3.6</td>
<td>10.4</td>
<td>7.9</td>
</tr>
</tbody>
</table>

2.2.2 Classification system based on engineering applications

Terzaghi (1946), Baron et al., (1960), and Coates (1964) attempted to classify rocks for different applications. Basically, each classification system is based on the strength of rocks and the features of geological structures such as joints. Coates (1964) described in detail the merits and demerits of each method and proposed a better approach. In Terzaghi’s (1946) rock classification for tunnels, he grouped rocks into 7 categories, as follows:

(a) Stratified rocks
(b) Intact rocks
(c) Moderately jointed rocks (e.g., vertical walls require no support)
(d) Blocky rocks (e.g., vertical walls require support)
(e) Crushed rocks
(f) Squeezing rocks (low swelling capacity)

(g) Swelling rocks (high swelling capacity)

As described by Coates (1964), one of the main deficiencies of this classification system is that it does not give any information on the strength of rocks. Also, at shallow depth, blocky rocks might not require any support for vertical walls, whereas moderately jointed rocks may require support in tunnels at greater depths.

Another classification system proposed by US Bureau Mines (1962) for underground openings, and recorded by Coates (1964) is described below. Rocks in underground mines have mainly been divided into two groups, namely: (a) competent rocks (i.e., no support is required) and (b) incompetent rocks (i.e., support is required to prevent failure of an opening). Competent rocks are further subdivided into three classes, as massive elastic rocks (e.g., homogeneous rocks), bedded-elastic rocks (e.g., homogeneous, isotropic beds with the bed thickness less than the span of the underground excavation) and massive-plastic rocks (i.e., rock may flow under even low stress).

The classification system suggested by Coates (1964) gives a better picture of the strength and failure characteristics of rocks. His method is based on 5 main characteristics of rock mass, as follows:

1. Uniaxial compressive strength

   (a) weak (<35 MPa)

   (b) strong (between 35 and 175 MPa - homogeneous and isotropic rocks)

   (c) very strong (> 175 MPa - homogeneous and isotropic rocks)
(2) Pre-failure deformation behaviour of rocks

(a) elastic

(b) viscous

(3) Failure characteristics of rocks

(a) brittle

(b) plastic

(4) Gross homogeneity

(a) massive

(b) layered (e.g. sedimentary rocks)

(5) Continuity of rock mass

(a) solid (joint spacing > 1.8m)

(b) blocky (joint spacing < 1.8m)

(c) broken (passes through a 75mm sieve).

It is clear that, while different classification systems available, one must employ geological and rock mechanics classification models in conjunction to obtain a better understanding of the nature of a particular type of rock. The classification system adopted by ISRM (1981) is given in Table 2.2.

2.3 FORMATION OF DISCONTINUITIES ON THE EARTH’S CRUST

Geological features in rock mass can be broadly divided into two categories, namely (a) primary and (b) secondary structures. Primary features are original features of rocks. whereas once they undergo subsequent deformation or metamorphism, they are referred to as secondary structures.
<table>
<thead>
<tr>
<th>GENETIC GROUP</th>
<th>ROCK TYPE</th>
<th>Metamorphic</th>
<th>Sedimentary rocks</th>
<th>Composition</th>
<th>Chemical/organic composition</th>
<th>Compositional type</th>
<th>Acid rocks</th>
<th>Intermediate rocks</th>
<th>Basic rocks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grain size</td>
<td>Description</td>
<td>Detrital</td>
<td>Grain shape</td>
<td>Chemical/organic composition</td>
<td>Compositional type</td>
<td>Acid rocks</td>
<td>Intermediate rocks</td>
<td>Basic rocks</td>
<td></td>
</tr>
<tr>
<td>60</td>
<td>Very coarse grained</td>
<td>Boulders</td>
<td>Rounded grains Conglomerate</td>
<td>Saline rocks Halite</td>
<td>Quartz, mica, feldspar</td>
<td>Gneiss</td>
<td>Pegmatite</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Coarse grained</td>
<td>Gravel</td>
<td>Angular grains Breccia</td>
<td>Gypsum</td>
<td>Gneiss</td>
<td>Granite</td>
<td>Diorite</td>
<td>Gabbro</td>
<td></td>
</tr>
<tr>
<td>Medium grained</td>
<td>Sand</td>
<td>Grains: mineral fragments. Contain more than 75% of quartz</td>
<td>Limestone Dolomite Chert Flint</td>
<td></td>
<td>Schist</td>
<td>Microgranite</td>
<td>Microdiorite</td>
<td>Dolerite</td>
<td></td>
</tr>
<tr>
<td>0.06</td>
<td>Fine grained</td>
<td>Silt</td>
<td>Fine grained particles</td>
<td>Peat</td>
<td></td>
<td>Phyllite</td>
<td>Rhyolite</td>
<td>Andesite</td>
<td>Basalt</td>
</tr>
<tr>
<td>0.002</td>
<td>Very fine grained</td>
<td>Clay</td>
<td>Very fine grained particles</td>
<td>Coal</td>
<td>Slate mylonite</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Rocks during the process of development and later formation of rock masses are usually subjected to a number of forces within the earth crust. These forces may be a single force or combined forces from ground stress, tectonic forces, hydrostatic forces, pore pressures, and temperature stresses. As a result of these forces and their magnitudes, rocks continuously undergo varying degree of deformation, resulting in formation of different kinds of structural features (Figure 2.4). For example, fractures or joints may initially develop within a rock mass, followed by the dislocation of fractured rock blocks. In some circumstances, these dislocated rock blocks move faster than the adjacent blocks, resulting in larger deformation between each other. Such structural features are referred to as faults. Both faults and joints are the result of the brittle behaviour of a rock mass. Joints and faults can easily be identified from the component of displacement parallel to the structure. Joints have usually very small normal displacement, referred to as the joint aperture. The estimation of joint aperture using various techniques under different stress conditions is discussed in detail in Chapter 5.

2.3.1 Joints and bedding planes

Joints in a rock mass are usually developed as families of cracks with regular spacing, and these joint families are referred to as joint sets. While the formation of joints is associated with the effect of differential stress, some joints are more prominent and well developed, extending to a considerable length (several kilometres), while others are minor joints having a length of only a few centimetres. In order to characterise joint sets, the properties such as joint spacing, their orientation, joint length, gap length and
Figure 2.4. Common geological structural features in a rock mass (after Hobbs et al., 1976; Hills, 1963).
Joint apertures must be considered. Relative movement of joints is negligible in comparison with fault movements. Joints in a rock mass may be open or closed or filled with some other materials like, clay and silt. Open joints often provide fluid flow paths to other connected joints in the rock mass, and the quantity of fluid carried by each joint depends on the separation between joint blocks (joint aperture), joint geometry, hydraulic gradient and fluid properties.

To describe a joint in the field, the inclination to the horizontal, referred to as the dip of the joint and the strike are used. Based on the dip angle and the strike, joints are grouped into three classes: (a) dip joints, (b) oblique joints, and (c) strike joints. However, the classification based on the origin of joints is often more useful in various engineering applications (Figure 2.5). According to the origin of joints, they are mainly grouped in two classes, as follows:

(a) tension joints

(b) shear joints

Figure 2.5. Genetic classification of joints.
Tension joints may develop either during the formation of rocks or after their formation, attributed to the tensile forces acting on the rock mass. Columnar, sheeting and mutually perpendicular joints are some common types of tension joints. Columnar joints generally exist in hexagonal shape and are often found in basalt rocks, whereas sheet joints may develop in granite. Figures 2.6a and 2.6b show columnar joints in basalt and sheet joints in granite rocks. The mechanism of formation of columnar joints in basalt is associated with the stress caused by the cooling of magma. Theoretical aspects of the development of joints and faults have been discussed in detail by Price (1966). He proposed that the joints in horizontally bedded rocks were mainly due to uplifting forces developed on the earth’s crust. It can sometimes create both shear and tension joints on either side of the limbs of fold, as shown in Figure 2.7. Apart from natural geological process, rock fracturing may also occur on the surface rocks because of human activities. The natural weathering process is accelerated by temperature

Figure 2.6a. Columnar joints at a basalt quarry in Kiama, NSW, Australia (taken by the writer).
Figure 2.6b. Closely packed sheet joints, National Park, Western Australia (taken by the writer).

Figure 2.7. Development of shear and tension joints on the limbs of fold (after Price, 1966).
change and the action of surface water. According to the definition given by Pettijohn et al., (1972), cross joints are defined as a structure confined to a single sedimentation unit and characterised by internal bedding, and inclined to the principle bedding plane.

The most common feature of sedimentary rocks is the occurrence of bedding planes, in which each bed is fairly homogeneous and differs in properties from the other beds, in relation to the texture and composition. However, in some circumstances, significant variations of material properties within a layer may occur because of the varied conditions of lithological process including random, uniform or systematic deposition. When the sediments are deposited uniformly, relatively homogenous layers are formed. Between each strata, a new discontinuity is developed, and the properties of this discontinuity depend upon the properties of the adjacent top and bottom layers. Typical bedding planes in a sedimentary rock are shown in Figure 2.8.

Figure 2.8. Bedding planes in sedimentary rock, NSW, Australia (taken by the writer).
2.3.2 Faults

When the shearing stress exceeds the shearing resistance of rocks, fractured rock blocks undergo a considerable displacement along a favourable shear plane resulting in the formation of a new discontinuity, which is referred to as a fault. Depending on the internal stress, and the properties of rocks, these relative displacements vary from few centimetres to several kilometres. Priest (1993) reported the extent of some major faults, such as the San Andreas Fault in California, the Great Glen Fault in Scotland and the Alpine Fault in New Zealand. The nature and the magnitude of the internal forces developed on the earth’s crust are not well known. These forces may give rise to tension, compression or rotation, or a combination of them. For example, thrust faults are believed to form as a result of compressive stresses. Faults can be broadly identified by analysing the lithological process (e.g., shear zone, gouge, abnormal behaviour of strata), landscape and escarpments in the vicinity. To describe a fault geometrically, the following terms (Figure 2.9) are usually used: (a) dip and the strike, (b) fault plane, (c) hanging wall and footwall, (d) hade, (e) heave and (f) throw. In the literature, different classification systems have been employed to identify various faults based on:

(a) The movement of the fault (e.g., thrust fault, normal fault, vertical fault), for example see Figure 2.10;

(b) The dip angle of the fault (e.g., low angle fault, high angle fault); and

(c) The direction of slip (e.g., strike fault, oblique fault).
Figure 2.9. Simplified schematic diagram of a typical fault.

Figure 2.10. Normal fault at Wombarra drainage tunnel, NSW, Australia.
Planar rock structures on the earth’s crust have been subjected to deformation, producing curved or non-planar structures. These new geological structures are referred to as folds, and usually these folded rocks behave as more ductile material. The extent of folding and the ultimate shape of fold depend on the intensity and the duration of the internal forces, as well as the properties of the rock material. In any type of rocks, folds may develop, however, in sedimentary and volcanic rocks, these structures are common. The study of the mechanism of folding is a complex subject, and it is not discussed within the scope of this write up. Briefly, folding of rock might occur due to the development of tectonic stresses (e.g. lateral compressive forces caused by shrinkage) and non-tectonic stresses caused by landslides, differential compaction, and glaciation.

Folds are broadly grouped into anticlines and synclines. A detailed classification of folds is given in Figure 2.11. For more details, the reader is referred to the work of Hobbs et al. (1976) and Biot (1957, 1959, 1961). Figure 2.12 shows a symmetric fold on the rock surface, Southern Freeway, NSW, Australia.

![Classification of various folds](image)

Figure 2.11. Classification of folds.
Figure 2.12. Symmetrical fold at top of the Southern Freeway, New South Wales, Australia (taken by the writer).

2.4 MEASUREMENTS OF DISCONTINUITY CHARACTERISTICS

For successful design and construction of engineering structures on the surface and underground, detailed investigation of the properties of soil or rock materials, planes of
weakness/discontinuity, frictional characteristics of discontinuities, groundwater conditions and existing stresses are required. Such comprehensive detailed study minimises extensive constructional delays, catastrophic failures, and loss of human lives and the cost of the overall project. As described by Franklin and Dusseault (1989), a detailed site investigation can be carried out in two phases (Figure 2.13).

Figure 2.13. Detailed site investigation for geological structures (after Franklin and Dusseault, 1989).

The information on discontinuity characteristics includes the identification of the different geological structures in the input data required for numerical modelling and for the development of analytical and empirical models. Analyses of groundwater flow, and the roof stability in longwalls on other underground structures are governed by the accuracy of field measurements of discontinuity characteristics. During the preliminary stage, published geological reports and topographic maps usually provide valuable
information on soil and rock types, their properties and the groundwater conditions. Air photos and remote sensing methods are also useful in mapping features such as faults, folds and surface topology. Remote sensing methods are now popular because of the improved resolution of photographs, and also a large area can be surveyed and data interpreted in a relatively short time, using modern computer technology. For the site investigation of nuclear power plants in USA, Mceldowney and Pascucci (1979) discussed different techniques of remote sensing methods and their use to identify geological features such as faults. Near surface rock can be explored by observing natural and man-made structures such as landslides, open pit mines, and slopes. Exposed rock faces provide direct measurement of discontinuity properties over a large area, despite the fact that the rock face may be badly damaged due to induced stresses. In addition, close to the ground surface, rocks are usually subjected to weathering, which results in increased discontinuity frequency, greater width of apertures, or the infilling of joints with materials like clay and silt.

Basically, two different techniques, i.e. scanline method and window method are widely used to map joints on the rock face (ISRM, 1978; Pahl, 1981; Priest and Hudson, 1981). These two methods are more or less similar in the way of measurement, except in the scanline method, only the discontinuities that intersect the scanline are mapped, whereas, all discontinuities in the defined (pre-determined) area are measured in the window method (Figure 2.14). In the window technique, as described by Pahl (1981), the discontinuities within the window are classified into 3 classes: (a) contained, (b) dissect and (c) transect discontinuities. If both ends of discontinuities are within the window, they are referred to as contained (Nos. 2, 4, 5, 9, and 10 discontinuities in Figure 2.14), whereas, when one end of the discontinuities is visible in the window.
they are referred to as dissect (discontinues 1, 6, 7, 8 in Figure 2.14). On some occasions, discontinuities may terminate outside the window, and these are called transect discontinuities.

Figure 2.14. Mapping techniques of discontinuities on an exposed rock surface.

Although an extensive view of rock structures and discontinuities is provided by the exploration of open pit and shafts, these methods are not feasible at greater depths. At such greater depths, the ground can be surveyed by other techniques such as the drill core method. Geophysical techniques (e.g. electrical, seismic, and magnetic) are also widely used to map geological structures and to find properties of soil and rocks to a certain extent. The mechanism of seismic techniques is usually based on the magnitude
of the velocity of sound waves travelling through the rock media. Geophysical methods often detect underground cavities such as faults, abandoned mines caverns, and joints with large apertures.

2.5 PHYSICAL PROPERTIES OF DISCONTINUITIES

It is important to distinguish the common terms used to describe the planes of weakness developed in rock surrounding an excavation. Joints often threaten the stability of rocks, however in some situations such as in oil recover process, interconnected joints accelerate the recovery process. Attewell and Woodman (1971), Priest (1993) and Goodman (1976) used the term discontinuities to describe a whole range of planes of weakness, such as joints, folds, faults, unconformities, outliers and inliers. According to the definition introduced by ISRM (1978), discontinuity is the general term for any mechanical fissure in a rock mass having zero or low tensile strength. It is the collective term used for most types of joints, weak bedding planes, weak schistocity planes, weakness zones and faults.

The main focus of this section is to describe the geometrical and physical properties of joints rather than the properties of other structural features such as folds and faults. In order to characterise a discontinuity in the field, the following aspects of joints are considered:

(a) Length,

(b) Spacing,

(c) Orientation,
(d) Aperture,
(e) Surface geometry,
(f) Mode of origin and the presence of infill material,
(g) Wall strength,
(h) Number of discontinuity sets, and
(i) Block sizes

Piteau (1970, 1973) and ISRM (1978) identified some of above properties as influencing parameters during the design of engineering structures such as slopes, underground structures and foundations. Aspects of discontinuity length, spacing, orientation and shape are discussed in the following sections, while the effects of surface geometry are discussed later in Chapter 5.

2.5.1 Discontinuity length

Length of joint or the areal extent of a joint can be observed at the exposure of rock surface, such as at a face of rock slope, tunnel roof or longwalls. The trace length or discontinuity length may vary from several centimetres to hundreds of metres. In two-dimensions, the trace length has two components, and they are the dip trace length and the strike trace length. Dip trace length is obtained by measuring the length in the direction of the dip, whereas if the length measurement is done in the direction of strike, it is referred to as the strike trace length. Every effort should be made to measure both the dip and strike length, whenever possible. Some discontinuities may terminate when the existing weak planes meet, while the others may extend beyond them.
In various engineering applications such as dam foundations and tunnelling, it is of importance to study the degree of interconnectivity of discontinuities. The degree of interconnectivity of fractures varies upon both the orientations and the trace lengths. The accurate mapping of trace length involves the measuring of the actual length of the weak planes as well as recording the type of termination at both ends of the discontinuity. The length of individual discontinuity at rock surface can be measured using a measuring tape, and the measurements are usually done in the direction of dip and in the direction of strike. According to ISRM (1978), based on the magnitude of trace lengths of discontinuity sets, they can be grouped as given in Table 2.3. As discussed by ISRM (1978), during the mapping process, it is important to note the type of joint termination as delineated below:

(a) Discontinuities, which terminate in the rock exposure (r),
(b) Discontinuities which extend to the outside rock exposure (x), and
(c) Discontinuities which terminate against the other discontinuities in the rock exposure (d).

The ends of some discontinuities may not be visible because of excavation, or the presence of vegetation or features extending beyond the limits the exposure. For example, when recording data, if the length of discontinuity is 12m and one end terminates in the rock surface (r) and the other end termination is outside the rock surface (x), the discontinuity is recorded as 12rx. It is also useful to calculate the termination index, which provides information such as the degree of blocks separation within the given area. Termination index for category (b) is represented as $T_x$ and expressed by (ISRM, 1978):
A large value of $T_x$ indicates that a large number of discontinuities terminate outside the rock surface. If $T_d$ takes a larger value, it is then expected that the rock exposure contains a large number of discrete blocks, which suggests that the exposure has a well interconnected fracture network. The category (a) listed above is usually smaller than the summation of category (b) and (c) in a rock mass (Piteau, 1973).

Table 2.3. Trace lengths of discontinuity sets (ISRM, 1978)

<table>
<thead>
<tr>
<th>Discontinuity trace length</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt;1m</td>
<td>Very low length</td>
</tr>
<tr>
<td>1-3m</td>
<td>Low length</td>
</tr>
<tr>
<td>3-10m</td>
<td>Medium length</td>
</tr>
<tr>
<td>10-20m</td>
<td>High length</td>
</tr>
<tr>
<td>&gt;20m</td>
<td>Very high length</td>
</tr>
</tbody>
</table>

The mean discontinuity length ($l$) is computed as follows:

$$l = \frac{1}{n} \sum_{i=1}^{n} l_i$$

where, $l_i$ = individual discontinuity length and $n$ = number of discontinuities.

As discussed by Cruden (1977), Priest & Hudson (1981) and Kulatilake (1988), if one uses the scanline method, the mean discontinuity length has several drawbacks: (a) the measurements depend upon the orientation and position of the scanline, (b) accuracy of data are based on the exposed area (large discontinuities may terminate beyond the limits of the exposure, hence it is impossible to measure the full length), and (c) it is
difficult to measure very small discontinuities. Because of these, one has to carry out a statistical analysis to increase the reliability of the mapped data. In the past, probability distribution methods such as lognormal, hyperbolic and exponential have been used for quantifying the trace length (Cruden, 1977; Priest & Hudson, 1981). Detailed discussion on different distribution functions is not within the scope here, and the reader can get more information from the references such as, Priest & Hudson (1981, 1983), and Lee & Farmer (1993).

2.5.2 Orientation of discontinuity

To define the orientation of a discontinuity on a rock surface, the dip angle and the dip direction at the point of intersection with scanline are needed. The dip is the steepest declination and measured from the horizontal, whereas the dip direction is measured clockwise from the true north. The dip angle is expressed in degrees. The overall strength and permeability of rock mass, and the stability of engineering structures are influenced by joint orientation. Higher the interconnectivity of fractures and the lower shear strength, the greater will be or become the risk of failure of rock mass. There are basically two methods suitable for measuring the orientation. They are: (a) compass and clinometer technique, and (b) photogrammetric technique. The detailed procedure involving mapping and analysing discontinuity orientation is given in Figure 2.15 (Priest, 1993).

The compass and clinometer technique is usually employed to measure the discontinuity orientation on exposed rock mass. The equipment such as compass, clinometer, a
measuring tape and protractor are usually required to measure the orientation for different field conditions.

![Diagram](https://via.placeholder.com/150)

**Figure 2.15.** Detailed procedure for mapping and analysis of discontinuity orientations.

The photogrammetric method is usually employed for mapping of a quite large number of discontinuities, or in the case of inaccessible places and at areas affected by high magnetic fields, where compass readings may not be readable. In order to determine the orientation of a given discontinuity, coordinates of at least 4 points are required. The measuring procedures involving these items of equipments are not described here, but are detailed elsewhere (ISRM, 1978; Priest, 1993).
Oriented rock coring technique is also used to explore properties of rocks including discontinuity orientations at greater depth. However, during the drilling process, rock cores tend to rotate, therefore, special precautions should be taken to obtain the correct alignment of fractures. This problem may be overcome using non-rotating scribing core barrels (e.g. Craelius Core Barrel). In this technique, a vertical line is scratched on one side of the core closed to the drilling bit, which facilitates the alignment of cores in the core box. According to Christensen-Huegel method, orientation of core is measured by the means of a hardened steel groove scriber and a compass photo device (ISRM, 1978).

As recorded by Franklin and Dusseault (1989), an alternative technique developed by National Civil Engineering Laboratory in Portugal (Rocha and Barroso, 1971), is called the Integral Coring method, which is suitable even for sheared rocks. In this method, a small diameter is cored first and then a reinforcing rod is grouted to the broken core together.

It is important to note that the borehole method is relatively expensive, and the information produced from a few number of boreholes may be misleading. Therefore, one has to carefully plan and execute core drilling exercises, including the use of hole inspection techniques such as borehole television cameras, photographic cameras and borehole periscopes. At greater depths, the usage of camera is limited because of high water pressures. For borehole lengths of up to 150m, cameras can be used with high level of confidence (ISRM, 1978). Cameras are practical in large boreholes (>76mm dia.), whereas the periscope may by employed in small diameter boreholes, but subjected to limited depth of up to 30m.
In order to examine the geological features, the bored core barrels should be kept on a core box with markers indicating the depths of geological horizons, and the start and end of each layer. Although quantifiable parameters such as Total Core Recovery (R), Rock Quality Designation, RQD (Vutukuri and Katsuyama, 1994) and Frequency (F) can be directly estimated, a closer look at the borehole wall is required for a quantitative description of orientation, infill, spacing and aperture. The dip angle (α) of individual discontinuities of cores can be measured relative to the core axis. Therefore, the true dip angle is calculated as 90-α. Using graphical techniques (e.g. stereographic projections), the dip angle and dip direction can also be obtained when data from non-parallel boreholes are available.

Once the preliminary data processing is completed, the next step is to analyse the data using statistical methods. The measured orientation data can be represented using (a) block diagrams, (b) rosette diagram, and (c) stereographic projection methods. Block diagrams are usually employed for a small number of discontinuities, and also when the orientations of discontinuities do not vary too much. Large numbers of discontinuity data are usually represented by graphical methods, such as the rosette diagram and stereographic projection. The simple technique “rose diagram” is usually employed when the dip angles of most discontinuities are larger than 60° (Attewell and Farmer, 1976; Cawsey, 1977 and Priest, 1993). In this method, data are plotted in a simple circular diagram in which the outward radius represents the frequency of the discontinuity orientation, whereas the horizontal angle represents the dip angle of each discontinuity. Before plotting the rose diagram (Figure 2.16), it is recommended to plot the histogram for suitable class interval depending on dip angles. The histogram shows directly the dip angle, whereas the rose diagram does not. In order to represent the
Orientation data in 3-D, stereographic technique is widely used (Phillips, 1971; Duncan, 1981 and Priest, 1993). In this method, the data are plotted on a special graph sheet with an equal angle net. The construction of the stereographic projections is not described here, but further information can be found elsewhere (e.g. Priest 1993).

Figure 2.16. Typical rosette diagram to represent the orientation data.

2.5.3 Discontinuity spacing

Spacing between joints is the perpendicular distance between a pair of discontinuities, and for a discontinuity set, it is the mean perpendicular distance. Comprehensive
definitions to discontinuity spacing are given by Priest (1993) as: “Total spacing is defined as the spacing between a pair of immediately adjacent discontinuities measured along a line at given location and orientation. A ‘set spacing’ is defined as the spacing between a pair of immediately adjacent discontinuities from a particular discontinuity set, measured along a line at a given orientation and location”. A “normal set spacing” is defined as the set spacing when measured along a line that is parallel to the mean normal to the set. Discontinuity spacing largely influences the stability of the structure and the mass permeability and seepage characteristics of rock mass. The spacing measurements can be simply carried out using a measuring tape, compass and clinometer. For an exposed rock surface, the perpendicular distance \( x_i \) between adjacent discontinuities may be calculated as follows:

\[
x_i = d \sin \alpha
\]

(2.3)

where, \( d \) = measured distance along the tape or scanline, and \( \alpha \) = the angle between the scanline and the discontinuity.

For borehole data, \( d \) is the length measured along the core axis between adjacent discontinuities and \( \alpha \) is the angle between the core axis and the individual joints (Figure 2.17). The reliability of measurements of spacing in boreholes can be increased by employing a periscope or TV camera. The measured values depend on both scanline orientation and location, however, for parallel discontinuities, measured spacing varies only as a function of the scanline. For measured \( n \) number of discontinuities along the scanline length of \( X \), the mean discontinuity spacing \( \bar{x} \) is given by:
\[ x = \frac{1}{n} \sum_{i=1}^{n} x_i \]  

(2.4)

where, \( x_i \) = measured discontinuity spacing along the scanline.

\[ n = \left[ \frac{z}{\varepsilon} \right]^2 \]  

(2.5)

where, \( z \) = standard normal variable, \( \varepsilon \) = proportionate error and \( n \) = sample size.

Figure 2.17. Measurement of discontinuity spacing from an exposed rock surface and borehole data.

In order to get a precise value for mean spacing, one needs to measure a large number of measurements. The sample size can be calculated for a proportionate error at a certain level of confidence as follows (Priest & Hudson, 1981):
For example, for a probable proportionate error of 5% at the 95% confidence level, the sample should contain at least 1530 discontinuities. The discontinuity spacing may also be expressed as a frequency, which equals the inverse of the mean spacing.

2.5.4 Discontinuity shape

The shape of discontinuity is important in determining the discontinuity size, which has not been thoroughly studied, because it is not easy to measure the entire length of the fracture surface in the field. The geometry of a natural fracture is in the form of a complex polygon and is difficult to identify, as the joint is not completely exposed. A simplifying assumption used to idealise the discontinuity shapes is the consideration of circular, rectangular, square or elliptic shapes (Robertson, 1970; Baecher et al, 1977 and Rasmussen et al., 1985). In some rocks such as basalt, joints have clearly visible hexagonal shapes. In orthogonal joint models, joint shapes are assumed as rectangular if joints terminate at the intersection of the other joint plane. As shown in Figure 2.18, if joints form two/three orthogonal sets of parallel joint network, it can be modelled as an orthogonal model for flow deformation characteristics (Snow, 1969).

![Figure 2.18. Simplified discontinuity shapes in a rock mass.](image)
Discontinuities may also be assumed as circular or elliptical discs in shape (Baecher et al., 1977; Long and Witherspoon, 1985; and Warburton, 1980). In this approach, a given discontinuity set is represented by a set of parallel discs, whose centres are in space, and the disc radii may be assumed to take a lognormal distribution.

2.5.5 Infill material in discontinuities

Infill material within discontinuities can influence the strength, deformation and permeability characteristics of jointed rock mass. The infilled rock joint behaviour is governed by material properties, water content, permeability, and the thickness of infill. Figure 2.19 shows a horizontal joint filled with some clay and silt. The infill material can typically be clay, silt, fault gouge, calcite or chlorite. During the mapping process of joints, it is important to collect some infill material for testing at a later stage.

Figure 2.19. A horizontal joint filled with other materials, Southern free way, Bulli, Australia (taken by the writer).
2.6 PHYSICAL AND MECHANICAL PROPERTIES OF INTACT ROCKS

The rock mass behaviour is a combination of both rock joint properties and intact material properties. In this section, properties of rock material are briefly described, and the reader is advised to refer to other texts for detailed information including test procedure and equipment used (e.g. ISRM, 1978; ASTM, 1995). Figure 2.20 summarises some typical physical properties of intact rock material and rock mass which need to be considered for geotechnical projects.

![Diagram of physical properties of rock]

Figure 2.20. Important physical properties of rocks.

2.6.1 Water content of rocks

In order to carry out non-destructive and rapid method to determine the water content of rock at shallow depths, the nuclear method as described in ASTM (1995) can be carried out. Unlike in destructive tests, using the nuclear method, several tests can be
conducted at a single location to get a series of test data, facilitating statistical analysis. The water content determined by this method depends on the chemical composition of the rock, sample heterogeneity, material density and the surface texture of rocks. Basically, in this technique, a radioactive material, fast neutron (e.g. americium or radium) is transmitted through the sample, which is then detected again at the surface. Subsequently, the water content in rock mass per unit volume is determined by comparing the detection rate of thermalized or slow neutron response with established calibration data. The water content of cored rock specimens can easily be carried out in the lab. Water content is defined as the ratio of the weight of water in the specimen and the weight of solid component. The weight of water in the specimen is the difference between the initial weight and the final weight once it is dried at 105°C for duration of 24hrs.

Porosity, defined as ratio of pore volume to the total volume, governs the strength, deformability and permeability characteristics of rocks. For example, uniaxial compressive strength decreases with increase in porosity, whereas the permeability is expected to increase with the increase in porosity. Depending on the formation history (geological process), the porosity of rocks varies from one rock to another. It is usually found that the porosity of igneous rocks (e.g. granite, basalt) does not often exceed 5%. The most common method to find the total pore volume is to determine the weight difference between fully water saturated and dried specimens. In the presence of joints in a rock mass, the rock material tends to become partially saturated, whereas joints may be fully saturated if they provide a favourable flow path.
2.6.2 Elastic modulus

The uniaxial compression apparatus is often used to determine the elastic moduli of the rock material (e.g. Young's modulus, shear modulus and bulk modulus) in the laboratory. However, in order to simulate the correct confining stress in the field, it is better to use the triaxial test method, in which the specimen is subjected to a given confining pressure, while applying the axial stress. The uniaxial compression test gives reliable values for fairly isotropic rock specimens. For a given loading rate, the change of axial and diametric deformation of the specimens is measured using a dial gauge and a clip gauge, respectively. For given test data, the Young's modulus ($E$), shear modulus ($G$), bulk modulus ($K$) and Poisson's ratio ($\nu$) are calculated using the following equations:

The axial strain ($\varepsilon_a$):

$$\varepsilon_a = \frac{\Delta a}{L} \quad (2.6a)$$

The diametric strain ($\varepsilon_d$):

$$\varepsilon_d = \frac{\Delta d}{D} \quad (2.6b)$$

where, $\Delta a$ and $\Delta d$ are axial and diametric deformation, and $L$ and $D$ are length and the diameter of the specimen, respectively. The shear modulus ($G$) and bulk modulus ($K$) are calculated using the following elastic formulae:

$$G = \frac{E}{2(1 + \nu)} \quad (2.7a)$$

$$K = \frac{E}{3(1 - 2\nu)} \quad (2.7b)$$
Depending on the engineering requirement, the Youngs' modulus can be calculated either as a secant modulus or as a tangent modulus using the plot of stress vs strain (e.g. as discussed later in Figure 5.9). Some typical moduli for various rocks are given in Table 2.4. The presence of joints within the specimens usually leads to a decrease in the moduli of rocks.

Table 2.4. Typical values of elastic moduli for different rocks (ISRM, 1978).

<table>
<thead>
<tr>
<th>Rock Type</th>
<th>Young's modulus (GPa)</th>
<th>Poisson Ratio</th>
<th>Bulk modulus (GPa)</th>
<th>Shear Modulus (GPa)</th>
<th>Friction angle (Degree)</th>
<th>Cohesion (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Granite</td>
<td>73.8</td>
<td>.22</td>
<td>43.9</td>
<td>30.2</td>
<td>51</td>
<td>55.1</td>
</tr>
<tr>
<td>Quartzize</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Basalt</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Shale</td>
<td>11.1</td>
<td>.29</td>
<td>8.8</td>
<td>4.3</td>
<td>14.4</td>
<td>38.4</td>
</tr>
<tr>
<td>Marble</td>
<td>55.8</td>
<td>.25</td>
<td>37.2</td>
<td>22.3</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Sandstone</td>
<td>19.3</td>
<td>.38</td>
<td>26.8</td>
<td>7.0</td>
<td>27.8</td>
<td>27.2</td>
</tr>
<tr>
<td>Siltstone</td>
<td>26.3</td>
<td>.22</td>
<td>15.6</td>
<td>10.8</td>
<td>32.1</td>
<td>34.7</td>
</tr>
<tr>
<td>Limestone</td>
<td>28.5</td>
<td>.29</td>
<td>22.6</td>
<td>11.1</td>
<td>42</td>
<td>6.72</td>
</tr>
</tbody>
</table>

Using the rigid or flexible loading method, the in-situ modulus of a rock mass can also be determined in underground cavities. The load is usually applied to the plate and the deformation is observed using LVDT'S. As the deformation is time dependant, it is important to record the final deformation for the particular load. The modulus \( E \) is calculated for a rigid plate as follows (ASTM, 1995):

\[
E = \frac{(1-\nu^2)P}{2W_oR} \tag{2.8}
\]

where, \( P = \) total load on the plate, \( W_o = \) average deflection of the plate, \( R = \) radius of the rigid plate.

Alternatively, a radial jacking test can be used to determine the deformation modulus and the anisotropic behaviour of rocks (ASTM, 1995; Vutukuri and Katsuyama, 1994).
This technique gives reliable test data than the other two techniques discussed above. This is because, in the field, a larger volume of rock is affected, so that the influence of discontinuities is taken into account. In-situ uniaxial compressive tests are also used to measure the deformability and the strength of rock mass with closely spaced joints, approximately 30mm to 500mm.

2.6.3 Tensile strength of intact rocks

The direct estimation of tensile strength of rocks using the uniaxial tensile test is not an easy task because of the difficulty involved in sample preparation. The Brazilian test seems to be desirable because of its simplicity and low cost. The description of Brazilian test procedures can be found in ISRM, (1978) and ASTM (1995). The tensile strength \( \sigma_t \) is calculated using the following formula.

\[
\sigma_t = \frac{2P}{\pi DL}
\]

(2.9)

where, \( P \) = failure load, \( D \) = diameter and \( L \) = thickness of the specimen. The tensile strength of rocks decreases with the increase in water content, the porosity of rocks and the interconnectivity of joints. For a single fractured specimen, the influence of fracture on tensile strength is minimum when the fracture orientation is parallel to the loading direction. Typical values of tensile strengths of rocks are given in Table 2.5.

2.6.4 Rock mass permeability

The permeability of rock mass is a combined function of both joint conductivity and rock matrix permeability. Permeability of a single discontinuity is given by:
\[ k = \frac{e^2}{12} \quad (2.10) \]

where, \( e \) = hydraulic aperture. The permeability deformation characteristics of rocks and permeameters will be described in Chapters 4 and 5.

Table 2.5. Tensile strength of rocks.

<table>
<thead>
<tr>
<th>Rock type</th>
<th>Tensile strength, MPa</th>
<th>Compressive strengths, MPa</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pikes Peak Granite</td>
<td>11.89</td>
<td>226</td>
<td>Miller, 1965</td>
</tr>
<tr>
<td>Nevada Test Site Granite</td>
<td>11.75</td>
<td>141</td>
<td>Stowe, 1969</td>
</tr>
<tr>
<td>Nevada Test Site Basalt</td>
<td>13.0</td>
<td>148</td>
<td>Stowe, 1969</td>
</tr>
<tr>
<td>John Day basalt</td>
<td>14.5</td>
<td>355</td>
<td>Miller, 1965</td>
</tr>
<tr>
<td>Dworshak Dam Gneiss</td>
<td>6.9</td>
<td>162</td>
<td>Miller, 1965</td>
</tr>
<tr>
<td>Micaceous shale</td>
<td>2.1</td>
<td>75.2</td>
<td>Blair, 1956</td>
</tr>
<tr>
<td>Flaming Gorge shale</td>
<td>0.2</td>
<td>35.2</td>
<td>Brandon, 1974</td>
</tr>
<tr>
<td>Tavernalle limestone</td>
<td>3.9</td>
<td>97.9</td>
<td>Miller, 1965</td>
</tr>
<tr>
<td>Bedford limestone</td>
<td>1.6</td>
<td>51</td>
<td>Miller, 1965</td>
</tr>
</tbody>
</table>

The average permeability of a rock mass depends on the porosity, number of fractures present and their interconnectivity, fracture aperture, hydraulic head and properties of fluid itself. For a relatively porous rock (e.g. coarse sandstone and limestone), it is found that significant flow takes place through the rock material (matrix), whereas, negligible flow is expected to take place through low porosity rocks, such as granite and slate. The permeability of rock mass is the summation of the permeability of rock matrix (material) and the discontinuities within the given rock block. Under steady state flow rate, for a given cylindrical rock specimen, the coefficient of matrix/intact rock permeability \( k_m \) can be written using Darcy's law as given below:

\[ k_m = \frac{4q\mu}{\pi D^2 (dp/\text{dx})} \quad (2.11) \]
where, \( q \) is the fluid flow rate through the specimen, \( \frac{dp}{dx} \) is the pressure gradient along the length (\( dx \)) of the specimen, \( \mu \) is the dynamic viscosity of the fluid and \( D \) is the diameter of the specimen. Apart from the hydraulic gradient and surrounding stress applied on the specimen, the matrix permeability depends on the properties of the matrix, characterized by the pore size (voids), shapes and the interconnectivity of voids. If the fluid traveling through the porous rock is gas, then the component of the matrix coefficient of air permeability is estimated according to the following equation:

\[
\begin{align*}
   k_m &= \frac{8q_\text{g}\mu L}{(p_i^2 - p_e^2)\pi D^2} \\
(2.12)
\end{align*}
\]

where, \( q = \) gas flow rate, \( \mu = \) dynamic viscosity of gas, \( L = \) length of the specimen, \( D = \) diameter of the specimen, \( p_i = \) inlet pressure of gas, and \( p_e = \) exit pressure of gas.

The matrix permeability coefficient by the transient method in which decay of pressure is observed and is given by Kranz et al., 1979:

\[
\begin{align*}
   k_m &= \frac{\alpha \beta \mu L V_t V_2}{A(V_t + V_2)} \\
(2.13)
\end{align*}
\]

where, \( \beta = \) isothermal compressibility of fluid, \( A = \) cross sectional area, \( V_t \) and \( V_2 = \) volume of the pore fluid at the top and the bottom of the sample, respectively, \( L = \) length of the specimen, and \( \alpha \) is an empirical constant for given initial pressure, and for the time period, \( t \).
CHAPTER 3

REVIEW OF SINGLE AND TWO-PHASE FLOWS THROUGH A ROCK MASS

3.1 INTRODUCTION

The following chapter reviews the literature relating to fluid flow characteristics in a jointed rock mass subjected to deformation. Four major parts associated with existing fluid flow simulation models i.e. flow through a single joint, laboratory testing equipment and limited studies on two-phase flow through rocks are discussed here, including numerical, experimental and analytical approaches. Depending on the available rock mass properties, the geology of the area concerned and the required accuracy, fluid flow simulation theories (small and large scale) and the expected accuracy have been reviewed. As flow through a single fracture is the basic building element of any fluid flow model in an interconnected network of fractures, the simple plane theory is reviewed in detail, in relation to stresses, variable aperture and joint roughness.

3.2 FLOW THROUGH A JOINTED ROCK MASS

As discussed in Chapter 1, fluid flow analysis plays a major role particularly in underground constructions because of possible catastrophic mine failures, loss of human lives and damage to mine equipment. When fractured rocks carry both water and gas together, mine inundation and gas outburst can develop simultaneously, resulting in extensive damage to the mine environment (Lama and Bodziony, 1996). Gas outbursts often take place in coal mining. Various definitions for gas outburst have been put
forward by various researchers based on observations and mechanisms of formation
(Hargraves, 1958; Thomas, 1963; Ryncarz, 1992). Figure 3.1 shows the result of
typical gas outbursts occurred in New South Wales, Australia.

(a) Consequence of an outburst which occurred at Tahmoor Colliery

(b) Damage due to an outburst at Tahmoor Colliery
All adverse situations and the risk of inundation can only be mitigated by correct evaluation of the preventable measures during the mine planning and design stages. Therefore, planning decisions concerning groundwater control measures such as grouting and dewatering should be implemented in advance, so that the whole operational system would contribute towards greater economies within a safer work environment. For rational analysis and design, it is essential to understand the hydraulic and mechanical behavior of a rock mass, the response of the natural joint system, and how the groundwater table responds to changes induced by proposed excavation sequence (Indraratna and Wong, 1995). In a comprehensive study of flow analysis, an array of geo-hydrological factors have to be considered, as illustrated in Figure 3.2. According to Figure 3.2, apart from geological features of rock formation, the type of flow (i.e. single or multiphase flow) plays an important role in the correct evaluation of
the flow parameters. It is an established fact that rock joints are usually unsaturated, although they often carry water and air together. Nevertheless, based on numerical, analytical, experimental and insitu tests, single-phase flow models are well established and large numbers of commercially available computer codes based on single-phase flow have been used in practice (ITASCA, 1996; Lee & Farmer, 1993; Oda, 1986; Louis, 1976). As discussed in Chapter 1, multiphase flow may occur in various engineering and non-engineering applications including power technology, human body, food industry, geotechnical, nuclear and petroleum engineering.

![Flow chart for fluid flow analysis through a rock mass.](image)

Figure 3.2. Flow chart for fluid flow analysis through a rock mass.
3.3 FLUID FLOW MODELS

It is of great interest to understand the correct flow mechanisms applied to porous or fractured media variety of disciplines including soil and rock engineering, petroleum engineering and chemical engineering. The porous medium is characterized by the manner in which the voids are distributed, their interconnectivity, size and shape. Usually, depending on the lattice structure, all natural and artificial materials can be divided into four main categories as follows: (a) porous, (b) non-porous, (c) fractured, and (d) combination of fractured and porous media (Figure 3.3). In the past, various theories have been developed for each application depending on how flow takes place through a porous or fractured media or a combination of both (Lomize, 1951; Bear, 1979; Long, et al., 1982; Shapiro & Anderson, 1983). For example, in soil mechanics, flow is usually modeled using a porous medium approach. Lomize (1951), Snow (1968a & b) and Louis, (1976) used modified theories developed for fluid flow through pipes or channels to simulate flow through fractured media, such as in jointed rock. The selection criterion for the type of flow analysis depends on the availability and extent of field data, computer resources, and the degree of the accuracy required for the particular application. The available rock mass properties and the geology of the area influence the initial adoption of the methodology. Techniques of flow analysis have been grouped as either one or more of the following (Snow, 1969; Bear, 1979; Long, 1983; Sharpio and Anderson, 1983; Long and Witherspoon, 1985):

(a) Continuum approach—porous media

(b) Discrete fracture flow theory—fractured media

(c) Dual porosity method—coupled porous and fractured media.
Fluid flow in porous media can either be single-phase or multiphase. Modeling of multiphase flow in deformable or rigid media requires quantification of the interaction between each fluid and the corresponding change in fluid properties (Celia & Binning, 1992; Schrefler & Xiaoyong, 1993). For single-phase flow, the most relevant parameter is the intrinsic permeability of the medium, whereas, both relative permeability and intrinsic permeability must be taken into account when modeling multiphase flow.

![Fluid flow mechanism in different materials.](image)

**Figure 3.3.** Fluid flow mechanism in different materials.

### 3.3.1 Continuum flow theory

From the point of view of civil engineers, porosity has been recognized as an influencing parameter not only for flow phenomena, but also for deformability and stability of the material under given loading conditions. In various applications, the flow mechanisms are different and complex, therefore, one has to carefully examine the pore structure of the medium, and how the pore network affects the flow distribution. The size and the shape of solid particles and their arrangement will govern the pore
volume. The pore structure and pore volume of the medium may be changed by external loading (e.g. degree of consolidation), chemical reactions, fluid pressures and the solubility of solid in fluid (Figure 3.4).

![Diagram of porous medium](image)

Figure 3.4. Change of pore structure of porous medium (Greenkorn, 1983).

A porous medium is broadly classified as either consolidated or unconsolidated. In the analysis of pore structure, there are two types of pores: (a) interconnected and (b) isolated. Although fluid flow takes place through interconnected pores, it is still important to consider isolated pores when they are subjected to load. This is because, with increasing deformation, isolated pores can subsequently become part of the interconnected pore network. Fluid flow takes place, only if the following conditions
are satisfied: (a) existence of an interconnected pore structure, and (b) fluid particle must pass through the smallest pore in the interconnected pore arrangement.

Pore volume is usually expressed in terms of porosity, which is defined as the ratio of interconnected pore volume divided by the total volume of the sample. The arrangement of particles and their shapes are important in developing any mathematical model for porous media. Spanne et al., 1994; Fredrich et al., 1995 investigated three dimensional geometry of porous media by various techniques including laser scanning confocal microscopy and magnetic resonance imaging microscopy. Doughtly and Toutsa (1997) employed magnetic resonance imaging technique which is based on the proton content of fluids saturating the pores within the rock specimen and therefore directly images the accessible pore space within the rock matrix. Fredrich et al. (1995) presented that the laser scanning confocal microscopy precisely imaged thin optical planes within thicker porous rock samples with high resolution. The accuracy of the method depends on general transparency of the sample. Therefore, the method is inappropriate for imaging larger specimens. The principal technique of this method is that the detection and illumination are focused on a single location on the porous sample. Doughtly and Toutsa (1997) observed the pore structure of sandstone as presented in Figure 3.5 in which black and white regions respectively represent the rock grains and pores.

The natural porous medium (e.g. soil) is anisotropic, because the permeability of the medium is not the same in all directions. Ferrandon (1948), Collins (1961) extended the Darcy’s law to three dimensions to model the fluid flow in an anisotropic medium as given below:
\[ q_i = A \frac{\rho g}{\mu} \left( k_{i1} \frac{\partial \phi}{\partial x_1} + k_{i2} \frac{\partial \phi}{\partial x_2} + k_{i3} \frac{\partial \phi}{\partial x_3} \right) \]  

where, \( i = 1, 2, \) and \( 3 \)

\( k \) = permeability in three orthogonal directions,

\( q \) = flow rate,

\( A \) = area,

\( \rho \) = density of fluid,

\( \mu \) = dynamic viscosity of fluid, and

\( \phi \) = hydraulic head

Figure 3.5. Image of pore structure of sandstones (Doughtly and Toutsa, 1997).

In the three-axis system, flow has 9 vectorial components for permeability, and usually this permeability tensor is symmetrical. Various researchers including Kozeny, (1927) and Scheidegger (1960) pointed out that for an orthogonal axis system, flow rate is a function of only three principle permeabilities, as given below:

\[ q_i = A \frac{\rho g}{\mu} k_i \frac{\partial \phi}{\partial x_i}, \quad i=1,2,or \ 3 \]  

(3.2)
The modified Darcy's expression proposed by Kozeny, (1927) and Scheidegger (1960) in three dimensions does not hold for every porous media, because the permeability tensor may not always be symmetric (Collins, 1961).

Bear (1979) modeled the single-phase fluid movement in a porous matrix under steady state flow using the following diffusion equation given in Equation 3.3.

\[ \nabla . (K \nabla h) + q(x, y) = 0 \]  
(3.3)

where, \( K \) = hydraulic conductivity tensor,

\[ \nabla = \text{operator} \left( \frac{\partial}{\partial x} i + \frac{\partial}{\partial y} j \right), \text{ } i \text{ and } j \text{ are unit vectors in } X \text{ and } Y \text{ directions,} \]

\( q = \text{flow rate of sources, and} \)

\( x, y = \text{cartesian co-ordinates} \)

Abu-EI-Sha'r & Abriola (1997), Amali & Rolston (1993), and Baehr & Bruell (1990) proposed different theoretical expressions for gas transportation in porous media including soil based on Fick's law, dusty gas model and Stefan-Maxwell equations. Fick's law is more commonly used to simulate gas diffusion in porous media (Abriola et al., 1992; Amali and Rolston, 1993). Evans et al. (1961) and Mason & Malinauskas (1983) proposed the dusty gas model to integrate the different flow mechanisms. However, this model is seldom used in soil mechanics. Abu-EI-Sha'r & Abriola (1997) proposed total flux \( (N^T) \) of gas phase as:

\[ N^T = \frac{D}{f} C \nabla x + x - \frac{p}{RT} \frac{k}{\mu} \nabla p \]  
(3.4)

where, \( f \) = obstruction factor to gas due soil particles,
\[ D = \text{free binary diffusivity of gasses}, \]
\[ x = \text{mole fraction of gas phase}, \]
\[ p = \text{pressure}, \]
\[ k = \text{intrinsic permeability}, \]
\[ \mu = \text{dynamic viscosity}, \text{ and } R = \text{gas constant} \]

In multiphase flow, Celia and Binning (1992) proposed the degree of saturation of each phase and change of fluid properties with time in the following fluid transportation equation as given below:

\[ \nabla \cdot (\rho_\beta k_i k_{r_\beta} (\nabla p_\beta - \rho_\beta g) \frac{1}{\mu_\beta}) + \frac{\partial (\phi \rho_\beta S_\beta)}{\partial t} \pm F_\beta = 0 \quad (3.5) \]

where, \( \beta = \text{fluid phase} \)
\[ k_i = \text{single phase permeability}, \]
\[ p = \text{pressure}, \]
\[ k_{r_\beta} = \text{relative permeability}, \]
\[ \mu_\beta = \text{dynamic viscosity}, \]
\[ \phi = \text{porosity}, \]
\[ F = \text{source or sink}, \]
\[ S = \text{degree of saturation}, \text{ and } g = \text{acceleration due to gravity}. \]

### 3.3.2 Discrete fracture flow theory

Fluid flow and solute transport through low porosity rocks has gained increasing interest in storage of nuclear waste and hazardous liquid toxic waste in deep underground cavities. In addition, in the recovery of petroleum products, particularly at
secondary recovery process, researchers have been continuously studying the flow process through tight rocks. Flow through tight rock mainly occurs through interconnected fracture networks. As pointed out by Sharpio & Anderson (1983), and Elsworth (1986) and Zhang et al. (1996), it is not possible to simulate such jointed rock mass using the continuum technique, which works well for homogeneous and continuous rock media. Therefore, when the fluid flow is dominated through a defined interconnected fracture network, the most realistic approach is to employ the discrete fracture flow theory. This approach provides several advantages over the continuum approach such as the consideration of:

(a) Properties of the fracture network

(b) Effect of fracture network on the fluid flow, and

(c) Detailed distribution of stress and deformations around fractures.

The distinct element method (ITASCA, 1996; Lemos et al., 1985), joint element method (Goodman, 1976) and block theory (Goodman and Shi, 1985) are based on the discrete fracture theory. Snow (1969), Long and Witherspoon (1985), and Anderson & Dverstorp, (1987) employed the discrete fracture theory for flow analysis in both laboratory work and in field studies. This approach requires detailed information of the distribution of fractures including their geometry, orientation, length, spacing, as well as individual relationships for describing flow in fractures when subjected to various stress fields (Lee and Farmer, 1993). Moreover, although probabilistic methods can be employed to generate fracture geometry and distribution, the assumed stochastic model may still not truly represent the actual case. In order to understand the mechanism of fluid flow through rock fractures, small scale (laboratory) and field experiments have been carried out in the past through single fractures and network of fractures. Earlier
studies of fluid flow through single fractures have been reported by Lomize (1951), Snow (1969), Louis (1968), and Wilson (1970). Since small scale experimental studies did not always represent the true picture, Abelin et al. (1985) and Neternieks (1985) used field experiments of solute migration in single fractures for obtaining more realistic data.

Discrete fracture flow models have been used as either in two dimensions (ITASCA, 1996 and Long and Witherspoon, 1985) or in three dimensions (ITASCA, 1997; Rasmussen, 1988; Dverstorp and Anderson, 1989). Two dimensional models are restricted in several ways because the connectivity of fractures cannot realistically represent the actual spatial joint pattern. Fractures which do not intersect with other fractures may intersect with each other in 3-D. As a result, flow and deformation characteristics will be significantly changed. As pointed out by Herbert (1996), unfortunately, three-dimensional joint flow models are often disadvantaged by the lengthy solution procedures, hence, large computing time and memory requirements.

The connectivity is governed by the fracture density, and it is often quantified using percolation threshold value. According to the definition given by Lee and Farmer (1993), percolation threshold is a factor at which flow through fractures just begin to occur if the fracture size or density is large enough to allow fracture flow. The percolation threshold depends on the accuracy of statistical properties of rock, and also on the type of network (i.e. 2-D or 3-D). In order to generate a fracture network based on field data, the following parameters were used and quantified using statistical methods (Herbert, 1996; Muhammad, 1995; Priest & Hudson, 1981):
(a) Fracture shape (e.g. rectangular, circular or elliptical),
(b) Fracture size based on trace length or radius,
(c) Fracture orientation,
(d) Fracture density, and
(e) Fracture aperture.

When it is not feasible to map all the fractures and obtain all the relevant geometrical parameters, a statistical approach is beneficial, where joints are seen as realizations of stochastic models. Sen & Eissa (1992), Priest (1993) and Wei et al., (1995) modeled the orientation of joints using a Fisher distribution or normal distribution depending on the spread of joint inclination. Similarly, for other parameters, different distributions are employed for a given rock mass, and possible distribution functions are used for various applications as listed in Table 3.1.

The fracture permeability (i.e. discrete approach) is estimated using the following Equation 3.6, (Snow, 1968b).

\[
k_d = \frac{1}{2.283} \left( \frac{q \mu}{d(dp/dx)} \right)^{2/3}
\]

where, \( q \) = discharge flow rate, \( \mu \) = dynamic viscosity, \( d \) = fracture width and \( p \) = permeating fluid pressure difference along a joint length of \( dx \).

As described by Muhammad (1995) in the discrete approach, the shape of the fracture is also important, especially when 3-D analysis is carried out. Fracture shape may be assumed as planar traces, rectangular, square, circular, elliptical or polygon (Figure 3.6).
Table 3.1. Distribution function used to describe joint density.

<table>
<thead>
<tr>
<th>REFERENCES</th>
<th>VARIOUS DISTRIBUTION FUNCTIONS</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Joint orientation</strong></td>
<td></td>
</tr>
<tr>
<td>Anderson &amp; Dverstorp (1987)</td>
<td>Fisher distribution, ( f(\theta, \phi) = \frac{ke^{k \cos \theta \sin \theta}}{4\pi k (\sinh)} )</td>
</tr>
<tr>
<td>Samaniego (1984), Wei et al. (1995)</td>
<td>Normal distribution, ( f(\theta, \phi) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{(\theta - \mu)^2}{2\sigma^2}} )</td>
</tr>
<tr>
<td>Priest (1993)</td>
<td>Negative exponential distribution, ( f(\theta, \phi) = \frac{1}{\mu} e^{-\frac{\theta}{\mu}} )</td>
</tr>
<tr>
<td><strong>Joint length/size (l)</strong></td>
<td></td>
</tr>
<tr>
<td>Samaniego (1984) and Wei et al. (1995)</td>
<td>Negative exponential distribution,</td>
</tr>
<tr>
<td>Priest and Hudson (1981)</td>
<td>Exponential</td>
</tr>
<tr>
<td>Bridges (1976), McMahon (1971)</td>
<td>Lognormal</td>
</tr>
<tr>
<td>Dershowitz (1984)</td>
<td>Gamma</td>
</tr>
<tr>
<td><strong>Fracture transmissivity</strong></td>
<td></td>
</tr>
<tr>
<td>Snow (1970)</td>
<td>Lognormal distribution,</td>
</tr>
<tr>
<td><strong>Spacing (s)</strong></td>
<td></td>
</tr>
<tr>
<td>Priest and Hudson (1981)</td>
<td>Normal</td>
</tr>
<tr>
<td>Sen and Eissa (1992)</td>
<td>Exponential and Lognormal</td>
</tr>
<tr>
<td>Bridges (1976)</td>
<td>Lognormal, ( f(s) = \frac{1}{\sqrt{2\pi} s \sigma} \exp \left[ -0.5 \frac{1}{s} \ln \frac{s}{m} \right] )</td>
</tr>
<tr>
<td>Wallis and King (1980)</td>
<td>Exponential</td>
</tr>
</tbody>
</table>

where, \( \theta \) = dip angle.

\( \phi \) = dip direction.

\( k \) = dispersion parameter about the mean direction, determined by the maximum
likelihood estimator (Mardia, 1972),

\[ \mu = \text{mean}, \]
\[ \sigma = \text{standard deviation}, \]
\[ s = \text{spacing}, \]
\[ m = \text{median}. \]

A numbers of authors have treated this problem entirely in two-dimensions by assuming discontinuities as linear traces in a planar rock surface (Priest & Hudson, 1981; Pahl, 1981). Long et al. (1985), Shapiro & Anderson (1983) and Anderson & Dvrstorp (1987) represented fractures as discs of finite radius in their 3-dimensional models.

In the discrete fracture approach, the fluid flow is assumed to take place via the passage of joint network. While this assumption is generally valid for hard crystalline rocks, continuum approach may still be exercised in sedimentary rocks such as sandstone, shales and other soft rocks.

### 3.3.3 Dual porosity theory

When fluid flow takes place through both the rock matrix and discontinuities, the flow behaviour is analyzed using the dual porosity model. Although this method is not as common as the previous methods, several researchers including Fidelibus et al. (1997), Rasmussen (1988) and Shapiro & Anderson (1983) have employed dual porosity method to represent the flow conditions, through the rock matrix and joints. For instance, the dual porosity technique may be applied to simulate fluid flow in fractured sandstone.
(a) Disk-shape fractures (Billaux et al., 1989)  

(b) 3-D fracture network (Anderson & Dverstorp, 1987)

(c) Rectangular-shape fractures (after Muhammad, 1995).

(d) Circular-shape fracture network (modified after Lee & Farmer, 1993)

Figure 3.6. Three-dimensional fracture network models.
In this method, flow at each intersection point is due to the summation of flow carried through the joints to the intersection plus the fluid transported through the porous rock matrix. Depending on the fluid pressure within the rock matrix and the fractures, flow may be conducted from the rock matrix to the joints or vice versa, and the total flow is mathematically expressed by Equation 3.3 (see Figure 3.7).

\[ Q_i = Q_1 + Q_2 + Q_3 + Q_4 \pm Q_m \]  \hspace{1cm} (3.7)

where, \( Q_i \) = flow at the intersection point \( i \),
\( Q_1 \) to \( Q_4 \) = flow at each joint, and
\( Q_m \) = flow through the rock matrix

Fidelibus et al. (1997) elaborated a numerical approach to simulate unsteady flow through fractured and porous rock matrix. The porous rock blocks were modeled implicitly by using boundary element and the rock fractures were simulated using the
finite element approach. Fluid flow rates through fractures and porous rocks were defined using the following expressions.

For fracture:

\[
\int_0^y \text{div} \, q \, dy = \frac{d}{dx} (K_f \frac{dh}{dx}) + q_b + q_t, \tag{3.8}
\]

where, \( q \) = flow rate along the joint,

\( q_b \) = flow rate entering from the bottom of the joint wall,

\( q_t \) = flow rate entering from the top of the joint wall,

\( x \) = distance along the joint,

\( y \) = distance perpendicular to the joint (joint aperture),

\( K_f \) = fracture transmissivity, and

\( h \) = hydraulic head.

Fluid flow in porous rock was modeled using the equation given below:

\[
K_s \frac{\partial^2 h}{\partial x^2} + K_p \frac{\partial h^2}{\partial x^2} + Q_s(w) = S \frac{\partial h}{\partial t}, \tag{3.9}
\]

where, \( S \) = specific storage

\( Q_s(w) \) = constant point source of fluid influx.

3.3.4 Comparison of continuum flow theory with discrete fracture flow theory

In field situations, a fractured medium can be modeled using a continuum approach, as long as the number of fractures are large and random (Brady and Brown, 1994). In this simulation, it has to define a suitable representative block with average properties to
represent the fractured rock media. While this process is never an easy task, the degree of confidence can be enhanced using experimental results based on two approaches. Neuzil and Tracy (1981), Long and Witherspoon (1985) and Oda (1986) applied the continuum approach to represent the discontinuous rock mass. In this approach, a suitable Representative Elementary Volume (REV) is required to represent sufficient number of fractures.

Snow (1969), Singh (1973), Long et al. (1982), Oda (1986), and Stietel et al (1996) studied the continuum representation of fractured rock media for fluid flow calculations. As discussed by Long et al. (1982), fractured media behave more like porous media, if the rock mass has the following characteristics: (a) large fracture density, (b) approximately constant fracture aperture, and (c) variable orientation of fractures. According to Neuzil and Tracy (1981) and Oda (1986), the equivalent porous medium concept can be applied to discontinuous rocks, if the number of discontinuities is sufficient to allow the determination of a statistical average value of flow paths. The main difficulty faced in such a simulation is the estimation of equivalent continuum properties to represent fractured rock. In other words, the exact mechanical and hydraulic behavior of fractured rock should be correctly represented in the continuum approach.

The work done in relation to the equivalent permeability tensor can be classified into two groups: assuming, (a) infinite length of joints and (b) finite joint length. Most reported work considers infinite length of joints, which is not always true in the field. The effects of joint length, orientation, fracture density, joint aperture distribution and their geometrical properties must be included when developing a comprehensive model
of the equivalent permeability tensor. Snow (1969) developed an equivalent permeability tensor for a single fracture and then used the method of superimposition to obtain the permeability tensor for a network of joints. The equivalent porosity \((\Phi_{eq})\) for a single fracture with an aperture \(e\), length \(l\) and area \(A\), was expressed by:

\[
\Phi_{eq} = \frac{el}{A}
\] (3.10)

where, \(A\) = area of the specimen.

On a macroscopic scale, the equivalent porosity of a large rock mass is the summation of individual porosity of each single fracture. Isolated fractures may be ignored as they do not contribute to flow. However, during the deformation stage, these isolated fractures may also influence the flow deformation characteristics of the joint network.

The equivalent permeability tensor \((k_{ij})\) of a joint set with spacing, \(s\) and mean orientation, \(\theta\), could be expressed by the equation given below (Snow, 1969; Stietel et al., 1996):

\[
k_{ij} = \left(\frac{\rho g}{12 \mu}\right) \sum \left(\frac{e^3}{s}\right) \delta_{ij} - \cos \theta, \cos \theta_j
\] (3.11)

where, \(\delta\) = Kroneker delta, \(i, j = 1, 2, 3\),

\(\theta\) = orientation, and

\(\rho\) = density of fluid.

From Equations 3.11, the equivalent permeability tensor in 2-dimensions is given by:

\[
k_{eq} = \begin{bmatrix}
k_{11} & k_{12} \\
k_{21} & k_{22}
\end{bmatrix}
\] (3.12)
While the joint aperture, \( e \) plays a major role in flow through fractured rocks, the joint aperture distribution is not included in Equation 3.11. The magnitude of joint deformation depends on the nature of joint asperities, applied stress level and their directions relative to the joint surface.

Apart from the equivalent hydraulic properties, it is also important to express the equivalent material properties of jointed rock mass to represent the porous media. Several researchers including Biot (1955), Oda (1986) and Pariseau (1993) discussed equivalent material properties such as shear and normal stiffness, Poisson's ratio and Young's moduli for different joint patterns. Oda (1986), assuming that joints are planar (parallel plate walls and connected by springs), proposed an equation for the equivalent shear stiffness over the domain \( \Omega \) as a function of friction angle, joint compressive strength \( (JCS) \), applied stress, joint size and the representative elementary volume of rock. The average equivalent shear stiffness was expressed by:

\[
G_{eq} = \frac{100}{\Omega} \left[ \frac{100}{r} \sigma_n n_j \tan(JRC \log_{10} (JCS / \sigma_n) + \phi) \right] E(\hat{n}) d\Omega 
\]

(3.13)

where, \( JRC \) = Joint roughness coefficient,

\( \sigma_n \) = normal stress,

\( r \) = size of the joint,

\( \phi \) = friction angle,

\( E(\hat{n}) \) = probability density function,

\( \hat{n} \) = unit normal vector, and

\( n_i \) = the component of vector \( \hat{n} \) with respect the axis \( x_i \), where \( i = 1, 2, 3 \).
Similarly, the average equivalent normal stiffness ($K_{eq}$) over the domain ($\Omega$) was given by Oda (1986) as:

$$K_{eq} = \int_{\Omega} \left( h + \frac{r}{e_0} \sigma_{ij} n_i n_j / r \right) E(\hat{n}) d\Omega$$

where, $h = \text{constant}$, depends on number of cycling loadings,

$e_0 = \text{initial aperture}$.

A comparison between the principal components of actual and equivalent permeability values for different joint patterns (Figure 3.8) is shown in Figure 3.9 (Stietel et al., 1996).

Figure 3.8. Fracture network used by Stietel et al., 1996.
3.4 FLUID FLOW THROUGH A SINGLE JOINT

According to ISRM (1978), a joint is defined as a break of geological origin in the continuity of a body of rock along which there has been no visible displacement. A group of parallel joints is called a set, and different sets intersect to form a joint system, which may include joints that are open, filled or healed. Joints frequently form parallel planes e.g., bedding planes, foliation and cleavage. The mechanical and geometrical characterisation of a single rock joint provides the basis to understand the fluid flow deformation behaviour in a fractured rock mass. It is difficult to give a comprehensive description of flow behaviour even in a single joint, because of the number of variables involved in a 3-dimensional situation. Therefore, much analysis is usually based on a
plane strain (2-dimensional) approach. Apart from external boundary conditions, the variable void geometry in shape and size determines the fluid flow through the fractures.

The main factors controlling fluid flow through a single rock joint is the magnitude of the joint aperture which is a function of external stress, fluid pressure and geometrical properties of the joint. In many early studies, flow through a single joint was simulated as flow through a channel or pipe, in which no deformation was considered due to external stress (Lomize, 1951). However, in reality, the deformation of fractures associated with external stress changes the flow rate of fluid, and the resulting pore pressures affect the subsequent deformation of the discontinuities. The historical work on flow characteristics through a single joint was conducted experimentally by Lomize (1951) and Louis (1968). In a single discontinuity, fluid flow is a function of surface roughness, variable aperture, the magnitude of external loads and their direction relative to the orientation of joint, as well as the infill materials (Brown, 1987; Neuzil & Tracy, 1981, and Tsang, 1984). Usually, the joint surface roughness plays a major role when the joint apertures are small or if the joints are sealed.

3.4.1 Effect of stress on permeability

Discontinuities in a rock mass are usually subjected to surrounding insitu stress, seismic loading and fluid pressure. In general, the resulting effective stress acting on a discontinuity consists of normal stress, shear stress and fluid pressure components. Depending on the magnitude and the direction of stress, a variation of mechanical behaviour of joint can be expected. As shown in Figure 3.10, the applied stress will
influence the joint to dilate or close, to create new contacts points, and even to crush the rock material depending on the surface geometry of the joint, magnitude of normal and shear strength and deformability of rock material.

Figure 3.10. Effect of normal and shear stress on a typical joint.

For example, normal stress perpendicular to a joint closes the fracture, whereas the shear stress causes mismatch between the joint surfaces, resulting in the change of void space within the joint. The deformability of discontinuities is of paramount importance in the design of large structures, such as tunnels, underground nuclear storage plants, dams and bridges. The deformability of a discontinuity is usually expressed in terms of stiffness, which is defined as the ratio of stress to displacement. As described by Brady
and Brown (1994), the stiffness has two components, based on the normal stress \( (\sigma_n) \) and shear stress \( (\tau_s) \), and their directional displacements, \( \delta_n \) and \( \delta_s \). The normal stiffness is defined as:

\[
K_n = \frac{\sigma_n}{\delta_n}
\]  
(3.15)

The shear stiffness is defined as:

\[
K_s = \frac{\tau_s}{\delta_s}
\]  
(3.16)

The study of the deformation of discontinuities and intact material is a vast subject area, which has been discussed extensively by many researchers over the past five decades. The stress dependency of fluid flow through fractured rocks has been investigated in the past (Gale, 1975; Iwai, 1976; Raven and Gale, 1985). The aim of this section is confined to evaluating the role of deformability on the permeability characteristics of a single joint.

Based on the initial hydraulic aperture and the closure of joint, Detoumay (1980) suggested the following relationship to determine the fracture permeability:

\[
k = \frac{e_0^2 \left(1 - \frac{v}{v_0}\right)^2}{12}
\]  
(3.17)

where, \( e_0 \) = hydraulic aperture at zero stress, \( v_0 \) = closure of the joint when the hydraulic aperture becomes zero, \( v \) = normal deformation of the joint. Snow (1968b) observed an
empirical model to describe the fracture fluid flow variation with the normal stress, as described by:

\[ k = k_0 + K_n \left( \frac{e^2}{s} (\sigma - \sigma_0) \right) \]  \hspace{1cm} (3.18)

where, \( k_0 \) = initial fracture permeability at initial normal stress(\( \sigma_0 \)), \( K_n \) = normal stiffness and \( s \) = fracture spacing, \( e \) = hydraulic aperture.

From the test results obtained for carbonate rocks, Jones (1975) suggested the following empirical relation between the fracture permeability and the normal stress:

\[ k = c_0 \left( \log(\sigma_{ch} / \sigma \epsilon) \right)^3 \]  \hspace{1cm} (3.19)

where, \( \sigma_{ch} \) = confining healing pressure in which the permeability is zero and \( \sigma_c \) = effective confining stress. The constant \( (c_0) \) depends on the fracture surface and the initial joint aperture.

Nelson (1975) proposed the following empirical relationship for the permeability of fractured sandstone:

\[ k = A + B \sigma_c^{-m} \]  \hspace{1cm} (3.20)

where, \( A, B \) and \( m \) are constants which are determined by regression analysis. These constants may vary from one rock to another, and even for the same rock type, depending on the topography of the fracture surface. Some values determined by Nelson (1975) are tabulated in Table 3.2. From this data, it can be clearly seen that there is no consistency of any of these parameters based on the above Equation.

By simulating a rock surface as a bed of nails, Gangi (1978) reported a theoretical
model for fracture permeability as a function of the confining pressure, as represented by:

\[ k = k_0 \left( 1 - \left( \frac{\sigma_e}{P_i} \right)^m \right)^3 \]  

(3.21)

where, \( P_i \) = effective modulus of the asperities, \( m \) = constant which describes the distribution function of the asperity length.

Table 3.2. Constants determined by regression analysis (Nelson, 1975).

<table>
<thead>
<tr>
<th>Sample No.</th>
<th>Constant, A</th>
<th>Constant, B</th>
<th>Constant, m</th>
</tr>
</thead>
<tbody>
<tr>
<td>9-13</td>
<td>1494.0</td>
<td>4311.0</td>
<td>0.1</td>
</tr>
<tr>
<td>11-10</td>
<td>101.07</td>
<td>35800.0</td>
<td>0.7</td>
</tr>
<tr>
<td>16-17</td>
<td>-434.4</td>
<td>3410.0</td>
<td>0.2</td>
</tr>
<tr>
<td>19-15</td>
<td>-1600.0</td>
<td>3780.0</td>
<td>0.1</td>
</tr>
</tbody>
</table>

The above expression gives a better prediction if the effect of surface roughness on flow is negligible, which of course is not reasonable in practice. Although Gangi (1978) could obtain a good fit for the flow data (Nelson, 1975), Tsang & Witherspoon (1981) encountered difficulties in fitting data from Iwai (1976) into Gangi’s model. Having identified the drawbacks in the existing models, Tsang & Witherspoon (1981) developed a physical model, incorporating the joint roughness to see the effect of normal stress on fracture permeability.

Considering the effects of surface roughness, Walsh (1981) derived an analytical expression for permeability against confining pressure as given by:

\[ k = k_0 \left[ 1 - \left( 2 \frac{h}{a_{o2}} \ln \left( \frac{\sigma_e}{\sigma_p} \right) \right)^{0.5} \right]^3 \left[ \frac{1 - b(\sigma_e - u_m)}{1 + b(\sigma_0 - u_m)} \right] \]  

(3.22)
where, \( b = \left[ \frac{3f}{E(1-\nu^2)h} \right]^{0.5} \)

\( f = \) auto correlation distance, \( E = \) elastic modulus, \( \nu = \) Poisson’s ratio, \( h = \) root mean square value of the height distribution of the fracture surface, \( k_0 = \) permeability at reference confining pressure \((\sigma_p)\), and \( a_{02} = \) half aperture at the reference pressure. Any of the permeability-stress relationships discussed above can be used for predicting the fracture permeability depending on the required accuracy and availability of data.

### 3.4.2 Effect of surface roughness on permeability

Flow through a single discontinuity is usually expressed in terms of the cubic law, which is based on smooth laminar flow for parallel plate walls (Long and Witherspoon, 1985; Iwai, 1976; Wilcock, 1996). Based on laboratory studies, Witherspoon et al. (1980) and Iwai (1976) suggested that the cubic law could still be used for rough, natural joints at low confining stress. However, in reality, rock joints are usually rough, thereby decreasing the head driving the flowing fluid. Having noticed the influence of roughness on flow, Louis (1976), Walsh (1981), Tsang & Witherspoon (1981) and Brown (1987) have studied the effects of roughness on permeability, in order to modify the existing cubic law formulations.

Louis (1976) extended the existing theories used to analyse of flow through a conduit, in order to investigate the effect of roughness. As the roughness increases, the driving head of the flowing fluid decreases, and a pressure drop coefficient \((\lambda)\) is introduced as a function of the Reynolds number \((R_e)\) and the joint aperture. For a given rock fracture, Louis (1976) experimentally found that, depending on the magnitude of
pressure drop coefficient ($\lambda$), different type of flow modes, such as laminar and transient flow could occur. In general, $\lambda$ is a function of the Reynolds number, joint roughness ($k$) and joint aperture ($e$). For a smooth joint, pressure drop is only a function of the Reynolds number, hence, the flow is considered to be laminar. For generalised flow (both laminar and turbulent), the pressure drop coefficient is normally represented by:

$$\lambda = f(R_e, k/2e)$$  \hspace{1cm} (3.23)

Table 3.3 gives the pressure drop coefficient and flow rate established by different researchers for a range of relative roughness, for both laminar and turbulent flows.

Brown (1987) extended the results of Patir and Cheng (1978) for the effects of surface roughness on fluid flow in their study of hydrodynamic lubrication of rough bearings. The rock surfaces were simulated appropriately using fractal models, and the laminar flow between rough surfaces was simulated using the Reynolds equation. In this analysis, the lowest order of surface roughness is described as follows:

$$\nabla \cdot \left[ \frac{d^3}{12\mu} \nabla p \right] = 0$$  \hspace{1cm} (3.24)

where, $d =$ the average fluid film thickness, and $p =$ fluid pressure.

Assuming the linear profiles of natural rock surfaces have power spectra of the form $G(k) \approx k^s$, in which $k = 2\pi/\lambda$ being the wave number, $\lambda$ the wave length and $s$ the slope of power spectrum, Brown (1987) generated the numerical surfaces for the solution of
Table 3.3. Pressure drop coefficient and the unit flow rate in single joint, from Thiel, 1989.

<table>
<thead>
<tr>
<th>Relative roughness, k/D_h ≤ 0.033</th>
<th>Flow Type</th>
<th>Pressure drop coefficient</th>
<th>Flow rate</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Parallel flow) Laminar</td>
<td>[ \lambda = \frac{96}{\text{Re}} ] Poiseuille</td>
<td>[ q = \frac{g}{12 \nu} e^3 J ]</td>
<td>Overestimates the flow volumes Widely used in numerical models</td>
<td></td>
</tr>
<tr>
<td>Turbulent</td>
<td>[ \lambda = 0.316 \text{Re}^{-0.25} \text{Blasius} ]</td>
<td>[ q = \left[ \frac{g}{0.079 \nu} (\frac{2}{\nu})^{0.25} e^3 J \right]^{4/7} ]</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>[ \frac{1}{\lambda} = -2 \log \frac{k}{3.7D_h} \text{Nikuradse} ]</td>
<td>[ q = 4 \sqrt{g} \left( \log \frac{3.7D_h}{k} \right) e^3 \sqrt{J} ]</td>
<td>Need to know roughness of each joint Results may be better than flow rate values computed by cubic law, if correct roughness is used Yields lower magnitude of flow rate than cubic law.</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Relative roughness, k/D_h &gt; 0.033</th>
<th>Flow Type</th>
<th>Pressure drop coefficient</th>
<th>Flow rate</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Non-parallel flow) Laminar</td>
<td>[ \lambda = \frac{96}{\text{Re}} \left[ 1 + 8.8 \left( \frac{k}{D_h} \right)^{1.5} \right] \text{Louis} ]</td>
<td>[ q = \frac{g e^3 J}{12 \nu \left[ 1 + 8.8(k/D_h)^{1.5} \right]} ]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Turbulent</td>
<td>[ \frac{1}{\sqrt{\lambda}} = -2 \log \frac{k}{1.9D_h} \text{Louis} ]</td>
<td>[ q = 4 \sqrt{g} \left( \log \frac{1.9D_h}{k} \right) e^{1.5} \sqrt{J} ]</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
the Reynolds equation. Figure 3.11 shows the flow rate ratio (i.e. actual flow rate through a rough surface/cubic law flow rate) against the mechanical apertures.

Only the upper and lower bounds for the data provided by Brown (1987) are illustrated. This reveals that for larger apertures, the deviation from cubic law theory is very small. However, at small apertures, the actual flow rate is 40 – 70 % of the flow predicted by the cubic law. The uncertainty of this model is that the Reynolds equation used to simulate flow may cover the lowest order of the roughness, but the actual joint roughness may vary over a larger range. Instead of plotting the direct flow rate ratio, Brown (1987) plotted the ratio \((d_h/d)^3\) against different mechanical apertures where, \(d_h\) is hydraulic aperture and \(d\) is the average aperture. When roughness is considered, it is important to use the correct value of \(\lambda\) which should be larger than that assumed in the cubic law.

![Figure 3.11. Comparison of flow rate from cubic law with flow rate computed for rough surface (Brown, 1987).](image)
The above results confirm that roughness is of paramount importance in an accurate prediction of flow through fractures. However, in the field, it is not feasible to incorporate roughness of each joint separately in numerical models, as there is a large number of fractures in a jointed rock mass.

3.4.3 Effect of variable apertures on permeability

Fracture aperture is the key parameter for determining the flow characteristics through jointed rock media. However, measuring the fracture aperture distribution is an extremely difficult task. Natural fractures are generally rough and irregular with uneven walls, which usually make contact at several discrete points. For simplicity, fractures have often been simulated as smooth and parallel joint walls to develop mathematical models (Engelder & Scholz, 1981; Brown, 1987; ITASCA, 1996). The determination of a joint aperture may be conducted either by direct or indirect measurements, and a variation of aperture along a joint length is generally expected. The mechanical aperture based on direct measurements is generally greater than the effective hydraulic aperture, which is back-calculated from steady state flow using the cubic law (Barton and Bakhtar, 1983; Bandis et al., 1985; Tsang, 1992).

Several researchers (Neuzil & Tracy, 1981; Tsang, 1984) have developed models incorporating the effect of variable aperture along joint walls into the well-known cubic law formulations discussed in this section. The natural irregular rock fractures could be modelled with small segments of parallel plate walls as shown in Figure 3.12 (Neuzil & Tracy, 1981). The total flow along the joint length L (Figure 3.12), can be expressed using the simplified Poiseuille’s (cubic) law as follows:
\[ Q = \sum_{i=1}^{N} J \frac{\gamma}{12 \mu} b^3_i l \]  

(3.25)

where, \( N \) = no of segments, \( J \) = Hydraulic gradient, \( l \) = segment length, \( b_i \) = segment aperture and \( \mu \) = dynamic viscosity.

Assuming the normalised aperture frequency distribution to be \( f(b) \), Neuzil & Tracy (1981) modified the Poiseuille equation for a given joint length \( L \), as stated below:

\[ Q = L J \frac{\gamma}{12 \mu} \int_0^\infty b^3 f(b) db \]  

(3.26)

In this equation, they have discussed the aperture distribution function, \( f(b) \) for different conditions of stress associated with the shear deformation of fractures. Neuzil & Tracy (1981) compared the conventional flow expression (Eqn. 3.25) with their modified flow data (Eqn. 3.26) for an aperture distribution proposed by Sharp (1970), based on laboratory tests (Figure 3.13). At small apertures, the deviation between the simplified
and modified Poiseuille's law is significant. Undoubtedly, at a low confining stress, Poiseuille's law can predict to a sufficient accuracy, the flow volume for a given hydraulic head and joint aperture. Nevertheless, at a high confining stress, the accuracy of flow according to Poiseuille’s law is affected by the development of new contact points between the joint walls, breakage of asperities, and the deposition of gouge material (infill) along the joint walls. These factors contribute to a variable aperture of the same joint. In order to incorporate the variable aperture in the cubic law, the distribution of the aperture $f(b)$, has to be accurate enough to represent the current fracture surfaces (Eqn. 3.26).

Figure 3.13. Experimental data from (Sharp 1970) compared to modified Poiseuille equation (Neuzil and Tracy, 1981).
Tsang (1984) investigated the effect of variable aperture on flow using an electrical circuit, employing the electrical current analogy to the flow rate, whereby the hydraulic head represented the electrical potential, and the resistors represented the flow paths. The irregular fracture surface was simulated as a 2-dimensional plane. The resistance assumes values of $1/b^3$ and the roughness is related using a typical asperity size. Similar to other studies, small apertures play a major role in decreasing the flow rate when both tortuosity and roughness are included. When the fractional contact area increases, the flow rate may decrease several orders of magnitude, if both the variable aperture and roughness are considered. From past studies, it is clear that the variable aperture and the surface roughness reduce the flow significantly in a single fracture. However, the effects of tortuosity and surface roughness are rarely modelled accurately when dealing with an interconnected fracture network.

3.4.4 Applicability of Darcy's law for fluid flow calculations

In the simplified form of Darcy's law, the hydraulic gradient is assumed to be linear along the fluid path, but this assumption is no longer valid when the fluid flow is non-linear. From the laboratory test results obtained for natural rock joints, it can be seen that the Darcy's law (i.e., linear relationship between the flow rate and the pressure gradient) does not hold at elevated confining pressures or at very high hydraulic gradients (Chitty and Blouin, 1995). Some have been made to find a relationship between the hydraulic gradient and the fluid velocity for non-linear flow (Elsworth and Doe, 1986; Witherspoon et al., 1980; Chitty and Blouin, 1995).
For linear flow:

\[ v = k \left( \frac{dh}{dx} \right) \]  \hspace{1cm} (3.27)

where, \( v \) = average velocity of fluid in the discontinuity, \( k \) = permeability coefficient, \( dh/dx \) = hydraulic gradient which varies linearly along the joint length, \( dx \).

For non-linear laminar flow, Sharp and Maini (1972) described that the velocity depends not only on the hydraulic gradient, but also on the geometry of the fluid flowing surface, as represented by the following equation:

\[ v = k \left[ \frac{dp}{dx} - B \left( \frac{dp}{dx} - \frac{dp}{dx_{lim}} \right)^n \right] \]  \hspace{1cm} (3.28)

where, \( B \) and \( n \) = constants, which are determined empirically. The values of \( n \) and \( B \) depend on the properties of the fluid and the geometry of the joint surface.

Lee and Farmer (1993) have reported the following relationships for hydraulic gradient when the flow is non-linear. According to the Forchheimer law, the hydraulic gradient takes a quadratic form of velocity given by:

\[ \frac{dh}{dx} = (av + bv^2) \]  \hspace{1cm} (3.29)

where, \( v \) = average fluid velocity, \( a \) and \( b \) are constants. Missbach’s Law says that the hydraulic gradient is proportional to the power of the velocity, as follows:
\[
\frac{dh}{dx} = Cv^m
\]  

(3.30)

where, \(C\) = a proportionality constant, and \(m\) = ranges between 1 and 2.

As discussed earlier, natural fractures are generally rough and irregular (uneven walls) making contact at random (discrete) points. Over the past four to five decades, for simplicity, fractures have often been represented as smooth and parallel joint walls in order to develop mathematical models and to analyse fluid flow data (Baker, 1955; Louis, 1969; Engelder & Scholz, 1981; Brown, 1987; ITASCA, 1996). Flow through a single discontinuity is often expressed in terms of the cubic law, which is based on smooth laminar flow between parallel plate walls (Figure 3.14). On the basis of cubic law, the velocity profile of fluid across the joint is simulated as a parabolic shape as elaborated later in this section. After extensive laboratory studies, Witherspoon et al. (1980) and Iwai (1976) suggested that the cubic law could still be used for rough, natural joints at low confining stress. However, at comparatively elevated normal stresses, a significant deviation from the cubic law was noted, because of the increased contact area of the joint surface.

![Image](a) Natural joint: fluid flow is not parallel due to rough surface  
(b) Idealised joint: fluid flow is parallel to the smooth walls

Figure 3.14. Comparison of natural rock joint having a rough surface with an idealised joint with smooth parallel walls.
For steady state flow through a parallel wall fracture with aperture \( e \), the flow rate \( q \) is usually expressed in terms of joint aperture and hydraulic head difference \( dh \) using Darcy's law for saturated, laminar and incompressible flow. In fluid mechanics, it is well known that the solution to Navier-Stokes equation for flow between parallel plates is referred to as the cubic law, which can be written as (Street et al., 1996):

\[
\frac{q}{dh} = Ce^3
\]  

(3.31)

where, \( C \) is a function of the fluid properties (e.g. Reynolds number) and the fracture length. The cubic law is sometimes referred to as Poiseuille's law or simply the parallel plate theory.

For linear (planar) flow:

\[
C = \frac{b}{12\mu L}
\]  

(3.32)

where, \( b \) = width of the fracture, \( \mu \) = dynamic viscosity of fluid, and \( L \) = length of the fracture.

Gale and Raven (1980) expressed radial flow to a control axial discontinuity (well) as given below:

\[
C = \frac{1}{12\mu} \frac{2\pi}{\ln(r_o/r_w)}
\]  

(3.33)

where, \( r_o \) is the outer radius and \( r_w \) is the well radius.

Taking the logarithm of Equation (3.31) yields the following:

\[
\ln(q/dh) = \ln C + 3 \ln e
\]  

(3.34)
The above relationship indicates that the plot of $\ln(q/dh)$ against $\ln(e)$ should have a gradient of 3 with the intercept giving the value of $C$.

If the aperture along the joint does not vary significantly, then the flow is well approximated by the cubic law. An attempt to investigate the validity of cubic law was carried out by Witherspoon et al. (1980) for artificial fractures created in granite, marble and basalt specimens. Under radial flow and planar flow conditions, these studies indicated that the cubic law was still reasonable for assessing flow through artificial fractures (Figure 3.15). Regardless of the loading cycles and rock type, the cubic law could be employed to estimate flow rates. This is because artificial fractures have less surface irregularities than the natural rock fractures. The effect of roughness was found to reduce the flow rate by a factor of 1.04 - 1.65.

![Figure 3.15. Deviation of cubic law from measured data for granite specimen (data from Witherspoon et al., 1980).](image)
Raven and Gale (1985) studied water flow through naturally fractured granite specimens under different levels of normal stress (up to 30 MPa), and they found significant deviation from the cubic law as shown in Figures 3.16 and 3.17. The results for specimen 1 indicate less deviation from the cubic law (Figure 3.16), probably because the joint walls are nearly planar. In contrast, Figure 3.17 indicates significant deviation from the cubic theory, mainly due to the highly irregular joint surfaces with many random contact points along the joint. Also, specimen 2 was approximately three times larger than specimen 1. Therefore, apart from joint surfaces irregularities, the sample size and the corresponding normal stress could influence the flow behaviour. Elevated normal stress increases the number of contacts points between joint surfaces as well as causing greater deposition of gouge material by shearing the asperities. As a result, a reduced flow rate is expected for the same aperture, correspond to the cubic law prediction.

Figure 3.16. Deviation of cubic law from measured data for granite specimen 1 (data from Raven and Gale, 1985).
Furthermore, the work carried out by Neuzil and Tracy (1981) and Tsang (1984) shows that cubic law does not hold for flow calculations in natural fractures, particularly at high normal stress levels and for large rough fractures (Figure 3.18). In spite of these drawbacks, the cubic model is still widely assumed in practical situations, because in the field, it is not feasible to incorporate the roughness of each joint in numerical models. Table 3.4 summarises some applications of cubic law, including its use in commercially available numerical models.
Figure 3.18. Experimental data from (Sharp 1970) compared to modified Poiseuille equation (Data from Neuzil and Tracy, 1981).

Table 3.4. Application of cubic law in theoretical and numerical models.

<table>
<thead>
<tr>
<th>Studies</th>
<th>Applications</th>
<th>comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Engelder and Scholz (1981)</td>
<td>Experimental investigation of flow through artificial fractures</td>
<td>Cubic law is found to be valid</td>
</tr>
<tr>
<td>Gale and Raven (1980)</td>
<td>Radial fluid flow through natural fractures –Experimental</td>
<td>Not in agreement</td>
</tr>
<tr>
<td>Pyrak-Nolte et al. (1987)</td>
<td>Flow through natural low permeability rocks – experimental</td>
<td>Cubic law is valid for effective stress below 20MPa</td>
</tr>
<tr>
<td>Brown (1987)</td>
<td>Effects of surface roughness on flow -Numerical</td>
<td>Cubic law estimates 40 –70 % larger flow rate</td>
</tr>
<tr>
<td>Amadei and Illangasekre (1992)</td>
<td>Transient flow in single joint</td>
<td>Cubic law was used but modified to account for surface roughness</td>
</tr>
<tr>
<td>Iwai (1976)</td>
<td>Fundamental studies of fluid flow through a single joint</td>
<td>Cubic law is followed</td>
</tr>
<tr>
<td>ITASCA (1996)</td>
<td>Universal Distinct Element Code (UDEC)</td>
<td>Cubic law is used for flow calculation</td>
</tr>
<tr>
<td>Wilcock (1996)</td>
<td>NAPSAC fracture network code</td>
<td>Cubic law is used for flow calculation</td>
</tr>
</tbody>
</table>
Having examined the works of past researchers, the validity of the cubic law for joints with different geometry is depicted in Figure 3.19. For instance, for open rough joints, cubic law may still hold with little deviation from the observed flow rates. However, with increase in normal stress, the turbulent flow may develop instead of laminar flow. Further increase in stress will result in greatly increased number of contact points, and ultimately crushing the asperities. Subsequently, the crushed material (gouge) will deposit within the joint, and under these circumstances, the cubic law cannot be assumed in flow calculations.

Figure 3.19. Validity of cubic law for different fractures.
3.5 NUMERICAL AND ANALYTICAL APPROACHES FOR FLOW RATE ESTIMATIONS

3.5.1 Analytical approach

Analytical methods may be used to estimate fluid flow quantities in a given rock mass, provided the rock mass contains simple fracture network formed with a small number of joints. Typically, there are two approaches based on analytical techniques (Priest, 1993; Thiel, 1989):

(a) for a given flow rate relationship, the flow is estimated;
(b) the hydraulic head at each intersection point of a given fracture network is first estimated, and then the flow is quantified.

The first approach is viable when the hydraulic conductivity for different boundary conditions including stresses or depths is known. The second approach is tedious and often involves lengthy calculation procedures, but it yields fairly accurate results for a given fracture network consisting of a few joint sets. Goodman et al. (1965) employed an analytical approach (Equation 3.35) for steady state flow into a horizontal drift, assuming the drawdown curve of the water table is negligible. The application of this method is not realistic in most cases, except for undersea tunnels. The steady state flow rate \( Q \) was given by:

\[
Q = \frac{2K_s \pi H_0}{\ln(4l/d)}
\]

(3.35)

where, \( K_s = \) constant,

\( H_0 = \) the depth of water table to the center of the tunnel,

\( d = \) diameter of the tunnel and
$l$ = depth of the ground surface to the center of the tunnel.

Zhang and Franklin (1993) used an analytical approach to estimate inflow to a tunnel assuming that the hydraulic conductivity decreases exponentially with depth. There are several empirical and theoretical relationships for estimating hydraulic conductivity as a function of depth or stress. Zhang and Franklin (1993) adopted the simple approach of Louis (1974) to derive:

$$K = K_s \exp(-Ah)$$  \hspace{1cm} (3.36a)

where, $K =$ hydraulic conductivity at depth, $h$

$K_s =$ constant,

$A =$ hydraulic conductivity gradient.

The corresponding flow rate ($Q$) represented by Equation (3.36b) derived by Zhang and Franklin does not account for the most important aspects related to excavation, such as the change of joint spacing, joint apertures and development of new fractures.

$$Q = \frac{2K_s \pi (\gamma_m - \gamma_w) \exp \left[ -\left( \frac{\gamma_m A}{\gamma_m - \gamma_w} - \frac{\gamma_w a_1}{\gamma_m - \gamma_w} \right) L \right] \exp \left( \frac{\gamma_w (A - a_1) H_0}{\gamma_m - \gamma_w} - 1 \right)}{\gamma_w (A - a_1) (K_0 Ad / 4 - K_0 AL)}$$  \hspace{1cm} (3.36b)

where,$\gamma_m =$ unit weight of rock

$\gamma_w =$ unit weight of water

$H_0 =$ the depth of water table to the center of the tunnel,

$K_0 =$ the hydraulic conductivity at the center of the tunnel,

$d =$ diameter of the tunnel,

$L =$ depth of the ground surface to the center of the tunnel,
\[ a_3 = \text{the stress-independent factor, which may be taken as approximately } 0.25A, \]
in which \( A \) is the hydraulic conductivity gradient.

In most analytical approaches employed by the various researchers, it is assumed that
the joints are of infinite length in a given rock volume, and that they are orientated in
perfectly parallel planes (Sharp, 1970; Maini, 1971).

### 3.5.2 Numerical modeling in rock mechanics

The usage of numerical modeling in soil and rock engineering has been expanded in
recent years, in order to handle complex problems efficiently. There have been
umerous computer programs (codes) developed for research work, as well as for
practicing engineers (Elsworth, 1985; Itasca, 1996; Wilcock, 1996). On the basis of the
type of flow problem dealt with, these computer codes can be classified into three
categories:

(a) domain method,

(b) boundary formulations, and

(c) lattice structure method.

The main difference between the domain and the boundary methods is that in the
former, the interior media is discretised, while in the latter, external surface is
discretised. Some of the widely used domain and boundary techniques in rock
mechanics are listed below (Neuman, 1973; Elsworth, 1987; Beer & Poulsen, 1994):

(a) Finite Element Method (FEM) - domain formulation,

(b) Finite Difference Method (FDM) - domain formulation,
(c) Boundary Element Method (BEM) - boundary formulation,
(d) Discrete Element Method (DEM), and
(e) Combination of above (e.g. coupled FEM and BEM)

The third category (i.e. lattice structure method) is not as popular as others among researchers. For example, the Lattice-gas and Lattice-Boltzmann methods (Muhammad, 1995) were used to model fluid flow through porous and fractured media. The conventional methods will continue to dominate until the lattice structure technique gains greater acceptance for handling complex problems.

Techniques such as FEM or FDM, can be used to model non-linear behavior and non-homogeneous materials (e.g. non-linear flow behavior, stress-deformation and unsaturated flows). The FEM has gained increased popularity in solving rock mechanics problems because of the capability of having finer mesh arrangements at the edges and corners. Also, in most of cases, the system matrices are symmetric, thereby, yielding an efficient solution approach (Neuman, 1973; Wangen, 1997). Basically in FEM, the region is subdivided into small elements and the equilibrium of each element is described in an implicit manner. FEM is efficient when the ratio of volume to surface area is small, and when the boundary stresses are not of primary importance (Elsworth, 1985).

There are a number of commercially available computer codes based on FEM and FDM. Fast Lagrangian Analysis of Continua (FLAC) developed by ITASCA (1993) is an explicit finite difference continuum code, which is commonly used for the analysis of soil and rock problems. The NAPSAC fracture network code developed by AEA
Technology, based on FEM, was used to simulate flow through interconnected fracture networks (Wilcock, 1996).

Boundary Element Method (BEM) is a relatively recent technique compared to FEM. It is also used for analysing problems in a rock mass by discretising the surface into boundary elements (Beer & Poulsen, 1994; Crouch & Starfield, 1983; and Crotty & Wardle, 1985). As discussed by Elsworth (1987), BEM is suited for analysing situations where the ratio of volume to surface area ratio is high, and for ensuring high accuracy of boundary stresses. One of the main disadvantages of BEM software is that the rock is often assumed to behave as a homogeneous and elastic medium, which is not realistic in practice, especially where fractured rock is encountered.

Discrete element method (DEM) is best suited for discontinuous media such as fractured rock mass, which is in direct contrast to continuum techniques such as FEM and FDM (Cundall, 1971). In DEM, there are two main advantages over the continuum approaches, as described below:

(a) Large deformations due to joint slip and block rotations are allowed;

(b) Both material and discontinuity (i.e., joints) properties are used to simulate the actual rock mass.

The shapes of rock blocks depend on the orientation of joints, discontinuity length and their spacing. Distinct element method was initially developed for mechanical analysis of solid blocks by Cundall (1971) and then further extended by co-workers (Cundall and Strack, 1979). The commercially available Universal Distinct Element Code (UDEC) is
a two-dimensional program based on distinct element approach, in which the rock blocks are assumed as deformable or rigid (ITASCA, 1996).

Coupled boundary element and Finite element methods (BEM-FEM) have been successfully used in rock mechanics to optimize the solution efficiency for complex problems (Elsworth, 1985; 1986; 1987; Zienkiewicz et al., 1977). Complexity in problems arises because, often the rock is inhomogeneous, non-linear, anisotropic and discontinuous (e.g. faults and fractures). Moreover, an infinite boundary is often assumed for modelling rock engineering problems. Under these circumstances, one may couple FEM and BEM to obtain high precision in modeling. Hybrid distinct element-boundary method (DEM-BEM) was used by Lorig et al. (1986) to study the stresses and displacements in highly jointed rock mass surrounding an underground cavity. The distinct element method was applied to model the jointed rock close to the cavity, while far field rock was modeled using the boundary element method. One advantage of this coupled technique is that the equilibrium conditions at the interface between the two domains are obtained explicitly.

3.6 TWO-PHASE FLOW THROUGH ROCKS

Particularly in mining and petroleum industry, research studies based on two-phase flow analysis through rock masses have gained increasing interest because of the need for prudence in design applications and risk assessment (Pruess and Tsang; 1990; Rasmussen 1991; Fourar and Bories 1995). Another practical application of two-phase flow relates to natural gas reservoirs where gas and water are trapped in fractured tight
rocks. The practical importance of two-phase flow in geotechnical, mining and petroleum engineering has been discussed by Pruess and Tsang (1990) who modeled real rock fractures as a two dimensional heterogeneous porous medium. The common approach employed by various researchers to describe steady state two-phase laminar flow through rock fractures is the extended Darcy’s law as given below (Peaceman, 1977; Fourar & Bories, 1995):

\[ v_a = \frac{k K_{ra} (\nabla p_a - \rho_a g)}{\mu_a} \]  \hspace{1cm} (3.37)

where, \( v \) = velocity of phase \( \alpha \)

\( k \) = intrinsic permeability

\( K_r \) = Relative permeability

\( \rho \) = density

\( \mu \) = dynamic viscosity

\( g \) = acceleration due to gravity

\( p \) = fluid pressure

From Equation 3.37, for a given phase pressure, the relative permeability of each phase can be predicted experimentally. However, the analytical approaches are more cumbersome because of time dependent interface for different boundary conditions. In addition, the applicability of Darcy’s law for fluid flow simulation in real rock fractures is uncertain. Pruess and Tsang (1990) used a different numerical approach to estimate capillary pressure between wetting and non-wetting phase. The capillary pressure was described by the following equation:
\[ p_c = 2\gamma \cos \alpha \frac{1}{b} \tag{3.38} \]

where, \( p \) = fluid pressure,

\( \gamma \) = surface tension between wetting and non-wetting phases

\( \alpha \) = contact angle between wetting phase meniscus and fracture wall, and

\( b \) = parallel plate joint aperture.

According to Pruess and Tsang (1990), the calculation was based on a defined "cut of aperture" which was used to estimate the capillary pressure and saturation and relative permeability of wetting and non-wetting phases. In this approach one may argue about the definition of the cut of aperture, which is time dependent due to the change of stresses and fluid pressures within rock fractures. However, for the assumptions made in their model, the observed relative permeability appeared to agree with previous two-phase experimental work on porous media (Corey, 1957; Touma and Vauclin, 1986).

In the case of nuclear/toxic waste disposal sites in unsaturated rocks, fluid travel time is important in the design stage of underground storage plants. Rasmussen (1991) investigated the travel time based on air-water interface in partially saturated fractures using the boundary integral method. The partial saturation of idealized fractures was modeled by filling part of the fracture with water and the remaining portion with air. The travel time between two points along the streamline was estimated using the following expression:

\[ t(s, d) = \int_{0}^{d} \frac{e_{s} \alpha_{f}}{q_{f}} \, dx \tag{3.39} \]

where, \( t \) = travel time along the stream line, \( s \).
\( e_v = \) effective volumetric porosity,

\( a_f = \) fracture aperture,

\( q_f = \) flow rate through fracture,

\( d x = \) distance along the stream line and

\( d = \) displacement from the source.

Figure 3.20 shows the calculated travel time against solute concentration ratio between the upstream and downstream ends of the fracture. When the solute concentration increases at the downstream end, as expected, the travel time increases for both vertical and horizontal fractures. The applicability of this method to real rock fractures is questionable, because real rock fractures have variable apertures with discrete contact points and also, one needs to incorporate the effect of interaction between the different fluids.

![Figure 3.20. Travel times through horizontal and vertical fractures (Data from Rasmussen, 1991).](image)
Many attempts to obtain visual observation of two-phase patterns can be found in the literature (Hewitt & Lovegrove, 1969; Arnold & Hewitt, 1967; Delhaye, 1979; Fourar & Bories, 1995). Fourar and Bories (1995) carried out an experimental study on two-phase flow of water and airflow through a narrow channel. Water and air were driven between two glass plates 1m long and 0.5m wide separated by 1mm, and the flow pattern was observed along the glass plates and photographed using a high speed camera (Figure 3.21). The two parallel glass plates were not subjected to external loads, and the observed flow pattern is presented in Figure 3.22. The observed flow patterns were bubble, complex and annular flow, depending on the inlet fluid flow rates of each phase.

![Figure 3.21. Flow pattern observed in a narrow glass channel (after, Fourar and Bories, 1995).](image)

Mishima & Hibiki (1996) studied two-phase flow through small diameter (1.05 to 4.08mm diameter) vertical tubes and they observed the kind of flow patterns as sketched in Figure 3.23.
Based on experimental work, Fourar and Bories (1995) concluded that the relative permeability of liquid phase becomes approximately equal to the liquid volume fraction for laminar flow, whereas for gas, the relative permeability shows a non-linear relationship against the gas volume fraction. The valuable experimental results obtained by Fourar and Bories (1995) may be directly applied for simplified two-phase flow analysis, but are not suitable to characterize the coupled hydro-air-mechanical flow that usually takes place within rock joints under stress.

Figure 3.22. Flow patterns observed in a narrow glass channel. The dark color and light show the liquid and the gas, respectively (after Fourar & Bories, 1995).

Based on laminar flow of two fluid layers in a horizontal channel, the average height fraction (see Figure 3.24) of each phase can be expressed as below (Coutris et al., 1989):
where, \( q \) = flow rate of phases \( \alpha \) and \( \beta \)

\[ \delta = \text{height fraction} \]

\[ \mu = \text{dynamic viscosity of fluid} \]
Various types of triaxial apparatus have been developed during the past 5-6 decades for testing soil and rock (Handin, 1953; Anderson & Simons, 1960; Hoek & Franklin, 1968; Hambly and Reik, 1969; Dusseault, 1981; Smart, 1995). Triaxial apparatuses may be classified depending on the (a) capacity of the triaxial cell (ie. high-pressure, or low-pressure), (b) loading system (ie. plain strain, quasi-static stress, polyaxial stress), and (c) use of single-phase flow or multiphase flows (Table 3.5).

Bishop & Henkel (1969) discussed in detail the features of their triaxial apparatus, application of triaxial tests to study the properties of soil, as well as the benefits and limitations of the equipment. The schematic diagram of the equipment is shown in Figure 3.25.

Figure 3.25. Schematic diagram of the Bishop and Henkel triaxial cell (1969).
Table 3.5. Classifications of triaxial apparatuses for soil and rock testing.

<table>
<thead>
<tr>
<th>CLASSIFICATION</th>
<th>APPLICATIONS</th>
<th>REFERENCES</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Pressure capacity of cell</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>High-pressure triaxial apparatus</td>
<td>For most type of rocks including hard rocks</td>
<td>Handin, 1953; Dusseault, 1981; Michelis, 1988; Crawford et al., 1995.</td>
</tr>
<tr>
<td><strong>Stress-State</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Quasi-static triaxial state</td>
<td>Two stresses are equal. $\sigma_1 \neq \sigma_2 = \sigma_3$</td>
<td>Anderson &amp; Simons, 1960; Hoek &amp; Franklin, 1968; Shibuya &amp; Mitachi, 1997.</td>
</tr>
<tr>
<td>Poly-triaxial state</td>
<td>Principle stresses are not equal $\sigma_1 \neq \sigma_2 \neq \sigma_3$</td>
<td>Hambly &amp; Reik, 1969; Airey &amp; Wood, 1988; Amadei &amp; Robison, 1986.</td>
</tr>
<tr>
<td><strong>Shape of specimen</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>State of fluid flows</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Single-phase flow</td>
<td>Fluid flow through the specimen is single phase (e.g. either water or gas).</td>
<td>Most work carried out in triaxial test is based on single-phase flow.</td>
</tr>
<tr>
<td>Two-phase flow</td>
<td>Two or more fluid phases through the specimen (e.g. water + gas or water + gas+oil).</td>
<td>Very few two-phase or multiphase triaxial apparatus are available.</td>
</tr>
</tbody>
</table>

The B & H cell was capable of withstanding a maximum confining pressure of around 1 MPa and was designed to test 38mm diameter soil specimens. The confining pressure was applied by the cell fluid (water) pressurised around the specimen.
The axial load was applied to the top of the specimen via a shaft. Instead of water, oil could also be used as a cell fluid because of its high viscosity, hence its enhanced resistance to leakage through the membrane. To measure the axial deformation, Bishop & Henkel (1969) used a vernier telescope, which was focused on top of the steel ball. The soil specimen was confined within a rubber membrane having a thickness of 0.26 mm. The commercially available transducers can measure pressure levels accurately, up to several decimal places. However, during the 1950s and 1960s, pressure transducers were not properly developed, hence other techniques were employed to measure cell and pore pressures in the triaxial tests. For example, Bishop and Henkel (1953) developed a self-compensating mercury control technique (Figure 3.26) for measuring cell pressures. Pore pressure was recorded using a null indicator which could measure it to an accuracy of 0.69 kPa (0.1 lb/sq.in). The equipment could conduct both drained and undrained tests for given radial and axial pressures.

![Self-compensating mercury control technique for measuring cell pressures.](image)

Figure 3.26. Self-compensating mercury control technique for measuring cell pressures.
Berre (1982) reported two types of triaxial cells: one for static loading and the other for cyclic loading designed by the Norwegian Geotechnical Institute. Both cells were identical, except the cyclic loading cell had an improved axial loading system. Basically, they are similar to the Bishop and Henkel (1969) triaxial cell, but equipped with traditional measuring devices for pressure measurement as well as electronic devices such as pressure transducers and linear variable differential transformers (LVDTs) for automatic datalogging. The cell could accommodate specimens having diameters of 54 and 80 mm, with a maximum cell pressure of 2 MPa. The triaxial cell was usually filled with liquid paraffin in order to reduce the friction between the piston through the cell and the top seating. Moreover, paraffin would minimise the leakage problems through the membrane due to its high viscosity. When a membrane is exposed to the paraffin medium, the membrane can undergo swelling. Consequently, the membrane wall may be damaged due to coarse particles on the surface of the specimen. As discussed by Berre (1982), during a longer period of testing, the membrane could expand laterally by absorbing water, and as a result, leakage could occur. The absorption of water by membrane with time is illustrated in Figure 3.27. When the sample diameter is not exactly the same as the membrane diameter, reducing the sample size to fit into the available membrane is not an easy task, particularly for very soft soil, which can develop fine cracks upon disturbance. To overcome these difficulties, Iversen and Moum (1974) and Berre (1982) proposed the use of paraffin coating around the specimen instead of membrane. Use of paraffin as a membrane is not widely accepted among researchers on triaxial testing due to its following limitations; (a) resistance to develop new cracks or preventing the opening of existing cracks or pore spaces, (b) small change of deformation of the specimens may develop
finer cracks within the paraffin, thereby initiating leakage, and (c) provides a low confining pressures to the specimen.

![Graph showing absorption of water for different membrane materials.](image)

Figure 3.27. Absorption of water for different membrane materials (from Berre, 1982).

Two-phase triaxial equipment for soil

The first kind of two-phase triaxial equipment was developed for the measurement of water and air phases within soil pores. Various laboratory methods have been used to investigate the permeability characteristics under steady state and unsteady state conditions (Klute, 1965; Barden and Pavlakis, 1971; Fredlund & Rahardjo, 1993). Some developments were based on the measurement of the permeability coefficients of water and air phases simultaneously, while the others were developed to measure only the coefficient of permeability with respect to water. According to the method described by Klute (1965), water was supplied to the specimen from an overhead water...
tank, and the water pressure was measured by two tensiometers as shown in Figure 3.28. The constant air pressure was measured using a manometer. The tests were carried out for different suction pressures \((p_a - p_w)\), where \(p_a\) is air pressure and \(p_w\) is the water pressure. In Klute’s apparatus, the change of permeability with respect to the change of axial stress has not been addressed.

![Figure 3.28. Apparatus used to measure permeability coefficient of unsaturated soil (from Klute, 1965).](image)

Barden and Pavlakis (1971) developed a more advanced triaxial equipment to measure both the water and air permeabilities of soil. In this apparatus, air and water pressures were applied to the specimen in a given cell pressure. This equipment was not capable of measuring the volume change of the specimen, and also, no strain measurement devices were attached to the specimen. Hamilton et al. (1981) studied the conductivity of partially saturated soil for different suction pressures. They found that the coefficient of water permeability decreases with the increase in the degree of air saturation in the
soil. Barden and Pavlakis (1971) reported similar test results of air and water conductivity on a boulder clay.

3.8 SUMMARY AND CONCLUSIONS

A thorough understanding of single-phase flow through rock fractures is required for the development of a theory for two-phase flow, which usually is encountered in rock fractures. As outlined above, fluid flow through a single fracture is governed by its joint aperture, joint variability, joint roughness, ground stresses, fluid pressures and properties of fluid and rock itself. In the implementation of any numerical or analytical modelling, joint aperture is important, which can be measured directly or using an indirect approach (e.g. hydraulic aperture). However, for a joint network, one has to use a proper joint distribution function, depending on the availability of joint aperture data. For flow computations, the applicability of cubic formula (Equation 3.31) is questionable particularly at high normal stress and high joint roughness.

In precise flow computation, an appropriate flow condition needs to be employed, depending on the availability of field data, computer resources, time, budget and the degree of accuracy required for the particular application. For example, if fluid flow is dominated through a network of fractures, the flow is best described by the discrete fracture theory. Numerical techniques such as Boundary Element Method or Finite Element Method are usually employed for flow-deformation modelling in jointed rock mass.
From previous research, two major inadequacies have been recognised and are addressed in this research. Firstly, rock fractures often carry more than one fluid (i.e., in multiphase flow), and this has not been clearly addressed in order to understand the complex flow phenomena and to develop a comprehensive mathematical model. Therefore, the research reported in this thesis aims to shed light on the relatively complicated two-phase flow system, and to provide a comprehensive mathematical model to compute the quantities of each fluid phase travelling in a given joint domain. Moreover, it examines the relative permeability of each phase for different degrees of saturation and varying inlet fluid pressure. Effects of joint deformations, joint roughness, interaction between each phase, change of fluid properties, changes of interface between two phases are described in Chapter 8.

Secondly, the triaxial test equipment for soils described in this chapter are not readily applicable to study two-phase flow through rock under high pressure. A number of available triaxial facilities can measure either the pore water pressure or pore air pressure within a fractured rock, but they are still incapable of measuring the relative permeability (air or water) of a fractured specimen. It is the relative permeability data that are most useful in the prediction of flow through a jointed rock mass. In order to study the two-phase flow behavior through fractured rock specimens, an attempt is made to design a new, Two-Phase, High-Pressure, Triaxial Apparatus (TPHPTA). The design of this equipment and the experimental study of two-phase flow through real rock fractures are described in Chapters 4 and 8 of this thesis, respectively.
CHAPTER 4

TRIAXIAL EQUIPMENT FOR TESTING SINGLE AND TWO-PHASE FLOW

4.1 INTRODUCTION

The development of a high pressure triaxial apparatus for two-phase flow was a significant part of this research project. Therefore, the discussion of other triaxial test equipment for rocks in this chapter is necessary to appreciate and recognise the unique features of the triaxial apparatus designed by the writer, described later in the chapter.

The knowledge of properties as well as stress-strain behaviour of engineering materials including soil and rocks are very useful for the design of various engineering structures on the surface and underground. For determining such properties of geological materials such as soil and rocks, the laboratory techniques are mainly used. In order to provide meaningful data from laboratory testing, the apparatus must be truly capable of simulating existing insitu field conditions, including the stress-strain behaviour and permeability characteristics. As discussed in Chapter 3, various types of triaxial apparatus have been developed during the past 5-6 decades for this purpose (Handin, 1953; Anderson & Simons, 1960; Hoek & Franklin, 1968; Hambly and Reik, 1969; Dusseault, 1981; Smart, 1995; Indraratna & Haque, 1999). Triaxial apparatus may be classified depending on the (a) use of single-phase flow or multiphase flows, (b) loading system (i.e. plain strain, quasi-static stress, polyaxial stress), and (c) capacity of the triaxial cell (i.e. high-pressure, or low-pressure), as illustrated in Figure 4.1.
Figure 4.1. Classification of various triaxial apparatuses.
The triaxial testing of cylindrical soil specimens and solids started in 1930s and the main aim of this equipment was limited to studying the stress-strain behaviour. Subsequently, the triaxial equipment was advanced to test harder materials like rocks. Bishop and Henkel (1969) have extensively discussed the use of triaxial apparatus to study the properties of soil for different boundary conditions. A rock or soil element below the ground surface is normally subjected to three principal ground stresses apart from the fluid pressures, as shown in Figure 4.2. In order to represent realistic boundary conditions, the testing apparatus must be capable of applying the ground stress independently of the additional fluid pressures. However, most types of triaxial cells have been designed to conduct research based on specialized testing or routine soil/rock property testing, considering two distinct stress states with fluid stress fields. For instance, the conventional triaxial equipment has the ability to use cylindrical specimens subjected to two distinct stress fields (i.e. major and minor principle stresses, $\sigma_1$ and $\sigma_3$) and the intermediate stress ($\sigma_2$) is assumed to be equal to $\sigma_3$, by the application of fluid pressures all round the cylindrical surfaces.

Figure 4.2. Rock element subjected to ground and fluid stresses where, $\sigma$ is the geological stress and $u$ is the fluid stress.
Before describing the appropriate triaxial equipment for single or two-phase flow through soil or rock specimens, it is important to appreciate the historical development of various forms of triaxial facilities. The following part of this chapter describes the main features of triaxial equipment developed for both soils and rock testing, indicating the main requirements governing their design.

4.2 TRIAXIAL APPARATUS FOR SINGLE-PHASE FLOW

4.2.1 Weak rock triaxial cells

For laboratory investigation of soil and rock deformational behaviour, the conventional axi-symmetric triaxial compression test, which can sufficiently represent field stresses, has become popular and versatile for many kinds of geotechnical applications, due to its simplicity of operation and for providing reliable results. One main advantage of the conventional triaxial cell is that no special preparation of specimens is necessary other than the two ends, because, the cylindrical shape of samples used in laboratory testing is the same as that of samples taken from field boreholes. As mentioned earlier, in the conventional triaxial tests, cylindrical samples are subjected to a major and two minor principal stresses (i.e. $\sigma_1$ and $\sigma_2 = \sigma_3$), i.e. axial pressure and lateral confining stress (cell pressure). The stress field on a typical specimen is shown in Figure 4.3. Original triaxial facilities were constructed mainly for testing soil samples, which were usually subjected to small all round pressures (less than 1 MPa), thus, the cell wall was usually constructed of low strength materials such as plastic and glass. In the following pages, the structure of several conventional triaxial cells will be discussed including their merits and demerits.
4.2.2 True triaxial cells

In the true triaxial tests, the samples are generally subjected to three different stresses (i.e. $\sigma_1$, $\sigma_2$ and $\sigma_3$). The main significance of the true triaxial concept is the recognition of intermediate stress ($\sigma_2$) not to be the same as $\sigma_3$ in the conventional triaxial apparatus. Depending on the boundary conditions, there are three distinct design features of the true triaxial apparatus: (a) all rigid boundaries, (b) all flexible boundaries and (c) combination of rigid and flexible boundaries. In the last two decades, a number of studies (Meier et al., 1985; Hambly and Reik, 1978; Smart, 1995; Crawford et al., 1995; Amadei and Robinson, 1986; Hight et al., 1983; Hon-Yam and Ronald, 1967; Michelis, 1988; Airey and Wood, 1988) have discussed the various forms of development of true triaxial testing. The
majority of the existing true triaxial devices can either be stress controlled or strain-controlled (Lade, 1973; Michelis, 1988; Smart, 1995). In the following section, three kinds of true triaxial apparatus based on stress-controlled, strain-controlled and both stress-strain controlled are discussed.

(a) *Cambridge True Triaxial Apparatus*

A strain controlled true triaxial apparatus, in which all six sides are made rigid, is illustrated in Figure 4.4 (Airey & Wood, 1988) which is mainly used for testing clay specimens. The clay sample is prepared by mixing as slurry at twice its liquid limit and then poured into the membrane. In this equipment, six platens are nested together, while the cubical size specimen is kept at the centre. Four stress transducers are attached to four platens to measure the stresses acting on the boundaries of the specimen. Load is applied to each platen via a motor driven ram. The relative movements of the pairs of opposite platens are measured by means of three LVDTs, and the results are recorded by a datalogger.

![Figure 4.4. Cambridge true triaxial cell (from Airey and Wood, 1988).](image)
Although this true triaxial cell is capable of applying an anisotropic stress state, it can only be used for clayey soils to determine strength parameters. Moreover, this is not suitable to directly obtain the permeability characteristics of specimens taken from the ground, because of the need for increasing the moisture content significantly during the sample preparation.

Figure 4.5. True triaxial cell for soil and rock (after Michelis, 1988).
(b) A true triaxial cell for soil and rock

Another version of the true triaxial cell (stress/strain controlled) is illustrated in Figure 4.5 (Michelis, 1988). In this equipment, the cell pressure is applied to the specimen through oil filled flexible membranes, and the axial load is applied via a rigid piston. The cell can withstand up to 250 MPa confining pressure and up to 1500 MPa axial stress. Lateral and axial deformations are recorded using LVDT's, and will be discussed later in the following chapter. The major advantage of this application is that both rock and soil specimens can be tested at low-to high pressures.

(c) A true triaxial cell for testing cylindrical rock specimens

As discussed above, most true triaxial testing cells have been designed for testing cubical or rectangular specimens. A novel true triaxial cell capable of testing cylindrical rock core plugs under realistic poly-axial stress state has been presented by Smart (1995). As in a conventional triaxial cell, confining pressure and axial stress are applied via a rubber membrane and rigid plate, respectively. The two distinct horizontal stresses (σ₂ and σ₃) are obtained by arranging 24 PVC tubes around the specimen as shown in Figure 4.6a. The axial cross section of the cell is also shown in Figure 4.6b. The tubes are encased in individual compartments machined in the body of the cell. Once pressurised, these tubes transmit load to the specimen via the rubber liner. Although the new apparatus ensures the application of three different stresses to a cylindrical core plug, it does not provide the answer to the question of how the permeability characteristics of fractured and unfractured rock specimens can be evaluated. In addition, the effect of increased membrane thickness on the strength-deformation behaviour is not clearly addressed.
Figure 4.6a. Stress state on true triaxial cell (from Smart, 1995).

Figure 4.6b. Axial cross section of the true triaxial cell (from Smart, 1995).
4.2.3. Hollow cylindrical triaxial-cells

The conventional and true triaxial cells are often employed significantly to determine a wide variety of stress paths, even though none of these are capable of rotating the principal stresses. Under the same field stress conditions, principle stresses can still rotate especially upon the application of a torque. There are two major types of hollow cylinder triaxial apparatus. They are characterised by: (a) without rotation of principle stress directions (Dusseault, 1981; and (b) with rotation of principle stress directions (Broms and Casbarian, 1965; Hight et al., 1983; Saada & Baah, 1967 and Lade, 1973). The objective of hollow triaxial devices is to obtain three different principle stresses or to rotate the principal stresses. Figure 4.7 shows a hollow cylindrical sample subjected to torque, axial and radial pressures. The torque, T, is applied about the vertical axis, and the external (\( \sigma_e \)) and internal (\( \sigma_i \)) radial confining stresses are applied to the specimen. The experimental procedure is a somewhat tedious task due to the need for preparing hollow specimens, practical difficulty of testing fractured rock specimens, and the possible development of new cracks during the specimen preparation.

The structural design of a typical hollow cylinder apparatus (Hight et al., 1983) is briefly discussed here. In this hollow cylinder device (Figure 4.8), a large hollow sample is subjected to combined axial, internal and external radial pressures plus a torque. Moreover, the magnitude and the direction of minor and major stresses can be controlled with the magnitude of the intermediate stress. The axial force and the torque are provided by pistons. Radial stresses are directly applied to the inner and outer cylindrical surfaces of the specimen by fluid pressure working through flexible membranes. Proximity
transducers are fixed to the inner and outer surfaces of the specimen to measure the radial deformations. A complete description of the apparatus is given by Hight et al., 1983.

Figure 4.7. Hollow cylindrical specimen subjected to torque, axial and radial stresses.

Figure 4.8. A hollow cylindrical triaxial device (from Hight et al., 1983).
4.2.4 High-pressure triaxial cells

In order to perform tests on hard materials, such as, concrete and rocks, as well as to achieve better understanding of fractured and intact rocks at higher loading conditions, various studies (Hoek and Franklin, 1968; Indraratna and Haque, 1999) have acknowledged the importance of high-pressure triaxial equipment. Permeability characteristics of fractured and intact rocks under elevated compressive stress are important in many applications, such as in the mining industry, nuclear plants and recovery of petroleum.

A simple and inexpensive triaxial cell for testing rock core plugs at high confining pressures was designed by Hoek and Franklin (1968). The section of the apparatus is shown in Figure 4.9. Because of the simplicity of the device, the tests can be carried out in the field as well as in the laboratory at a maximum cell pressure of 70 MPa.

Figure 4.9. High pressure triaxial cell (modified after Hoek & Franklin, 1968).
The cell pressure is provided by a hydraulic pump connected to an oil inlet in the cell. Deformation is recorded using strain gauges attached to the surface of the rock. Rock specimens are usually covered with 1.6mm thick rubber sealing sleeves. Although the equipment provides stress-strain behaviour of rock for preliminary investigation, however, the permeability characteristics of rocks cannot be measured using this facility.

For rock specimens, the provisions for measuring the permeability together with strength parameters under drained and undrained conditions associated with volume changes and pore pressures have been fully accommodated in the high-pressure triaxial apparatus initially developed at the University of Wollongong (Indraratna and Haque, 1999). The high-pressure triaxial system described herein comprises five major components, namely: (i) high pressure cell assembly, (ii) volume change device, (iii) pore pressure measurement system, (iv) axial loading device and (v) the digital display unit (Figure 4.10).

The cell is made from high yield steel having a 100 mm internal diameter and a 120 mm height. The cell walls can withstand a maximum pressure of 150 MPa with a factor of safety 2.0. Sample sizes up to 54 mm in diameter (NX core size) and 120 mm in height can be tested. The cell is confined at the top and bottom by a thick, stepped steel plate, which is firmly held in place by six steel bolts. The water pressure inlet and outlet valves as well as the strain meter connections (i.e. for clip gauge reading) are attached to the bottom plate. The outlet at the bottom of the cell wall is connected to a hydraulic jack for pumping oil prior to testing. The overflow valve for expelling air from the cell is located at the top plate of the cell. Once the specimen is set up inside the cell, oil is manually poured from top of the cell up to the level of the overflow valve. The top
plate is then mounted and all the bolts are tightened. Subsequently, using the hydraulic jack, oil is further pumped into the cell via the cell outlet until all entrapped air is expelled through the overflow valve, which is then closed. Two transducers, one at the inlet and the other fixed to the cell wall are provided to measure the pore pressure and confining pressure, respectively.

Figure 4.10. High pressure triaxial cell (Indraratna and Haque, 1999).
4.2.5 *Volumetric and lateral deformation devices used in the triaxial equipment*

The change of specimen volume is related to the change of cell pressure and or axial load. In order to estimate the deformation of the ground and the displacements of adjacent structures, accurate information on the deformation properties of soils and rocks is needed. Bishop and Henkel (1969) reported that for over-consolidated or compacted soils, the volume changes can vary from 5 to 10% of the initial value, and that the volume change can be as high as 25% or more for normally consolidated or loose soils.

In the triaxial test, the strain state is normally expressed by axial, lateral (diametric) and volumetric strains. There are basically three categories to measure deformation: (a) contact measurements- strain gauges, (b) Linear Variable Differential Transformers (LVDTs) and (c) non-contact devices such as, optical and inductive instruments. It is common practice to record axial deformation using dial gauges or LVDT’s and volumetric changes using volume change devices, such as burettes. However, in some cases, it is important to measure the diametric deformations. For example, in a fractured rock media, permeability is a function of the joint aperture, thus the change in joint apertures in lateral and axial directions of fractured specimens in triaxial tests is important to assess. Various techniques, such as fixing strain gauges directly on specimens, indirect method of measuring volume change (Wawersik, 1975), cantilever devices (Hobbs, 1970), winding a peripheral wire around the specimen (Attinger and Köppel, 1983) and clip gauge transducers have been employed to measure lateral...
deformations in the triaxial and uniaxial tests conditions. In the following section, distinct volumetric and lateral deformation devices are discussed.

(a) **Volumetric deformation devices used in triaxial equipment**

Basically, there are three techniques of measuring the volume change in triaxial tests; (a) the volume of fluid entering the cell to compensate for the change in volume of the sample, (b) the volume of fluid expelled from the pore space of the soil, and (c) the direct measurement of the change in length and diameter of the sample. For partially saturated specimens, volume changes are due to the compressibility and solubility of air in water in the pore spaces. During undrained tests of partially saturated specimens, the volume change can be measured with the aid of mercury 'U' tube method, as shown in the Figure 4.11a. It is important to note here that a correction should be applied to the volume change measurements, because the increase in cell pressure can itself increase the cell volume depending on the wall thickness. Typical cell expansion at various cell pressures is shown in Figure 4.11b. This correction is not applicable for high-pressure cells as the thick wall of the cell is usually high strength steel. For a drained test, the volume change of the saturated specimen is purely a function of the axial load or the cell pressure. Therefore, the volume of water drained from the specimen is indeed the change of volume of specimen, which can be directly measured using a burette or an electronic weighing scale.

(b) **Lateral deformation techniques used in the triaxial equipment**

An indirect method of estimating the radial strain based on the volume change of the cell fluid was proposed by Wawersik (1975). In the conventional triaxial test, the change of the volume of the specimen is replaced by an equal volume of the cell fluid. Using
Figure 4.11a. Schematic diagram for measuring volume change (from Bishop and Henkel (1969).

Figure 4.11b. Typical expansion of cell for different cell pressures.
this concept, Wawersik (1975) used an experimental setup (Figure 4.12) with the intention of measuring the volume of the cell fluid displaced, as a function of specimen deformation. For a homogeneous sample, the radial strain is given by the following expression:

\[
\varepsilon_2 = \varepsilon_1 \left[ \frac{1}{1 - \varepsilon_1} \left( \sum \Delta V \left(C_2 \varepsilon_1 + C_3 \right)F + C_4 \left(\varepsilon_1 + C_5 F \right) \right) \right]
\]

where, \(C_1 = 2A_L\), \(C_2 = \frac{v_s}{E_s A_r}\), \(C_3 = \frac{v_s}{E_r A_r} \left(\frac{L_t}{L_p - L_t} \right)\),

\[
C_4 = \left[ \frac{A_t - A_s}{2A_t} \right] \quad \text{and} \quad C_5 = \frac{L_c}{E_s A_r L_t}.
\]

\(A_t\) = cross sectional area of test sample end caps that are commonly placed between the specimen ends and the loading pistons.

\(L_t\) = Length of test sample

\(L_e\) = combined length of end caps

\(E_s, v_s\) = elastic constants of loading piston and end-cap material

\(A_s\) = cross sectional area of loading pistons

\(L_p\) = effective internal length of pressure vessel

\(F\) = axial force and

\(\sum \Delta V\) = cumulative, incremental volume adjustments of confining pressure medium

Accuracy of calculated strain values entirely depends on the measured volume change of the specimen. Wawersik (1975) discussed the disadvantages of strain gauges for direct strain measurements. It is possible to eliminate such problems using proper strain gauges with appropriate fixing materials, such as compatible strain glues and clip gauges. However, the Equation (4.1) can still be used to estimate lateral strains in order to perform direct and indirect comparisons. The drawbacks of Wawersik (1975)
method include the inability to obtain volume change measurements at varying changing cell pressures.

![Schematic diagram of the strain device](image)

Figure 4.12. Schematic diagram of the strain device (Wawersik, 1975).

Attinger and Koppel (1983) used a technique, in which a wire was wound three times around the specimen with a prestress of about 100MPa. The strain measurements are based on the principle of the change of electrical resistance of a stretched resistance wire. The electric current is applied to the resistance wire via two copper conductors. The resistance wires and the copper wires are glued to the specimen as shown in Figure 4.13. The actual change of resistance is determined by the Wheatstone bridge principle. An amplifier is used to amplify the output signal, which is proportional to the lateral strain. The method described here gives the circumferential strain rather than the local strain, which is usually obtained by gluing a strain gauge to the specimen. The
application of this method to triaxial apparatus is not an easy task, because leakage can occur due to the two copper conducting wires at the one end of the specimen. The uncertainty of measuring large deformations is also regarded as a problem.

![Diagram of lateral deformation of rock using resistance wire method](image)

Figure 4.13. Lateral deformation of rock using resistance wire method (modified after Attinger and Koppel. 1983).

In a true triaxial cell, displacement of all four sides of the cubical specimen can be recorded using thin rods and Linear Variable Differential Transformers (LVDT’s), as illustrated in Figure 4.14 (Michelis, 1988). The cumulative deformation is transmitted through rods to two pistons, and then through tubes (filled with mercury) to one piston, where the LVDT is connected. The measured strain represents local strain in two directions. In order to obtain an average strain, one has to install at least three strain devices to one side of the specimen. However, in this technique, practical difficulties arise due to the presence of several pistons and LVDT’s.
Figure 4.14. Deformation measuring device for a specimen subjected to tension (Michelis, 1988).

Lateral deformation can be measured using two micrometers mounted on the exterior of the cell chamber as shown in Figure 4.15 (Silvestri et al., 1988). Accuracy of the local strain entirely depends on the accuracy of the micrometer. Mochizuki et al. (1988) designed a new technique to measure deformation using no-contact gap sensors as well as evaluating the shape of the deformed rectangular specimen. Figure 4.16 shows a rectangular specimen with markers of aluminum foil (10 x 20 x 0.1 mm) on its sides together with gap sensors set in position for testing. The no-contact gap sensors induce the variation of outlet voltage when conductors such as iron or aluminum change the distance to the markers on the specimen sides. The main advantage of this method is that the deformation pattern of each side can be mapped. Although it is suitable for uniaxial conditions, this approach is not practical for triaxial situations.
4.3 Triaxial equipment for two-phase flows

In order to study stress/strain behaviour and permeability characteristics of rock or soil, one has to incorporate actual fluid flow in the testing equipment. Soil/rock can either be fully saturated or unsaturated states. If fully saturated soil or rock carries a single fluid, a single-phase flow analysis can be carried out. If two or more fluids are found, then the specimen is considered to be in an unsaturated state. Under these circumstances, a
two-phase flow analysis should be carried out. Figure 4.17 clearly shows whether single or multiphase flows analysis is carried out depending on the number of fluids present in the media. The stress state acting on an unsaturated and fully saturated rock/soil element are shown in Figure 4.18. For an example, in two-phase flow of water-gas, capillary pressure (i.e. $P_w - P_a$) acts in addition to the ground stresses. However, if $P_a > P_w$, then suction pressure (i.e. $P_a - P_w$) acts on the solid element to increase the apparent strength.

![Flow Diagram](image)

Figure 4.17. State of fluid in a rock mass.
(a) Single-phase (water flow only).

(b) Two-phase (water and air flow).

Figure 4.18. Comparison of stress state under two-phase and single-phase flow conditions.
4.3.1 Apparatus for unsaturated soils

For unsaturated soils, various laboratory methods (Klute, 1965 and Barden et al., 1969) have been used to investigate permeability characteristics under steady state and unsteady state conditions. The constant air pressure is measured using a manometer. The tests have been carried out for different suction pressures \((P_a - P_w)\), where \(P_a\) is the air pressure and \(P_w\) is the water pressure. In Klute’s (1965) apparatus, the change of permeability with respect to change of confining pressure or axial stress has not been addressed. An advanced triaxial equipment was developed to measure both water and air permeability of soil (Barden et al., 1969), in which the effect of lateral pressure was incorporated. In this apparatus (Figure 4.19), air and water pressures are applied to the specimen from top and bottom respectively, for a given lateral pressure. The fluid flow direction is not properly modelled in the apparatus, because fluid flow in reality is in one direction. Moreover, this equipment is not capable of applying axial stress, and also, no strain measurement devices are attached to the specimen.

Figure 4.19. Triaxial testing apparatus for unsaturated soil (from Barden et al., 1969).
4.3.2 Two-phase triaxial equipment for rocks

Although much experimentation has been carried out to understand the complete two-phase (air-water) flow behaviour in the field of chemical and mechanical engineering, the proper understanding of two-phase flow behaviour in jointed rocks still remains at infancy. As discussed earlier, some triaxial facilities are capable of measuring either the pore water pressure or pore air pressure or both within a fractured rock, but are not capable of measuring the relative permeability (air or water) of a fractured specimen. However, a fractured rock mass is generally associated with a multi-phase flow system, such as, water +air, water +air+solid or water +air+oil. In order to conduct an experimental study of two-phase flows through fractured rock specimens, a novel triaxial equipment “Two-Phase, High-Pressure Triaxial Apparatus” (TPHPTA) was designed by the author of this thesis.

The following section describes the salient features of the two-phase triaxial apparatus, which can measure the relative permeability characteristics, as well as the stress-strain behaviour of rocks subjected to axial and confining pressure conditions. The TPHPTA is capable of investigating both single and two-phase flows through soft and hard rocks. The schematic diagram and a photograph of the TPHPTA are illustrated in Figures 4.20a & 4.20b. The cell is made from high yield steel having a 100mm internal diameter and 120mm height. The cell walls can withstand a maximum pressure of 150 MPa with a factor of safety 2.0. The cell is confined at the top and bottom by a thick, stepped steel plate which, is firmly held in place by six steel bolts. The modified cell can accommodate a range of specimens from 45mm to 60 mm in diameter. In two-phase flow, both water and air simultaneously flow through the specimen.
Figure 4.20a. Schematic diagram of the Two-Phase, High-Pressure Triaxial Apparatus (TPHPTA).

Figure 4.20b. Experimental setup for TPHPTA.
In this equipment, water and air phases are carried by two separate lines to the bottom end of the specimen. In order to prevent water and air interaction before entering the specimen, the separate lines which carry water and air are integrated with several on/off valves and check valves as show in the Figure 4.21. These valves attached to the bottom plate to ensure that there is no back flow of one phase through the line of the other phase. In order to measure the pressure of each phase, a pressure transducer is attached to each phase line.

![Schematic diagram of the triaxial base with water and air phase lines.](image)

Noël. Schematic diagram of the triaxial base with water and air phase lines.

The outlet at the bottom of the cell wall is connected to a hydraulic jack for pumping oil prior to testing. The overflow valve for expelling air from the cell is located at the top plate of the cell. Once the specimen is set up inside the cell, oil is manually poured from top of the cell up to the level of the overflow valve. The top plate is then mounted and all the bolts are tightened. Subsequently, using the hydraulic jack, oil is further pumped into the
cell via the cell outlet until all entrapped air is expelled through the overflow valve, which is then closed.

The readings of the inlet water pressure transducer, inlet air pressure transducer, outlet pressure transducer of both air and water, volume change device, axial and lateral deformations are monitored continuously, and displayed digitally on the instrumentation display unit. In order to measure the quantity of the each phase, the mixture from the specimen has to be separated into water and gas. Using the gravity separation technique, water and air which migrate through the rock specimen are separated by a dreschel bottle as shown in Figure 4.22.

Different separation techniques for gas/liquid, liquid/liquid, liquid/solid are discussed later. The water flow measurement is recorded using an electronic weighing scale, and an electronic film flow meter is used to monitor the continuous airflow. Details of different airflow meters including the film flow meter are described at the end of this chapter.

In two-phase flows, the time taken to reach the steady state condition is much longer than the single phase condition for given boundary conditions. Therefore, a datalogger is employed for acquisition of measurements from all transducers, flow measurement devices, strain meters and LVDTs. When the output voltage of some transducer is not large enough, it is first amplified by an amplifier before transmission to the datalogger.
4.3.3 Separation techniques for two-phase mixture

The most common types of fluid phases that are encountered in hydro-mechanics are (a) gas-liquid, (b) liquid-liquid, (c) gas-solid, and (d) liquid-solid. The separation techniques of these mixtures are listed in Figure 4.23. However, no attempt is made here to describe all the separation techniques, except for gas-liquid mixtures.
Figure 4.23. Separation techniques for different two-phase mixtures.
The gas in liquid can be generally of two types: (a) unstable bubbles and (b) stable gas bubbles, which are difficult to separate. The unstable gases in water are normally separated by the action of gravity. The water-gas flow is allowed to move in a laminar flow path until the gas bubbles come to the surface. Depending on the gas flow rates, different sizes of separators (e.g. static tank and continuous flow tanks) can be used. The gravity action may not be enough for gases in high viscosity fluids, and in this case, a centrifugal action (e.g. using Versator machine by Cornell Machine Co.) may be employed. Techniques such as, thermal, electrical and mechanical methods are applied for stable gas in liquids. In the TPHPTA, the mixture contains both gas and water, where gas is in an unstable form. The fluid mixture from the triaxial specimen is allowed to flow through a dreschel bottle, as shown in the Figure 4.22. The unstable gas in the water separates from the bottle and passes to the film flow meter, where the water stays in the dreschel bottle sitting on an electronic balance.

In the TPHPTA, an alternative device is used to measure the volume change of the specimens, which consists of a cylindrical chamber having an internal diameter of 25 mm and a height of 90 mm. A piston is attached co-axially to the cylindrical chamber, in which the piston moves up or down depending on the volume increase or decrease of the specimen resulting in the displacement of oil applying confining pressure (Figures 4.24a & 4.24b). The movement of the piston is continuously monitored by a LVDT. The top chamber of the volume change device is connected to a hydraulic jack and the bottom is connected to the cell. Once the cell is filled with oil, the hydraulic jack is disconnected and the volume change device is connected to the cell. The required cell pressure is applied by another hydraulic jack.
Figure 4.24a. Volume change device based on change of cell fluid.

Figure 4.24b. Volume device installed in TPHPTA.
Wires from strain gauge to the strain metre
Strain gauge glued to inner side of the ring
Membrane fits to the specimen
Mild steel ring (2mm x 8mm)
Gap (5mm) of the ring

Figure 4.25a. Clip gauge for lateral deformation of rocks

Figure 4.25b. Clip gauges mounted on the membrane.
The clip gauge transducer is calibrated using an internal micrometer and a strain meter. The internal micrometer is usually placed along the diameter of the clip gauge, and the diameter of the micrometer can be adjusted to touch the clip gauge at both ends of it. The corresponding strain meter reading with a gauge factor of say 2.14 would be recorded.

Figure 4.26 indicates that the clip gauge transducer produces a linear relationship between the diameter change and the strain meter readings. The relationship between the strain meter readings with deformation is thereby established, which is ultimately used for the back calculation of aperture changes in a regular joint under various confining and driving pressures. The advantages of clip gauges over other methods are listed below:

(a) No difficulty involved in mounting transducer on a metal ring. In most other devices, the transducers are mounted on the surface of the rock specimen, which becomes difficult on coarse grained, porous, fractured and saturated rock or soil specimens,

(b) Measures circumferential strain rather than local strains,

(c) Accuracy, simplicity and re-usability,

(d) Possibility of using several clip gauges around a single specimen,

(e) Water proofing of the strain gauge is guaranteed, and

(f) Applicable in triaxial, uniaxial and polyaxial stress states.
4.3.4 Membranes used in triaxial apparatus

This is certainly one of the most critical aspects in conventional triaxial testing, because the role of membrane on sample behaviour cannot be ignored. If confining pressure is increased significantly, then the membrane can penetrate into the sample surface. The membrane penetration is significant for coarse-grained soil or fractured rock specimens. Therefore, the thickness, type of material and easy manufacturing process are essential requirements in the design of membranes. In the past, various materials such as natural and synthetic rubber and annealed copper jackets of thickness ranging from 0.2 to 3.0 mm have been used for manufacturing membranes. The magnitude of cell pressures, surface of specimen (i.e. fractured/coarse grained), shape and size of the specimen, change of temperature, duration of tests and types of cell fluid govern the required thickness and type of membrane. Hoek and Franklin (1968) used two kinds of synthetic rubber for manufacturing membranes: (a) silicone rubber - for low cell pressures and (b)
urethane rubber to withstand high pressures. A 1.6 mm thick silicone rubber membrane and urethane membrane can withstand up to 70 MPa confining pressure. Bicycle inner tubes, which come in a variety of diameters, are often recommended for membranes when they are subjected to low cell pressures. According to Dusseault (1981), 1.5 mm thick latex rubber can withstand up to 6 MPa cell pressure, and 3 mm thick Neoprene membrane can withstand even greater pressures. For testing granite specimens under high pressure, Brace et al. (1968) used a 3 mm thick polyurethane rubber jacket, which was strengthened by clamping with several loops of steel wire. The drawbacks of thick membranes are that they provide excessive confinement which leads to an increase in compressive strength, as well as influencing the failure mode.

(a) Membranes for TPHPTA

For the TPHPTA, the writer has carefully designed a series of moulds using Perspex material to accommodate different specimen sizes (field cores), as shown in Figure 4.27. Having considered the important role of the membrane, polyurethane has been selected as the membrane material for TPHPTA. The membrane should not be too thin or soft as it can get damaged either at high lateral pressures or during extended testing periods. On the other hand, harder material influences the stress-strain behaviour of rocks. The polyurethane has a wider range of hardness than other materials such as, rubber and plastic (Figure 4.28). Under triaxial test conditions, the membrane is often surrounded by an oil medium and the inner surface is in contact with gases or water. It is important to note that the membrane material should be water resistant and it should not react with chemicals such as oil and kerosene. The water absorption of polyurethane is as low as 0.3% by weight, and swelling is negligible. Also, polyurethane does not react with oil or kerosene.
Era Polymers Pty Ltd (1998) supplies the polyurethane (TU 801 and TU 901) in two parts: (a) part A-resin and (b) part B-hardener. The corresponding mixing ratios by
weight are 100/51 for TU 801 and 100/44 for TU 901, respectively. The mixing could be carried out using a special type of resin gun (dispenser), dual foil pack (two tubes) and 0.3m long nozzles with internal static mixer, as shown in Figure 4.29.

Figure 4.29. Casting of membranes for TPHPTA.

The dispenser enables two parts of material A and one part of material B to flow into the 0.3m long nozzle, before the material is thoroughly mixed. This mixture is then pumped into a mould (pre-coated with a release agent) from the bottom until it overflows, ensuring no entrapped air. The mould is kept for 24 to 48 hrs for curing purposes under room temperature. The stress-strain behaviour of the cast membrane is shown in Figure 4.30.
Figure 4.30. Stress/strain behaviour of new and used membranes.

\[(b)\] **Membrane correction for lateral deformation**

This correction is necessary when recording the lateral deformation of joints associated with confining pressures. In TPHPTA, the deformation is measured using 2 clip gauges, which are mounted to the membrane. Therefore, the total deformation indicated by the strain meters include the deformations of both membrane and specimen. The following procedure was undertaken to estimate the membrane deformation only:

(a) Consider different specimens (i.e. 44, 51 and 54 mm φ) made of high strength steel;

(b) Each steel specimen was covered with the membrane and two clip gauges (mounted to the membrane);

(c) The clip gauges were then connected to the two strain meters;

(d) The triaxial cell was filled with oil, and a small cell pressure (10kPa) was applied to record the initial strain meter readings;
(e) For all specimens at different cell pressures, the strain meter readings were recorded (Figure 4.31(a) & 4.31(b)). According to Figure 4.31, more than 90% of membrane deformation is attained below confining pressures of 500kPa;

(f) The following average correction factors are then estimated for different range of cell pressures, based on the tests conducted on all specimens (Table 4.1).

![Figure 4.31a. Deformation of membrane at different cell pressures (0 - 0.6MPa)](image1)

![Figure 4.31b. Deformation of membrane at different cell pressures (0.5 - 12MPa)](image2)
Table 4.1. Membrane correction factors for different cell pressures.

<table>
<thead>
<tr>
<th>Cell pressure ranges (kPa)</th>
<th>Correction factors (strain rate)</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-50</td>
<td>-</td>
<td>Unstable region</td>
</tr>
<tr>
<td>50-200</td>
<td>-</td>
<td>Unstable region</td>
</tr>
<tr>
<td>200-600</td>
<td>1.5</td>
<td>Unstable region</td>
</tr>
<tr>
<td>600-1500</td>
<td>1.0</td>
<td>Nearly constant</td>
</tr>
<tr>
<td>1500-4000</td>
<td>2.0</td>
<td>Nearly constant</td>
</tr>
<tr>
<td>4000-8000</td>
<td>2.0</td>
<td>Nearly constant</td>
</tr>
<tr>
<td>8000-12000</td>
<td>1.0</td>
<td>Nearly constant</td>
</tr>
<tr>
<td>12000-15000</td>
<td>1.0</td>
<td>Nearly constant</td>
</tr>
</tbody>
</table>

4.3.5. **Determination of cracking pressure for membrane**

Cracking pressure can be defined as the minimum inlet fluid pressure, which permits flow between the specimen wall and the membrane for a given confining pressure. In the past, various estimates of the difference between the fluid pressure and the cell pressure have been stated in the range of 100-200 kPa (i.e. within this range, no fluid flow takes place between the specimen and membrane). However, cracking pressure depends on the confining stress, fluid viscosity, the material properties of membrane (e.g. thickness, hardness) as well as their end conditions, including the effect of clamps or ‘o’-rings. The following procedure is suggested for estimating the cracking pressure:

(a) For example, consider a high strength steel specimen, 54mm in diameter. To ensure ‘zero’ permeability of the steel specimen needs to be coated with a paint;
(b) After mounting the steel specimen in the TPHPTA, the fluid pressure is varied for a constant cell pressure. For instance, at 250 kPa cell pressure, observe whether any flow takes place at an inlet water pressure of 75 kPa;

(c) If not, increase the inlet water pressure gradually (cell pressure kept constant) until some flow is observed at the outlet;

(d) This inlet water pressure causing water flow between the sample and the membrane is now defined as the cracking pressure at the corresponding cell pressure;

(e) Similarly, for both water and air, the cracking pressure is observed for different cell pressures.

Figure 4.32 shows the normalized inlet fluid pressure against the confining pressure. The normalized fluid pressure is the inlet pressure divided by the confining pressure. The points on the graph represent the minimum normalized inlet pressure causing flow.
between the specimen and the membrane. At low confining pressures (< 500kPa), the normalized cracking pressure for air is between 0.6 to 0.8, and for water it is between 0.8 to 0.9. Clearly, the cracking pressure for low viscosity fluids, such as air would be lower than for fluids such as water and oil. It is expected that at elevated confining pressures, the cracking pressure for high viscosity fluids can approach the magnitude of the applied confining pressures.

4.3.6 Flow meters

For routine practical flow measurements, there exists a number of flow meters and flow measuring techniques. Flow meters are separated into displacement type and velocity type. Venturi meters, orifices, nozzles and elbow devices are the most common velocity type instruments. The velocity type flow meters are normally employed to measure large quantities of flow, such as in large pipes, rivers and streams. Displacement type flow devices indicate flow rate directly, by recording the volume and the time. Weighing scales and rotary types are examples of displacement type flow meters. These are normally employed to record small flow rates, and mainly used in laboratory environment and domestic water distribution systems.

Apart from the cost, the selection of flowmeters for a particular application depends on the type of flow (e.g. gas, liquid or multi-phase), special fluid constraints (e.g. corrosive), design constraints (e.g. precision and flow range) and environmental considerations (e.g. humidity and temperature). Generally, orifice, venturi meters and vortex devices are used for both gas and water flow measurements. Thermal flow
devices, shielded microflowmeter and bubble meters are used only for gas flow measurements. For two-phase flows, electromagnetic flow meters may be useful.

Flow measurements through soil or rock usually involves very small quantities of gas or water or both. At high confining pressures, flow rate can be smaller than say 0.2 ml/min. In general, for small flow volumes of air, shielded micro flow meters or bubble meters can be employed, whereas for small flow rates of water, a precision weighing scale is usually sufficient. For TPHPTA, a typical bubble meter, 'Electronic Film Flow meter' (STEC, 1998) was found to be most appropriate, and these flow meters are designed for automatic measurement with high accuracy. Such electronic flow meters are also equipped with a high precision sensor to incorporate atmospheric pressure changes. This equipment basically consists of two parts: (a) measuring unit and (b) measuring tubes, for typical flow rates of 0.2 ml/min to 10 l/min. The operating principle of the film flow meter is shown in Figure 4.33.

![Diagram of film flow meter](image)

Figure 4.33. Operating principle of the film flow meter (STEC, 1998).
When non-soluble gas in water, such as, N₂, Air, O₂, H₂, CO, CH₄ and Ar flows, a soap water film formed at the mouth of the measuring tube by a film ring will move up in the measuring tube, which is calibrated to a high precision. The equipment calculates the flow rate when it measures the time taken by the soap film to move between the 'start' and 'stop' detection points. The microcomputer processes the temperature and atmospheric pressure to correct the measured flow rate.

4.4 CONCLUSIONS

The significant efforts to study the stress-strain and permeability characteristics of soil and rocks under laboratory conditions are clearly evident from the numerous triaxial apparatus described in this Chapter. Some triaxial rigs were designed purely for studying stress-strain behaviour of a material, while others were capable of performing coupled fluid flow deformation of soil and rocks. Based on the stress-strain concept, all triaxial equipment can be grouped as: (a) 2-D stress field in which σ₁ ≠ σ₂ = σ₃ and (b) 3-D stress field in which σ₁ ≠ σ₂ ≠ σ₃. 3-D stress field triaxial units are not very popular among researchers, due to their practical difficulties involving specific specimen shapes, specimen preparation and difficulties of accommodating measuring devices such as volume change and diametric deformation gauges. It is important to note that 2-D stress field does not always represent realistic ground stresses. However, the application of this method is common in both academia and industry, because it is relatively easy to apply circumferential confining pressure (σ₃) and an axial stress (σ₁).

Stress-strain and permeability properties of rocks can be predicted reasonably well, if
actual fluid flow in rocks is simulated under the laboratory conditions. When rock fractures carry more than a single fluid, multi phase triaxial equipment is necessary to model the actual fluid flow field system. The two-phase triaxial apparatus (TPHPTA) capable of accommodating fluid flow through soft and hard rocks under 2-D stress fields was described in detail. This specially designed apparatus can accommodate a range of specimens from 45mm to 60mm in diameter, and is capable of conducting both single and two-phase flows under different confining pressures, axial stresses and capillary pressures. In addition, both transient and steady state flow can be observed for given boundary conditions. Tests on natural fractured rock specimens under two-phase flow using the TPHPTA are described later in Chapter 8.
5.1 INTRODUCTION

The aim of this chapter is to discuss saturated fluid flow through intact and fractured rock specimens using the designed triaxial equipment. The effect of confining pressure, axial stress, inlet fluid pressure and loading/unloading behaviour on permeability are investigated. If the rock mass is saturated with a single fluid, then a single-phase flow analysis is carried out. Fully saturated flow techniques have been widely investigated during the past four decades for small scale and large scale conditions, and the relevant theories were discussed in Chapter 2. Permeability is an important parameter for various applications including fluid flow analysis, stability of rock slopes and underground tunnels. Permeability is simply the ability to conduct fluids, such as water and gas flows through porous or fractured media, such as soil and rocks.

For crystalline rocks, fluid flow through the rock matrix is much less than that through any fractures, because, the extent of interconnected pores and the pore sizes in hard rocks are generally small. Permeability can greatly influence the mechanical behaviour of rocks, thereby increasing or decreasing the stability (failure mode) of rock structures. As shown in Figure 5.1, fluid flow within a rock specimen can take place either through the rock matrix or interconnected discontinuities or combination of both. Depending on the nature of flow, the corresponding permeability term must be used in the flow analysis. For example, if the fluid flow is governed by the rock matrix (e.g. intact coarse sandstone), the matrix permeability needs to be identified, whereas in the
presence of joints particularly in low permeability rocks (e.g. granite, slate), the fracture permeability governs the fluid flow behaviour within the rock mass. Under single-phase fluid flow, permeability can be divided into three main categories as given below:

(a) Matrix permeability,
(b) Fracture permeability, and
(c) Dual permeability.

![Intact rock with voids, where possible flow occurs through interconnected voids](image1)

![Specimen with a major discontinuity, where flow occurs through discontinuity and any interconnected voids](image2)

Figure 5.1. Fluid flow paths in intact and fractured rock specimens.

In hydromechanics, the coefficient of permeability, $k$ has the dimensions of $L^2$ (units in $m^2$ or $cm^2$). In conventional soil mechanics, the permeability coefficient has different units, i.e. m/sec. In order to avoid any confusion, it is important to express the conductivity ($K$) in terms of the coefficient of permeability, where $K$ has dimensions of $LT^{-1}$. Common units to express the permeability are listed in Table 5.1.

$$K = \frac{\rho g k}{\mu}$$

(5.1)

<table>
<thead>
<tr>
<th>Description</th>
<th>Units</th>
<th>Equivalent units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coefficient of permeability (rocks) - $k$</td>
<td>$1 m^2$</td>
<td>$10^4 cm^2$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$10^{12}$ Darcy</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$10^{15}$ millidarcy</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$10^{18}$ microdarcy</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$10^{21}$ nannodarcy</td>
</tr>
<tr>
<td>Conductivity of rock ($K$)</td>
<td>$m/sec$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$1K (m/sec) \equiv 10^7 m^2 k$ (for water)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$1K (m/sec) \equiv 10^6 m^2 k$ (for air)</td>
</tr>
</tbody>
</table>

Table 5.1: Common units used in permeability of rocks
In this thesis, the permeability of discontinuities is referred to as the fracture permeability or the intrinsic permeability, whereas, intact rock permeability is referred to as the matrix permeability. Therefore, the combined permeability of rock mass is given by:

\[ k = k_f + k_m \]  \hspace{1cm} (5.2)

where, \( k \) is the combined permeability of rock mass, \( k_f \) and \( k_m \) relationships are the fracture permeability and matrix permeability, respectively. The application of fractured and matrix permeability for typical cases is shown in Figure 5.2.

![Diagram of permeability application](image)

Figure 5.2. Application of fractured and Matrix permeability in different occasions (after, Brady and Brown, 1994).

The permeability of rocks is measured using small-scale standard core specimens, or it can be carried out in the field using Packer tests in boreholes. Although the matrix
permeability is relatively easy to determine from laboratory tests or insitu tests, the fracture permeability measured from insitu tests can vary depending on the test zone. This is because, it entirely depends on the number of fractures, which intersect the borehole. Also, the existing fractures may deform due to stress release or new fractures may open due to excessive hydraulic pressure or due to vibration of the drilling tools. In spite of the effect of sample size, laboratory test results on fractured rock specimens can provide the permeability of each individual fracture system for a given stress condition. The aim of this chapter is to examine the permeability in small scale i.e., laboratory conditions.

Darcy's law is often employed for fluid flow calculation in soil and rock mechanics when the media is fully saturated with a single fluid phase. In a simplified form of Darcy's law, the hydraulic gradient is linear along the fluid flow path and given by the total energy difference between two points (Eqn. 5.3). In this study, the effects of velocity and gravity on hydraulic gradient were neglected for the estimation of permeability. This is because, the lengths of the tested specimens were less than 0.13m, hence the effects of gravity is negligible compared to the inlet fluid pressure. The velocity of the fluid phase within rock matrix or rock joint is very small, hence the term \( \frac{v^2}{2g} \) can be neglected. In general, the hydraulic gradient along a \( dx \) length can be expressed as:

\[
\frac{dh}{dx} = \frac{h_1 - h_2}{dx} = (z_1 - z_2) + \left( \frac{p_1 - p_2}{\rho g} \right) + \left( \frac{v_1^2 - v_2^2}{2g} \right) - \text{losses}
\]

(5.3)

where, \( z \) is elevation head, \( p \) is fluid pressure and \( v \) is velocity, \( h_1 \) and \( h_2 \) are total heads at point 1 and 2 in the direction of flow.
5.2 FLOW THROUGH INTACT ROCK MATERIAL

'Rock material' is the term used to describe the intact rock between discontinuities; it might be represented by a hand specimen or drill core examined in the laboratory (Brady and Brown, 1994). In other words, there are no fractures existing within the rock matrix. However, in reality, microfractures can still exist in rock material, although they may not contribute to any reduction in the strength properties.

In this section, an attempt is made to discuss the important role of rock material on fluid flow behaviour. For certain types of rock (e.g. coarse sandstone and limestone), it is found that significant flow takes place though the rock material (matrix), whereas, negligible flow is expected to take place through low permeability rocks, such as granite and slate. Typical permeability values for various rocks are tabulated in Table 5.2 (Brace et al. 1968). The permeability measurements are based on either (a) steady state flow or (b) transient state, in which decay of pressure is observed. In the past, the transient method has been widely used by several researchers including Brace et al. (1968) and Kranz et al. (1979). The steady state flow method was adopted for flow measurements in this study.

Table 5.2. Typical values of porous (matrix) permeability (Brace et al., 1968).

<table>
<thead>
<tr>
<th>Rock</th>
<th>Permeability, nannodarcy</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fine-grained dolomite, Tennessee</td>
<td>80</td>
<td>Ohle, 1951</td>
</tr>
<tr>
<td>Fine-grained limestone, Tennessee</td>
<td>30</td>
<td>Ohle, 1951</td>
</tr>
<tr>
<td>Coarse-grained dolomite, Tennessee</td>
<td>6000</td>
<td>Ohle, 1951</td>
</tr>
<tr>
<td>Granite, Barriefield, Ontario</td>
<td>50</td>
<td>Ohle, 1951</td>
</tr>
<tr>
<td>Granite, Quincy, Mass</td>
<td>4600</td>
<td>Ohle, 1951</td>
</tr>
<tr>
<td>Diabase, Hudson, N.Y.</td>
<td>0.8</td>
<td>Ohle, 1951</td>
</tr>
</tbody>
</table>
Under steady state flow rate approach, for a given cylindrical rock specimen, the coefficient of matrix/intact rock permeability \( k_m \) was estimated using Darcy's law as given below.

\[
k_m = \frac{4q\mu}{\pi D^2 (dp/dx)}
\]  

where, \( q \) is the fluid flow rate through the specimen, \( dp/dx \) is the pressure gradient along the length \( (dx) \) of the specimen, \( \mu \) is the dynamic viscosity of the fluid and \( D \) is the diameter of the specimen. Apart from the hydraulic gradient and surrounding stress applied on the specimen, the matrix permeability depends on the pore size (voids), shapes and the interconnectivity of voids. If the fluid travelling through the porous rock is gas, then the component of the matrix coefficient of air permeability is estimated according to the following equation:

\[
k_m = \frac{2qp_e\mu L}{(p_i^2 - p_e^2)A}
\]

where, \( q = \) gas flow rate, \( \mu = \) dynamic viscosity of gas, \( L = \) length of the specimen, \( A = \) cross section area of specimen, \( p_i = \) inlet pressure of gas, and \( p_e = \) exit pressure of gas.

5.2.1 Test procedure

Laboratory tests on granite were carried out using the designed high pressure triaxial equipment (TPHPTA) for different boundary conditions. Granite specimens were selected to investigate fluid flow through fractures rather than through rock matrix. Intact and fractured rock specimens were supplied by Strata Control Technology (Wollongong) for testing. Single tension fractures were induced on intact rock.
specimens, in order to obtain a fairly regular joint. The rock samples used in this study were granite cylinders of 55mm in diameter, and 110mm in length. Typical tests specimens (Figure 5.3) were cored using a small scale coring machine in the laboratory, for obtaining intact rock specimens.

![Intact rock specimen](image1)

![Fractured specimen (induced tension fracture)](image2)

![Fractured specimen (natural)](image3)

![Fractured specimen (natural)](image4)

Figure 5.3. Some typical tested specimens.

Having smoothened both end surfaces after making them parallel, the rock specimens were covered with a 2mm thick polyurethane rubber jacket. In order to measure the diametric deformations, two clip gauges 30mm apart were mounted to the jacket. After placing two porous discs, one at each end, the specimen was placed in between a pair of steel caps using horseshoe clamps. Using two horseshoe clamps, the membrane was
tightened to the top and bottom seating so that no fluid flow through the membrane and the specimen could take place. The spiral tube was fixed to carry fluid flow from the specimen to the outlet. Oil was used as the hydraulic pressure medium, and the permeating fluid (either water or air) was made to flow within the sample joints and pores by applying a hydraulic head to the bottom of the sample. The mass and volume flows of water or air that permeated through the sample were monitored using an electronic weighing scale or an airflow meter, respectively. The required cell pressure was applied to the sample via a hydraulic pump, and the axial load was applied through a piston using an Instron machine. Flow was measured under transient and steady state conditions, and the continuously monitored data were stored by a datalogger. Flow measurements, pore pressures, diametric and axial deformations, were taken at different cell pressures and axial stresses, once the system reached equilibrium in steady state flow. Steady state flow was observed after a certain time period. Figure 5.4 shows the time taken to fully saturate the specimen with air for a given confining pressure of 1.5MPa, and at inlet air pressures of 0.25MPa.

Figure 5.4. Observation of steady state flow.
The flow rate increases during the first 15 minutes, followed by a nearly constant flow rate after about 30 minutes. At this constant flow (steady state), the rock specimen has attained its maximum interconnectivity of pore structure. The data on the following diagrams are based on the steady state flow condition.

For a given inlet fluid pressure (0.25MPa), flow rates against varied confining pressures are presented in Figure 5.5. The corresponding matrix permeability of water and air based on Equations 5.4 and 5.5 is shown in Figure 5.6. Increase in confining pressure results in a decrease in permeability due to the continuous deformation of pores, which subsequently results in a reduced interconnectivity of the flow paths.

Figure 5.5. Effect of confining pressure on flow rate through intact granite specimens.
As seen in Figure 5.6, when the effective confining pressure exceeds 10MPa, the variation of permeability becomes negligible. This shows that the rock matrix has attained its residual permeability. The matrix permeability at relatively large confining pressure is very small, and it is usually neglected when fractures dominate flow conditions.

![Figure 5.6. Effect of confining pressure on matrix permeability of intact granite specimens.](image)

Brace et al., 1968 have also shown that the matrix permeability of hard rocks at elevated confining pressure is negligible, and can be in the order of $10^{-21} \text{ m}^2$ (Figure 5.7).
Tests were also conducted to study how the matrix permeability varies with axial stress. For a given confining and inlet fluid pressure, the axial stress was increased step by step, and the steady state flow was recorded for each value of axial stress. In the case of intact granite, the flow volume of water hardly changes during the initial increase of the axial load, as illustrated in Figure 5.8 (i.e. negligible permeability). This is because, the initial applied stress range is not sufficient to cause any subsequent change in the existing pore structure of the specimen. However, once a critical axial stress of 50MPa is reached for water saturated specimen, increase in permeability is observed until failure. The increased permeability beyond 50MPa is the result of new crack formation and pore structure changes. In the case of air saturated specimen, sudden increase of
flow is observed due to the possible occurrence of an initial crack, that conducts air. The air permeability remains relatively constant until such time the permeability increases again approaching failure (i.e. initiation of further cracking).

Figure 5.8. Effect of axial stress on matrix permeability (tests on intact granite).

Water saturated specimens show higher axial strain at lower axial stress (Figure 5.9). The opposite trend was observed for air saturated specimens. It is evident that a higher failure load and lower strain is expected for an intact rock sample, if the permeating fluid is air (Curve A). This is because of the increased air content that makes the rock specimen more brittle. In contrast, for water-saturated samples (Curve B), a lower ultimate strength is attained at approximately two times the axial strain of an intact rock sample subjected to internal air flow (Curve A). This suggests that (a) ductility of rock mass increases and (b) deformation modulus decreases, when the water content is
increased. It is important to note that compressible gas phase generates less pore pressure, in comparison with relatively incompressible fluids which induce higher pore pressure.

![Graph](image)

**Figure 5.9.** Stress-strain behaviour of saturated intact granite specimens.

### 5.3 Fluid Flow Through a Single Joint

The mechanical and geometrical characterisation of a single rock joint provides the basis to understand the fluid flow-deformation behaviour in a fractured rock mass. It is difficult to give a comprehensive description of flow behaviour even in a single joint, because of the number of variables involved in three-dimensions. Therefore, much analysis is based on plane strain (2-dimensional condition). The main factors
controlling fluid flow through a single rock joint are shown in Figure 5.10. Out of these controlling factors, the magnitude of the joint aperture is the major parameter, which is a function of external stress, fluid pressure and geometrical properties of the joint.

In early studies, flow through a single joint was simulated as flow through a channel or pipe, in which no deformation due to external stress was considered (Lomize, 1951). However, in reality, the deformation of fractures associated with external stress changes the flow rate of fluid, and the resulting pore pressures affect the subsequent deformation of the discontinuities. In a single discontinuity, fluid flow is a function of surface roughness, variable aperture, the magnitude of external loads and their direction relative to the orientation of joint, as well as the infill materials. Usually, the joint surface roughness plays a major role when the joint apertures are small or if the joints are sealed.

Figure 5.10. Factors which control permeability of a single joint in rock.
5.3.1 Influence of stress-stain behavior of a single joint on its permeability

As shown in Figure 5.11, the stresses including fluid pressures will influence the joint to dilate or close, to create new contacts points or to form a new joint network, depending on the surface geometry of the joint, magnitude of normal and shear stress, fluid pressures and deformability of rock material.

Figure 5.11. Effect of stress on a single rock specimen.

At given stress conditions, if the geometry of a fracture is defined by \( F_f(x, y) \) and \( F_b(x, y) \), then the fracture permeability for laminar fluid flow is given by:

\[
 k = \frac{(F_f(x, y) - F_b(x, y))^2}{12} \quad (5.6)
\]
where, \( F_T(x, y) \) and \( F_B(x, y) \) are the profiles of the two surfaces at the discontinuity relative to the cartesian coordinate axis system, and \( k \) is called coefficient of fracture permeability, having units of \( m^2 \) or \( \text{cm}^2 \). It is usual to employ Equation (5.6) for fluid flow analysis when flow takes place within smooth planar walls (i.e. \( F_T(x, y) - F_B(x, y) = \text{constant} \)). For a smooth, planar joint having an aperture of magnitude \( e \), the fracture permeability for laminar flow is given by:

\[
k = \frac{e^2}{12}
\]  

(5.7)

As discussed in Chapter 3, there are various permeability-stress relationships for predicting fracture permeability which can be found in the literature. Equation 5.7 was used in estimating fracture permeability in this study.

The test procedure for the TPHPTA was discussed earlier in Section 5.2.1. The fractured rock specimens were saturated either with water or air. Figure 5.12 shows the plot of axial stress against the axial strain for natural/artificial-fractured samples, in which the flow medium was either water or air. It is evident that a higher failure load at a lower strain is expected for the fractured rock sample, if the permeating fluid is air (Curve A). As explained earlier (Figure 5.9), this is because of the increased air content that makes the rock specimen more brittle. In contrast, for water-saturated samples (Curves B), a lower ultimate strength is attained at larger axial strain. As explained earlier, water saturated specimens will experience higher pore pressure than the specimens saturated with relatively compressible air. This confirms that (a) ductility of rock mass increases and (b) deformation modulus decreases, when the water content is increased. These results carry important implications on the stability of fractured tunnel
roofs and mine longwalls under different fluid pressures. For instance, in a gas flow
dominated fractured rock mass, sudden instability of a mine roof can be expected at a
critical gas pressure, contributing to a dramatic decrease in effective shear strength. In
contrast, if a jointed rock mass is saturated with water, the failure mechanism will be
more gradual (ductile) and predictable.

![Chart showing stress-strain behavior of saturated fractured granite specimens.](image)

Figure 5.12. Stress - strain behaviour of saturated fractured granite specimens.

### 5.3.2 Effects of axial stress

Using the triaxial test, the effect of axial stress on fracture permeability was investigated
experimentally. In the past, several researchers including Walsh, (1981) and Singh
(1997) have investigated the effects of axial stress on permeability. All laboratory
results show that the permeability is greatly affected by the stress. The increased
permeability is due to the formation of new cracks. From the laboratory study carried
out, it was observed that there could be three possible stages of permeability variation caused by an increase in axial compression (Figure 5.13), which are as follows:

(a) Constant permeability,

(b) Decreasing permeability, and

(c) Increasing permeability.

The permeability values shown in Figure 5.13, were calculated using Equation (5.7). Some specimens undergo all three stages while the others undergo combinations of either (a) & (b) or (a) & (c) or (b) & (c). A rock specimen saturated with air shows an almost constant permeability at the beginning of the application of axial stress. The permeability decreases with the increased stress combination acting perpendicular to the joint. The constant permeability is dependent on the magnitude of the effective stress applied to the rock specimen. The decrease in permeability associated with the increase in axial stress is attributed to possible crack closure. With further increase in axial load, micro cracks may start to develop and existing fractures may begin to dilate, thereby forming a new interconnected fracture network within the sample. As a result, a higher permeability is expected to occur until the sample fails. Depending on the magnitude of stress levels and the relative orientation of fractures, increased or decreased permeability can be expected.

Singh (1997) found that the permeability decreases markedly at the beginning of the cycle due to crack closure, and then followed by increasing permeability (Figure 5.14).
Figure 5.13. Effect of axial stress on permeability of fractured granite specimens.

Figure 5.14. Effect of axial stress on permeability (data from Singh, 1997).
5.3.3 Effects of confining pressure

As shown in Figures 5.15, laboratory test results have clearly indicated that the permeability through jointed rock specimens decreases as a function of increased effective confining stress. Similar laboratory test results were also observed by Kranz et al., 1979; Walsh, 1981; Raven and Gale, 1985. According to Figure 5.15, when the confining stress increases from 0 to 8 MPa, the average permeability decreases by more than 90%. Beyond an effective confining pressure of 8 MPa or so, further reduction in permeability is marginal for both air and water. This is attributed to joint apertures attaining their residual values, which are unaffected by further increase in confining stress. The residual aperture is a function of external stress conditions, the initial rock surface profile and the material and geometrical properties.

![Figure 5.15. Effect of confining stress on permeability of fractured granite specimens](image)
It was also observed that greater the roughness, the lower was the rate of permeability reduction under a given confining pressure. In other words, for smooth planar joints, the permeability decreases more significantly with the increase in confining pressure in comparison to rough joints. Tests conducted on Barre granite specimens by Kranz et al. (1979) also show that the permeability decreases significantly with the increase in confining pressure.

5.3.4 Effect of loading and unloading behavior on permeability

It is important to look at the flow changes due to loading and unloading behaviour. Underground constructions, for instance twin tunnels and adjacent roadways are subjected to such loading stages during non-simultaneous excavation. Figure 5.16 shows that the flow rate decreases significantly during the increase in confining stress for the 1st loading cycle. When confining stress exceeds 10MPa, little or no decrease in flow occurs, irrespective of the type of permeating fluid, air or water. It is relevant to note that the 2nd and 3rd loading and unloading cycles do not contribute to any significant change in flow volumes. This is because, once the fractures attain their residual apertures, subsequent dilation (due to unloading) and compression (due to reloading) seem to be insignificant. In other words, plastic, irrecoverable strains have set in once the critical load of confining pressure has been exceeded. Hence, reduced permeability will always prevail once the joints are loaded beyond this critical value. The accumulation of gouge due to crushing or shearing of joint asperities will also contribute to subsequent reduction in flow within joints. Redistribution of stress during and after excavation leads to loading and unloading stress paths affecting the adjacent areas in the rock mass. Therefore, the subsequent flows are influenced by the ultimate
air or water permeability resulting after several loading and unloading cycles. The zones of fractured rock once loaded substantially will always retain reduced permeabilities even after they undergo stress relief at a later stage. Therefore, the measured permeability associated with loading-unloading behaviour can be subsequently incorporated in a numerical procedure to analyse the groundwater flow in a jointed rock mass, subject to multiple or sequential excavation such as driving twin tunnels or enlargement of existing cavities.

Figure 5.16. Effects of loading and unloading on flow characteristics of fractured granite specimens.
5.4 MEASUREMENT OF JOINT APERTURES

Fluid flow through low permeability rocks is mainly dominated by interconnected fractures. Apart from the fluid properties and applied hydraulic head, the critical factor which controls the flow quantity is the joint aperture. The size and interconnectivity of fractures influence the volume and rate of flow through jointed rock. The accurate estimation of joint apertures is difficult because of their irregular surfaces, that are characterized by many contact points between joint surfaces.

As shown in Figure 5.17, aperture measurements can be mainly categorized as direct and indirect measurements. Direct measurements can provide the local mechanical aperture at a particular location. Mechanical apertures can develop due to shear displacement along highly irregular joint surfaces. Joint apertures can extend from say 1 micron to 0.2m in a fractured rock mass. The dramatic opening of fractures is often due to the development of local tensile stresses within the rock mass. Once a discontinuity is created, its aperture can increase or decrease depending on the stress history, such as deposition of infill, washing out of infill, erosion or change of adjacent stresses due to blasting, or increased fluid pressures surrounding the rock mass. Figure 5.18 shows various forms of discontinuities in a rock mass, such as planar uniform aperture, variable/rough aperture with or without contacts points between joint walls, and joints filled with foreign materials. In practice, although joints with uniform apertures rarely exist, for simplicity of flow analysis through a single joint or network of joints, smooth open joints are usually simplified as shown in Figure 5.18a.
Based on mechanical properties of joint (e.g., JRS)

Measurement of joint apertures

Direct measurement
- Feeler Gauge
- Fluorescent dye method
- Impression packer method
  - Mechanical aperture

Indirect measurement
- Tracer test
- Hydraulic test
  - Equivalent Mass balance aperture
  - Equivalent Frictional loss aperture
  - Equivalent Cubic law aperture or hydraulic aperture
  - Barton and Bandis model
  - Mechanical aperture

Figure 5.17. Techniques for measuring joint aperture.
According to the method of Barton (1973), classification of apertures by size has been recorded by Lee and Farmer (1993), as given in Table 5.3. From the insitu permeability tests conducted in Colorado, Snow (1968a) found that the fracture aperture could range from 50 to 350 μm to a depth of 10m from the surface. At greater depths exceeding 30m, the apertures decrease to values of 40 to 100 μm.

Table 5.3. Classification of apertures by its magnitude (after Barton, 1973)

<table>
<thead>
<tr>
<th>Class</th>
<th>Aperture (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Very tight</td>
<td>&lt;0.1</td>
</tr>
<tr>
<td>Tight</td>
<td>0.1-0.25</td>
</tr>
<tr>
<td>Partly open</td>
<td>0.25-0.50</td>
</tr>
<tr>
<td>Open</td>
<td>0.50-2.5</td>
</tr>
<tr>
<td>Moderately wide</td>
<td>2.5-10.0</td>
</tr>
<tr>
<td>Wide</td>
<td>10</td>
</tr>
</tbody>
</table>
5.4.1 Direct measurement

The physical measurement of joint apertures which are exposed to the surface, give a rough but quick indication of the separation between joint walls at local points. The measured opening is termed as the mechanical aperture of the discontinuity at a given stress level. In fluid flow calculations, the mechanical aperture is useful to determine the hydraulic aperture for a given joint profile. A tapered feeler gauge was used to measure the mechanical aperture of cored specimens. This approach has been used by many researchers including Bandis (1980).

A correct size feeler gauge was inserted to the joint at different locations and the average aperture was estimated (Figure 5.19a). Typical test results on mechanical apertures of naturally fractured specimens under a range of normal stress are shown in Figure 5.19b.

Figure 5.19a. Mechanical aperture using feeler gauge.
It was observed that joint aperture could not be measured beyond 600 kPa normal stress, because the smallest feeler gauge could not be inserted into a very small joint. Although this direct measurement is simple, the tapered feeler gauge must be small enough to be inserted into the smallest joints which are in the order of $10^{-5}$ - $10^{-9}$ m. Nevertheless, this technique can be easily employed in the field to measure the apertures of exposed discontinuities in a rock mass. Particularly, for the joint aperture measurement of roof tunnels or walls in the field, the feeler gauge method can easily be implemented with little cost.

### 5.4.2 Indirect measurement

Under the indirect approach, the joint apertures may be estimated either by the mechanical properties of the discontinuities or by fluid flow measurements through
discontinuities. Depending on the techniques employed, the mechanical aperture or the hydraulic aperture of the joint is computed, and the mechanical aperture is not the same as the hydraulic aperture as indicated later. Usually, the mechanical aperture is larger than the hydraulic aperture, except when the joint has reached its residual aperture at high stress levels.

5.4.3 Measurement of hydraulic aperture based on the fluid flow

There are two other common indirect techniques to estimate the hydraulic aperture based on fluid flow through a rock mass. They are: (a) steady state flow measurement in the laboratory under triaxial test conditions and, (b) insitu tests (e.g. borehole pumping test/tracer tests). The study described here is based on laboratory work to evaluate the hydraulic aperture for different boundary conditions. Under laboratory conditions, rock specimens with a single fracture or multiple fractures were tested using a triaxial apparatus, under given confining pressure, inlet fluid pressure and axial stress. Steady state flow rates were used to calculate the hydraulic aperture using the Darcy's (cubic) law. It is important to note that the hydraulic aperture is not the same as the mechanical aperture, because the natural fractures are dissimilar to ideal parallel plates. However for plane, smooth joint surfaces with JRC =0, the magnitude of mechanical aperture tends to approach the hydraulic aperture. When interpreting flow data for estimating joint aperture, various kinds of fluid flow theories may yield different values of hydraulic apertures for the same joint. These are often termed as: equivalent mass balance aperture, equivalent frictional loss aperture, equivalent cubic law aperture (i.e. hydraulic aperture) and tracer joint aperture as stated in Figure 5.17.
From laboratory flow measurements data, the equivalent aperture \( (e_c) \) for laminar fluid flow through parallel joint walls was calculated using the cubic law theory, as given in Equation (5.8):

\[
e_c = \left[ \frac{12q\mu}{b(dp/dx)} \right]^{1/3}
\]  

(5.8)

where, \( q = \) steady state flow rate;
\( b = \) width of the fracture;
\( dp/dx = \) pressure gradient between two ends of the specimen, and
\( \mu = \) dynamic viscosity of fluid.

Frequently, for simplicity, the equivalent cubic law aperture is referred to as the hydraulic aperture or mean aperture. For a given axial stress and confining pressure, the measured steady state flow rate and estimated joint apertures at different inlet water pressures are presented in Figure 5.20. The curves A\(_1\), and A\(_2\) in Figures 5.20a and 5.20b show that the flow rate linearly varies with the inlet fluid pressures, confirming the applicability of Darcy's law for fluid flow analysis through naturally fractured rock samples. The data represented by curves B\(_1\) and B\(_2\) in Figures 5.20a and 5.20b are plotted using Equation (5.8). It is noted that the flow rate increases with the increasing inlet fluid pressure, but the estimated joint aperture decreases (non-linearly) against the inlet fluid pressure. This is not surprising because, according to Equation (5.8), the fluid flow is directly proportional to the inlet fluid pressure, whereas, the joint aperture is proportional to the cube root of the flow rate and inversely proportional to the inlet fluid pressure.
(a) At confining pressure = 0.5MPa

![Graph showing hydraulic aperture vs. inlet fluid pressure at 0.5 MPa confining pressure.]

(b) At confining pressure = 1.0MPa

![Graph showing hydraulic aperture vs. inlet water pressure at 1.0 MPa confining pressure.]

Figure 5.20. Equivalent hydraulic aperture at different confining pressure.
The confining pressure can significantly influence the magnitude of the joint aperture (Figure 5.21). As expected, steady state flow rate and the estimated joint apertures decrease with the increase in confining stress. When the confining pressure exceeds 6MPa, the change of aperture becomes very small. This relatively constant aperture is called the residual aperture of the joint. Therefore, even at high stress levels, a minimum flow corresponding to the residual aperture is expected, provided the fluid pressure is large enough to cause the flow through the fracture. It is relevant to note that the residual aperture exists only when the fractures are irregular, but not in the case of planar and smooth joints.

Figure 5.21. Equivalent hydraulic aperture based on cubic law.

The effect of deviator stress ($\sigma_1 - \sigma_3$) on the joint aperture is different to what one may observe with the confining pressure (Figure 5.22). The deviator stress may close or dilate or create new fractures depending on the magnitude of stress levels and the orientation of fractures in a rock mass. When the deviator stress level exceeds
130 MPa, the increased aperture size or the created new fractures (approaching failure stress) dramatically increase the flow rate (Figure 5.22)

Figure 5.22 Equivalent hydraulic aperture based on cubic law.

5.5 PERMEABILITY OF ROCKS WITH MULTIPLE FRACTURES: LABORATORY INVESTIGATION

Little effort has been made so far to study the effect of multiple fractures on permeability experimentally (Elsworth and Piggot, 1987; Shimo and Iihoshi, 1993). Shimo and Iihoshi (1993) used a large block of rock (30cm cube) with natural fractures in their investigation. Figure 5.23 shows the mapped surface fractures on the chert rock cube used for testing and the flow rate corresponding to the sub-panel number is plotted in Figure 5.24. Different flow volumes indicate that the interconnectivity of flow paths is different even for a small block of 30cm sides.
Figure 5.23. Mapped fractures on the surface of a chert rock cube (Shimo & Iihoshi, 1993).

Figure 5.24. Flow rate distribution along sides of the sample (data from Shimo and Iihoshi, 1993).
The hydraulic measurement of individual fluid flux or pressure can be carried out at several locations, once the fluid pressure is applied via small tubes to one side of the cube. Each side has 25 sub panels. One advantage of this test is that fluid boundaries can be effectively controlled than in a conventional triaxial testing method. However, the effects of confining pressure and axial stress, which govern fluid flow through the sample cannot be studied using this equipment.

Air and water flow measurement through standard size granite core specimens (45mm dia. to 55mm dia.) with multiple fractures (supplied by Strata Control Technology, New South Wales, Australia) was carried out (Figure 5.25). Some fractures within the specimens were found to be open, while the others were sealed. Closed joints may not contribute to any flow changes under low inlet fluid pressures depending on the relative orientation of joints to the principal stresses. As expected, specimens with multiple fractures carry a larger flow rate than a single fracture, because of the increased connectivity of fluid flow paths (Figure 5.26).

Figure 5.25. Naturally fractured granite rock specimens with multiple fractures.
Irrespective of the number of flow paths, the well-known Darcy's law can be employed here, as the flow rate against the inlet fluid pressure shows an approximately linear relationship, for both single and multiple fracture specimens (Figure 5.26). According to Figure 5.27, the fracture permeability coefficient of both single and multiple fracture specimens diminish almost exponentially with the increased confining pressure. The fracture permeability \( k \) is calculated using the expression, \( k = e^2 / 12 \) where, \( e \) = hydraulic aperture based on the cubic law for single fracture. In the case of multiple fractures, the value of \( e \) can be evaluated as an average of all fractures carrying fluid. Apart from the external boundary conditions (e.g. applied stress), the permeability of the multiple fracture specimen is a function of the interconnectivity of joints, their individual apertures, joint orientations, and the specimen size.
The measurement of joint roughness and its effect on fluid flow will be discussed in Chapter 6. Findings of this study are employed for the developing mathematical model (described in Chapter 7) for two-phase flow in a single rock joint.

5.6 CONCLUSIONS

Studying the stress-strain and permeability characteristics of a single joint forms the basic step for developing a comprehensive flow model for jointed rock mass. The influencing parameters on fluid flow through a single rock joint were evaluated both
experimentally and theoretically. The important aspects related to the permeability and joint aperture are discussed and summarized below.

(a) The permeability of intact rock material can significantly drop due to the increase in confining pressure, because of the decrease in interconnectivity of the pore structures. As the confining pressure approaches 10MPa, the matrix permeability of granite becomes insignificant (Figure 5.6). Axial stress usually increases the permeability of matrix rock, due to the formation of secondary fluid paths, facilitated by the interconnected fracture network.

(b) From the stress-strain curves of intact and fractured rocks, it is evident that a higher failure load at a lower strain is expected, if the permeating fluid is air (Figures 5.9 and 5.12). In contrast, for water-saturated samples, a lower ultimate strength is attained. This is because the compressible gas phase generates less pore pressure, in comparison with relatively incompressible fluids which induce higher pore pressure. This verifies that (a) ductility of rock mass increases and (b) deformation modulus decreases, when the water content is increased in relation to air. It is important to mention here that only air is used in all the tests. It is envisaged the other gases such as CH₄ or CO₂ may have different influence. However, this aspect is not a subject of this thesis and should be addressed as a future research topic.

(c) From a practical point of view, (e.g. the stability of tunnel roofs and mine longwalls under different fluid pressures), when the gas flow is dominant through rock joints, sudden instability of a mine roof can occur at a
critical gas pressure. In contrast, if a jointed rock mass is saturated with water, the failure is gradual and more predictable.

(d) Flow through a single joint is a function of the magnitude of the joint aperture, external stresses and their loading and unloading behaviour, fluid pressure, joint surface roughness and relative orientation of the joint (Figures 5.13, 5.15 and 5.16). It is also observed that there are three possible stages of permeability caused by an increase in axial compression (a) constant permeability, (b) decreasing permeability, and (c) increasing permeability, depending on the dilation or compression of fractures.

(e) When the confining stress (i.e. normal stress) increases from 0 to 8MPa, the average permeability of a fractured specimen may decrease by more than 90%. Beyond an effective confining pressure of 8MPa or so, further reduction in permeability is usually marginal for both air and water. The constant aperture attained at this confining pressure is called the residual aperture (Figure 5.15).

(f) The loading and unloading plots indicate that significant reduction of the flow rates takes place during the 1st loading cycle (Figure 5.16). The contribution of the 2nd and 3rd loading and unloading cycles on flow is small. This confirms that plastic, irrecoverable strains have set in, once the critical load has been exceeded. Therefore, this reduced permeability will always govern the flow conditions, once the joints are loaded beyond a critical value to attain residual aperture.
CHAPTER 6

EFFECT OF JOINT SURFACE ROUGHNESS ON FLUID FLOW

6.1 INTRODUCTION

Discontinuity properties such as joint openings, roughness, dip direction and infill material influence the behaviour of a rock mass. During the past 2-3 decades, much focus has been given to surface roughness estimation (Patton, 1966; Barton, 1973; Barton and Choubey 1977; Brown and Scholz, 1985; Xie and Pariseau, 1995; Kwafniewski and Wang, 1997; Barton and Quadros, 1997). This is because, the joint roughness directly affects the fluid flow characteristics as well as the shear strength of rock mass. Roughness is defined as the surface waviness (i.e. large scale undulations) and unevenness (i.e. small scale undulations) of a given discontinuity relative to its mean plane. The geometrical roughness of a joint is often represented by the joint roughness coefficient, which is purely a numerical index and not the same as effective friction angle of the joint. Factors such as, the mineral properties of rocks and mode of fracture inception (e.g. attributed to tensile stresses) govern the magnitude of surface roughness of rock joints. The effect of roughness varies with the change of joint aperture, thickness of infill material and the relative displacement of joint planes. Also, in the presence of water, the shear strength of rough joints diminishes, which results in a reduced joint roughness due to the shear deformation. ISRM (1978a) has quantified the degree of roughness, based on unevenness and waviness of joints, as shown in Figure
6.1. Rough joints are normally classified as planes having surface irregularities with a wavelength of less than 100mm (Priest, 1993).

![Classification of roughness based on unevenness and waviness of joints](after ISRM, 1978).

Barton (1973) introduced the term 'Joint Roughness Coefficient' (JRC) to express the roughness of a joint, which was assumed to vary between 0 and 20. The higher the JRC value, greater the surface irregularities. For an ideally planar (smooth) joint surface, the JRC value can be assumed to be zero. Because of irregularities (asperities), a rough
joint shows a greater friction angle than a smooth joint for the same rock type. The increased friction angle associated with the surface roughness is called the 'effective roughness angle' whose magnitude depends on the surface profile (e.g. geometry of asperities), contact area and the degree of indentation (Schneider, 1974). Different methods of measuring surface roughness have been proposed, for instance by Fecker and Rengers (1971), Barton (1973), and Weissbach (1978). In order to study the effect of roughness on fluid flow through jointed rocks and mechanical behaviour of rock mass, the first step is to map the correct surface texture of discontinuities. For simplicity, some researchers have idealised the rock joint surfaces profiles as saw tooth profiles or sinusoidal profiles. In reality, one may question whether such idealised profiles behave as natural joints or not, and if not, how accurate the subsequent analysis will be. The following section describes the technique used by the writer for mapping joint surfaces.

6.2 MAPPING OF SURFACE PROFILES

Surface texture is perhaps the most important variable governing the magnitude of friction, which controls the relative motion between surfaces and fluid flow. Although the investigation of surface texture is of paramount importance in a variety of engineering applications, the discussion presented here is confined to the surface of rock joints only. Particularly in mining and rock engineering, the surface texture of rocks significantly influences the stability of mine roofs, jointed slopes and fluid flow through discontinuities. The techniques of modelling joint surfaces generally adopt one or more of the following alternatives.
(a) graphical representation of the surface profile,
(b) mathematical representation of the mapped surface profile, and
(c) statistical features.

Various techniques have been used in the past to obtain the graphical representation of surface profiles at macroscopic and microscopic levels. On the basis of measurement options, such methods can be divided into four categories: (a) mechanical, (b) hydraulic, (c) optical and (d) laser techniques. Figure 6.2 describes different types of surface profilometers. In the past, mechanical methods such as brush gauges (Barton, 1973) were used, however, the laser and optical techniques have currently become the most popular, because of both convenience and accuracy. Some digitised rock surfaces mapped by the writer using the digital co-ordinates equipment are presented in Figure 6.5.

All 55mm diameter cores used for testing contained a single fracture. The measurements were made using the mechanical profilometer, which could digitise the joint surface in three dimensions by taking a series of parallel lines along the joint, where the X-Y-Z system of axes is illustrated in Figure 6.3. A typical mechanical profilometer is shown in Figure 6.4. In the course of measurements, the digital pointer was lowered to the surface to obtain x, y and z co-ordinates, and by moving the pointer systematically, many points of the surface were mapped. The readings of each point are automatically analysed by the computer to map the joint surface. Typical surface profiles of jointed cores that were mapped using this method are shown in Figure 6.5.
Figure 6.2. Available profilometers based on modes of measurement.
Figure 6.3. Plan view of selected grids for joint surface mapping.

Figure 6.4. Digital profilometer.
Figure 6.5. Typical mapped joint surface profiles.
The individual joint profiles along each line segment (e.g. $A_1B_1$ and $A_2B_2$ in Figure 6.3) were also plotted in order to estimate the JRC values for each profile (Figures 6.6a-c). This enables a better representation of the joint geometry.

Figure 6.6a. Individual profiles of specimen IF 16.

Figure 6.6b. Individual profiles of specimen IF 17.
For insitu measurement, the simple string-line method was used to map the joint profiles as shown in the Figure 6.7. In this method, a string is placed approximately parallel to the general surface of joint, and at certain intervals (x), the perpendicular distance from the line to the fracture surface is recorded. Depending upon the degree of irregularities, the number of measurements can be increased or decreased.
Such mapped joint profiles on a basalt rock slope (New South Wales, Nowra) are shown in Figure 6.8. Measurements were taken every 0.5m along the string line. The purpose of this type of joint mapping was to get a thorough understanding of prototype joints.

![Figure 6.8. Measured insitu joint profiles.](image)

### 6.3 MEASUREMENT OF SURFACE ROUGHNESS COEFFICIENT

There are several techniques that can be used to measure the surface roughness of a joint to quantify the Joint Roughness Coefficient (JRC), which include,

(a) visual comparison of joint profiles with the standard profiles (Figure 6.9) proposed by Barton (1973),

(b) tilt tests to determine the basic friction angle,

(c) use of maximum roughness amplitude, $r_a$,
(d) calibration with the mechanical and hydraulic joint apertures, 

(e) analytical approach based on mathematical functions, and 

(f) fractal models. 

Various techniques yield different magnitudes of joint roughness values for the same joint. Therefore, the writer used four approaches (i.e. (a), (c), (d) and (e) mentioned above) to compare the estimated values.

<table>
<thead>
<tr>
<th>Description of joint</th>
<th>Standard joint profiles</th>
<th>JRC range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Smooth, planar: cleavage joints</td>
<td></td>
<td>0 - 2</td>
</tr>
<tr>
<td>Smooth, planar: tectonic joints</td>
<td></td>
<td>2 - 4</td>
</tr>
<tr>
<td>Undulating, planar: foliation joints</td>
<td></td>
<td>4 - 6</td>
</tr>
<tr>
<td>Rough, planar: tectonic joints</td>
<td></td>
<td>6 - 8</td>
</tr>
<tr>
<td>Rough, planar: tectonic joints</td>
<td></td>
<td>8 - 10</td>
</tr>
<tr>
<td>Rough, undulating: bedding joints</td>
<td></td>
<td>10 - 12</td>
</tr>
<tr>
<td>Rough, undulating: tectonic joints</td>
<td></td>
<td>12 - 14</td>
</tr>
<tr>
<td>Rough, undulating: relief joints</td>
<td></td>
<td>14 - 16</td>
</tr>
<tr>
<td>Rough, irregular: bedding joints</td>
<td></td>
<td>16 - 18</td>
</tr>
<tr>
<td>Rough, irregular: artificial tension</td>
<td></td>
<td>18 - 20</td>
</tr>
</tbody>
</table>

Figure 6.9. Standard profiles proposed by Barton (1973).
6.3.1 Matching of joint profiles with standard profiles

For preliminary design, a quick estimation of JRC can be carried out by matching the joint surfaces with the standard profiles as proposed by Barton (1973). The standard profiles and the JRC values are given in Figure 6.9. For a smooth planar joint, JRC tends to become zero, whereas, for highly irregular surface, JRC values tend to approach 16 - 20. This method of roughness determination has been supported by ISRM (1978), and due to its practicality, Barton's method is probably one of the most widely used in preliminary design in jointed rock strata. In order to use this method, the measured profiles (Figures 6.6a-c) were drawn to the standard scale, and subsequently, these scaled profiles were compared with the standard profiles. The predicted values are tabulated in Table 6.1

Table 6.1. Estimation of JRC using standard profiles

<table>
<thead>
<tr>
<th>Profile location (mm)</th>
<th>JRC based on standard profiles</th>
<th>Scaled joint profiles</th>
</tr>
</thead>
<tbody>
<tr>
<td>Specimen - IF20</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Y = 0</td>
<td>11</td>
<td></td>
</tr>
<tr>
<td>Y = 10</td>
<td>8</td>
<td></td>
</tr>
<tr>
<td>Y = 20</td>
<td>9</td>
<td></td>
</tr>
<tr>
<td>Y = 30</td>
<td>7</td>
<td></td>
</tr>
<tr>
<td>Y = 40</td>
<td>8</td>
<td></td>
</tr>
<tr>
<td>Y = 50</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>Y = 55</td>
<td>8</td>
<td></td>
</tr>
<tr>
<td>Mean JRC</td>
<td>7.8</td>
<td></td>
</tr>
</tbody>
</table>
Table 6.1. Estimation of JRC using standard profiles (continued).

<table>
<thead>
<tr>
<th>Profile location (mm)</th>
<th>JRC based on standard profiles</th>
<th>Scaled joint profiles</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Specimen - IF16</td>
<td></td>
</tr>
<tr>
<td>Y = 0</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Y = 10</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>Y = 20</td>
<td>7</td>
<td></td>
</tr>
<tr>
<td>Y = 30</td>
<td>13</td>
<td></td>
</tr>
<tr>
<td>Y = 40</td>
<td>7</td>
<td></td>
</tr>
<tr>
<td>Y = 50</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>Y = 55</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>Mean JRC</td>
<td>6.2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Specimen - IF17</td>
<td></td>
</tr>
<tr>
<td>Y = 0</td>
<td>7</td>
<td></td>
</tr>
<tr>
<td>Y = 10</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>Y = 20</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>Y = 30</td>
<td>8</td>
<td></td>
</tr>
<tr>
<td>Y = 40</td>
<td>9</td>
<td></td>
</tr>
<tr>
<td>Y = 50</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>Y = 55</td>
<td>11</td>
<td></td>
</tr>
<tr>
<td>Mean JRC</td>
<td>8</td>
<td></td>
</tr>
</tbody>
</table>
6.3.2 Maximum roughness amplitude ($r_a$)

Barton (1982) suggested an alternative empirical approach to estimate JRC based on the maximum amplitude ($a$) of a given profile length ($L$). On a 100mm scale, an empirical relationship to estimate JRC is defined by:

$$JRC = 400 \frac{a}{L}$$ \hspace{1cm} (6.1)

For given three specimens (Figures 6.6a-c), JRC values computed using Equation (6.1) are listed in Table 6.2. The computed JRC values vary from 7.0 to 9.3 for all these granite specimens. Barton and Choubey (1977) reported that for granite specimens, in general, JRC could vary from 6.7 to 9.5. The results of this analysis agree with this methodology.

<table>
<thead>
<tr>
<th>Distance along the Y-axis, shown in Figure 6.3</th>
<th>Joint Roughness Coefficient based on Equation 6.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Specimen 1 (IF16)</td>
<td>Specimen 2 (IF17)</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td>8.8</td>
</tr>
<tr>
<td>20</td>
<td>8</td>
</tr>
<tr>
<td>30</td>
<td>17.78</td>
</tr>
<tr>
<td>40</td>
<td>12.8</td>
</tr>
<tr>
<td>50</td>
<td>6.6</td>
</tr>
<tr>
<td>55</td>
<td>11.1</td>
</tr>
<tr>
<td>Average</td>
<td>9.3</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>5.5</td>
</tr>
</tbody>
</table>
6.3.3 Mechanical and hydraulic joint apertures

Although this method is theoretically sound, in practice, its application is limited. For a given mechanical aperture ($E$) and hydraulic aperture ($e$) of joint, $JRC$ can be predicted by the following equation (Barton et al., 1985)

$$JRC = \left( \frac{E^2}{e} \right)^{-2.5}$$

(6.2)

The units of $E$ and $e$ are given in microns.

The mechanical aperture was estimated at different locations along the joint using a tapered feeler gauge. The hydraulic aperture ($e$) was back calculated for steady state flow using the cubic formula. $JRC$ values computed using Equation (6.2) for specimens IF 16, IF 17 and IF 20 are 10, 11 and 12, respectively. These values deviate from the preceding measurements, discussed in Sections 6.3.1 and 6.3.2, because in the triaxial apparatus, the joint is subjected to some confining pressure resulting in a reduced hydraulic aperture.

6.3.4 Empirical approach

Based on the values of joint surface parameters, Tse and Cruden (1979) proposed two linear relationships to estimate $JRC$ for relatively rough joints, subjected to low normal stress, as follows:

$$JRC = 32.2 + 32.47 \log \left[ \frac{1}{L} \int_{x=0}^{x=L} (dy/dx)^2 \right]$$ and

$$JRC = 37.28 + 16.58 \log \left[ \int_{x=0}^{x=L} (f(x) - f(x + \Delta x))^2 \right]$$

(6.3a)  (6.3b)
where, \( y \) = the amplitude of the roughness about the centre line,

\[ f(x) = \text{amplitude of asperity height at distance } x, \text{ along the length } L \]

The parameter within large brackets in Equation (6.3a) is calculated as follows.

\[
Z = \frac{1}{L} \int_{x=0}^{x=L} (dy/dx)^2 = \left[ \left( \frac{1}{M(dx)^2} \sum_{i=1}^{M} (y_{i+1} - y_i)^2 \right) \right]^{0.5} \quad (6.3c)
\]

where, \( M = \text{number of intervals over the length of the joint and } dx = \text{interval length} \).

Based on Equations (6.3a) and (6.3c), the calculated JRC values are listed in Table 6.3.

When the logarithm of \( Z \) is higher than \(-1\), the estimated value of JRC takes negative values. This occurs when the asperity height difference between the adjacent intervals is small. In other words, for relatively smooth joints, the validity of the equation is uncertain.

Table 6.3. Estimated JRC based on Tse and Cruden (1979) for specimens shown in Figures 6.6a-c.

<table>
<thead>
<tr>
<th>Joint profile location (Figure 6.3)</th>
<th>Log ( Z )</th>
<th>Estimated JRC</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Specimen 1 (IF 16)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( Y = 0 )</td>
<td>-1.07</td>
<td>-2.78</td>
</tr>
<tr>
<td>( Y = 10 )</td>
<td>-1.21</td>
<td>-7.1856</td>
</tr>
<tr>
<td>( Y = 20 )</td>
<td>-0.9</td>
<td>2.7</td>
</tr>
<tr>
<td>( Y = 30 )</td>
<td>-1.07</td>
<td>-2.8</td>
</tr>
<tr>
<td>( Y = 40 )</td>
<td>-1.06</td>
<td>-2.5</td>
</tr>
<tr>
<td>( Y = 50 )</td>
<td>-1.3</td>
<td>-10</td>
</tr>
<tr>
<td><strong>Specimen 2 (IF 17)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( Y = 0 )</td>
<td>-1.18</td>
<td>-6.1</td>
</tr>
<tr>
<td>( Y = 10 )</td>
<td>-1.26</td>
<td>-8.8</td>
</tr>
<tr>
<td>( Y = 20 )</td>
<td>-1.28</td>
<td>-9.4</td>
</tr>
<tr>
<td>( Y = 30 )</td>
<td>-1.18</td>
<td>-6.1</td>
</tr>
<tr>
<td>( Y = 40 )</td>
<td>-1.02</td>
<td>-0.9</td>
</tr>
<tr>
<td>( Y = 50 )</td>
<td>-1.01</td>
<td>-0.7</td>
</tr>
<tr>
<td>( Y = 55 )</td>
<td>-0.97</td>
<td>0.6</td>
</tr>
</tbody>
</table>
Table 6.3. Estimated JRC based on Tse and Cruden (1979) for specimens (Contd.)

<table>
<thead>
<tr>
<th>Joint profile location (Figure 6.3)</th>
<th>Log Z</th>
<th>Estimated JRC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Specimen 3 (IF 20) (Contd. Table 6.3)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Y = 0</td>
<td>-1.39</td>
<td>-13</td>
</tr>
<tr>
<td>Y = 10</td>
<td>-1.39</td>
<td>-12.9</td>
</tr>
<tr>
<td>Y = 20</td>
<td>-1.35</td>
<td>-11.8</td>
</tr>
<tr>
<td>Y = 30</td>
<td>-1.39</td>
<td>-13</td>
</tr>
<tr>
<td>Y = 40</td>
<td>-1.27</td>
<td>-9.1</td>
</tr>
<tr>
<td>Y = 50</td>
<td>-1.25</td>
<td>-8.4</td>
</tr>
<tr>
<td>Y = 55</td>
<td>-1.81</td>
<td>-16.8</td>
</tr>
</tbody>
</table>

The various techniques discussed above yield different values of JRC for the same joint pattern. However, methods (a) in Section 6.3.1 and (c) in Section 6.3.2 are easy to employ and they provide approximately similar JRC values except for specimen IF16. In the following section, the effect of joint roughness on fluid flow is discussed.

6.4 EFFECT OF SURFACE ROUGHNESS ON FLUID FLOW

Analytical, experimental and numerical studies have shown that the joint roughness and aperture are the most important factors governing fluid flow through a single joint or a network of joints. In this section, attention is given to the effects of roughness on the permeability characteristics of a jointed rock. It is expected that a minimum amount of fluid flow through a fracture occurs even at very high normal stress levels. Corresponding to this minimum flow, there is a nominal aperture characterising every rough joint. This nominal aperture is referred to as the residual aperture. The joint roughness can be incorporated into fluid flow equations via:

(a) Mechanical joint roughness coefficient (JRC),

(b) Relative (hydraulic) joint roughness coefficient (F), and
The relative joint roughness is a function of the hydraulic aperture and the asperity height of the joint surface. Lomize (1951), Louis (1969) and Quadros (1982) have described the effect of surface roughness on fluid conductivity, in terms of the relative joint roughness. The relative roughness \( F \) is defined as the ratio of the difference between maximum and minimum asperity height \( k \) divided by twice the hydraulic diameter Robert et al. (1996):

\[
F = \frac{k}{2e}
\]  

(6.4)

Depending on the magnitude of \( F \), Louis (1969) found experimentally that the flow within joints could either be parallel or non-parallel. When \( F \leq 0.033 \), the flow is assumed to be parallel, which is expected probably when JRC <8. Depending on the viscosity of fluid, joint aperture and velocity of fluid within a joint, the flow may be either laminar or turbulent. The flow pattern can be determined by considering the Reynolds number. If the Reynolds number is found to be more than 2000, the flow will most probably be turbulent.

### 6.4.1 Role of relative roughness coefficient in flow equations

For a smooth planar joint, the pressure drop coefficient \( \lambda \) depends on the Reynolds number \( R_e \) only, such that:

\[
\lambda = \frac{96}{R_e}
\]  

(6.5)
For an irregular joint, the pressure drop is expected to increase by a factor \( C \) due to roughness of the joint wall, as represented by:

\[
C = (1 + m \pi^{1.5})
\]  \hspace{1cm} (6.6)

The magnitude of \( m \) has been evaluated by several researchers (Lomize, 1951; Louis, 1969; and Quadros, 1982) for various rough surfaces, where \( m \) varies from 8.8 to 20.5.

For a given joint aperture, the variation of \( C \) with the difference in asperity heights \( k \) between the maximum and minimum of joint surface is shown in Figure 6.10. When \( k \) approaches the magnitude of joint aperture \( e \), the factor \( C \) associated with roughness becomes as high as 8. As seen in Figure 6.10, a large pressure drop coefficient \( \lambda \) is expected when \( m = 20.5 \), whereas, the least value of \( \lambda \) is given for \( m = 8.8 \) (Quadros, 1982).

![Figure 6.10. Effect of roughness on pressure drop coefficient.](image)

The effect of roughness is now introduced into the flow equation, and represented by:

\[
q = \frac{be^3}{12\mu} \left( \frac{1}{1 + m \pi^{1.5}} \right) \left( \frac{dp}{dx} \right)
\]  \hspace{1cm} (6.7)
where, $b$ is the width of the joint, $dp$ is the pressure variation along length of $dx$, $\mu$ is the dynamic viscosity of fluid, $e$ is the joint aperture and $F$ is the relative joint roughness coefficient.

At given inlet fluid pressure and confining pressure (1.0MPa) used in the experiment work, normalised flow rate against relative roughness is shown in Figure 6.11. The normalised roughness is defined as the flow rate divided by the cube of the joint aperture. For a given joint aperture, it is evident that the increase in joint roughness decreases the flow rate almost exponentially.

![Figure 6.11. Effect of relative roughness on fluid flow](image)

6.4.2 Role of JRC in flow equation

In the previous section, the introduction of relative roughness into fluid flow equations was discussed. The aim of this section is to look at how JRC can be coupled with the
governing flow equations. Assuming the parameter, $a$ (see Equation 6.1) approximately equals to $k$ (see Equation 6.4), Barton and Quadros (1997) arrived at the following correlation between the relative roughness ($F$) and $JRC$ by combining Equations (6.1) and (6.2):

$$ F = \left( \frac{L}{800} \right) \left( \frac{JRC^{3.5}}{E^2} \right) $$  \hspace{1cm} (6.8)

where, $E$ is the mechanical aperture of the joint, $L$ is the length of the joint.

Instead of the relative roughness ($F$) in Equation (6.7), it is beneficial to introduce $JRC$ because of the practical relevance of $JRC$ as discussed earlier. Incorporating $JRC$ into the flow equation yields:

$$ q = \frac{bE^3}{12\mu \left( \frac{E^3}{E^3 + nL^{1.5}JRC^{5.25}} \right)} \frac{dp}{dx} $$  \hspace{1cm} (6.9)

where, the values of $n$ become 0.000751, 0.000375 and 0.000906 based on $m = 17, 8.8$ and 20.5 respectively (Thiel, 1989).

The flow rate $q$ computed from Equation (6.9) versus $JRC$ is plotted in Figure 6.12. The mechanical aperture of the joint at zero stress level was taken as 0.01mm. Due to the increased $JRC$, the flow rate is significantly decreased. In a practical point of view, one can directly incorporate $JRC$ in the cubic formula using Equation (6.1), which requires the mechanical aperture ($E$) to compute the flow rate for a given hydraulic head. The modified cubic law incorporating $JRC$ is given below:

$$ q = \frac{b}{12\mu} \left( \frac{E^6}{JRC^{7.5}} \right) \left( \frac{dp}{dx} \right) $$  \hspace{1cm} (6.10)
Figure 6.12. Effect of joint roughness on fluid flow rate.

The above formula (Eqn. 6.10) is very useful in a practical sense. For instance, consider the need for a quick estimate of flow rate through a fracture that is exposed at a tunnel roof. After mapping the fracture surface using the string line method, the fracture surface is matched with the standard profiles to obtain the appropriate JRC value. The next step involves measuring the mechanical aperture of the fracture, which can be carried out using a feeler gauge. The magnitude of the mechanical joint aperture depends on the stress redistribution due to excavation. For a given hydraulic head, it is now a matter of substituting these values into Equation (6.10) to compute the flow quantity.
The above results confirm that roughness is of paramount importance in the accurate prediction of flow through fractures. However, in reality, it is not feasible to incorporate roughness of each joint separately in numerical models, as there is a large number of fractures in a jointed rock mass. Having identified these limitations, the cubic law should be used with caution in practical situations.

6.5 SUMMARY

Experimental and theoretical measurement of surface roughness and effects of them on fluid flow were presented in detail, and the important aspects are summarized below.

(a) Roughness of joint surface varies from point to point, and the average value is usually assigned to represent the entire surface. Different roughness values at different locations indicate that a fracture can be highly irregular, and that it may not be possible to represent the discontinuity in a simplified manner (e.g. parallel plates) in order to compute the flow conditions correctly.

(b) For practical purposes, standard profile method (Barton, 1973) based on the maximum amplitude of the joint can be used to estimate the roughness coefficient conveniently and fairly accurately.

(c) The joint roughness can be incorporated in fluid flow equation via: (a) mechanical joint roughness, (b) hydraulic joint roughness, and (c) mathematical function describing the joint surface. The first two methods are relatively easy to employ.
(d) When the asperity height ($k$) between the maximum and minimum of joint surface approaches the magnitude of joint aperture ($e$), the pressure drop coefficient is reduced by 8 times, in relation to the corresponding pressure drop coefficient of the smooth joint.

(e) Increase in JRC will result in a decrease in flow rate almost exponentially. Even at very high JRC values and high normal stresses, there is a minimum flow that corresponds to the residual aperture of the joint.

(f) Roughness is important in accurate fluid flow estimations. However, it is not feasible to incorporate roughness of each joint separately in numerical models, especially when there is a large number of fractures in a rock mass. Nevertheless, it is important to characterise the roughness of major joints that are expected to carry a significant fluid flow.
CHAPTER 7

DEVELOPMENT OF AN ANALYTICAL MODEL FOR TWO-PHASE FLOW THROUGH A ROCK JOINT

7.1 INTRODUCTION

A critical review of flow analysis applied to fractured rock was made in Chapter 3. The flow models based on analytical and experimental methods for determining permeability characteristics with the deformation of rocks have been developed since 1950 (Lomize, 1951; Louis, 1968; Iwai, 1976; Witherspoon et al., 1980). In general, these models deal with the single fluid flow analysis in rock fractures, which are usually in unsaturated condition carrying both water and air. As discussed in Chapter 3, there have been a few attempts to analyse two-phase flow through rock fractures, however none of these models have included the effects of one phase on the other. Two-phase flow in a rock mass involves a set of complex phenomena, and to date, no comprehensive model has been developed that can consider the factors such as the interaction between the two phases, changes of properties of the fluids and associated joint deformation. To the knowledge of the writer, inter-relations of multiphase flows, variations of relative permeability with pressure through rock fractures have not been properly investigated. Therefore, this chapter aims to shed light on the relatively complicated two-phase flow system, and to provide a comprehensive mathematical model to compute the quantities of each fluid travelling in a given joint domain.
In order to minimize the complexity of multiphase flow, it may be simulated as a homogeneous model in which average fluid properties are used. In true multiphase flow analysis, each phase should be modeled separately, and subsequently, phase change equations should be written to represent interaction between the phases. In the following section, two-phase stratified flow through a single rock joint is considered, and the governing equations are developed based on energy, mass and momentum principles.

7.2 FLOW STRUCTURES IN A SINGLE ROCK JOINT

The accurate determination of flow structures is very important in developing mathematical models for multiphase flow analysis. The knowledge of the particular flow regime is also important in the study of transient flows, and parameters such as change of fluid properties and wall shear stresses in various computer codes also needs to be incorporated. The patterns commonly encountered in pipe flows are of the type: stratified, bubble, droplet, annular, complex, plug and churn.

The main difficulty involved in determining the flow patterns through rock is that the flow is neither visible nor easily accessible. Unlike the case of flow through transparent pipes, no visual observations or photographic techniques can be employed in a convenient manner in the instance of rock joints. The photographic method may however, be used to map the flow pattern in artificial fractures made out of transparent plastics. For example, a natural rock joint surface may first be represented a transparent
plastic surface, and water and airflow can be induced through the joint to observe the flow type.

As shown in Figure 7.1, two-phase flow regimes can mainly be divided into two groups (a) continuous two-phase flow, and (b) discontinuous two-phase flow. In continuous flow, both phases prevail all along the flow path, whether stratified or annular. Stratified flow is the simplest flow pattern in multiphase flows. In such stratified flows, the main difficulty is in defining the interface level between the phases, as the interface location is time dependent and subjected to changes, depending upon various external conditions such as flow path geometry, fluid pressure and fluid properties. The theoretical development presented here is based on stratified flow in a rock joint, which is subjected to deformation under boundary stresses. More general equations for homogeneous and multi-fluid flow models are given in Appendix A.

(a). Discontinuity of air phase on a continuous water phase (Fluid)

(b). Continuity of both phases

Figure 7.1. Possible two-phase gas-water flow patterns in a given rock joint.
7.3 FORMULATION OF AN ANALYTICAL MODEL FOR TWO-PHASE STRATIFIED FLOW WITHIN A ROCK JOINT

7.3.1 Introduction

The analysis is based on a single joint filled with water and air as stratified layers, which is the simplest flow pattern that can occur (Figure 7.1). Unlike in two-phase flow through pipes, flows through rocks involve additional variables including continuous deformation of joint fractures associated with changes in ground stresses and fluid pressures. Therefore, the use of a simplified stratified flow model to simulate complex flows in rock joints is an initial step to understand the fundamental mechanisms of two-phase flow. The stratified flow is characterised by the liquid flowing as a layer at the bottom section of the discontinuity with gas flowing above it. This kind of flow generally occurs at low gas velocities. An increase in gas flow volume leads to the generation of waves on the gas-liquid interface, giving a stratified but undulating (wavy) flow. Further increases of gas velocity will form different kinds of flow structures, including mixed flow with no continuous interface.

7.3.2 Development of equations governing two-phase flow

The joint shown in Figure 7.2, carrying gas and water as layers is initially subjected to confining and vertical stresses. Due to increment of stress with time, the top surface of the joint deforms by a certain amount, and consequently, the interface level also alters. The flow properties of air and water are time dependent due to the occurring deformation of the joint, as indicated by a function of time ‘t’ as shown in Figure 7.2. The mechanical deformation of a joint wall due to the changes in external forces
gravitational stress, e.g. \( \sigma_1 \) and \( \sigma_3 \)) and the fluid pressure itself are combined in the model, with factors such as compressibility of fluids, solubility of air in water and phase change of fluids are shown significantly to influence the behaviour of the interface between air and water. The general procedure used to analyse the two-phase flow is described by the flow chart given in Figure 7.3.

Figure 7.2. A typical inclined joint filled with two-phase flows indicating change of interface and deformation of joint wall caused change of stress level.

The application of momentum conservation in the flow along a given rock joint yields the following governing equations (Graham, 1969).
For the water phase,

\[(\rho_w v_n A_w) \frac{dv_n}{dx} - (\rho_w A_w) \frac{dv_w}{dt} - (A_w) \left( \frac{dp}{dx} \right)_w - (\rho_w g \sin \beta A_w) + (F_{wa} S_{wa}) - (F_{jw} S_{jw}) - \]

\[(v_n - v_w) (1-\eta) M \frac{dR}{dx} + [(\sigma_1 /\sin \beta \cos \beta - \sigma_3 /\cos \beta \sin \beta) + \sigma_3 h_w \cos^2 \beta] = 0 \quad (7.1)\]

where,

- \(a\) = the subscript represents air phase
- \(w\) = the subscript represents water phase
- \(A\) = area of the cross-section of the fluid (m²)
- \(l\) = length of fluid element considered (m)
- \(\rho\) = density of fluid (kg/m³)
- \(v\) = velocity of fluid at time \(t\) (m/s)
- \(S_{wa}\) = perimeter of the interface between the phases (m)
- \(S_{jw}\) = perimeter of bottom joint wall (m)
- \(F_{wa} = \frac{f_i (v_n - v_b)^2 \rho_s}{2}\) = shear stress acting on the water-air interface (N/m²). It is convenient to express the interfacial and joint wall shear stress in terms of friction factors (Taitel et al., 1976), where \(f_i\) is the friction factor between two phases.
- \(F_{jw} = \frac{f_w \rho_w v_w^2}{2}\) = shear stress acting on the wall (N/m²), where \(f_w\) is the friction factor between the wall and water
- \(\sigma_1\) = vertical stress applied on the discontinuity (N/m²)
- \(\sigma_3\) = horizontal stress applied on the discontinuity (N/m²)
- \(M\) = total mass rate (kg/s) = \((Q_a \rho_a + Q_w \rho_w)\)
- \(Q_a\) = volumetric flow rate of air-phase (m³/s)
Two-phase flow (Air-Water)

Annular flow

Droplet flow

Mechanical deformation of joints

Solubility of air in water

Different two-phase flow patterns

Identify flow structure — Stratified flow —

Governing equations based on principles of conservation mass and momentum

General equation for interface level

Interface equation for unsteady state flow

Numerical technique is required

Interface equation for steady state flow.

Closed form solution

Flow rate for each phase

Figure 7.3. Simplified analysis of two-phase stratified flow.
Q_w = volumetric flow rate of water-phase (m^3/s)

R = air fraction of the total mass flow across the interface = M_a/M = Q_a ρ_a / (Q_a ρ_a + Q_w ρ_w)

η = fraction of the force taken by air phase associated with phase change

β = orientation of the joint relative to the horizontal (degrees)

p = fluid pressure inside the joint (N/m^2)

x = joint length (m)

Similarly, for the air-phase:

\( (\rho_a \cdot v_a \cdot A_a) \frac{dv_a}{dx} - (\rho_a \cdot v_a \cdot A_a) \frac{dv_a}{dt} - (A_a) \left( \frac{dp}{dx} \right) a - (\rho_a \cdot g \cdot \sin \beta \cdot A_a) - (F_{wa} \cdot S_{wa}) - (F_{ja} \cdot S_{ja}) = (V_a - V_w) \eta M \frac{dR}{dx} + (\sigma_3 \cdot h_a \cdot \cos \beta \cdot \sin \beta - \sigma_1 \cdot l \cdot \sin \beta \cdot \cos \beta + \sigma_3 \cdot l \cdot \cos^2 \beta) = 0 \) (7.2)

where,

S_{ja} = perimeter of the top joint wall (m)

\( F_{ja} = \frac{f_a \cdot \rho_a \cdot v_{a}^2}{2} \) = shear stress acting on the wall (N/m^2), where \( f_a \) is the friction factor between wall and air. The other notations are similar to water phase notations with the subscript “a” representing the air-phase.

The pressure drop \( dp/dx \) needs to be eliminated from the above two equations in order to attain a solution for the interface level. The pressure drop along the joint length \( dp/dx \) is given by the following equation, after a mathematical rearrangement of Equation (7.1).
\[
\left( \frac{dp}{dx} \right)_w = (\rho_w v_w) \frac{dv_w}{dx} - (\rho_w) \frac{dv_w}{dt} - (\rho_w g \sin \beta) + \frac{(F_{w_a} S_{w_a})}{A_w} - \frac{(F_{jw} S_{jw})}{A_w} - \frac{1}{A_w} (v_w - v_a) (1-\eta) M \frac{d\tau}{dx} + \frac{1}{A_w} \left( \frac{l \sigma_1 \sin \beta \cos \beta - l \sigma_3 \cos \beta \sin \beta + \sigma_3 h_w \cos^2 \beta}{l h_w(t)} \right) - \frac{(v_a - v_w)}{(1-\eta)} M \frac{d\tau}{dx} + \frac{1}{h_w(t)} \left( \sigma_1 \sin \beta \cos \beta - \sigma_3 \cos \beta \sin \beta \right) + \frac{1}{l} \sigma_3 \cos^2 \beta
\]  

(7.3)

The cross-sectional area of the water phase may be written in terms of the phase level and the length of wetted perimeter of the joint wall. The wetted perimeter of the interface and the joint walls (i.e. \(S_{wa}, S_{ja}\) and \(S_{jw}\)) can be assumed to be same, since the joint aperture is very small compared to the length of the wetted perimeter. Therefore, the pressure drop may be re-written as:

\[
\left( \frac{dp}{dx} \right)_w = (\rho_w v_w) \frac{dv_w}{dx} - (\rho_w) \frac{dv_w}{dt} - (\rho_w g \sin \beta) + \frac{(F_{w_a} S_{w_a})}{h_w(t)} - \frac{(F_{jw} S_{jw})}{h_w(t)} - \frac{1}{l h_w(t)} (v_a - v_w) (1-\eta) M \frac{dR}{dx} + \frac{1}{h_w(t)} \left( \sigma_1 \sin \beta \cos \beta - \sigma_3 \cos \beta \sin \beta \right) + \frac{1}{l} \sigma_3 \cos^2 \beta
\]  

(7.4)

When capillary forces are negligible (compared to viscous forces), the pressure gradient in both phases is equal \([ie \frac{dp}{dx}_w = \frac{dp}{dx}_a = \frac{dp}{dx}]\). The pressure drop \(\frac{dp}{dx}\) needs to be eliminated from the above two equations in order to attain a solution for the interface level. Having written a similar expression (as Eqn. 7.4) to the air-phase the following equation can be obtained after eliminating the \(\frac{dp}{dx}\) term to give:
\[
\frac{1}{h_w(t)} \left[ \sigma_1 \sin \beta \cos \beta - \sigma_3 \cos \beta \sin \beta + F_{wa} - F_{jw} - \frac{1}{l} (v_a - v_w) (1-\eta) \frac{dR}{dx} \right] + \frac{1}{h_a(t)} \left[ \sigma_1 \sin \beta \cos \beta - \sigma_3 \cos \beta \sin \beta + F_{wa} + F_{ja} + \frac{1}{l} (v_a - v_w) \eta \frac{dR}{dx} \right] = \rho_w \frac{dv_w}{dx} - \rho_a v_a \frac{dv_a}{dx} + \rho_w \frac{dv_w}{dt} - \rho_a \frac{dv_a}{dt} - g \sin \beta [\rho_a - \rho_w]
\]

The mechanical and hydraulic behaviour of discontinuities in rocks depend strongly on the topography of the joint surfaces and the degree of correlation between them. The topography of joint walls has been studied using various techniques including profilometers (Pyrak-Notle et al., 1987; Brown et al., 1985). Brown (1987) studied the effects of surface roughness of rock joints on fluid flow along discontinuities using the Reynolds equation and a fractal model of surface topology. The effects of surface roughness on flow were in detail discussed in Chapter 6. The topography of fracture surfaces are quantitatively described here by specifying the location of the top and bottom fracture surfaces by \( F_T (x, y)_t \) and \( F_B (x, y)_t \) respectively, relative to some co-ordinate system as shown in Figure 7.2. Unlike in single-phase flow through discontinuities, for two-phase flow, the geometry of the interface between the two phases can be defined by \( F_I (x, y)_t \) relative to the same co-ordinate system. The initial surfaces of top and bottom joint walls profiles are taken as \( F_T (x, y)_0 \) and \( F_B(x, y)_0 \), respectively. When they are subjected to deformation associated with external stresses and fluid pressure after time \( t \), they are assumed to take the form of \( F_T (x, y)_t \) and \( F_B (x,y)_t \), respectively. The interface between the two phases is assumed to be \( F_I (x,y)_t \) at the beginning and after time \( t \), respectively.

The water-phase level (height), \( h_w(t) \), can be represented as:
\[ h_w(t) = F_I(x,y)_t - F_B(x,y)_t \]  \hspace{1cm} (7.6)

If \( \xi_{wc} \) is the change of interface level between two phases, due to compressibility of water, then,

\[ h_w(t) = F_I(x,y)_0 - F_B(x,y)_0 - \xi_{wc}, \text{ or} \]

\[ h_w(t) = F_I(x,y)_0 - F_B(x,y,\Delta_B)_t \]  \hspace{1cm} (7.7)

where, \( F_B(x,y,\Delta_B)_t \) is given by the expression, \( F_B(x,y)_0 + \xi_{wc} \).

In the same manner, the air-phase level, \( h_a(t) \), at time \( t \) is given by:

\[ h_a(t) = F_I(x,y)_t - F_I(x,y)_t \]  \hspace{1cm} (7.8)

Factors which control the air-phase level are (a) mechanical deformation of joint, (b) compressibility of air, (c) rate of solubility of air in water, and (d) effects of change of fluid properties and temperature. Hence, Eqn. (7.8) can be linked to the initial condition by:

\[ h_a(t) = F_I(x,y)_0 - F_I(x,y)_0 - \Delta_T \]  \hspace{1cm} (7.9)

where, \( \Delta_T \) is the deformation of wetted wall in contact with the air-phase.

The total deformation, \( \Delta_T \), includes the effects of compressibility of water (\( \xi_{wc} \)), compressibility of air (\( \xi_{ac} \)), solubility of air in water (\( \xi_{ad} \)), and the elastic deformation of the joint wall (\( \delta_n \)) on the air-phase level \( h_a(t) \). Hence,

\[ \Delta_T = \xi_{ac} + \xi_{ad} + \delta_n - \xi_{wc} \]  \hspace{1cm} (7.9a)

The evaluation of the functions \( \xi_{ac}, \xi_{ad}, \delta_n \) and \( \xi_{wc} \) will be discussed later.
If \( F_T(x,y,\Delta_T) \) is represented by the expression \([F_T(x,y) - \Delta_T]\), Eqn 7.9 can now be rewritten as:

\[
h_a(t) = F_T(x,y,\Delta_T) - F_1(x,y) \tag{7.10}
\]

Substituting Equations (7.7) and (7.10) into the Equation (7.5) yields:

\[
\left\{\sigma_1 \sin \beta \cos \beta \left[ F(x,y,\Delta) + F_wa \left[ F(x,y,\Delta) \right] \right] - [F_{jw} + \sigma_3 \cos \beta \sin \beta] F_T(x,y,\Delta_T) + [\sigma_3 \cos \beta \sin \beta - F_{jw}] F_B(x,y,\Delta_B) - \frac{1}{l} (v_a - v_w) (1-\eta) M \frac{dR}{dx} \left[ (1-\eta) F_T(x,y,\Delta_T) + \eta F_B(x,y,\Delta_B) \right] + F_1
\]

\[
(x,y) \left\{ F_{jw} + F_{ja} + \frac{1}{l} (v_a - v_w) M \frac{dR}{dx} \right\} \} = \left\{ F_1(x,y) \left[ F_T(x,y,\Delta_T) + F_B(x,y,\Delta_B) \right] - F_1^2
\]

\[
(x,y) \cdot F_T(x,y,\Delta_T) F_B(x,y,\Delta_B) \} \{ A - g \sin \beta (\rho_a - \rho_w) \} \tag{7.11}
\]

where, \( F(x,y,\Delta) = F_T(x,y,\Delta_T) - F_B(x,y,\Delta_B) \) \tag{7.11a}

The above expression may be presented in a simplified form as:

\[
[F_1(x,y) \sum_{k=1}^{2} \Delta_k - F_1^2 (x,y) - \Delta_1 \Delta_2 ] [A] = F(x,y,\Delta) [\sigma_1 \sin \beta \cos \beta - \sigma_3 \cos \beta \sin \beta + \]

\[
F_{wa} \left\{ \left[ \sum_{k=1}^{2} F_{j_1} \Delta_k \right] - \sum_{h=1}^{2} (1-\eta) C N \Delta_k + F_1(x,y) \left[ \sum_{h=1}^{2} F_{j_1} + C \right] \right\} \]

\[
A = [B_1 - g \sin \beta (\rho_a - \rho_w)] \tag{7.12a}
\]

where, \( B_1 = [\rho_w v_w \frac{dv_a}{dx} - \rho_a v_a \frac{dv_a}{dx} + \rho_w \frac{dv_w}{dt} - \rho_a \frac{dv_a}{dt}] \) = force per unit area

235
associated with unsteady effects of flow.

\[ C = \left[ \frac{1}{l} (v_a - v_w) M \frac{dR}{dx} \right] \]  

(7.12c)

\[ \Delta_1 = F_T(x,y,\Delta_T), \text{ and} \]  

(7.12d)

\[ \Delta_2 = F_B(x,y,\Delta_B) \]  

(7.12e)

Alternately, Equation 7.12 may be presented as:

\[
F_1(x,y) \left[ A \sum_{k=1}^{2} \Delta_k - \sum_{i=a,w} F_{ji} - C \right] - A F_1^2(x,y) = F(x,y,\Delta) \left[ \sigma_1 \sin \beta \cos \beta - \sigma_3 \cos \beta \sin \beta + F_{wa} \right] - \left[ \sum_{k=1}^{2} F_{ji} \Delta_k \right] - \sum_{N=-\eta,\eta}^{2} C N \Delta_k + A \Delta_1 \Delta_2
\]  

(7.13)

Equation (7.13) can also be simplified to write the main governing equation as follows:

\[
F_1(x,y) \Delta_3 - A F_1^2(x,y) - D = 0
\]  

(7.14)

where, \[ \Delta_3 = \left[ A \sum_{k=1}^{2} \Delta_k - \sum_{i=a,w} F_{ji} - C \right], \]  

(7.14a)

and \[ D = F(x,y,\Delta) \left[ \sigma_1 \sin \beta \cos \beta - \sigma_3 \cos \beta \sin \beta + F_{wa} \right] - \left[ \sum_{k=1}^{2} F_{ji} \Delta_k \right] - \sum_{N=-\eta,\eta}^{2} C N \Delta_k + A \Delta_1 \Delta_2 \]  

(7.14b)
Equation 7.14 in quadratic form, provides a definite solution to represent the interface level, which facilitates the computation of both water and air levels at any time, incorporating the water and air levels given earlier by Equations 7.7 and 7.10.

In two-phase flow, the main difficulty is the location of the interface, which if known will enable the evaluation of other parameters such as the flow quantity of each phase at a given time. Although the solution of the governing Equation 7.14 is somewhat cumbersome, the apparent complexity will be lessened under steady state flow conditions, on which most practical engineering solutions are based. For steady state flow, the acceleration term \( B_1 \) in Eqn. 7.12b vanishes. Moreover, the gravity effects due to air can be neglected, noting that the density of air at atmospheric pressure (1.23 kg/m\(^3\) at 15 °C) is insignificant compared to that of water density (1000 kg/m\(^3\) at 15 °C). Further simplification will provide a new expression for the term A in Equation 7.12a, as given by:

\[
A = [\rho_w g \sin \beta] \quad (7.15)
\]

By assuming that no sudden temperature change occurs, it is then reasonable to neglect any change of properties within the phases. This will make the term \( \frac{dR}{dx} \) associated with phase change to be zero in Eqns. 7.2-7.5 and 7.11-7.14.

One could seek a very simple form of expression for the interface level, if one would ignore the gravity effects of fluid. Such assumptions are not unreasonable given that, when computing flow rates using Darcy's law it is customary to ignore the gravity effects.
effects in conventional soil mechanics. The effects of horizontal stress \((\sigma_3)\) acting on
the face of a horizontal joint segment can be neglected, as the effective area of the face
on which the horizontal stress acts, is insignificant. Considering the same assumptions,
for the interface, Eqn.7.14 will take the following simplified from, for steady state flow
through a horizontal joint:

\[
F(x,y,\Delta) [F_{wa}] - \sum_{k=1}^{2} F_{j_i} \Delta_k + F_1 (x,y) \sum_{i=a,w} F_{j_i} = 0
\tag{7.16}
\]

\[
\sum_{k=1}^{2} F_{j_i} \Delta_k - F(x,y,\Delta)[F_{wa}]
F_1 (x,y) = \frac{\sum_{i=a,w} F_{j_i}}{\sum_{i=a,w} F_{j_i}}
\tag{7.17}
\]

\[
F_1 (x,y) = \frac{F_{ja} F_B(x,y,\Delta_B) + F_{jw} F_T(x,y,\Delta_T) - F_{wa} (x,y,\Delta)}{F_{ja} + F_{jw}}
\tag{7.18}
\]

where, \(F(x,y,\Delta) = F_T(x,y,\Delta_T) - F_B(x,y,\Delta_B)\)

\[
\Delta_2 = F_T(x,y,\Delta_T), \text{ and } \Delta_1 = F_B(x,y,\Delta_B)
\]

\(F_{ji} = F_{ji}\) where 'i' takes subscripts 'a' and 'w' for air and water, respectively.

For the conditions of steady state flow through a horizontal joint and with the
assumptions that the effect of horizontal stress on joint segment is insignificant, the
pressure drop \(\left(\frac{dp}{dx}\right)\) given by general Equation 7.14 can now be simplified to:

\[
\frac{dp}{dx} = \frac{F_{wa}}{h_w(t)} - \frac{F_{jw}}{h_w(t)}
\]
The Equation 7.18 represents the surface of discontinuity between phases for steady state flow through a single horizontal joint. The above solution confirms that the following parameters govern the position of the interface level for two-phase flow through a horizontal discontinuity with negligible gravity effects on the fluids is directly proportional to:

1. External stress (e.g. gravitational stress, fluid pressure)
2. Surface topography of joint walls,
3. Initial discontinuity aperture,
4. Compressibilities of the fluids and solubility of air in water, and
5. Pressure causing fluid flow.

7.4 EFFECTS OF JOINT DEFORMATION, SOLUBILITY OF AIR AND COMPRESSIBILITY OF FLUIDS ON FLUID FLOW RATE

This section now focuses on developing a mathematical expression for flow rate of each phase, using the general Equation 7.18. The deformation of a joint associated with the compressibilities of air and water, solubility of air in water, fluid pressure itself and external forces such as gravitational stresses are discussed below. The changes in the properties of both the phases and their effects on the flow rates are also discussed.

7.4.1 Effects of solubility of air on water

The water phase initially has an amount of dissolved air, which is in equilibrium with
the free air. As a result of pressure increase in the air-phase due to the mechanical
deformation of joint walls, the properties governing the flow such as density and fluid
pressure will change. The temperature variation will also influence these properties.
Because of the pressure difference between the air-phase and the water-phase, some air
(in air-phase) will dissolve in the water-phase according to Henry’s law described
below. This process will continue until a new equilibrium is attained (i.e. new pressure
in the air-phase is the same as the new pressure of dissolved air in the water-phase).
The dissolved quantity of air in water can be predicted by Henry’s law assuming the air
to be as ideal gas: \( p = m c \), where \( p \) is the partial pressure in the air phase (atm) and \( c \) is
the concentration of soluble component in liquid (lb moles/cuft) and \( m \) is the Henry’s
constant (atm cuft/mole). The mass transfer from the air phase to water phase takes
place in two stages: (a) air phase to interface and (b) interface to water phase. The
solubility of air in water at the equilibrium state is best described by the ideal gas law
and Henry’s law at given pressure and temperature conditions (Fredlund and Rahardjo,
1993):

\[
V_{dt} = \left( \frac{M_d}{p_a} \right) \left( \frac{RT}{W_a} \right) 
\]

(7.19)

In the above equation, \( V_{dt} \) is the volume of dissolved air in water at time \( t \), \( M_d \) and \( p_a \) are
the mass of dissolved air in water and absolute pressure of the air respectively. \( W_a \) is
the molecular mass of air (kg/kmol), \( R \) is the universal gas constant (8.314 J/mol.k) and
\( T \) is the absolute temperature (k) \((T=273+ t^\circ)\) where \( t^\circ \) is the temperature in \(^\circ C\).

For constant temperature, the ratio of the mass of dissolved air to absolute air pressure
can be given as follows (Universal gas law):
\[
\frac{M_{dt}}{P_0} = \frac{M_{dt}}{P_at}
\]  

(7.20)

where, the subscripts "0" and "t" represent the initial conditions \((t=0)\) and at any given time, \(t\). At a constant temperature, the volume of dissolved air in water-phase is a constant value at different pressures. The change of equivalent air phase level \((\xi_{ad})\) associated with dissolved air may be calculated for a given joint length \((l)\) per unit width, as follows:

\[
\xi_{ad} = \frac{V_{dt}}{lx1}
\]  

(7.21)

Air can dissolve in water and can occupy approximately 2% by volume of water (Dorsey, 1940). The coefficient of solubility of a gas in liquid is defined as the volume of the gas which is measured at the same phase conditions contained in a unit volume of the saturated solution. The coefficient of solubility and volumetric coefficient of solubility of different gases for various temperatures have been discussed by Dorsey (1940). At 1 atmospheric pressure, Figure 7.4 shows the solubility of air in water at various temperatures.

![Figure 7.4. Solubility of air in water at different temperatures (data from Dorsey, 1940).](image)
This shows a decrease in solubility with an increase in temperature. At elevated temperatures, the rate of solubility is very small. The effect of pressure on solubility of some gases in water is presented in Figure 7.5. Increase in pressure does not show an increase in solubility, because at a particular pressure, water is saturated with the gas depending on the molecular structure of gas and water.

![Figure 7.5. Effect of pressure on the solubility of gases in water (data from Dorsey, 1940).](image)

7.4.2 Effects of compressibility of air

The compressibility of a fluid is a measure of the change in density due to the specified change in pressure. Generally, pressure change will induce density changes, which will influence other flow parameters, such as the flow rate and velocity. A knowledge of the compressible fluid flow theory is required in various engineering applications, such as...
gas turbines, steam turbines, combustion chambers and natural transmission pipe lines. More significantly, in this study, the compressibility of fluid, particularly air/gas, is an essential consideration in coupled water-gas flow through rock joints.

In a simplified compressible flow theory, the following assumptions are made:

(a) The gas is a perfect fluid (i.e., PV = mRT),
(b) Gas is a continuous phase,
(c) Gravity effects on flow are negligible, and
(d) No chemical changes occur between the phases.

Under such conditions, the general equations for gas flow are based on (a) continuity equation, (b) momentum equation and (c) law of thermodynamics. Compared to water, gases are highly compressible, and the changes in gas density are directly related to the changes in the pressure and temperature through Charles’ equation:

\[ p = \rho R T. \quad (7.22) \]

where, \( p \) is absolute the pressure, \( \rho \) is the density of gas, \( R \) and \( T \) are the gas constant and the absolute temperature, respectively.

The compressibility of air at a constant temperature with respect to pressure is described by (Fredlund and Rahardjo, 1993):

\[ C_a = \frac{1}{V_a} \left( \frac{dV_a}{dp_a} \right), \quad (7.23) \]

where, \( C_a \) =compressibility of air (m\(^2\)/N), \( V_a \) = volume of air, \( p_a \) = pressure of air. t is time and \( dp_a \) =change of air pressure
For a joint length, \( l \), the change of equivalent air-phase level due to compressibility of air is given by:

\[
\xi_{ac} = \frac{C_a V_a}{l} \frac{dp_a}{l}
\]  

(7.24)

The effect of \( \xi_{ac} \) on the air-phase level \( h_a(t) \) was discussed earlier via Eqn. 7.9a.

### 7.4.3 Effects of compressibility of water

The compressibility of water due to the deformation of joint walls is discussed in this section. Water has a very low value of compressibility \([4.58 \times 10^{-7} \text{ (1/kPa)}]\) compared to gas \([4.94 \times 10^{-3} \text{ (1/kPa)}]\). A property that is commonly used to characterise the compressibility of water is the bulk modulus \([K = dp/(dv/V)]\). For the analysis presented here, the term compressibility coefficient \((C_w)\) is adopted, where \( C_w = \frac{1}{K} \).

Similar to Eqn. 7.24, the change of equivalent water-phase level due to the compressibility effects of water is given by:

\[
\xi_{wc} = \frac{C_w V_w}{l} \frac{dp_w}{l}
\]  

(7.25)

The effect of \( \xi_{wc} \) on the water-phase level \( h_w(t) \) and air-phase \( h_a(t) \) was introduced earlier through Eqn. 7.7 and Eqn. 7.9a, respectively. The accuracy of Eqn. 7.25 may be less valid for air-dissolved water, which inevitably occurs in real discontinuities. However, Dorsey (1940) has shown that there is no significant difference between the compressibility of air-free water and that of air-saturated water. Figure 7.6 presents the effect of temperature on the compressibility of water for different pressures.
Figure 7.6. Effect of temperature on compressibility of water at different pressures (data from Dorsey, 1940).

7.4.4 Influences of water and air densities

In two-phase flow, on one hand, the pressure changes caused by deformation of joint walls or inlet velocity of air will generally induce density changes, which in turn will affect the flow rate. On the other hand, the temperature changes in the flow, which arise due to the kinetic energy changes associated with the velocity changes will also influence the flow quantity. In this study, the effects of temperature changes are omitted for simplicity, in which case, the changed density of air-phase at time $t$ is determined by (Fredlund and Rahardjo, 1993):

$$\rho_a(t) = \frac{p_a(t)}{p_a(0)} \rho_a(0)$$ (7.26)
where, \( \rho_a(t) \) = final density of air corresponding to final pressure \( p_a(t) \), and \( p_a(0) \) = initial pressure of air in air-phase, and \( \rho_a(0) \) = initial density of air at \( t = 0 \).

As a result of dissolved air in water, the water-phase will be characterised by the following density term:

\[
\rho_w(t) = \frac{M_w(0) + M_{da}(t)}{V_w(t)}
\]  

(7.27)

where, \( M_w(0) \) = initial mass of water, \( M_w(t) \) = final mass of dissolved air in water and \( V_w(t) \) = final volume of water (i.e. water + dissolved air).

Equation 7.27 may be presented in a different way by incorporating the Boyle’s law, and assuming that the final volume of water and initial volume will be the same, hence.

\[
\rho_w(t) = \rho_w(0) + \left[ \frac{V_{da}(t)}{V_w(0)} \right] \left[ \frac{p_{da}(t)}{p_a(0)} \right] [\rho_a(0)]
\]  

(7.28)

The deviation of the density of water from standard conditions (1atm, 20\(^{0}\)C) to extreme conditions such as high pressure and low temperature (300atm, 0\(^{0}\)C), or high temperature and low pressure (100\(^{0}\)C, 0atm), can be ignored in most geotechnical applications. Therefore, in this analysis the second term of Eqn. 7.28 has been neglected [i.e. \( \rho_w(t) = \rho_w(0) \)].
7.4.5 Deformation of joint due to external stress

The aperture variations of discontinuities due to stress changes are examined in this section. The rock material is considered as impermeable and the flow is assumed to be confined to the discontinuities. Moreover, the rock matrix is assumed to be isotropic and linear elastic, obeying Hooke's law.

The aperture of the discontinuity, $e_t$, at any time is given by:

$$e_t = e_0 \pm \delta_n$$  \hspace{1cm} (7.29)

where, $e_0 =$ aperture at time $t=0$

$e_t =$ aperture at time $t$ and

$\delta_n =$ aperture increment during time interval $t$

In conventional rock mechanics, the normal and shear deformation components are given by:

$$\delta_n = \frac{1}{k_n} \left[ \sigma_1 \cos^2 \beta + \sigma_3 \sin^2 \beta \right] \text{ and } (7.30a)$$

$$\delta_t = \frac{1}{k_s} \left[ \sigma_3 \cos^2 \beta - \sigma_1 \sin^2 \beta \right] \text{ and } (7.30b)$$

Considering water pressure acting perpendicular to the joint surface, Eqn. 7.30a can be modified to give:

$$\delta_n = \frac{1}{k_n} \left[ \sigma_1 \cos^2 \beta + \sigma_3 \sin^2 \beta - p_w \right] \text{ and } (7.30c)$$

$\delta_t$ remains unchanged
where, $\sigma_1 =$ vertical stress applied to the discontinuity,

$\sigma_3 =$ horizontal stress applied to the discontinuity,

$k_n =$ normal stiffness of discontinuity,

$k_s =$ shear stiffness of discontinuity,

$p_w =$ water pressure within the discontinuity,

$\beta =$ orientation of discontinuity relative to the horizontal

$\delta_n =$ normal aperture of discontinuity, and

$\delta_t =$ tangential displacement of discontinuity

In reality, $\delta_n$ may be a function of both water and air pressures ($p_w$ and $p_a$), but it is assumed for simplicity that a critical value for $\delta_n$ will be associated with $p_w$ only. One may describe the elasto-plastic behaviour in the mechanical deformation of rock the joint by the Mohr-Coulomb theory. However, it is assumed that yielding of rock with respect to the Mohr-Coulomb failure envelope will not occur, and that elastic conditions will prevail.

Knowing the individual components, $\xi_{ad}$, $\xi_{asc}$, $\xi_{wc}$ and $\delta_n$ from Equations 7.21, 7.24, 7.25 and 7.30, both Eqn. 7.7 representing $h_w(t)$ and Eqn. 7.9 representing $h_s(t)$ can be solved.

7.5 FLOW LAWS RELATED TO TWO-PHASE FLOW

Once the governing equations for interface level, and phase levels of air and water have
been evaluated, the next task is to evaluate the flow rates of each phase through a given discontinuity. Unlike in single-phase flow, there can be more than one driving potential for the flow rate in two-phase flow. For example, air-water stratified flow can have two components: (a) for air the phase which is governed by the pressure gradient with negligible gravity effects, and (b) the water phase which is usually caused by hydraulic gradient determined on the basis of elevation head, pressure head and the velocity head, which constitute the total pressure head.

The Darcy’s law is commonly applied to flow of water in saturated rock joints, which is described by:

\[ v_w = k_w i_w \]  \hspace{1cm} (7.31)

where, \( v_w \) = flow rate of water, \( k_w \) = coefficient of permeability and \( i_w \) = hydraulic gradient. Darcy’s law have also been applied to unsaturated rocks (Richard, 1931; Buckingham, 1907), with a variable coefficient of permeability. For a saturated discontinuity, \( k_w \) is the hydraulic conductivity of water, whereas, if the fracture is unsaturated, then the hydraulic conductivity of the fracture becomes a function of the frictional drag forces generated by the fluids acting along the joint walls.

Poiseuille’s law is best suited for describing flow (both gas and air) through a single smooth fracture (parallel plate), which is represented mathematically by:

\[ v_t = \frac{e i g}{12 \nu} \nabla \left( \frac{p}{\rho g} + z \right) \]  \hspace{1cm} (7.32)

where, \( v_t \) = the average velocity vector of fluid inside the fracture.
et = the aperture of discontinuity,
g = the gravitation of acceleration,
ν = the kinematics viscosity,
P = fluid pressure,
z = elevation head,
∇ = partial differential operator

The effects of gravity on fluid flow through a vertical fracture are negligible over the small length of the specimen (i.e. 120mm) compared to the high applied inlet pressure. Therefore, the elevation head (z) is assumed to be negligible compared to the inlet fluid pressure. Equation 7.32 is then given by the simplified cubic law:

\[
q_w = \frac{e \cdot g \left( \frac{dp}{dx} \right)}{12\nu} \tag{7.33}
\]

The aperture, e in Equation (7.33) can be replaced by the height of water phase \( h_w(t) \) to give:

\[
q_w(t) = \frac{h_w^3(t) g \left( \frac{dp}{dx} \right)}{12\nu_w} \tag{7.34}
\]
in which, \( h_w(t) = F_i(x,y) - F_B(x,y) - \xi_{wc} \)

p = pressure of the fluid

In the same manner, for the air-phase:

\[
q_a(t) = \frac{h_a^3(t) g \left( \frac{dp}{dx} \right)}{12\nu_a} \tag{7.35}
\]
where, $h_s(t) = F_T(x,y)_0 - F_l(x,y)_0 - (\delta_n + \xi_{ad} + \xi_{ac} - \xi_{wc})$, and

$$\frac{dp}{dx} = \frac{F_{wa}}{h_w(t)} - \frac{F_{jw}}{h_w(t)}$$

For the given water and air pressure of 0.25MPa inlet pressures, the phase heights of air and water for different joint apertures are shown in Figure 7.7. The material properties of the rock and the fluids are presented in Appendix B. Increased joint aperture results in more air entering into the joint than water. This causes increases in the solubility of air in water and compressibility of air. A significant increase in the solubility and compressibility of air occurs at small apertures. An increase in the joint aperture does not increase the solubility and compressibility linearly, if the solubility and compressibility of fluid reach their maximum values. As expected, the phase heights of both water and air increase with the increase with joint apertures.

Figure 7.7. Phase height of air and water flow through a rock joint.
According to Figure 7.8, flow rates follow the same trend as phase heights against joint apertures. However, the airflow rates are significantly higher than the water flow rates because, water has a greater viscosity than air. At the larger apertures, more air enters the joint in comparison with water, resulting in increased airflow. It also follows that air will occupy most part of the joint when the joint has a larger opening. In other words, similar to multiphase flow in pipes, a higher airflow rate is usually expected at larger apertures in the case of rock joints.

At a given confining pressure and axial stress, the two-phase flow rate against the inlet fluid pressure ratio is plotted in Figure 7.9. The flow rates (vertical axis) are plotted on a log scale, in order to improve the clarity of the relatively small water flow rates in relation to the air flow rates. The corresponding phase heights are presented in Figure 7.10. At the crossover point, the air phase height becomes equal to water phase height,
which indicates that fracture permeability of both phases are similar at 0.8MPa inlet fluid pressure. It is obvious that airflow through a rock specimen is much greater than the water flow rate for the same boundary conditions.

![Figure 7.9. Effects of inlet fluid pressures on two-phase flow rates.](image)

![Figure 7.10. Effects of inlet fluid pressures on phase heights.](image)
For different inlet fluid pressures and joint apertures, the estimated values of the parameters in Equations 7.7, 7.10, 7.18, 7.21, 7.24, 7.30c, 7.34 and 7.35 are given in Appendix B. The comparison of mathematical model with experimental results using designed two-phase high pressure triaxial apparatus will be discussed in Chapter 8.

7.6 SUMMARY

This study was concerned with the theoretical investigation of two-phase flow of water and air in a single rock joint. In a multiphase flow analysis, it is almost impossible to develop a general equation incorporating any type of flow, given the extreme variabilities associated with joint geometry and roughness. In this study, the flow pattern within the joint was assumed as stratified, in order to develop a mathematical model while recognising the limitations from such an assumption. However, this is an initial effort for describing the poorly understood phenomenon of two-phase flow through rock joints and the future necessity to extend this model for more complex flow analysis is appreciated.

In the analytical approach, governing equations were developed to model the behaviour of a joint filled with water and air, incorporating the effects of the normal deformation of joint, compressibilities of air and water, and the solubility of air in the water phase. The proposed mathematical formulation can simulate two-phase flow through a smooth or rough joint. Explicit solutions were determined for a joint with parallel walls and inclined to the horizontal, within which the air and water phases travel as stratified layers.
CHAPTER 8

LABORATORY STUDY OF TWO-PHASE FLOW THROUGH
FRACTURED ROCK SPECIMENS

8.1 INTRODUCTION

Although much experimentation has been carried out to understand two-phase (air-water) flow behaviour in the field of chemical and mechanical engineering, the proper understanding of two-phase flow behavior in jointed rocks still remains at infancy, because of the complexity of geological variabilities. A number of available triaxial facilities can measure either the water pressure or air pressure or both within a fractured rock, but most of them are still incapable of measuring the relative permeability (air or water) of a fractured specimen. It is the relative permeability data that are most useful in the numerical analysis of flow through jointed rock mass. In order to study the two-phase flow behaviour through fractured rock specimens, a novel equipment, namely, Two-Phase, High-Pressure Triaxial Apparatus" (TPHPTA) was employed. The features of commonly available single-phase triaxial equipment and those of TPHPTA for soil and rock testing were discussed in Chapter 4. This chapter concentrates on the possible two-phase flow patterns in a rock joint under different boundary conditions, effects of boundary conditions on the flows, intrinsic and relative permeabilities, validity of Darcy’s law and finally, validation of the mathematical model developed in Chapter 7.
Fractured granite rock specimens having diameters from 45 to 55mm and length of twice the diameter with low matrix permeability (order of $10^{-19}$ m$^2$) were selected for the laboratory investigation. The tests were carried out on both artificially and naturally fractured specimens, having either a single fracture or multiple fracture networks. The test specimens were cored from large samples, using a small scale coring machine. Subsequently, artificial (tension) fractures were induced on the intact rock cores using the Brazilian test. Artificially fractured specimens have single fracture while naturally fractured specimens exhibit single or multiple fractures. Naturally fractured granite specimens were supplied by Strata Control Technology (SCT, Wollongong) from different mines in Australia. Some typical fractured specimens for testing are shown in Figure 8.1.

Figure 8.1. Typical fractured granite rock specimens used for testing.
Table 8.1. Testing procedures for single-phase and two-phase flow measurements.

<table>
<thead>
<tr>
<th>Stages</th>
<th>Cell pressure</th>
<th>Axial stress</th>
<th>Fluid pressures</th>
<th>Comments</th>
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<td></td>
<td></td>
<td></td>
<td>Water ($p_w$)</td>
<td>Air ($p_a$)</td>
</tr>
<tr>
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<td>Constant</td>
<td>Constant</td>
<td>Variable</td>
<td>No air flow</td>
</tr>
<tr>
<td>Stage 2</td>
<td>Variable</td>
<td>Constant</td>
<td>Constant</td>
<td>No air flow</td>
</tr>
<tr>
<td>Stage 3</td>
<td>Constant</td>
<td>Variable</td>
<td>Constant</td>
<td>No air flow</td>
</tr>
</tbody>
</table>

**Two-phase flow analysis (water + air)**

<table>
<thead>
<tr>
<th>Stages</th>
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<tbody>
<tr>
<td>Stage 1</td>
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<td>Constant</td>
<td>Constant</td>
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<tr>
<td>Stage 3</td>
<td></td>
<td>Variable</td>
<td>Constant</td>
<td>Constant</td>
</tr>
</tbody>
</table>
The basic test procedure was the same as for single-phase flow illustrated in Chapter 5. Table 8.1 shows a combination of tests carried out using TPHPTA. As discussed in Chapter 6, prior to testing, all the fractured specimens were mapped using the digital coordinate profilometer to estimate the roughness of the fractured surfaces.

8.3 DETERMINATION OF FLOW REGIMES

The knowledge of the flow regimes is important in the study of transient flows, and also in steady state flows in order to incorporate the flow parameters such as changes of fluid properties and wall shear stresses in various computer based analyses. The inlet pressures of each phase, their physical properties, interactions between the phases and the geometries of flow paths determine the flow patterns in a pipe or channel or in a rock joint.

In the past, various approaches have been employed to identify flow structures as listed below (Fiori & Bergles, 1966; Hewitt, 1978; Fourar & Bories, 1995):

(a) Visual observations supported by photographic method, and

(b) Selected fluid flow parameters such as superficial velocities.

From the work carried out in two-phase flow study, the flow regimes can mainly be divided into two groups (Figure 8.2):

(a) Continuous two-phase flow, and

(b) Discontinuous two-phase flow.
In continuous flow, both phases are usually continuous through the flow path (e.g. stratified, annular). In discontinuous flow, either one or both the phases can be discontinuous (e.g. bubble, droplet, slug and complex flows). These flow patterns may develop due to the effect of one or more of the following factors:

(a) Pressure/velocity of each phase,
(b) Viscosity of gas and the liquid,
(c) Density difference and buoyancy,
(d) Flow path geometry, and
(e) Surface tension and surface contamination.

In this study, the flow patterns were determined based on fluid flow parameters such as superficial velocities, because visual observations supported by photographic method could not be directly applied to real rock fractures.
8.3.1 Flow pattern based on fluid flow parameters

The technique based on the study of the fluid flow parameters is an indirect method of analysing the flow pattern of gas-liquid two-phase flow system. This technique is particularly suitable when the flow is not visible or when the flow takes place at high-speed. In multiphase flow analysis through pipes, the common procedure is to plot the liquid superficial velocity against the gas superficial velocity. Such a plot on water-gas flow through horizontal glass plates is shown in Figure 8.3 (Fourar & Bories, 1995). Golan & Stenning (1969) reported a flow regime map for a vertical pipe, as shown in Figure 8.4. They observed annular, oscillatory and slug flow patterns for elevated gas and liquid superficial velocities. Both plots show that bubble flow, in which liquid flow is dominant, occurs at low gas velocities. In contrast, annular flow prefers to develop at elevated gas velocities.

Figure 8.3. Possible water-gas flow maps (data from Fourar & Bories, 1995).
Extensive laboratory work on water-gas flow through fractured rocks was conducted using the newly designed Two-Phase, High-Pressure Triaxial Apparatus (TPHPTA, described in Chapter 4). The test procedure was as follows. The specimen was first saturated with water, and then the air phase was forced through the specimen. Once the water and air mixture passed through the dreschel bottle, airflow rates and water flow quantities were recorded by the film flow meter and electronic weighing scale, respectively. These individual flow rates were used to calculate the superficial velocity \( v_a \) of each phase, using Equation 8.1. It is important to note here that the effect of gravity on fluid flow is negligible when compared to the magnitude of applied fluid pressure.

\[
v_a = \frac{k_a K_{rs}}{\mu_a} \left( \frac{dp}{dx} \right)_a = \frac{q_a}{A_a}
\] (8.1)
where, $\alpha = \text{phase,}$

$\mu = \text{dynamic viscosity,}$

$k = \text{intrinsic permeability,}$

$K = \text{relative permeability,}$

$dp = \text{pressure difference along the joint length, } dx,$

$q = \text{flow rate, and } A = \text{area perpendicular to the flow}$

Using this technique, the flow patterns within natural rock fractures were studied for different boundary conditions including confining pressure and inlet fluid pressures. The typical mapped flow patterns are shown in Figure 8.5. For the results in Figures 8.5a & b, the inlet water pressure was maintained at a given constant value of 0.25MPa and the air pressure was increased gradually from zero to till the specimen was fully saturated with air. For the results shown in Figures 8.5c & d, the inlet air pressure ($p_a$) and water pressure ($p_w$) were made to be equal ($p_a = p_w$). A similar flow study for water-gas flow through a pipe was recorded by Golan & Stenning (1969). Although these plots do not clearly distinguish the different flow regimes, they indicate the changes from bubble flow patterns to annular or complex flow regimes.

One of the possible flow mechanisms, which can occur within a joint, is explained below. A simplified flow chart for the formation of different flow regimes is shown in Figure 8.6, and the corresponding flow regimes are also represented in Figure 8.7. When the joint is fully saturated with water, it is assumed that the joint has no air (Figure 8.7a). With the subsequent injection of the air-phase, tiny air bubbles enter the joint and stay along the joint walls (Figure 8.7b). Further increase in inlet air pressure results in the development of a string of larger bubbles along the joint walls (Figure 8.7c). The continuous flow of air begins once a film of air develops along the joint
walls (Figure 8.7d).

(a) \textit{Inlet water pressures \neq inlet air pressures} \((p_a \neq p_w = 0.25\text{MPa})\)

(a) Artificially induced rock joint

![Graph showing flow relationship in rock joint with artificially induced joint.](image)

(b) Natural rock joint

![Graph showing flow relationship in rock joint with natural joint.](image)

Figure 8.5(a-b). Possible flow relationship in rock joint when \(p_a \neq p_w = 0.25\text{MPa}\)

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(b) Inlet water pressures = inlet air pressures ($p_a = p_w$)

(c) Artificially induced rock joint

Figure 8.5 (c-d). Possible flow relationship in rock joint when $p_a = p_w$
Saturated flow (e.g. water)

Air flow (increased gradually from zero)

Unsaturated flow (water+gas)

Few air bubbles along the joint wall

A thin film of air along the joint

Annular flow

Stratified flow

Randomly displaced bubbles in water phase

Bubble flow

Large air pockets in water phase

Complex flow

Figure 8.6. Simplified flow chart for formation of flow regimes in a joint.

(a) Fully saturated water flow

$P_w =$ water pressure

$P_a =$ no air pressure

(b) Two-phase flow

$P_w >> P_a$

(c) Two-phase flow

$P_w > P_a$

(f) Single-phase flow

$P_a >> P_w$

(e) Two-phase flow

$P_a > P_w$

(d) Two-phase flow

$P_w = P_a$

Figure 8.7. Possible flow mechanisms within a joint.
For elevated capillary pressures ($p_a \gg p_w$), the initial flow pattern may be smooth-stratified or wavy stratified (Figure 8.7e), and ultimately, single-phase airflow may result once the water phase is totally replaced by air (Figure 8.7f). The latter flow patterns, which may occur in a single rock joint are shown in Figure 8.8.

(a) Possible stratified flow patterns (Both phases continuous)

(i) Air core is covered by thin film of water
(ii) Stratified wavy flow
(iii) Simplified stratified flow (assumed in authors' model)

(b) Mixed flows (One phase is continuous)

(i) Water droplets in continuous air phase
(ii) Air bubbles in continuous water phase
(iii) Air pockets in continuous water phase
(iv) Bubble flow
(v) Complex flow

Figure 8.8. Possible flow patterns in a typical rock joint.
It is well established that for single-phase flow, the flow rate is a function of confining pressure, axial stress, inlet fluid pressure, properties of fluid and surface geometry (Chapters 5 and 6). Apart from these influencing parameters, for two-phase flow of water and air, the effects of solubility of air in water and the compressibility effects of the fluids have to be considered. The following section illustrates the effects of changes in inlet fluid pressure, confining pressure, axial stress on two-phase flow rate and the comparison of single-phase flow with the two-phase flow for above mentioned conditions.

**8.4.1 Effect of inlet fluid pressure**

Flow rates through rock specimens were observed under both steady state and transient conditions. Figure 8.9 indicates the change of flow rate against time, where a steady state water flow rate has been attained after 30 minutes of applying the inlet fluid pressure. During the initial 10 minutes or so, the flow rate increased, probably due to increased fracture connectivity, facilitating the flow. The subsequent decrease in flow rate was due to the joint deformation associated with the confining pressure. Beyond 30 minutes or so, the flow rate became constant (steady state) as the joints attained their residual apertures.

The observation of steady state and transient flow in the specimens of two-phase flow were carried out in two distinct ways:

(a) Air flow through an initially water saturated specimen, and
(b) Water flow through an initially air saturated specimen.

Figure 8.9. Transient conditions of water flow for different inlet pressures at confining pressure of 0.5MPa.

In (a), for given confining pressure and axial stress, the inlet water pressure was applied to the specimen and the steady state water flow was monitored. Once the water flow became constant, the air phase was introduced, as shown in Figure 8.10a. As an example, in Figure 8.10b, the specimen was first saturated with water, at a pressure of 0.25MPa, and then the air phase was introduced into the specimen. The inlet air pressure was gradually increased up to 0.25MPa, and the corresponding flow rates of both the phases were observed. After 240mins, water flow rate decreased to a constant value of 0.04ml/min, but the airflow increased to a constant value of above 7ml/min (note that the two flow rate scales are different for air and water). Therefore, under the above conditions, steady state two-phase flow through the specimen occurred.
Steady state water flow, and Inlet water pressure held constant at 0.25MPa.

Air pressure held constant at 0.25MPa

Inlet air pressure increment

Air phase is introduced

Cumulative time, min

Figure 8.10a. Applied water and air pressure to the specimen.

Inlet water pressue = 0.25MPa
Inlet air pressure = 0.25MPa

Two-phase steady state condition

Water flow, ml/min

Air flow, ml/min

Cumulative time, min

Figure 8.10b. Two-phase flow for through initially water saturated specimen at a confining pressure of 2MPa
In the case of water flow through an initially saturated air specimen, the inlet air pressure was first applied to the specimen until the steady state airflow was observed. At this stage, the water phase was introduced to the specimen, however, no flow of water could be observed for a long time. Only after a period of 675 minutes, the water flow began to increase, while the airflow decreased. Comparison with Figure 8.10b indicates that the response time for airflow through the specimen is much shorter because the specimen was initially saturated with water. The long time delay shown in Figure 8.11 suggests that if the specimen is initially saturated with a low viscosity fluid such as air, the introduction of a higher viscosity fluid will not be able to penetrate the specimen for a long period of time.

Figure 8.11. Two-phase flow for a specimen, initially saturated with air.
The comparison of the measured laboratory data with theoretical data is described in the following sections. The theoretical predictions for steady state two-phase laminar flow through fractured rock specimen were carried out based on the mathematical model described in Chapter 7. The material properties of the rock and fluids are presented in Appendix B. The heights of the air phase (Equation 7.9 in Chapter 7) and water phase (Equation 7.7 in Chapter 7) strongly depend on the normal deformation of the joint. For different normal stresses, the predicted normal deformation (Equation 7.30c) and the measured normal deformation are shown in Figure 8.12. The Curve A shows the normal deformation based on Equation 7.30c, where the joint normal stiffness was calculated from uniaxial test data, in which the axial load was applied normal to the horizontal fracture surface, and the deformations of the mechanical aperture were measured for various axial loads. In contrast, for the Curve B also based on Equation 7.30c, the joint normal stiffness was determined from triaxial test data in which, the circumferential confining pressure (i.e. normal stress to the joint) and axial load were applied to the specimen and the deformation of the fracture for various confining pressure was measured by clip gauges (mentioned in Section 4.3.2). The data indicate a significant reduction in normal deformation in Curve B in comparison with Curve A. This is because, in the triaxial test, where there is confining pressure, the normal deformation is expected to be less. Also, from the measured fluid flow data, the hydraulic apertures of the joint could be back calculated for a given confining stress, using the cubic relation (Equation 7.33). Each data point plotted (Curve C) represents the average of at least 3 tests. The sudden drop in normal deformation at 1.5MPa is probably due to some experimental error. In general, the Curve C based on cubic relation is in agreement with the triaxial data, while the normal deformation represented by Curve A is over-
estimated. Therefore, in this study, the behaviour indicated by Curve B has been incorporated in the verification of the mathematical model.

Irrespective of single-phase or multiphase flow, increasing the inlet fluid pressure of one phase will result in an increased flow rate for the same phase, while decreasing the flow rate of the other. If the inlet fluid pressures of both the phases are increased simultaneously, (either $p_a = p_w$ and $p_a \neq p_w$) then the relative fluid flow rate of each phase is governed by the properties of the individual phases. For given boundary conditions and fluid pressures, once the two-phase mixture was collected from the specimen and the individual phases separated, the respective flow rates were plotted against the inlet fluid pressures. For $p_a = p_w$, Figure 8.13a-c illustrates the flow behaviour at three different confining pressures. At relatively low inlet fluid pressures, the flow rate varies linearly with the fluid pressure.

![Figure 8.12. Measured and predicted normal deformation of jointed rock specimen for various normal stress levels.](image)

Curves A, B: based on mechanical deformation
Curve C: based on hydraulic tests
Inlet water pressure = inlet air pressure \( (p_a = p_w) \)

(a) At 0.55 MPa confining pressure

(b) At 1.0MPa confining pressure.

Figure 8.13. Two-phase flow rates at different confining pressure and at \( p_a = p_w \).
(c) At 2.0MPa confining pressure.

The predictions obtained from the proposed two-phase model (Equations 7.34 & 7.35 in Chapter 7) are also compared with the experimental data in Figure 8.13. The dotted lines represent the predicted flow from the used model. While the arrangement is acceptable, the experimental results show some deviation from the theoretical values, which is attributed to some air and water still being trapped within the pores of the specimen. At elevated inlet fluid pressure, the flow rates become less linear, because, Darcy’s law \( q = av \) is not valid for elevated fluid pressures. Such non-linear flow can be simulated, using the following expression:

\[
q = av + bv^2 
\]

where, \( a \) and \( b \) are constant, \( v \) is velocity vector.
The non-linearity may be probably due to the formation of non-parallel laminar or turbulent flow within the joint.

(b) Inlet water pressure ≠ inlet air pressure \((p_a \neq p_w)\)

For a given confining pressure and axial stress and increasing inlet air pressures (when \(p_a \neq p_w\), while \(p_w\) held constant), the flow rate of the water phase decreases, while the air flow rate increases. According to Figure 8.14, further increase in inlet air pressure results in single-phase flow (i.e. water flow rate approaches zero).

(a) At 0.55MPa confining pressure.

![Figure 8.14. Two-phase flow rates at different confining pressures and at \(p_a \neq p_w\).](image)
(b) At 1.0MPa confining pressure

![Graph showing two-phase flow rates at 1.0MPa confining pressure.]

(c) At 2.0MPa confining pressure

![Graph showing two-phase flow rates at 2.0MPa confining pressure.]

Figure 8.14. Two-phase flow rates at different confining pressures and at \( p_a \neq p_w \) (Contd.).
8.4.2  Effect of confining pressure and axial stress

As discussed in Chapter 5, for fully saturated water flow through rock joints, various studies have shown that the flow rates decrease with the increase in confining pressure ($\sigma_3$) due to the closure of apertures. Two-phase flow is also affected by the confining pressure in a way similar to single-phase flow. For given inlet water and air pressures, the effect of confining pressure on the permeability of two-phase flow is illustrated in Figure 8.15, for $p_a = p_w$. The dotted lines represent the predicted flow from the proposed model (Equations 7.34 & 7.35). The measured flow rate is slightly smaller than the calculated values at low confining pressures which is probably due to some fluid being trapped in pores or along the joint walls as a thin layer of film. At low confining pressures, the air occupies the main volume of the joint, hence, water is expected to flow as an unstable film along the joint walls, thereby contributing to a significantly reduced water flow rate. As expected, fluid flow decreases with the increase in confining pressure, however, beyond a confining pressure of 6MPa, the rate of change of permeability becomes marginal (Figure 8.15). This is an indicative of joint apertures attaining their residual values.

Figure 8.16 shows that for constant inlet water pressures ($p_w =125$, 200 and 250kPa), the two-phase flow rate decreases with the increasing confining pressure (i.e., shift of curves to the right). For a constant confining pressure, when the inlet air pressure ($p_a$) is increased, the airflow is expected to increase with an associated decrease in the water flow. Figure 8.16 illustrates that at increasing confining pressures, even with the increase in inlet air pressure, the airflow decreases. This verifies the dominant role of confining pressures.
Figure 8.15. Effect of confining pressure on two-phase fracture permeability when $p_a = p_w$. 

(a) Specimen 1 – multiple fractures

(b) Specimen 2 – single fracture
Equivalent phase heights of water $h_w(t)$ (Equation 7.7) and air $h_a(t)$ (Equation 7.9), joint deformation due to normal stress, $\delta_n$ and the equivalent height components of air, $\zeta_{ad}$ and $\zeta_{ac}$ (based on solubility and compressibility, respectively) are plotted in Figure 8.17. The total deformation of the joint wall is composed of these equivalent phase heights as represented earlier by Equations 7.6 (Chapter 7). Almost 95% of the magnitude of $h_a(t)$ and $h_w(t)$ is due to the normal joint deformation ($\delta_n$), the rest being the combined effect of $\zeta_{ac}$ and $\zeta_{ad}$. As expected, the contribution of the air compressibility term, $\zeta_{ac}$ is more significant than the solubility component $\zeta_{ad}$. While $\zeta_{ac}$ amounts to about 4-5% of the value of $\delta_n$, the magnitude of $\zeta_{ad}$ is not more than 0.01% of $\delta_n$. At large confining pressures (> 5MPa), further decrease in $\delta_n$ may be
marginal, once the joint aperture reaches its residual values. Beyond this stage, the role $\zeta_{ac}$ and $\zeta_{ad}$ will become increasingly pronounced.

Figure 8.17. Equivalent phase heights of water and air in two-phase flow in a jointed granite specimen.

Apart from the confining pressure, inlet fluid pressures and the degree of saturation, the deviator stress ($\sigma_1 - \sigma_3$) can also influence the permeability, strength and deformation properties of the rock specimens. Continued increase in strain results in an increased flow through the specimen, either because of the formation of new fractures or dilation of existing fractures or both. Figure 8.18 illustrates the variation of flow rate with the deviator stress at a constant cell pressure of 1MPa. The initial decrease in air and water flow is associated with the closure of joint apertures upon initial loading. However, with increased deviator stress, the air and water flow rates start to increase, probably due to the dilation of some existing fractures and the formation of new cracks. In Figure 8.18, a sudden drop in water flow occurs at a deviator stress of 50MPa, which is
accompanied by an increase in airflow, as expected. This is because at these stress levels, airflow dominates when the dilation of joint takes place.

Figure 8.18. Relationship of two-phase flow and deformation of the specimen for different axial stress.

8.5 DETERMINATION OF FLOW TYPE WITHIN A ROCK JOINT

As illustrated in Section 8.4.1, further increase in the inlet fluid pressure does not increase the fluid flow rate linearly, and a non-linear flow relationship against inlet fluid pressure develops. The aim of this section is to investigate whether the flow rate through real rock fractures under different boundary conditions assumes laminar or
turbulent flow patterns. This is studied using the Reynolds number. The expression for estimating the Reynolds number in flow through pipes is given by:

\[ R_e = \frac{v d}{v} \]  \hspace{1cm} (8.3a)

where, \( v \) = average velocity, 
\( d \) = diameter of the pipe, and 
\( v \) = kinematic viscosity of fluid. At temperature, 20°C, the kinematic viscosity of water and air are approximately 1.12x10^{-6} and 1.56x10^{-5} m²/sec, respectively.

If the hydraulic aperture is 'e', then the 'd' is replaced by 2e. For a single joint with a width 'w', and flow rate of 'q' through the joint, the Reynolds number for the joint is expressed in terms of the flow rate, width of the joint and kinematic viscosity as:

\[ R_e = \frac{2q}{vw} \]  \hspace{1cm} (8.3b)

(a) For single-phase flow

For a given confining pressure and axial stress, the calculated Reynolds numbers for single-phase flow using Equation 8.3b for typical specimens are indicated in Figure 8.19. As expected in fluid flow through pipes, the increase in fluid pressure results in an increase in the Reynolds number. For single-phase water flow, the calculated Reynolds numbers are below 100, whereas, for the air phase, Reynolds numbers are as high as 500 because of the low viscosity of air (Figure 8.19b). The Reynolds number depends mainly on the joint surface roughness, the fluid properties and the inlet fluid pressure. The magnitude of critical Reynolds number representing
the flow regimes between parallel walls is 1000 (Street et al., 1996). This indicates that the single-phase flow derived in this study can be considered as laminar.

(a) Single-phase water flow

(b) Single-phase air flow

Figure 8.19. Average Reynolds number of single-phase flow within the joint.
For two-phase flow \((p_a = p_w)\)

The Reynolds numbers for two-phase flow were also estimated using Equation 8.3b, in which \(q\) is the individual flow rate of each phase. The estimated Reynolds numbers for the two-phase flow are shown in Figure 8.20 for different confining pressures. In Figures 8.20a-b the applied fluid pressures of each phase is approximately the same. As in single-phase flow, Reynolds numbers increase with the increase in inlet fluid pressure. However, Reynolds number for air in two-phase flow is lower than the Reynolds number for the water phase. This is because, in two-phase flow, the difference between the flow rates of water and air is not as high as in single-phase flow, although the difference in the viscosities is considerable.

(a) At confining pressure 0.5MPa

![Figure 8.20a. Average Reynolds number of two-phase flow for \(p_a = p_w\)](image)
(b) At confining pressure 1.0MPa

Figure 8.20b. Average Reynolds number of two-phase flow for $p_a = p_w$.

Figure 8.21 shows the corresponding Reynolds numbers for different inlet air pressures when the inlet water pressure was held constant and not equal to the inlet air pressure ($p_a \neq p_w$). Reynolds numbers corresponding to the air-phase increase with the increasing inlet air pressure, whereas the corresponding Reynolds numbers based on water flow decrease.

At low confining pressures (i.e. when the joint aperture is relatively larger), the difference between air and water phase Reynolds numbers is much higher. For very small joint apertures (i.e. at larger confining pressures), the Reynolds numbers are small.
and they seem to be independent of the elevated confining pressures (Figure 8.22).

![Figure 8.21. Average Reynolds number for two-phase when $p_a \neq p_a$.](image1)

![Figure 8.22. Effect of confining pressure on Reynolds number of two-phase flow.](image2)

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It is evident from the above data that the likelihood of the development of turbulent flow within a rock joint is remote. As all the estimated Reynolds numbers in this study are well below 1000, it can be concluded that the flow within joints remains as laminar flow, even for the rough joints considered here.

8.6 FRACTURE PERMEABILITY OF TWO-PHASE FLOW

In the presence of more than one fluid, one has to consider the relative permeabilities of the two phases apart from the fracture permeabilities. The study in this section is limited to the discussion of absolute permeability of fractured granite rocks for different boundary conditions including inlet water and air pressures.

For laminar flow conditions \((Re<1000)\), the fracture permeability is expressed by:

\[
k = \frac{e^2}{12}
\]

(8.4)

where, \(e\) = hydraulic aperture or phase height, and \(k\) = fracture permeability.

From above expression, it is clear that the fracture permeability is independent of fluid properties and depends only on the aperture. When a fractured specimen is subjected to various loading conditions, it is relatively easy to estimate the hydraulic aperture from the triaxial test data, using the following expression, assuming that the joint walls are smooth and parallel. In Eqn. 8.5, the effect of gravity on fluid flow is not included because it is negligible over a small length of the specimen when compared to the inlet fluid pressure.
\[ e = \left( \frac{12 \mu q}{d (dp/dx)} \right)^{1/3} \]  

(8.5)

where, \( q \) = total flow rate,

\( \mu \) = dynamic viscosity of fluid,

\( d \) = width of fracture, and

\( dp \) = pressure difference along the length of \( dx \)

If there are multiple fractures in the tested specimen, the estimated 'e' using Equation 8.5 represents the net or equivalent hydraulic aperture. In the following section, the fracture permeability based on Equation 8.4 for two-phase steady state flow is examined using the TPHPTA. The individual flow rates of water and air mixture were measured separately, in order to incorporate values for \( q \) in Equation 8.5. Figure 8.23 shows the permeability of each phase against inlet fluid pressures based on individual flow rates. For a given confining pressure and axial stress, the inlet fluid pressures could be applied in two different ways:

(a) Inlet air pressure= inlet water pressure (\( p_a = p_w \)),

(b) Water pressure held constant; inlet air pressure is increased (\( p_a \neq p_w \))

(a) \textit{Two-phase fracture permeability when } \( p_a = p_w \)

When the inlet fluid pressure was increased (e.g. \( p_a = p_w = 0.2, 0.3 \text{MPa etc} \)), the permeability of the water phase decreased with the increase in inlet fluid pressures, whereas, a slight increase in permeability of the air phase was observed (Figure 8.23). In contrast, flow rate of both phases increased with the increase in fluid pressure as shown earlier in Figure 8.12. This is because, the flow rate is usually linearly proportional to the inlet fluid pressure (in the laminar region), however, the fracture
permeability is estimated using \( k = e^2/12 \), in which the hydraulic aperture \( 'e' \) is based on Equation 8.5, which does not linearly vary as the flow rate. It is the term \( e^3 \) that is linearly proportional to the flow rate.

(a) At 0.5MPa confining pressure

![Graph showing two-phase fracture permeabilities for different inlet fluid pressures at 0.5MPa confining pressure.]

(b) At 2MPa confining pressure

![Graph showing two-phase fracture permeabilities for different inlet fluid pressures at 2MPa confining pressure.]

Figure 8.23. Two-phase fracture permeabilities for different inlet fluid pressures, when \( p_a = p_w \).
(b) Two-phase fracture permeability when $p_a \neq p_w$

The effect of inlet fluid pressure on permeability is also analysed when $p_a \neq p_w$.

Figure 8.24 shows the permeabilities in two-phase flow of each phase at different confining pressures when the water pressure is held constant while increasing the inlet air pressure. As an example, in Figure 8.24a, the applied water pressure ($p_w$) of 0.1 MPa, held constant during the test. Once the air phase is introduced to the specimen, the permeability of water phase decreases, and after a certain magnitude of the inlet air pressure, the air permeability begins to increase. If the magnitude of the inlet air pressure ($p_a$) is sufficient to replace all the water in the joint, then the water permeability tends to become zero. The permeability of both phases become equal when the air pressure is approximately 1.1 times the inlet water pressure. In other words, the equivalent water phase height tends to become equal to the air phase height. When $p_a/p_w$ becomes 1.1, the permeability of both phases is approximately 33% of the single-phase water permeability (Figure 8.24a). Table 8.2 shows the magnitude of single-phase permeability for given confining pressures and inlet fluid pressures. The values of $p_a/p_w$ are also given, when the permeability of both phases become equal (cross-over point). The ratio of $p_a/p_w$ varies from 0.8 to 1.2 for the tested specimens, at constant inlet water pressure. When $p_a/p_w$ ranges from 0.8 to 1.2, the 'cross-over' two-phase permeability varies 20-70% from the single-phase water permeability.

The inlet fluid pressure ratio at which the permeability of both phases become equal depends on the joint aperture and its surface profile. It can be concluded that when $p_a/p_w$ exceeds this limit value, air phase dominates the flow, hence, the effect of water permeability can be neglected.
(a) At 0.5MPa confining pressure

Figure 8.24. Effect on varying inlet air pressure on fracture permeability of two-phase flow through initially water saturated specimen.

(b) At 2.0MPa confining pressure
Table 8.2. Two-phase fracture permeability for different confining and inlet fluid pressures.

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Confining pressure, MPa</th>
<th>Inlet water pressure, MPa</th>
<th>Single-phase water permeability x 10^{-11}, m^2</th>
<th>Permeability of both phase become equal when p_a/p_w</th>
<th>Two-phase Permeability x 10^{-11}, m^2</th>
<th>% reduction of permeability from single phase flow</th>
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</tr>
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<td>42</td>
<td></td>
</tr>
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<td>0.38</td>
<td>1.0</td>
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<td>52</td>
<td></td>
</tr>
</tbody>
</table>
8.7 Validity of Darcy's law for two-phase flow

In the simplified form of Darcy's law, the hydraulic gradient is assumed to be linear along the fluid path. This assumption is no longer valid when the fluid flow is non-linear. Therefore, in general, the flow can be expressed in terms of hydraulic gradient as follows:

\[ q = AKi \quad \text{for laminar flow,} \quad (8.6a) \]
\[ q = AKi^\beta \quad \text{for turbulent flow or non-linear laminar flow,} \quad (8.6b) \]

where, \( q \) = flow rate, \( K \) = conductivity, \( i \) = hydraulic gradient, \( A \) = cross section area normal to the flow, \( \beta \) = constant, which varies between 0 and 1. For laminar flow, \( \beta \) becomes 1.

For negligible elevation head and the velocity head, the hydraulic gradient in Equation 8.6 is replaced by the pressure gradient \( (dp/dx) \) as given below:

\[ q = AK \frac{dp}{dx} \quad (8.7) \]

where, \( dp \) = pressure difference along the length of the fluid flow path, \( dx \).

From the laboratory test results obtained assuming fully saturated flow through natural rock joints, it can be seen that the Darcy's law (i.e. linear relationship between the flow rate and the pressure gradient) is valid at relatively low confining pressures and at low hydraulic gradients (Figure 8.25). However, at high confining stress, linearity is not followed which can be attributed to the formation of irregular rough joint walls and the
changing contact area between joint walls. In the following section, the validity of Darcy’s law for two-phase flow through natural rock specimens is examined.

![Graph showing applicability of Darcy's law for single phase flow in natural rock joints.](image)

**Figure 8.25.** Applicability of Darcy's law for single phase flow in natural rock joints.

### 8.7.1 Comparison of single-phase flow rates with two-phase flows

Figure 8.26 shows the experimental single-phase and two-phase flow rates against inlet fluid pressures. During the two-phase flow, the applied inlet air and water pressures are equal i.e. $p_a = p_w$. The Curves A & B (Figure 8.26a) represent single-phase flow rates, which vary approximately linearly with the inlet fluid pressure. The individual two-phase water flow (Curve C) and air flow (Curve D) rates also exhibit a linear relationship between flow rates and fluid pressures. A significant reduction of two-phase flow rate (compared to single-phase) is experienced due to the influence of one phase on the other.
(a) At 0.5MPa confining pressure

![Graph showing flow rates](image)

Figure 8.26. Comparison of two-phase with single-phase flow rates for varied inlet fluid pressures.

(a) At 1.0MPa confining pressure

![Graph showing flow rates](image)
For example, at 0.2MPa inlet fluid pressure, the individual components of water and airflow rates of two-phase flow have respectively decreased by 50 % and 95 % from the single-phase flow rates (Figure 8.26a). The variation of the observed air and water flows with inlet pressure (two-phase condition) can still be approximated to a linear relationship as in the case of single-phase flow. These findings confirm that two-phase flows also follow Darcy’s law for the treated range of confining and inlet air and water pressures.

Darcy’s law can be extended to model unsaturated flow through jointed rocks by introducing the factor, ‘relative permeability’ ($K_r$) in Equation 8.8 as follows:

$$q = AK_r K \frac{dp}{dx}$$  \hspace{1cm} (8.8)

where, $K_i$ = relative permeability, the subscript ‘i’ represents the phase and it takes $a$ and $w$ for air and water, respectively. $K_r$ depends on the properties of permeating fluids such as solubility and compressibility at different stress levels, and the driving (inlet) pressures and temperatures of each phase.

8.8 Relative permeability of two-phase flow

The relative permeability is an important concept in multiphase flow analysis, because it provides a more realistic coefficient representing the relative dominance of the individual fluid phases. Factors such as, properties of fluids, fluid pressures and joint apertures govern the magnitude of the relative permeability, which ranges between 0 and 1. In order to examine the behaviour of unsaturated flow through a rock mass, one may need to incorporate the relative permeability factor in the general flow equations.
For negligible elevation and velocity head, when two-phase flow occurs through a joint with an aperture ‘e’, the relative permeability is given by the following expression:

For the water phase,

\[ K_{rw} = \frac{q_w \mu_w}{A \ k_{rw} (dp/ \ dx)_w} \] (8.9a)

Similarly, for the air-phase:

\[ K_{ra} = \frac{q_a \mu_a}{A \ k_{ra} (dp/ \ dx)_a} \] (8.9b)

where, \( k_s \) = single-phase permeability = \( e^2/12 \).

\( K_{ra} \) and \( K_{rw} \) = relative permeability of air and water, respectively. The subscripts \( a \) and \( w \) represent air and water, respectively.

When the relative permeability with respect to one phase becomes unity, the relative permeability of the other phase becomes zero. For instance, if the medium is considered to be fully saturated with air, then the ‘relative permeability of air’ is unity, and the ‘relative permeability of water’ is zero. At intermediate values of saturation, the relative permeability of each fluid phase will be reduced from the corresponding saturated value, due to the reduction in cross section area of the fluid phase (Figure 8.27). At a specific degree of saturation, the relative permeability of both phases become equal. Theoretically, the relative permeability of each phase should add up to unity (\( K_{ra} + K_{rw} = 1 \)), but the validity of this result was not confirmed experimentally (Fourar et al., 1993; Pyrak-Nolte et al. 1992; Pruess & Tsang, 1990).
Figure 8.27. The theoretical relationship between relative permeability and the degree of saturation.

(a) Two-phase flow when $p_a = p_w$

When the inlet air pressure and water pressure are held approximately equal, the relative permeability coefficients for water and air (Figure 8.28) are calculated using Equations 8.9a & b. When the relative permeability of one phase increases, the relative permeability of the other phase should decrease. For example, at 0.4MPa inlet fluid pressure (i.e. $p_a = p_w = 0.4$MPa), the relative permeability of air decreases while increasing the relative permeability of the water phase. This is probably due to fact that the water phase dominates when the inlet fluid pressure is below a certain value. A continuous increase in relative permeability of air phase is experienced once the inlet fluid pressure exceeds 0.1MPa (Figure 8.28a) and 0.5MPa (Figure 8.28b), respectively.
(a) At 0.5MPa confining pressure

(b) At 1.0MPa confining pressure

Figure 8.28. Relative permeability of two-phase flow when inlet air pressure equals water pressure.
(b) **Two-phase flow when \( p_a \neq p_w \)**

The change in relative permeability of two-phase flow at different inlet fluid pressure ratios is shown in Figures 8.29 and 8.30. For an initially water saturated specimen, the inlet fluid pressure ratio is defined as the inlet air pressure divided by the inlet water pressure (i.e. \( p_a/p_w \)). However, for the initially air saturated specimen, the inlet fluid pressure ratio is defined as inlet water pressure divided by the inlet air pressure (i.e. \( p_w/p_a \)). When the ratio of \( p_a/p_w \) increases, the relative permeability of air increases while the relative permeability of water decreases (Figure 8.29). The opposite trend occurs when the \( p_w/p_a \) ratio is increased (Figure 8.28). When the relative permeability of one phase approaches 1, then the joint becomes fully saturated with that phase. For a given confining pressure and axial stress, the relative permeability of both phases tends to become equal when the \( p_a/p_w \) ratio is between 0.8 and 1.2. Table 8.3 shows the variation of the relative permeability with the inlet fluid pressure ratios for different fluid pressures and confining pressures.

It is important to note that the range of confining pressures applied in this study is moderate (1-8 MPa), hence, the findings of this study may not be extrapolated to predict the two-phase flow behaviour at much greater confining pressures.
(a) At 0.5MPa confining pressure

(b) At 2.0MPa confining pressure

Figure 8.29. Relative permeability for water saturated specimen.
(a) At 0.35MPa confining pressure

(b) At 1.0MPa confining pressure

Figure 8.30. Relative permeability for air saturated specimen.
Table 8.3. Relative permeability factors at different inlet fluid pressure ratios.

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Confining pressure, MPa</th>
<th>Inlet fluid pressure, MPa</th>
<th>$K_{ra} = K_{rw}$</th>
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<td>(Water saturated)</td>
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<td>0.26</td>
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<td>0.32</td>
</tr>
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<td></td>
<td>2.0</td>
<td>0.26</td>
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<td></td>
<td>1.0</td>
<td>0.2</td>
<td>0.21</td>
</tr>
<tr>
<td></td>
<td>0.5</td>
<td>0.1</td>
<td>0.12</td>
</tr>
<tr>
<td>Specimen 2</td>
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</tr>
<tr>
<td>(Artificially</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>fractured)</td>
<td>1.0</td>
<td>0.3</td>
<td>0.3</td>
</tr>
<tr>
<td></td>
<td>1.0</td>
<td>0.16</td>
<td>0.13</td>
</tr>
<tr>
<td></td>
<td>0.5</td>
<td>0.19</td>
<td>0.16</td>
</tr>
<tr>
<td></td>
<td>0.5</td>
<td>0.1</td>
<td>0.12</td>
</tr>
</tbody>
</table>

8.9 SUMMARY

Two-phase (water and air) flow through fractured rock specimens were carried out using the newly designed TPHPTA for different boundary conditions. Tests were carried out for both naturally fractured specimens with single and multiple fractures, and for specimens with artificial fractures induced using the Brazilian test. The laboratory investigation program included investigation of (a) effect of inlet fluid pressure, confining pressure and axial stresses on two-phase flow rates, (b) possible flow pattern and flow type, (c) validity of Darcy’s law for two-phase flow and (d) validation of the mathematical model described in Chapter 7. The important aspects related to the two-phase flow behaviour in fractured rock are discussed and summarized below.

- Two-phase flow pattern in a single rock joint can be in the form of either bubble flow, annular flow or complex flow. At low inlet fluid pressures,
the flow within rock joints is best described by bubble flow. Further increase in inlet fluid pressure may result in annular flow. At very high inlet fluid pressures and confining pressures, two-phase flow in highly irregular joints should be modeled using complex flow patterns. This area of research was beyond the scope of this study.

• Steady state flow of single-phase flow is generated in a shorter period, whereas, a longer period is required for the occurrence of steady state two-phase flow. Steady state water flow through naturally fractured specimen was attained after about 40 minutes at 0.5MPa confining pressure and 0.1MPa inlet water pressure. In contrast, for two-phase flow at 2MPa cell pressure and 0.25MPa inlet pressure, more than 240 minutes were taken to reach steady state flow.

• At relatively small inlet fluid pressure and confining pressure, both experimental and predicted results show that two-phase flow rates vary linearly with inlet fluid pressures when inlet pressures of both phases are approximately equal. A linear relationship between flow rate and fluid pressures does not exist at elevated fluid pressures. Increases in inlet pressure of one phase results in the increase of flow rate of the same phase, while a significant decrease in flow rate of the other phase is observed simultaneously. Flow rate of the air phase is always higher than that of the water phase.

• Reynolds numbers measured in this study verified that the assumption of laminar flow was appropriate for both single and two-phase flows observed under the applied boundary conditions.
• Increases in confining pressure results in a decrease of joint aperture. At elevated confining pressures, the change in the flow rate becomes marginal once the joints have attained their residual apertures. It is the normal joint deformation that contributes most to the change in phase heights of air and water layers, while the effects of air solubility and compressibility components are relatively small. Nevertheless, at significantly elevated confining pressures, where the joint apertures have reached their residual values, the effects of compressibility and solubility of air in water become increasingly more pronounced.

• In this study, the fracture permeability of both phases becomes equal when the ratio of $p_a/p_w$ is between 0.8 to 1.2. For water saturated specimens, air permeability plays the most dominant role, at high $p_a/p_w$ values.

• In both single and two-phase flow conditions, the air flow rate is always higher than the water flow rate.

• Darcy's law has been modified to represent two-phase flow, based on the relative permeability concept. The summation of the relative permeability of all phases should be unity, theoretically. The relative permeability of the air phase increases almost exponentially with the increase in $p_a/p_w$ ratio, while decreasing the relative permeability of the water phase, at the same time. The opposite trend occurs when the $p_w/p_a$ ratio is increased.

• The experimental data agree well with the mathematical model (Chapter 7) for the boundary conditions and test conditions appropriate to this
study. The proposed model can be used to predict two-phase flow in a natural rock joint for a given insitu stress and two-phase flow filed.
CHAPTER 9

FLOW THROUGH AN INTERCONNECTED FRACTURE NETWORK

NUMERICAL MODELLING

9.1 INTRODUCTION

This chapter discusses flow through an interconnected fracture network using numerical modeling techniques. The main objective of the numerical analysis is to demonstrate the use of saturated flow concepts in the prediction of groundwater ingress to underground cavities. The numerical analysis described in this chapter is for hypothetical underground situations, i.e. worked examples with well defined joint patterns. The effects of boundary block dimensions, insitu stress ratios, varied fluid boundary conditions and orientation of joint sets on water ingress towards an underground cavity are investigated.

9.1.1 Analytical approaches

The first part of this chapter describes an analytical approach for water flow through a simple fracture network, followed by different numerical techniques, which can be used for flow analysis.

Analytical methods may be used to estimate fluid flow quantities in a given rock mass, provided the rock mass contains a simple fracture network formed with a small number of joints. Typically, there are two approaches based on the analytical techniques:
(a) Flow estimation, for a given hydraulic conductivity relationship,
(b) Estimate the hydraulic head at each intersection point of the given
fracture network and then quantify the flow

In most analytical approaches employed by various researchers, it is assumed that the
joints are of infinite length in a given volume, and that they are orientated in perfectly
parallel planes (Sharp, 1970; Maini, 1971). Let us consider the following simple
fracture network intersecting a circular tunnel periphery (Figure 9.1). It is assumed that
the two joint sets completely extend over the given area in order and the simplified
‘pipe network’ theory is applicable. The network has basically three types of nodal
points depending on the interconnectivity and the boundary conditions, as listed below:

(a) Internal nodal points in which two joint sets are intersected,
(b) Tunnel boundary nodal points, intersected by the joints at the tunnel
periphery, and
(c) Nodal points on the top of the bedrock, where the hydraulic pressures
are known.

![Figure 9.1. A circular tunnel in a rock mass with two sets of regular joints.](image-url)
Assuming that the rock material is impermeable, the fluid remains incompressible and continuous along the joint, the net inflow to an internal node which equals the net outflow may be, as written below:

At node $i$, for laminar steady state flow:

$$ q_{i-1,i} + q_{i+2,i} = q_{i+1,i} + q_{i+3,i} \quad (9.1) $$

where, $q_{i-1,i}$ = flow rate from node $(i-1)$ to node $i$

Assuming parallel plate flow theory, the flow rate can be expressed in terms of the hydraulic gradient and the joint aperture to yield:

$$ q_{i-1,i} = \frac{e^3_{i-1,i}}{12\mu} \left[ \frac{h_{i-1} - h_i}{l_{i-1,i}} \right] \quad (9.2) $$

where, $e_{i-1,i}$ = joint aperture of the joint length, $l_{i-1,i}$,

$$ h_{i-1} - h_i = \text{hydraulic head difference along the length of } l_{i-1,i} $$

$\mu$ = dynamic viscosity of the fluid.

Similarly, the flow rates $q_{i+1,i}, q_{i+2,i},$ and $q_{i+3,i}$ can be expressed in terms of the hydraulic head and the aperture of each segment. Using Equations 9.1 and 9.2, the following equation is derived for the flow rate at node $i$.

$$ \frac{e^3_{i-1,i}}{12\mu} \left[ \frac{h_{i-1} - h_i}{l_{i-1,i}} \right] + \frac{e^3_{i+2,i}}{12\mu} \left[ \frac{h_{i+2} - h_i}{l_{i+2,i}} \right] = \frac{e^3_{i+1,i}}{12\mu} \left[ \frac{h_i - h_{i+1}}{l_{i+1,i}} \right] + \frac{e^3_{i+3,i}}{12\mu} \left[ \frac{h_i - h_{i+3}}{l_{i+3,i}} \right] \quad (9.3) $$

Taking $c_{i-1,i} = \frac{e^3_{i-1,i}}{12\mu} \left[ \frac{1}{l_{i-1,i}} \right]$, Equation (9.3) may be rewritten as follows:
\[ c_{i-1,j} (h_{i-1} - h_j) + c_{i+1,j} (h_{i+1} - h_j) = c_{i+1,j} (h_j - h_{i+1}) + c_{i+3,j} (h_j - h_{i+3}) \]  

(9.4)

Making \( c_{i,j} = -\left( c_{i-1,j} + c_{i+1,j} + c_{i+3,j} \right) \), Equation 9.4 may be rearranged in a simplified form:

\[ c_{i-1,j} h_{i-1} + c_{i,j} h_j + c_{i+1,j} h_{i+1} + c_{i+2,j} h_{i+2} + c_{i+3,j} h_{i+3} = 0 \]  

(9.5)

At a given node \( i \), therefore, the hydraulic head can be written as:

\[ h_i = -\left( \frac{c_{i+1,j} h_{i+1} + c_{i+2,j} h_{i+2} + c_{i+3,j} h_{i+3} + c_{i-1,j} h_{i-1}}{c_{i,j}} \right) \]  

(9.6)

If there are \( n \) numbers of nodes in the model, it is feasible to arrange Equation 9.6 in matrix form, hence, the solution for hydraulic head at each node is obtained by the following matrix iteration.

\[
\begin{bmatrix}
  c_{2,1} & c_{2,2} & c_{2,3} & c_{2,4} & c_{2,5} & 0 & 0 & 0 & 0 & 0 \\
  0 & c_{3,2} & c_{3,3} & c_{3,4} & c_{3,5} & c_{3,6} & 0 & 0 & 0 & 0 \\
  0 & 0 & c_{4,3} & c_{4,4} & c_{4,5} & c_{4,6} & c_{4,7} & 0 & 0 & 0 \\
  & & & & & & & & & \\
  & & & & & & & & & \\
  & & & & & & & & & \\
  & & & & & & & & & \\
  0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
  & & & & & & & & & \\
  & & & & & & & & & \\
  & & & & & & & & & \\
  0 & 0 & 0 & 0 & 0 & 0 & 0 & c_{n,n} & 0 & 0 \\
  0 & 0 & 0 & 0 & 0 & 0 & c_{n+1,n} & c_{n,n+1} & c_{n,n+2} & c_{n,n+3} \\
  0 & 0 & 0 & 0 & 0 & 0 & 0 & c_{n+2,n} & c_{n+1,n+1} & c_{n+1,n+2} & c_{n+1,n+3} \\
  & & & & & & & & & & ...
\end{bmatrix}
\begin{bmatrix}
  h_1 \\
  h_2 \\
  h_3 \\
  h_4 \\
  h_5 \\
  h_6 \\
  & & & & & & & & & & \\
  h_n
\end{bmatrix}
= 0
\]  

(9.7)
The usage of numerical modeling in soil and rock engineering has been expanded in recent years, in order to handle complex problems efficiently. There have been numerous computer programs (codes) developed for research work, as well as for practicing engineers. On the basis of the type of flow problem dealt with, these computer codes can be classified into three categories:

(a) Domain method,

(b) Boundary formulations, and

(c) Lattice structure method.

The above three categories were in detail discussed in Chapter 3. In this section, the application of the discrete element method is discussed. The discrete element method (DEM) is best suited for discontinuous media such as fractured rock mass and contrasts with continuum techniques such as FEM and FDM. In DEM, there are two main advantages over the continuum approaches, as described below:

(a) Large deformations due to joint slip and block rotations are allowed,

(b) Both material and discontinuity (i.e. joints) properties are used to simulate the actual rock mass.

The shapes of the rock blocks depend on the orientation of joints, discontinuity length and their spacing. The distinct element method was initially developed for mechanical analysis of solid blocks by Cundall (1971) and then was further extended by co-workers (Cundall and Strack, 1979). The commercially available Universal Distinct Element Code (UDEC) is a two-dimensional program based on the distinct element approach, in which the rock blocks are assumed as deformable or rigid (ITASCA, 1996).
A hybrid distinct-element-boundary method (DEM-BEM) was used by Lorig et al. (1986) to study the stresses and displacements in highly jointed rock mass surrounding underground cavity. The distinct element method was applied to model the jointed rock close to the cavity, while far field rock was modeled using the boundary element method. One advantage of this coupled technique is that the equilibrium conditions at the interface between the two domains are obtained explicitly.

9.2 FLUID FLOW THROUGH FRACTURED NETWORK IN A ROCK MASS

In a comprehensive study of flow analysis, one has to consider an array of geo-hydrological factors as discussed in Chapter 3. Depending on the availability of the geological data and the required accuracy of flow estimation corresponding to the availability of time, numerical techniques and computer resources, the most appropriate flow model can be selected for the particular study (i.e. discrete, continuum or combination of both). The discrete method is preferred for fractured rock, in which fluid flow mainly takes place through a network of fractures. Once the flow approach is chosen, the next step is to identify whether the flow is in a saturated or unsaturated state of the medium, based on the field data. Generally, flow through most discontinuous media will be in an unsaturated form (e.g. water + air, water + solid and water +air +solid). However, most numerical techniques currently available are based on saturated flow, comprehensive unsaturated flow models for jointed rock are absent as yet. The scope of this chapter is limited to saturated water flow, based on UDEC based numerical analysis.
When fluid flow is governed by fractures, the interconnectivity and the density of fractures play a very important role, since they provide the multiple flow paths that conduct water through the rock mass (Lee and Farmer, 1993; Brady and Brown 1994; Tsang and Stephansson, 1996; and Herbert, 1996). Apart from this, the stability of the rock mass decreases with the increase in degree of interconnectivity. Particularly in underground constructions, catastrophic mine roof may occur, if the fractures transport abundant water to generate excess internal water pressures that substantially reduce the effective stresses at the boundaries of the mine opening. In order to describe a pattern of fracture interconnectivity, one needs to know the fracture lengths, their orientation and location. The trace length, orientation, joint apertures and joint roughness were discussed in Chapters 2, 3, 5 and 6. A network of fractures is formed by connecting several fractures. Some fractures may be isolated because their length, orientation and location are not suitable to connect with the existing network. The volume of water conducted in the jointed rock mass is a function of the degree of joint interconnectivity, the joint apertures and the magnitude of driving (fluid) pressures. Long and Witherspoon (1985) investigated how the permeability varies in a fracture network with the degree of interconnectivity. Recent studies have shown that permeability increases with the increase of fracture connectivity (Zhang et al., 1996).

An excavation induces stress relief and stress re-distribution in the surrounding rock mass, forming new fractures or opening up of existing fractures along the cavity surface. The extent of these fractures depends on the magnitudes of the stresses, fluid pressures within the rock mass and the rock properties. The excavation technique itself controls the fracture patterns and the magnitudes of joint apertures. For example, rock blasting using explosives generates high magnitude stress waves and significant gas
Figure 9.2. Factors affecting the water ingress to an underground cavity.
pressures, which change the initial fracture pattern significantly. However, the effect on fracture pattern due to mechanical excavation (e.g. tunnel boring machines) is much less. Therefore, as shown in Figure 9.2, one also needs to consider the prior and post-conditions of the rock mass associated with the type of excavation, in order to simulate the fracture patterns correctly before any numerical modelling technique can be implemented.

9.3 INTRODUCTION TO UDEC

The study described here was an attempt to investigate how the total inflow towards a mine cavity in a jointed rock media changes with the boundary conditions such as block dimensions, joint properties, effect of excavation and ground stress ratio. In this study, UDEC (ITASCA, 1996) was employed to simulate water flow through joints adopting a fully coupled hydro-mechanical analysis. UDEC is a two-dimensional numerical program based on the distinct element method (DEM) for discontinuum modeling. For coupled hydro-mechanical flow analysis, UDEC code is suitable when the flow is mainly governed by a well defined network of fractures.

9.3.1 Block motion theory

The motion of loaded blocks is due to the magnitude and direction of the resultant out-of-balance moments and forces. The type of motion includes both linear and angular movement. Considering the Newton’s second law, the motion of a body under a given force $F$ is given by:
\[
\frac{du}{dt} = \frac{u^{t+\Delta t/2} - u^{t-\Delta t/2}}{\Delta t} = \frac{F}{m}
\]

(9.8)

where, \( u \) = velocity,

\( t \) = time, and \( \Delta t \) = small time increment

\( m \) = mass of the block

Considering the effect of gravity and other external forces on the blocks, the angular and linear velocity can be used to estimate the linear displacement and the rotation of each block as described below:

The linear displacement at time \( t + \Delta t \), is given by

\[
x_{t+\Delta t} = x_t + u_{t-\Delta t/2} \Delta t + \left( \sum \frac{F_i}{m} + g \right) \Delta t^2
\]

(9.9)

where, \( x_t \) = the initial displacement of the block, and \( g \) is the acceleration due to the gravity.

The rotation at \( t + \Delta t \) is given by, \( \theta_{t+\Delta t} \):

\[
\theta_{t+\Delta t} = \theta_t + \theta_{t-\Delta t/2} \Delta t + \left( \sum \frac{M_i}{I} \right) \Delta t^2
\]

(9.10)

where, \( \theta \) = rotation of the block

\( M \) = total moment of the block due to the external forces and gravity,

\( I \) = second moment of inertia.

### 9.3.2 Joint models

UDEC assigns two types of models separately for the behaviour of rocks: (a) joint model and (b) block model. The block model may be rigid or deformable depending on
the situation. In practice, the degree of deformation of intact rock depends on the magnitude and direction of stress, the porosity of rock and the existing fluid flow condition. There are five block models which are included in UDEC, and the practical applications of these models are listed in Table 9.1. Out of these models, the Mohr-Coulomb model is more popular mainly because of the ease of obtaining the relevant strength parameters by simple laboratory tests, such as the direct shear box.

Table 9.1. Different block models used in UDEC (ITASCA, 1996).

<table>
<thead>
<tr>
<th>Block Models</th>
<th>Applications</th>
</tr>
</thead>
<tbody>
<tr>
<td>Null Model</td>
<td>To represent material either removed or excavated.</td>
</tr>
<tr>
<td>Elastic Model</td>
<td>For homogeneous and isotropic material (e.g., steel, soil layers)</td>
</tr>
<tr>
<td>Drucker-Pager Plasticity</td>
<td>Soft clay with low friction</td>
</tr>
<tr>
<td>Mohr-Coulomb Plasticity</td>
<td>Used for rock/soil mechanics applications (e.g., Coarse grain sand stone, soil, rock)</td>
</tr>
<tr>
<td>Strain-Hardening</td>
<td>Progressive failure of structures (e.g., concrete beam)</td>
</tr>
<tr>
<td>Double-Yield</td>
<td>Hydraulically placed backfill.</td>
</tr>
</tbody>
</table>

In order to represent the joint characteristics, the user has the choice of five joint models, as given in Table 9.2. The application of point contact method is remote in reality, as joints have several discrete points along the joint path. The area contact method is preferable because any two blocks defining a particular joint are usually in contact due to the external loads. The main advantage of using the residual strength is that this approach is capable of simulating displacement-weakening of joints due to the loss of friction or cohesion. The analysis of a single joint including variable apertures associated with different stress conditions can be investigated thoroughly using the Barton-Bandis model.
Table 9.2. Joint models incorporated in UDEC (ITASCA, 1996).

<table>
<thead>
<tr>
<th>Joint Models</th>
<th>Applications</th>
</tr>
</thead>
<tbody>
<tr>
<td>Point contact-Coulomb slip</td>
<td>For highly fractured and unstable rock</td>
</tr>
<tr>
<td>Joint area contact - Coulomb slip</td>
<td>General rock mechanics</td>
</tr>
<tr>
<td>Joint area contact - Coulomb slip with residual</td>
<td>General rock mechanics</td>
</tr>
<tr>
<td>strength</td>
<td></td>
</tr>
<tr>
<td>Continuously yielding</td>
<td>For dynamic loadings</td>
</tr>
<tr>
<td>Barton &amp; Bandis model</td>
<td>To estimate variable hydraulic apertures</td>
</tr>
</tbody>
</table>

Several researchers including Herbert (1996), Zhang et al., 1996, and Liao & Hencher (1997) have used the UDEC code to investigate fluid flow through jointed rock media. In this study, the blocks surrounded by the discontinuities were modelled as deformable material. Fluid flow analysis was performed in which the joint conductivity was directly related to the mechanical deformation associated with the joint (domain) water pressures. Each domain (filled with water) was separated by contact points at which mechanical interaction between blocks was established.

Depending on the type of contacts, UDEC employs two types of flow equations.

\[ q = k_c \Delta p \]  \hspace{1cm} (9.11)

where, \( k_c \) = point contact permeability factor, and

\[ \Delta p = p_2 - p_1 + \rho_w g (y_2 - y_1) \]  \hspace{1cm} (9.12)

where, \( p_1 \) = pressure in joint domain 1

\( p_2 \) = pressure in joint domain 2

\( \rho_w \) = density of water.

\( y_2, y_1 = y \) (vertical) co-ordinates of domain centers, and \( g \) is the acceleration due to gravity.
The domain pressures are updated by taking into account of the net flow into each domain, as well as the changes in domain volume due to the incremental motion of the surrounding blocks. As a result, the new domain pressure becomes:

\[ p = p_0 + K_w Q \frac{\Delta t}{V} - K_w \frac{\Delta V}{V_m} \Delta t \]  \hspace{1cm} (9.13)

where, \( p_0 \) is the domain pressure in the proceeding time step, \( Q \) is the sum of the flow rate into the domain from all surrounding contacts and \( K_w \) is the bulk modulus of the fluid. \( \Delta V = V - V_0 \) and \( V_m = (V + V_0)/2 \), where \( V \) and \( V_0 \) are the new and old domain areas, respectively.

In the case of edge to edge contact, the cubic law for flow in a planar fracture was used for estimating flow rate, as given by the following expression:

\[ q = - k_j a^3 \left( \frac{\Delta p}{1} \right) \]  \hspace{1cm} (9.14)

where, \( k_j \) = joint permeability factor,
\( a \) = hydraulic conductivity aperture,
\( l \) = length of the joint,
\( q \) = flow rate per \( \text{m width} \),

The simple relationship between the mechanical and hydraulic apertures of the joints used in this analysis was as follows:

\[ a = a_0 + u_n \]  \hspace{1cm} (9.15)

where, \( a_0 \) = joint aperture at zero normal stress, and
\( u_n = \) joint normal displacement.

At each time step in the mechanical calculation, UDEC employs updated geometry of the system, thus prescribing new values of apertures for all domain contacts and volumes. It is postulated that the discontinuities which do not form connectivity with the main fracture network may not contribute to any flow. Consequently, UDEC ignores isolated fractures for fluid flow calculations, but these fractures still contribute to a reduced overall modulus.

9.4 ROCK MASS PROPERTIES AND BOUNDARY CONDITIONS

A square boundary block size \((a \times a)\) was selected to analyze the deformation and permeability characteristics of the jointed rock mass, associated with the induced stresses. A boundary block contained regular joints, irregular joint networks and isolated joints depending on their location, orientation and the lengths. The analysis presented here was based on the regular joints (e.g. horizontal bedding planes with cross joints, continuous joints) with various boundary conditions (Figures 9.3-9.5). The joint pattern shown in Figure 9.5 is particularly applicable to a variety of sedimentary rock types with systematic bedding planes intersecting with cross-joints.

In order to simulate the jointed rock mass, joint geometrical parameters such as, the orientation, spacing, joint aperture, gap length and spacing were assigned for all joint sets and for isolated joints, separately. One of the most difficult parameters to measure, is the joint aperture. The geometrical properties of discontinuities can be incorporated
using two different approaches: (a) direct technique and (b) stochastic method.

Figure 9.3. Boundary conditions applied in the model (Case 1).
Case 2a: Hydraulic Boundary Conditions (Conventional)

Case 2b. Porous medium grid is wrapped around the external boundary block to represent hydraulic boundary conditions (i.e. large scale mesh discretisation).

Figure 9.4. Boundary conditions applied in the analysis (Case 2).

The direct approach is suitable for small fracture network, such as fractures in laboratory (small) scale specimens. However, this approach is not feasible when there is a large number of fracture sets, making it difficult to predict the correct fracture flow path within the rock mass. Alternatively, the stochastic modeling method, based on a statistical description of fracture network may be used to generate the fracture network.
This method does not describe the location, spacing and orientation of joints deterministically. Also, the stochastic technique does not represent the actual fracture network as described by the direct or conventional approach. As discussed earlier, in reality, fractures may have complex geometry and variable apertures. However, for the current analysis, the geometry of fractures was simplified, and the joint geometrical properties of Cases 1, 2 and 3 (Figures 9.3-9.5) are given in Table 9.3.

Case 3a: Static water pressures are applied on all four sides as shown above.
Case 3b: Same as case 3a, except the ground water table is made to coincide with top of the boundary block.
Case 3c: Same as case 3a, except bottom boundary is made fully permeable, such that excess pore pressures are dissipated to zero. This practically simulates a sand aquifer or sand lens underlying the fractured rock.

Figure 9.5. Horizontal and vertical stresses associated with gravity and pore pressure due to static water head applied into the model (Case 3).
Once the joint geometrical properties were assigned, the next task was to incorporate the material properties of intact rock, discontinuities and fluid, as presented in Table 9.4. For simplicity, the variation of the material properties within the boundary block was not taken into account. For each joint set, mean value of material properties and their standard deviations (e.g. friction angle, normal and shear stiffness) were incorporated in the model separately.

Table 9.3. Joint parameters used in Cases 1, 2 and 3.

| CASE 1 |  |
| Parameters | Units | Joint set 1 | Joint set 2 | Joint set 3 |
| Orientation | deg. | 0 | 90 | 90 |
| Spacing | m | 3.0 | 3.5 | 3.5 |
| Gap length | m | 0 | 3.0 | 3.0 |
| Trace length | m | 5.0 | 3.0 | 3.0 |

| CASE 1B |  |
| Parameters | Units | Joint set 1 | Joint set 2 |
| Orientation | deg. | 30 | 150 |
| Spacing | m | 4.0 | 3.5 |
| Gap length | m | 0 | 0 |
| Trace length | m | 5.0 | 3.0 |

| CASE 2 |  |
| Parameters | Units | Joint set 1 | Joint set 2 | Permeability tensor – case 2b |
| Orientation | deg. | 45 | 135 | $K_{11}$ m³/(Pa·sec) | $K_{12}$ m³/(Pa·sec) | $K_{22}$ m³/(Pa·sec) |
| Spacing | m | 3.5 | 4.0 | $2.33 \times 10^{-10}$ | $1.33 \times 10^{-10}$ | $2.33 \times 10^{-10}$ |
| Gap length | m | 0 | 0 |  |  |  |
| Trace length | m | 4.5 | 3.5 |  |  |  |

| CASE 3 |  |
| Parameters | Units | Joint set 1 | Joint set 2 | Joint set 3 |
| Orientation | degrees | 0 | 90 | 0 |
| Spacing | m | 3.0 | 2.5 | 4.5 |
| Gap length | m | 0 | 3.0 | 2.8 |
| Trace length | m | 5.0 | 3.0 | 4.5 |
Properties such as density, cohesion and bulk modulus of the intact rock were used to characterize the rock matrix. In order to quantify the effect of fluid flow on the deformation characteristics, the density, dynamic viscosity and bulk modulus of the fluid were also incorporated in the model. Naturally, the bulk modulus of the fluid plays a much bigger role for highly compressible fluids, such as CO₂, CH₄, and air, in underground coal mining.

The assumed boundary conditions (for both fluid and ground stresses) are shown in Figures 9.3-9.5. Basically, four different hydraulic boundary conditions were considered for Cases 1-3, as described below:

(a) Constant water pressure,
(b) Linearly varying water pressure,
(c) Permeable/impermeable boundaries, and
(d) Wrapping a porous medium around the boundary block.

Constant water pressures act along the top and bottom boundary surfaces, while linearly varying fluid pressure acts along the left and right vertical surfaces (see Cases 1, 2a & 3). Permeable boundary conditions may arise naturally when some boundary surfaces coincide with two faults of highly permeable nature (Case 1a-Figure 9.3). Impermeable boundary surfaces can include fluid filled joints but with no flow. The constant and linearly varying pressures are respectively given by the following expressions:

\[ p_{yy} = \rho_w g h_w \]  
\[ p_{xx} = \rho_w g h_w + p_{y-grad} y \]

where, \( p_{xx} \) = fluid pressure along the vertical boundaries 
\( p_{yy} \) = fluid pressure along the top and bottom boundaries 
\( \rho_w \) = density of water 
\( h_w \) = depth of the water table 
\( p_{y-grad} \) = hydraulic gradient in Y-direction and, 
\( y \) = vertical depth from the ground surface.

A porous medium was wrapped around the boundary block in order to simulate flow on a large scale. Having created the radial mesh around the block (Figure 9.4), then the fluid pressure was imposed (Case 2b-Figure 9.4) on the created mesh.

Darcy's law applied to fluid flow in an anisotropic medium was represented as given below:
\[ v_i = K_{ij} \left( \frac{\partial p}{\partial x_j} \right) \]  

(9.18)

where, \( v_i \) is the velocity vector, \( p \) is the pressure and \( K_{ij} \) is the permeability tensor given by

\[
\begin{bmatrix}
K_{11} & K_{12} \\
K_{21} & K_{22}
\end{bmatrix}
\]

The permeability tensor for a continuous joint set was calculated using the following equations

\[ K_{11} = K_j \cos^2 \alpha \]  

(9.19a)

\[ K_{22} = K_j \sin^2 \alpha \]  

(9.19b)

\[ K_{12} = K_j \cos \alpha \sin \alpha \]  

(9.19c)

\[ K_j = \frac{a^3}{12 \mu s} \]  

(9.19d)

where, \( \alpha \) = orientation of joint set

\( a \) = aperture of joint set

\( s \) = spacing of joint

\( \mu \) = dynamic viscosity of water

In order to ascertain the permeability tensor, the contribution of permeability of each joint set is summed together.

Initially, there was no flow into the region, and the hydrostatic pressure prevailed all around the excavation. Once the tunnel was excavated, a constant atmospheric (zero) pressure was applied around the tunnel surface. Therefore, the resulting inward hydraulic gradient causes fluid to flow from the boundary towards the underground cavity.
The initial compressive stress was defined by the isotropic stress associated with gravity, and is represented by the following equations:

\begin{align}
\sigma_{yy} &= \rho_r g y \\
\sigma_{xx} &= \alpha \rho_r g y + \sigma_{y-\text{grad}} y
\end{align}

(9.20a) \hspace{2cm} (9.20b)

where, \( \rho_r \) = density of rock,

\( y \) = vertical distance (depth), measured downward from the ground surface,

\( \sigma_{yy} \) and \( \sigma_{xx} \) = insitu stress components in Y and X directions,

\( \sigma_{y-\text{grad}} \) = insitu stress gradient, and \( \alpha \) = insitu stress ratio factor.

The vertical component of compressive stress (\( \sigma_{yy} \)) is applied along the top and bottom boundary surfaces, while the horizontal component (\( \sigma_{xx} \)) is applied to the vertical surfaces. In order to prevent the boundary being displaced, the bottom boundary was fixed in the X and Y directions (i.e. \( v_x = v_y = 0 \)). The hydraulic and insitu stress boundary conditions are listed in Table 9.5.

Table 9.5. Stresses and hydraulic boundary conditions applied in the model.

<table>
<thead>
<tr>
<th>Cases</th>
<th>Vertical stress</th>
<th>Horizontal stress</th>
<th>Fixed boundaries</th>
<th>Hydraulic stress</th>
<th>Permeable boundaries</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>( \sigma_{yy} = \rho_r g h )</td>
<td>( \sigma_{xx} = \alpha \rho_r g h )</td>
<td>Bottom boundary ( v_x = v_y = 0 )</td>
<td>( p_{yy} = p_w g h_w )</td>
<td>Left and Right boundaries</td>
</tr>
<tr>
<td>Figure 9.3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Case 2a</td>
<td>( \sigma_{yy} = \rho_r g h )</td>
<td>( \sigma_{xx} = \alpha \rho_r g h )</td>
<td>Bottom boundary ( v_x = v_y = 0 )</td>
<td>( p_{yy} = p_w g h_w )</td>
<td>No</td>
</tr>
<tr>
<td>Figure 9.4</td>
<td></td>
<td></td>
<td></td>
<td>( p_{xx} = p_w g h_w )</td>
<td></td>
</tr>
<tr>
<td>Case 2b</td>
<td>( \sigma_{yy} = \rho_r g h )</td>
<td>( \sigma_{xx} = \alpha \rho_r g h )</td>
<td>( \sigma_{yy} = \rho_r g h )</td>
<td>Porous medium is wrapped around the block</td>
<td>No</td>
</tr>
<tr>
<td>Figure 9.4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Case 3</td>
<td>( \sigma_{yy} = \rho_r g h )</td>
<td>( \sigma_{xx} = \alpha \rho_r g h )</td>
<td>( \sigma_{yy} = \rho_r g h )</td>
<td>( p_{yy} = p_w g h_w )</td>
<td>No</td>
</tr>
<tr>
<td>Figure 9.5</td>
<td></td>
<td></td>
<td></td>
<td>( p_{xx} = p_w g h_w )</td>
<td></td>
</tr>
</tbody>
</table>
9.5 MODEL DESCRIPTION AND BEHAVIOUR

For a given boundary block size (e.g. 100x100m), let us assume that the centre of the boundary block is 50m below the ground surface and the water table coincides with the top of the boundary block. Having defined the co-ordinates and the joint geometrical parameters of this boundary block, a 6m diameter tunnel was simulated in the jointed rock. It is important to note that the excavation of the tunnel is simulated at a later stage, i.e. once the model has reached the initial equilibrium condition. Assuming the rock was deformable, the rock blocks formed by the joints were discretised into smaller triangular blocks for a finer analysis (Figure 9.6). Having assigned the material properties of intact rock, joints and fluid (Table 9.4), the fluid and ground stresses were then imposed, and the model behaviour under these loading conditions were observed. A typical UDEC code is given in Appendix C.
Figure 9.7 shows the ground stress (principal stress levels) and fluid pressure variation within the model. Fluid pressures within joints greater than 0.1MPa are shown in Figure 9.7b. Not surprisingly, the magnitudes of these stresses increase with depth. The observation may be carried out for different stages as follows:

(a) Undrained conditions, i.e. mechanical deformation of rock mass only;

(b) Drained conditions with no mechanical deformation of rock mass;

(c) Coupled flow and mechanical deformation of rock mass.

The total unbalanced force, the total net fluid flow and the deformation at several locations are essential parameters to be considered for equilibrium.

Figure 9.7a. Initial principal stress distribution.
Forces are accumulated at each grid point of the deformable blocks, and the algebraic summation of these forces should approach zero at static equilibrium. However, in practice, the total unbalanced force may never reach zero, hence an engineering judgement is required for the acceptable magnitude of unbalanced forces at given time steps. Under coupled flow-mechanical deformation stage, the excavation induces deformation of the rock mass and affects the magnitude of joint apertures, which in turn affects the flow rate and the re-distribution of stresses. In the next iterative time step, the current stress levels are employed in the coupled flow-deformation analysis.
9.5.1 Effect of excavation on the flow and deformation characteristics

The concept of a disturbed zone around the excavation is important in the design of piers, evaluating the tunnel stability and for predicting mine inundation and gas outbursts. As described earlier, the degree of deformation depends on the excavation technique, material properties of the rock, the presence of geological features, ground stresses and fluid flow conditions.

In UDEC, once the model was brought to equilibrium under the initial field conditions, the excavation of the tunnel was made instantaneously, and again the equilibrium of the model was attained. Any disequilibrium of the system was reflected by large unbalanced nodal forces and a significant discrepancy between the total inflow and outflow of the model. In this regard, one needs to ensure that the total net flow to several randomly selected internal nodal points is compatible with the principle of conservation of energy. In order to study the effects of fluid pressure on joints deformation after the excavation of the cavity, basically two analyses were carried out for 

(a) undrained flow and

(b) drained flow (i.e. steady state flow).

9.5.2 Undrained flow analysis.

During the undrained flow analysis method, only the mechanical deformation of the model was executed with the flow mode switched off. The deformation of the joints was due to the tunnel excavation, and a high hydraulic gradient developed towards the
tunnel boundary. Undrained displacement vectors show that a different state of deformation close to the tunnel periphery occurred, as presented in Figure 9.8.

The arrow-heads represents the direction, and the length of arrow indicates the magnitude of the displacement vectors of the rock blocks. The rock mass above and below the tunnel appears to have undergone significant deformation. It is evident that close to the tunnel periphery, deformation is initiated and subsequently propagates toward the top and bottom of the boundary block. As expected, the tunnel periphery undergoes large deformations. The largest deformations occur along the joint planes (e.g. BD, BE and AC) as indicated in Figure 9.9. By considering the general pattern of the displacement vectors, and identifying the probable deformed zone, the potential
unstable rock blocks can be interpreted (see Figure 9.8). These unstable rock blocks are more vulnerable when the rock joints carry water.

The fluid pressure distribution along the joints is shown in Figure 9.9, in which the line thickness shows the magnitude of fluid pressures within joints. The fluid pressure at point A is approximately 5 times the static fluid pressure (i.e. $pgh$, where $h$ depth of water table to the point A). The effective stress within the joints is the difference between normal component of initial total stress in the joint and the initial hydrostatic pressure. The elevated fluid pressure exceeds the normal stress in the joint, thereby resulting with a negative effective pressure. The negative effective pressure at point A

![Figure 9.9. Pore pressure distribution after undrained deformation.](image)
and B (see Figure 9.9) results in dilation of the joints. These effective stresses are used to calculate the flow deformation parameters in the model.

9.5.3 Drained flow analysis (steady state flow)

During drainage, a coupled hydro-mechanical analysis was carried out under the steady state flow to observe deformation due to drainage of groundwater. The drainage of water flow through joint network causes a large deformation close to the tunnel periphery. As seen in Figure 9.10, a significant deformation has occurred above the tunnel. This is because the reduced shear strength due to the flow of water has increased the movement of rock blocks towards the tunnel periphery. According to Figure 9.11, a large shear stress concentration has developed close to the tunnel. The magnitude of shear stress less than 0.8MPa is not plotted in Figure 9.11.

![Figure 9.10. Displacement vectors during steady state flow.](image-url)
Because of the excavation, the increased hydraulic gradient towards the tunnel causes fluid flow into the cavity, and as a result, pore pressure within the joints close to the tunnel is dissipated. During steady state flow, the pore pressure distribution within the joints is given in Figure 9.12, in which the line thickness shows the magnitude of the fluid pressure. The minimum fluid pressure in the plot is 0.1MPa. After excavation, fluid pressure variation within a typical joint BD and CA is shown in Figure 9.13.
Fluid pressure increases away from the excavation during the drain, whereas a large fluid pressure was observed close to the tunnel, during undrained analysis.

Figure 9.12. Pore pressure distribution during the steady state flow.

Figure 9.13a. Pore pressure variation along typical joint, CA during drained and undrained conditions.
It is important to note how the effective stress varies along the joints which intersect the tunnel boundary, as the effective stress controls the quantity of fluid carried along these joints towards the tunnel. Before and after excavation, the effective stress in two typical joints (i.e. AC and BD) which intersect the tunnel boundary is shown in Figure 9.14. As expected, the effective stress before the excavation at points C and B is greater than that at points A and D, respectively. However, after the excavation, the effective stress at A and B has significantly increased due to dissipation of pore pressure at the tunnel boundary. As an example, the effective stress at point A after the excavation of the tunnel is approximately 7 times that of the initial effective stress at A before the excavation. The increased effective stress causes change of joint apertures and in turn the flow rate is altered.

Figure 9.13b. Pore pressure variation along typical joint, DB during drained and undrained conditions.
If one compares the drained and undrained analyses, it is seen that the presence of fluid greatly influences the deformation of joints. During the undrained analysis, the maximum fluid pressure and the least value of the effective stress (can be negative) exists close to the tunnel, while the opposite trend is observed in the drained analysis. Table 9.6 summarizes the drained and undrained flow analysis.

Table 9.6. Effects of drained and undrained analysis.

<table>
<thead>
<tr>
<th>Undrained analysis</th>
<th>Drained analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td>An elevated fluid pressure close to the tunnel periphery</td>
<td>Fluid pressure in joints close to the tunnel periphery</td>
</tr>
<tr>
<td>Negative effective pressure may develop close to the tunnel,</td>
<td>Effective stress close to the tunnel periphery is high.</td>
</tr>
<tr>
<td>due to high pore pressure</td>
<td></td>
</tr>
<tr>
<td>A large joint dilation can occur due to negative effective</td>
<td>A large flow rate within the joints close to the tunnel</td>
</tr>
<tr>
<td>stress.</td>
<td>can be seen.</td>
</tr>
<tr>
<td>Large displacement vectors all around the tunnel periphery.</td>
<td>large displacement vectors the tunnel roof, but not at the</td>
</tr>
<tr>
<td></td>
<td>invert.</td>
</tr>
</tbody>
</table>
The induced stresses associated with stress relief significantly influence the pore pressures and flow rates within the joints. In this section, the following parameters, which influence the flow rates to the tunnel, are discussed. They are: (a) different sizes of representative blocks, (b) orientation of joint sets, and (c) vertical and horizontal stresses.

9.6 DETERMINATION OF FLOW RATES BASED ON EFFECTIVE STRESS ANALYSIS.

The effective stresses on the joints influence the change of joint aperture and the permeability. For given boundary conditions, the flow quantity in each joint close to the tunnel is given in Figure 9.15, in which the line thickness shows the magnitude of flow rates and the magnitude of flow rates. The joints with flow rates below $2 \times 10^{-4} \text{m}^3/\text{sec}$ is not plotted in the figure. For a given joint pattern, boundary block size and boundary conditions, the total water ingress is estimated by the summation of the flow quantity of each joint intersecting the tunnel periphery. The flow results are based on steady state flow only. As an example, the total inflow to the tunnel is the summation of the flow rates at points B and A, which are the intersections of the joints AC, BD, and BE with the tunnel boundary (Figure 9.15). It is clearly seen that the flow rate at the excavation boundary has increased in all joints because of the high hydraulic gradient developed towards the tunnel.
9.6.1 Effect of representative block sizes

For the given joint pattern and boundary conditions in Figure 9.3, the relationship between the total water ingress and various block sizes is shown in Figure 9.16, which indicates that increasing the block sizes will result in a decreasing flow rate. On one hand, it may be argued that the flow rate should increase with the increasing block size, because, the discontinuities which intersect the tunnel periphery have a greater degree of intersection with other discontinuities away from the cavity. On the other hand, if the block size is increased, the effects of water pressure on discontinuities that are...
intersected by the tunnel boundary become less, consequently, a lower value of water flow can then be expected due to a decreased hydraulic head.

![Graph](image)

Figure 9.16. Effects of boundary block size on flow rate for different insitu stress.

Figure 9.17 illustrates the effect of joint surface area on the flow rate for the joint model shown in Figure 9.3. The joint surface area increases when the boundary block size increases, if the joint spacing is kept the same. For a larger block size, as the length of joints increases, the hydraulic gradient will decrease for the same external fluid pressure applied to the block boundary. Therefore, the flow rate to the excavation is expected to decrease with the increasing joint surface area or increasing block size. This is because, the larger the representative block size (i.e. fracture area), the smaller the hydraulic gradients applied towards the opening and hence, the smaller the inflow to the cavity. It
is more useful to develop a relationship between the total water ingress and a dimensionless ratio defined by total joint surface area /excavation area \((A_c/A_e)\). It can be demonstrated that the total water ingress is very high when the ratio \(A_c/A_e\) is between 10 to 40. It can be also shown that very low water ingress can be expected when \(A_c/A_e\) varies from 80 to 200, as illustrated in Figure 9.17.

![Figure 9.17. Results of total flow against the total crack area/excavation area ratio.](image)

The hydraulic boundary conditions can also play a major role on the water ingress to the subsurface cavity as shown in Figures 9.18 and 9.19. A very high flow rate can be expected when the block size lies between 15 and 50m. If one side of the boundary is treated as permeable (Case 3c- Figure 9.5), the flow rate increases significantly (Figure 9.18b). Irrespective of the fluid boundary conditions or insitu stress ratios (i.e. insitu horizontal stress/vertical stress), the increase in boundary block size will result in
a decreasing flow rate. The flow rate becomes marginal when the boundary block size exceeds 50m in all three cases. Therefore, this sensitivity analysis provides a most appropriate block size to be selected for a numerical flow analysis. Based on the current distinct element method, the optimum boundary block size is 10-12 times the maximum width of excavation considered.

Figure 9.18a. Effects of boundary block size on flow rate for different insitu stress conditions (Case 3a - Figure 9.5).

9.6.2 Effects of orientation of discontinuities.

Figure 9.20 shows the relationship between the total flow rate to the tunnel and the orientation ratio of joint set 1 to joint set 2. In this analysis, orientation of joint set 1
Figure 9.18b. Effects of boundary block size on flow rate for different insitu stress (Case 3c – Figure 9.5).

Figure 9.19a. Effects of boundary block size on flow rate for different insitu stress (Cases 2a – Figure 9.4)
was kept constant ($\theta_1 = 30^\circ$ relative to the X-axis), while the orientation of joint set 2 ($\theta_2$) was varied for an insitu stress ratio ($\sigma_h/\sigma_v$) of 2.0, which is the typical of the Wollongong region, NSW, Australia. Figure 9.20 clearly shows that the maximum water flow to the tunnel occurs when $\theta_2/\theta_1$ equals 3.0. Not surprisingly, this indicates that the vertical discontinuities ($\theta_2 = 90^\circ$), which intersect the tunnel periphery carry more water than any other discontinuity in the model. Naturally, the effects of gravity flow are optimized in this situation. The maximum flow rate occurs when the angle between the two joint sets is around $60^\circ$ ($\theta_2/\theta_1 = 3$), whereas Zhang et al. (1996) show that permeability is maximum if the angle between the joint sets is around $30^\circ$. This is not surprising because the joint pattern considered by Zhang et al. (1996) was more interconnected. Therefore, depending on fracture orientation and density, the extent of
connectivity of fluid flow paths in a given joint pattern, the resulting flow rate can be significantly different even for the same boundary stress levels.

![Mean orientation angle between two joint sets (θ₂ - θ₁)](image)

<table>
<thead>
<tr>
<th>θ₂ - θ₁</th>
<th>30</th>
<th>60</th>
<th>90</th>
<th>120</th>
<th>150</th>
</tr>
</thead>
<tbody>
<tr>
<td>Max flow rate</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Joint model 2
Hori. stress/veri. stress = 2.0

Figure 9.20. Effects of orientation of joint sets on total inflow towards the tunnel (Case 1b – Figure 9.3).

9.6.3 Effects of horizontal stress and vertical stress

The three different figures (9.21a-9.21c) demonstrate the changing flow rates for various joint models. Figure 9.21 presents the relationship of the total flow ingress to the tunnel versus \( \sigma_h / \sigma_v \) ratio. As expected, the flow rate decreases with the increasing horizontal stress. However, for large boundary block sizes, the decrease in flow rate is insignificant with the increase in insitu ratio. As the ratio of horizontal stress to vertical stress increases from 0.5 to 2.25, the reduction in the percentage of water ingress to the mine cavity lies in the range 40 – 60% (Figure 9.21a). When the
stress distribution ratio exceeds 1.2, the change of joint flow rate is marginal, because, most conducting fractures have reached their residual apertures (Figure 9.21b).

These results are generally in agreement with Indraratna & Wang (1996) who observed that a flow reduction of up to 70% is possible for a systematic joint pattern, such as Case 1a (Figure 9.3). Using UDEC, Liao & Hencher (1997) investigated the effect of horizontal stress, boundary conditions and block sizes on the permeability characteristics of jointed rock mass. They found that the overall permeability decreases with the increase in horizontal to vertical stress ratio. These findings are also in accordance with the analysis presented here, the difference being that this analysis has concentrated more on the variation of flow rates rather than the permeability variations.

(a) Case 1

Figure 9.21a. Effects of insitu stress ratios on water inflow into tunnel (Case 1).
According to the numerical flow analysis carried out by Zhang et al. (1996), permeability decreases significantly with the increase in vertical stress for a given joint pattern and horizontal stress. In the current study, the predicted inflow decreases with the increase in horizontal stress for a given joint pattern. This is because the near vertical joints that conduct significant amount of water ($\theta = 90^\circ$) become compressed (reduced aperture) when horizontal stress is increased. This shows that the influence of horizontal stress on the closure of near vertical joints is greater than that of vertical stress, resulting in reduced flow through a fracture network.

![Figure 9.21b. Effects of insitu stress ratios on water inflow into tunnel (Case 2).](image)
A fully coupled hydro-mechanical analysis was carried out in order to determine water ingress to an underground cavity with various geo-hydraulic and joint parameters, which influence the flow rate. The numerical analysis was carried out for three different joint models with different boundary conditions (Figures 9.3-9.5). The flow through the rock mass was simulated by using the discrete model as the fluid flow was mainly dominated by a network of discrete fractures. The important aspects of this analysis are summarized below:

- For flow analysis, analytical solutions are useful when flow takes place through a small number of fractures. Numerical approaches such as
FEM, BEM have been employed for various rock and soil engineering problems. The coupled BEM - FEM or DEM - BEM are becoming increasingly popular because of their ability to model rock joints and material explicitly for larger volume of rock.

- The water ingress towards a cavity is a function of the geometrical and material properties of the joints, insitu stresses, location of groundwater table, the dilation or closure of joint apertures due to stress relief caused by the excavation of cavity, selected boundary block size and the chosen technique of flow analysis.

- The representative boundary block size for flow analysis plays a major role, therefore, a sensitivity analysis should be carried out to establish the appropriate size of the boundary block. Irrespective of fluid boundary conditions or insitu stress ratios (i.e. insitu horizontal stress/vertical stress), increase in boundary block size will result in a decreasing flow rate. Flow rate becomes marginal when the boundary block size exceeds 50m, in all the three cases considered here. Based on the distinct element method, the optimum block size is 10-12 times the maximum width of excavation (Figures 9.16-9.19).

- It is demonstrated that the total water ingress is very high when the ratio \( \frac{A_c}{A_e} \) (total crack area/excavation area) is between 10 to 40 (Figure 9.17). This is because, the larger the representative block size (i.e. fracture area), the smaller the hydraulic gradients applied towards the opening, hence the smaller the inflow to the cavity. Very low water ingress is expected when \( \frac{A_c}{A_e} \) varies from 80 to 200.
• The hydraulic boundary conditions play a major role on the water ingress to the subsurface cavity. The flow rate towards the cavity significantly increases when the excavation boundary is treated as permeable (Figures 9.16 and 9.18b).

• The flow rate decreases with increasing horizontal stress for the joint patterns considered here. For large boundary blocks, the decrease in flow rate is insignificant with the increase in insitu stress ratios. The reduction in the percentage of water ingress to the mine cavity is at the order of 40 –60% when the ratio of horizontal stress to vertical stress increases from 0.5 to 2.25 (Figure 9.21). When the insitu stress ratio exceeds 1.6, then the rate of change of flow becomes very small.

• Based on the numerical analysis, the maximum water flow to the tunnel occurs when joint orientation ratio, \(\theta_2/\theta_1\) equals 3.0. The vertical discontinuities \(\theta_2 = 90^\circ\), which intersect the tunnel periphery carry more water than any other discontinuity in the model (Figure 9.20). Depending on fracture orientation and density, and the extent of connectivity of fluid flow paths in a given joint pattern, the resulting flow rate can be significantly different even for the same boundary stress levels.
CHAPTER 10

CONCLUSIONS AND RECOMMENDATIONS

10.1 CONCLUSIONS

Two-phase flow behavior through fractured rock was studied analytically and experimentally. The experimental programme included (a) design of a novel, two-phase flow testing triaxial equipment, (b) study of fully saturated flow through fractured and intact rock and (c) two-phase flow measurement and analysis through natural and artificially created rock joints. The naturally fractured rock specimens were supplied by Strata Control Technology (Wollongong) for the tests. Fractured granite was conveniently used in the study because of its availability and to simulate flow behaviour through rock fractures. The analytical work included the development of a mathematical model to estimate two-phase flow through a single joint, and the validation of the model was carried out experimentally for at least 10 fractured granite specimens.

The design concept of the two-phase triaxial equipment was presented in Chapter 4. A laboratory study of single-phase flow analysis through fractured rock was presented in Chapter 5. An analytical procedure based on mass, momentum and energy balance principles for estimating individual flow rates in two-phase fluid flow through rock joints for different boundary conditions was presented in Chapter 7. The predictions for flow by the analytical model for various boundary conditions were compared with the experimental data in Chapter 8. The laboratory findings verified the acceptable accuracy of the mathematical model for two-phase flow developed by the writer. In
order to determine water ingress to an underground cavity, a fully coupled, hydromechanical analysis was also carried out using UDEC, with various geo-hydraulic and joint parameters in Chapter 9.

The following main conclusions have been drawn on the basis of this doctoral study:

- Significant efforts to study the strain-stress and permeability characteristics of soil and rocks under laboratory conditions are clearly evident from the numerous types of triaxial testing apparatus. However, none of the existing triaxial facilities were capable of modelling realistic fluid flow through jointed rock mass. The newly designed triaxial apparatus is capable of simulating the actual fluid flow field in jointed rock mass (Section 4.3.2). In this apparatus, two-phase (water-air) fluid flow through soft and hard rocks can be simulated (i.e. saturated or unsaturated flow conditions) under triaxial stress state.

- The failure stress of water and air saturated intact granite were respectively at 150MPa with 0.035 strain, and 190MPa with 0.02 strain (Section 5.2.1). For axially fractured specimens, the failure of water and air saturated specimens occurred respectively at 140MPa with 0.04 strain, and 155MPa with 0.03 strain (Section 5.3.1). It is evident from this experimental data, that a higher failure load at a lower strain is expected, if the permeating fluid is air. In contrast, for water-saturated samples, a lower ultimate strength is attained. It is important to note that compressible gas phase generates less pore pressure, in comparison with relatively incompressible fluids which
induce higher pore pressure. This concludes that, (a) ductility of rock mass increases and (b) deformation modulus decreases, when the water content is increased in relation to the air content.

- From a practical point of view, (e.g. the stability of tunnel roofs and mine longwalls under different fluid pressures), when gas flow dominates, sudden instability of mine roof can be expected at a critical gas pressure. In contrast, if the jointed rock is saturated with water, the failure process is more gradual and also predictable.

- The flow through a single joint is a function of the magnitude of the joint aperture, external stress field and its loading and unloading history, applied fluid pressures, the joint surface roughness and the relative orientation of the joint in relation to the principal stresses. The effect of loading and unloading on fractured rock permeability is significant. There is a marked reduction of flow rate during the 1st loading cycle with stress increases normal to the joint, but such effect in the 2nd and 3rd loading and unloading cycles is very small (Section 5.3.4). Based on this study, it is concluded that once fractures attain their residual apertures at 8MPa effective confining pressure, subsequent dilation (due to unloading) and compression (due to reloading) seem to be insignificant. Above 8MPa effective confining pressure, the average permeability decreases by almost 90% from the coefficient of permeability at zero confining pressure. This reduced permeability associated with the residual aperture will always prevail once the joints are loaded normal to the joint above the threshold value.
The matrix permeability of intact granite is in the order of $10^{-19}\text{m}^2$, whereas fracture permeability can vary from $10^{-12}$-$10^{-15}\text{m}^2$ depending on the magnitude of joint aperture and the interconnectivity of fractures (Sections 5.2 and 5.3). For numerical analysis, the matrix permeability of granite can be neglected, in relation to the fracture permeability.

From the experimental work, it is verified that the joint roughness coefficient varies considerably from one location to another along the same specimen ($\text{JRC} = 3 - 12$, Section 6.3). This indicates that field fractures can be highly irregular, and it is not always appropriate to model them as parallel plates. For practical purposes, Barton’s standard profile method (1973) and alternative technique based on the maximum amplitude of the joint yield roughness coefficients reasonably accurately.

When the asperity heights ($k$) between the maximum and minimum amplitudes of rough joint surfaces approach the magnitude of joint aperture ($e$), the pressure drop coefficient is reduced by 8 times from the pressure drop coefficient for a smooth joint. Increase in JRC will result in an exponential decrease in flow rate. Even at very high JRC values (e.g. 15) and high normal stresses, there is always a minimum flow corresponding to the residual aperture. Although roughness is important in fluid flow estimations, in reality, it is not feasible to incorporate roughness of each joint separately. Having identified these limitations, the cubic law is still found to be applicable with caution in numerical modelling applied to practical situations.
Assuming that the fluid flow pattern in a rock joint is stratified, a mathematical model was formulated for two-phase flow incorporating joint deformation, effect of solubility of air in water, compressibility of water and air associated with joint deformation, and change of fluid properties such as density. For the above conditions, the proposed model can predict equivalent phase heights of water $h_w(t)$ (Equation 7.7) and air $h_a(t)$ (Equation 7.9 in Section 7.3.2). Subsequently, these phase heights can be used to estimate flow rates, permeability and relative permeability of each phase. The model shows that almost 95% of the magnitude of $h_a(t)$ and $h_w(t)$ is due to the normal joint deformation ($\delta_n$), the rest being the combined effect of $\zeta_{ac}$ (air compressibility) and $\zeta_{ad}$ (solubility of air in water). The term, $\zeta_{ac}$ is more significant than the component $\zeta_{ad}$, and $\zeta_{ac}$ amounts to about 4-5% of the value of $\delta_n$. Once the joint aperture reaches its residual value (confining pressure exceeding 6MPa), the role $\zeta_{ac}$ and $\zeta_{ad}$ becomes increasingly pronounced.

Based on this study, it is verified that at low inlet fluid pressures, the two-phase flow within rock joints can be best described by bubble flow (Section 8.3.1). Further increase in inlet fluid pressure may result in annular flow. At very high inlet fluid pressures, two-phase flow in highly irregular joints should be modeled using complex flow patterns. It is also verified that for flow of low viscosity fluid through a high viscosity medium (e.g. air injected to water saturated joints) steady state conditions result in a shorter period of time (<4hrs). As shown in Section 8.4.1, a significantly larger
period of time is taken to observe steady state flow, when water is injected to air-saturated joints (>10hrs).

The Reynolds numbers measured for two-phase flow in this study are well below 1000 (Section 8.5). Therefore, laminar flow is considered appropriate for both single and two-phase flows observed under the boundary conditions applied in the experimental work.

As shown in Section 8.4, at relatively small inlet fluid pressure (<0.5MPa), both experimental and predicted results show that two-phase flow rates vary almost linearly with inlet fluid pressures, when the inlet pressures of both phases are approximately equal ($p_a = p_w$). However, non-linear changes take place when the inlet fluid pressure is increased beyond 0.5MPa or when $p_a \neq p_w$. Increase in inlet pressure of one phase usually results in the increase of flow rate of the same phase. Nevertheless, the flow rate of the air phase is always higher than that of the water phase.

Increase in confining pressure leads to a decrease in the two-phase flow rates. At elevated confining pressures exceeding 6MPa, change in flow rate becomes marginal, because the joints have attained their residual aperture (Section 8.4.2).

Based on this study, it is verified that the fracture permeability of both air and water phases becomes equal when the ratio of $p_a/p_w$ is between 0.8 to 1.2
(Section 8.6). For initially water saturated specimens, the air permeability plays the more dominant role at high $p_a/p_w$ ratios.

- The Darcy’s law can be extended to represent two-phase flow, based on the relative permeability concept. The relative permeability of the air phase increases almost exponentially with the increase in $p_a/p_w$ ratio, while decreasing the relative permeability of the water phase, at the same time. The opposite trend occurs when the $p_w/p_a$ ratio is increased (Section 8.8).

- In the distinct element (UDEC) modelling of groundwater ingress to underground cavity, the flow rate becomes marginal when the boundary block size exceeds 50m for selected joint patterns. Based on this distinct element analysis, the optimum block size for such flow investigation should be 10-12 times the maximum width of excavation to obtain numerical convergence (Section 9.6).

- Numerical modelling results confirm that the flow rate decreases with the increasing horizontal stress. The reduction in the percentage of water ingress to the mine cavity lies in the range of 40–60%, when the ratio of horizontal stress to vertical stress ($\sigma_h/\sigma_v$) increases from 0.5 to 2.25 (Section 9.6.1). When $\sigma_h/\sigma_v$ ratio exceeds 1.6, then the rate of flow reduction becomes insignificant.
The mathematical model developed in this study can be applied to characterize two-phase flow in an inclined joint using a numerical procedure. It is suggested that a coupled finite element and boundary element method should be applied to model the interface between the fluids, when the joint is subjected to deformations. In such numerical analysis, the roughness profiles of joint surfaces should be considered to determine the effects of joint surface irregularities on the flow. Moreover, the changing geometry of the joint during shear must be modelled.

Pore pressure distribution within joints cannot be measured precisely under laboratory conditions, unless very small pressure transducers can be installed within a joint. The writer is aware of the existence of micro-transducers for hydraulic applications, but the budget limitations have prevented the use of such expensive instrumentation in this study.

At given boundary conditions, it is of interest to study the relationships between joint permeability and the fracture (void) volume. This information can then be used to extend the current mathematical model and the scope of numerical analysis.

The current developed theory by the writer may be extended to model the flow through fracture networks by considering energy losses and change of fluid properties at each joint intersection. Special attention should also be
given to the change of the interface conditions at the intersection, which will probably lead to complex flows that have not been modelled in this study.

- At very high inlet fluid pressures, fluid flow within fractures may be turbulent flow. Under such conditions, possible flow pattern may take complex forms. General flow equations should be written for each phase to model such complex flows (Appendix A). Subsequently, flow equations are written for the boundary to model the interaction between the phases. By designing artificial joints in transparent material (e.g. perspex) and using coloured fluids, the occurrence of complex flow patterns may be studied more comprehensively. These complex flow patterns can then be contrasted with the simplified mathematical models.

- The study should extend to gases other than air. Gases such as CH₄, CO₂ and mixtures of CO₂/CH₄ are of particular significance to coupled flow in coal measure rocks. The properties of gases are significantly different to the air, and therefore the coupled water/gas flow should provide a new avenue for investigation.

- For future work, the current two-phase high pressure triaxial apparatus can be modified in the following ways, if a greater budget can be secured.

  (1) To include high inlet fluid pressures (exceeding 2MPa) and high velocity flows, more reliable electronic devices should replace the existing instrumentation that is appropriate for low-medium range of pressures.
The sensitivity of the current volume change device should be increased by further reducing the friction between the piston and drum. Alternately, a new volume change device using the same concept may be built employing 'teflon' based materials withstanding high pressure. Again the cost of such a device is at least a factor of 2-3 greater than the currently used stainless steel volume change device.
REFERENCES


Toronto, pp.168-173.


APPENDIX A

GENERAL EQUATIONS IN TWO-PHASE FLOW

Similar to single-phase flows, two-phase flows also obey the basic laws of fluid mechanics. However, the mathematical formulation of two-phase flow is more difficult than the case of single-phase flow. This is because, different flow patterns, such as stratified and mixed flows may develop within a pipe or rock joint in different locations at different times, depending on the change of pressure or velocity of each phase, their interaction and surface geometry of fluid flow path. Under these circumstances, it is not easy to transfer the mathematical equations directly from one flow pattern to a newly formed flow pattern. Basically, there are two main approaches to analyse two-phase flow:

(a) Homogeneous flow model (i.e. considering only a single phase flow)
(b) Multi-fluid flow models (i.e. treating the fluid phases separately)

A.1 HOMOGENEOUS STEADY STATE FLOW MODEL

From an analytical or numerical point of view, this is the most convenient way of modelling two-phase flows. However, the accuracy of results are subjected to variation, based on estimated properties of the homogeneous fluid. The crudest point in this method is the determination of average fluids properties. For some flow patterns (e.g. stratified flow), it may be easier to estimate the average properties of fluid such as
density, velocity, temperature and viscosity. Nevertheless, more rigorous efforts supported by experimental work are certainly needed to determine properties of homogeneous flow models, particularly when the flow pattern takes a mixed or complex form. Once the average properties are determined with a certain degree of confidence, the mixture is treated as an equivalent single-phase flow, in which all-basic fluid flow laws are considered to be applicable.

The density of the mixture can be expressed by either considering the volume fraction or the mass fraction. Based on the volume fraction, the mean density of the mixture is given by:

\[\rho_m = \frac{\rho_a q_a + \rho_p q_p}{q_a + q_p}\]  

where, \(\rho\) = density, \(q\) = flow rate and the subscripts \(m\), \(\alpha\) and \(\beta\) represent mean, phase \(\alpha\) and phase \(\beta\), respectively.

Dukler et al. (1964), Mcadams et al. (1942) and Cicchitti et al., (1960) suggested different expressions for the viscosity of the mixture, based on the volume fraction and the mass fraction. According to Mcadams et al. (1942), the viscosity of the mixture of water-gas flow depends on the individual component of viscosity of each phase and the mass fraction as follows:

\[\frac{1}{\mu_m} = \frac{x}{\mu_\alpha} + \frac{1-x}{\mu_\beta}\]  

where, \(\mu\) = dynamic viscosity of fluid phase, \(x\) = mass fraction factor and given by \(M_\alpha/(M_\alpha + M_\beta)\) and \(M\) = mass flow rate.
Based on volume flow rate fraction, Dukler et al. (1964) proposed the mixture viscosity as given below:

\[ \mu_m = \frac{\mu_a q_a + \mu_p q_p}{q_a + q_p} \]  

(A.2b)

where, all the parameters have been described above.

Based on the average viscosity and density of the mixture, the Reynolds number \((Re_m)\) of the mixture can now be formulated as given below:

\[ Re_m = \frac{2hV_m \rho_m}{\mu_m} \]  

(A.3)

where, \(V_m = \) mixture velocity and given by \((q_a + q_p)/A\). \(A\) is the cross section area and \(h\) is the hydraulic diameter.

Using the momentum equation, the pressure gradient of the mixture \((dp/dx)_m\) for an inclined fluid flow surface is the summation of friction, acceleration and gravity terms. Therefore, pressure gradient of the mixture \((dp/dx)_m\) is as follows:

\[ \left( \frac{dp}{dx} \right)_m = \left[ \frac{dp_f}{dx} + \frac{dp_a}{dx} + \frac{dp_g}{dx} \right] \]  

(A.4)

where, \(p = \) pressure and the subscripts \(f\), \(a\) and \(g\) represent friction, acceleration and gravity, respectively.

By substituting corresponding expressions in Equation A.4, the following equation represents for the pressure drop of the mixture in terms of fluid properties and geometrical properties of the fluid flow path.

\[ \left( \frac{dp}{dx} \right)_m = \left[ \frac{S}{A} \tau_w + \left( \frac{\rho q dV}{A} \right)_m + \rho g \sin \theta \right] \]  

(A.5)
where, $S =$ perimeter of the mixture of flow path

$A =$ cross sectional area

$\theta =$ inclination of the fluid flow surface to the horizontal

$\tau_w =$ wall shear stress

The average wall shear stress may be expressed in terms of velocity, friction factor ($C_f$) and velocity of the mixture, as follows:

$$\tau_w = \frac{1}{2} C_f \rho m V_m^2 \tag{A.6}$$

where, $C_f = \frac{A}{\text{Re}_m^\alpha}$. In this equation $A$ and $\alpha$ are variables. From experimental work carried out by Fourar et al. (1993) for gas and water flow through two horizontal glass plates, the following values for $A$ and $\alpha$ were obtained for smooth and rough fractures.

For smooth fracture and $\text{Re}_m < 1000$, $A = 6806$

$\alpha = 1.1$

For smooth fracture and $\text{Re}_m > 1000$, $A = 0.75$

$\alpha = 0.45$

For rough fracture, $A = 6.46$

$\alpha = 0.61$

A.2 MULTI-FLUID STEADY STATE FLOW MODELS

In a comprehensive flow model, theories are first developed based on the two following classes: (a) individual equations for each phase and (b) equations at the boundary of the discontinuity. Subsequently, those two sets of equations are combined to develop the
final expression. In the first and second categories, a set of equations are written for each phase and at the boundary of the discontinuity phases, considering the mass, momentum, enthalpy and energy theories. It is feasible to develop flow theories for flow patterns, if the given flow pattern remains constant along the flow path.

In the following section, general equations for two-phase flow analysis based on mass, momentum, energy and enthalpy theories are presented.

Based on the mass conservation theory, for a control volume of two-phase flow, the net mass flux should be equal to zero. The complete derivations of the following equations are found elsewhere (Hetsroni, 1982).

\[
\frac{d}{dt} \int_{V_{\alpha}} \rho_{\alpha} dV + \frac{d}{dt} \int_{V_{\beta}} \rho_{\beta} dV + \int_{A_{\alpha \rightarrow \beta}} \rho_{\alpha} v_{\alpha} n_{\alpha} dA + \int_{A_{\beta \rightarrow \alpha}} \rho_{\beta} v_{\beta} n_{\beta} dA = 0
\]

(A.7)

where, \( V \& A = \) volume and area, respectively,

- \( v = \) velocity of fluid,
- \( n = \) normal unit vector,
- \( t = \) time, and

\( \alpha \) and \( \beta = \) two phases in the control volume

Due to the external forces, torques, and the velocity of the fluid flow, the control volume is subjected to linear and angular momentum. The linear momentum in the control volume equals the summation of the momentum of influx and the external forces, as given below:
\[
\begin{align*}
\left( \frac{d}{dt} \int_{V_a(t)}^d \rho_a \nu_a dV + \frac{d}{dt} \int_{V_\beta(t)}^d \rho_\beta \nu_\beta dV \right) + \left[ \int_{A_a(t)}^d \rho_a \nu_a (v_a \cdot n_a) dA + \int_{A_\beta(t)}^d \rho_\beta \nu_\beta (v_\beta \cdot n_\beta) dA \right] - \\
\left[ \int_{V_a}^d \rho_a f dV + \int_{V_\beta}^d \rho_\beta f dV + \int_{A_a}^d n_a T_a dA + \int_{A_\beta}^d n_\beta T_\beta dA \right] = 0
\end{align*}
\]

(A.8)

where, \( f = \) external forces

\( T = \) torque

The kinematics and internal energy contribute to the total energy of the control volume. This energy is equal to the summation of the energy of the net influx to the control volume and energy due to the external forces. Assuming that no heat transfer is involved, the total energy of the system can be written as follows:

\[
\begin{align*}
\left( \frac{d}{dt} \int_{V_a(t)}^d \rho_a \left( \frac{1}{2} v_a^2 + u_a \right) dV + \frac{d}{dt} \int_{V_\beta(t)}^d \rho_\beta \left( \frac{1}{2} v_\beta^2 + u_\beta \right) dV \right) + \left[ \int_{A_a(t)}^d \rho_a \left( \frac{1}{2} v_a^2 + u_a \right) (v_a \cdot n_a) dA + \int_{A_\beta(t)}^d \rho_\beta \left( \frac{1}{2} v_\beta^2 + u_\beta \right) (v_\beta \cdot n_\beta) dA \right] - \\
\left[ \int_{V_a}^d \rho_a f v_a dV + \int_{V_\beta}^d \rho_\beta f v_\beta dV + \int_{A_a}^d (n_a T_a) v_a dA + \int_{A_\beta}^d (n_\beta T_\beta) v_\beta dA \right] = 0
\end{align*}
\]

(A.9)

where, \( u = \) internal energy

### A.2.1 Phase equations

Once the general equations are developed, the next step is to develop the equations for each phase. As shown in Figure A.1, the primary and secondary theories involving each phase are derived from the above equations, based on Gauss and Leibniz theorems.
The complete derivations of the following equations are found elsewhere (Hetsroni, 1982). For two-phase flow, the mass conservation theory is represented by:

\[
\frac{\partial (\rho_a + \rho_\beta)}{\partial t} + \nabla \cdot (\rho_a \mathbf{v}_a + \rho_\beta \mathbf{v}_\beta) = 0 \quad (A.10)
\]

From the momentum conservation theory, the following equation can be written:
Considering the total energy of the two-phase flow system, the governing equation is given by:

\[
\frac{\partial (\rho_a v_a + \rho_\beta v_\beta)}{\partial t} + \nabla \cdot (\rho_a v_a^2 + \rho_\beta v_\beta^2 - T_a - T_\beta) - (\rho_a + \rho_\beta) f = 0
\]  

(A.11)

By combining primary equations, secondary phase equations are developed for mechanical energy, internal energy and enthalpy.

A.2.2 Equations at the discontinuity boundary between phases

In two-phase flow, one has to describe the boundary between each phase, in order to account for the interaction between the phases. Basic physical laws of conservation of mass, momentum, energy and enthalpy are imperative to describe the characteristics of the discontinuity boundary or the interface. If the interface between phase \( \alpha \) and phase \( \beta \) (Figure A.2) moves at a velocity of \( v_d \), the mass transfer from the phase \( \alpha \) via the discontinuity should be equal to the mass transfer from the phase \( \beta \) via the discontinuity surface. The mass, momentum and energy conservation equations are thereby expressed as follows:

\[
\rho_a (v_a - v_d) n_a + \rho_\beta (v_\beta - v_d) n_\beta = 0
\]  

(A.13)
The momentum at the interface is due to the effect of velocity of each phase and surface forces, as modelled by:

\[
\rho_a (v_a - v_d) n_a v_a + \rho_\beta (v_\beta - v_d) n_\beta v_\beta - n_a T_a - n_\beta T_\beta = 0
\]  

(A.14)

Kinematics energy, internal energy, surface forces and heat transfer contribute to the total energy along the interface between each phase, as expressed below:

\[
\rho_a (v_a - v_d) n_a \left[\frac{1}{2} v_a^2 + u_a\right] + \rho_\beta (v_\beta - v_d) n_\beta \left[\frac{1}{2} v_\beta^2 + u_\beta\right] - (n_a T_a) v_a - (n_\beta T_\beta) v_\beta = 0
\]

(A.15)

Figure A.2. Two-phase flow in a given control volume.
### APPENDIX B

**TYPICAL INPUT DATA FOR THE ANALYTICAL SOLUTION**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial aperture</td>
<td>0.0000062</td>
<td>m</td>
</tr>
<tr>
<td>Length</td>
<td>0.091</td>
<td>m</td>
</tr>
<tr>
<td>Density of the granite rock</td>
<td>2500</td>
<td>kg/m³</td>
</tr>
<tr>
<td>Vertical stress</td>
<td>5E+7</td>
<td>N/m²</td>
</tr>
<tr>
<td>Density of air at 20 C</td>
<td>1.23</td>
<td>kg/m³</td>
</tr>
<tr>
<td>Initial gas pressure</td>
<td>250000</td>
<td>N/m²</td>
</tr>
<tr>
<td>Change pressure</td>
<td>1008000</td>
<td>N/m²</td>
</tr>
<tr>
<td>Density of water</td>
<td>1000</td>
<td>kg/m³</td>
</tr>
<tr>
<td>Joint normal Stiffness</td>
<td>2.8 x 10¹¹</td>
<td>N/m²</td>
</tr>
<tr>
<td>Horizontal Stress</td>
<td>7480000</td>
<td>N/m²</td>
</tr>
<tr>
<td>Water pressure in the joint</td>
<td>250000</td>
<td>N/m²</td>
</tr>
<tr>
<td>Shear stiffness</td>
<td>9 x 10¹¹</td>
<td>N/m²</td>
</tr>
<tr>
<td>Coefficient of compressibility of air, Ca</td>
<td>2.857143 x 10⁻⁶</td>
<td>N/m²</td>
</tr>
<tr>
<td>Vol. coefficient at 1 atm and 20 C</td>
<td>0.02918</td>
<td></td>
</tr>
<tr>
<td>Water velocities</td>
<td>0.000018</td>
<td>m/sec</td>
</tr>
<tr>
<td>Air velocities</td>
<td>0.0013</td>
<td>m/sec</td>
</tr>
<tr>
<td>Interfacial friction factor</td>
<td>0.014</td>
<td></td>
</tr>
<tr>
<td>Coefficient, C for laminar flow</td>
<td>16</td>
<td></td>
</tr>
<tr>
<td>Coefficient, n for laminar flow</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Viscosity of water</td>
<td>0.00112</td>
<td>Pa.sec</td>
</tr>
<tr>
<td>Viscosity of air</td>
<td>0.0000179</td>
<td>Pa.sec</td>
</tr>
</tbody>
</table>
Table B.1. Analytical solutions for each equation for different apertures at given fluid pressure of 0.25MPa applied to the jointed granite specimen.

<table>
<thead>
<tr>
<th>Initial aperture, m</th>
<th>6.2 x 10^{-6}</th>
<th>6.2 x 10^{-5}</th>
<th>1.55 x 10^{-4}</th>
<th>5.0 x 10^{-4}</th>
<th>1.0 x 10^{-3}</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normal displacement, m</td>
<td>3.6 x 10^{-6}</td>
<td>3.6 x 10^{-6}</td>
<td>3.6 x 10^{-6}</td>
<td>3.6 x 10^{-6}</td>
<td>3.6 x 10^{-6}</td>
</tr>
<tr>
<td>Phase level due to compressibility of air, m (Eqn. 7.24)</td>
<td>4.468 x 10^{-6}</td>
<td>3.397 x 10^{-7}</td>
<td>1.12e-6</td>
<td>3.6 x 10^{-6}</td>
<td>7.21 x 10^{-6}</td>
</tr>
<tr>
<td>Phase level due to solubility of air, m (Eqn. 7.21)</td>
<td>2.898 x 10^{-10}</td>
<td>2.94 x 10^{-9}</td>
<td>7.25 x 10^{-9}</td>
<td>4.67 x 10^{-4}</td>
<td>9.34 x 10^{-4}</td>
</tr>
<tr>
<td>Interface level, m (Eqn. 7.18)</td>
<td>2.547 x 10^{-6}</td>
<td>2.83 x 10^{-5}</td>
<td>7.11 x 10^{-5}</td>
<td>2.29 x 10^{-4}</td>
<td>4.58 x 10^{-4}</td>
</tr>
<tr>
<td>Phase level of air, m (Eqn. 7.10)</td>
<td>2.918 x 10^{-6}</td>
<td>3.24 x 10^{-5}</td>
<td>8.16 x 10^{-5}</td>
<td>2.65 x 10^{-4}</td>
<td>5.31 x 10^{-4}</td>
</tr>
<tr>
<td>Phase level of water, m (Eqn. 7.7)</td>
<td>2.547 x 10^{-6}</td>
<td>2.83 x 10^{-5}</td>
<td>7.11 x 10^{-5}</td>
<td>2.29 x 10^{-4}</td>
<td>4.58 x 10^{-4}</td>
</tr>
<tr>
<td>Water flow rate, m^3/sec (Eqn. 7.34)</td>
<td>2.081 x 10^{-8}</td>
<td>2.84 x 10^{-7}</td>
<td>4.52 x 10^{-6}</td>
<td>1.52 x 10^{-4}</td>
<td>1.21 x 10^{-3}</td>
</tr>
<tr>
<td>Air flow rate, m^3/sec (Eqn. 7.35)</td>
<td>1.081 x 10^{-8}</td>
<td>1.48 x 10^{-4}</td>
<td>2.37 x 10^{-3}</td>
<td>8.08 x 10^{-2}</td>
<td>6.53 x 10^{-1}</td>
</tr>
</tbody>
</table>
Table B.2. Analytical solutions for each equation for different fluid pressures and at given joint aperture 0.001m.

<table>
<thead>
<tr>
<th>Fluid pressures, MPa</th>
<th>0.25</th>
<th>0.5</th>
<th>1</th>
<th>1.5</th>
<th>2.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normal displacement, m (Eqn. 7.30c)</td>
<td>$3.6 \times 10^{-6}$</td>
<td>$3.6 \times 10^{-6}$</td>
<td>$3.61 \times 10^{-6}$</td>
<td>$3.62 \times 10^{-6}$</td>
<td>$3.64 \times 10^{-6}$</td>
</tr>
<tr>
<td>Phase level due to compressibility of air, m (Eqn. 7.24)</td>
<td>$7.21 \times 10^{-6}$</td>
<td>$4.2 \times 10^{-6}$</td>
<td>$2.29 \times 10^{-6}$</td>
<td>$1.5 \times 10^{-6}$</td>
<td>$1.2 \times 10^{-6}$</td>
</tr>
<tr>
<td>Interface level, m (Eqn. 7.18)</td>
<td>$4.58 \times 10^{-4}$</td>
<td>$4.6 \times 10^{-4}$</td>
<td>$4.61 \times 10^{-4}$</td>
<td>$4.62 \times 10^{-4}$</td>
<td>$4.63 \times 10^{-4}$</td>
</tr>
<tr>
<td>Phase level of air, m (Eqn. 7.10)</td>
<td>$5.31 \times 10^{-4}$</td>
<td>$5.32 \times 10^{-4}$</td>
<td>$5.33 \times 10^{-4}$</td>
<td>$5.34 \times 10^{-4}$</td>
<td>$5.34 \times 10^{-4}$</td>
</tr>
<tr>
<td>Phase level of water, m (Eqn. 7.7)</td>
<td>$4.58 \times 10^{-4}$</td>
<td>$4.6 \times 10^{-4}$</td>
<td>$4.61 \times 10^{-4}$</td>
<td>$4.62 \times 10^{-4}$</td>
<td>$4.63 \times 10^{-4}$</td>
</tr>
<tr>
<td>Water flow rate, m$^3$/sec (Eqn. 7.34)</td>
<td>$1.21 \times 10^{-3}$</td>
<td>$2.45 \times 10^{-3}$</td>
<td>$4.96 \times 10^{-3}$</td>
<td>$7.46 \times 10^{-3}$</td>
<td>$9.97 \times 10^{-3}$</td>
</tr>
<tr>
<td>Air flow rate, m$^3$/sec (Eqn. 7.35)</td>
<td>0.653</td>
<td>0.785</td>
<td>1.05</td>
<td>1.31</td>
<td>1.57</td>
</tr>
</tbody>
</table>
APPENDIX C

A TYPICAL UDEC CODE FOR FLUID FLOW ANALYSIS THROUGH A JOINTED ROCK MASS

TITLE FLOW THROUGH JOINTED ROCK MEDIA

; For a typical model, the following assumptions are used

; Boundary block size is 50m x 50m

; In situ stress ratio, (i.e. Horizontal Stress/Vertical stress) = 0.5

; Groundwater table is 10m

; Tunnel diameter = 6m

; The origin of the tunnel is 100 m below from the ground surface

;-------------------------------------------------------------------------------------------------------------------------------------
CONFIG TFLOW

SET EDGE 0.16

RO 0.07

; Creating the boundary block and joint pattern

BLOCK 0 100 0 150 50 150 50 100
TITLE
MODEL SIZE OF 50 x 50 m
PL BL HOLD

; Creating the tunnel

TUNNEL 50 125 3 40
TITLE
A CIRCULAR TUNNEL WITH A DIAMETER 6m
PL BL HOLD

; Generating joints

; Joint set 1

JSET 45 0 4.5 0 0 0 3.5 0 0 0

; Joint set 2
JSET 135 0 3.5 0 0 4 0 75 0
PL BL HOLD

; Rock blocks are assumed as deformable
GEN EDGE 5 range 0 50 0 150
;Generated zones for deformable blocks
PL BOUN FILL YELL ZON LMAGE BL NC HOLD

; The rock material is modelled using Mohr Coulomb plastic theory
CHANGE CONS 3

; The rock joint is modelled using the joint area contact Coulomb slip theory
CHA JCONS = 2 RANGE ANGLE 44 46
CHA JCONS = 2 RANGE ANGLE 134 136

; Assign of material numbers to rock material
CHA MAT=1

; Assign of material numbers to joints
; Joint set 1
CHA JMAT=1 RANGE ANGLE 44 46

; Joint set 2
CHA JMAT=2 RANGE ANGLE 134 136

; Assign of material properties to rock material. Assume rock type as granite.
; Note: B = bulk modulus; COHE = cohesion; DE = density; FRI = friction;
; g = shear modulus
PROP MAT=1 B=4.39e10 COHE= 5.51e7 DE = 2500 FRI = 51 G = 2.8e10

; Assign of material properties to joints
; Note: ARES = Residual joint aperture; AZER = Joint aperture at zero stress level;
; JKS = joint normal stiffness; JKN = joint normal stiffness, JPER = joint permeability
; factor.
; Joint set 1
PROP JMAT=1  ARES=1e-6 AZER= 8e-4
PROP JMAT=1 JFR 51 JKS 2E10 JKN 2.2E12 JPERM 80

; Joint set 2
PROP JMAT=2 ARES=4e-6 AZER=5.0e-4
PROP JMAT=2 JFR1=48 JKS=1E10 JKN=1.2E12 JPERM=81

; Assume hydraulic aperture is 6 times the residual aperture
SET CAPRATIO 6

FLUID DE 1000  BULK=2e9

SET GRA 0 -10

DAMP AUTO

; Assign of boundary conditions to the model

; Ground stress
BOUN STRESS -4.5E6 0 0 RANGE -0.1 50 149.9 150.1
BOUN STRESS -4.05E6 0 0 YGRAD 2.7E4 0 0 RANGE -0.1 0.1 99.9 150.1
BOUN YVEL 0 RANGE -0.1 50.1 99.9 100.1
BOUN XVEL 0 RANGE 49.9 50.1 99.9 150.1

; Fluid stress field
BOUN PP 1.5E6 PYGRAD -1.E4 RANGE -0.1 0.1 99.9 150.
BOUN PP 0.5E6 RANGE -0.1 50.1 99.9 100.1
BOUN IMPERM RANGE 49.9 50.1 99.9 150.1
BOUN IMPERM RANGE -0.1 50.1 149.9 150.1
BOUN PP 1.5E6 RANGE -0.1 0.1 99.9 150.1

; Application of in situ stress in order to bring the model to initial equilibrium condition
INSITU STRESS -4.05E6 0 -4.05E6&
SZZ-1.563e6 &
YGRAD 2.7e4 0 2.7e4 &
YWTABLE 150
; Fluid flow analysis

; The model is allowed to settle under gravity

SET FLOW STEADY

; To monitor the model behaviour

HIST UNBALANCE

HIST FLOWTIME

HIST YDISP 50 150 47.62 126.84 48.48 122.72

HIST FLOW 47.62 126.84 48.48 122.72

HIST PP 47.62 126.84 48.48 122.72

SOLVE

SAVE FLOW1.SAV

PRINT MAX

; Undrained flow analysis

RESET FLOW1.SAV

RESET JDISP DISP

SET FLOW COMP

FLUID BULK 200E6

; No fluid flow

SET FLOW OFF

SET CAPRAT 3.

; Excavation of the tunnel is now carried out. The effects of induced stress on joint deformation is studied with no flow condition.

DELETE ANN 50 125 0 3

RESET HIST

HIST UNBAL
HIST YDISP 50 150 47.62 126.84 48.48 122.72
HIST FLOW 47.62 126.84 48.48 122.72
HIST PP 47.62 126.84 48.48 122.72
SOLVE
SAVE FLOW2.SAV
; Steady state flow is now activated
RESET HIST TIME JDISP DISP
SET FTIME 0.0
HIST NCYC 100
HIST FLOWTIME
HIST UNBAL
HIST YDISP 50 150 47.62 126.84 48.48 122.72
HIST FLOW 47.62 126.84 48.48 122.72
HIST PP 47.62 126.84 48.48 122.72
HIST JOINT 48.48 122.72 25.97 100 FLOW
HIST JOINT 48.48 122.72 25.97 100 PP
SET FLOW COMP
FLUID BULK 200E6
SET NFMECH 3
CYCLE TIME 3
SET NFMECH 2
CYCLE TIME 4
SAVE FLOW2A.SAV
SET NFMECH 1
CYCLE TIME 10
SAVE FLOW2B.SAV
SET NFLOW 5
CYCLE TIME 10
SAVE FLOW2C.SAV

; Effect of joint deformation due to fluid flow

SET MECH OFF

SET FLOW STEADY

SET GRAVITY 0 - 10

HIST YDISP 50 150 47.62 126.84 48.48 122.72

HIST FLOW 47.62 126.84 48.48 122.72

HIST PP 47.62 126.84 48.48 122.72

HIST JOINT 48.48 122.72 25.97 100 FLOW

HIST JOINT 48.48 122.72 25.97 100 PP

SOLVE

SAVE FLOW3.SAV

; Coupled hydro-mechanical flow analysis

REST FLOW1.SAV

SET CAPRAT 3.

; Remove the tunnel

DELETE ANN 50 125 0 3

SET FLOW STEADY

SET MECH ON

HIST YDISP 50 150 47.62 126.84 48.48 122.72

HIST FLOW 47.62 126.84 48.48 122.72

HIST PP 47.62 126.84 48.48 122.72

HIST JOINT 48.48 122.72 25.97 100 FLOW

HIST JOINT 48.48 122.72 25.97 100 PP

HIST JOINT 47.62 126.84 24.68 150 PP
HIST JOINT 47.62 126.84 24.68 150 FLOW

SOLVE

PRINT MAX

SAVE FLOW4.SAV

; To observe the model behaviour

PL BL NC PLAS HOLD

PL BOUN STRESS HOLD

PL BOUN SDIF INTERVAL 8E4 HOLD

PL BOUN VFLOW HOLD

PL BOUN MAPE HOLD

PL BOUN SEPAR HOLD

RETURN