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Keywords

mobile, efficient, pitfalls, security, scheme, signature, proxy, agents, threshold

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Security Pitfalls of an Efficient Threshold Proxy Signature Scheme for Mobile Agents

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Abstract

A (t, n) threshold proxy signature scheme enables an original signer to delegate his/her signing power to n proxy signers such that any t or more proxy signers can sign messages on behalf of the original signer, but $t-1$ or less of them cannot produce a valid proxy signature. Based on the RSA cryptosystem, Hong proposed an efficient (t, n) threshold proxy signature for mobile agents. Cai et al. found that the scheme due to Hong is proxy-unprotected, meaning that the original signer can generate a valid proxy signature by himself. However, it is unclear whether the scheme can be used in reality after fixing the security problem discovered by Cai et al.. In this letter, we provide a detailed analysis on Hong's scheme and show that the scheme fails to achieve the properties of secrecy, proxy protected, undeniability, identifiability and even time constraint and thus adopted of this efficient construction in practice is not recommended.

Key words: Cryptography, Digital signature, Proxy signature, RSA cryptosystem, Security analysis

1 Introduction

The notion of proxy signature [1] was invented by Mambo et al., which enables a proxy signer to sign messages on behalf of an original signer in case of say, temporal absence, lack of computational power etc. After validating the correctness of a proxy signature following a given verification algorithm, a verifier

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can be convinced of the original signer's agreement on the signed message. In the last years, fruitful achievements [2–9] of proxy signatures have been seen, including novel constructions, analysis, improvements and applications. Proxy signatures have found extensive uses in numerous practical applications such as in distributed computing, e-commerce, e-cash, and grid computing where delegation of rights is quite common [2,3]. Mambo et al. [1] classified this kind of cryptographic primitive into two categories, namely, proxy-unprotected and proxy-protected. A proxy-protected scheme, where only the proxy signer is able to generate valid proxy signatures, is more practical since it accommodates some highly desirable properties such as fairness and signature ownership. In the latest research, delegation with warrant is popular because of its high security and flexible delegation policy for a proxy signature scheme.

Among all the variants of proxy signature schemes, threshold proxy signature is one of the useful cryptographic primitives. In a (t, n) threshold proxy signature scheme, the original signer delegates and distributes the signing power to n proxy signers such that collaborative effort of at least t proxies is required to the creation of a valid proxy signature, while $t - 1$ or less of them cannot complete a signing operation. Threshold proxy signature is a promising primitive for it allows the original signer to control the delegation of his signing capability. Not only does it allow the original signer to choose the group of proxies, but also the selection of the threshold value. Thus, to some extent, threshold proxy signatures are more flexible and practical than traditional proxy signature schemes. Based on the tricks of secret sharing and threshold cryptography, Zhang [10] and Kim [11] proposed threshold proxy signature schemes for the first time independently. Subsequently, Sun et al. [12] extended the construction due to Kim et al. and presented a non-repudiable threshold proxy signature scheme with known signers. Unfortunately, Hsu et al. [13] found the scheme in [12] suffers from the conspiracy attack, namely, any t or more proxy signers can get the secret keys of other proxy signers. Hwang et al. [14] proposed a new threshold proxy signature scheme with known signers and claimed their construction can achieve all the desirable security properties of a proxy signature scheme. However, Wang et al. [15] identified several security weaknesses in the scheme and concluded that the scheme is not secure. Observing there are few secure (t, n) threshold proxy signature schemes based on the RSA cryptosystem, Hong [16] proposed a novel and practical (t, n) threshold proxy signature from RSA mechanism by applying the traditional RSA cryptosystem without using additional cryptographic techniques, and suggested to apply the proposal to mobile agent systems. They claimed that the scheme satisfies all the desirable security requirements. Unfortunately, Cai et al. [17] demonstrated a concrete attack against Hong's construction in which a malicious original signer can forge a valid threshold proxy signature.

Our contributions: It is not an easy task to construct a secure threshold proxy signature scheme from the well-studied RSA problem since sharing the

private key of the RSA system [20] among multiple members is difficult and the Euler phi function of the modulus cannot be leaked to any proxy signer. Hong's scheme [16] has many advantages over other schemes in the same style such as it shares the proxy signing key using the simple Lagrange formula; the proxy signature generation and combination are completely non-interactive; the sizes of both the partial proxy signing key and partial proxy signature are independent of the number of the proxy signers. It is interesting to find out whether we can use Hong's scheme in reality after fixing the security problem identified by Cai et al. [17]. Unfortunately, in this letter, after giving a detailed analysis of Hong's scheme, we find that the construction fails to achieve all the security properties of a secure proxy signature scheme, including secrecy, proxy protected, undeniability, identifiability and even time constraint (prevention of misuse).

Organization: Section 2 reviews Hong's threshold proxy scheme. Section 3 describes our security analysis of the scheme. Section 4 concludes the paper.

2 Review of Hong's scheme

The following notations are used in Hong's scheme [16]. The original signer is denoted by U_0 , the n proxy signers are denoted by $U_1, \dots, U_n$, and a combiner is denoted by C . H is a secure hash function; m_w denotes a warrant, which specifies the identities of the original signer and the proxy signers, the parameters (t, n) , the valid delegation period and the kind of messages being delegated etc. Q_N denotes the subgroup of squares in Z_N^* . The following five phases are involved in the scheme.

Setup: The original signer U_0 picks two large secure primes of equal length p_0, q_0 and computes $N_0 = p_0 q_0$, where $p_0 = 2p'_0 + 1, q_0 = 2q'_0 + 1$ with p'_0, q'_0 themselves prime. Let $M_0 = p'_0 q'_0$, which is the order of the group Q_{N_0} . U_0 computes her RSA exponents e_0 and d_0 such that $e_0 d_0 \equiv 1 \pmod{M_0}$. The private key of U_0 is (d_0, M_0) and the public key is (e_0, N_0) . Each proxy signer $U_i (i = 1, 2, \dots, n)$ chooses two random large secure primes of equal length p_i and q_i , and computes $N_i = p_i q_i$, $\phi(N_i) = (p_i - 1)(q_i - 1)$, e_i and d_i where $e_i d_i \equiv 1 \pmod{\phi(N_i)}$. The private key and the public key of U_i are d_i and (e_i, N_i) respectively.

Proxy sharing: U_0 firstly generates the threshold proxy signing key $D \equiv d_0 \cdot H(m_w) \pmod{M_0}$, and shares the signing key among the proxy signer group as follows.

- (1) Set $a_0 = D$ and for $1 \leq i < t$, pick at random a_i from $\{0, \dots, M_0 - 1\}$, and define a $t - 1$ degree polynomial

$$f(x) \equiv a_0 + a_1 x + \dots + a_{t-1} x^{t-1} \pmod{M_0}.$$

- (2) Compute partial proxy signing key $k_i \equiv f(i) \pmod{M_0}$ for each proxy signer U_i .
- (3) For the purpose of share validation, U_0 picks a random element $v \in Q_{N_0}$ and computes $v_i = v^{k_i}$ for $1 \leq i \leq n$. U_0 makes $(v, v_1, \dots, v_n)$ public and sends k_i to U_i in a secure manner.

Proxy signature generation: Assume t different proxy signers $U_i (i = 1, \dots, t)$ would like to generate a proxy signature of message m on behalf of U_0 cooperatively. Let $x = H(m, m_w)$ and $\Delta = n!$. Each proxy signer computes the partial proxy signature $x_i = x^{2\Delta \cdot k_i} \in Q_{N_0}$. Then, U_i computes $\Delta\sigma_i = \lfloor x_i/N_i \rfloor$, $\sigma_i \equiv x_i^{d_i} \pmod{N_i}$. To guarantee soundness, U_i produces a proof that the discrete log of x_i^2 to the base $\hat{x} = x^{4\Delta}$ equals to the discrete log of v_i to base v . Specifically, U_i chooses a random $r \in \{0, \dots, 2^{|N_0|+2L_1} - 1\}$, where L_1 is a secondary security parameter, and a secure hash function $H'(\cdot)$, then computes $v' = v^r$, $x' = \hat{x}^r$, $c_i = H'(v, \hat{x}, v_i, x_i^2, v', x')$, $z_i = k_i c + r$. The final partial proxy signature due to U_i is $(i, \Delta\sigma_i, \sigma_i, c_i, z_i)$.

proxy signature combining: The combiner C can be one of the proxy signers or a secretary who does not possess any secret parameter. Upon receiving the partial proxy signature $(i, \Delta\sigma_i, \sigma_i, c_i, z_i)$ from U_i , C computes $x_i = \Delta\sigma_i \times N_i + (\sigma_i^{e_i} \pmod{N_i})$ and then checks if

$$c = H'(v, \hat{x}, v_i, x_i^2, v^z v_i^{-c}, \hat{x}^z x_i^{-2c}).$$

If the equation holds, the partial proxy signature is valid; Otherwise, invalid.

Assume t partial proxy signatures are valid and without losing generality, the corresponding proxy signers set is $s = \{1, \dots, t\} \subset \{1, \dots, n\}$. The proxy signature w of the message m under the warrant m_w is $w = x_1^{2\lambda_{0,1}^S} \dots x_t^{2\lambda_{0,t}^S} \pmod{N_0}$, where $\lambda_{i,j}^S = \Delta \frac{\prod_{j' \in S_j (i-j')}}{\prod_{j' \in S_j (j-j')}$.

Since $w^{e_0} \equiv x^{4\Delta^2 \cdot H(m_w)} \pmod{N_0}$ and $\gcd(4\Delta^2, e_0) = 1$, it is easy to find out the final proxy signature y such that $y^{e_0} \equiv x^{H(m_w)} \pmod{N_0}$ by using a standard algorithm $y = w^a x^{b^1}$, where a, b are integers such that $4\Delta^2 a + e_0 b = 1$.

Proxy signature verification: Receiving a (t, n) threshold proxy signature y of the message m under the warrant m_w , a verifier checks that if $y^{e_0} \equiv x^{H(m_w)} \pmod{N_0}$ holds, where $x = H(m, m_w)$.

3 Security analysis of the scheme

A comprehensive security analysis was provided in [16] and the scheme was claimed to achieve all the desirable security requirements. Unfortunately, in this section, we present several security weaknesses in the scheme and thus show that the claim is not valid.

¹ This is a typo in [16], and the correct one is $y = w^a x^{bH(m_w)}$

3.1 Secrecy

This property says that even if all the proxy signers collude, they cannot derive the original signer's private key. In [16], it was claimed that both signing and verification algorithms are based on the RSA problem, one cannot derive the proxy signing key $D = d_0 \cdot H(m_w) \pmod{M_0}$. Moreover, even if t out of n proxy signers conspire to obtain the group proxy signature key D , they cannot derive the original signer's private key d_0 . Below we demonstrate that the claim does not hold.

Firstly, for any set $\Omega \subseteq \{1, 2, \dots, n\}$ satisfying $|\Omega| \geq t$, we can get $\Delta \cdot D \equiv \sum_{j \in \Omega} \lambda_{0,j}^S \cdot f(j) \pmod{M_0}$ using the Lagrange formula. Thus, any t or more colluding proxy signers are able to compute the proxy signing key D . Worse still, since $D \equiv d_0 \cdot H(m_w) \pmod{M_0}$, we can get

$$e_0 \cdot D \equiv e_0 \cdot d_0 \cdot H(m_w) \equiv H(m_w) \pmod{M_0},$$

which means that $e_0 \cdot D - H(m_w)$ is a nonzero multiple of M_0 , where $M_0 = p'_0 q'_0 = \phi(N_0)$. However, following the well-known conclusion due to Miller [18], knowing such a multiple of $\phi(N_0)$ is equivalent to factoring N_0 . Consequently, the t colluding proxy signers, after obtaining the value D , is capable of factorizing N_0 and thus computing the secret key of the original signer. Specifically, with the factors of N_0 , the colluding proxy signers can compute the Euler's phi function $\phi(N_0)$, and they can get the private key d_0 from $e_0 d_0 \equiv 1 \pmod{M_0}$ by employing extended Euclidean algorithm [19]. Therefore, the property of secrecy is violated in the scheme.

3.2 Proxy protected and identifiability

Proxy protection states that only at least t out of n proxy signers can cooperate to generate valid proxy signatures. Any third party, including the original signer, cannot masquerade as proxy signers group to generate a valid proxy signature. As identified by Cai et al. [17], since the final proxy signature is y and the verification equation is $y^{e_0} \equiv H(m, m_w)^{H(m_w)} \pmod{N_0}$, nobody can recognize the identities of the t actual proxy signers upon receiving the final proxy signature. That is to say, the property of identifiability cannot be achieved in the scheme [16]. Moreover, they showed that an malicious original signer can forge a valid proxy signature in polynomial time. Thus, the scheme fails to provide the property of proxy protection.

3.3 Unforgeability

According to the analysis above, we can conclude that the scheme is not secure against the attacks due to the malicious original signer and malicious proxy signers, namely, it is insecure against the insider attacks. Here we also demonstrate that it is even possible for an outsider adversary to mount an universal forgery.

- (1) Choose an arbitrary warrant m_w^* which records the delegation policy including limits of authority, valid periods of delegation and proxy signature, the threshold value t and the identities and the public keys of the original signer and the proxy signers group.
- (2) Compute $H(m_w^*)$ and checks if $e_0 | H(m_w^*)$ holds. If it does not hold, repeats (1) and (2).
- (3) Choose a message m^* that conforms to the the warrant m_w^* , and compute $y^* \equiv H(m^*, m_w^*)^k \pmod{N_0}$ where $H(m_w^*) = ke_0$.
- (4) Publish y^* as the threshold proxy signature of message m^* under the warrant m_w^* .

It's easy to check the correctness of the forged proxy signature. Next we assess the success probability of the adversary. The RSA parameters should be thoroughly evaluated for security and efficiency [21, 22] in some applications. To reduce the time of signature verification or encryption in RSA, one may wish to use a small e . The smallest value for e is 3, but the value $e = 2^{16} + 1$ is recommended to resist certain attacks [23]. Assume the recommended RSA exponent $e = 2^{16} + 1$ and the standard SHA1 hash algorithm, whose output is of 160-bit, are employed. For a random m_w^* , the probability that $e_0 | H(m_w^*)$ is at least 2^{-16} , which is not negligible. Thus, the expected time for the attacker to find a warrant m_w^* such that $e_0 | H(m_w^*)$ is within evaluating 2^{16} hash values on different warrants m_w^* . This is highly feasible since modern computer can handle hashing of a file over a 100 MB in less than 1 second² and the warrant m_w^* is supposed to be much shorter.

3.4 Undeniability

From the analysis of unforgeability, we can conclude that the scheme can resist neither insider attacks nor outsider attacks. As a consequence, given a threshold proxy signature, both the original signer or the proxy signer group can deny having signed the message.

² <http://www.cryptopp.com/benchmarks.html>.

3.5 Time constraint

According to the universal forgery attack described above, an outsider adversary can arbitrarily modify the warrant m_w^* including changing the valid periods of delegation, replacing the identities of the original signer and proxy signers, modifying and scope of signing message, altering the threshold value t , and then forge threshold proxy signature as long as $e_0|H(m_w^*)$ holds. Thus, the scheme fails to achieve the property of prevention of misuse as well.

4 Conclusion

In this letter, we presented a detailed security analysis of the threshold proxy signature proposed in [16], and showed that it fails to achieve the properties of secrecy, proxy protected, undeniability, identifiability as well as prevention of misuse. Thus, its adoption in practice is not recommended. Hopefully the analysis is helpful for the future constructions of threshold proxy signature from RSA assumption.

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